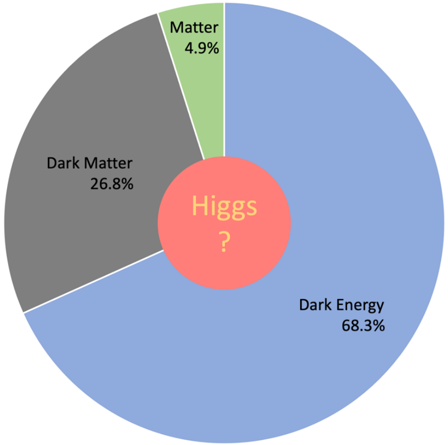
中山大學天琴中心
TIANQIN CENTER FOR GRAVITATIONAL PHYSICS, SYSU

Precise calculations of electroweak phase transition dynamics

Fa Peng Huang
Sun Yat-sen University

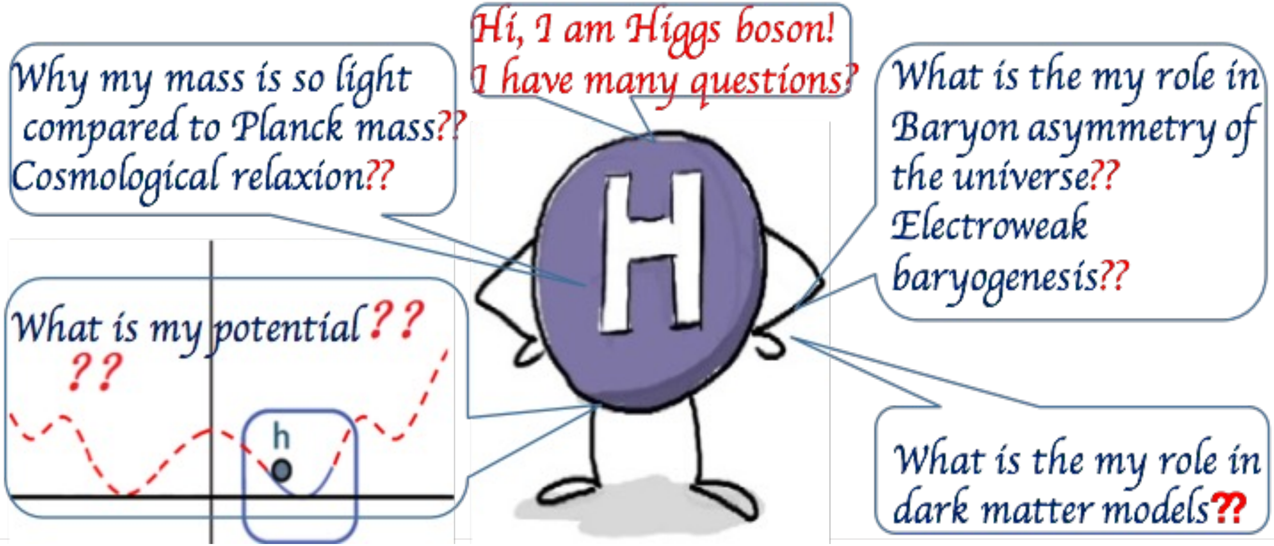
The 2023 international workshop on the Circular Electron Positron Collider (CEPC)
@University of Edinburgh, UK, 3-6 July, 2023

Motivation



What is the role of Higgs in the early universe ?

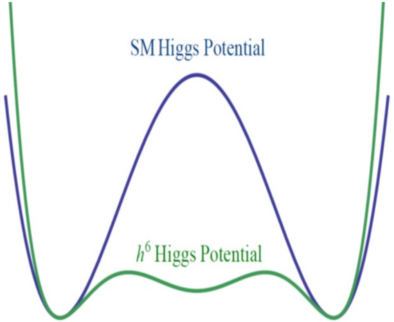
After the discovery of 125 GeV scalar, it becomes a realistic portal to study the fundamental physics and its deep connections to cosmology



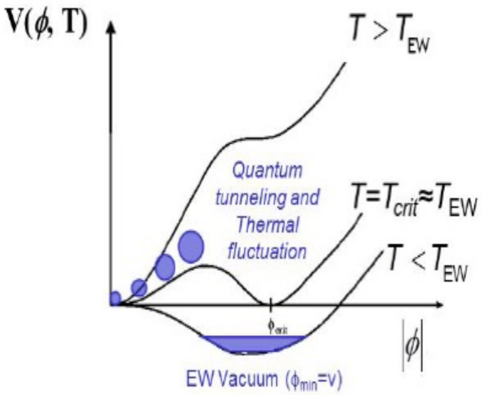
How to test?

Motivation

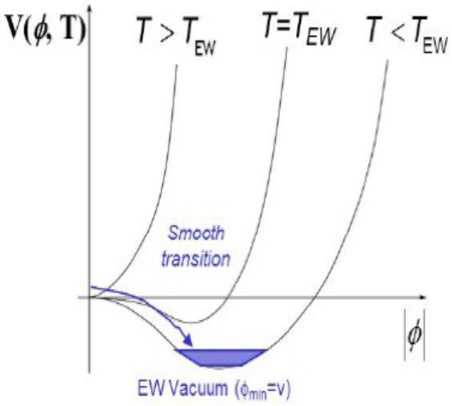
From lattice simulation



SFOPT for $m_H < 75$ GeV



Cross over for $m_H > 75$ GeV



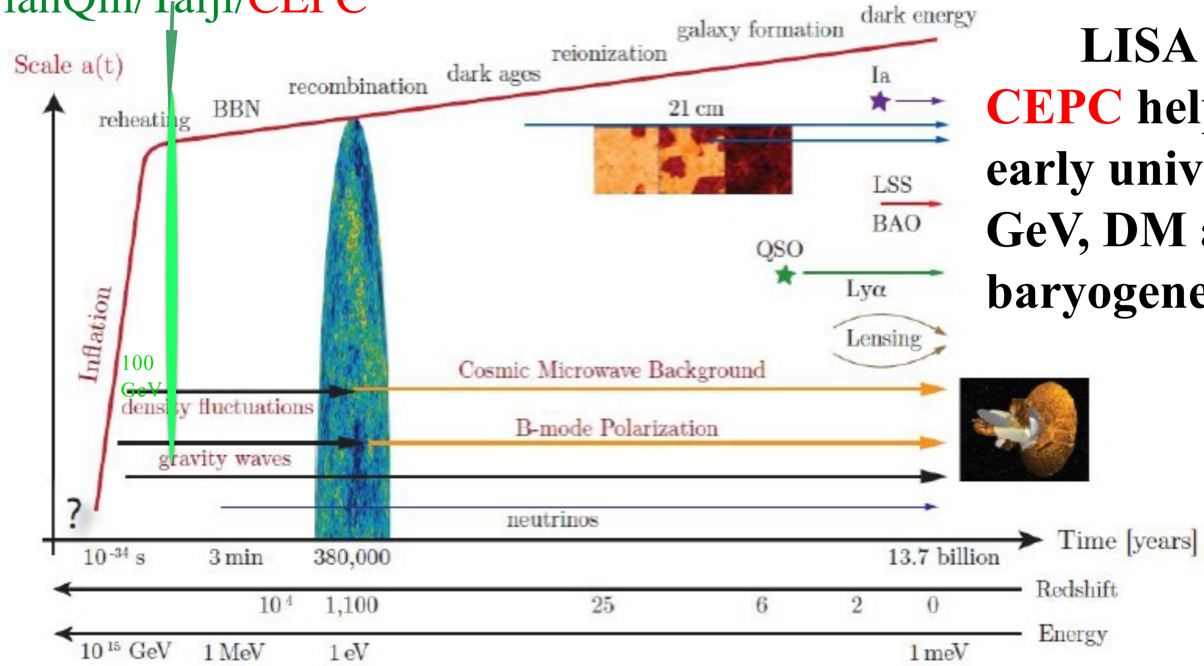
Extension of the Higgs sector is needed to SFOPT for 125 GeV Higgs boson.

We discuss well-motivated extensions (baryogenesis, DM…) of Higgs section to realize strong first-order phase transition (SFOPT) with abundant cosmological effects.

EW phase transition and its GW signals becomes realistic after the discovery of Higgs by LHC and GW by LIGO.

Motivation

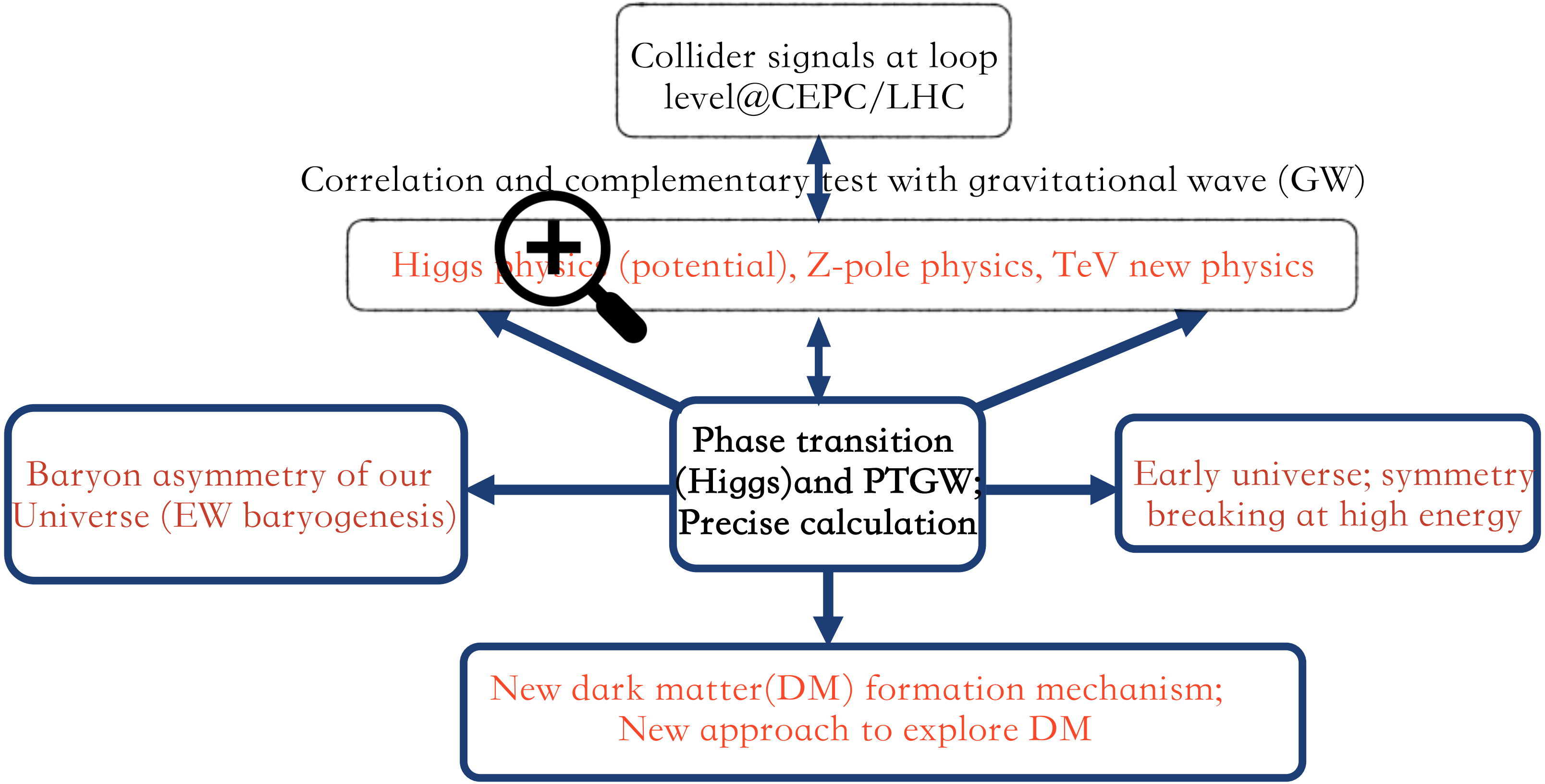
EW phase transition/
baryogenesis:
LISA/TianQin/Taiji/CEPC



LISA in synergy with CEPC helps to explore the early universe around 100 GeV, DM and baryogenesis.

credit:D.Baumann

Motivation



Collider signals at loop level@CEPC/LHC

Correlation and complementary test with gravitational wave (GW)

Higgs physics (potential), Z-pole physics, TeV new physics

Phase transition (Higgs)and PTGW; Precise calculation

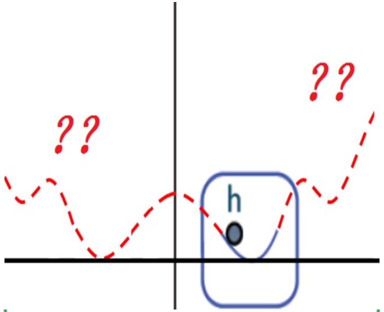
Baryon asymmetry of our Universe (EW baryogenesis)

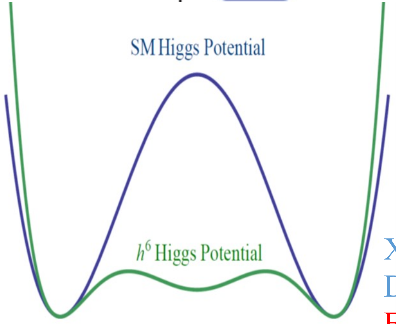
Early universe; symmetry breaking at high energy

New dark matter(DM) formation mechanism; New approach to explore DM

Higgs potential

What is the shape of Higgs potential?





Current data tells us nothing but the quadratic oscillation around the VEV 246 GeV with 125 GeV mass.

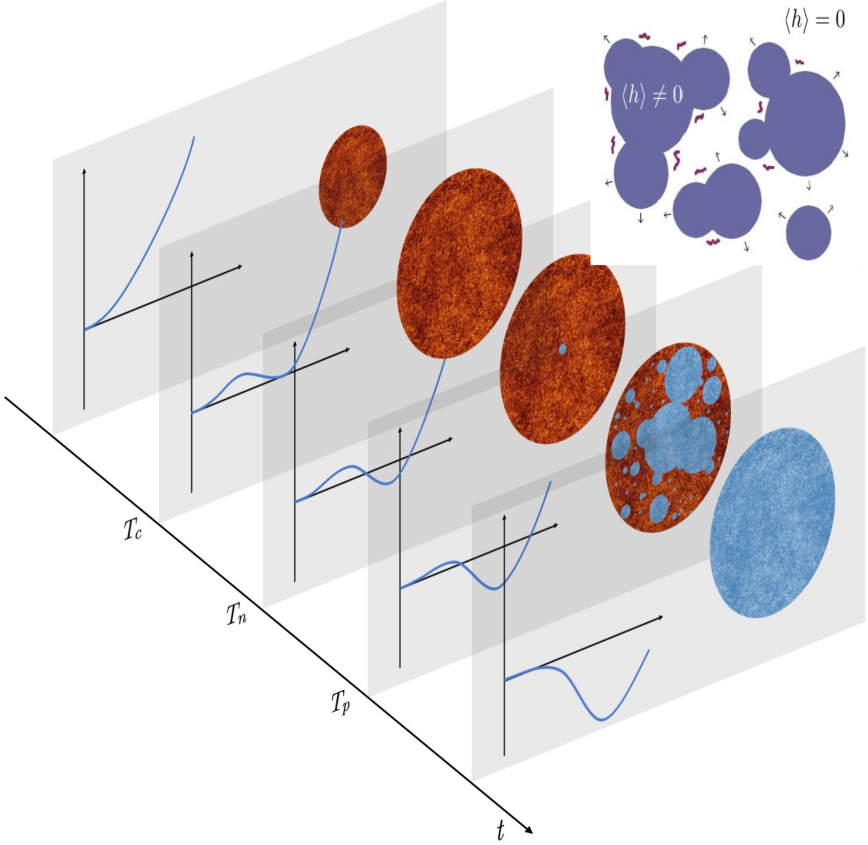
$$V(h) = \frac{1}{2}\mu^2 h^2 + \frac{\lambda}{4} h^4$$

or $$V(h) = \frac{1}{2}\mu^2 h^2 - \frac{\lambda}{4} h^4 + \frac{1}{\Lambda^2} h^6$$

Produce a SFOPT, large deviation of Higgs trilinear coupling and GW

Xinmin Zhang Phys.Rev. D47 (1993) 3065-3067;C. Grojean, G. Servant, J. Well PRD71(2005)036001
D.J.H. Chung, A. J. Long, Lian-tao Wang Phys.Rev. D87(2013) 023509
FPH, et.al, Phys.Rev.D94(2016)no.4,041702 ; **FPH**, et.al, Phys.Rev.D93 (2016) no.10,103515
arXiv:1511.06495, Nima Arkani-Hamed et. al.; PreCDR of CEPC; arXiv: 1811.10545,CDR of CEPC

Phase transition GW in a nutshell



calculate the finite-temperature effective potential using the thermal field theory: free energy density.

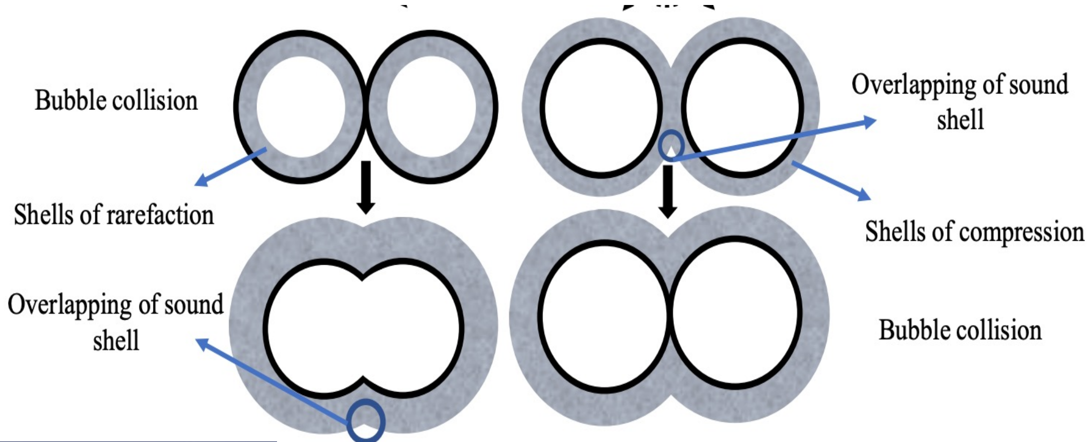
$$V_{\rm eff}^{(1)}(\bar{\phi}) = \sum_i n_i \Big[\int \frac{{\rm d}^D p}{(2\pi)^D} \ln\big(p^2 + m_i^2(\bar{\phi})\big) + J_{\rm B,F}\Big(\frac{m_i^2(\bar{\phi})}{T^2}\Big)\Big]$$

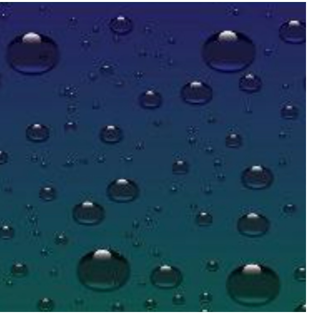
$$S(T) = \int d^4x \left[\frac{1}{2}\left(\frac{\partial\phi}{\partial x}\right)^2 + V_{\rm eff}(\phi, T)\right]$$

$$\Gamma = \Gamma_0 e^{-S(T)}$$

Xiao Wang, FPH, Xinmin Zhang, JCAP05(2020)045

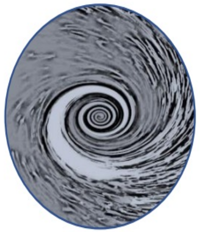
Phase transition GW in a nutshell





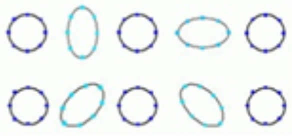
Detonation mode

Deflagration mode



$$R_{\mu\nu} - \frac{1}{2} R\, g_{\mu\nu} + \Lambda\, g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

$$h_{ij} \simeq \frac{2G}{c^4 r} \ddot{Q}_{ij}^{TT}(t - r/c)$$



E. Witten, Phys. Rev. D 30, 272 (1984)

C. J. Hogan, Phys. Lett. B 133, 172 (1983);

M. Kamionkowski, A. Kosowsky and M. S. Turner, Phys. Rev. D 49, 2837 (1994))

EW phase transition GW becomes more interesting and realistic after the discovery of Higgs by LHC and GW by LIGO.

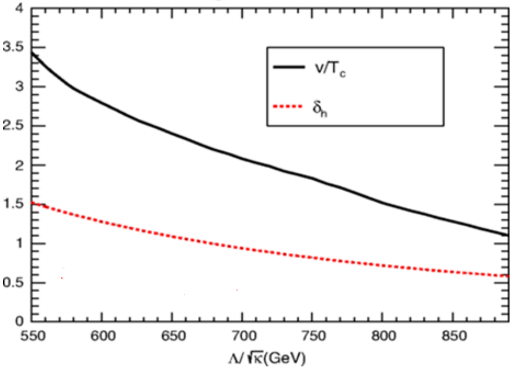
Xiao Wang, FPH, Xinmin Zhang, JCAP05(2020)045

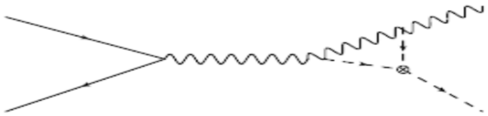
Higgs potential

Correlate collider and GW: double test on Higgs potential from particle to wave

SFOPT leads to obvious deviation of the tri-linear Higgs coupling

$$\mathcal{L}_{hhh} = -\frac{1}{3!}(1+\delta_h)A_h h^3$$





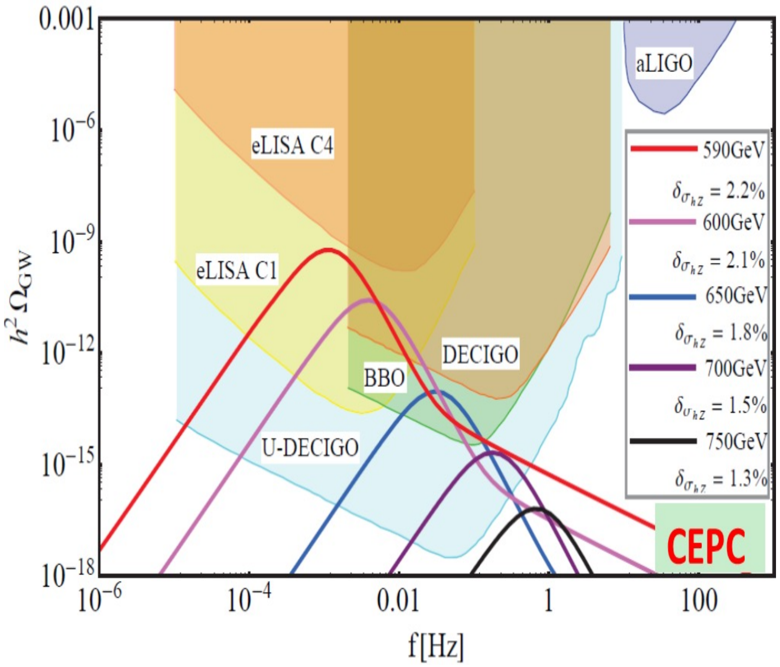
At one-loop level, deviation of the tri-linear Higgs coupling

$$\delta_h \in (0.6, 1.5)$$

The Circular Electron Positron Collider (CEPC), ILC, FCC-ee can precisely test this scenario by precise measurements of the hZ cross section (e- e+ ⟶ hZ).

SM NNLO hZ cross section recently by Lilin Yang, et al 2016, Yu Jia et at 2016

$$\delta_\sigma = \frac{\sigma_{hz,\delta_h \neq 0}}{\sigma_{hz,SM}} - 1$$



FPH, et.al, Phys.Rev.D94(2016)no.4,041702 ; **FPH**, et.al, Phys.Rev.D93 (2016) no.10,103515

Higgs potential

SM EFT

$$\mathcal{L} \supset -\mu^2|H|^2 - \lambda|H|^4 + c_6|H|^6$$
$$+ c_T\mathcal{O}_T + c_{WW}\mathcal{O}_{WW} + \text{other dimension-six operators}$$

$$\delta_{\sigma(hZ)} \approx (0.26c_{WW} + 0.01c_{BB} + 0.04c_{WB} - 0.06c_H - 0.04c_T + 0.74c_L^{(3)\ell}$$
$$+0.28c_{LL}^{(3)\ell} + 1.03c_L^{\ell} - 0.76c_R^{e}) \times 1\ \text{TeV}^2 + 0.016\delta_h,$$

SFOPT produces large
modification
of trilinear Higgs coupling δ_h

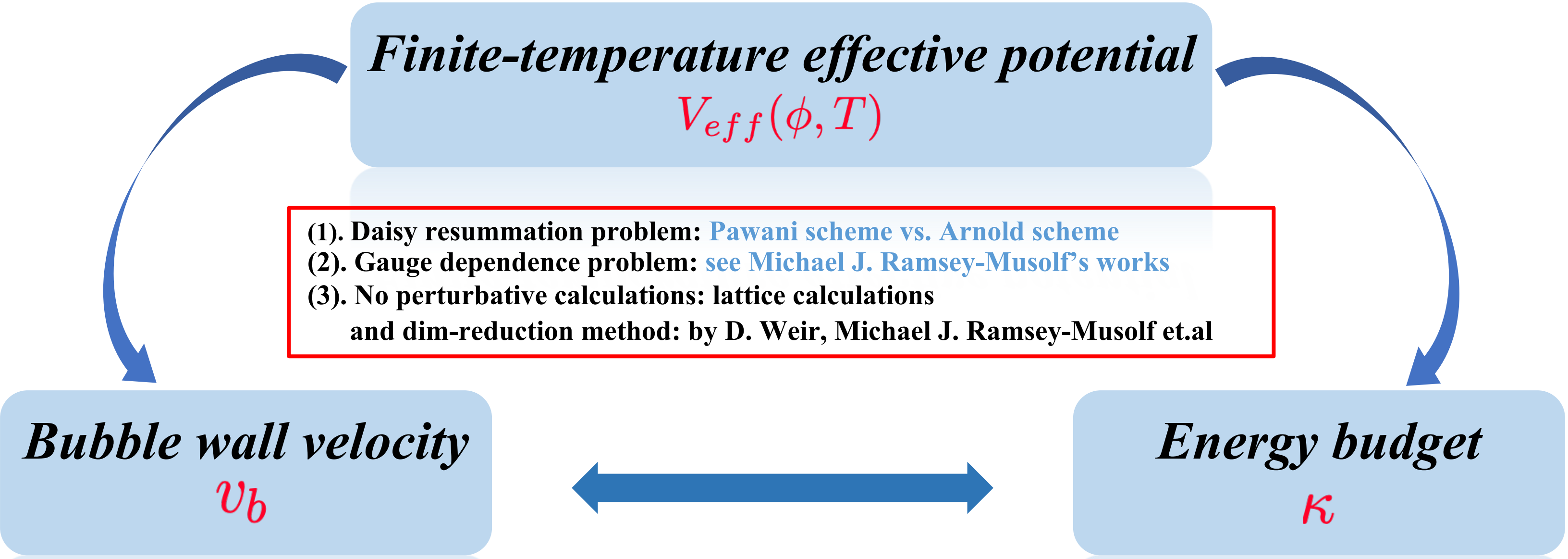
c_6 dominates the hZ cross section
deviation

Taking a general study of the scalar extended models and the composite Higgs model as examples, we find that the Higgs sextic scenario still works well after considering all the dim-6 operators and the precise measurements.

Qing-Hong Cao, FPH, Ke-Pan Xie, Xinmin Zhang, arXiv:1708.0473.

Phase transition Dynamics

Precise predictions on the phase transition dynamics and its GW signals



Finite-temperature effective potential

$V_{eff}(\phi, T)$

(1). Daisy resummation problem: Pawani scheme vs. Arnold scheme
(2). Gauge dependence problem: see Michael J. Ramsey-Musolf's works
(3). No perturbative calculations: lattice calculations
and dim-reduction method: by D. Weir, Michael J. Ramsey-Musolf et.al

Bubble wall velocity

v_b

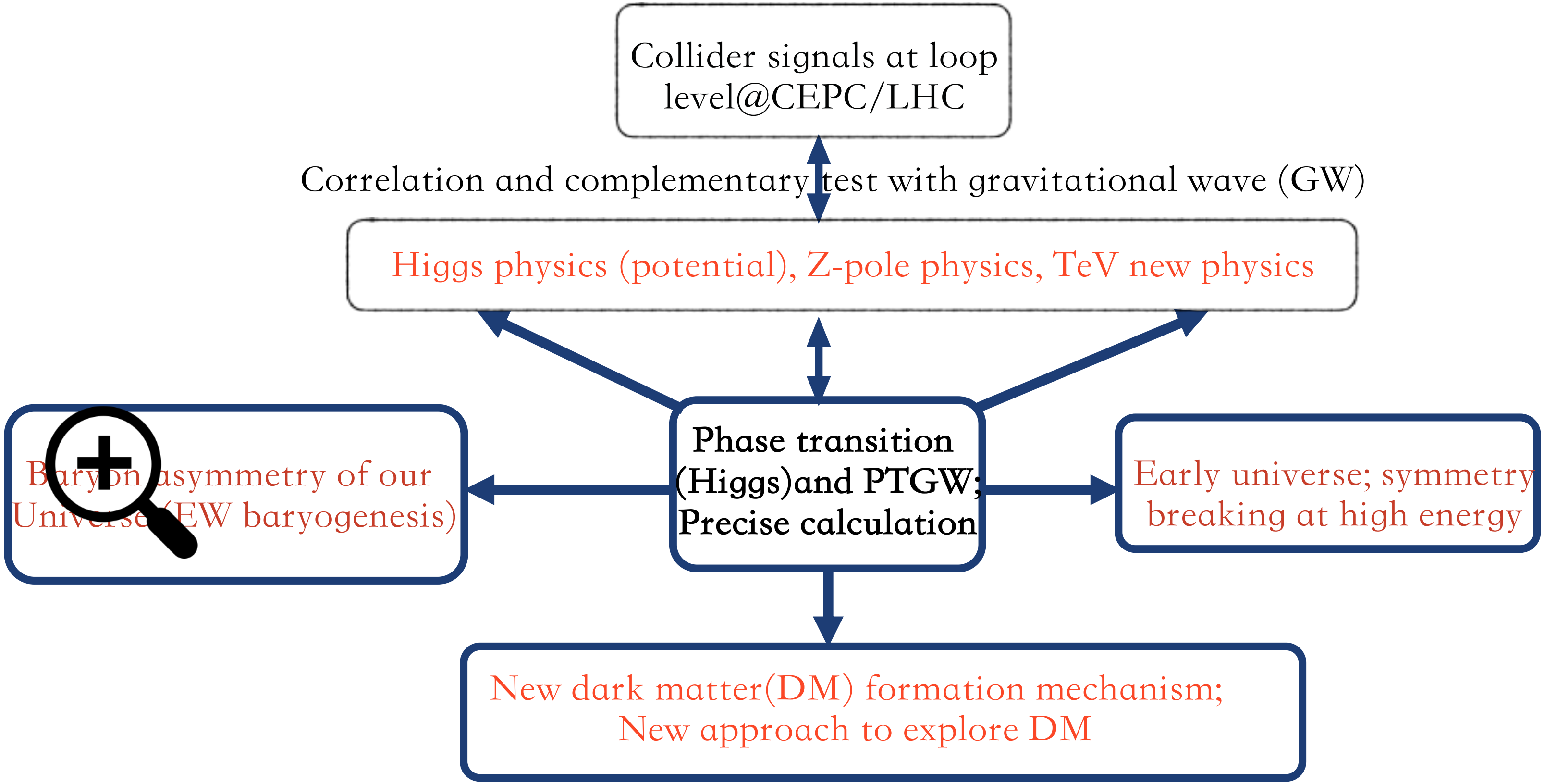
Energy budget

κ

S. Hoche, J. Kozaczuk, A. J. Long, J. Turner and Y. Wang, arXiv:2007.10343,
Avi Friedlander, Ian Banta, James M. Cline, David Tucker-Smith,
arXiv:2009.14295v2
Xiao Wang, **FPH,** Xinmin Zhang,arXiv:2011.12903
Siyu Jiang, **FPH**, xiao wang, Phys.Rev.D 107 (2023) 9, 095005

F. Giese, T. Konstandin, K. Schmitz and J. van de Vis,arXiv:2010.09744
Xiao Wang, **FPH** and Xinmin Zhang, Phys.Rev.D 103 (2021) 10, 103520
Xiao Wang, Chi Tian, **FPH**, arXiv: 2301.12328

Motivation



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Early universe; symmetry breaking at high energy

New dark matter(DM) formation mechanism;
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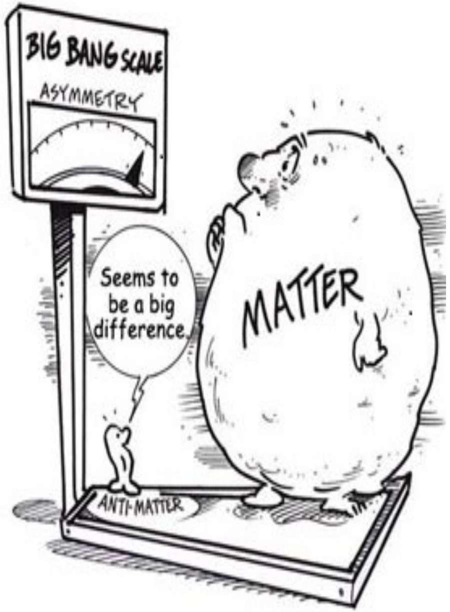
SFOPT and EW baryogenesis



$\eta_B = n_B/n_\gamma = 5.8 - 6.6 \times 10^{-10}$

(CMB, BBN)

A long standing problem in particle cosmology is the origin of baryon asymmetry of the universe.

After discovery of Higgs boson@LHC & GW @aLIGO, EW baryogenesis becomes a timely and testable scenario.

SM technically has all the 3 elements for baryogenesis(Sakharov conditions)

- **B violation from anomaly in B+L current;**
- **C and CP-violation: CKM matrix, but too weak, need new CP-violating sources;**
- **Departure from thermal equilibrium: SFOPT with expanding Higgs bubble wall**

SFOPT and EW baryogenesis

Question: How to alleviate the tension of EW baryogenesis?

Answer: Dynamical CP-violating source

Large enough CP-violating source for successful EW baryogenesis

Alleviate by assuming the CP-violating source is time dependent

Dynamical/cosmological evolve

Negligible CP-violating source at current time to avoid strong EDM constraints

Effective field theory: **FPH, Zhuoni Qian, Mengchao Zhang,Phys.Rev. D98 (2018) no.1, 015014,FPH,Chong Sheng Li, Phys. Rev. D 92, 075014 (2015);**

Renormalizble model: **Complex 2HDM,Xiao Wang, FPH, Xinmin Zhang, arXiv: 1909.02978, work in progress with Eibun Senaha, Xiao Wang in an extended IDM model**

Baldes, T. Konstandin and G. Servant, arXiv:1604.04526;I. Baldes, T. Konstandin and G. Servant, JHEP 1612, 073 (2016);

S. Bruggisser,T.Konstandin and G. Servant, JCAP 1711, no. 11, 034 (2017)

Matthew Reece recently has more deep study on the dynamical cp-violation.

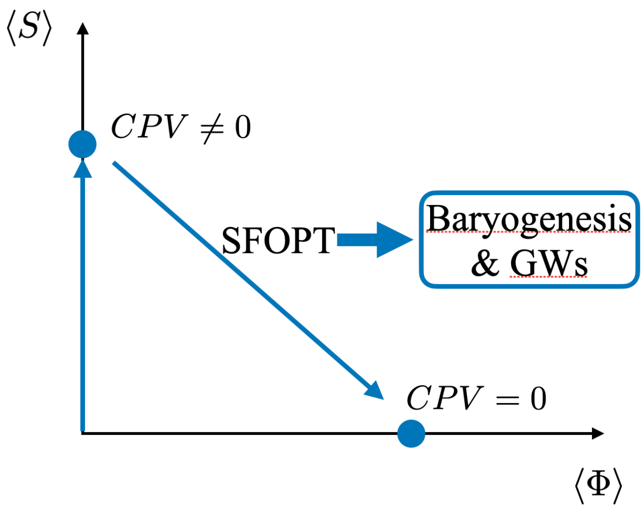
SFOPT and EW baryogenesis

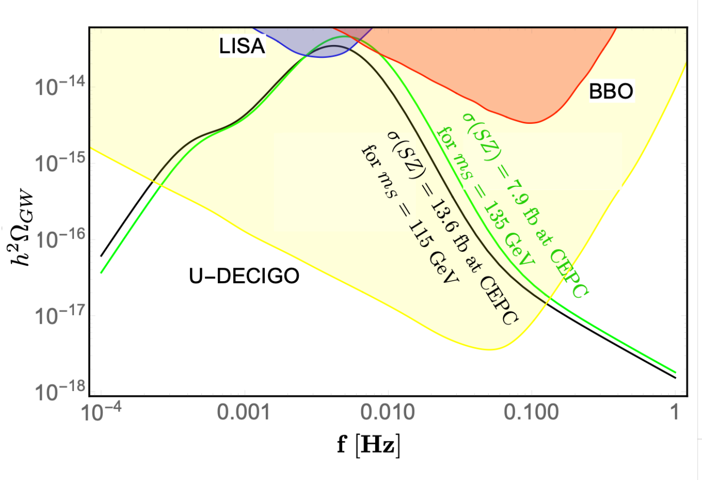
Taking the effective scenario as a representative example:

$$\mathcal{L}_{\rm SM} - y_t \frac{\eta}{\Lambda} S \bar{Q}_L \tilde{\Phi} t_R + \text{H.c} + \frac{1}{2} \partial_\mu \text{S} \partial^\mu \text{S} + \frac{1}{2} \mu^2 \text{S}^2 - \frac{1}{4} \lambda \text{S}^4 - \frac{1}{2} \kappa \text{S}^2 (\Phi^\dagger \Phi)$$

The singlet and the dim-5 operator can come From many types composite Higgs model, arXiv:0902.1483 , arXiv:1703.10624 ,arXiv:1704.08911, J. M. Cline and K. Kainulainen, JCAP **1301**, 012 (2013)

J. R. Espinosa, B. Gripaios, T. Konstandin and F. Riva, JCAP **1201**, 012 (2012)





Phys.Rev. D98 (2018) no.1, 015014 , **FPH**, Zhuoni Qian, Mengchao Zhang
lots of new works unmentioned here

EW baryogenesis & Z-pole physics

Renormalizable DM and EW baryogenesis with dynamical CP-violating source.

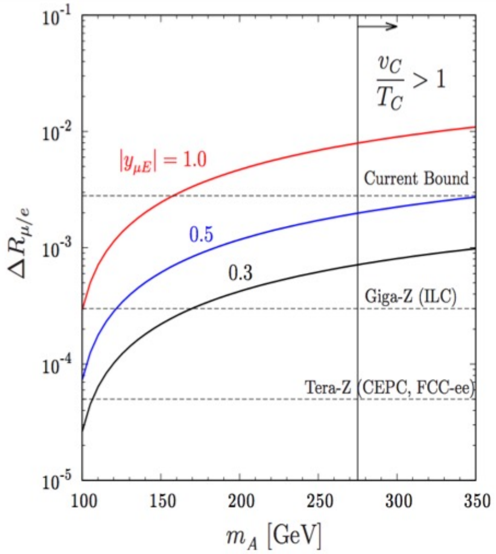
arXiv:1905.10283, Phys.Rev.D 100 (2019) 3, 035014, FPH, Eibun Senaha
and work in progress with Eibun Senaha

$$V_0(\Phi,\eta) = \mu_1^2\Phi^\dagger\Phi + \mu_2^2\eta^\dagger\eta + \frac{\lambda_1}{2}(\Phi^\dagger\Phi)^2 + \frac{\lambda_2}{2}(\eta^\dagger\eta)^2 + \lambda_3(\Phi^\dagger\Phi)(\eta^\dagger\eta) + \lambda_4(\Phi^\dagger\eta)(\eta^\dagger\Phi) + \left[\frac{\lambda_5}{2}(\Phi^\dagger\eta)^2 + \text{h.c}\right],$$

The new lepton Yukawa interaction is

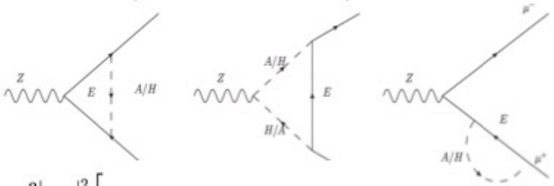
$$-\mathcal{L}_Y \ni y_{ij}\bar{\ell}_{iL}\eta E_{jR} + m_{E_i}\bar{E}_{iL}E_{iR} + \text{h.c.}$$

vector-like lepton (E_i)

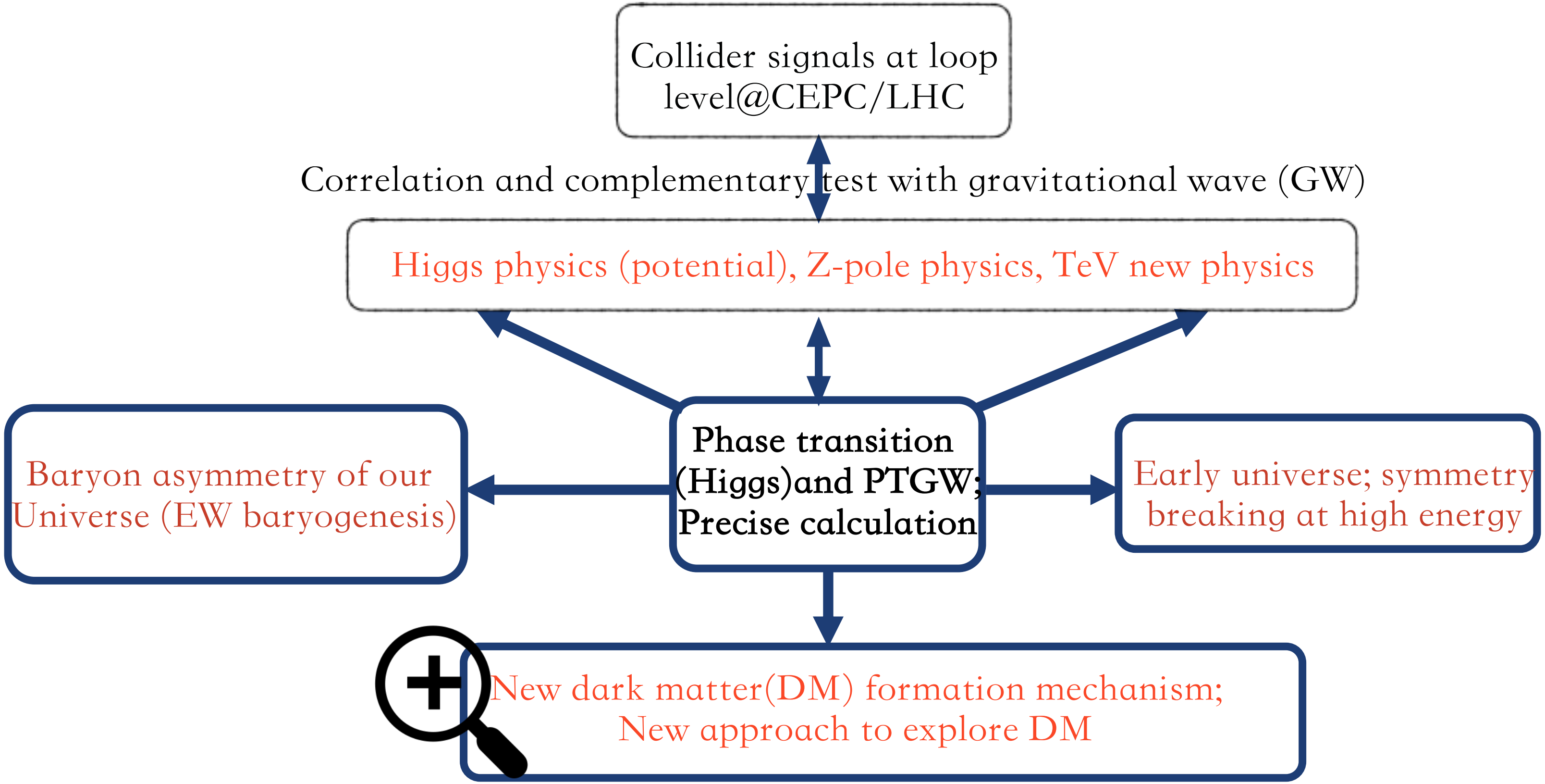


D. Borah, S. Sadhukhan and S. Sahoo, Phys. Lett. B **771**, 624 (2017).
L. Calibbi, R. Ziegler and J. Zupan, JHEP **1807**, 046 (2018).
D. Borah, P. S. B. Dev and A. Kumar, Phys. Rev. D **99**, no. 5, 055012 (2019).

Indirect search by $Z \to \mu^+\mu$



Motivation



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SFOPT and new DM mechanism/signal

- **New The observation of GW by LIGO has initiated a new era of exploring DM by GW.**
- **DM can trigger a SFOPT in the early universe, which can leads to detectable GW signals.**

Hearing the signal of dark sectors with gravitational wave detectors
J.Jaeckel, V. V. Khoze, M. Spannowsky, Phys.Rev. D94 (2016) no.10, 103519
Zhaofeng Kang,et.al. arXiv:2101.03795
Zhaofeng Kang, et. al. arXiv:2003.02465
Yan Wang, Chong Sheng Li, and **FPH**, arXiv:2012.03920
FPH, Eibun Senaha Phys.Rev. D100 (2019) no.3, 03501
FPH PoS ICHEP2018 (2019) 397
FPH, Chong Sheng Li, Phys.Rev. D96 (2017) no.9, 095028
FPH, Jiang-Hao Yu, Phys.Rev. D98 (2018) no.9, 095022
FPH, Xinmin Zhang, Phys.Lett. B788 (2019) 288-29

SFOPT and new DM mechanism/signal

Inert Doublet Models

$$\begin{aligned} V_0 &= M_D^2 D^\dagger D + \lambda_D (D^\dagger D)^2 + \lambda_3 \Phi^\dagger \Phi D^\dagger D \\ &+ \lambda_4 |\Phi^\dagger D|^2 + (\lambda_5/2)[(\Phi^\dagger D)^2 + h.c.], \end{aligned}$$

mixed singlet-doublet model

$$\begin{aligned} V_0 = {} & \frac{1}{2} M_S^2 S^2 + M_D^2 H_2^\dagger H_2 + \frac{1}{2}\lambda_S S^2 |\Phi|^2 + \lambda_3 \Phi^\dagger \Phi H_2^\dagger H_2 \\ & + \lambda_4 |\Phi^\dagger H_2|^2 + \frac{\lambda_5}{2}[(\Phi^\dagger H_2)^2 + \mathrm{H.c.}] + A[S\Phi H_2^\dagger + \mathrm{H.c.}]. \end{aligned}$$

mixed singlet-triplet model

$$\begin{aligned} V_0 = {} & \frac{1}{2} M_S^2 S^2 + M_\Sigma^2 \mathrm{Tr}(H_3^2) + \kappa_\Sigma \Phi^\dagger \Phi \mathrm{Tr}(H_3^2) \\ & + \frac{\kappa}{2} |\Phi|^2 S^2 + \xi S \Phi^\dagger H_3 \Phi. \end{aligned}$$

provide natural DM candidate

produce SFOPT and phase transition GW

FPH, Jiang-Hao Yu, Phys.Rev. D98 (2018) no.9, 095022
Yan Wang, Chong Sheng Li, and **FPH**, *Phys.Rev.D* 104 (2021) 5, 053004;

SFOPT and new DM mechanism/signal

New dark matter (DM) production scenario filtered by the bubbles of the SFOPT in the early universe.

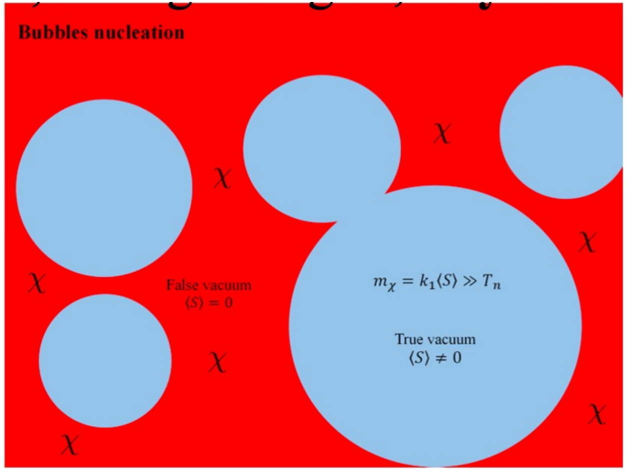
The cosmic phase transition with Q-balls production can explain baryogenesis and DM simultaneously, where constraints on DM mass and reverse dilution are significantly relaxed. We study how to probe this scenario by GW signals and collider signals at QCD NLO.

FPH, Chong Sheng Li, Phys. Rev. D96 (2017) no.9, 095028

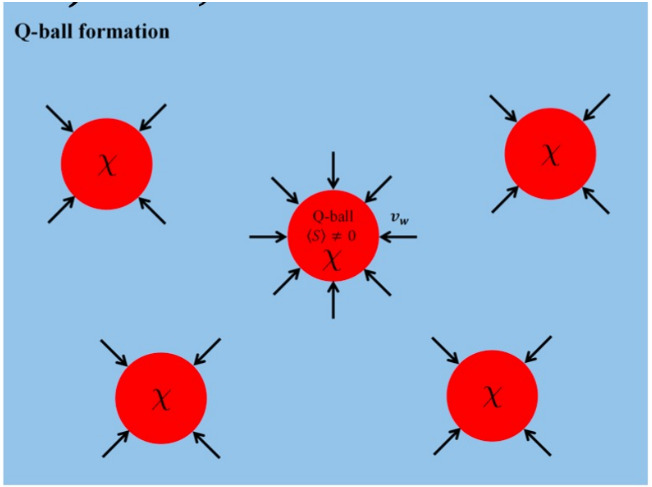
$$\rho_{\rm DM}^{4} v_b^{3/4} = 73.5(2\eta_B s_0)^3 \lambda_S \sigma^4 \Gamma^{3/4}$$

SFOPT and new DM mechanism/signal

SFOPT naturally correlates DM, baryogenesis, particle collider and GW signals.



(a) Bubble nucleation:χ particles trapped in the false vacuum due to Boltzmann suppression



(b) Q-ball formation:After the formation of Q-balls, they should be squeezed by the true vacuum

FPH, Chong Sheng Li, Phys.Rev. D96 (2017) no.9, 095028

SFOPT and new DM mechanism/signal

In the recent two years, this dynamical DM formed by phase transition has became a new idea and attracted more and more attentions.

Namely, bubbles in SFOPT can be the “filters” to packet your needed heavy DM.

FPH, Chong Sheng Li, Phys.Rev. D96 (2017) no.9, 095028
arXiv:1912.04238, Dongjin Chway, Tae Hyun Jung, Chang Sub Shin
arXiv:1912.02830, Michael J. Baker, Joachim Kopp,and Andrew J. Long
arXiv:2012.15113, Wei Chao, Xiu-Fei Li, Lei Wang
arXiv:2101.05721, Aleksandr Azatov, Miguel Vanvlasselaer, Wen Yin
arXiv:2103.09827, Pouya Asadi , Eric D. Kramer, Eric Kuflik, Gregory W. Ridgway, Tracy R. Slatyer, J. Smirnov
arXiv:2103.09822, Pouya Asadi , Eric D. Kramer, Eric Kuflik, Gregory W. Ridgway, Tracy R. Slatyer, J. Smirnov
arXiv:2008.04430 Jeong-Pyong Hong， Sunghoon Jung, Ke-pan Xie
arXiv:2305.02218 Siyu Jiang, FPH, Chong Sheng Li

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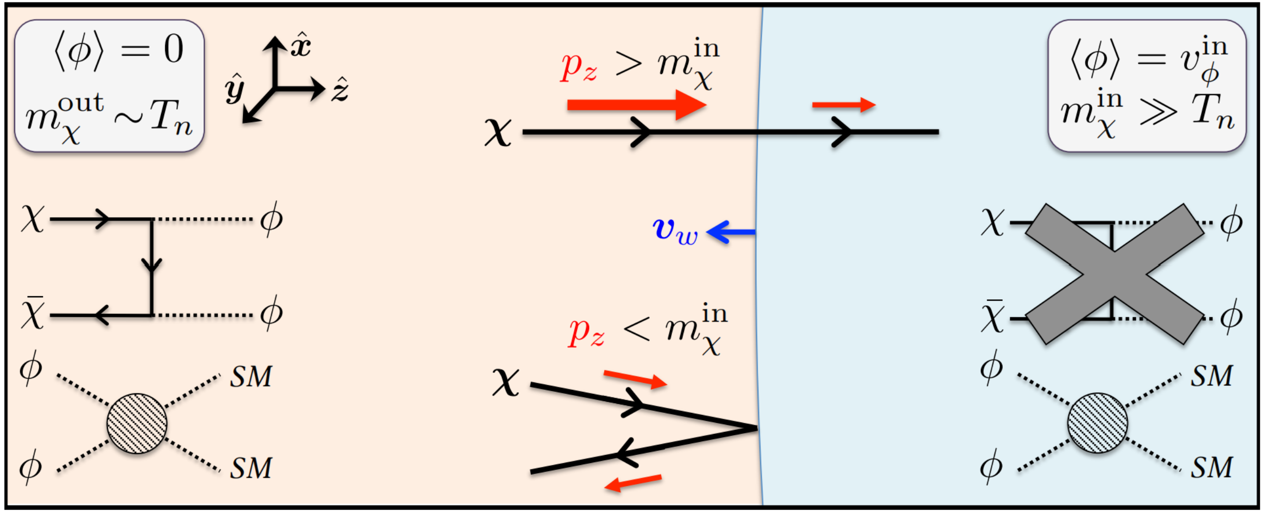
SFOPT and new DM mechanism/signal

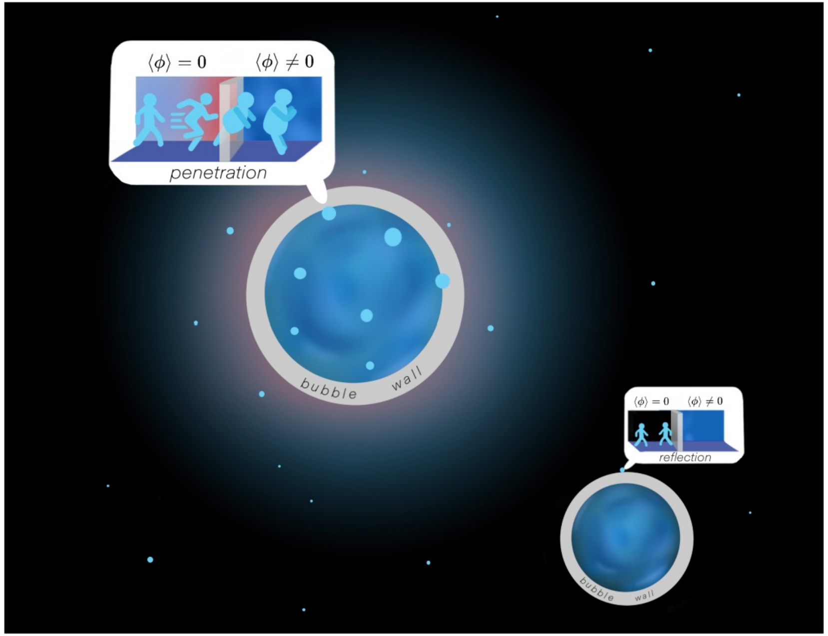
Michael J. Baker, Joachim Kopp, Andrew J. Long,Phys.Rev.Lett. 125 (2020) 15, 151102



$$\Omega_{\rm DM} h^2 \approx 0.17 \left(\frac{T_n}{\rm TeV}\right)\left(\frac{m_\chi^\infty}{30 T_n}\right)^{-\frac{5}{2}} \exp\left(-\frac{m_\chi^\infty}{30 T_n}\right)$$

SFOPT and new DM mechanism/signal

Bubble wall dynamics play an essential role in the filtered dark matter mechanism



Siyu Jiang, **FPH**, Chong Sheng Li, arXiv:2305.02218

SFOPT and new DM mechanism/signal

outside the bubble wall $$f_\chi^{\rm eq} = \frac{1}{e^{\tilde{\gamma}_{\rm pl}\left(\sqrt{(p^w)^2+m_0^2}-\tilde{v}_{\rm pl}p_z^w\right)/T} \mp 1} ,$$

$$J_\chi^w = g_\chi \int \frac{d^3p^w}{(2\pi)^3}\frac{p_z^w}{E^w} f_\chi^{\rm eq} \Theta\left(p_z^w - \sqrt{\Delta m^2}\right) ,$$

$$n_\chi^{\rm in} = \frac{J_\chi^w}{\gamma_w v_w} .$$

$$n_\chi^{\rm in} \simeq \frac{g_\chi T^3}{\gamma_w v_w}\left(\frac{\tilde{\gamma}_{\rm pl}\left(1-\tilde{v}_{\rm pl}\right) m_\chi^{\rm in}/T+1}{4\pi^2\tilde{\gamma}_{\rm pl}^3\left(1-\tilde{v}_{\rm pl}\right)^2}\right) e^{-\frac{\tilde{\gamma}_{\rm pl}\left(1-\tilde{v}_{\rm pl}\right)m_\chi^{\rm in}}{T}} ,$$

original works: $$\boxed{\tilde{v}_{\rm pl} = v_w, \quad T = T' = T_n \quad \to \quad \Omega_{\rm DM}^{(0)}h^2 \ [2] .}$$

our works: $$\boxed{\tilde{v}_{\rm pl} = \tilde{v}_+, \quad T = T_+, \quad T' = T_- \qquad \text{(this work with hydrodynamic effects)} .}$$

SFOPT and new DM mechanism/signal

$$\mathscr{L} = -y_\chi \overline{\chi} \Phi \chi - V(\Phi) - \kappa \Phi^\dagger \Phi H^\dagger H + h.c. \ ,$$

toy model

$$V_{\text{eff}}(\phi, T) = \frac{\mu^2 + DT^2}{2}\phi^2 - CT\phi^3 + \frac{\lambda}{4}\phi^4 - \frac{g_\star \pi^2 T^4}{90} \ ,$$

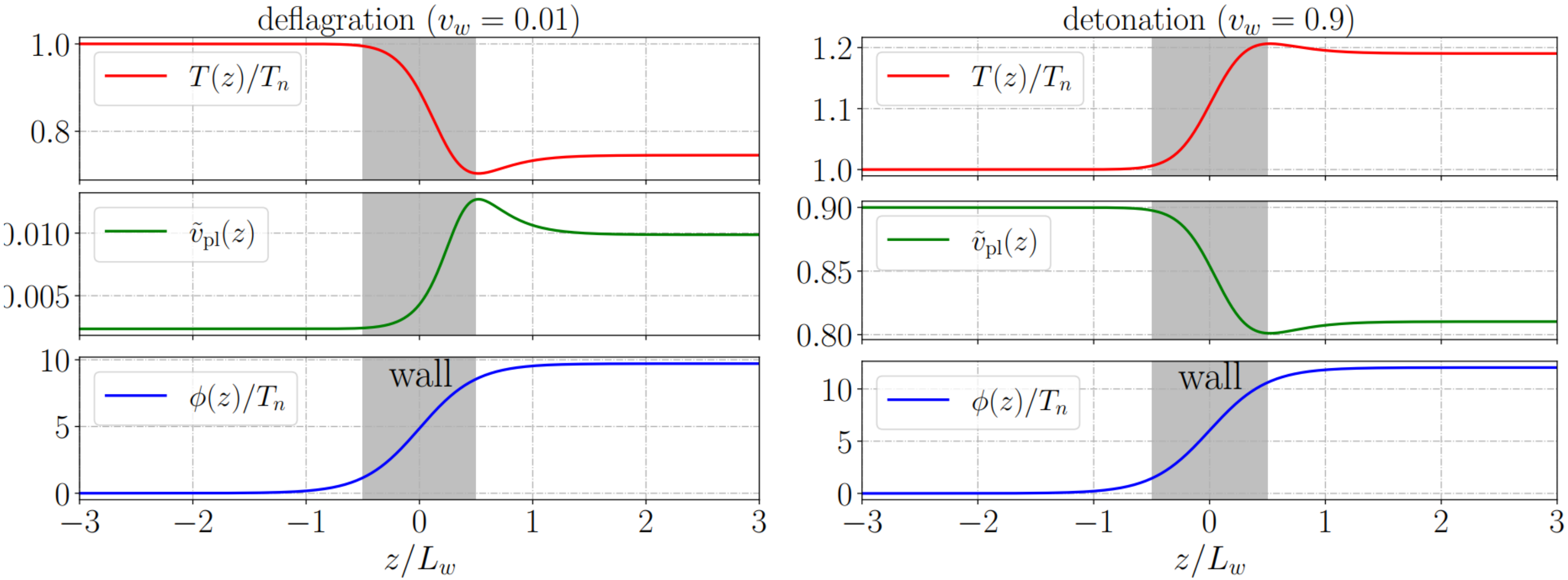
$$\alpha \equiv \frac{\left(1 - \frac{T}{4}\frac{\partial}{\partial T}\right) \Delta V_{\text{eff}}}{\pi^2 g_\star T^4/30} \ ,$$

$$\beta = -\frac{d}{dt}\frac{S_3(T)}{T} = H(T) T \frac{d}{dT}\frac{S_3(T)}{T} \ .$$

SFOPT and new DM mechanism/signal

bubble wall with non-zero width

$$\phi(z) = \frac{\phi(T_-)}{2}\left(1 + \tanh\frac{2z}{L_w}\right) .$$



SFOPT and new DM mechanism/signal

Boltzmann equation $\mathbf{L}\left[f_\chi\right] = \mathbf{C}\left[f_\chi\right]$

$$\boxed{f_\chi = \mathcal{A}\left(z, p_z\right) f^{\rm eq}_{\chi,+} = \mathcal{A}\left(z, p_z\right) \exp\left(-\frac{\tilde{\gamma}_+\left(E - \tilde{v}_+ p_z\right)}{T_+}\right),}$$

$$p_z^2 + m_\chi(z)^2 = \text{const.} \quad \longrightarrow \quad \mathbf{L}\left[f_\chi\right] = \frac{p_z}{E}\frac{\partial f_\chi}{\partial z} - \frac{m_\chi}{E}\frac{\partial m_\chi}{\partial z}\frac{\partial f_\chi}{\partial p_z}\,.$$

$$g_\chi \int \frac{dp_x dp_y}{(2\pi)^2}\mathbf{L}\left[f_\chi\right] \approx$$

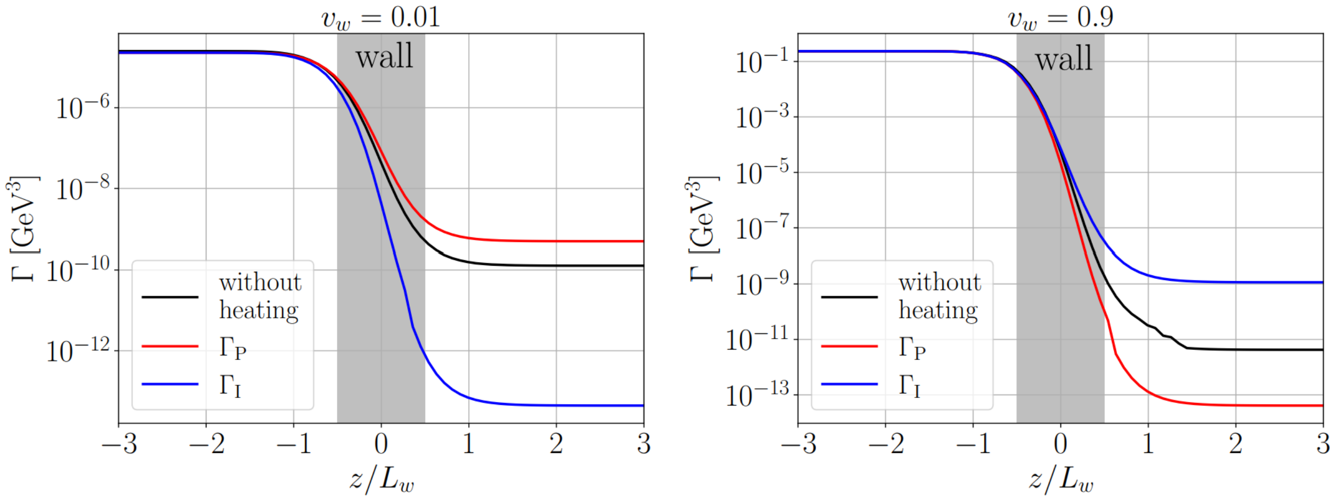
$$\left[\left(\frac{p_z}{m_\chi}\frac{\partial}{\partial z} - \left(\frac{\partial m_\chi}{\partial z}\right)\frac{\partial}{\partial p_z} - \left(\frac{\partial m_\chi}{\partial z}\right)\frac{\tilde{\gamma}_+\tilde{v}_+}{T_+}\right)\mathcal{A}\left(z, p_z\right)\right]\frac{g_\chi m_\chi T_+}{2\pi\tilde{\gamma}_+} e^{\tilde{\gamma}_+\left(\tilde{v}_+ p_z - \sqrt{m_\chi^2 + p_z^2}\right)/T_+}\,.$$

$$\mathbf{C}\left[f_\chi\right] = -\sum_{\text{spins}} d\Pi_{q^p} d\Pi_{k^p} d\Pi_{l^p}\frac{(2\pi)^4}{2E_p^p}\delta^{(4)}\left(p^p + q^p - k^p - l^p\right)|\mathcal{M}|^2\mathcal{P}[f_\chi]$$

$$\mathcal{P}[f_\chi] = f_{\chi_p} f_{\bar{\chi}_q}\left(1 \pm f_{\phi_k}\right)\left(1 \pm f_{\phi_l}\right) - f_{\phi_k} f_{\phi_l}\left(1 \pm f_{\chi_p}\right)\left(1 \pm f_{\bar{\chi}_q}\right)$$

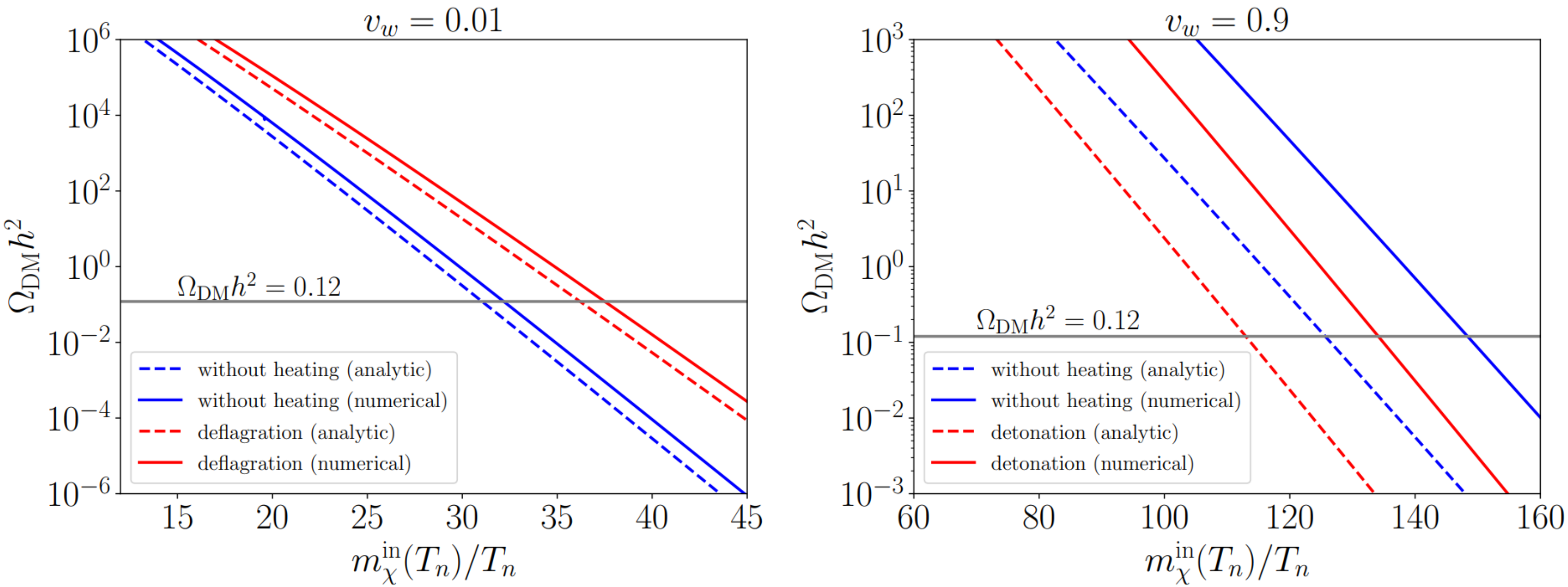
SFOPT and new DM mechanism/signal

$$
\begin{aligned}
g_\chi \int \frac{dp_x dp_y}{(2\pi)^2} \mathbf{C}\left[f_\chi\right] &= -g_\chi g_{\bar{\chi}} \int \frac{dp_x dp_y}{(2\pi)^2 2E_p^{\mathcal{P}}} d\Pi_{q^{\mathcal{P}}} 4F\sigma_{\chi\bar{\chi}\to\phi\phi} \left[f_{\chi_p} f^{\rm eq}_{\bar{\chi}_q,+} - f^{\rm eq}_{\chi_p} f^{\rm eq}_{\bar{\chi}_q}\right] \\
&= -g_\chi g_{\bar{\chi}} \int \frac{dp_x dp_y}{(2\pi)^2 2E_p^{\mathcal{P}}} d\Pi_{q^{\mathcal{P}}} 4F\sigma_{\chi\bar{\chi}\to\phi\phi} \left[\mathcal{A} f^{\rm eq}_{\chi_p,+} f^{\rm eq}_{\bar{\chi}_q,+} - f^{\rm eq}_{\chi_p} f^{\rm eq}_{\bar{\chi}_q}\right] \\
&\equiv \Gamma_{\rm P}(z,p_z)\mathcal{A}\left(z,p_z\right) - \Gamma_{\rm I}(z,p_z) \ ,
\end{aligned}
$$



SFOPT and new DM mechanism/signal

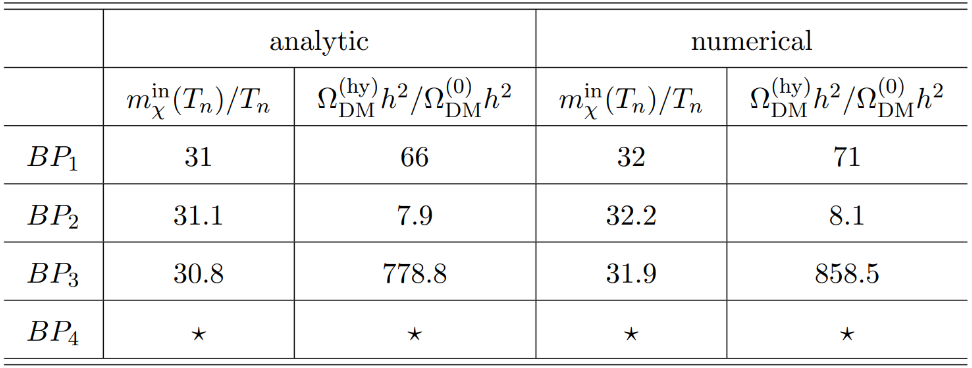
$$n_\chi^{\rm in} = \frac{T_+}{\gamma_w \tilde{\gamma}_+} \int_0^\infty \frac{dp_z}{(2\pi)^2} \mathcal{A}(z \gg L_w, p_z) \exp\left[\tilde{\gamma}_+ \left(\tilde{v}_+ p_z - \sqrt{p_z^2 + (m_\chi^{\rm in})^2}\right)/T_+\right] \left(\sqrt{p_z^2 + (m_\chi^{\rm in})^2} + \frac{T_+}{\tilde{\gamma}_+}\right)$$



SFOPT and new DM mechanism/signal

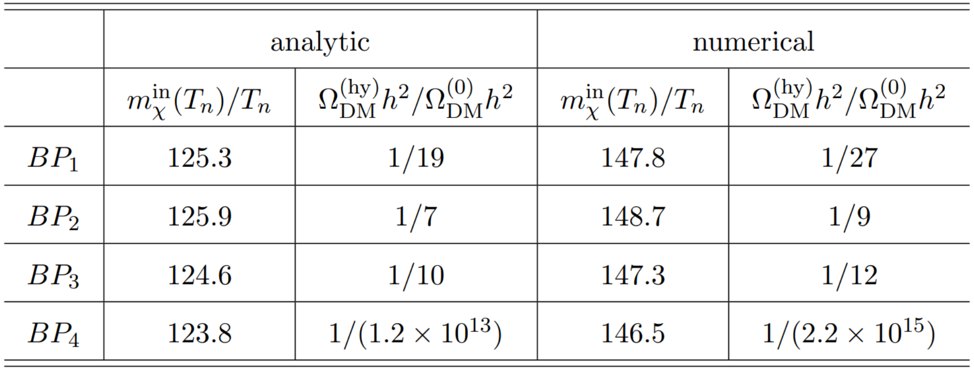
Ratio of dark matter relic density with and without hydrodynamic effects

$$v_w = 0.01$$



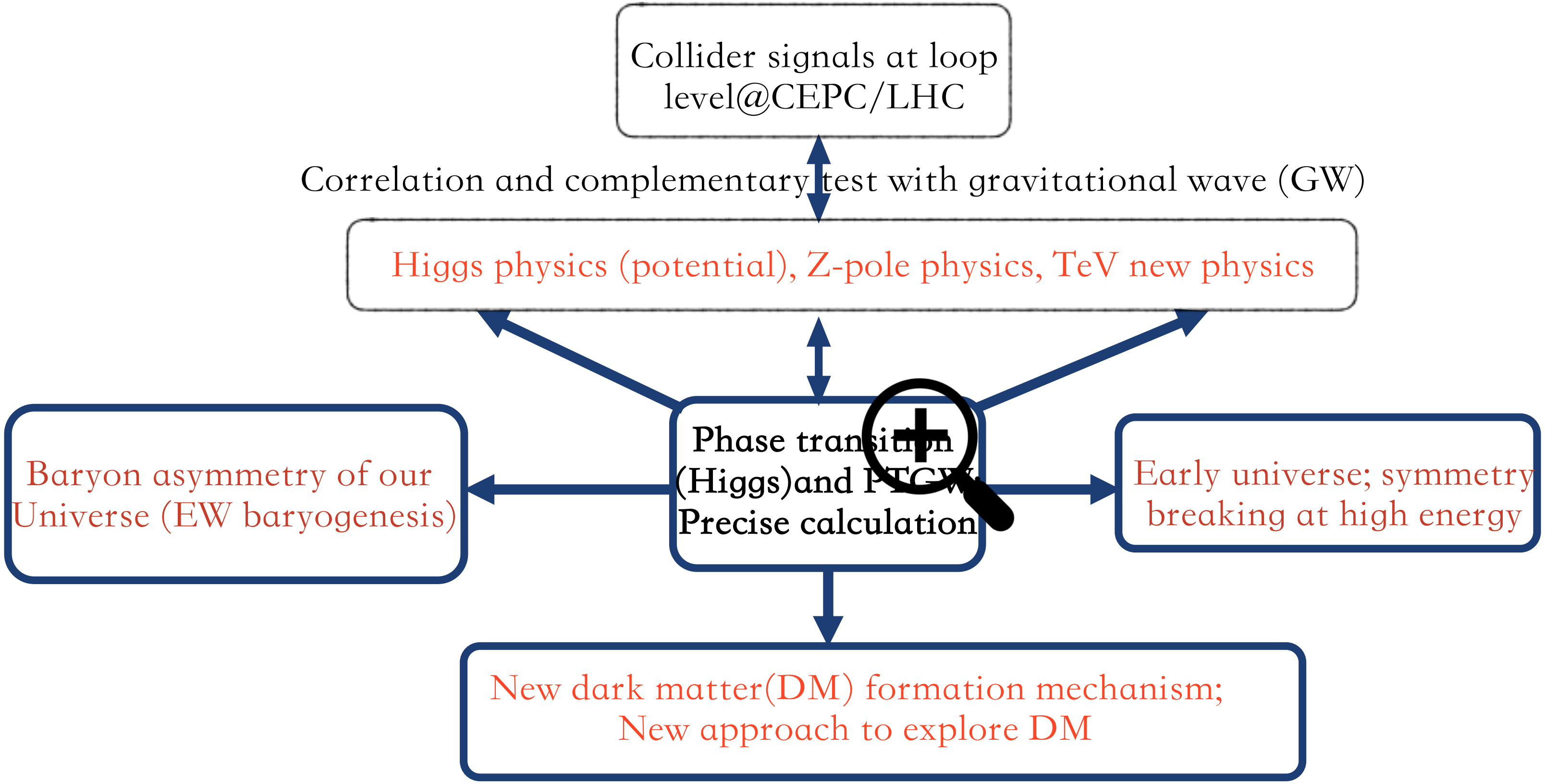
	analytic		numerical	
	$m_\chi^{\rm in}(T_n)/T_n$	$\Omega_{\rm DM}^{\rm (hy)}h^2/\Omega_{\rm DM}^{(0)}h^2$	$m_\chi^{\rm in}(T_n)/T_n$	$\Omega_{\rm DM}^{\rm (hy)}h^2/\Omega_{\rm DM}^{(0)}h^2$
BP_1	31	66	32	71
BP_2	31.1	7.9	32.2	8.1
BP_3	30.8	778.8	31.9	858.5
BP_4	$\star$	$\star$	$\star$	$\star$

$$v_w = 0.9$$



	analytic		numerical	
	$m_\chi^{\rm in}(T_n)/T_n$	$\Omega_{\rm DM}^{\rm (hy)}h^2/\Omega_{\rm DM}^{(0)}h^2$	$m_\chi^{\rm in}(T_n)/T_n$	$\Omega_{\rm DM}^{\rm (hy)}h^2/\Omega_{\rm DM}^{(0)}h^2$
BP_1	125.3	$1/19$	147.8	$1/27$
BP_2	125.9	$1/7$	148.7	$1/9$
BP_3	124.6	$1/10$	147.3	$1/12$
BP_4	123.8	$1/(1.2\times10^{13})$	146.5	$1/(2.2\times10^{15})$

Motivation



Collider signals at loop level@CEPC/LHC

Correlation and complementary test with gravitational wave (GW)

Higgs physics (potential), Z-pole physics, TeV new physics

Baryon asymmetry of our Universe (EW baryogenesis)

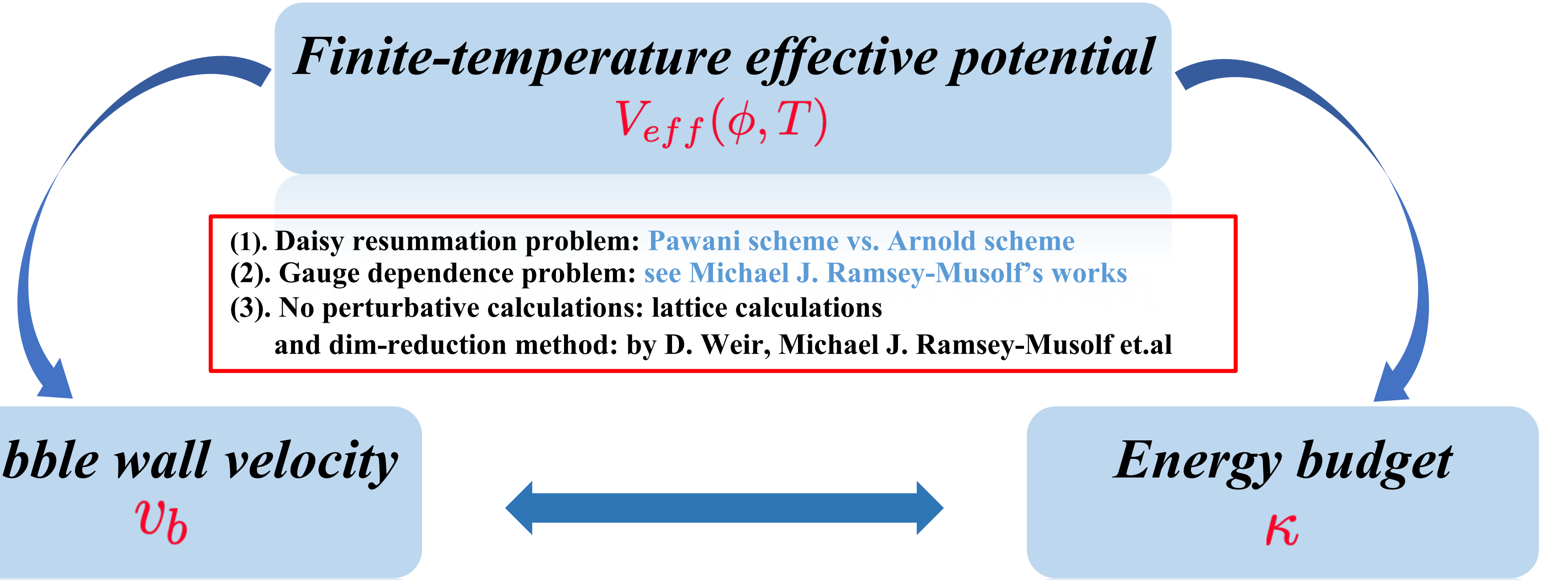
Phase transition (Higgs)and PTGW; Precise calculation

Early universe; symmetry breaking at high energy

New dark matter(DM) formation mechanism; New approach to explore DM

Phase transition Dynamics

Precise predictions on the phase transition dynamics and its GW signals



Finite-temperature effective potential

$V_{eff}(\phi, T)$

(1). Daisy resummation problem: Pawani scheme vs. Arnold scheme
(2). Gauge dependence problem: see Michael J. Ramsey-Musolf's works
(3). No perturbative calculations: lattice calculations
and dim-reduction method: by D. Weir, Michael J. Ramsey-Musolf et.al

Bubble wall velocity

v_b

Energy budget

κ

S. Hoche, J. Kozaczuk, A. J. Long, J. Turner and Y. Wang, arXiv:2007.10343,
Avi Friedlander, Ian Banta, James M. Cline, David Tucker-Smith,
arXiv:2009.14295v2
Xiao Wang, **FPH,** Xinmin Zhang,arXiv:2011.12903
Siyu Jiang, FPH, xiao wang, Phys.Rev.D 107 (2023) 9, 095005

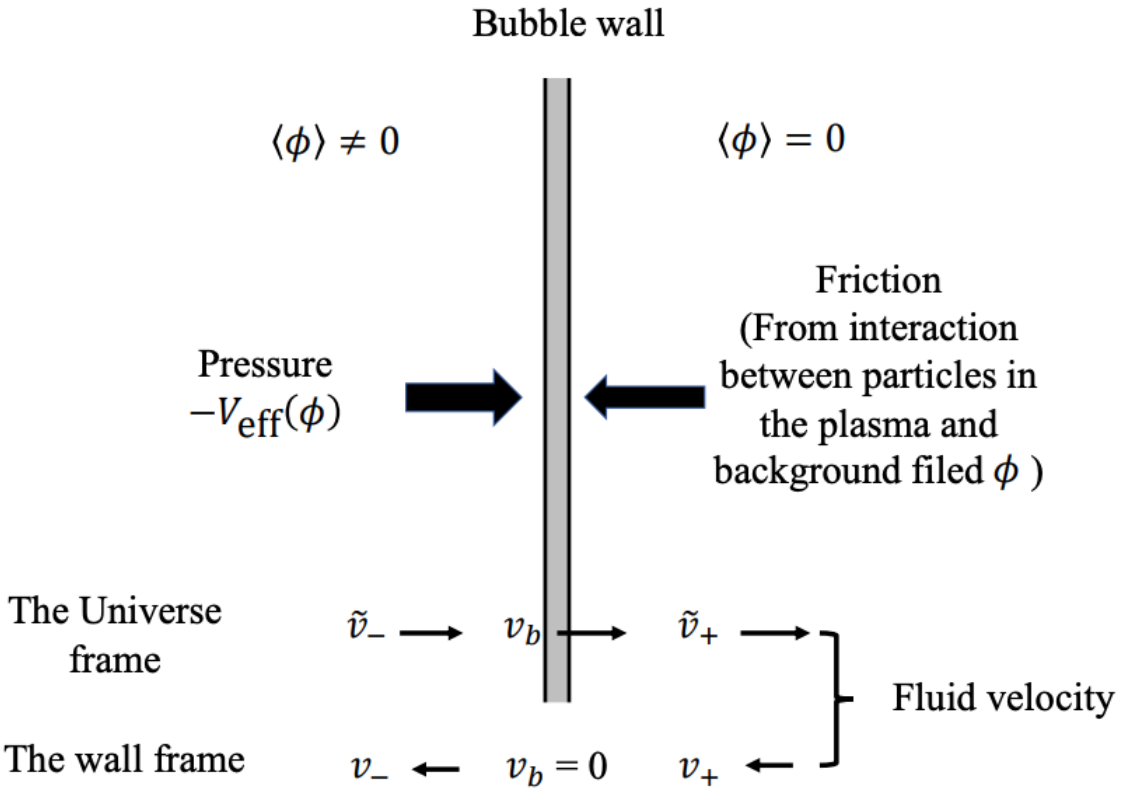
F. Giese, T. Konstandin, K. Schmitz and J. van de Vis,arXiv:2010.09744
Xiao Wang, **FPH** and Xinmin Zhang, Phys.Rev.D 103 (2021) 10, 103520
Xiao Wang, Chi Tian, **FPH**, arXiv: 2301.12328

Phase transition Dynamics

bubble wall velocity:

most difficult； most important for EW baryogenesis and GW experiments

See James cline and Hindmarsh's recent work;
lots of unsolved problems, eg. resummation



Bubble wall

$\langle\phi\rangle \neq 0$

$\langle\phi\rangle = 0$

Pressure
$-V_{\text{eff}}(\phi)$

Friction
(From interaction between particles in the plasma and background filed ϕ)

The Universe frame

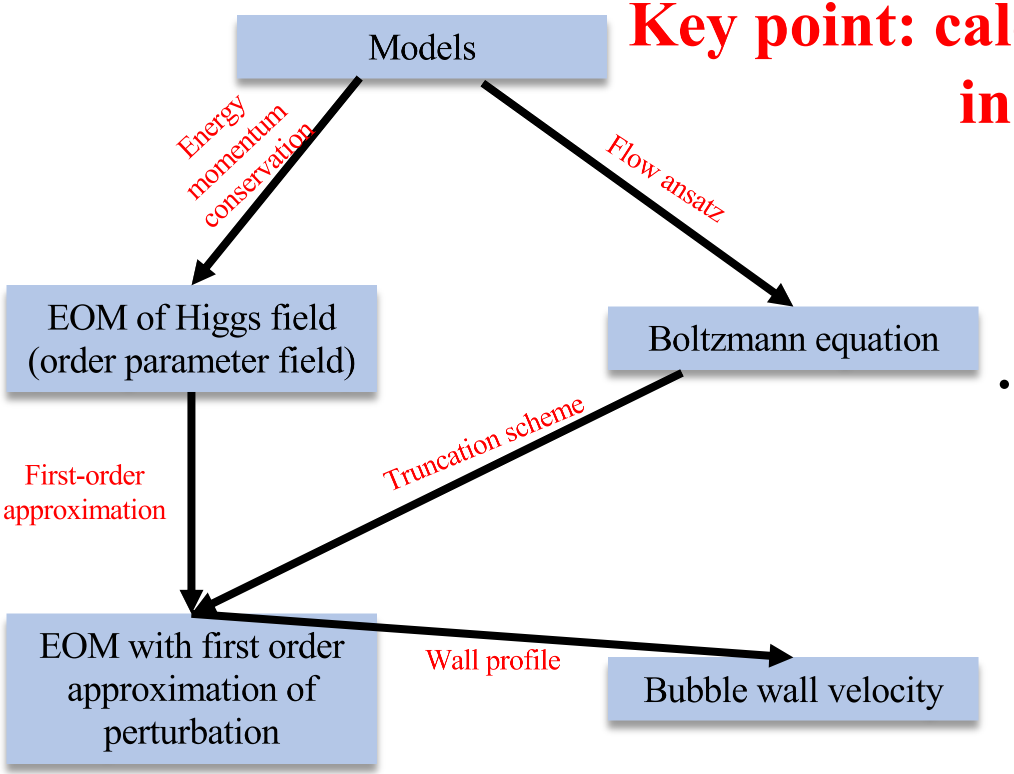
$\tilde{v}_- \longrightarrow$ $v_b \longrightarrow$ $\tilde{v}_+ \longrightarrow$

The wall frame

$v_- \longleftarrow$ $v_b = 0$ $v_+ \longleftarrow$

Fluid velocity

bubble wall velocity in SM EFT



Models

Energy momentum conservation

Flow ansatz

EOM of Higgs field (order parameter field)

Boltzmann equation

First-order approximation

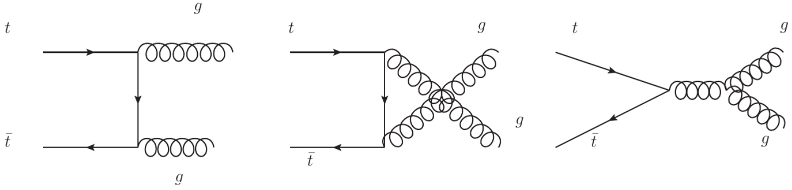
Truncation scheme

EOM with first order approximation of perturbation

Wall profile

Bubble wall velocity

Key point: calculate the particle scattering in thermal plasma

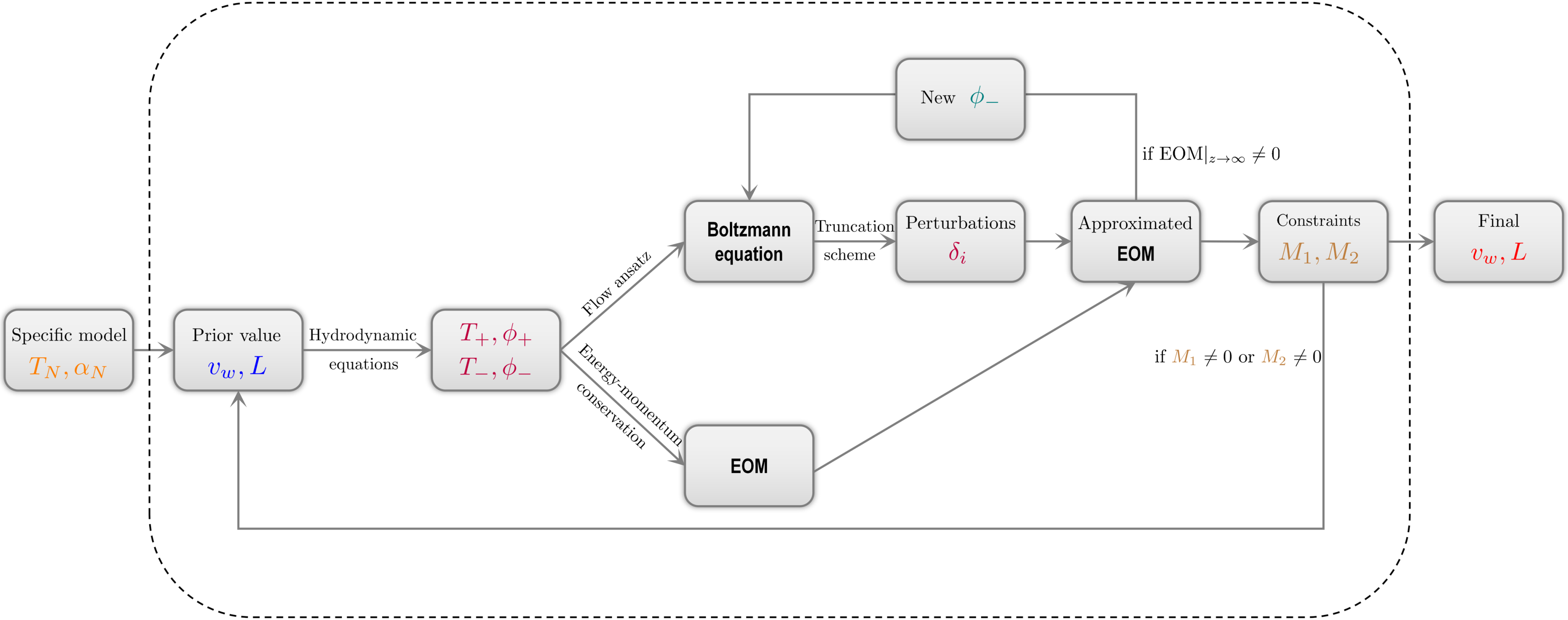


- For a specific benchmark point, the results of beyond leading-log approximation with the contribution of Higgs boson

	$\Lambda/\sqrt{\kappa}$ [GeV]	v_w	L_wT	v_c/T_c	T_n [GeV]
Two-particle	780	0.3382	20.1863	1.1044	100.977
Three-particle	780	0.2499	18.1759	1.1044	100.977

Xiao wang, FPH, Xinmin Zhang, Bubble wall velocity beyond leading-log approximation in electroweak phase transition, arXiv:2011.12903

bubble wall velocity in inert 2HDM



Siyu Jiang, FPH, xiao wang, Phys.Rev.D 107 (2023) 9, 095005

Phase transition Dynamics

$$
\begin{aligned}
V_0 = & \mu_1^2|\Phi|^2 + \mu_2^2|\eta|^2 + \frac{1}{2}\lambda_1|\Phi|^4 + \frac{1}{2}\lambda_2|\eta|^4 \\
& + \lambda_3|\Phi|^2|\eta|^2 + \lambda_4|\Phi^\dagger\eta|^2 + \frac{1}{2}\{\lambda_5(\Phi^\dagger\eta)^2 + \mathrm{H.c.}\} \ ,
\end{aligned}
$$

$$
\Phi = \begin{pmatrix} G^+ \\ \frac{1}{\sqrt{2}}(h + v + iG^0) \end{pmatrix} , \ \eta = \begin{pmatrix} H^+ \\ \frac{1}{\sqrt{2}}(H + iA) \end{pmatrix} ,
$$

$$
V_{\mathrm{CW}}(\phi, T = 0) = \sum_i \frac{n_i}{64\pi^2}\left[m_i^4(\phi)\left(\ln\frac{m_i^2(\phi)}{\overline{m}_i^2} - \frac{3}{2}\right) + 2\overline{m}_i^2\bar{m}_i^2(\phi)\right] ,
$$

$$
V_{\mathrm{T}}(\phi, T > 0) = \sum n_i \frac{T^4}{2\pi^2} I_b\left(\frac{M_i^2}{T^2}\right) ,
$$

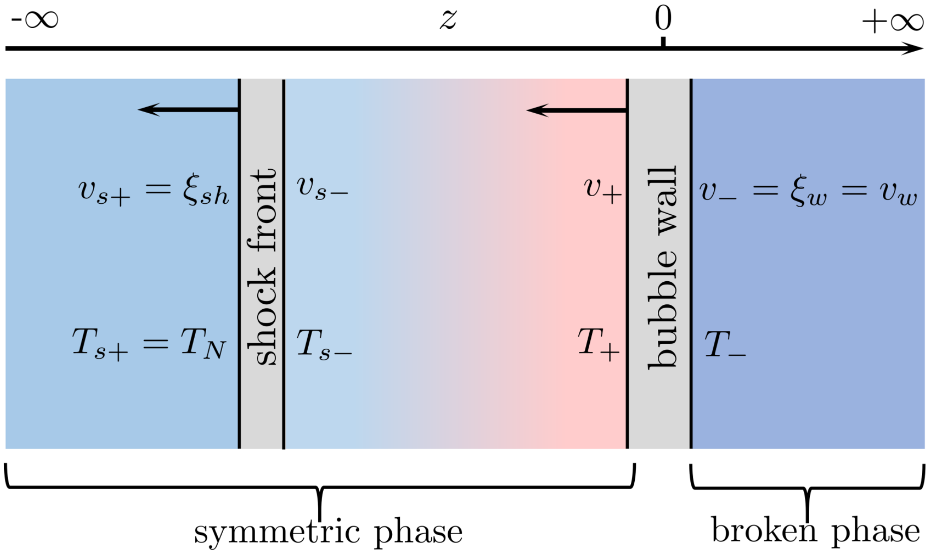
$$
V_{\mathrm{eff}}(\phi, T) = V_0(\phi) + V_{\mathrm{CW}}(\phi) + V_{\mathrm{T}}(\phi, T) \ .
$$

Phase transition Dynamics

Hydrodynamic equations

$$(\xi - v)\frac{\partial_\xi e_f}{\omega_f} = 2\frac{v}{\xi} + \left[1 - \gamma^2 v(\xi - v)\right]\partial_\xi v,$$

$$(1 - v\xi)\frac{\partial_\xi p_f}{\omega_f} = \gamma^2(\xi - v)\partial_\xi v \ ,$$



Phase transition Dynamics

Evolution of the perturbations

source term

$$\hat{A}\delta' + \boxed{\Gamma}\delta = \boxed{\Sigma},$$

collision term

$$\delta = (\mu_t, \delta T_t, T\delta v_t, \mu_W, \delta T_W, T\delta v_W \boxed{, \mu_A, \delta T_A, T\delta v_A)},$$

New particles introduced by inert-doublet model

$$\Sigma = \frac{v_w}{2T}\left(c_1^t(m_t^2)', c_2^t(m_t^2)', 0, c_1^W(m_W^2)', c_2^W(m_W^2)', 0, c_1^A(m_A^2)', c_2^A(m_A^2)', 0\right) \;,$$

$$\hat{A} = \begin{pmatrix} \hat{A}_t & 0 & 0 \\ 0 & \hat{A}_W & 0 \\ 0 & 0 & \hat{A}_A \end{pmatrix}, \quad \text{where} \quad \hat{A}_i = \begin{pmatrix} v_w c_2^i & v_w c_3^i & \frac{1}{3}c_3^i \\ v_w c_3^i & v_w c_4^i & \frac{1}{3}c_4^i \\ \frac{1}{3}c_3^i & \frac{1}{3}c_4^i & \frac{1}{3}v_w c_4^i \end{pmatrix},$$

Phase transition Dynamics

Perturbations for background

$$\tilde{c}_4 \left(v_w \delta T'_{bg} + \frac{\delta v'_{bg}}{3} T \right) = N_t(\mu_t \Gamma_{\mu 2,t} + \delta T_t \Gamma_{T2,t}) + \sum_{\text{bosons}} N_b(\mu_b \Gamma_{\mu 2,b} + \delta T_b \Gamma_{T2,b}) \ ,$$

$$\frac{\tilde{c}_4}{3}(\delta T'_{bg} + v_w T \delta v'_{bg}) = N_t T \delta v_t \Gamma_{v,t} + \sum_{\text{bosons}} N_b T \delta v_b \Gamma_{v,b} \ , \quad \mu_{bg} = 0 \ ,$$

Phase transition Dynamics

Collision terms

$$\Gamma_{\mu 1,t} \simeq \left(5.0\times 10^{-4} g_s^4 + 5.8\times 10^{-4} g_s^2 y_t^2\right) T \ ,$$

$$\Gamma_{T1,t} \simeq \Gamma_{\mu 2,t} \simeq \left(1.1\times 10^{-3} g_s^4 + 1.3\times 10^{-3} g_s^2 y_t^2\right) T \ ,$$

$$\Gamma_{T2,t} \simeq \left(1.1\times 10^{-2} g_s^4 + 4.0\times 10^{-3} g_s^2 y_t^2\right) T \ ,$$

$$\Gamma_{v,t} \simeq \left(2.0\times 10^{-2} g_s^4 + 1.8\times 10^{-3} g_s^2 y_t^2\right) T \ ,$$

$$\Gamma_{\mu 1,W} \simeq \left(2.3\times 10^{-3} g_s^2 g_w^2 + 2.0\times 10^{-3} g_w^4\right) T \ ,$$

$$\Gamma_{T1,W} \simeq \Gamma_{\mu 2,W} \simeq \left(4.7\times 10^{-3} g_s^2 g_w^2 + 4.1\times 10^{-3} g_w^4\right) T$$

$$\Gamma_{T2,W} \simeq \left(1.5\times 10^{-2} g_s^2 g_w^2 + 1.5\times 10^{-2} g_w^4\right) T \ ,$$

$$\Gamma_{v,W} \simeq \left(5.7\times 10^{-2} g_s^2 g_w^2 + 1.5\times 10^{-2} g_w^4\right) T \ ,$$

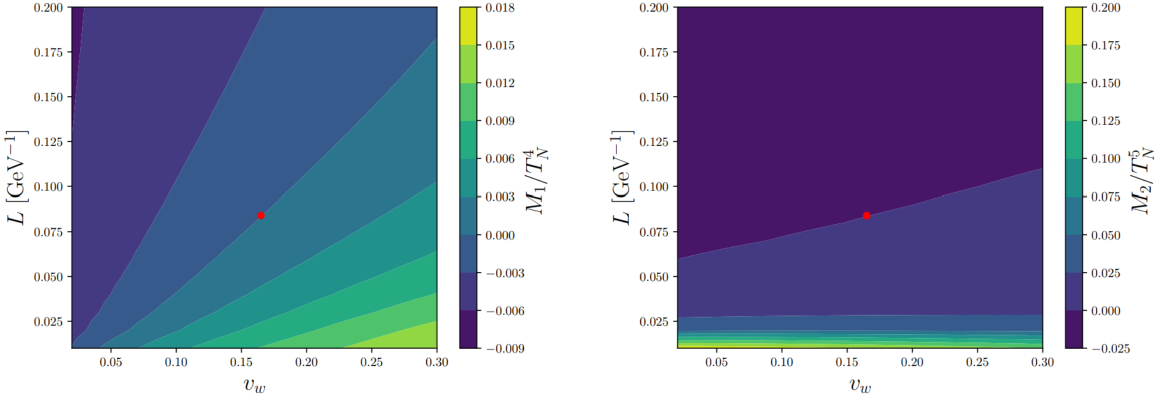
$$\Gamma_{\mu 1,A} \simeq 1.0\times 10^{-2} \lambda_3^4 T \ ,$$

$$\Gamma_{T1,A} \simeq \Gamma_{\mu 2,A} \simeq 4.9\times 10^{-3} \lambda_3^4 T \ ,$$

$$\Gamma_{T2,A} \simeq 5.1\times 10^{-3} \lambda_3^4 T \ ,$$

$$\Gamma_{v,A} \simeq 1.8\times 10^{-3} \lambda_3^4 T \ .$$

Phase transition Dynamics



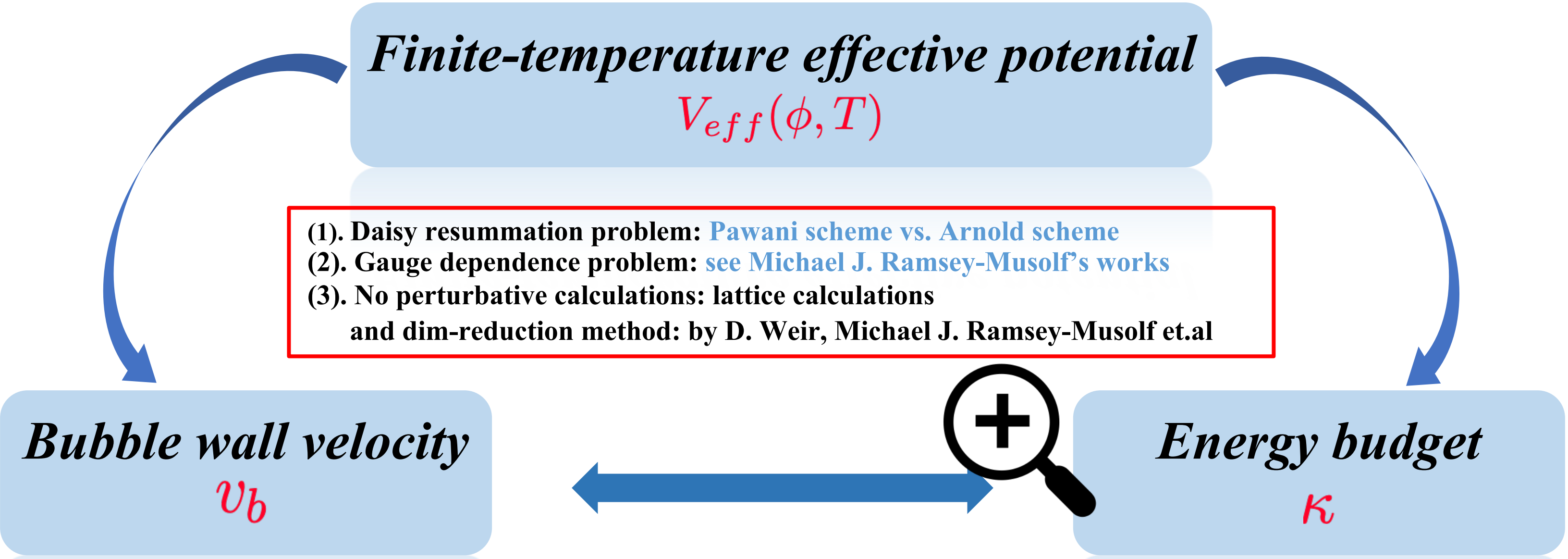
	T_c [GeV]	T_N [GeV]	v_w	L [GeV^{-1}]
Benchmark A	118.3	117.1	0.165	0.084
Benchmark B	118.6	117.5	0.164	0.085
Benchmark C	119.4	118.4	0.164	0.088

In the allowed parameter spaces of IDM, the bubble wall velocity varies slightly around 0.165.

Siyu Jiang, **FPH**, Xiao Wang, Phys.Rev.D 107 (2023) no.9, 095005

Phase transition Dynamics

Precise predictions on the phase transition dynamics and its GW signals



Finite-temperature effective potential

$V_{eff}(\phi,T)$

(1). **Daisy resummation problem:** **Pawani scheme vs. Arnold scheme**
(2). **Gauge dependence problem:** **see Michael J. Ramsey-Musolf's works**
(3). **No perturbative calculations: lattice calculations**
and dim-reduction method: by D. Weir, Michael J. Ramsey-Musolf et.al

Bubble wall velocity

v_b

Energy budget

κ

S. Hoche, J. Kozaczuk, A. J. Long, J. Turner and Y. Wang, arXiv:2007.10343,
Avi Friedlander, Ian Banta, James M. Cline, David Tucker-Smith,
arXiv:2009.14295v2
Xiao Wang, **FPH,** Xinmin Zhang,arXiv:2011.12903
Siyu Jiang, FPH, xiao wang, Phys.Rev.D 107 (2023) 9, 095005

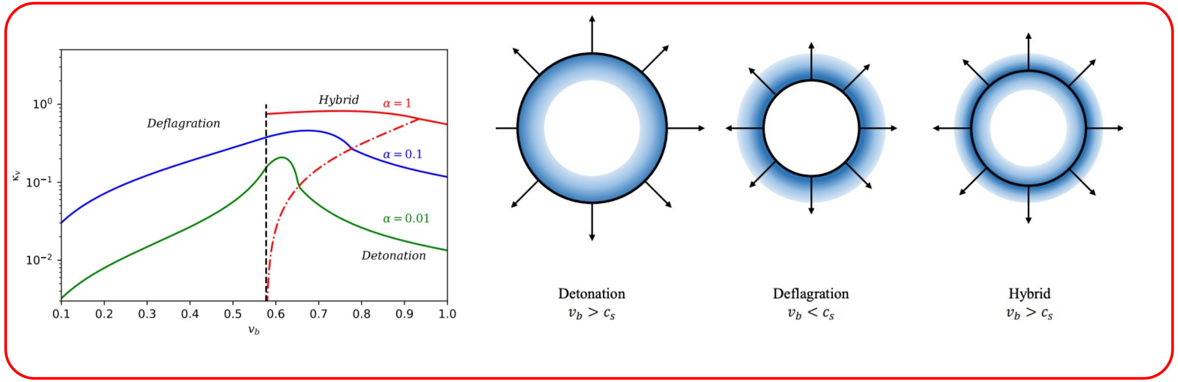
F. Giese, T. Konstandin, K. Schmitz and J. van de Vis,arXiv:2010.09744
Xiao Wang, **FPH** and Xinmin Zhang, Phys.Rev.D 103 (2021) 10, 103520
Xiao Wang, Chi Tian, **FPH**, arXiv: 2301.12328

Energy budget

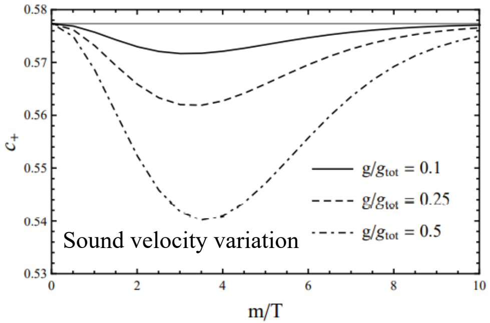
Energy budget (to measure the efficiency of the energy released by a SFOPT converting to the kinetic energy of sounding plasma)

LISA Cosmology working group *JCAP* 03 (2020) 024

$$h^2\Omega_{\rm sw}(f) \simeq 1.64\times10^{-6}(H_*\tau_{\rm sw})(H_*R_*)\left(\frac{\kappa_v\alpha}{1+\alpha}\right)^2\left(\frac{100}{g_*}\right)^{1/3}(f/f_{\rm sw})^3\left(\frac{7}{4+3(f/f_{\rm sw})^2}\right)^{7/2}$$



J. R. Espinosa, T. Konstandin, J. M. No, and G. Servant, JCAP 06 (2010) 028.



L. Leitao and A. Megevand, Nucl. Phys. B **891**, 159-199 (2015)

But for a realistic SFOPT, particle can obtain the mass, hence, the sound velocity can deviate from pure radiation phase. The energy budget/efficiency parameter beyond the bag model are study in our recent work:

Xiao Wang,**FPH** and Xinmin Zhang, PRD 103 (2021) 10, 103520
Xiao Wang, **FPH** and Xinmin Zhang, JCAP 05, 045 (2020)
Xiao Wang, Chi Tian, **FPH**, arXiv: 2301.12328

Phase transition Dynamics

Energy budget for phase transition GW

- Matching condition

$$w_-v_-^2\gamma_-^2 + p_- = w_+v_+^2\gamma_+^2 + p_+, \quad w_-v_-\gamma_-^2 = w_+v_+\gamma_+^2. \longrightarrow v_+v_- = \frac{p_+ - p_-}{e_+ - e_-}, \frac{v_+}{v_-} = \frac{e_- + p_+}{e_+ + p_-}.$$

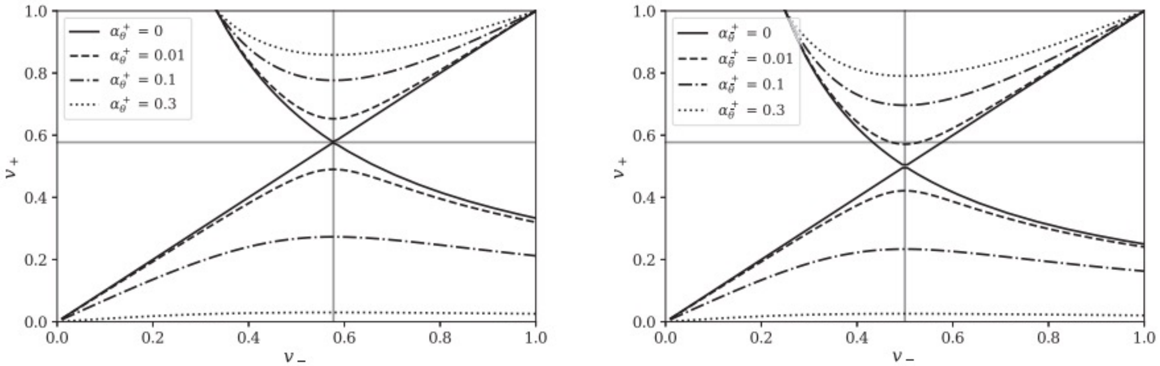


FIG. 1. The fluid velocities v_+ and v_- in the reference frame of bubble wall for different definitions and values of phase transition strength parameter. The horizontal and vertical gray lines indicate the sound velocities of symmetric and broken phase. Left panel: the bag model. Right panel: the DSVM with $c_+^2 = 1/3$ and $c_-^2 = 0.25$.

X. Wang, F. P. Huang and X. Zhang, PRD 103 (2021) 10, 103520

Phase transition Dynamics

Energy budget for phase transition GW

GW spectrum and SNR for different EoS with different parameter combination:

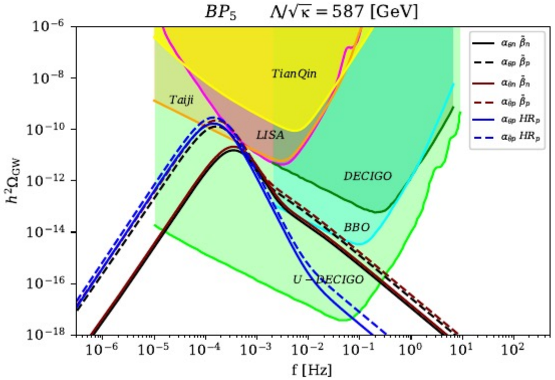


TABLE II. The SNR of BP_5 for different experiment configurations with different combinations of phase transition parameters and models of EOS.

	$\alpha_{\theta n}\,\tilde\beta_n$	$\alpha_{\theta p}\,\tilde\beta_p$	$\alpha_{\bar\theta n}\,\tilde\beta_n$	$\alpha_{\bar\theta p}\,\tilde\beta_p$	$\alpha_{\theta p}\,HR_p$	$\alpha_{\bar\theta p}\,HR_p$
$\mathrm{SNR}_{(\mathrm{LISA})}$	7.949	16.930	10.913	28.836	16.009	27.468
$\mathrm{SNR}_{(\mathrm{Taiji})}$	14.760	58.607	20.271	100.343	66.216	113.609
$\mathrm{SNR}_{(\mathrm{TianQin})}$	0.452	1.506	0.620	2.576	1.629	2.794

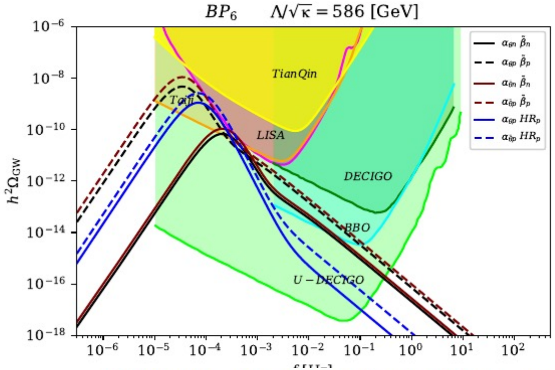


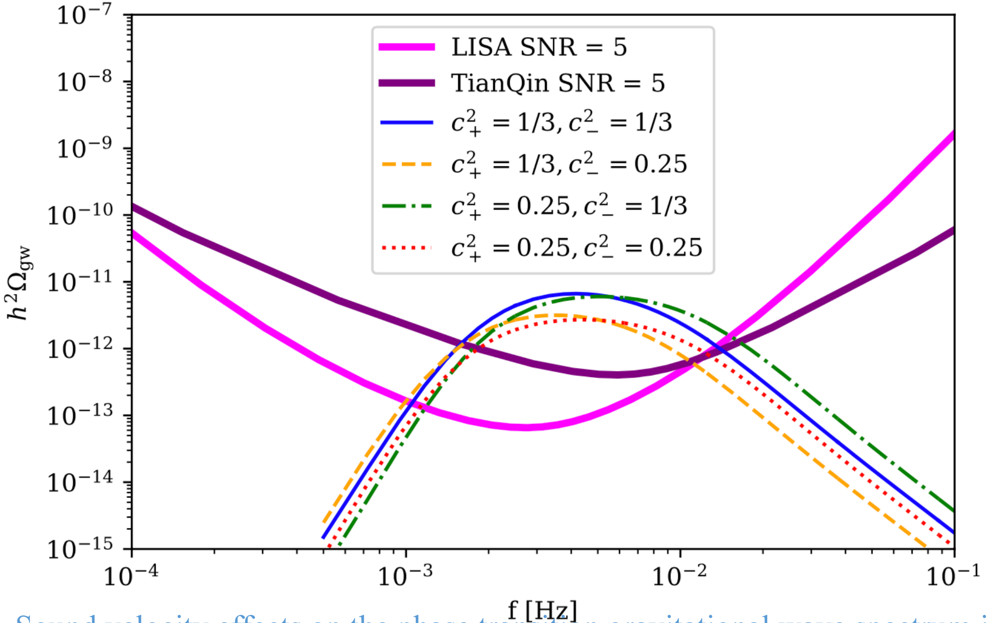
TABLE III. The SNR of BP_6 for different experiment configurations with different combinations of phase transition parameters and models of EOS.

	$\alpha_{\theta n}\,\tilde\beta_n$	$\alpha_{\theta p}\,\tilde\beta_p$	$\alpha_{\bar\theta n}\,\tilde\beta_n$	$\alpha_{\bar\theta p}\,\tilde\beta_p$	$\alpha_{\theta p}\,HR_p$	$\alpha_{\bar\theta p}\,HR_p$
$\mathrm{SNR}_{(\mathrm{LISA})}$	14.230	15.368	22.470	26.382	17.367	40.816
$\mathrm{SNR}_{(\mathrm{Taiji})}$	38.666	427.813	61.208	1000.501	213.123	500.668
$\mathrm{SNR}_{(\mathrm{TianQin})}$	1.060	5.569	1.678	12.934	3.973	9.333

X. Wang, F. P. Huang and X. Zhang, PRD 103 (2021) 10, 103520

Phase transition Dynamics

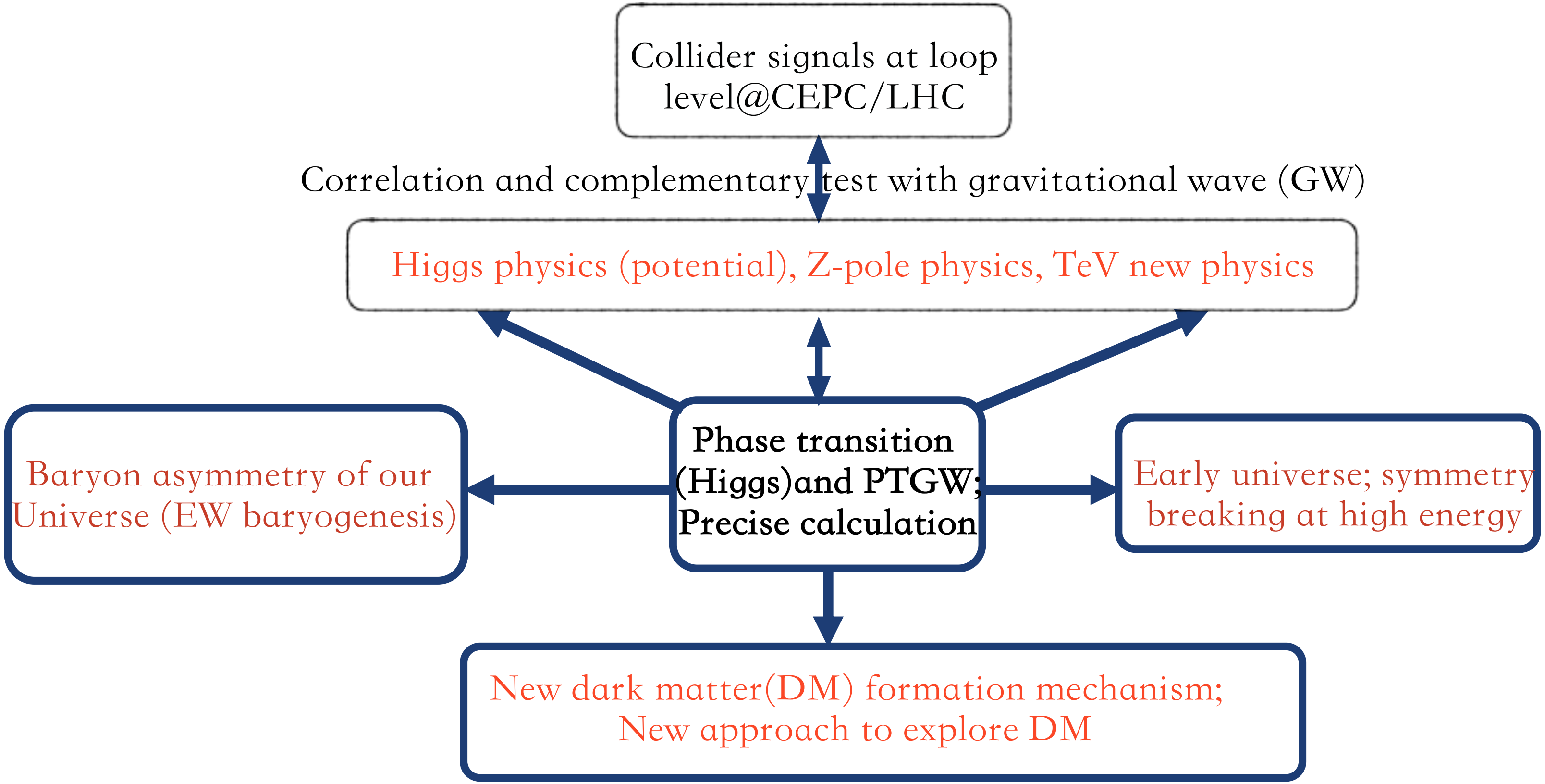
Sound velocity effects on the phase transition gravitational wave spectrum in the Sound Shell Model



Xiao Wang, **FPH**, Yongping Li, Sound velocity effects on the phase transition gravitational wave spectrum in the Sound Shell Model, arXiv:2112.14650

Xiao Wang, **FPH,** Xinmin Zhang, Energy budget and the gravitational wave spectra beyond the bag model ,Phys.Rev.D 103 (2021) 10, 103520

Summary



Collider signals at loop level@CEPC/LHC

Correlation and complementary test with gravitational wave (GW)

Higgs physics (potential), Z-pole physics, TeV new physics

Phase transition (Higgs)and PTGW; Precise calculation

Baryon asymmetry of our Universe (EW baryogenesis)

Early universe; symmetry breaking at high energy

New dark matter(DM) formation mechanism; New approach to explore DM

Summary and Outlook

- **EW first-order phase transition has abundant collider and cosmological effects in baryogenesis, dark matter, GW…**

- **The correlation between GW and collider signals at CEPC can make complementary test on the Higgs nature, baryogenesis, dark matter and the cosmic evolution history at 100 GeV.**

- **More precise study are needed: resummation， non-perturbation, bubble dynamics (wall velocity, energy budget).**

Thanks! Comments and collaborations are welcome!

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