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Circular Electron Positron Collider (CEPC)

中國科學院高能物理研究所
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BSM Opportunities in Triangle Singularity

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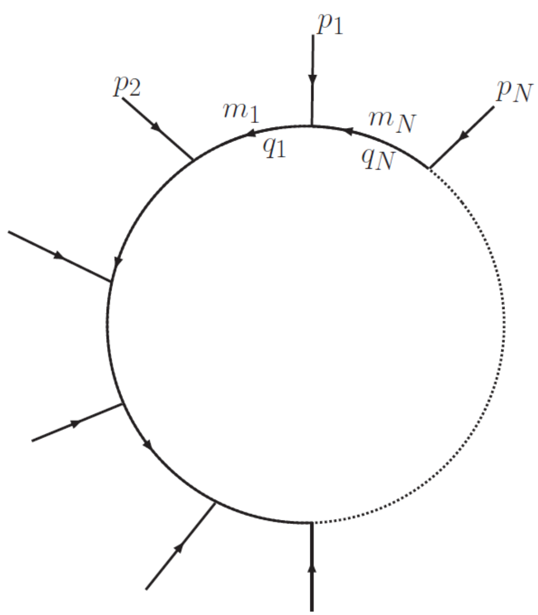
Based on work with Yu Gao, Yu Jia & Jia-Yue Zhang
arXiv: 2211.12920

Landau Singularity

- A situation when all internal particles go on shell inside a loop



One loop Feynman diagram with N external particles

The scalar N-point loop integral

$$\int \frac{d^D q}{(2\pi)^D i} \frac{1}{D_1 D_2 \cdots D_N} \qquad D_i = q_i^2 - m_i^2 + i\varepsilon$$

$$\int_0^\infty dx_1 \cdots dx_N \delta(\sum_{i=1}^{N} x_i - 1) \int \frac{d^D q}{(2\pi)^D i} \frac{1}{(x_1 D_1 + x_2 D_2 + \cdots x_N D_N)^N}$$

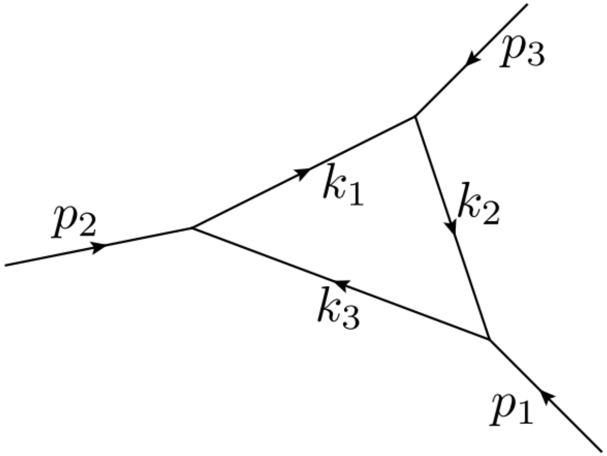
The **leading** Landau singularities are given by

$\sum_i x_i$=1 and x_i>0; $\qquad \sum_i x_i \, q_i^\mu = 0;$

$x_i(q_i^2 - m_i^2) = 0 \qquad$ *L.D.Landau 1958'*

Such singularity is corresponding to kinematic pole of S-matrix, and its location is determined completely by kinematical variables.

N=3: Triangle Singularity (TS)



$$\begin{cases} \alpha_1 k_1^\mu + \alpha_2 k_2^\mu + \alpha_3 k_3^\mu = 0, \\ k_1^2 - m_1^2 = k_2^2 - m_2^2 = k_3^2 - m_3^2 = 0. \end{cases}$$

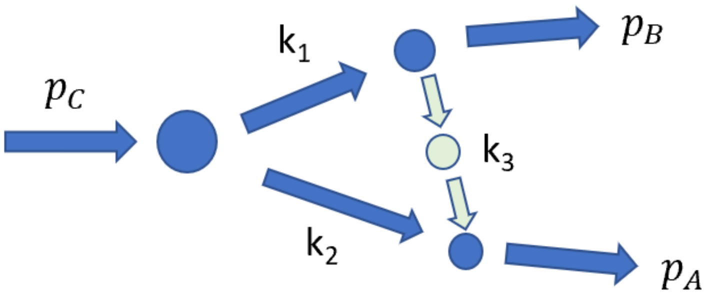
$$\Downarrow$$

$$\begin{cases} \beta_1 + \beta_2 y_{12} + \beta_3 y_{13} = 0 \\ \beta_1 y_{12} + \beta_2 + \beta_3 y_{23} = 0 \\ \beta_1 y_{13} + \beta_2 y_{23} + \beta_3 = 0 \end{cases}$$

$$y_{ij} = \frac{k_i \cdot k_j}{m_i m_j} = \frac{m_i^2 + m_j^2 - p_k^2}{2 m_i m_j} \quad (i \neq j \neq k) \qquad \beta_i = \alpha_i m_i$$

- The above equations build the mathematical relationship between internal mass and external momentum for triggering TS.
- However, only the singularity in physical region will emerge in amplitude.

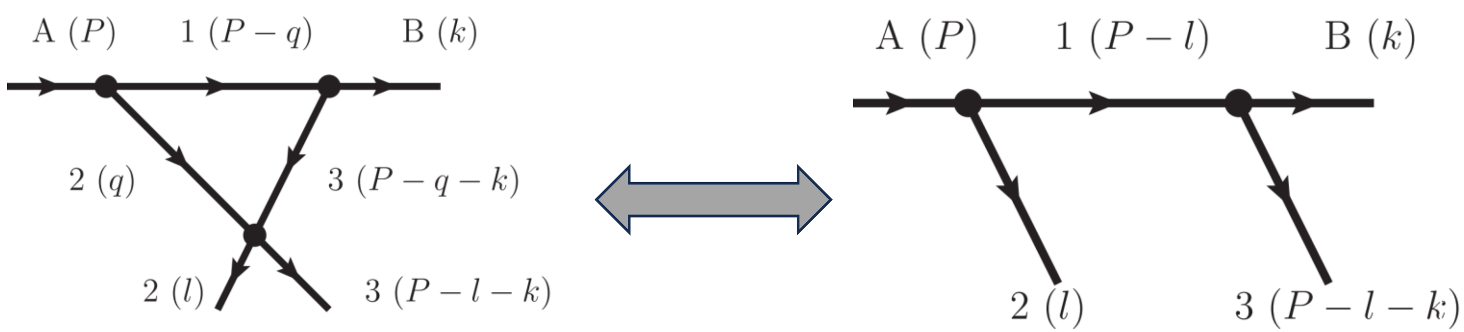
Physical picture for TS



When 3 moves faster than 2, it can catch 2 and fuse to A.

In this case, singularity locate in the physical region.

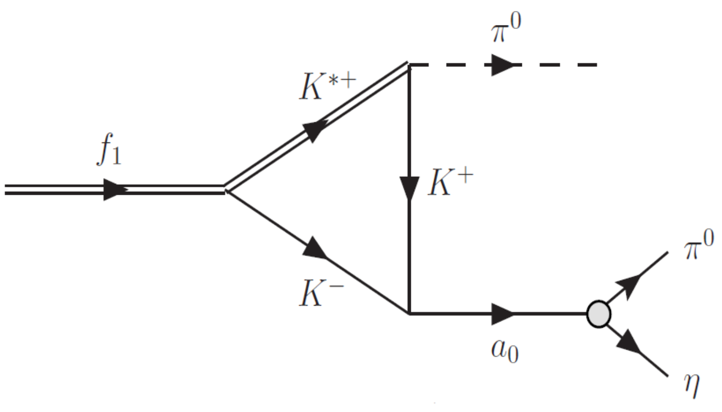
- **Coleman Norton theorem**: the singularity is on the physical boundary if and only if the diagram can be interpreted as a classical process in spacetime.



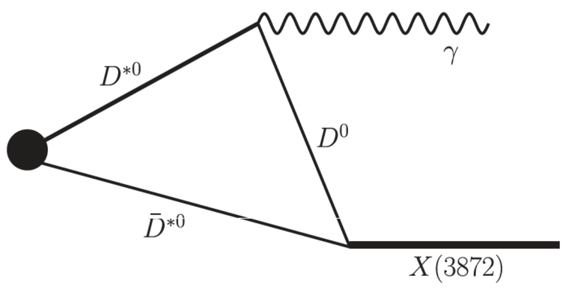
- **Schmid theorem**: $t_t^{(0)} + t_L = t_t^{(0)} e^{2i\delta}$

- When rescattering is **inelastic**, the Schmid theorem does not hold. The Singularity can be observed (due to the loop contribution) in the 2 + 3 invariant mass distribution.

Application in hadron physics



One TS diagram in f(1285) decay
Aceti, Dias, Oset, 1501.06505



X(3872) production with a TS diagram
F.K.Guo, 1902.11221

- Large number of hadronic states, make it easier to satisfy singularity conditions.
- Singularity may be mis-identified as new resonances. It can be also used to make precise measurements and enhance the production of hadronic molecules.
- For TS in hadron spectroscopy, see review Guo, Liu, Sakai, 1912.07030.

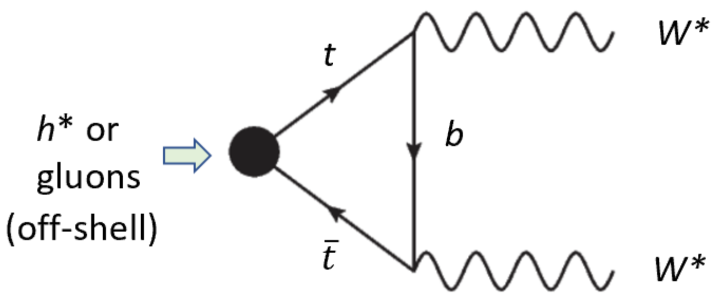
TS in hadron physics

Structures	Processes	Loops	I/F	Refs.
$\rho(1480)$ [78, 79]	$\pi^- p \to \phi\pi^0 n$	$K^*\bar{K}K$	I	[80, 81]
$\eta(1405/1475)$ [82–86]	$\eta(1405/1475) \to \pi f_0$	$K^*\bar{K}K$	I	[87–91][a,b]
$f_1(1420)$ [92]	$f_1(1285) \to \pi a_0/\pi f_0$	$K^*\bar{K}K$	I	[89, 93–95][b]
$a_1(1420)$ [96, 97]	$a_1(1260) \to f_0\pi \to 3\pi$	$K^*\bar{K}K$	I	[97–99]
1.4 GeV [100]	$J/\psi \to \phi\pi^0\eta/\phi\pi^0\pi^0$	$K^*\bar{K}K$	I	[101][b]
1.42 GeV	$B^- \to D^{*0}\pi^- f_0(a_0), \tau \to \nu_\tau\pi^- f_0(a_0)$	$K^*\bar{K}K$	I	[102, 103]
	$D_s^+ \to \pi^+\pi^0 f_0(a_0), \bar{B}_s^0 \to J/\psi\pi^0 f_0(a_0)$	$K^*\bar{K}K$	I	[104, 105]
$f_2(1810)$ [10]	$f_2(1640) \to \pi\pi\rho$	$K^*\bar{K}^*K$	I	[106]
1.65 GeV	$\tau \to \nu_\tau\pi^- f_1(1285)$	$K^*\bar{K}^*K$	I	[107]
1515 MeV	$J/\psi \to K^+K^- f_0\,(a_0)$	$\phi\bar{K}K$	I	[108]
2.85 GeV, 3.0 GeV	$B^- \to K^-\pi^- D_{s0}^*/K^-\pi^- D_{s1}$	$K^{*0}D^{(*)0}K^+$	I	[109, 110]
5.78 GeV	$B_c^+ \to \pi^0\pi^+ B_s^0$	$\bar{K}^{*0}B^+\bar{K}$	F	[111]
[4.01, 4.02] GeV	$[\bar{D}^{*0}D^{*0}] \to \gamma X$	$D^{*0}\bar{D}^{*0}D^0$	I	[112]
4015 MeV	$e^+e^- \to \gamma X$	$D^{*0}\bar{D}^{*0}D^0$	I	[113, 114]
4015 MeV	$B \to KX\pi$, $pp/p\bar{p} \to X\pi$+anything	$D^{*0}\bar{D}^{*0}D^0$	I	[115, 116]
$\Upsilon(11020)$ [117, 118]	$e^+e^- \to Z_b\pi$	$B_1(5721)\bar{B}B^*$	I	[119, 120]
3.73 GeV	$X \to \pi^0\pi^+\pi^-$	$D^{*0}\bar{D}^0D^0$	F	[121]
[4.22, 4.24] GeV	$e^+e^- \to \gamma J/\psi\phi/\pi^0 J/\psi\eta$	$D_{s0(s1)}^*\bar{D}_s^{(*)}D_s^{(*)}$	F	[122]
[4.08, 4.09] GeV	$e^+e^- \to \pi^0 J/\psi\eta$	$D_{s0(s1)}^*\bar{D}_s^{(*)}D_s^{(*)}$	F	[122]
$Z_c(3900)$ [31, 32]	$e^+e^- \to J/\psi\pi^+\pi^-$	$D_1\bar{D}D^*$	F	[119, 123–127][c]
		$D_0^*(2400)\bar{D}^*D$	F	[128, 129]
$Z_c(4020, 4030)$ [33, 130]	$e^+e^- \to \pi^+\pi^- h_c(\psi')$	$D_{1(2)}\bar{D}^{(*)}D^{(*)}$	F	[125]
$X(4700)$ [131, 132]	$B^+ \to K^+J/\psi\phi$	$K_1(1650)\psi'\phi$	F	[133]
$Z_c(4430)$ [30, 134]	$\bar{B}^0 \to K^-\pi^+J/\psi$	$\bar{K}^{*0}\psi(4260)\pi^+$	F	[135]
$Z_c(4200)$ [136, 137]	$\bar{B}^0 \to K^-\pi^+\psi(2S)$	$\bar{K}_2^*\psi(3770)\pi^+$	F	[135]
	$\Lambda_b^0 \to p\pi^- J/\psi$	$N^*\psi(3770)\pi^-$	F	[135]
$X(4050)^\pm$ [138]	$\bar{B}^0 \to K^-\pi^+\chi_{c1}$	$\bar{K}^{*0}X\pi^+$	F	[139]
$X(4250)^\pm$ [138]	$\bar{B}^0 \to K^-\pi^+\chi_{c1}$	$\bar{K}_2^*\psi(3770)\pi^+$	F	[139]
$Z_b(10610)$ [34]	$e^+e^- \to \Upsilon(1S)\pi^+\pi^-$	$B_J^*\bar{B}^*B$	F	[128]

Guo, Liu, Sakai, 1912.07030

LS at electroweak energy scale

- Known example in the SM: h*-> ttb -> W*W*

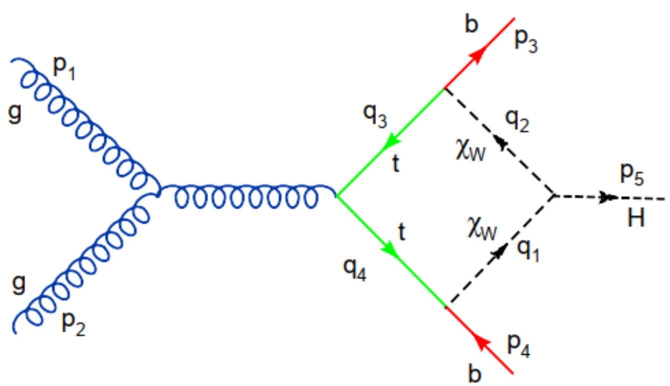


Virtual final-state *W* bosons:
Can trigger T.S. for $350 < \sqrt{s} < 750$ GeV
No physical solution for two on-shell Ws.

Leads to an anomalous threshold:
finite correction in cross-section.

- N=4 (box) Landau singularity in gg-> h bb

Boudjema and Duc Ninh, 0806.1498



Virtual final-state *H* bosons:
Can trigger L.S. for $\sqrt{s} > 2m_t$ and $\sqrt{p_5^2} > 2m_W$

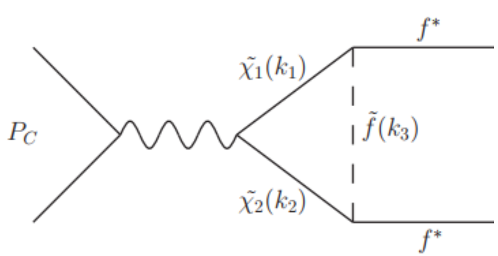
All the four particles in the loop can be simultaneously on-shell.

TS in BSM

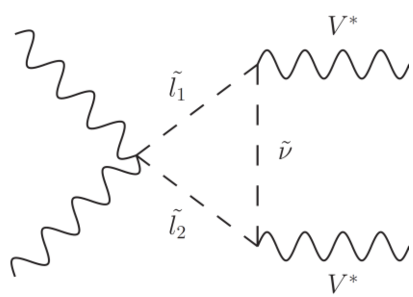
- A kaleidoscope of new particles in BSM, also make it easy to satisfy Landau singularity conditions.

- BSM particles fill in the internal lines and couple to the SM, which can produce a **purely visible SM final states** that carry BSM scale energies.

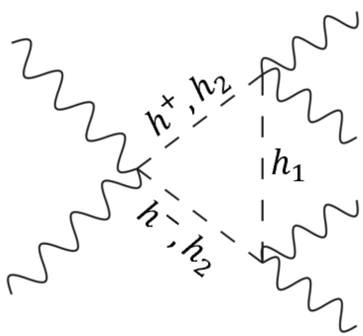
- Extra bosons in BSM to provide four-particles vertices to **evade large virtuality suppression**, which does **not realize** in the electroweak-scale SM.



Drell-Yan like
(gaugino+sfermion loop, MSSM)

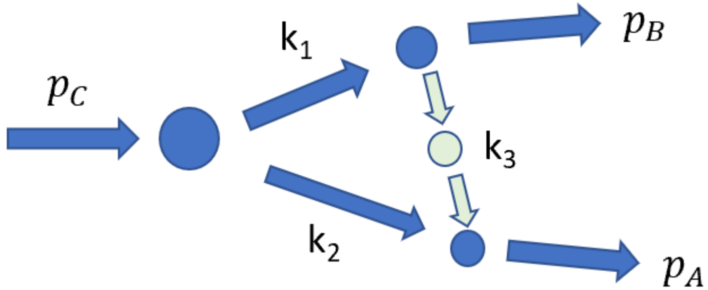


VBF diagram
(slepton loop, MSSM)



VBF – all 4-boson coupling
(2HDM)

Kinematic region of TS



$$m_1^2 \in \left[\frac{m_c^2 m_3 + m_B^2 m_2}{m_2+m_3} - m_2 m_3, (m_c - m_2)^2\right]$$

In term of the invariant mass of 2+3 / 1+2 system

$$p_C^2 \in \left[(m_1+m_2)^2, m_1^2 + m_2^2 + m_2 m_3 + \frac{m_2}{m_3}\left(m_1^2 - p_B^2\right)\right]$$

$$p_A^2 \in \left[(m_2+m_3)^2, m_2^2 + m_3^2 + m_1 m_2 + \frac{m_2}{m_1}\left(m_3^2 - p_B^2\right)\right]$$

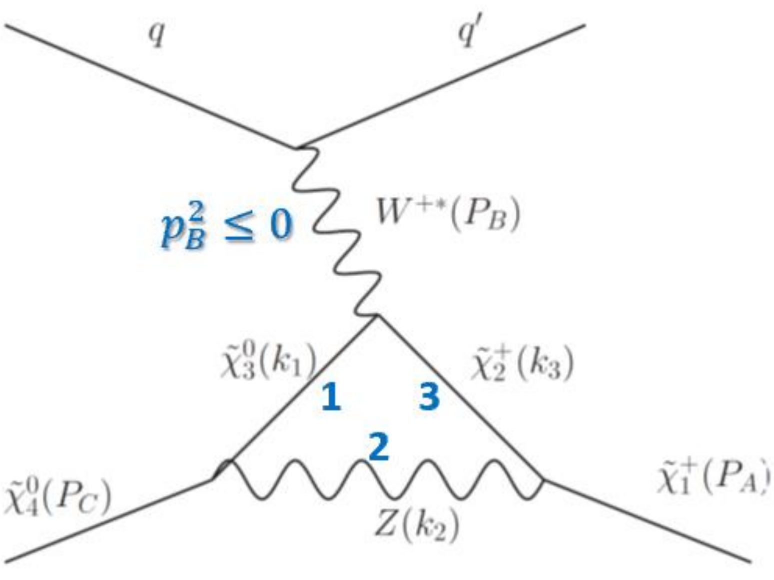
Assume k_3 direction:
$B \rightarrow A$

From above range we know, the external invariant momentum-square p_A^2, p_C^2 must be positive and p_B^2 is free. This leads to **two physical scenarios**:

- All three invariant momentum-squares are positive, which typically corresponds to a decay process or an s-channel collision process into two final-state momentum systems.
- One negative invariant momentum-square, i.e. $p_B^2 < 0$, that can occur in a *t*-channel scattering process with p_B as a virtual momentum exchange.

TS in t-channel

A massive system C exchanges momentum during a collision process and converts to particle system A.

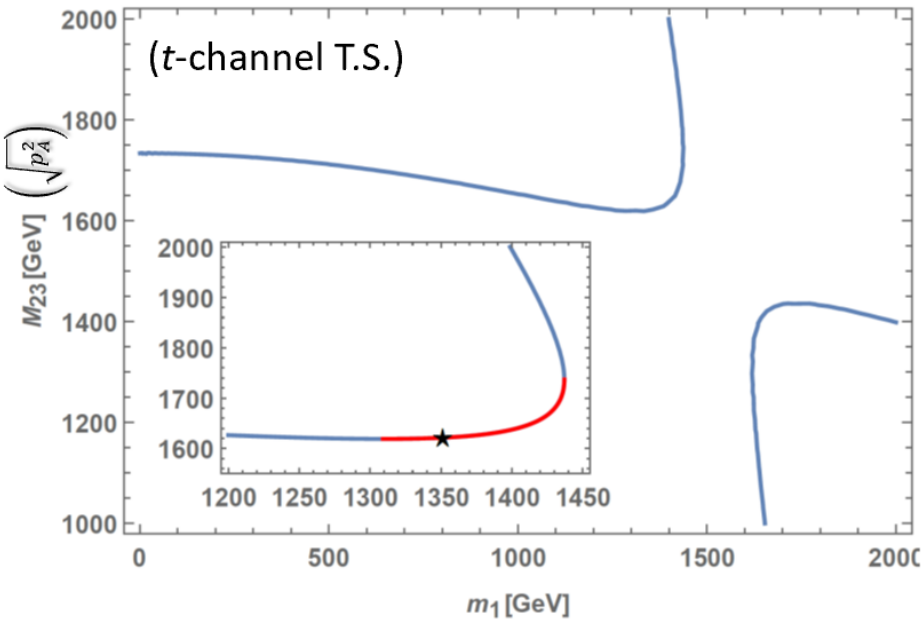


The t-channel process refers to an incited conversion with momentum transfer from the environment.

- We choose $m_3 > m_1$ so that spontaneous decay would not occur and particle 1 must receive external momentum to realize 1+B ->3 process.
- Initial state can not be the lightest stable state of a decay-able particle spectrum (like a LSP dark matter) .
- A negative p_B^2 can be extended to (soft) $\sqrt{|p_B^2|} \ll m_{BSM}$ region.

$$\begin{cases} m_1 \to m_3 \\ p_C^2 \to p_A^2 \end{cases} \quad \text{for } p_B^2 \to 0$$

Dalitz plot in t-channel



Landau equation equiv. as

$$\beta_i + \sum_j^{j\neq i} \beta_j y_{ij} = 0,$$

$$\text{where} \quad y_{ij} \equiv \frac{m_i^2 + m_j^2 - p_k^2}{2 m_i m_j},$$

$$\text{and } \beta_i \equiv \alpha_i m_i$$

Solutions require

$$det \begin{vmatrix} 1 & y_{12} & y_{13} \\ y_{12} & 1 & y_{23} \\ y_{13} & y_{23} & 1 \end{vmatrix} = 0$$

$$1 + 2 y_{12} y_{23} y_{13} - y_{12}^2 - y_{23}^2 - y_{13}^2 = 0$$

containing 6 kinematic parameters.

Conventional choice:

fix m_2, m_3 and p_B^2, p_C^2

m_1 and $m_{23} = \sqrt{p_A^2}$ as variables

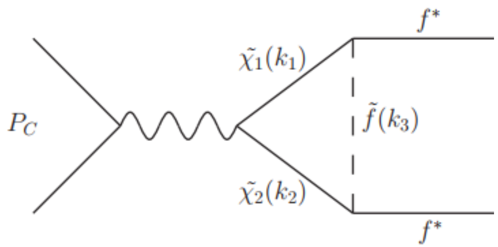
blue: trajectory of det$|y_{ij}|$=0 (*t*-channel)

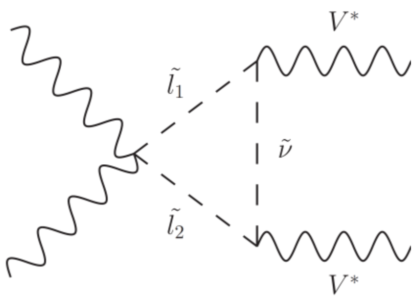
red: physical solutions

asterisk: MSSM benchmark

$\alpha_i > 0$ select a small section (red) of physical solutions.

TS in s-channel

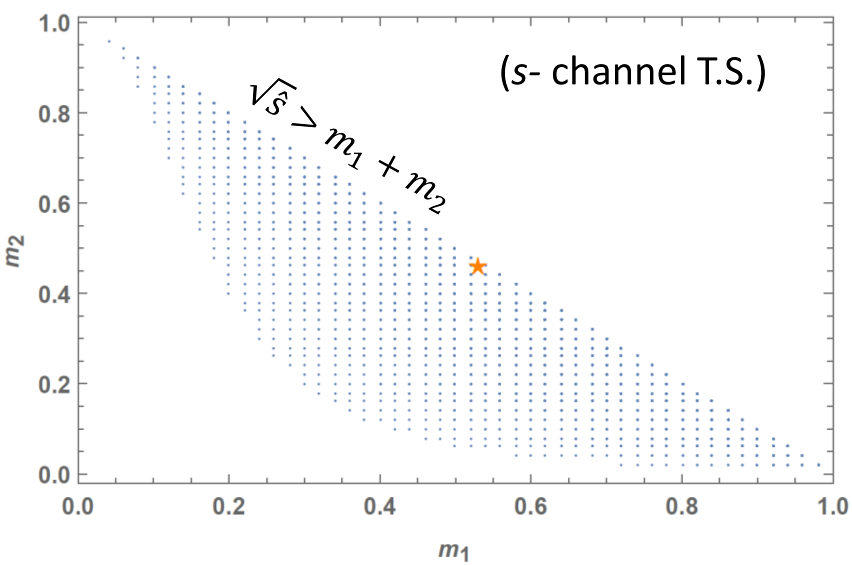




For physical solutions, the external invariant momentum-square p_C^2, p_A^2 satisfy

$$p_C^2 \in \left[(m_1+m_2)^2 , m_1^2 + m_2^2 + m_2 m_3 + \frac{m_2}{m_3}\left(m_1^2 - p_B^2\right) \right]$$

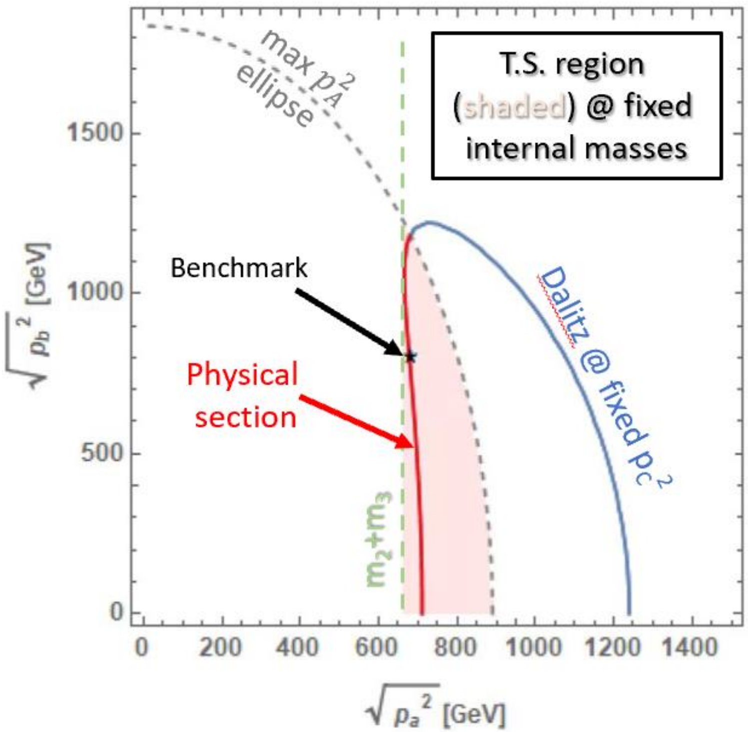
$$p_A^2 \in \left[(m_2+m_3)^2 , m_2^2 + m_3^2 + m_1 m_2 + \frac{m_2}{m_1}\left(m_3^2 - p_B^2\right) \right]$$

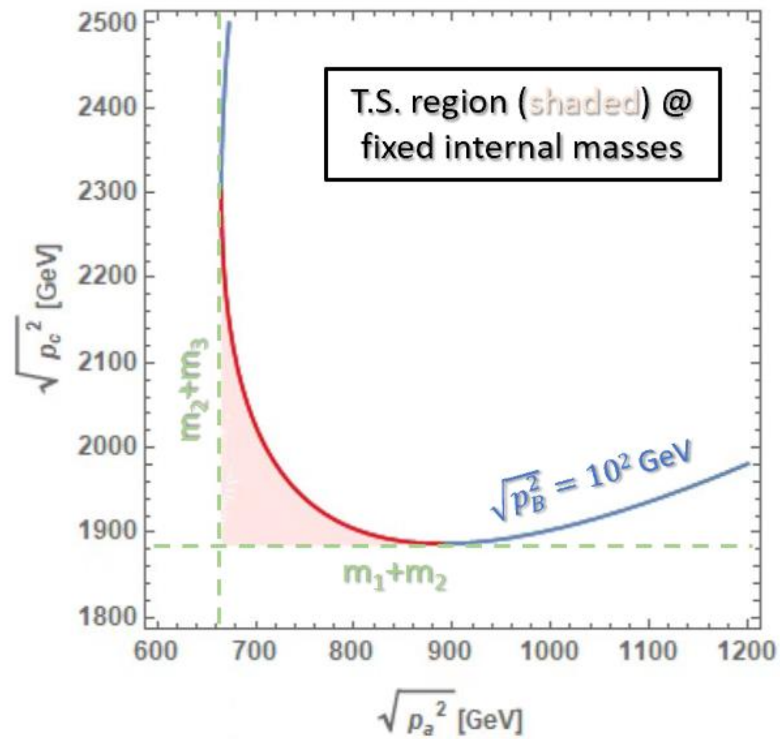


T.S. region on $\{m_1, m_2\}$ plane at given $\sqrt{p_C^2}$, and $p_A^2, p_B^2 > 0$

(1) above pair-production threshold
(2) Satisfy physical boundary ($\alpha_i > 0$)

Dalitz plot in s-channel



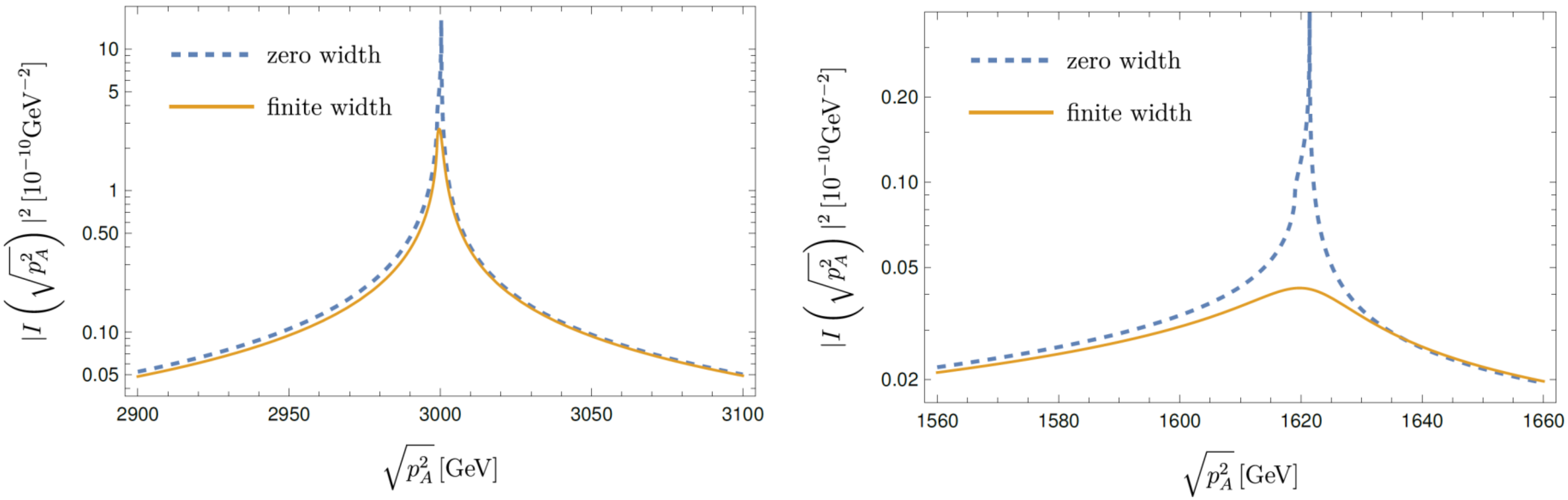


- Fixing all three internal BSM masses and one external momentum, the relation between the two remaining external invariant momenta is a Dalitz curve.

- Don’t fix external momentum and only fix internal mass, the physical Dalitz curve sweep across the parameter plane and covers a ‘total’ shaded region.

Peak in physical region

Singularity encoded in $I\left(\sqrt{p_A^2}\right) = \int \frac{d^4 l}{i\pi^2}\left[\frac{1}{l^2 - m_3^2}\cdot\frac{1}{(l+p_A)^2 - m_2^2}\cdot\frac{1}{(l+p_A+p_C)^2 - m_1^2}\right]$

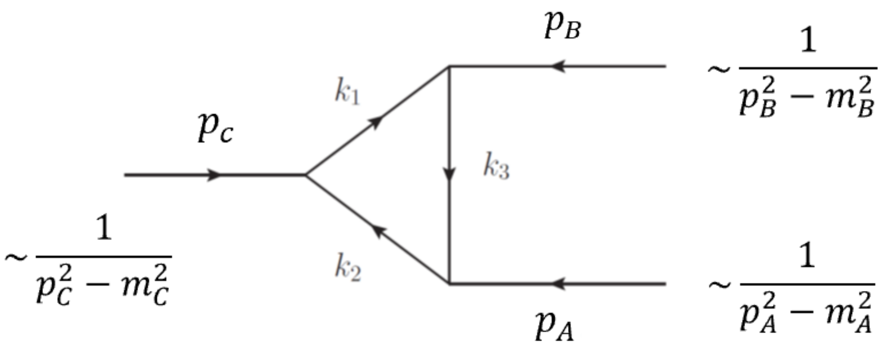


(a) WW fusion

(b) t-channel

Plots: VBF & t-channel at MSSM benchmark scenarios, with p_C^2 fixed.
Finite width of internal particles gives a small Im part.
Singularity -> a finite peak (broaden with the particle widths) .

Question: visibility



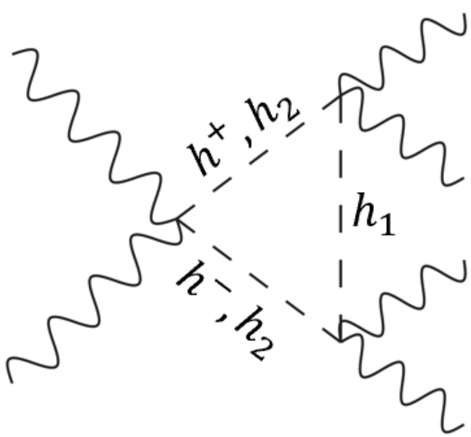
A not small virtuality suppression as

$$\sqrt{p_C^2} > m_1 + m_2$$

$$\sqrt{p_A^2} > m_2 + m_3$$

when BSM particles are heavy.

Less suppressed scenario



IS: 4-particle vertex allows for TS in a *boson-fusion* process, avoids a large 1/s.

FS: 4-particle vertices replaces a highly virtual propagator with two *more identifiable,* on-shell particles.

Summary

- The diverse particle spectrum in BSM theories can , and *often*, provide candidate particles to fill in a triangle loop diagram and satisfy triangular singularity at a high energy collider.
- TS with BSM loops can lead to a fully identifiable SM final state that carry BSM scale energies. It also helps solving the issue with the elusive `compressed scenario' and offer a unique opportunity to search for new physics at colliders.
- A t-channel scattering also triggers TS with virtual momentum exchange, different from traditional s-channel decay processes, and potentially extends to a soft-collision regime.
- BSM four-point vertices can play a role of evading large virtuality suppression which is unlikely to realize in the Standard Model. The complete calculation (xsec and bkg comparison) should be studied deeply, and we leave it to the future work.