Gravitational radiation from classical QCD

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based on arXiv:1611.03493
w/ A. Ridgway (Caltech);
arXiv:1705.09263
w/ S. Prabhu (Yale)
J. Thompson (Yale/Stanford),

Motivations:

- Theoretical: Does the BCJ double copy (or something like it) relate classical solutions in YM and GR? This was first raised by:

  Monteiro, O'Connell, White (2014)

  Luna, Monteiro, O'Connell, White (2015)

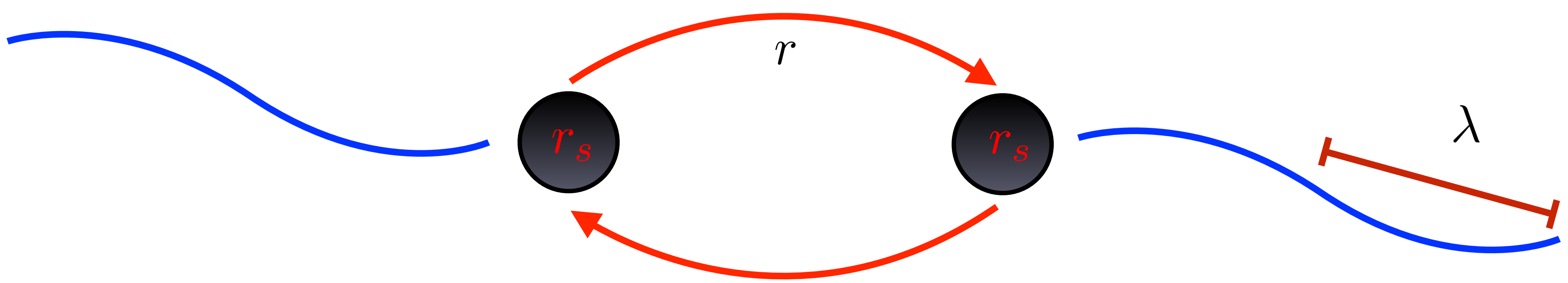
  Luna, Monteiro, Nicholson, O'Connell, White (2016)

  who provided non-perturbative examples.

- Phenomenological: If so, applications to gravitational radiation from binary black holes (LIGO physics)? (speculative).

Binary Black Hole Inspirals

Gravitational dynamics of radiating classical BH (or NS) binary systems in the non-relativistic limit are experimentally relevant (LIGO/VIRGO, LISA,…)



Experiments will be sensitive to at least v^6 corrections beyond Newtonian gravity (Thorne et al 1994). Numerical GR results also motivate computing higher order corrections.

(GW150914, GW151226, GW170104 BH mergers entirely in the strong field regime…)

For this system, the radiation field measured by observers at infinity

$$\lim_{|\vec{x}|\to\infty} h_{ij}^{TT}(t,\vec{x})$$

encodes all the relevant physical info about this system (masses,spins, multipole moments,QNM frequencies,...). In perturbation theory, it is most conveniently computed by recasting Einstein's equations in the form (eg Weinberg 1972)

$$h_{\mu\nu}(x) = \frac{1}{2m_{Pl}} \int_k \frac{e^{-ik\cdot x}}{k^2} \left[\tilde{T}^{\mu\nu}(k) - \frac{1}{2}\eta_{\mu\nu}\tilde{T}^{\sigma}{}_{\sigma}\right]$$

(deDonder gauge)

$$\tilde{T}^{\mu\nu} = T^{\mu\nu}_{pp} + T^{\mu\nu}_{g} =$$ EM pseudotensor $$\partial_\mu \tilde{T}^{\mu\nu} = 0$$

$$T^{\mu\nu}_{g} \sim h\partial^2 h + h^2\partial^2 h + \cdots$$

$$S_{pp} = -m\int d\tau + c\int d\tau R^2_{\mu\nu\alpha\beta} + \cdots$$

$$c_{NS} \sim mR^4 \qquad c_{BH,d=4} = 0$$

(Damour et al; Poisson et al;Kol+Smolkin 2010)

The radiation field at infinity has a simple relation to the pseudo-tensor evaluated on-shell

$$h_\pm(t,\vec{n}) = \frac{4G_N}{r}\int \frac{d\omega}{2\pi} e^{-i\omega t} \epsilon^{*ij}_\pm(k)\tilde{T}_{ij}(k)$$

(Weinberg 1972)

w/

$$k^\mu = \omega(1, \vec{n} = \frac{\vec{x}}{r})$$

$$k^2 = 0$$

In practice computing higher order terms in perturbation theory $(v \ll 1)$ is difficult for two reasons:

Many terms in the expansion of $\tilde{T}^{\mu\nu}(x)$ at high orders in $h_{\mu\nu}$

Many physically relevant scales

Gravitational radius: $r_g = 2G_N M$

Physical radius: $r_s (= r_g \text{ for BH})$

Orbital scale: r

Radiation wavelength λ

$$r_g \sim r_s \gg r \gg \lambda$$

all correlated to the perturbative expansion parameter

$$r \sim r_g/v^2 \qquad \lambda \sim r/v \sim r_g/v^3$$

We (WG+I. Rothstein, 2005) found that these challenges can be ameliorated by employing some 20th century tools from QFT:

Many terms in the expansion of $\tilde{T}^{\mu\nu}(x)$ at high orders in $h_{\mu\nu}$

→ Organize the expansion in terms of Feynman diagrams

Many physically relevant scales

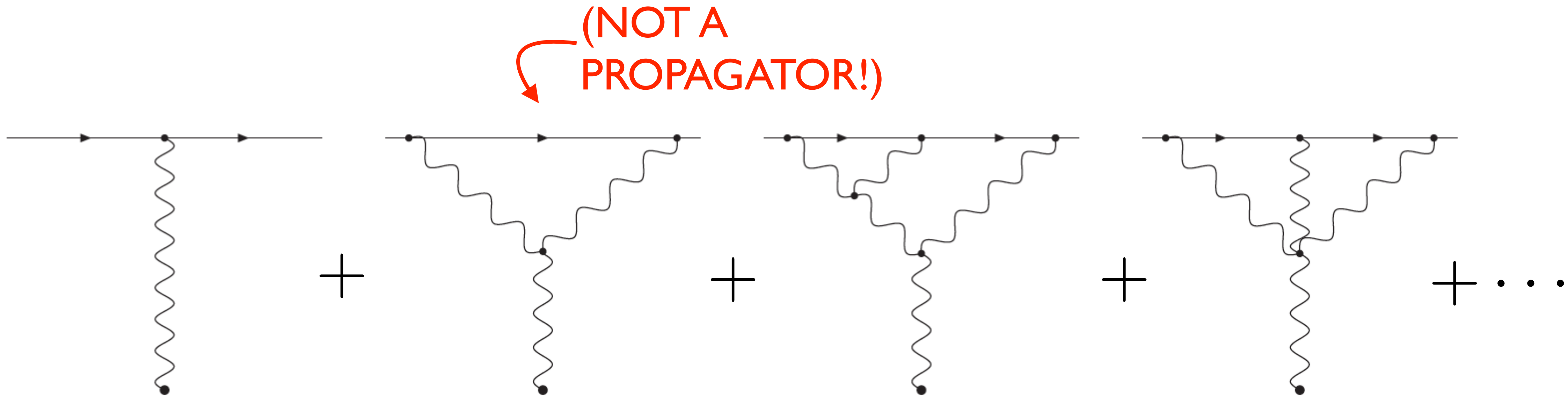
→ Treat each scale separately, by constructing a tower of gravity Effective Field Theories

$$h_{\mu\nu} = h_{\mu\nu}^{potential} + h_{\mu\nu}^{rad}$$

The focus of this talk is the Feynman diagram expansion.

The types of Feynman diagrams that are relevant are of the same type as in Duff's (1973) perturbative construction of the Schwarzschild solution:

$h_{\mu\nu} =$

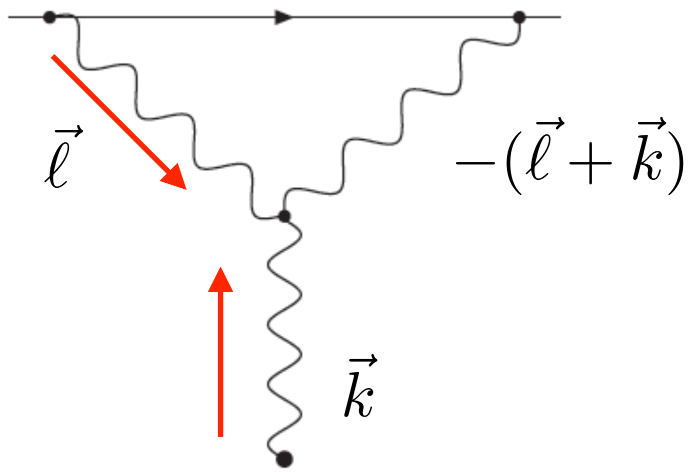


$$g_{00} = 1 - \frac{2G_N m}{r} + 2\left(\frac{G_N m}{r}\right)^2 + 2\left(\frac{G_N m}{r}\right)^3 + \cdots$$

$$g_{ij} = -\delta_{ij}\left[1 + \frac{2G_N m}{r} + 5\left(\frac{G_N m}{r}\right)^2 - \frac{2}{3}\left(\frac{G_N m}{r}\right)^3 + \cdots\right]$$

$$+ \frac{x_i x_j}{r^2}\left[7\left(\frac{G_N m}{r}\right)^2 - \frac{4}{3}\left(\frac{G_N m}{r}\right)^3 + \cdots\right]$$

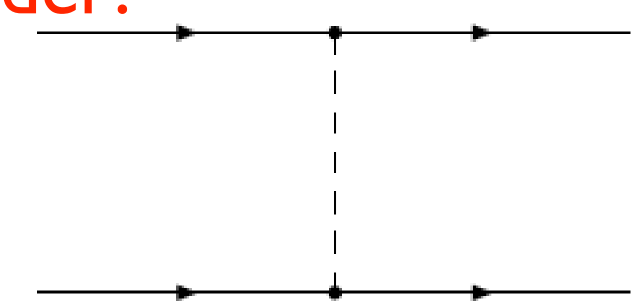
These are tree diagrams coupled to classical (particle sources). Despite being tree, they have the same structure as loop Feynman integrals in QFT. E.g.,



$$\sim I = \int \frac{d^{d-1}\ell}{(2\pi)^{d-1}} \frac{1}{\vec{\ell}^2} \frac{1}{(\vec{k}+\vec{\ell})^2} [\text{Numerator}]$$

The same sort of diagrams can be used to construct two-body solutions, by attaching another particle source. In the NR limit, these can be used to read off a point-particle Lagrangian:

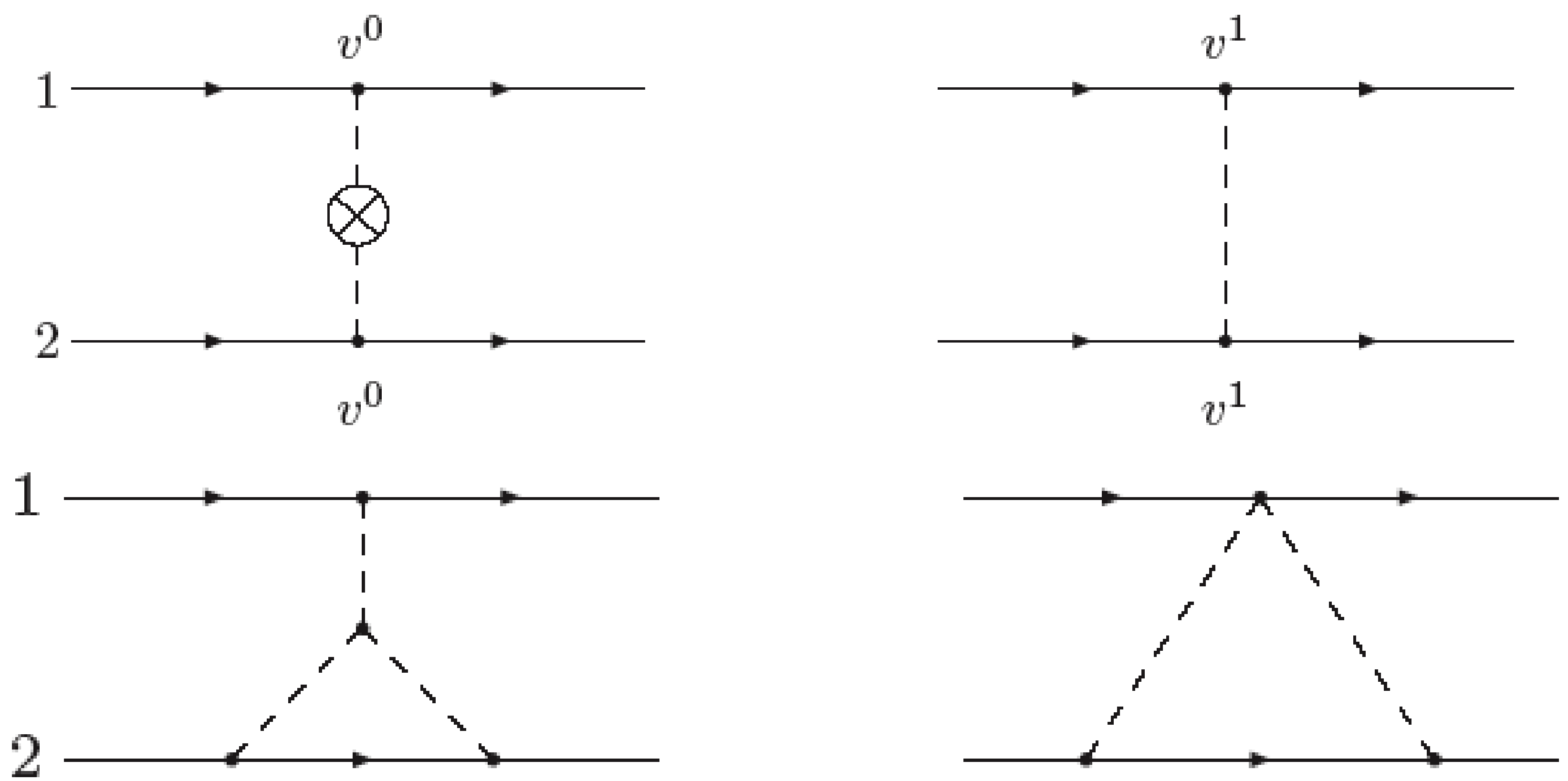
Leading order:
Newton
(1687)



(a)

$$L = \frac{1}{2}\sum_a m_a \vec{v}_a^2 + \frac{G_N m_1 m_2}{r}$$

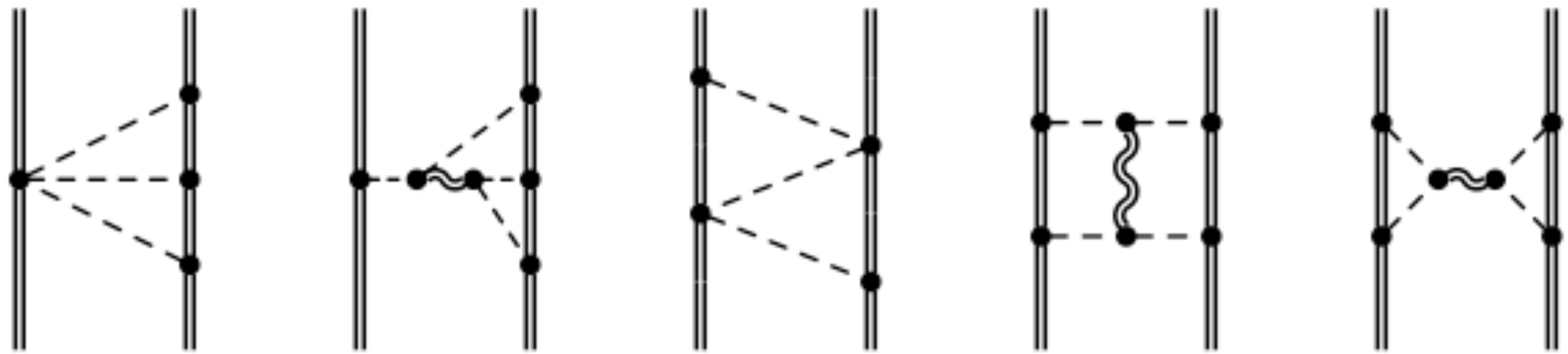
Next-to-leading (1PN): Einstein-Infeld Hoffman Lagrangian (1938)



$$L_{EIH} = \frac{1}{8}\sum_a m_a \vec{v}_a^4 + \frac{G_N m_1 m_2}{2r}\left[3(\vec{v}_1^2 + \vec{v}_2^2) - 7\vec{v}_1\cdot\vec{v}_2 - (\vec{v}_1\cdot n)(\vec{v}_1\cdot n)\right] - \frac{G_N^2 m_1 m_2}{2r^2}$$

2PN (1981-2002): Diagram topologies

(Gilmore+Ross, PRD 2008)



$$L_{2PN} = \frac{m_1\mathbf{v}_1^6}{16}$$
$$+\frac{Gm_1m_2}{r}\left(\frac{7}{8}\mathbf{v}_1^4 - \frac{5}{4}\mathbf{v}_1^2\mathbf{v}_1\cdot\mathbf{v}_2 - \frac{3}{4}\mathbf{v}_1^2\mathbf{n}\cdot\mathbf{v}_1\mathbf{n}\cdot\mathbf{v}_2 + \frac{3}{16}\mathbf{v}_1^2\mathbf{v}_2^2 + \frac{1}{8}(\mathbf{v}_1\cdot\mathbf{v}_2)^2\right.$$
$$\left. - \frac{1}{8}\mathbf{v}_1^2(\mathbf{n}\cdot\mathbf{v}_2)^2 + \frac{3}{4}\mathbf{n}\cdot\mathbf{v}_1\mathbf{n}\cdot\mathbf{v}_2\mathbf{v}_1\cdot\mathbf{v}_2 + \frac{3}{16}(\mathbf{n}\cdot\mathbf{v}_1)^2(\mathbf{n}\cdot\mathbf{v}_2)^2\right)$$
$$+Gm_1m_2\left(\frac{1}{8}\mathbf{a}_1\cdot\mathbf{n}\mathbf{v}_2^2 + \frac{3}{2}\mathbf{a}_1\cdot\mathbf{v}_1\mathbf{n}\cdot\mathbf{v}_2 - \frac{7}{4}\mathbf{a}_1\cdot\mathbf{v}_2\mathbf{n}\cdot\mathbf{v}_2 - \frac{1}{8}\mathbf{a}_1\cdot\mathbf{n}(\mathbf{n}\cdot\mathbf{v}_2)^2\right)$$
$$+Gm_1m_2r\left(\frac{15}{16}\mathbf{a}_1\cdot\mathbf{a}_2 - \frac{1}{16}\mathbf{a}_1\cdot\mathbf{n}\mathbf{a}_2\cdot\mathbf{n}\right)$$
$$+\frac{G^2m_1m_2^2}{r^2}\left(\frac{7}{4}\mathbf{v}_1^2 + 2\mathbf{v}_2^2 - \frac{7}{2}\mathbf{v}_1\cdot\mathbf{v}_2 + \frac{1}{2}(\mathbf{n}\cdot\mathbf{v}_1)^2\right)$$
$$+\frac{G^3m_1m_2^3}{2r^3} + \frac{3G^3m_1^2m_2^2}{2r^3} + (1\leftrightarrow 2),$$

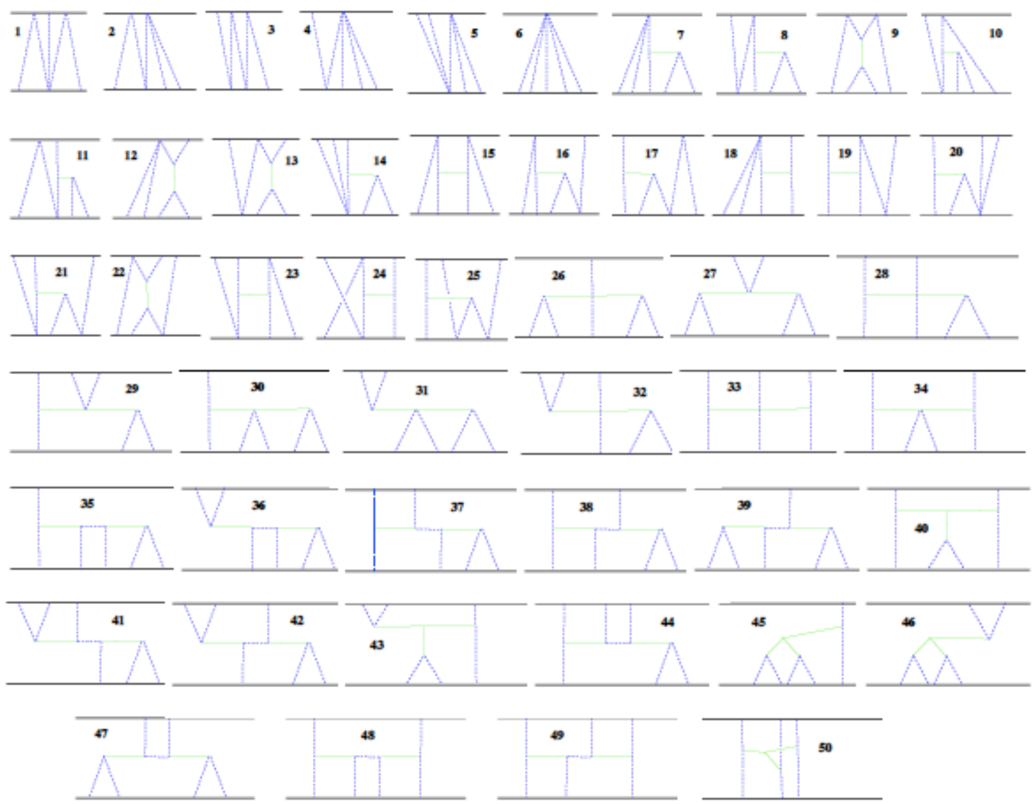
reducible to one-loop integrals via IBP:

$$\int \frac{d^{d-1}\mathbf{k}}{(2\pi)^{d-1}}\frac{1}{[(\mathbf{k}+\mathbf{p})^2]^\alpha[\mathbf{k}^2]^\beta}$$

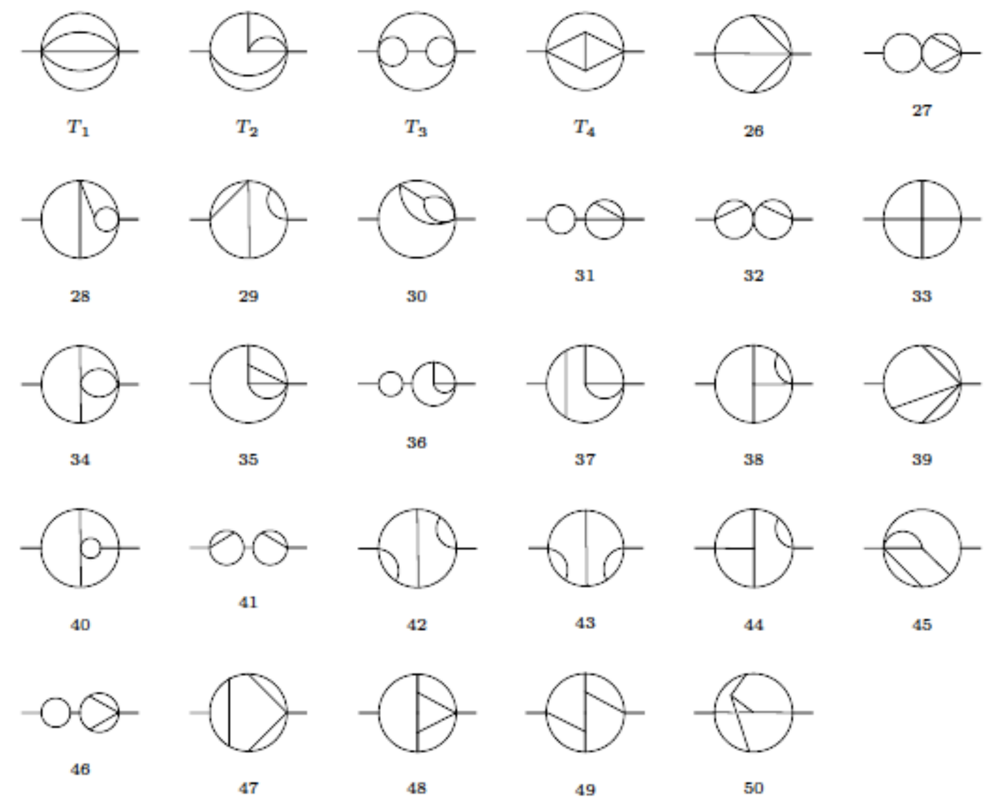
(simplification of PT via field redefs: B. Kol+M. Smolkin, 2007-2008.)

Current state of the art is 4PN order (Foffa, Mastrolia,Sturani,Sturm;2016) (Damour et al; Blanchet et al 2015)

Reduces to 5 master integrals via IBP identities:



2-body graviton exchange diagrams

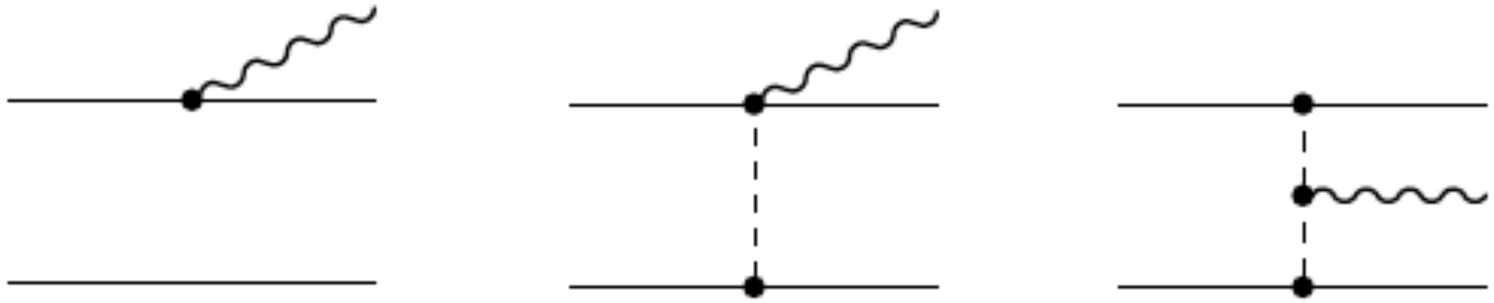


Equivalent 2-pt fns.

Static 2-body Lagrangian:

$$\mathcal{L}_{4PN}^{G_N^5} = \frac{3}{8}\frac{G_N^5 m_1^5 m_2}{r^5} + \frac{G_N^5 m_1^4 m_2^2}{r^5}\left[\frac{1690841}{25200} + \frac{105}{32}\pi^2 - \frac{242}{3}\log\frac{r}{r_1'} - 16\log\frac{r}{r_2'}\right]$$
$$+ \frac{G_N^5 m_1^3 m_2^3}{r^5}\left[\frac{587963}{5600} - \frac{71}{32}\pi^2 - \frac{110}{3}\log\frac{r}{r_1'}\right] + (m_1 \leftrightarrow m_2)\ ,$$

Radiation sector: Feynman diagrams with one external graviton (WG+Ross,2010)

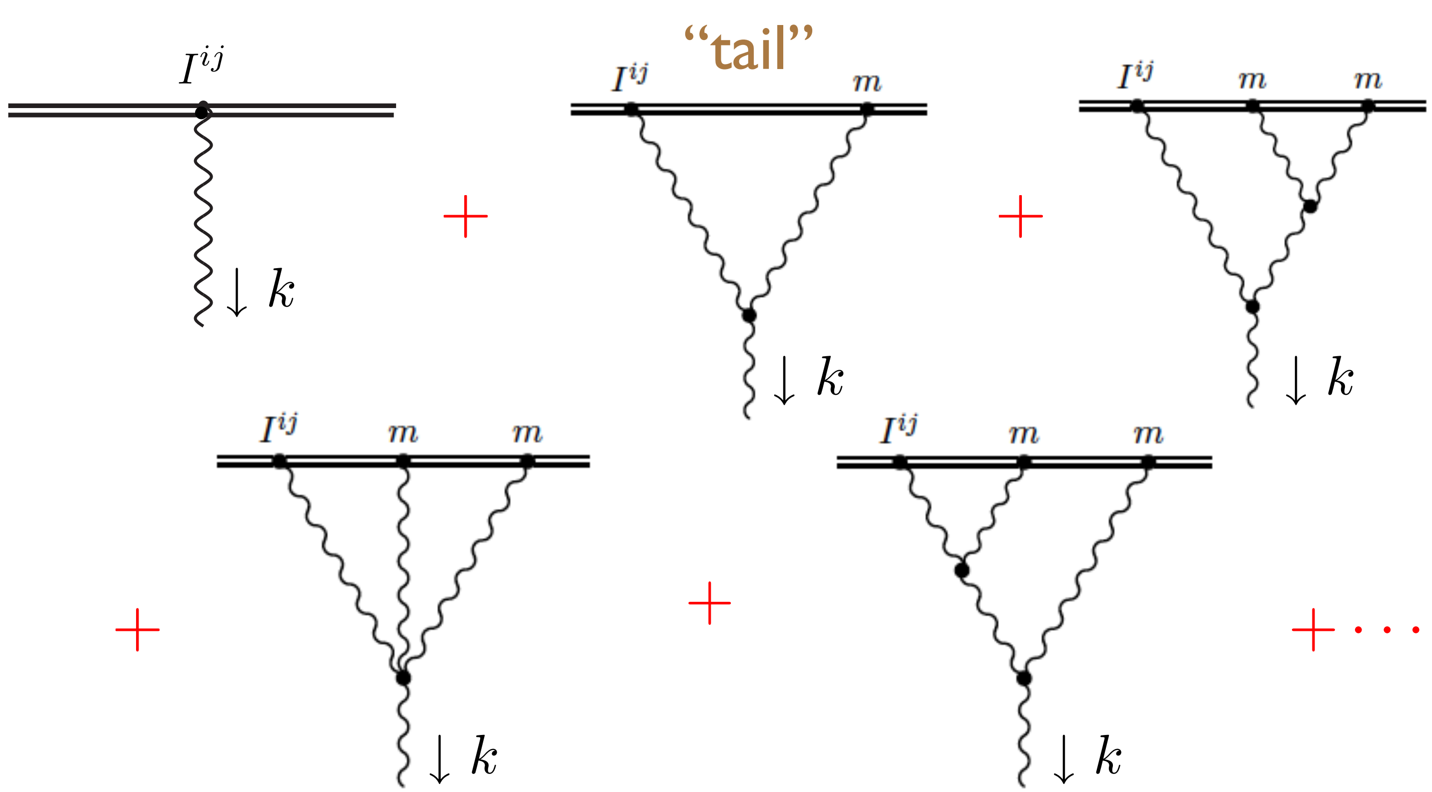


Quadrupole moment

$$I_{ij} = \sum_a m_a \mathbf{x}_a^i \mathbf{x}_a^j \left[1 + \frac{3}{2}\mathbf{v}_a^2 - \sum_b \frac{G_N m_b}{|\mathbf{x}_a - \mathbf{x}_b|}\right] + \frac{11}{42}\sum_a m_a \frac{d^2}{dt^2}(\mathbf{x_a}^2 \mathbf{x}_a^i \mathbf{x}_a^j)$$
$$- \frac{4}{3}\sum_a m_a \frac{d}{dt}(\mathbf{x_a}\cdot \mathbf{v}_a \mathbf{x}_a^i \mathbf{x}_a^j) - \text{traces} + \mathcal{O}(v^4)$$

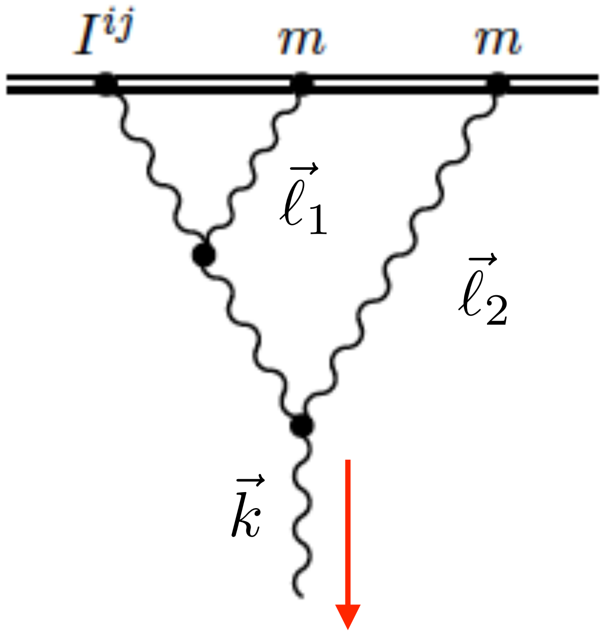
Quadrupole Radiation:

$$h_{ij}^{TT} =$$



$$\frac{\dot{E}_{3PN,LL}}{\dot{E}_{LO}} = -\frac{1712}{105}v^6 \ln v$$

In the radiation sector, also get Feynman integrals corresponding to QFT 2-point functions (one external momentum). E.g., at two-loops, get integrals of the form



$$I = \int_{\vec{\ell}_1,\vec{\ell}_2} \frac{1}{\vec{\ell}_1^2}\frac{1}{\vec{\ell}_2^2}\frac{1}{\vec{k}^2-(\vec{k}+\vec{\ell}_2)^2}\frac{1}{\vec{k}^2-(\vec{k}+\vec{\ell}_1+\vec{\ell}_2)^2}[\mathrm{Numerators}]$$

$$\sim \left[\frac{1}{\epsilon_{UV}}+\frac{1}{\epsilon_{IR}^2}+\mathrm{finite}\right]\left(\vec{k}^2\right)^{d-4}$$

(Reduction to master integrals + Mellin-Barnes)

These contain interesting IR and UV log divergences, encoded as $1/\epsilon$ poles in dim. reg.

- IR : Interaction of outgoing graviton with Newton potential
- UV : Short distance singularities in multipole expansion. → RG evolution

At yet higher orders, encounter five-pt graviton vertex and beyond (see eg 1612.00482) .   Do modern amplitude methods help?

Q: Can one use the color kinematics (BCJ?) to streamline gravity wave calculations?

Classical radiation from a double copy (WG+A. Ridgway, 1611.03493)

Does the BCJ double copy relate other observables, not just the S-matrix? Can it be used to obtain classical perturbative solutions in gravity from (computationally simpler) solutions in YM?

Will now provide evidence at lowest non-trivial order in perturbation theory, by explicit calculation on both sides of the color-kinematics correspondence.

Gauge Theory Solutions

Solve the classical Yang-Mills equations coupled to classical point color charges.

$$D_\nu F_a^{\nu\mu}(x) = g J_a^\mu(x)$$

By classical color charge we mean an object whose degrees of freedom are

$x^\mu(\tau)$ =wordline coordinate

$c^a(\tau)$ =color d.o.f in adjoint at $x(\tau)$

(Sikivie and Weiss, 1978)

Current for a collection of charges

$$J_a^\mu(x) = \sum_\alpha \int d\tau c_\alpha^a(\tau) v_\alpha^\mu(\tau) \delta^d(x - x_\alpha(\tau)) + \cdots$$

(finite size terms and other color moments)

The particle equations of motion follow from conservation laws. Color conservation:

$$D_\mu J^\mu_a = 0 \Rightarrow v \cdot D c^a = 0$$

or in terms of adjoint rep. Wilson line $c^a_\alpha(\tau) = W_\alpha{}^a{}_b(\tau) c^b_\alpha(-\infty)$

$$W_\alpha{}^a{}_b(\tau) = \left[P \exp \left\{ -ig \int_{-\infty}^{\tau} dx^\mu_\alpha A_\mu \cdot T_{\text{adj}} \right\} \right]^a{}_b .$$

The orbital motion is fixed by energy-momentum and gives the non-Abelian Lorentz force law:

$$\partial_\mu T^{\mu\nu} = 0 \Rightarrow \frac{dp^\mu}{d\tau} = m \frac{d^2 x^\mu}{d\tau^2} = g c^a F^\mu_a{}_\nu v^\nu$$

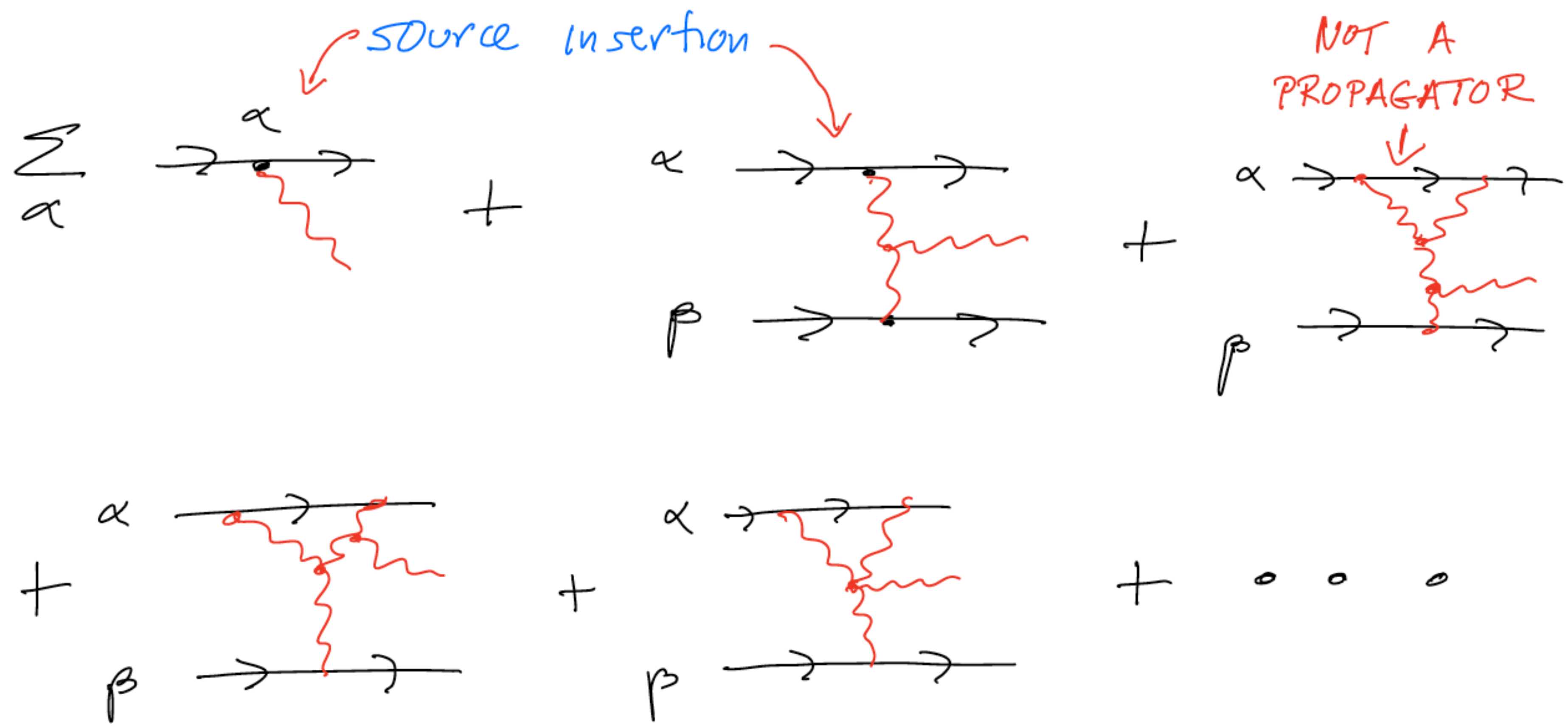
We find it convenient to solve the coupled equations in Lorentz gauge, in the form

$$\Box A_a^\mu = g\tilde{J}_a^\mu \qquad \tilde{J}_a^\mu = J_a^\mu + f^{abc} A_\nu^b (\partial^\nu A_c^\mu - F_c^{\mu\nu})$$

$$\partial_\mu \tilde{J}_a^\mu = 0$$

The classical solution is an off shell one-point function in the presence of (self-consistent) sources:

$\langle A \rangle_J =$



Our focus in this talk is on the radiation field measured by observers at $r \to \infty$ and fixed retarded time t (i.e."asymptotic null future infinity"). This is related to the on-shell momentum space current

$$\tilde{J}^{\mu}_{a}(k) = \int d^d x e^{ik\cdot x} \tilde{J}^{\mu}_{a}(x)$$

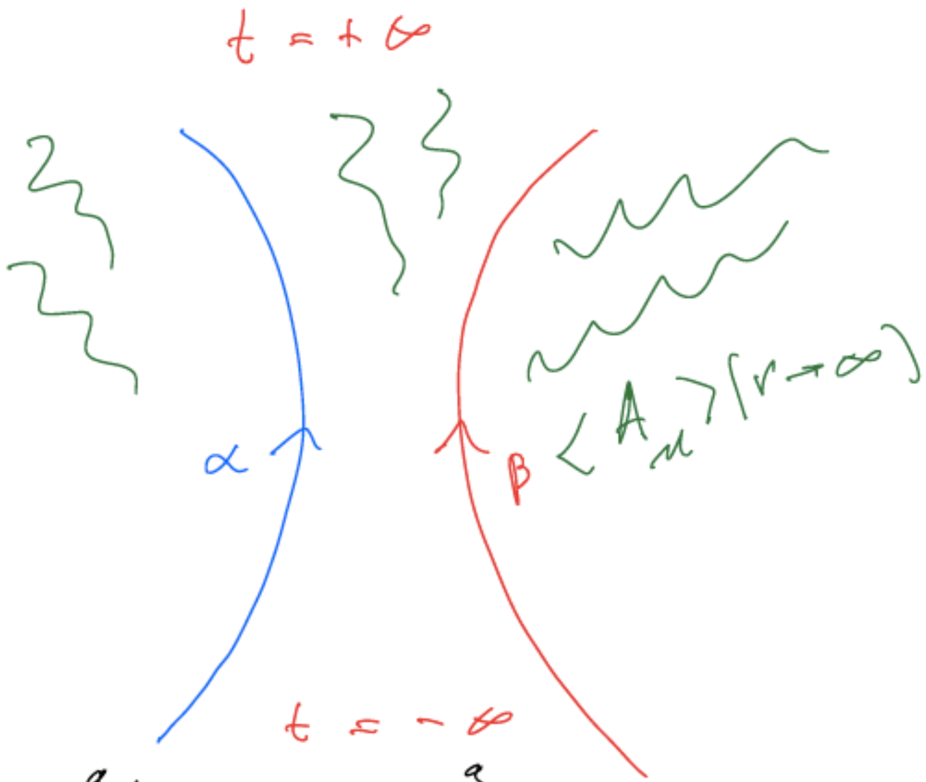
w/ $k^2 \to 0$. E.g., in four dimensions, the asymptotic gauge field is

$$\lim_{r\to\infty} \langle A^{a}_{\mu} \rangle (x) = \frac{g}{4\pi r} \int \frac{d\omega}{2\pi} e^{-i\omega t} \tilde{J}^{\mu}_{a}(k)$$

$$k^{\mu} = (\omega, \vec{k}) = \omega(1, \vec{x}/r)$$

This determines all observables measured by detectors at $r \to \infty$

For concreteness, we consider a bunch of color charges coming in from $r \to \infty$ as $t \to -\infty$
These particles then scatter and emit classical radiation out to $r \to \infty$, $t \to \infty$



$$c^a_\alpha(\tau \to -\infty) \equiv c^a_\alpha$$

$$x^\mu_\alpha(\tau \to -\infty) = b^\mu_\alpha + v^\mu_\alpha \tau$$

$$(v_\alpha^2 = 1)$$

$$\alpha, \beta = 1, \cdots N$$

$$\omega b \sim \mathcal{O}(1)$$

$$E/m \sim \mathcal{O}(1)$$

Formally, the perturbative solution for the radiation field can be constructed as an expansion in powers of the gauge coupling g .

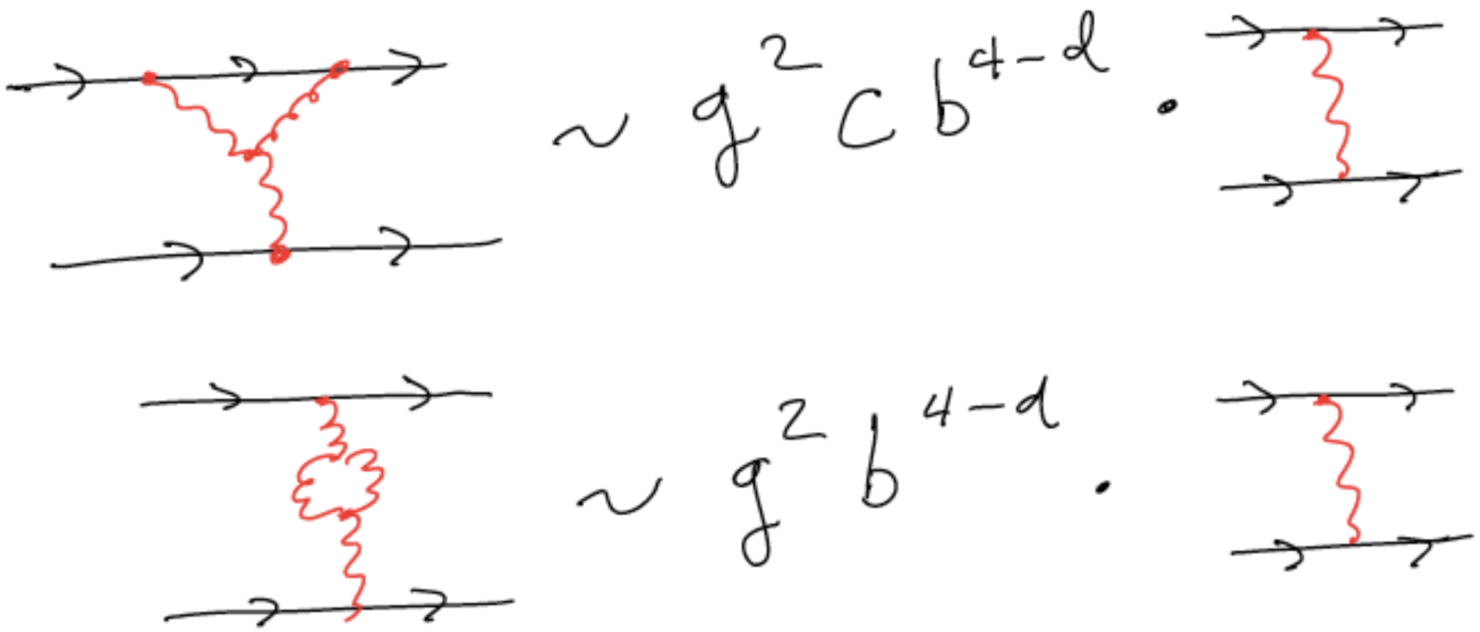
Less formally, for generic E/m and ωb, in the classical limit,

$$L \sim Eb \gg \hbar$$

perturbation theory is controlled by two independent expansion parameters. Non-linear corrections due to YM interactions are small as long as

$$g^2 c b^{4-d} \ll 1$$

E.g., corrections to the two-body potential:



2nd diagram is a quantum effect (one-loop beta fn.) which is suppressed as long as the color charges are sufficiently large $c^a \gg 1$. As we'll see, double copy works in the regime

$$c^a \sim Eb \gg 1$$

The other small expansion is in the corrections to the particle orbital and color deflections. The classical equations of motion

$$\frac{dp^\mu}{d\tau} = gc^a F_a^\mu{}_\nu v^\nu \qquad\qquad \frac{dc^a}{d\tau} = gf^{abc} v^\mu A_\mu^b(x(\tau))c^c(\tau)$$

are perturbative in the kinematic regime

$$\boxed{g^2c^2/Eb^{d-3} \ll 1}$$

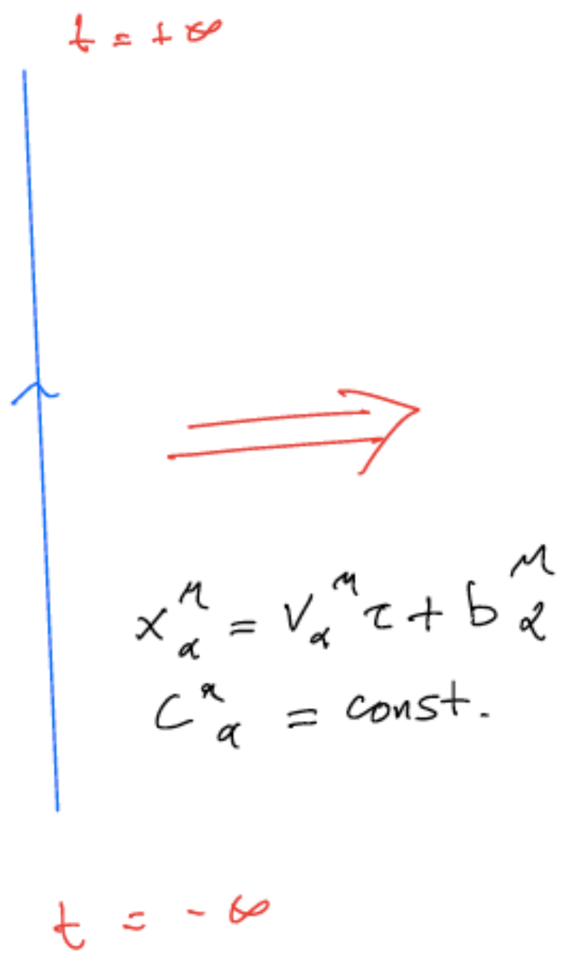
where deflections away from straight line motion are small.

But note, for $c^a \sim Eb$

$$g^2c^2/Eb^{d-3} \sim g^2cb^{4-d}$$

the two expansion coincide, and perturbation theory is equivalent to an expansion in powers of the YM gauge coupling.

Perturbative solution: Leading order



Superposition of static Coulomb fields

$$\langle A^\mu_a \rangle(x) = -g \sum_\alpha \int_\ell (2\pi)\delta(\ell \cdot v_\alpha) \frac{e^{-i\ell\cdot(x-b_\alpha)}}{\ell^2} v^\mu_\alpha c^a_\alpha .$$

gets fed back into equations of motion to determine color and orbital deflections:

$$x^\mu_\alpha(\tau) = b^\mu_\alpha + v^\mu_\alpha \tau + z^\mu_\alpha(\tau) \qquad z^\mu_\alpha(\tau \to -\infty) = 0$$

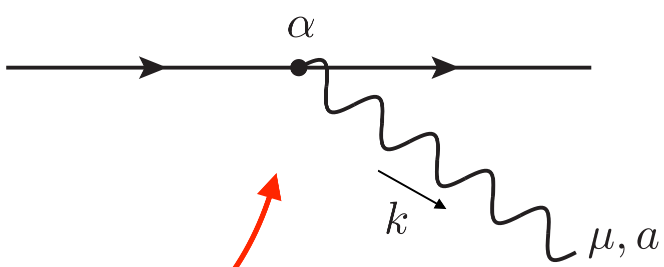
$$c^a_\alpha(\tau) = c^a_\alpha + \bar{c}^a_\alpha(\tau) \qquad \bar{c}^a_\alpha(\tau \to -\infty) = 0$$

$$\frac{d\bar{c}^a_\alpha(\tau)}{d\tau} = -g^2 \sum_{\beta\neq\alpha} (v_\alpha \cdot v_\beta) f^{abc} c^b_\beta c^c_\alpha \int_\ell (2\pi)\delta(\ell\cdot v_\beta) \frac{e^{i\ell\cdot(b_{\alpha\beta}+v_\alpha\tau)}}{\ell^2}$$

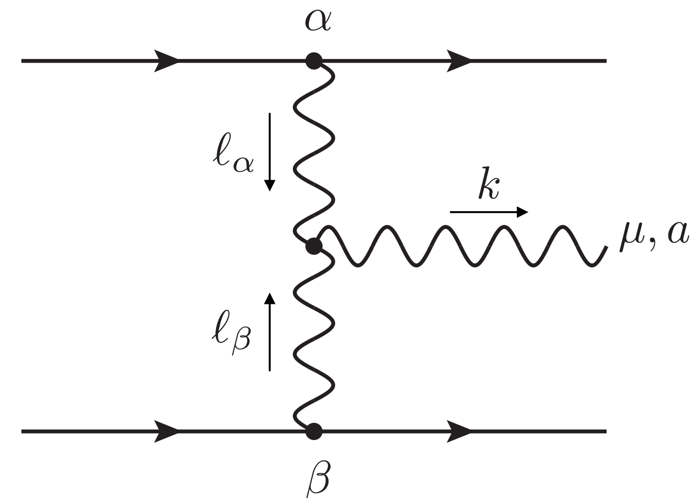
$$m_\alpha \frac{d^2 z^\mu_\alpha}{d\tau^2} = -ig^2 \sum_{\beta\neq\alpha} (c_\alpha \cdot c_\beta) \int_\ell (2\pi)\delta(\ell\cdot v_\beta) \frac{e^{i\ell\cdot(b_{\alpha\beta}+v_\alpha\tau)}}{\ell^2} \left[(v_\alpha \cdot v_\beta)\ell^\mu - (v_\alpha \cdot \ell) v^\mu_\beta \right]$$

Perturbative corrections and radiation:

The time dependent particle deflections source radiation. The off-shell current at order ϵ beyond LO:



$$\sim \epsilon^0 + \epsilon^1 + \cdots$$



$$\sim \epsilon^1 + \cdots$$

Includes NLO correction to particle deflections

Zero deflection

Sum of diagrams:

$$\tilde{J}^\mu_a(k)\Big|_{\mathcal{O}(g^2)} = \quad g^2 \sum_{\substack{\alpha,\beta \\ \alpha\neq\beta}} \int_{\ell_\alpha,\ell_\beta} \mu_{\alpha,\beta}(k) \Bigg[\frac{c_\alpha \cdot c_\beta}{m_\alpha} \frac{\ell_\alpha^2}{k\cdot v_\alpha} c^a_\alpha \left\{ -v_\alpha \cdot v_\beta \left(\ell^\mu_\beta - \frac{k\cdot \ell_\beta}{k \cdot v_\alpha} v^\mu_\alpha \right) + k\cdot v_\alpha v^\mu_\beta - k \cdot v_\beta v^\mu_\alpha \right\}$$
$$+ i f^{abc} c^b_\alpha c^c_\beta \left\{ 2(k\cdot v_\beta) v^\mu_\alpha - (v_\alpha \cdot v_\beta) \ell^\mu_\alpha + (v_\alpha \cdot v_\beta) \frac{\ell^2_\alpha}{k\cdot v_\alpha} v^\mu_\alpha \right\} \Bigg]$$

$$\mu_{\alpha,\beta}(k) = \left[(2\pi)\delta(v_\alpha \cdot \ell_\alpha) \frac{e^{i\ell_\alpha \cdot b_\alpha}}{\ell_\alpha^2} \right] \left[(2\pi)\delta(v_\beta \cdot \ell_\beta) \frac{e^{i\ell_\beta \cdot b_\beta}}{\ell_\beta^2} \right] (2\pi)^d \delta^d(k - \ell_\alpha - \ell_\beta)$$

As a check, the current obeys the Ward identity $k_\mu \tilde{J}^\mu{}_a(k) = 0$, but only after self-consistent particle eqns. of motion are plugged in to the diagrams.

Gravitating Sources and Radiation:

We now consider the analogous classical problem in the graviton/dilaton system defined by:

$$S = S_g + S_{pp}$$

$$S_g = -2m_{Pl}^{d-2} \int d^d x \sqrt{g} \left[R - (d-2) g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi \right]$$

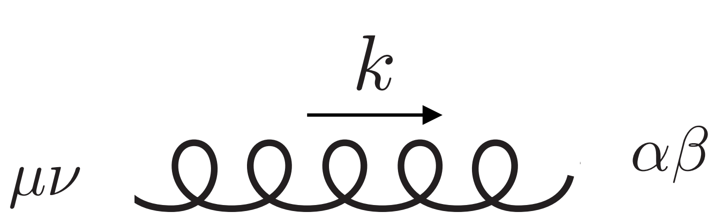
with point particle sources that couple to the dilaton

$$S_{pp} = -m \int d\tau e^{\phi} + \cdots ,$$

$$\text{“}e^{\phi} = 1 + \phi + \frac{1}{2!}\phi^2 + \cdots\text{”}$$

Inclusion of dilaton motivated by BCJ: Pure YM $\to h_{\mu\nu}, \phi, B_{\mu\nu}$

Choice of worldline interactions motivated by Bern+Grant, PLB (1999). Necessary in order to cancel explicit dependence on the dimensionality in the gravity Feynman rules:



$$= \frac{i}{2m_{Pl}^{d-2}} \frac{1}{k^2} \left[\eta_{\mu\rho}\eta_{\nu\sigma} + \eta_{\mu\sigma}\eta_{\nu\rho} - \frac{2}{d-2} \eta_{\mu\nu}\eta_{\rho\sigma} \right]$$

The strategy for finding perturbative solutions is the same as in gauge theory. Write the full non-linear Einstein eqns. as

$$\langle h_{\mu\nu}\rangle(x) = \frac{1}{2m_{Pl}^{d-2}}\int_k \frac{e^{-ik\cdot x}}{k^2}\left[\tilde{T}_{\mu\nu}(k) - \frac{1}{d-2}\eta_{\mu\nu}\tilde{T}^{\sigma}{}_{\sigma}(k)\right]$$

(deDonder gauge)

The source on the LHS is a conserved energy-momentum pseudo-tensor that depends on all the field and particle d.o.fs

$$\tilde{T}^{\mu\nu} = T^{\mu\nu}_{\phi} + T^{\mu\nu}_{pp} + T^{\mu\nu}_{h} \longleftarrow \sim h\partial^2 h + h^2\partial^2 h + \cdots$$

(see Weinberg, 1972)

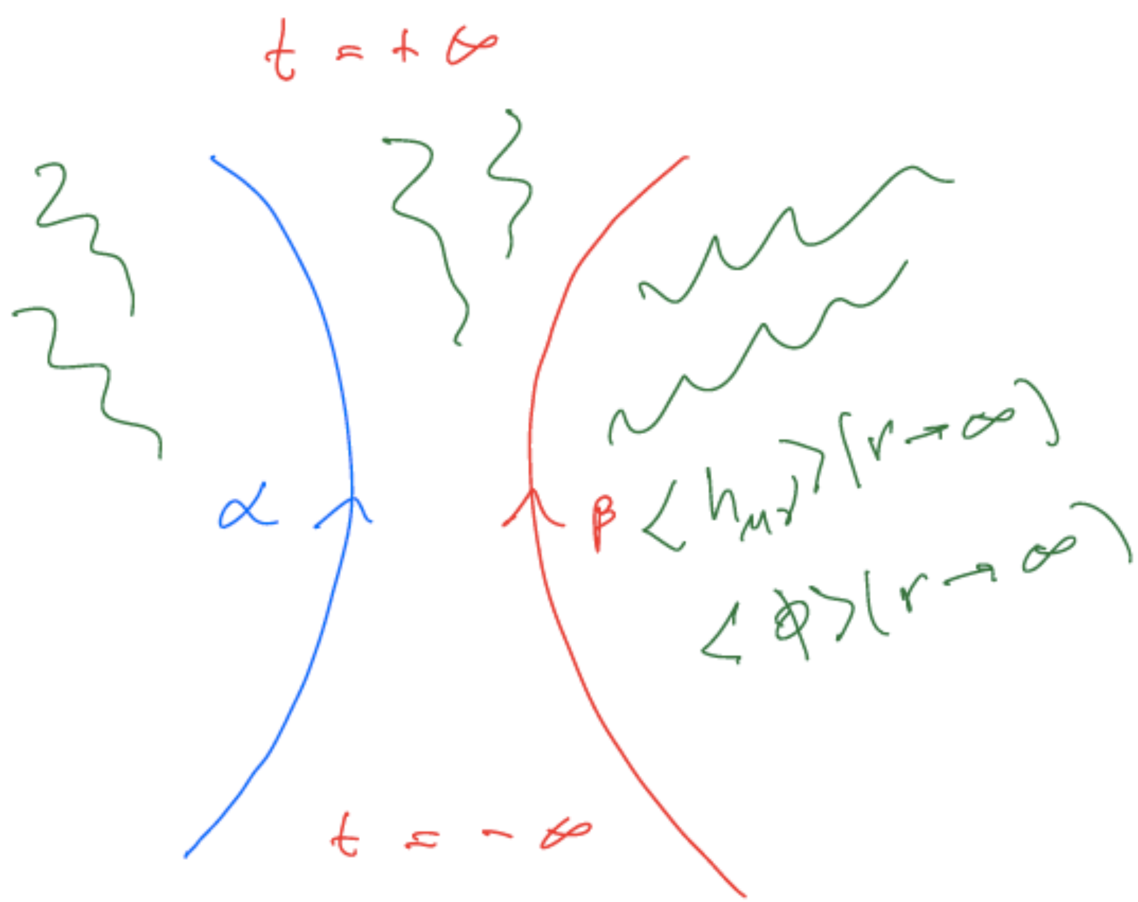
$$\partial_\mu \tilde{T}^{\mu\nu} = 0$$

On-shell, it directly measures the radiation field seen by detectors at $r\rightarrow\infty$. Eg

$$h_{\pm}(t,\vec{n}) = \frac{4G_N}{r}\int \frac{d\omega}{2\pi} e^{-i\omega t}\epsilon^{*ij}_{\pm}(k)\tilde{T}_{ij}(k)$$

in four dimensions.

Now we look at a similar setup of interacting particles as in the YM case:



$$x^\mu_\alpha = b^\mu_\alpha + v^\mu_\alpha \tau + z^\mu_\alpha(\tau)$$

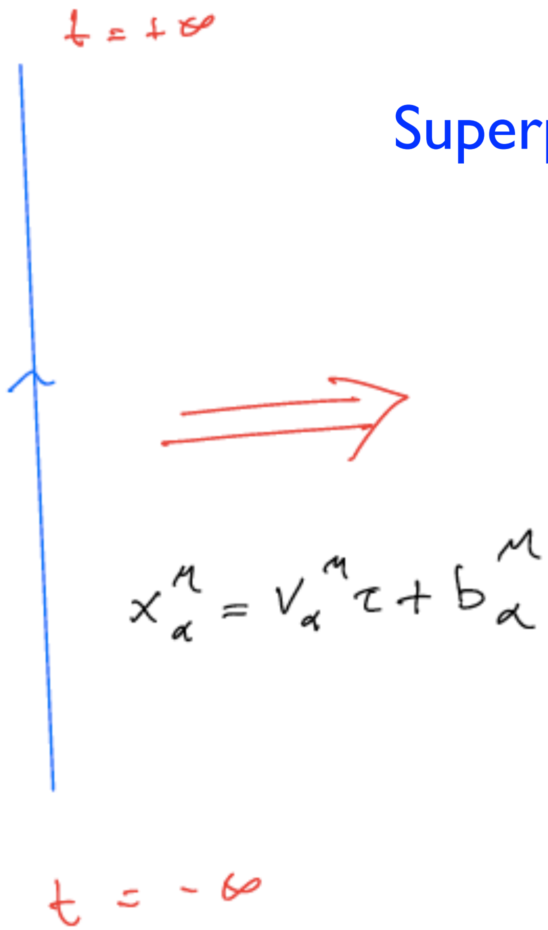
$$z^\mu_\alpha(\tau \to -\infty) = 0$$

The perturbative expansion parameter in the gravitational case is:

$$\epsilon = \frac{\Gamma(d/2 - 3/2)}{(4\pi)^{(d-1)/2}} \frac{E}{m_{Pl}^{d-2} b_{\alpha\beta}^{d-3}} \ll 1$$

which controls both non-linear and orbital corrections

Trivial leading order solution



Superposition of boosted Newtonian potentials

$$\langle h_{\mu\nu}\rangle(x) = \frac{1}{2m_{Pl}^{d-2}} \sum_\alpha m_\alpha \int_\ell \frac{e^{-i\ell\cdot(x-b_\alpha)}}{\ell^2}(2\pi)\delta(\ell\cdot v_\alpha)\left(v_{\alpha\mu}v_{\alpha\nu} - \frac{1}{d-2}\eta_{\mu\nu}\right)$$

$$\langle \phi\rangle(x) = \frac{1}{4m_{Pl}^{d-2}(d-2)} \sum_\alpha m_\alpha \int_\ell \frac{e^{-i\ell\cdot(x-b_\alpha)}}{\ell^2}(2\pi)\delta(\ell\cdot v_\alpha)$$

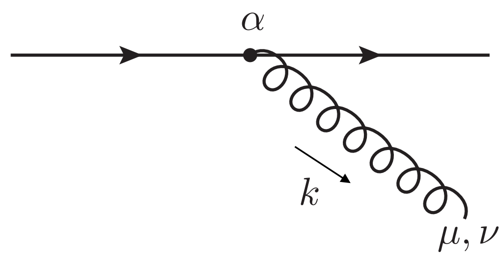
gets fed back into equations of motion to determine orbital deflections:

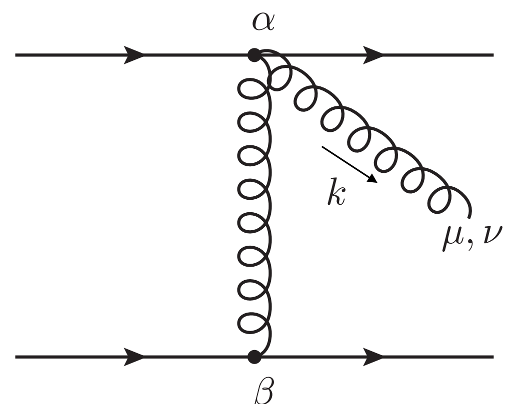
$$\frac{d^2 z^\mu_\alpha}{d\tau^2} = \frac{i}{2m_{Pl}^{d-2}} \sum_{\beta\neq\alpha} m_\beta \int_\ell (2\pi)\delta(\ell\cdot v_\beta)\frac{e^{i\ell\cdot(b_{\alpha\beta}+v_\alpha\tau)}}{\ell^2}\left[\frac{1}{2}(v_\alpha\cdot v_\beta)^2\ell^\mu - (\ell\cdot v_\alpha)\left((v_\alpha\cdot v_\beta)v^\mu_\beta - \frac{v^\mu_\alpha}{2(d-2)}\right)\right]$$

Perturbative corrections and radiation in the graviton channel:

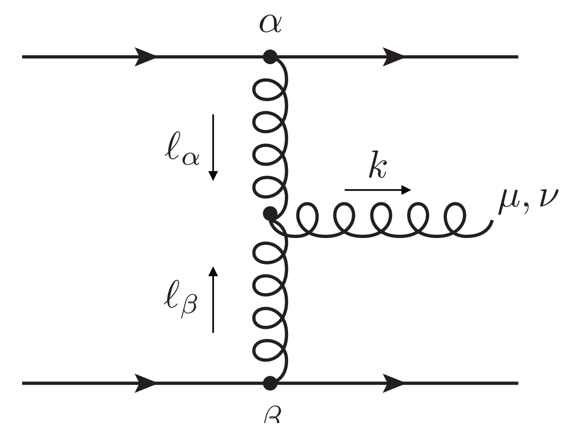
The time dependent particle deflections source radiation. The off-shell EM tensor at order ϵ beyond LO:

Pure gravity contributions:





$$\sim \epsilon^0 + \epsilon^1 + \cdots$$



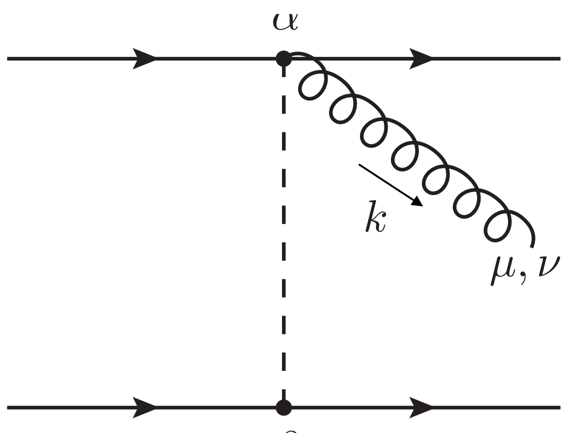
$$\sim \epsilon^1 + \cdots$$

Includes NLO correction to particle deflections

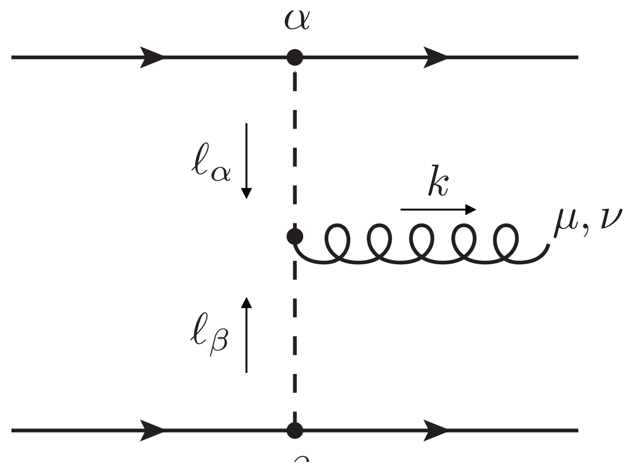
Zero deflection

d=4: Thorne+Kovacs (1970s); D’Eath (1970’s-1980’s)

Scalar exchange terms



$\sim \epsilon^1 + \cdots$



$\sim \epsilon^1 + \cdots$

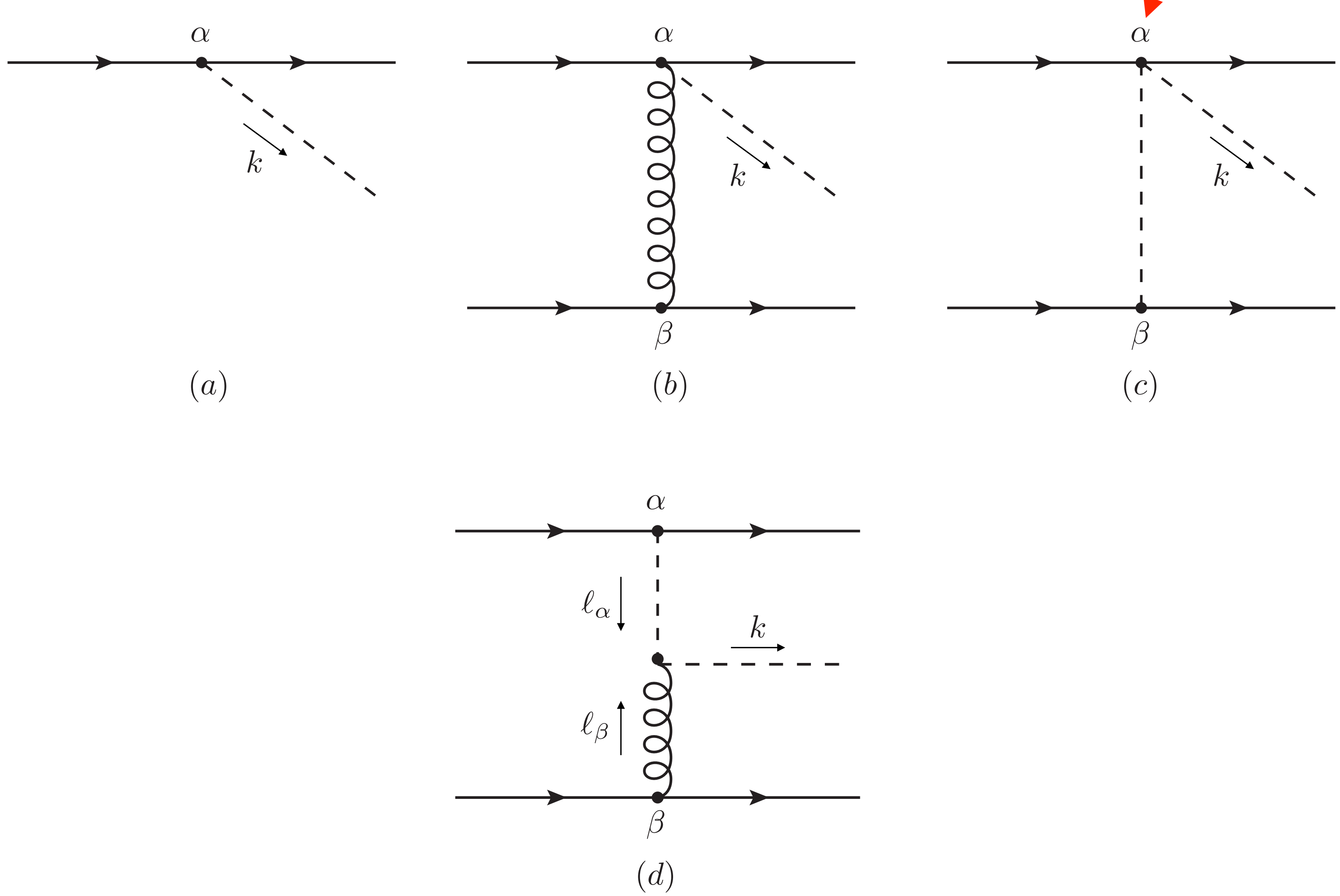
Combining terms and taking $k^2 = 0$

$$
\begin{aligned}
-\frac{1}{2m_{Pl}^{(d-2)/2}}\epsilon^*_{\mu\nu}(k)\tilde{T}^{\mu\nu}(k) = -\frac{\epsilon^*_{\mu\nu}(k)}{8m_{Pl}^{3(d-2)/2}} \sum_{\substack{\alpha,\beta \\ \alpha\neq\beta}} m_\alpha m_\beta \int_{\ell_\alpha,\ell_\beta} \mu_{\alpha,\beta}(k) \Big[(v_\alpha\cdot v_\beta)^2 \ell_\alpha^\mu \ell_\alpha^\nu \\
+(v_\alpha\cdot v_\beta)\eta^{\mu\nu}\left\{\frac{1}{2}(v_\alpha\cdot v_\beta)^2\ell_\alpha^2 + (k\cdot v_\alpha)(k\cdot v_\beta)\right\} \\
-2(v_\alpha\cdot v_\beta)\left((v_\alpha\cdot v_\beta)\frac{\ell_\alpha^2}{k\cdot v_\alpha} + 2k\cdot v_\beta\right)\ell_\alpha^\mu v_\alpha^\nu - 2\left((k\cdot v_\alpha)(k\cdot v_\beta) + (v_\alpha\cdot v_\beta)\ell_\alpha^2\right) v_\alpha^\mu v_\beta^\nu \\
+\left\{(v_\alpha\cdot v_\beta)\frac{\ell_\alpha^2}{(k\cdot v_\alpha)^2}\left((v_\alpha\cdot v_\beta)k\cdot\ell_\alpha + 2(k\cdot v_\alpha)(k\cdot v_\beta)\right) + 2(k\cdot v_\beta)^2\right\} v_\alpha^\mu v_\alpha^\nu\Big],
\end{aligned}
$$

all explicit dependence on spacetime dimension has cancelled.

Radiation in the scalar channel:

$$e^{\phi} = 1 + \phi + \frac{1}{2!}\phi^2 + \cdots$$



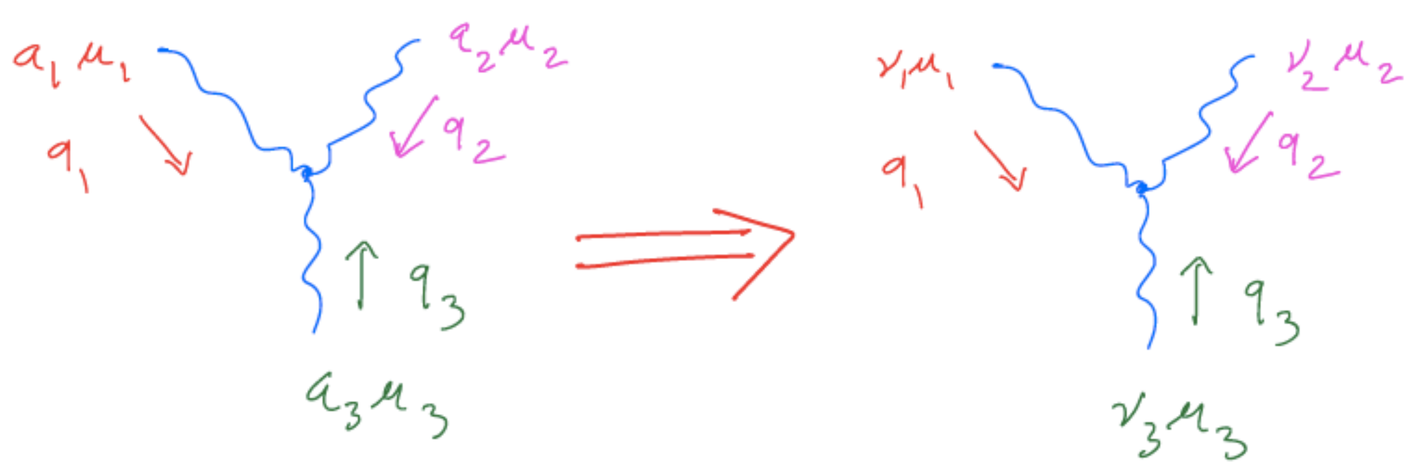
canonically normalized radiation field at future null infinity

$$\mathcal{A}_s(k) \;=\; -\frac{1}{8m_{Pl}^{3(d-2)/2}(d-2)^{1/2}} \sum_{\substack{\alpha,\beta \\ \alpha\neq\beta}} m_\alpha m_\beta \int_{\ell_\alpha,\ell_\beta} \mu_{\alpha,\beta}(k) \left[\frac{(v_\alpha \cdot v_\beta)\ell_\alpha^2}{(k\cdot v_\alpha)^2} \left\{ (v_\alpha\cdot v_\beta) k\cdot\ell_\alpha + 2(k\cdot v_\alpha)(k\cdot v_\beta) \right\} + 2(k\cdot v_\alpha)^2 \right]$$

Double copy relations between solutions

Following BCJ, make the following double copy substitutions on the gauge theory side, at the level of the integrand

$$c_\alpha^a \to p_\alpha^\mu$$



$$if^{a_1 a_2 a_3} \to \Gamma^{\nu_1\nu_2\nu_3}(q_1,q_2,q_3) = -\frac{1}{2}\left[\eta^{\nu_1\nu_3}(q_1-q_3)^{\nu_2} + \eta^{\nu_1\nu_2}(q_2-q_1)^{\nu_3} + \eta^{\nu_2\nu_3}(q_3-q_2)^{\nu_1}\right],$$

$$\epsilon_\mu^a(k) \to \epsilon_\mu(k)\tilde{\epsilon}_\nu(k)$$

$$g \to \frac{1}{2m_{Pl}^{d/2-1}}$$

applying these substitutions to the YM radiation field yields an on-shell amplitude

$$\epsilon^a_\mu(k)\left[g\tilde{J}^a_\mu(k)\right] \to \epsilon_\mu(k)\tilde{\epsilon}_\nu(k)\hat{\mathcal{A}}_{\mu\nu}(k)$$

RHS is defined up to terms that that vanish on-shell. Using this freedom we obtain

$$\begin{aligned}\hat{\mathcal{A}}^{\mu\nu}(k) \;=\; & -\sum_{\substack{\alpha,\beta\\ \alpha\neq\beta}} \frac{m_\alpha m_\beta}{8m_{Pl}^{3(d-2)/2}} \int_{\ell_\alpha,\ell_\beta} \mu_{\alpha,\beta}(k)\left[\frac{(v_\alpha\cdot v_\beta)\ell_\alpha^2}{k\cdot v_\alpha} v_\alpha^\nu \left\{(v_\alpha\cdot v_\beta)\left(\frac{1}{2}(\ell_\beta-\ell_\alpha)^\mu - \frac{k\cdot\ell_\beta}{k\cdot v_\alpha}v_\alpha^\mu\right) + (k\cdot v_\beta)v_\alpha^\mu - (k\cdot v_\alpha)v_\beta^\mu\right\}\right.\\ & \left. +\frac{1}{2}\left\{2(k\cdot v_\beta)v_\alpha^\nu - 2(k\cdot v_\alpha)v_\beta^\nu + (v_\alpha\cdot v_\beta)(\ell_\beta-\ell_\alpha)^\nu\right\}\left\{2(k\cdot v_\beta)v_\alpha^\mu - (v_\alpha\cdot v_\beta)\ell_\alpha^\mu + \frac{(v_\alpha\cdot v_\beta)\ell_\alpha^2}{k\cdot v_\alpha}v_\alpha^\mu\right\}\right].\end{aligned}$$

which has been defined such that

$$k^\mu \hat{\mathcal{A}}_{\mu\nu}(k) = k^\nu \hat{\mathcal{A}}_{\mu\nu}(k) = 0$$

By decomposing the (unitary gauge) polarization into little group irreps

$$\epsilon_i(k)\tilde{\epsilon}_j(k) = \epsilon_{ij}(k) + a_{ij}(k) - \frac{\epsilon(k)\cdot\tilde{\epsilon}(k)}{d-2}h_{ij}(k),$$

$\epsilon_{ij}(k) = \epsilon_{ji}(k) =$ TT graviton mode

$a_{ij}(k) = -a_{ji}(k) =$ Transverse antisymmetric mode

$h_{ij}(k) = \delta_{ij} - \dfrac{k_i k_j}{\vec{k}^2} =$ Scalar mode

we are able to reproduce the results in gravity from the double copy YM radiation field $\hat{\mathcal{A}}^{\mu\nu}$:

$$-\frac{h^{ij}(k)\hat{\mathcal{A}}_{ij}(k)}{\sqrt{h_{mn}h^{mn}(k)}} = \frac{\eta^{\mu\nu}\hat{\mathcal{A}}_{\mu\nu}(k)}{(d-2)^{1/2}} = \mathcal{A}_s(k)$$

$$a_{ij}(k)\hat{\mathcal{A}}^{ij}(k) = 0$$

$$\epsilon_{ij}(k)\hat{\mathcal{A}}^{ij}(k) = -\frac{1}{2m_{Pl}^{(d-2)/2}}\epsilon_{\mu\nu}(k)\tilde{T}^{\mu\nu}(k)$$

$\propto r h_{\pm}(\omega, \vec{n})$
(in 4D)

Bi-adjoint double copy and classical solutions

(WG, Prabhu, Thompson arXiv:1705.09263)

Same methods as above can be used to construct radiating solutions in scalar bi-adjoint theory. This is a theory with global internal symmetries

$$G \times \tilde{G}$$

field content

$$\phi^{a\tilde{a}} =$$ bi-adjoint scalar

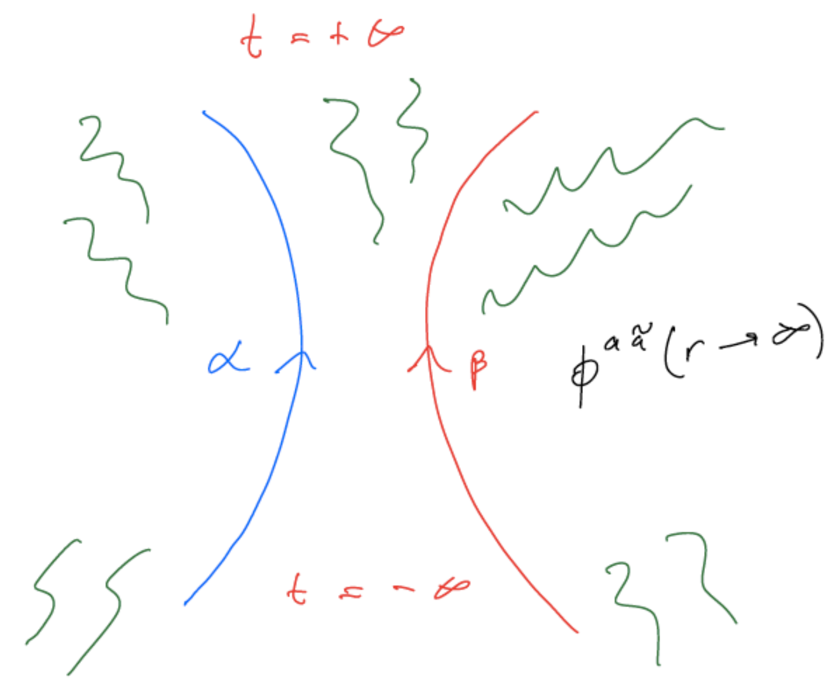
restricted to purely cubic interactions

$$\mathcal{L}_{int} = -y f_{abc} \tilde{f}_{\tilde{a}\tilde{b}\tilde{c}} \phi^{a\tilde{a}} \phi^{b\tilde{b}} \phi^{c\tilde{c}}$$

Color-kinematics relates all tree-level amplitudes in this theory to gluon scattering in pure Yang-Mills (Cachazo, He, Yuan (2013))

We find that the color-kinematics also relates classical solutions of this theory coupled to bi-color charges

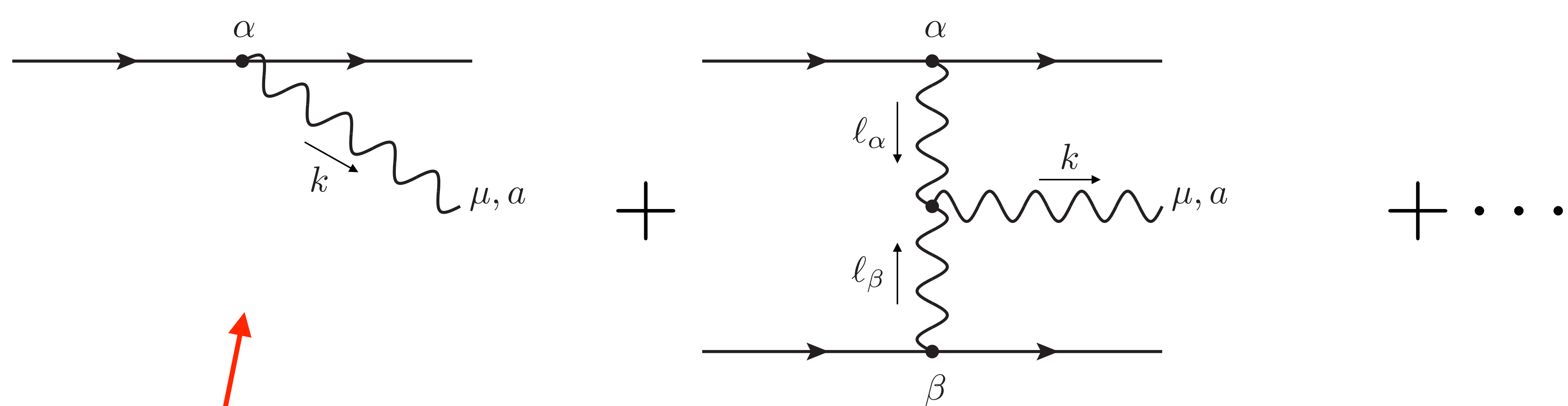
$$S_{pp} = \int d\tau c_a \tilde{c}_{\tilde{a}} \phi^{a\tilde{a}} + \cdots$$



to the corresponding classical gauge theory solutions discussed earlier

$$\phi^{a\tilde{a}}(x) = -y \int_k \frac{e^{-ik\cdot x}}{k^2} \mathcal{J}^{a\tilde{a}}(k)$$

$$\mathcal{J}^{a\tilde{a}}(k) =$$



$$-y^2 \sum_{\alpha\beta} \int_{\ell_\alpha,\ell_\beta} \mu_{\alpha\beta}(k) \frac{\ell_\alpha^2}{k\cdot p_\alpha} \left[\frac{k\cdot \ell_\beta}{k\cdot p_\alpha} ((c_\alpha \cdot c_\beta)c_\alpha)^a ((\tilde{c}_\alpha \cdot \tilde{c}_\beta)\tilde{c}_\alpha)^{\tilde{a}} \right.$$
$$\left. + [c_\alpha, c_\beta]^a ((\tilde{c}_\alpha \cdot \tilde{c}_\beta)\tilde{c}_\alpha)^{\tilde{a}} + ((c_\alpha \cdot c_\beta)c_\alpha)^a [\tilde{c}_\alpha, \tilde{c}_\beta]^{\tilde{a}} \right].$$

$$y^2 \sum_{\alpha,\beta} \int_{\ell_\alpha,\ell_\beta} \mu_{\alpha\beta}(k) [c_\alpha, c_\beta]^a [\tilde{c}_\alpha, \tilde{c}_\beta]^{\tilde{a}}$$

Applying color-kinematics to $\tilde{G}$:

$$\tilde{c}^a_\alpha \to p^\mu_\alpha$$

$$[\tilde{c}_\alpha, \tilde{c}_\beta]^a \mapsto \Gamma^{\mu\nu\rho}(-k, \ell_\alpha, \ell_\beta) p_{\nu\alpha} p_{\rho\beta}$$

and $y \to g$, we reproduce the classical radiation gauge field

$$\mathcal{J}^{a\tilde{a}}(k) \to \epsilon^a_\mu(k) \tilde{J}^\mu_a(k)$$

A further color-kinematics transformation then yields the gravity solutions

$$\mathcal{J}^{a\tilde{a}}(k) \to \epsilon^a_\mu(k) \tilde{J}^\mu_a(k) \to \epsilon_\mu(k) \tilde{\epsilon}_\nu(k) \tilde{T}^{\mu\nu}(k)$$

directly from the much simpler Feynman rules in a scalar field theory.

Conclusions:

Feynman integral + EFT methods are a powerful tool in gravitational wave calculations. Can they be simplified using the double copy?

Color-kinematics relations between classical scalar, YM and gravity radiation solutions:

$$\mathcal{A}^{a\tilde{a}} \to \mathcal{A}^{a\mu} \to \mathcal{A}^{\mu\nu}$$

Under color-kinematics:

Bi-adjoint charge → YM color charge → $S_{pp} = -m \int d\tau e^{\phi} + \cdots ,$

(at least to quadratic order in the scalar)

Open questions:

Classical $B_{\mu\nu}$ radiation? Spin couplings

$$\int d\tau S^{\mu\nu} c^a F^a_{\mu\nu} \longrightarrow \int d\tau S^{\mu\nu} v^\sigma H_{\mu\nu\sigma}$$

(Ridgway+Prabhu+WG, in progress)

Higher orders in PT? Relation to BCJ?

See D. O'Connell's talk at KITP (6/2017)

Assuming it persists at higher orders in PT, is the classical double copy useful for gravity wave calculations?

Dilaton decoupling at large boost: "burst pipeline" BH+BH mergers at LIGO in pure gravity (also see Johansson, Ochirov (2015); Bern,Davies,Nohle, (2015); Luna, Monteiro, Nicholson, Ochirov, O'Connell,Westerberg,White (2016))

Possible simplification of Feynman rules (no need for vertices beyond 3 pt)

But only at the level of the integrand. Doing the integrals is still hard, even in the relevant kinematic limits…