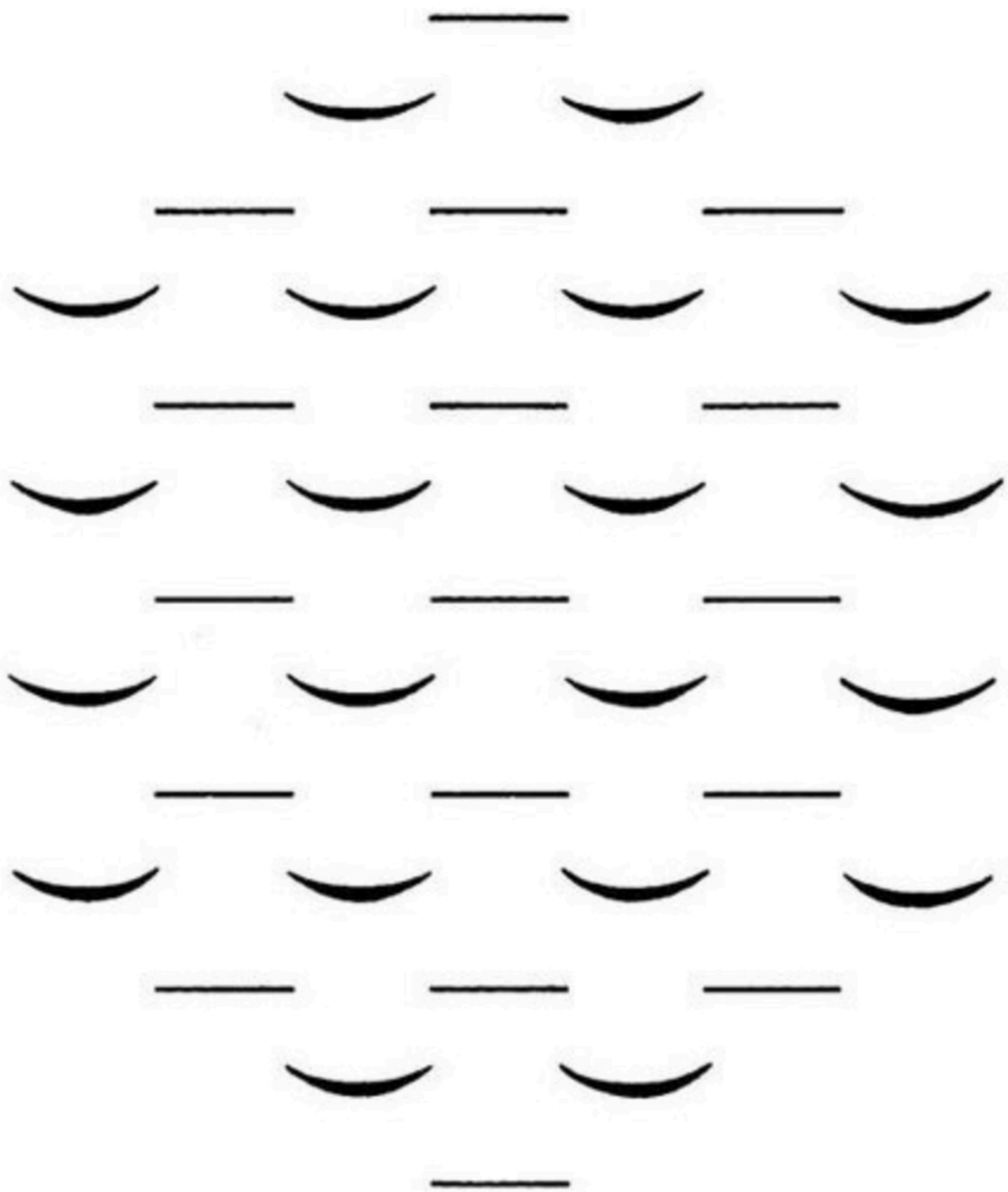


Gravity and supergravity perturbation theory: a bird's eye view

Radu Roiban
Pennsylvania State University

Based in part on work with
Zvi Bern, John Joseph Carrasco, Wei-Ming Chen,
Henrik Johansson, Mao Zeng


© Jason Hawkes / Barcroft Media

A plan

- An outline of some of recent successes and limitations of color/kinematics duality and the double-copy

A plan

- An outline of some of recent successes and limitations of color/kinematics duality and the double-copy
- Diff. invariance, double-copy and relaxation of color/kinematics duality
- Higher-loop scattering amplitudes, contact terms and novel methods for their determination
- An application and a word on integration

A duality between color and kinematics

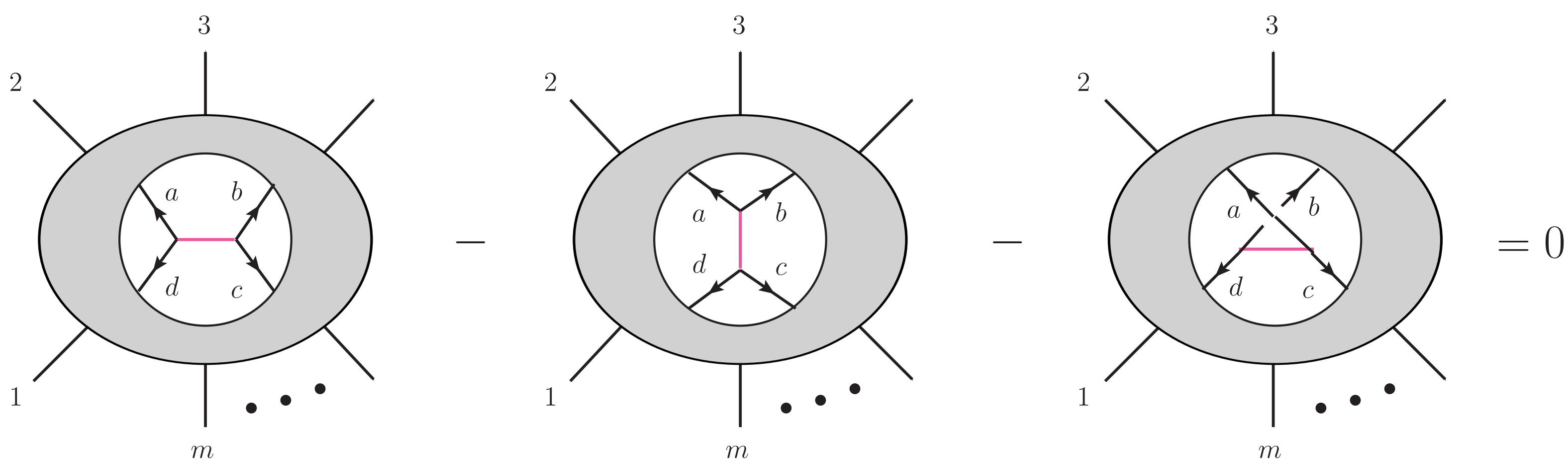
Bern, Carrasco, Johansson

non-adjoint rep: Johansson, Ochirov; Chiodaroli, Jin, RR; Chiodaroli, Gunaydin, Johansson, RR

- For qfts with certain additional matter

$$\mathcal{A}_m^{L-\text{loop}} = i^L g^{m-2+2L} \sum_{i\in\mathcal{G}_3} \int \prod_{l=1}^{L} \frac{d^D p_l}{(2\pi)^D} \frac{1}{S_i} \frac{n_i C_i}{\prod_{\alpha_i} p_{\alpha_i}^2} \qquad n_i = n_i(p_\alpha \cdot p_\beta, \epsilon \cdot p_\alpha, \dots)$$

$$C_i + C_j + C_k = 0 \longleftrightarrow n_i + n_j + n_k = 0$$



- Present in many theories: YM+matter, QCD, Coulomb branch, ϕ^3, NLSM, *Z*-theory, BLG, ABJM, $(DF)^2$, ... as well as certain form fcts and corr. fcts.

Color/kinematics

Bern, Carrasco, Johansson

$$\mathcal{A}_m^{L-\text{loop}} = i^L g^{m-2+2L} \sum_{i\in\mathcal{G}_3} \int \prod_{l=1}^{L} \frac{d^D p_l}{(2\pi)^D} \frac{1}{S_i} \frac{n_i C_i}{\prod_{\alpha_i} p_{\alpha_i}^2}$$

$$n_i = n_i(p_\alpha \cdot p_\beta, \epsilon \cdot p_\alpha, \dots)$$

$$C_i = \dots f^{a_1 bc} f^{ca_2 d} \dots$$

n_i are not gauge-invariants

$$C_i + C_j + C_k = 0 \quad \leftrightarrow \quad n_i + n_j + n_k = 0$$

- 5-point 2-loop all-plus amplitude — Mogull, O'Connell
  remarkably-complicated expression; remarkably bad powercounting
- 2-loop 4-point amplitudes in $\mathcal{N}$=2 SQCD — Johansson, Kaelin, Mogull
- Explicit color/kinematics-satisfying numerators for NLSM — Du, Fu; Chen, Du
- Color/kinematics-satisfying Feynman rules from a NLSM action — Cheung, Shen
- Suggestion for a(nother) symmetry behind BCJ amplitudes relations — Brown, Naculich
  momentum-dependent shift of color factors
- Can be defined for form factors of certain operators; — Boels, Kniehl, Tarasov, Yang
  first 5-loop computation – the form factor of the 20' operator in $\mathcal{N}$=4 sYM — Yang

  Can be defined for correlation functions of certain operators — cf. Engelund, RR
- $(DF)^2$ theory — Johansson, Nohle
- Generalization of BCJ amp. rel's at higher loops — Vanhove, Tourkine; also He, Schlotterer; Stieberger, Hohenegger; Chiodaroli, Gunaydin, Johansson, RR; earlier Boels, Isermann

and the double copy

Bern, Carrasco, Johansson

$$\mathcal{M}_m^{L-\text{loop}} = i^{L+1}\left(\frac{\kappa}{2}\right)^{m-2+2L}\sum_{i\in\mathcal{G}_3}\int\prod_{l=1}^{L}\frac{d^Dp_l}{(2\pi)^D}\frac{1}{S_i}\frac{n_i\tilde{n}_i}{\prod_{\alpha_i}p_{\alpha_i}^2}$$

- Property of many pure & YM/Maxwell-Einstein SGs w/ further matter, open string theory, self-dual gravity, $R+R^3$, EYM+SSB, Conformal SG, …
- 5-loop double copy of $\mathcal{N}$=4 sYM Sudakov form factor — Gang Yang
  - physical interpretation is under debate; not necessarily a form factor of local op.
- 1- & 2-loop 4-point amp's in $\mathcal{N}$=2 SG + matter — Johansson, Kaelin, Mogull, Ochirov
- double-copy for Conformal SG — Johansson, Nohle
- New perspective on "enhanced cancellations" — Bern, Enciso, Para-Martinez, Zeng
- Progress in the identification of SG symmetries i.t.o. YM operations — Anastasiou, Borsten, Duff, Hughes, Marrani, Nagy, Zoccali
- YM classical solutions $\longrightarrow$ (S)G classical solutions — O'Connell et al; Goldberger, Ridgeway
- First example of 3-point scattering amplitude in curved space from double-copy — Adamo, Casali, Mason, Nekovar
- EYM amp's from gauge inv. w/o manifest c/k — Chiodaroli, Gunaydin, Johansson, RR; Feng, Teng
- KLT at 1 loop — He, Schlotterer
- New techniques for SG amplitudes when c/k is expected but not manifest — Bern, Carrasco, Chen, Johansson, RR

A word on classical gravity solutions from YM classical solutions

- Kerr-Schild-type solutions

Monteiro, O'Connell, White

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + \kappa h_{\mu\nu} \qquad h_{\mu\nu} = -\frac{\kappa}{2}\phi k_\mu k_\nu \qquad \bar{g}_{\mu\nu}k^\mu k^\nu = 0 \qquad (k\cdot D)k = 0$$

$$A^\mu = g\frac{1}{4\pi r}k^\mu \quad \longrightarrow \quad h^{\mu\nu} = -\frac{M\kappa}{2}\frac{1}{4\pi r}k^\mu k^\nu$$

Schwarzschild $\longleftrightarrow$ (Coulomb field of point charge)^2

- Other solutions:

Kerr black hole, some higher dimensional black holes, supersymmetric black holes, Taub-NUT spaces, spaces w/ cosmological constant, radiation from accelerating b.h.

Luna, Monteiro, Nicholson, O'Connell, White; Goldberger, Ridgway;
Cardoso, Nagy, Nampuri; Ridgway, Wise

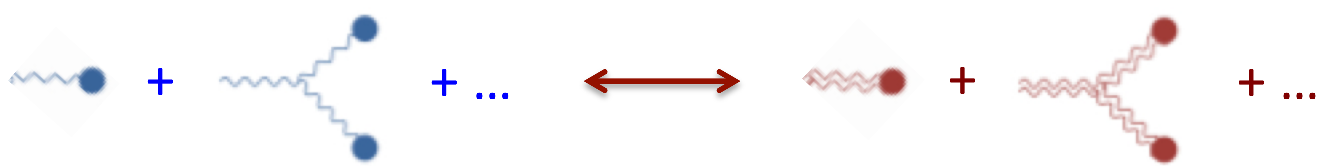
- Perturbative gravitational radiation for colliding masses/b.h. from gluon radiation

Goldberger, Ridgway

possible applications to LIGO (in the early stages of a merger)

- Algorithm for perturbative construction of gravity sol/s i.t.o. gauge theory sol's

Luna, Monteiro, Nicholson, O'Connell, Ochirov, Westerberg, White



A word on enhanced cancellations

Extensive work on understanding the UV behavior of (super)gravity

- Supersymmetry constraints — Green, Bjornsson, Bossard, Howe, Stelle, Nicolai, Elvang, Kiermaier, Ramond, Kallosh, Vanhove, Bern, Davies, Dennen, etc
- Duality constraints — Beisert, Elvang, Freedman, Kiermaier, Morales, Stieberger; Kallosh, etc

Consensus: poor UV behavior unless new cancellations between diag's exist that are "not consequences of supersymmetry in any conventional sense" — Green, Bjornsson

Such "enhanced cancellations" are known to exist

$\mathcal{N}$=4 SG does not diverge at 3 loops in D=4 — Bern et al.

$\mathcal{N}$=5 SG does not diverge at 4 loops in D=4

So... Does $\mathcal{N}$=8 SG diverge at 7 loops in D=4 ?

Does $\mathcal{N}$=8 SG diverge at 5 loops in D=24/5 ?

Suggestion: 0. Manifesting the enhanced cancellations requires integration

In critical dimension they follow from — Bern, Enciso, Para-Martinez, Zeng

1. Lorentz invariance
2. $SL(L)$ reparam. symmetry of L-loop integrals
3. Checks at 2, 3 and 4 loops

Important to check this suggestion and adjust it if necessary

Color/kinematics and the double-copy

- Many open questions:

  - Develop 4-, 5- and higher-loop integration technology
    generalize 2 & 3-loop integration and reduction strategy of Henn, Mistlberger, Smirnov
    Zhang et al; Johansson, Kosower, Larsen
  - All double copies of gauge th's are (super)gravities, but
    are all (super)gravities double copies?
  - Is there a criterion for when a qft can/cannot be a double-copy?
  - Color/kinematics vs. fundamental principle?
  - Complete explicit solution for the tree-level S matrix in <theory of your choice>
  - Complete identification of (U-duality) symm's and of their physical consequences
  - Understand the kinematic algebra and its off-shell realization
  - Is there a direct link between c/k and the UV properties of double copy?
  - Can all classical solutions of (super)gravity be expressed as double-copies?
  - Find explicit (tree-level *n*-point) S-matrix of a perturbative QFT in curved space
  - …

Color/kinematics and the double-copy

Bern, Carrasco, Johansson

- Progress on some of them hinges on several technical issues
  - + can be difficult to find manifest c/k-satisfying rep's at higher loops
    - large ansatze $\longrightarrow$ large linear systems -- $\mathcal{O}(10^6)$ unknowns
  - + the result can have unexpectedly high powers of loop mom. Mogull, O'Connell
    - larger ansatze than one might expect
  - + going straight for (super)gravity amplitudes is even worse
  - + classical solution construction slightly different from scattering amp's; c/k duality needs some reanalysis at higher points
- What is needed:
  - + keep the idea of the double copy
  - + avoid large ansatze $\longleftrightarrow$ construct amplitudes one piece at a time
    - may address possible difficulties with construction of classical solutions

      (projection onto pure gravity states is a separate issue)
  - + some kind of structure should be present

Diff inv. from gauge inv. and what to expect w/o manifest c/k duality:

If c/k is manifest, all double-copy theories are diffeomorphism-invariant:

BCJ; JO;BDHK; Chiodaroli, Gunaydin, Johansson, RR

1. Linearized YM gauge transformations: $\epsilon^\mu(p) \mapsto p^\mu$

$$\mathcal{A} = \sum_\Gamma \frac{n_\Gamma(\epsilon_1(p_1), \epsilon_2, \dots) c_\Gamma}{D_\Gamma} \quad \longrightarrow \quad 0 = \sum_\Gamma \frac{n_\Gamma(p_1, \epsilon_2, \dots) c_\Gamma}{D_\Gamma}$$

1. structure of n_Γ 2. Jacobi identities for c_Γ

2. Linearized diffeomorphisms: $\epsilon^{\mu\nu}(p) \mapsto p^{(\mu} q^{\nu)}$

$$\epsilon^{\mu\nu}(p) \mapsto \epsilon^{(\mu}(p)\epsilon'^{\nu)}(p) \mapsto p^{(\mu}\epsilon'^{\nu)}(p) + p^{(\nu}\epsilon^{\mu)}(p)$$

- follow from YM linearized gauge symmtry

$$\mathcal{M} = \sum_\Gamma \frac{n_\Gamma(\epsilon_1(p_1), \epsilon_2, \dots) \tilde{n}_\Gamma(\epsilon'_1(p_1), \epsilon'_2, \dots)}{D_\Gamma}$$

$$\delta\mathcal{M} = \sum_\Gamma \frac{n_\Gamma(p_1, \epsilon_2, \dots) \tilde{n}_\Gamma(\epsilon'_1(p_1), \epsilon'_2, \dots)}{D_\Gamma} + (n \leftrightarrow \tilde{n})$$

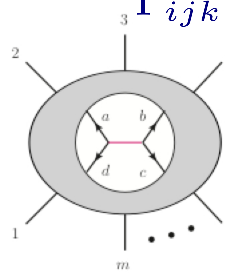
$n_\Gamma, \tilde{n}_\Gamma \& c_\Gamma$ have the same properties $\Longrightarrow$ $\delta\mathcal{M} = 0$ for the same reasons as in YM theory

Just how bad is a double copy without manifest c/k duality?

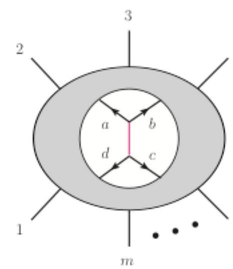
Closest analog: gauge theory in which we formally relax the color Jacobi relations

$$\delta\mathcal{A} \sim \sum_{\Gamma_{ijk}} \frac{f_{\Gamma_{ijk}}(\hat{\epsilon}_1, \epsilon_2, \ldots, p_1, \ldots)(c_{\Gamma_i} + c_{\Gamma_j} + c_{\Gamma_k})}{D_{\Gamma_{ijk}}}$$

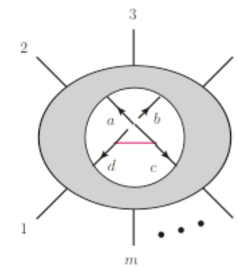
Γ_i :



Γ_j :



Γ_k :



On to gravity:

$$\delta\mathcal{M} \sim \sum_{\Gamma_{ijk}} \frac{f_{\Gamma_{ijk}}(\hat{\epsilon}_1, \epsilon_2, \ldots, p_1, \ldots)(\tilde{n}_{\Gamma_i} + \tilde{n}_{\Gamma_j} + \tilde{n}_{\Gamma_k})}{D_{\Gamma_{ijk}}} + (n \leftrightarrow \tilde{n})$$

$$\sim \sum_{\Gamma} \sum_{\lambda \in \Gamma} \frac{f_{\{\Gamma,\lambda\}}(\hat{\epsilon}_1, \epsilon_2, \ldots, p_1, \ldots)\, \tilde{J}_{\{\Gamma,\lambda\}}}{D_\Gamma} + (n \leftrightarrow \tilde{n})$$

Conclusions:
0. It should be possible to correct a naïve double-copy
1. Breaking of diff. inv. of naïve double-copy is itself a double copy
2. Correction terms restoring diff. inv. should also be double-copies
3. Relevant factors are $J_{\{\Gamma,\lambda\}}$ and $\tilde{J}_{\{\Gamma,\lambda\}}$ -- violations of the kinematic Jacobi relations in the two gauge theory factors

Most straightforward test of these ideas is at tree level

↓

Should be equally straightforward to use them to find generalized cuts

KLT: too many terms, too many spurious poles, not organized in terms of graphs

↓

More efficient methods always come in handy

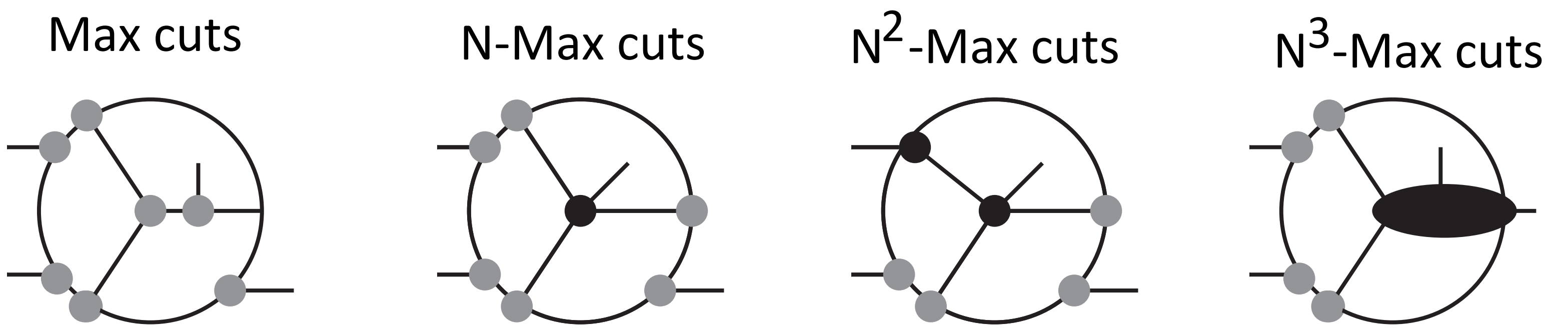
Ansatz-based generalized unitarity/method of maximal cuts:

1. Organize integrand in terms of graphs of φ^3 theory; each graph gets an ansatz for numerator with some desired properties

$$\mathcal{A}^{YM} = \sum_{\Gamma} \int \frac{n_\Gamma c_\Gamma}{D_\Gamma} \qquad\qquad \mathcal{M}^{(S)G} = \sum_{\Gamma} \int \frac{N_\Gamma}{D_\Gamma}$$

2. Fix numerators by fitting them onto cuts

E.g.



► Leads to (obscure) large linear systems at sufficiently high loop order

To avoid this…

Generalized unitarity/the contact term method:

Bern, Carrasco, Dixon, Johansson, RR
Bern, Carrasco, Chen, Johansson, RR

-focus on (super)gravity

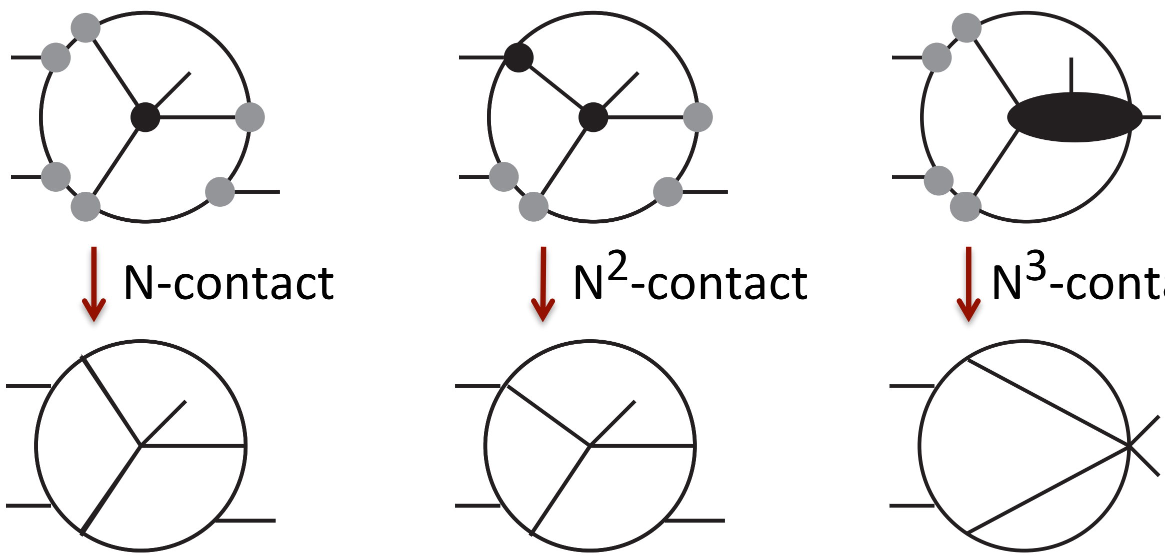
1. Start with some approximation of the supergravity amplitude, organized in terms of the graphs of φ^3 theory, which has the correct maximal cuts, e.g.

a naïve double-copy: $$\mathcal{M}^{(S)G} = \sum_{\Gamma} \int \frac{n_\Gamma \tilde{n}_\Gamma}{D_\Gamma}$$

2. Iteratively correct it w/ graphs w/ higher-pt. vert's to satisfy such that N^k-Max cuts

N^k-contact = N^k-max cut – (N^k-max cut of approximation of amp.)

E.g.



- Each cut gives an independent contrib. to amplitude
- Freedom in choosing each of them
- Lots of cuts
- But a finite number!
- $\mathcal{N}$=8 SG: N^{2L-4}-contact

- Effectively a tree-level calculation
- Ideal if cuts are organized in terms of cubic tree graphs

A few unexpected but useful features

$$\text{N}^k\text{-contact} = \text{N}^k\text{-max cut} - (\text{N}^k\text{-max cut of approximate amplitude})$$

0. A naïve double-copy has the correct maximal and next-to-maximal cuts

$M_4^{\rm tr}(1,2,3) = i A_3^{\rm tr}(1,2,3) A_4^{\rm tr}(1,2,3)$ & 4-pt amp's obey c/k duality

Using KLT to construct SG cuts:

1. Contact terms are much simpler than one has the right to expect
   - In $\mathcal{N}$=8 SG most of them vanish (at least through 5 loops)

2. Four-point double-contact terms factorize; each factor has features resembling gauge theory quantities

3. Higher-contact terms no longer factorize but, in hindsight, can be written as sums of products of factors with features resembling gauge theory quantities

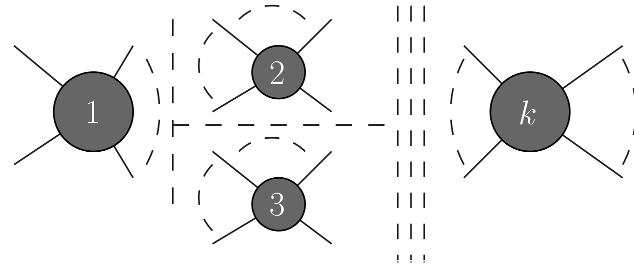
4. These observations match the expected features of the conclusions we drew from the diff. invariance constraints on corrections to a naïve double copy.

Expect that it should be possible to express cuts and contacts in terms of BCJ discrepancy functions, $J_{\Gamma,\lambda}$ and $\tilde{J}_{\Gamma,\lambda}$, using solely gauge theory information

Key for using this is the generalized gauge symmetry

$$\mathcal{C}^{\text{naive 2-copy}} = \sum_{i_1,\dots,i_k} \frac{n_{i_1,i_2,\dots i_k}\tilde{n}_{i_1,i_2,\dots i_k}}{D_{i_1}\dots D_{i_k}}$$

$$\mathcal{C}_{\mathrm{G}} = \sum_{i_1,\dots,i_k} \frac{n^{\mathrm{BCJ}}_{i_1,i_2,\dots i_k}\tilde{n}^{\mathrm{BCJ}}_{i_1,i_2,\dots i_k}}{D_{i_1}\dots D_{i_k}}$$



The generalized gauge transf. relating n and n^{BCJ}:

$$n_{i_1,i_2,\dots i_k} = n^{\mathrm{BCJ}}_{i_1,i_2,\dots i_k} + \Delta_{i_1,i_2,\dots i_k}$$

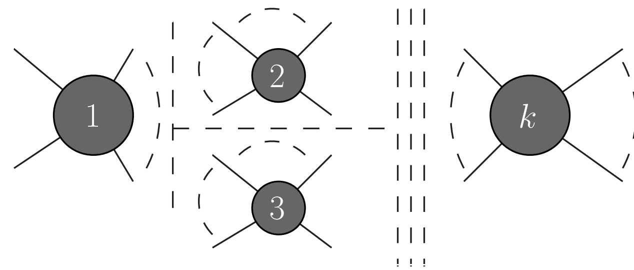
It leaves the gauge theory amplitude invariant

$$\sum_{i_1,\dots,i_k} \frac{\Delta_{i_1,i_2,\dots i_k} c_{i_1,i_2,\dots i_k}}{D_{i_1}\dots D_{i_k}} = 0$$

A solution: the numerators of each graph of triplet of graphs related by Jacobi transformation on edges i, j, k is shifted by the inverse propagator of its Jacobi edge multiplied by an arbitrary function common to all members of the triplet

$$\mathcal{C}^{\text{naive 2-copy}} = \sum_{i_1,\dots,i_k} \frac{n_{i_1,i_2,\dots i_k}\tilde{n}_{i_1,i_2,\dots i_k}}{D_{i_1}\dots D_{i_k}}$$

$$\mathcal{C}_{\text{G}} = \sum_{i_1,\dots,i_k} \frac{n^{\text{BCJ}}_{i_1,i_2,\dots i_k}\tilde{n}^{\text{BCJ}}_{i_1,i_2,\dots i_k}}{D_{i_1}\dots D_{i_k}}$$



The generalized gauge transf. relating n and n^{BCJ}:

$$n_{i_1,i_2,\dots i_k} = n^{\text{BCJ}}_{i_1,i_2,\dots i_k} + \Delta_{i_1,i_2,\dots i_k}$$

It leaves the gauge theory amplitude invariant

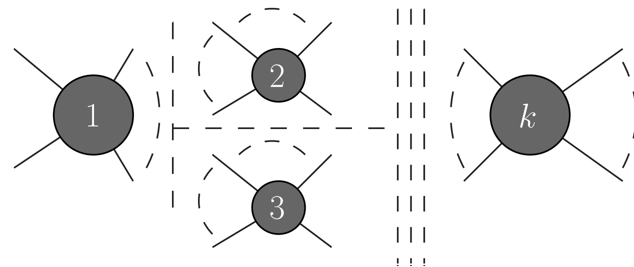
$$\sum_{i_1,\dots,i_k} \frac{\Delta_{i_1,i_2,\dots i_k} c_{i_1,i_2,\dots i_k}}{D_{i_1}\dots D_{i_k}} = 0 \longrightarrow \sum_{i_1,\dots,i_k} \frac{\Delta_{i_1,i_2,\dots i_k} n^{\text{BCJ}}_{i_1,i_2,\dots i_k}}{D_{i_1}\dots D_{i_k}} = 0$$

The gravity (double-copy theory) cut:

$$\mathcal{C}_{\text{G}} = \sum_{i_1,\dots,i_k} \frac{n_{i_1,i_2,\dots i_k}\tilde{n}_{i_1,i_2,\dots i_k}}{D_{i_1}\dots D_{i_k}} - \sum_{i_1,\dots,i_k} \frac{\Delta_{i_1,i_2,\dots i_k}\tilde{\Delta}_{i_1,i_2,\dots i_k}}{D_{i_1}\dots D_{i_k}}$$

$$\mathcal{C}^{\text{naive 2-copy}} = \sum_{i_1,\dots,i_k} \frac{n_{i_1,i_2,\dots i_k}\tilde{n}_{i_1,i_2,\dots i_k}}{D_{i_1}\dots D_{i_k}}$$

$$\mathcal{C}_{\text{G}} = \sum_{i_1,\dots,i_k} \frac{n^{\text{BCJ}}_{i_1,i_2,\dots i_k}\tilde{n}^{\text{BCJ}}_{i_1,i_2,\dots i_k}}{D_{i_1}\dots D_{i_k}}$$



The generalized gauge transf. relating n and n^{BCJ}:

$$n_{i_1,i_2,\dots i_k} = n^{\text{BCJ}}_{i_1,i_2,\dots i_k} + \Delta_{i_1,i_2,\dots i_k}$$

It leaves the gauge theory amplitude invariant

$$\sum_{i_1,\dots,i_k} \frac{\Delta_{i_1,i_2,\dots i_k} c_{i_1,i_2,\dots i_k}}{D_{i_1}\dots D_{i_k}} = 0 \longrightarrow \sum_{i_1,\dots,i_k} \frac{\Delta_{i_1,i_2,\dots i_k} n^{\text{BCJ}}_{i_1,i_2,\dots i_k}}{D_{i_1}\dots D_{i_k}} = 0$$

The gravity (double-copy theory) cut:

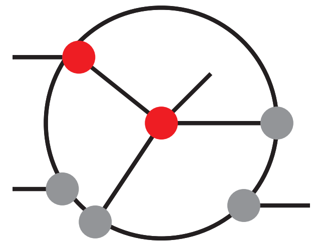
$$\mathcal{C}_{\text{G}} = \sum_{i_1,\dots,i_k} \frac{n_{i_1,i_2,\dots i_k}\tilde{n}_{i_1,i_2,\dots i_k}}{D_{i_1}\dots D_{i_k}} - \sum_{i_1,\dots,i_k} \frac{\Delta_{i_1,i_2,\dots i_k}\tilde{\Delta}_{i_1,i_2,\dots i_k}}{D_{i_1}\dots D_{i_k}}$$

Express Δ in terms of J

$$\begin{aligned} J_{i_1,\dots,i_{p-1},\{A,l\},i_{p+1},\dots,i_k} &= n_{i_1,\dots,i_{p-1},A,i_{p+1},\dots,i_k} \pm n_{i_1,\dots,i_{p-1},B,i_{p+1},\dots,i_k} \pm n_{i_1,\dots,i_{p-1},C,i_{p+1},\dots,i_k} \\ &= \Delta_{i_1,\dots,i_{p-1},A,i_{p+1},\dots,i_k} \pm \Delta_{i_1,\dots,i_{p-1},B,i_{p+1},\dots,i_k} \pm \Delta_{i_1,\dots,i_{p-1},C,i_{p+1},\dots,i_k} \end{aligned}$$

Many more equations than $\Delta_{i_1,\dots,i_k}$ $\longrightarrow$ Why is there a solution at all?!

All double-4-point cut and contact terms from gauge theory data



Bern, Carrasco, Chen, Johansson, RR

Gauge theory cut:

$$\mathcal{C}^{4\times 4}_{\rm YM} = \sum_{i_1,i_2} \frac{n_{i_1 i_2} c_{i_1 i_2}}{d^{(1)}_{i_1} d^{(2)}_{i_2}}$$

Generalized gauge transformation:

$$\sum_{i_1} c_{i_1 i_2} = 0 = \sum_{i_2} c_{i_1 i_2} \qquad \Delta_{i_1 i_2} \equiv n_{i_1 i_2} - n^{\rm BCJ}_{i_1,i_2}$$

$$\sum_{i_1,i_2} \frac{\Delta_{i_1 i_2} c_{i_1 i_2}}{d^{(1)}_{i_1} d^{(2)}_{i_2}} = 0 = \sum_{i_1,i_2} \frac{\Delta_{i_1 i_2} n^{\rm BCJ}_{i_1 i_2}}{d^{(1)}_{i_1} d^{(2)}_{i_2}}$$

The gauge parameters:

$$\Delta_{i_1 i_2} = d^{(1)}_{i_1} \alpha^{(2)}(i_2) + d^{(2)}_{i_2} \alpha^{(1)}(i_1)$$

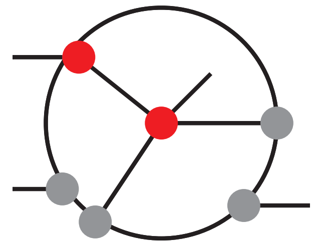
BCJ discrepancy functions: $(\sum_{i_1} d^{(1)}_{i_1} = 0 = \sum_{i_2} d^{(2)}_{i_2})$

$$J_{\bullet,i_2} \equiv \sum_{i_1} n_{i_1 i_2} = d^{(2)}_{i_2} \sum_{i_1} \alpha^{(1)}(i_1) \qquad J_{i_1,\bullet} \equiv \sum_{i_2} n_{i_1 i_2} = d^{(1)}_{i_1} \sum_{i_2} \alpha^{(2)}(i_2)$$

Supergravity cut:

$$\mathcal{C}^{4\times 4}_{\rm SG} = \sum_{i_1,i_2} \frac{n_{i_1 i_2} \tilde{n}_{i_1 i_2}}{d^{(1)}_{i_1} d^{(2)}_{i_2}} - \sum_{i_1,i_2} \frac{\Delta_{i_1 i_2} \tilde{\Delta}_{i_1 i_2}}{d^{(1)}_{i_1} d^{(2)}_{i_2}}$$

All double-4-point cut and contact terms from gauge theory data



Bern, Carrasco, Chen, Johansson, RR

Gauge theory cut:

$$\mathcal{C}^{4\times 4}_{\rm YM} = \sum_{i_1,i_2} \frac{n_{i_1 i_2} c_{i_1 i_2}}{d^{(1)}_{i_1} d^{(2)}_{i_2}}$$

The gauge parameters:

$$\Delta_{i_1 i_2} = d^{(1)}_{i_1} \alpha^{(2)}(i_2) + d^{(2)}_{i_2} \alpha^{(1)}(i_1)$$

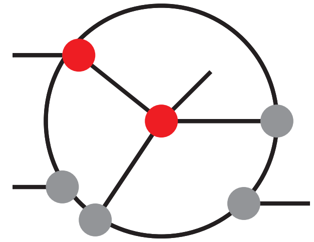
BCJ discrepancy functions: $(\sum_{i_1} d^{(1)}_{i_1} = 0 = \sum_{i_2} d^{(2)}_{i_2})$

$$J_{\bullet,i_2} \equiv \sum_{i_1} n_{i_1 i_2} = d^{(2)}_{i_2} \sum_{i_1} \alpha^{(1)}(i_1) \qquad J_{i_1,\bullet} \equiv \sum_{i_2} n_{i_1 i_2} = d^{(1)}_{i_1} \sum_{i_2} \alpha^{(2)}(i_2)$$

Supergravity cut:

$$\begin{aligned}
\mathcal{C}^{4\times 4}_{\rm SG} &= \sum_{i_1,i_2} \frac{n_{i_1 i_2} \tilde{n}_{i_1 i_2}}{d^{(1)}_{i_1} d^{(2)}_{i_2}} - \sum_{i_1,i_2} \frac{\Delta_{i_1 i_2} \tilde{\Delta}_{i_1 i_2}}{d^{(1)}_{i_1} d^{(2)}_{i_2}} \\
&= \sum_{i_1,i_2} \frac{n_{i_1 i_2} \tilde{n}_{i_1 i_2}}{d^{(1)}_{i_1} d^{(2)}_{i_2}} - \sum_{i_1,i_2} \frac{(d^{(1)}_{i_1} \alpha^{(2)}(i_2) + d^{(2)}_{i_2} \alpha^{(1)}(i_1))(d^{(1)}_{i_1} \tilde{\alpha}^{(2)}(i_2) + d^{(2)}_{i_2} \tilde{\alpha}^{(1)}(i_1))}{d^{(1)}_{i_1} d^{(2)}_{i_2}} \\
&= \sum_{i_1,i_2} \frac{n_{i_1 i_2} \tilde{n}_{i_1 i_2}}{d^{(1)}_{i_1} d^{(2)}_{i_2}} - \sum_{i_1,i_2} (\alpha^{(2)}(i_2) \tilde{\alpha}^{(1)}(i_1) + \alpha^{(1)}(i_1) \tilde{\alpha}^{(2)}(i_2))
\end{aligned}$$

All double-4-point cut and contact terms from gauge theory data



Bern, Carrasco, Chen, Johansson, RR

Gauge theory cut:

$$\mathcal{C}^{4\times 4}_{\rm YM} = \sum_{i_1,i_2} \frac{n_{i_1 i_2} c_{i_1 i_2}}{d^{(1)}_{i_1} d^{(2)}_{i_2}}$$

The gauge parameters:

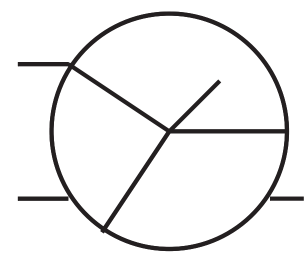
$$\Delta_{i_1 i_2} = d^{(1)}_{i_1} \alpha^{(2)}(i_2) + d^{(2)}_{i_2} \alpha^{(1)}(i_1)$$

BCJ discrepancy functions: $(\sum_{i_1} d^{(1)}_{i_1} = 0 = \sum_{i_2} d^{(2)}_{i_2})$

$$J_{\bullet,i_2} \equiv \sum_{i_1} n_{i_1 i_2} = d^{(2)}_{i_2} \sum_{i_1} \alpha^{(1)}(i_1) \qquad J_{i_1,\bullet} \equiv \sum_{i_2} n_{i_1 i_2} = d^{(1)}_{i_1} \sum_{i_2} \alpha^{(2)}(i_2)$$

Supergravity cut (there are several equivalent variants):

$$\mathcal{C}^{4\times 4}_{\rm SG} = \sum_{i_1,i_2} \frac{n_{i_1 i_2} \tilde{n}_{i_1 i_2}}{d^{(1)}_{i_1} d^{(2)}_{i_2}} \boxed{- \frac{1}{d^{(1)}_1 d^{(2)}_1} \left(J_{\bullet,1} \tilde{J}_{1,\bullet} + J_{1,\bullet} \tilde{J}_{\bullet,1} \right)}$$

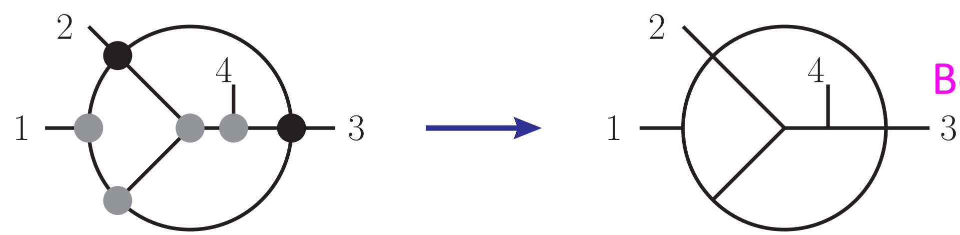


- Correct in any double-copy quantum field theory

$\longrightarrow$ Complete 1-loop 4-point amp's without massive matter

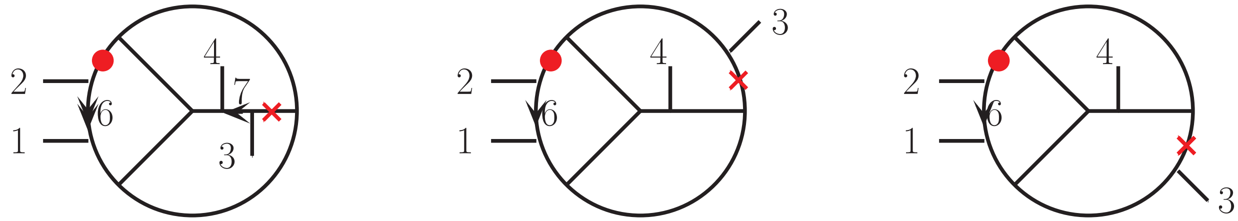
- Only particular combinations of gauge parameters appear in the SG cut

Detailed 3-loop example:

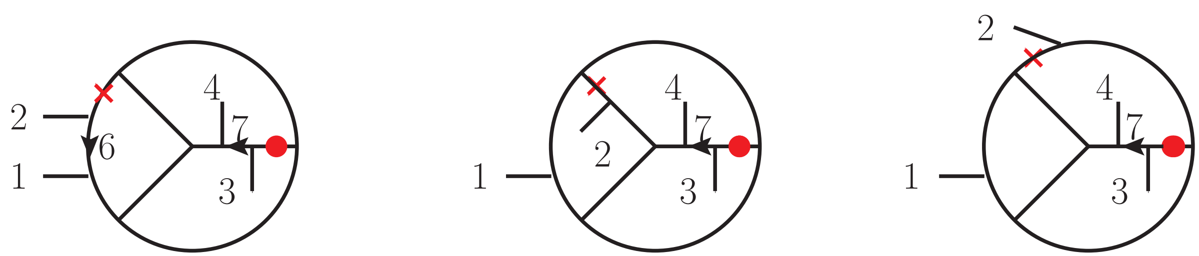


Bern, Carrasco, Chen, Johansson, RR

Relevant YM numerators (from 0808.4112)



$$n_{1,1} = s^2\ , \quad n_{1,2} = s(t + \tau_{26} + \tau_{26})\ , \quad n_{1,3} = s(u - \tau_{26}) \longrightarrow J_{\bullet,1} = s\tau_{26}$$



$$n_{1,1} = s^2\ , \quad n_{1,2} = s(t + \tau_{37} + \tau_{27})\ , \quad n_{1,3} = s(u - \tau_{27}) \longrightarrow J_{1,\bullet} = s\tau_{27}$$

$$\mathcal{N}_{(l)}^{\mathcal{N}=8} = -2\frac{J_{\bullet,1}J_{1,\bullet}}{\tau_{26}\tau_{37}} = -2s^2$$

- Reproduces known SG contact term

Bern, Carrasco, Dixon, Johansson, RR

- All other nonzero double-four-point contacts are relabelings of this one
- Five-point contact terms are also present; more formulae are needed

So why are there $J(\Delta) \longleftrightarrow \Delta(J)$ solutions:

$$J_{i_1,\dots,i_{p-1},\{A,l\},i_{p+1},\dots,i_k} = n_{i_1,\dots,i_{p-1},A,i_{p+1},\dots,i_k} \pm n_{i_1,\dots,i_{p-1},B,i_{p+1},\dots,i_k} \pm n_{i_1,\dots,i_{p-1},C,i_{p+1},\dots,i_k}$$
$$= \Delta_{i_1,\dots,i_{p-1},A,i_{p+1},\dots,i_k} \pm \Delta_{i_1,\dots,i_{p-1},B,i_{p+1},\dots,i_k} \pm \Delta_{i_1,\dots,i_{p-1},C,i_{p+1},\dots,i_k}$$

A. Not all discrepancy functions are independent

- Defined for each propagator of each graph $\longrightarrow$ each appears three times
- (n-3)(2n-5)!!/3 vs. (2n-5)!! $\longrightarrow$ linear relations similar in spirit to KK
- when evaluated on parameters leaving amp's invariant, the rhs is of the form $K\Delta^{\text{independent}}$ with K noninvertible: $v_0 \cdot K = 0 \longrightarrow v_0 \cdot J = 0$

E.g. single-blob: $\sum_{i,\lambda,\lambda'} c_{i,\lambda,\lambda'} \frac{J_{\{i,\lambda\}}}{d_i^{(\lambda')}} = 0$

5-&6-blob: Tye, Zhang
Bjerrum-Bohr, Damgaard, Sondergaard, Vanhove

B. Not all generalized gauge parameters are important

- K noninvertible $\longrightarrow$ not all $\Delta^{\text{independent}}$ are determined i.t.o. $J^{\text{independent}}$
- Remaining ones drop out of SG amplitude

C. Global picture:

1. some gauge parameters are functions of $J^{\text{independent}}$
2. remaining gauge parameters drop out from the SG amplitude
3. re-expressing in terms of overcomplete set of J restores locality

Generalization: cuts have (fairly) closed-form structured expressions i.t.o. cubic graphs

Bern, Carrasco, Chen, Johansson, RR

$$\mathcal{C}_{\rm SG}^{4\times4\times4} = \sum_{i_1,i_2,i_3} \frac{n_{i_1 i_2 i_3}\tilde{n}_{i_1 i_2 i_3}}{d^{(1)}_{i_1} d^{(2)}_{i_2} d^{(3)}_{i_3}} - T_1 - T_2$$

$$\begin{aligned} T_1 =& \sum_{i_3} \frac{J_{\bullet,1,i_3}\tilde{J}_{1,\bullet,i_3} + J_{1,\bullet,i_3}\tilde{J}_{\bullet,1,i_3}}{d^{(1)}_{1} d^{(2)}_{1} d^{(3)}_{i_3}} + \sum_{i_2} \frac{J_{\bullet,i_2,1}\tilde{J}_{1,i_2,\bullet} + J_{1,i_2,\bullet}\tilde{J}_{\bullet,i_2,1}}{d^{(1)}_{1} d^{(2)}_{i_2} d^{(3)}_{1}} \\ &+ \sum_{i_1} \frac{J_{i_1,\bullet,1}\tilde{J}_{i_1,1,\bullet} + J_{i_1,1,\bullet}\tilde{J}_{i_1,\bullet,1}}{d^{(1)}_{i_1} d^{(2)}_{1} d^{(3)}_{1}} \end{aligned}$$

$$\begin{aligned} T_2 =& -\frac{J_{\bullet,1,1}\tilde{J}_{1,\bullet,\bullet} + J_{1,\bullet,\bullet}\tilde{J}_{\bullet,1,1}}{d^{(1)}_{1} d^{(2)}_{1} d^{(3)}_{1}} - \frac{J_{1,\bullet,1}\tilde{J}_{\bullet,1,\bullet} + J_{\bullet,1,\bullet}J'_{1,\bullet,1}}{d^{(1)}_{1} d^{(2)}_{1} d^{(3)}_{1}} \\ &- \frac{J_{1,1,\bullet}\tilde{J}_{\bullet,\bullet,1} + J_{\bullet,\bullet,1}\tilde{J}_{1,1,\bullet}}{d^{(1)}_{1} d^{(2)}_{1} d^{(3)}_{1}} \end{aligned}$$

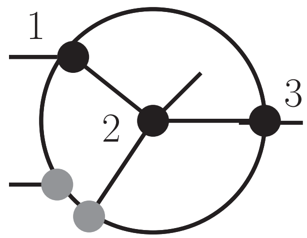
$$\mathcal{C}_{\rm SG}^{4\times\cdots\times4} = \ldots$$

- Subtraction of the cuts of the approximate amplitude is straightforward
- Built-in verification: difference must be local (i.e. all denominators should cancel)

An(other) example: a class of N^3 contact terms

N^k-contact = N^k-max cut – (N^k-max cut of approximate amplitude)

N^3-Max cut:



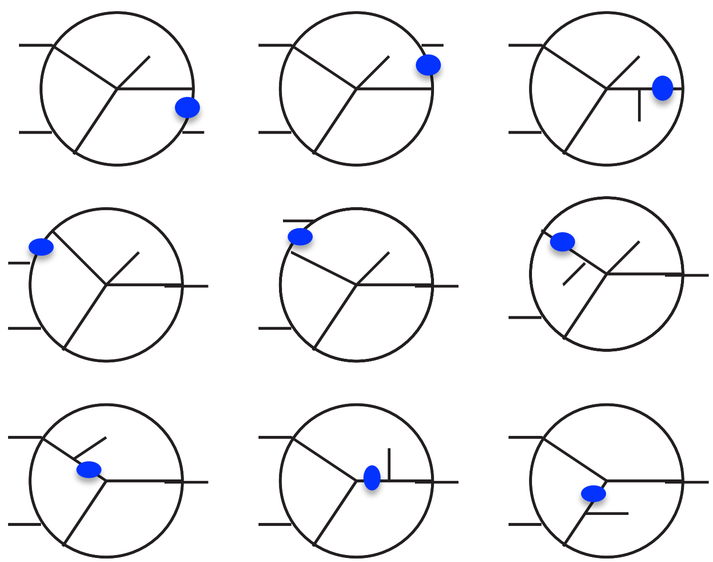
$$\sum_{i_1,i_2,i_3} \frac{n_{i_1 i_2 i_3}\tilde{n}_{i_1 i_2 i_3}}{d^{(1)}_{i_1} d^{(2)}_{i_2} d^{(3)}_{i_3}} - T_1 - T_2$$

N^3-Max cut of (naïve double copy + N^2-Max contacts)

- naïve double copy:

$$\sum_{i_1,i_2,i_3} \frac{n_{i_1 i_2 i_3}\tilde{n}_{i_1 i_2 i_3}}{d^{(1)}_{i_1} d^{(2)}_{i_2} d^{(3)}_{i_3}}$$

- N^2-max ct's:



$$-\frac{1}{d^{(1)}_1 d^{(2)}_1 d^{(3)}_k}\left(J_{\bullet,1,k}\tilde{J}_{1,\bullet,k} + J_{1,\bullet,k}\tilde{J}_{\bullet,1,k}\right)$$

$$-\frac{1}{d^{(1)}_k d^{(2)}_1 d^{(3)}_1}\left(J_{k,\bullet,1}\tilde{J}_{k,1,\bullet} + J_{k,1,\bullet}\tilde{J}_{k,\bullet,1}\right)$$

$$-\frac{1}{d^{(1)}_1 d^{(2)}_k d^{(3)}_1}\left(J_{\bullet,k,1}\tilde{J}_{1,k,\bullet} + J_{1,k,\bullet}\tilde{J}_{\bullet,k,1}\right)$$

Difference is a linear combination of J bilinears; all denominators cancel out

Many generalized cuts have (fairly) closed-form structured expressions

$$\mathcal{C}^{5}_{\mathrm{SG}} = \sum_{i=1}^{15} \frac{n_i \tilde{n}_i}{d^{(1)}_{i,1} d^{(1)}_{i,2}} - \frac{1}{6} \sum_{i=1}^{15} \frac{J_{\{i,1\}} \tilde{J}_{\{i,2\}} + J_{\{i,2\}} \tilde{J}_{\{i,1\}}}{d^{(1)}_{i,1} d^{(1)}_{i,2}}$$

$$\mathcal{C}^{5\times 4}_{\mathrm{SG}} = \sum_{i} \frac{n_i \tilde{n}_i}{d^{(1)}_{i,1} d^{(1)}_{i,2} d^{(2)}_{i,1}} + \text{more complicated}$$

$$\mathcal{C}^{5\times 4\times \cdots \times 4}_{\mathrm{SG}} = \ldots$$

Others, e.g. $\mathcal{C}^{6}_{\mathrm{SG}}$ have currently a less… pleasant appearance

These formulae hold in any adjoint double-copy field theory

The 5-point formula is similar to (though prettier than) a known 5-point tree formula, written in a basis of discrepancy functions Bjerrum-Bohr, Damgaard, Sondergaad, Vanhove

Some features:

- Starting point can be any graph-based representation of amplitudes, including Feynman diagrams

- Novel way to find gravity tree-level amplitudes adapted to cubic graphs

- Cuts are naturally in a cubic graph-based form; identification of the new contact term is straightforward

- Highest contact terms depend on the power counting of the theory; top levels are very simple – linear in momentum invariants. Numerical approach – rather than analytic simplification – may be more efficient

- But…

Some features:

- Starting point can be any graph-based representation of amplitudes, including Feynman diagrams

- Novel way to find gravity tree-level amplitudes adapted to cubic graphs

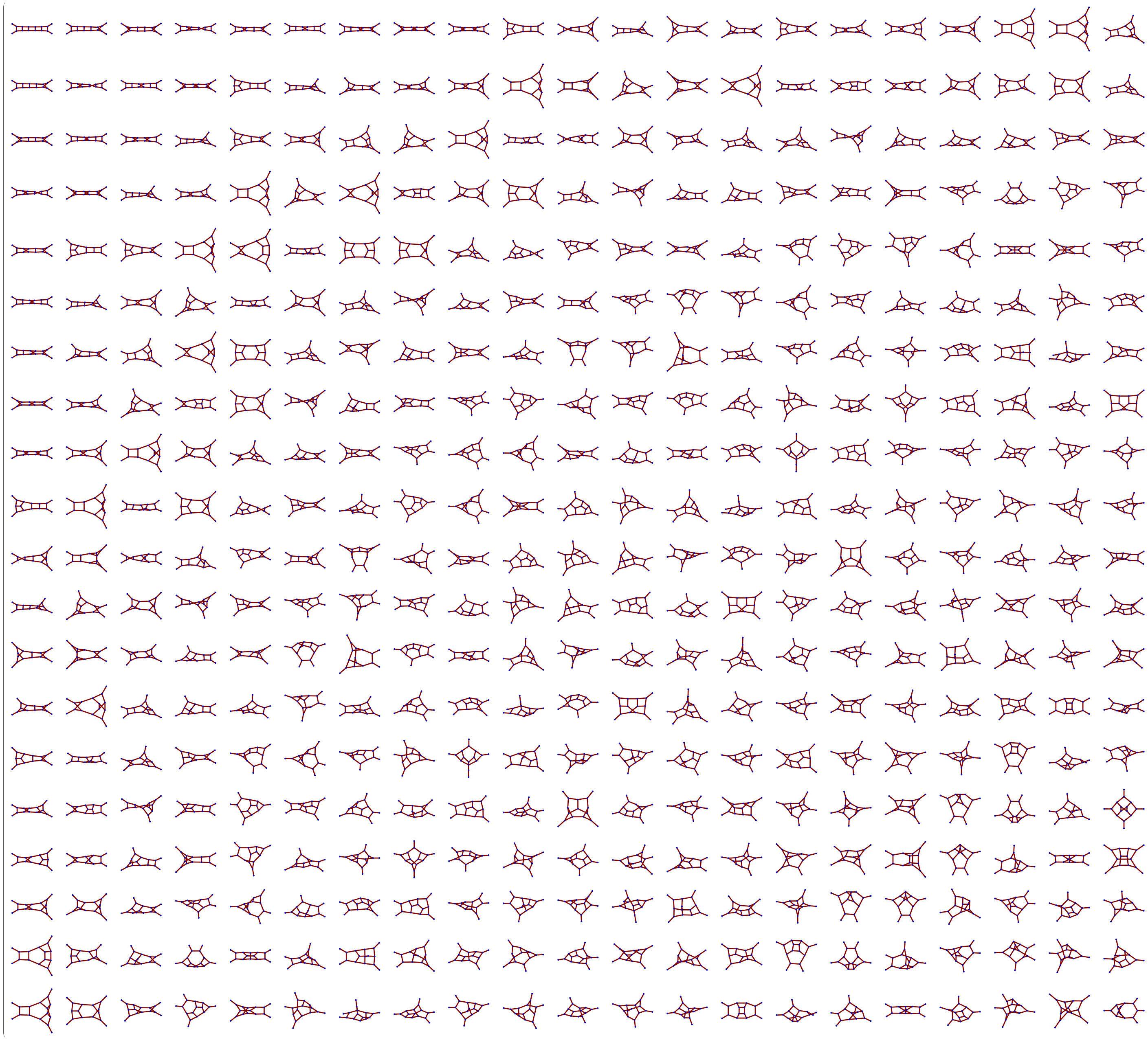
- Cuts are naturally in a cubic graph-based form; identification of the new contact term is straightforward

- Highest contact terms depend on the power counting of the theory; top levels are very simple – linear in momentum invariants. Numerical approach – rather than analytic simplification – may be more efficient

- But the proof is in the pudding…

Allowed us to construct the 4-point 5-loop integrand of $\mathcal{N}$=8 supergravity

Bern, Carrasco, Chen, Johansson, RR, Zeng – to appear



together with
2-, 3-, 4-, 5-, and
6-collapsed
propagator graphs:

N^2: 9159
N^3: 17935
N^4: 23996
N^5: 24198
N^6: 17110

- only about 20% of which are nonzero

Explicit power ct is poor because of poor rep. of $\mathcal{N}$=4 sYM amplitude

On integration

Chetyrkin, Kataev, Tkachov; Laporta; A.V. Smirnov; V. A. Smirnov; Vladimirov; Marcus, Sagnotti; Czakon; Laporta; Kosower; v. Manteuffel, Schabinger; Henn et al; Larsen, Zhang; Zeng, etc

- General structure of the amplitude:

$$\mathcal{M}_4^{(5)} \sim (stu\mathcal{M}_4^{(0)})\ s^2 \int d^{5D}l \sum_{k=0}^{6} \frac{\mathcal{N}_{6-k}(p^2, l\cdot p, l_i\cdot l_j)}{((l+p)^2)^{16-k}}$$

$$\sim (stu\mathcal{M}_4^{(0)})\ s^2 \int d^{5D}l \left[F_{-10}(l_i\cdot l_j) + sF_{-11}(l_i\cdot l_j) + s^2 F_{-12}(l_i\cdot l_j) + \ldots\right]$$

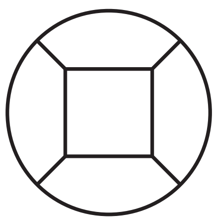
Critical dimension: 4 22/5 24/5

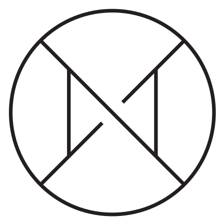
- 5-loop vacuum integrals are state of the art in QCD

- QCD beta function: one needs to expand to second order in external momenta; Here second order (6 external momenta) checks convergence in D = 22/5
  - constrained by supersymmetry
  - checks our construction of the integrand

Observations: 1. All linear relations among integrals are IBPs ($\sim SL(L)$ symmetry)
2. Lower loops suggest that integrals with maximal cuts have highest transcendentality Kosower, Larsen; Abreu, Britto, Duhr, Gardi; Bosma, Sogaard, Zhang; Schabinger et al; Tancredi, Primo; Zeng; etc

Two such integrals; through IBPs, they receive contributions from many terms





On integration

Chetyrkin, Kataev, Tkachov; Laporta; A.V. Smirnov; V. A. Smirnov; Vladimirov; Marcus, Sagnotti; Czakon; Laporta; Kosower; v. Manteuffel, Schabinger; Henn et al; Larsen, Zhang; Zeng, etc

- General structure of the amplitude:

$$\mathcal{M}_4^{(5)} \sim (stu\mathcal{M}_4^{(0)})\, s^2 \int d^{5D}l \sum_{k=0}^{6} \frac{\mathcal{N}_{6-k}(p^2, l\cdot p, l_i \cdot l_j)}{((l+p)^2)^{16-k}}$$

$$\sim (stu\mathcal{M}_4^{(0)})\, s^2 \int d^{5D}l \left[F_{-10}(l_i\cdot l_j) + sF_{-11}(l_i\cdot l_j) + s^2 F_{-12}(l_i\cdot l_j) + \ldots\right]$$

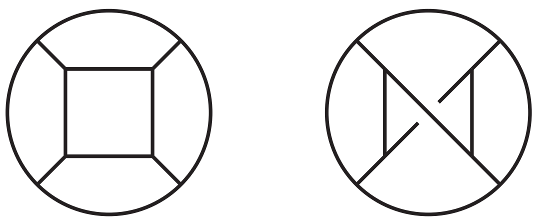
Critical dimension: 4 22/5 24/5

- 5-loop vacuum integrals are state of the art in QCD

- QCD beta function: one needs to expand to second order in external momenta;
  Here second order (6 external momenta) checks convergence in D=22/5
  - constrained by supersymmetry
  - checks our construction of the integrand

Observations: 1. All linear relations among integrals are IBPs ($\sim SL(L)$ symmetry)
2. Lower loops suggest that integrals with maximal cuts have highest transcendentality Kosower, Larsen; Abreu, Britto, Duhr, Gardi; Bosma, Sogaard, Zhang; Schabinger et al; Tancredi, Primo; Zeng; etc

Two such integrals; through IBPs, they receive contributions from many terms



coefficients vanish, as expected

Chetyrkin, Kataev, Tkachov, Laporta, A.V. Smirnov, V. A. Smirnov,
Vladimirov; Marcus, Sagnotti; Czakon; Laporta; Kosower;
v. Manteuffel, Schabinger; Henn et al; Larsen, Zhang; Zeng, etc

On integration

- General structure of the amplitude:

$$\mathcal{M}_4^{(5)} \sim (stu\mathcal{M}_4^{(0)})\; s^2 \int d^{5D}l \sum_{k=0}^{6} \frac{\mathcal{N}_{6-k}(p^2, l\cdot p, l_i\cdot l_j)}{((l+p)^2)^{16-k}}$$

$$\sim (stu\mathcal{M}_4^{(0)})\; s^2 \int d^{5D}l \left[F_{-10}(l_i\cdot l_j) + sF_{-11}(l_i\cdot l_j) + s^2 F_{-12}(l_i\cdot l_j) + \ldots\right]$$

Critical dimension: 4 22/5 24/5

- 5-loop vacuum integrals are state of the art in QCD
- QCD beta function: one needs to expand to second order in external momenta; Here second order (6 external momenta) checks convergence in D=22/5
  - constrained by supersymmetry
  - checks our construction of the integrand

  $\longrightarrow$ further strong indication that integrand is correct

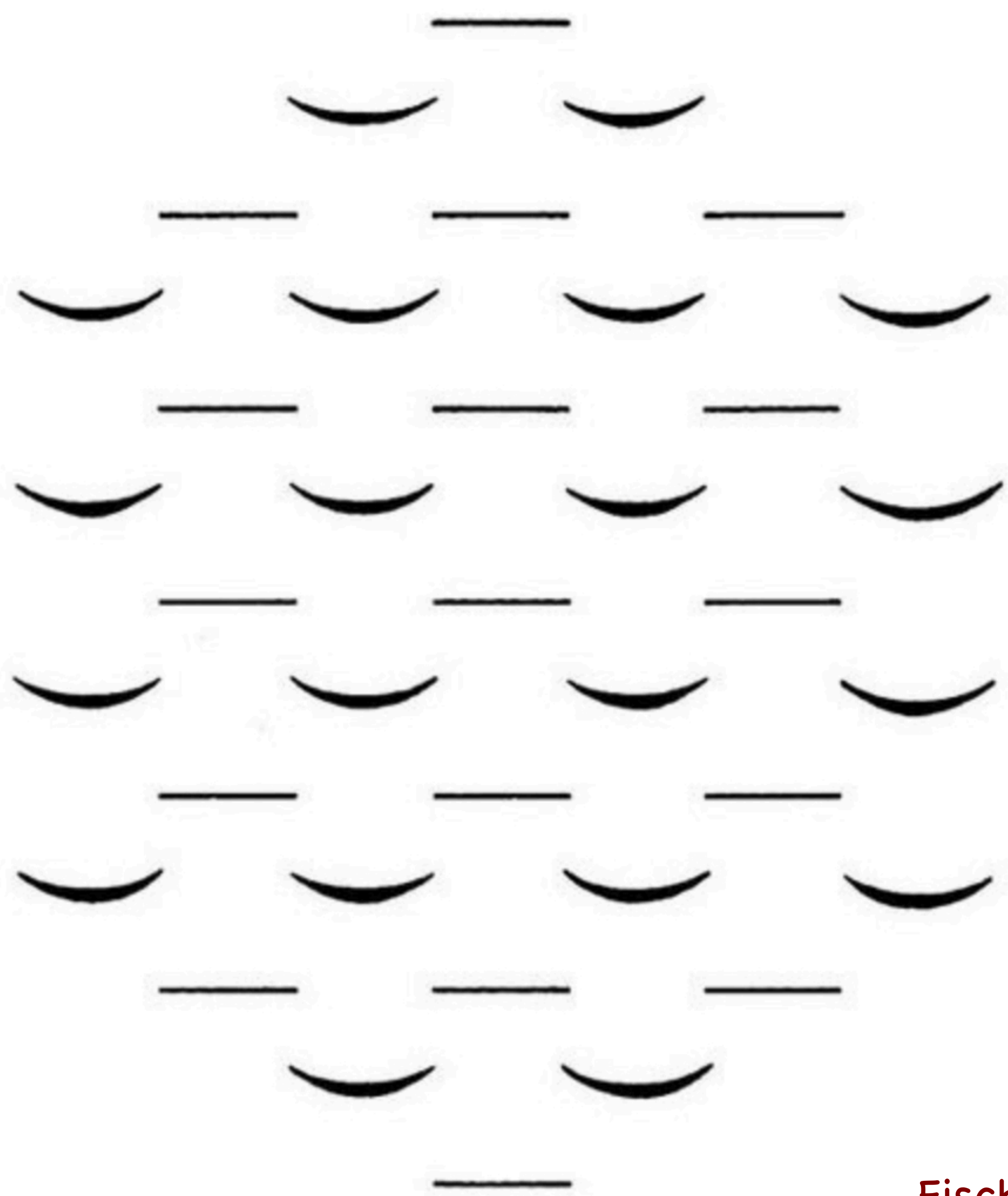
- Enhanced cancellations probed at fourth order -- $\mathcal{O}(10^8)$ terms in $F_{-12}(l_i\cdot l_j)$

  Stay tuned!

Bern, Carrasco, Chen, Johansson, RR, Zeng – in progress

An outlook

- Reviewed recent developments and illustrated some of them
  - Focused on color/kinematics and double-copy
- Many open questions, some computational, some conceptual
- New method for constructing supergravity amplitudes:
  can convert any representation of gauge theory amp's into supergravity amp's
  - Takes over when c/k duality is for some reason impractical; algorithmic construction of amplitudes' contact terms in terms of the breaking of kinematic Jacobi relations
  - Terms in amplitudes are constructed one by one
  - Allows the construction of the 5-loop 4-graviton integrand of $\mathcal{N}$=8 SG
  - May have applications to construction of classical solutions of SG eom
  - Full potential is to be explored, as is the physics of the 5-loop SG amplitude



Fisches Nachtgesang
– Christian Morgenstern