Scattering in Conformal Gravity

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Based on work with:

Josh Nohle [1707.02965];

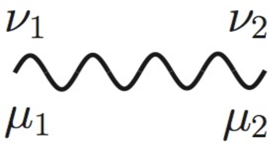
Marco Chiodaroli, Murat Gunaydin, Radu Roiban

[1408.0764, 1511.01740, 1512.09130, 1701.02519];

Gregor Kälin, Gustav Mogull [1706.09381].

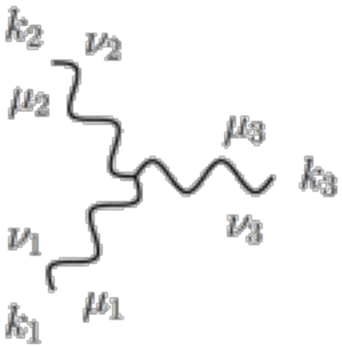
Perturbative Einstein gravity (textbook)

$$\mathcal{L} = \frac{2}{\kappa^2}\sqrt{g}R, \qquad g_{\mu\nu} = \eta_{\mu\nu} + \kappa h_{\mu\nu}$$



$$= \frac{1}{2}\left[\eta_{\mu_1\nu_1}\eta_{\mu_2\nu_2} + \eta_{\mu_1\nu_2}\eta_{\nu_1\mu_2} - \frac{2}{D-2}\eta_{\mu_1\mu_2}\eta_{\nu_1\nu_2}\right]\frac{i}{p^2 + i\epsilon}$$

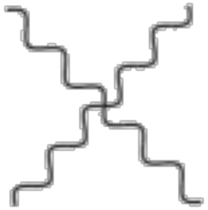
de Donder gauge



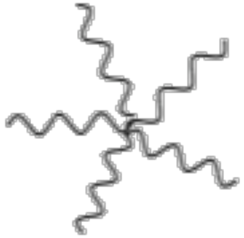
$$\begin{aligned}
= \mathrm{sym}[&-\frac{1}{2}P_3(k_1\cdot k_2\eta_{\mu_1\nu_1}\eta_{\mu_2\nu_2}\eta_{\mu_3\nu_3}) - \frac{1}{2}P_6(k_{1\mu_1}k_{1\nu_2}\eta_{\mu_1\nu_1}\eta_{\mu_3\nu_3}) + \frac{1}{2}P_3(k_1\cdot k_2\eta_{\mu_1\mu_2}\eta_{\nu_1\nu_2}\eta_{\mu_3\nu_3}) \\
&+P_6(k_1\cdot k_2\eta_{\mu_1\nu_1}\eta_{\mu_2\mu_3}\eta_{\nu_2\nu_3}) + 2P_3(k_{1\mu_2}k_{1\nu_3}\eta_{\mu_1\nu_1}\eta_{\nu_2\mu_3}) - P_3(k_{1\nu_2}k_{2\mu_1}\eta_{\nu_1\mu_1}\eta_{\mu_3\nu_3}) \\
&+P_3(k_{1\mu_3}k_{2\nu_3}\eta_{\mu_1\mu_2}\eta_{\nu_1\nu_2}) + P_6(k_{1\mu_3}k_{1\nu_3}\eta_{\mu_1\mu_2}\eta_{\nu_1\nu_2}) + 2P_6(k_{1\mu_2}k_{2\nu_3}\eta_{\nu_2\mu_1}\eta_{\nu_1\mu_3}) \\
&+2P_3(k_{1\mu_2}k_{2\mu_1}\eta_{\nu_2\mu_3}\eta_{\nu_3\nu_1}) - 2P_3(k_1\cdot k_2\eta_{\nu_1\mu_2}\eta_{\nu_2\mu_3}\eta_{\nu_3\mu_1})]
\end{aligned}$$

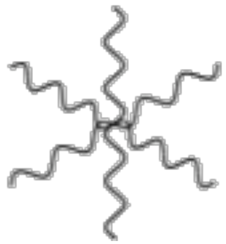
After symmetrization
$\sim$ 100 terms !

higher order vertices...



$\sim 10^3$ terms





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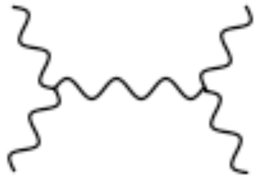
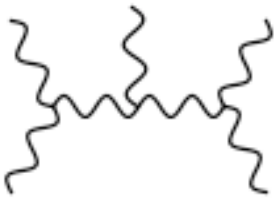
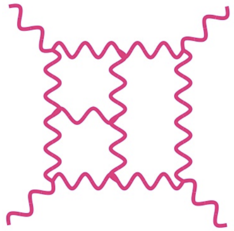
complicated diagrams:



$\sim 10^4$ terms



$\sim 10^7$ terms



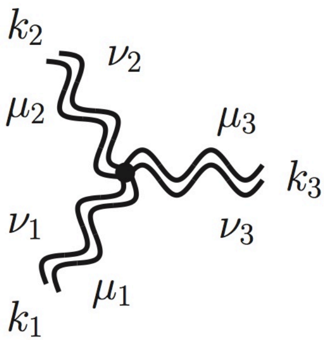
$\sim 10^{21}$ terms

On-shell simplifications

Graviton plane wave: $\varepsilon^{\mu}(p)\varepsilon^{\nu}(p)\, e^{ip\cdot x}$

Yang-Mills polarization

On-shell 3-graviton vertex:

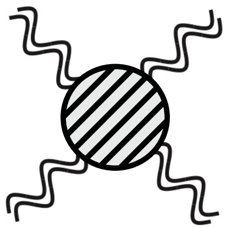


$$= \; i\kappa\Big(\eta_{\mu_1\mu_2}(k_1-k_2)_{\mu_3} + \text{cyclic}\Big)\Big(\eta_{\nu_1\nu_2}(k_1-k_2)_{\nu_3} + \text{cyclic}\Big)$$

Yang-Mills vertex

Gravity scattering amplitude:

Yang-Mills amplitude



$$M^{\rm GR}_{\rm tree}(1,2,3,4) = \frac{st}{u} A^{\rm YM}_{\rm tree}(1,2,3,4) \otimes A^{\rm YM}_{\rm tree}(1,2,3,4)$$

Gravity processes = "squares" of gauge theory ones

$$\text{gravity} = (\text{gauge th}) \otimes (\text{gauge th})$$

Generic gravities are double copies

Amplitudes in familiar theories are secretly related, for example:

- $(\mathcal{N}=4\ \mathrm{SYM})\otimes(\mathcal{N}=4\ \mathrm{SYM})=(\mathcal{N}=8\ \mathrm{SUGRA})$
- $(\mathcal{N}=4\ \mathrm{SYM})\otimes(\text{pure YM})=(\mathcal{N}=4\ \mathrm{SUGRA})$
- $(\text{pure YM})\otimes(\text{pure YM})=\mathrm{GR}+\phi+B^{\mu\nu}$ (dilaton-axion)
- $\mathrm{QCD}\otimes\mathrm{QCD}=\mathrm{GR}+\text{matter}$ (Maxwell-Einstein)
- $(\mathrm{YM})\otimes(\mathrm{YM}+\phi^3)=\mathrm{GR}+\mathrm{YM}$ (Yang-Mills-Einstein)

and many more...

$\longrightarrow$ $(\text{gauge sym})\otimes(\text{gauge sym})=\text{diffeo sym}$

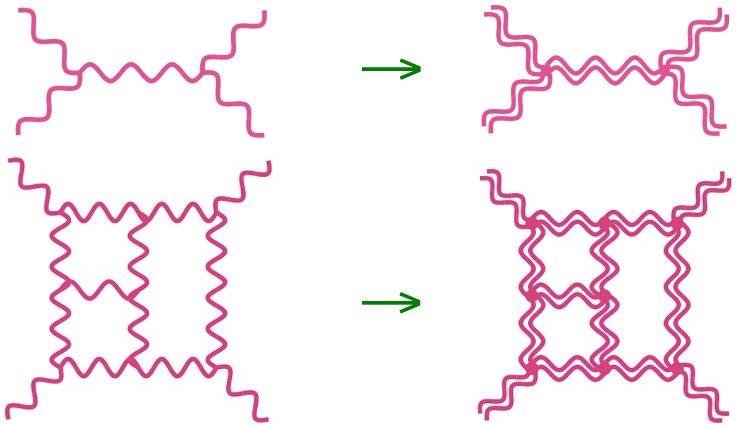
Generality of double copy

Gravity processes = product of gauge theory ones - entire S-matrix

Gauge theory Gravity Bern, Carrasco, HJ ('10)

trees:

loops:



"squared" numerators

Recent generalizations:

→ Theories that are not truncations of *N*=8 SG HJ, Ochirov; Chiodaroli, Gunaydin, Roiban

→ Theories with fundamental matter HJ, Ochirov; Chiodaroli, Gunaydin, Roiban

→ Spontaneously broken theories Chiodaroli, Gunaydin, HJ, Roiban

→ Form factors Boels, Kniehl, Tarasov, Yang → talks by Yang & Boels

→ Gravity off-shell symmetries from YM Anastasiou, Borsten, Duff, Hughes, Nagy

→ Classical (black hole) solutions Luna, Monteiro, O'Connell, White; Ridgway, Wise; Goldberger,...

→ Amplitudes in curved background Adamo, Casali, Mason, Nekovar

→ New double copies for string theory Mafra, Schlotterer, Stieberger, Taylor, Broedel, Carrasco...

→ CHY scattering eqs, twistor strings Cachazo, He, Yuan, Skinner, Mason, Geyer, Adamo, Monteiro,..

→ Conformal gravity HJ, Nohle see talks by Goldberger; Mason; Schlotterer

Motivation: (super)gravity UV behavior

Old results on UV properties:

- SUSY forbids 1,2 loop div. ~~R^2, R^3~~ Ferrara, Zumino, Deser, Kay, Stelle, Howe, Lindström, Green, Schwarz, Brink, Marcus, Sagnotti
- Pure gravity 1-loop finite, 2-loop divergent Goroff & Sagnotti, van de Ven
- With matter: 1-loop divergent 't Hooft & Veltman; (van Nieuwenhuizen; Fischler..)

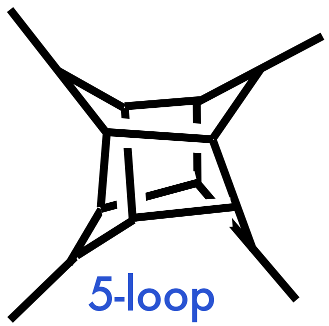
New results on UV properties:

- $\mathcal{N}$=8 SG and $\mathcal{N}$=4 SG 3-loop finite! Bern, Carrasco, Dixon, HJ, Kosower, Roiban; Bern, Davies, Dennen, Huang
- $\mathcal{N}$=8 SG: no divergence before 7 loops Beisert, Elvang, Freedman, Kiermaier, Morales, Stieberger; Björnsson, Green, Bossard, Howe, Stelle, Vanhove, Kallosh, Ramond, Lindström, Berkovits, Grisaru, Siegel, Russo, and more....
- First $\mathcal{N}$=4 SG divergence at 4 loops Bern, Davies, Dennen, Smirnov, Smirnov
  (unclear interpretation, U(1) anomaly?)
- Evanescent effects: Einstein gravity Bern, Cheung, Chi, Davies, Dixon, Nohle

$\mathcal{N} = 1, 2, 3$ SG

need calculations:

finite ???



5-loop
N=8 SG

→ talk by Roiban

Outline

- On-shell diffeomorphisms from gauge symmetry
- Color-kinematics duality & double copy
  - examples: SQCD; magical SG; YM-Einstein.
- Generalization to conformal (super-)gravity
- New dimension-six gauge theory
- Deformations and extensions → more gravities
- Conclusion

Gauge & diffeomorphism symmetry

kinematic numerators

color factors

cubic diagram form: $$\mathcal{A}^{\text{tree}} = \sum_{i \in \text{cubic}} \frac{n_i c_i}{D_i}$$

propagators

$n_i \equiv \varepsilon_\mu(p)\, n_i^\mu$ Consider a gauge transformation $\varepsilon \to \varepsilon + \alpha p$

$$n_i \to n_i + \Delta_i \qquad \Delta_i = \alpha\, p_\mu n_i^\mu$$

Invariance of $\mathcal{A}^{\text{tree}}$ requires that c_i are linearly dependent

$$c_i - c_j = c_k$$ [Jacobi id. or Lie algebra]

thus the combination $$\sum_{i \in \text{cubic}} \frac{\Delta_i c_i}{D_i} = 0$$ vanishes.

"Double copy always gravitates"

Assume: gauge freedom can be exploited to find color-dual numerators

$$c_i - c_j = c_k \quad \Leftrightarrow \quad n_i - n_j = n_k$$

Then the double copy $\quad \mathcal{M}^{\text{tree}} = \sum_{i \in \text{cubic}} \frac{n_i \tilde{n}_i}{D_i} \quad \rightarrow$ Gravity

describes a spin-2 theory $\quad \varepsilon_{\mu\nu} = \varepsilon_\mu \varepsilon_\nu$

invariant under (linear) diffeos $\quad \varepsilon_{\mu\nu} \to \varepsilon_{\mu\nu} + p_\mu \xi_\nu + \xi_\mu p_\nu$

$$\mathcal{M}^{\text{tree}} \to \mathcal{M}^{\text{tree}} + \underbrace{\sum_{i \in \text{cubic}} \frac{\Delta_i \tilde{n}_i}{D_i} + \sum_{i \in \text{cubic}} \frac{n_i \tilde{\Delta}_i}{D_i}}_{=0}$$

$$(\text{gauge sym}) \otimes (\text{gauge sym}) = \text{diffeo sym}$$

Chiodaroli, Gunaydin, HJ, Roiban

Color-kinematics duality

Color-kinematics duality

Gauge theories are controlled by a hidden kinematic Lie algebra

→ Amplitude represented by cubic graphs:

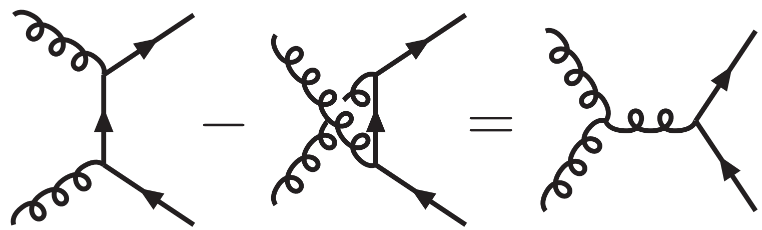
numerators

color factors

$$\mathcal{A}_m^{(L)} = \sum_{i \in \Gamma_3} \int \frac{d^{LD}\ell}{(2\pi)^{LD}} \frac{1}{S_i} \frac{n_i c_i}{p_{i_1}^2 p_{i_2}^2 p_{i_3}^2 \cdots p_{i_l}^2}$$

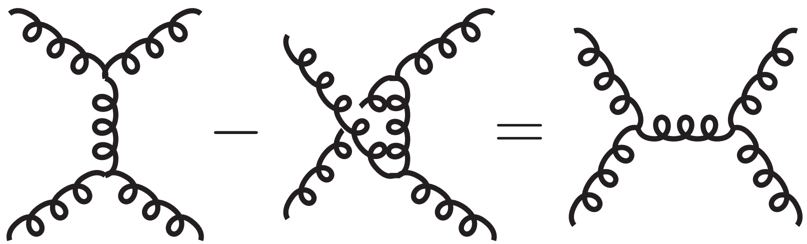
propagators

Color & kinematic numerators satisfy same relations:



commutation identity

$$T^a T^b - T^b T^a = f^{abc} T^c$$



Jacobi identity

$$f^{dac} f^{cbe} - f^{dbc} f^{cae} = f^{abc} f^{dce}$$

$$n_i \quad - \quad n_j \quad = \quad n_k$$

color	c_i
⬍	⬍
kinematics	n_i

Bern, Carrasco, HJ

Gauge-invariant relations (pure adjoint theories)

$$A(1,2,\ldots,n-1,n) = A(n,1,2,\ldots,n-1)$$ cyclicity → (n-1)! basis

$$\sum_{i=1}^{n-1} A(1,2,\ldots,i,n,i+1,\ldots,n-1) = 0$$ U(1) decoupling

$$A(1,\beta,2,\alpha) = (-1)^{|\beta|} \sum_{\sigma\in\alpha\sqcup\beta^T} A(1,2,\sigma)$$ Kleiss-Kuijf relations ('89)

$\left.\right\}$ (n-2)! basis

$$\sum_{i=2}^{n-1}\Big(\sum_{j=2}^{i} s_{jn}\Big) A(1,2,\ldots i,n,i+1,\ldots,n-1) = 0$$

$$A(1,2,\alpha,3,\beta) = \sum_{\sigma\in S(\alpha)\sqcup\beta} A(1,2,3,\sigma)\prod_{i=1}^{|\alpha|}\frac{\mathcal{F}(3,\sigma,1|i)}{s_{2,\alpha_1,\ldots,\alpha_i}}$$

$\left.\right\}$ BCJ relations ('08) (n-3)! basis

BCJ rels. proven via string theory by Bjerrum-Bohr, Damgaard, Vanhove; Stieberger ('09)

and field theory proofs through BCFW: Feng, Huang, Jia; Chen, Du, Feng ('10 -'11)

Relations used in string calcs: Mafra, Stieberger, Schlotterer, et al. ('11 -'17)

Loop-level generalizations: Tourkine, Vanhove; Hohenegger, Stieberger; Chiodaroli et al.

→ talk by Tourkine; Schlotterer

Gravity is a double copy of YM

Gravity amplitudes obtained by replacing color with kinematics

$$\mathcal{A}_m^{(L)} = \sum_{i\in\Gamma_3} \int \frac{d^{LD}\ell}{(2\pi)^{LD}} \frac{1}{S_i} \frac{n_i c_i}{p_{i_1}^2 p_{i_2}^2 p_{i_3}^2 \cdots p_{i_l}^2}$$

double copy

Bern, Carrasco, HJ

$$\mathcal{M}_m^{(L)} = \sum_{i\in\Gamma_3} \int \frac{d^{LD}\ell}{(2\pi)^{LD}} \frac{1}{S_i} \frac{n_i \tilde{n}_i}{p_{i_1}^2 p_{i_2}^2 p_{i_3}^2 \cdots p_{i_l}^2}$$

- The two numerators can differ by a generalized gauge transformation
  → only one copy needs to satisfy the kinematic algebra
- The two numerators can differ by the external/internal states
  → graviton, dilaton, axion (*B*-tensor), matter amplitudes
- The two numerators can belong to different theories
  → give a host of different gravitational theories

Equivalent to KLT at tree level for adj. rep.

→ talk by Schlotterer

Which gauge theories obey C-K duality

- Pure $\mathcal{N}$=0,1,2,4 super-Yang-Mills (any dimension) — Bern, Carrasco, HJ ('08) Bjerrum-Bohr, Damgaard, Vanhove; Stieberger; Feng et al. Mafra, Schlotterer, etc ('08-'11)
- Self-dual Yang-Mills theory O'Connell, Monteiro ('11)
- Heterotic string theory Stieberger, Taylor ('14)
- Yang-Mills + F^3 theory Broedel, Dixon ('12)
- QCD, super-QCD, higher-dim QCD HJ, Ochirov ('15); Kälin, Mogull ('17)
- Generic matter coupled to $\mathcal{N}$= 0,1,2,4 super-Yang-Mills — Chiodaroli, Gunaydin, Roiban; HJ, Ochirov ('14)
- Spontaneously broken $\mathcal{N}$= 0,2,4 SYM Chiodaroli, Gunaydin, HJ, Roiban ('15)
- Yang-Mills + scalar ϕ^3 theory Chiodaroli, Gunaydin, HJ, Roiban ('14)
- Bi-adjoint scalar ϕ^3 theory — Bern, de Freitas, Wong ('99), Bern, Dennen, Huang; Du, Feng, Fu; Bjerrum-Bohr, Damgaard, Monteiro, O'Connell
- NLSM/Chiral Lagrangian Chen, Du ('13)
- *D*=3 BLG theory (Chern-Simons-matter) — Bargheer, He, McLoughlin; Huang, HJ, Lee ('12 -'13)
- (Non-)Abelian Z-theory Carrasco, Mafra, Schlotterer ('16)
- Dim-6 gauge theories: $(DF)^2 + F^3 + \ldots$ HJ, Nohle ('17)

Which gravity theories are double copies

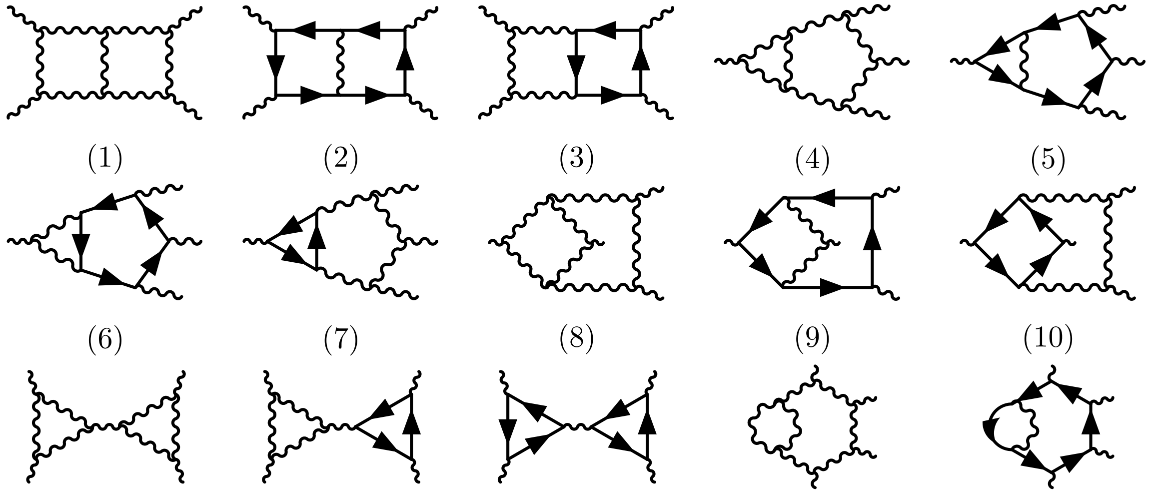
- Pure $\mathcal{N}$=4,5,6,8 supergravity (2 < D < 11) KLT ('86), Bern, Carrasco, HJ ('08 -'10)
- Einstein gravity and pure $\mathcal{N}$=1,2,3 supergravity HJ, Ochirov ('14)
- *D*=6 pure $\mathcal{N}$=(1,1) and $\mathcal{N}$=(2,0) supergravity HJ, Kälin, Mogull ('17)
- Self-dual gravity O'Connell, Monteiro ('11)
- Closed string theories Mafra, Schlotterer, Stieberger ('11); Stieberger, Taylor ('14)
- Einstein + R^3 theory Broedel, Dixon ('12)
- Abelian matter coupled to supergravity Carrasco, Chiodaroli, Gunaydin, Roiban ('12); HJ, Ochirov ('14 - '15)
- Magical sugra, homogeneous sugra Chiodaroli, Gunaydin, HJ, Roiban ('15)
- (S)YM coupled to (super)gravity Chiodaroli, Gunaydin, HJ, Roiban ('14)
- Spontaneously broken YM-Einstein gravity Chiodaroli, Gunaydin, HJ, Roiban ('15)
- *D*=3 supergravity (BLG Chern-Simons-matter theory)^2 Bargheer, He, McLoughlin; Huang, HJ, Lee ('12 -'13)
- Born-Infeld, DBI, Galileon theories (CHY form) Cachazo, He, Yuan ('14)
- $\mathcal{N}$=0,1,2,4 conformal supergravity HJ, Nohle ('17)

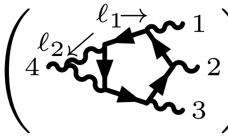
Super-QCD at two loops

HJ, G. Kälin, G. Mogull

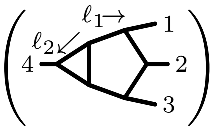
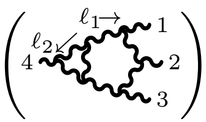
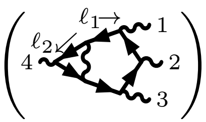
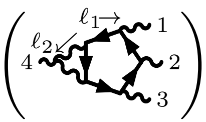
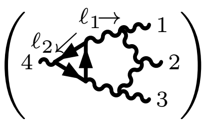
→ talk by Mogull

- two-loop SQCD amplitude
- color-kinematics manifest
- planar + non-planar
- N_f massless quarks
- integrand valid in $D \leq 6$



e.g. simple SQCD numerator: $n\Bigg($  $\Bigg) = \frac{1}{15}\big(p_4\cdot(\ell_1 - \ell_2) - 2\ell_1\cdot\ell_2\big)$

half-maximal supergravity numerator:

$N^{[\mathcal{N}=4\ \mathrm{SG}]}\Bigg($  $\Bigg) = \Bigg|n\Bigg($  $\Bigg)\Bigg|^2 + (D_s - 6)\Bigg(\Bigg|n\Bigg($  $\Bigg)\Bigg|^2 + \Bigg|n\Bigg($  $\Bigg)\Bigg|^2 + \Bigg|n\Bigg($  $\Bigg)\Bigg|^2\Bigg)$

pure supergravities given by $D_s = D = 4, 5, 6$ cf. HJ, Ochirov ('14)

(the $D = 6$ theory is $\mathcal{N}$=(1,1) supergravity)

$\mathcal{N}$=(1,1) vs $\mathcal{N}$=(2,0) supergravity

In six dimensions there are two half-maximal supergravities:

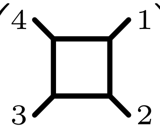
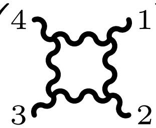
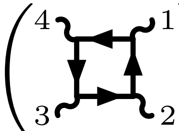
$$\Big(\mathcal{N}=(1,1)\ \mathrm{SG}\Big) = \Big(\mathcal{N}=(1,0)\ \mathrm{SQCD}\Big) \otimes \Big(\mathcal{N}=(0,1)\ \mathrm{SQCD}\Big)$$

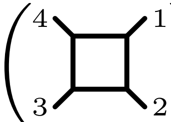
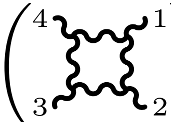
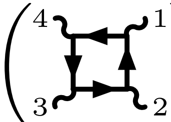
(graviton + vector multiplets)

$$\Big(\mathcal{N}=(2,0)\ \mathrm{SG}\Big) = \Big(\mathcal{N}=(1,0)\ \mathrm{SQCD}\Big) \otimes \Big(\mathcal{N}=(1,0)\ \mathrm{SQCD}\Big)$$

(graviton + tensor multiplets)

In terms of gravity numerators:

(1,1) → $N^{[\mathcal{N}=(1,1)\ \mathrm{SG}]}\Bigg($  $\Bigg) = \Bigg| n\Bigg($  $\Bigg)\Bigg|^2 + (D_s - 6)\Bigg| n\Bigg($  $\Bigg)\Bigg|^2$

(2,0) → $N^{[\mathcal{N}=(2,0)\ \mathrm{SG}]}\Bigg($  $\Bigg) = \Bigg(n\Bigg($  $\Bigg)\Bigg)^2 + (N_T - 1)\Bigg(n\Bigg($  $\Bigg)\Bigg)^2$

same simple relation at two loops!

HJ, G. Kälin, G. Mogull

Magical and homogeneous SUGRAs

Maxwell-Einstein 5d supergravity theories — Gunaydin, Sierra, Townsend

$$e^{-1}\mathcal{L} = -\frac{R}{2} - \frac{1}{4}\mathring{a}_{IJ}F^I_{\mu\nu}F^{J\mu\nu} - \frac{1}{2}g_{xy}\partial_\mu\varphi^x\partial^\mu\varphi^y + \frac{e^{-1}}{6\sqrt{6}}C_{IJK}\epsilon^{\mu\nu\rho\sigma\lambda}F^I_{\mu\nu}F^J_{\rho\sigma}A^K_\lambda$$

Lagrangian is entirely determined by constants C_{IJK}

- Homogenous scalar manifold

$$C_{IJK} \sim \Gamma^a_{\alpha\beta}$$

de Wit, van Proeyen

- Double copy:

$$(\mathcal{N}=2\ \mathrm{SQCD}) \otimes (D, N_f\ \mathrm{QCD})$$

	$\#f_D=0$	$\#f_D=1$	$\#f_D=2$	$\#f_D=3$	$\cdots$
$D=4$	— Luciani model (?) —				
$D=5$		— Generic non-Jordan Family —			
$D=6$		— Generic Jordan Family —			
$D=7$		$\mathbb{R}$			
$D=8$		$\mathbb{C}$			
$D=9$					
$D=10$		$\mathbb{H}$			
$\cdots$					
$D=14$		$\mathbb{O}$			
$\cdots$					

- Magical theories

$$(\mathcal{N}=2\ \mathrm{SQCD}) \otimes (D=7,8,10,14\ \mathrm{QCD})$$
$$= \mathrm{Magical}\ \mathcal{N}=2\ \mathrm{Supergravity}$$
$$(\mathbb{R},\mathbb{C},\mathbb{H},\mathbb{O}\ \mathrm{type})$$

Chiodaroli, Gunaydin, HJ, Roiban ('15)

cf. HJ, Ochirov ('15)

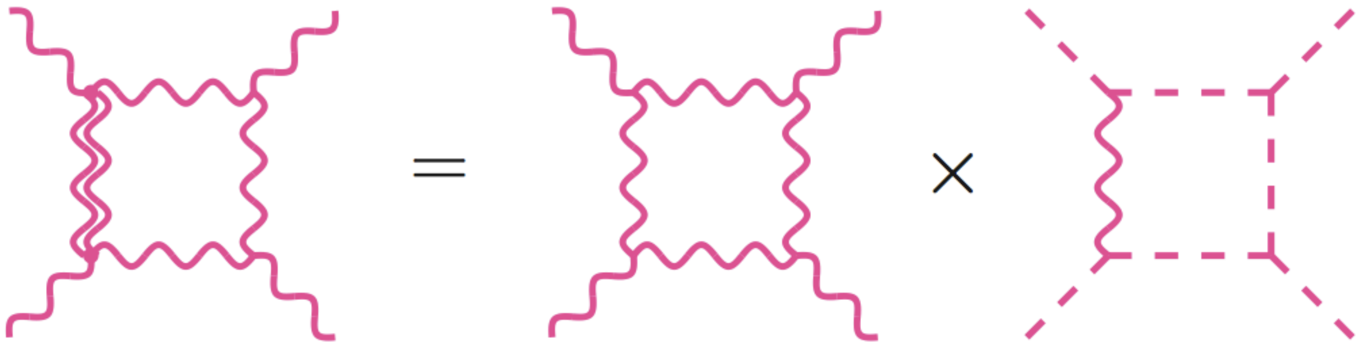
Yang-Mills-Einstein theory (GR+YM)

Chiodaroli, Gunaydin, HJ, Roiban ('14)

$$\mathrm{GR} + \mathrm{YM} = \mathrm{YM} \otimes (\mathrm{YM} + \phi^3)$$

GR+YM amplitudes are "heterotic" double copies

$$h^{\mu\nu} \sim A^\mu \otimes A^\nu$$
$$A^{\mu a} \sim A^\mu \otimes \phi^a$$



$\mathcal{N}$= 0,1,2,4 YME supergravity

$\mathcal{N}$= 0,1,2,4 SYM

YM + ϕ^3

- simplest type of gauged supergravity; R-symmetry gauged is more complicated
- construction extends to SSB (Coulomb branch) Chiodaroli, Gunaydin, HJ, Roiban ('15)

$$(\mathrm{SSB}) = (\mathrm{SSB}) \otimes (\mathrm{expl.\,SB})$$

Conformal Gravity

Conformal gravity

Some properties of conformal gravity:

4-derivative action $$\int d^4x\sqrt{-g}\,W^2$$

(Weyl)^2 $$(W_{\mu\nu\rho\sigma})^2 = (R_{\mu\nu\rho\sigma})^2 - 2(R_{\mu\nu})^2 + \frac{1}{3}R^2$$

invariant under local scale transformations: $g_{\mu\nu} \to \Omega^2(x) g_{\mu\nu}$

propagator $\sim \dfrac{1}{k^4} \quad \sim \quad \dfrac{1}{k^2} - \dfrac{1}{k^2 - m^2}$

States: 2 + 5 − 1 (Weyl invariance removes one state):
- graviton: 2
- graviton ghost: 2
- vector ghost : 2

However: physical S-matrix trivial in flat space
Maldacena; Adamo, Mason

→ negative-norm states → non-unitary

→ R^2 gravities renormalizable K. Stelle ('77)

Conformal supergravity

Some properties of conformal supergravity:

supersymmetric extensions of $\int d^4x\sqrt{-g}\,f(\phi)W^2 + \text{mess}$

invariant under local superconformal transformations

$$\mathcal{N}=1,\quad \mathcal{N}=2 \quad \text{and} \quad \mathcal{N}=4 \quad \text{models}$$

	$\mathcal{N}=4$	$\mathcal{N}=2$	$\mathcal{N}=1$
Total # on-shell states:	192	40	16
- graviton multipet:	32	8	4
- graviton ghost multipet :	32	8	4
- gravitino ghost multiplets:	4×32	2×8	4
- vector ghost multiplets:	0	8	4

"minimal" N=4 at one-loop: conformal anomaly present unless coupled to four *N*=4 vector multiplets. Fradkin, Tseytlin

"non-minimal" *N*=4 (e.g. arise in twistor string) – may remove anomaly.

Double copy for conformal supergravity?

How to obtain conformal gravity and supersymmetric extensions?

Double copy ? $$\frac{n_s\tilde{n}_s}{s}+\frac{n_t\tilde{n}_t}{t}+\frac{n_u\tilde{n}_u}{u}$$

Problems: 1) simple poles → double poles
2) many strange states to account for
3) non-unitary theory (unitarity method?)

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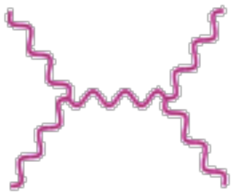
dimensional analysis: Ampl. has 4 derivatives: $M \sim \partial^4$



$$n_s \sim \partial^2$$
$$\tilde{n}_s \sim \partial^4$$

at any multiplicity:

$$n_i \sim \partial^{(m-2)}$$
$$\tilde{n}_i \sim \partial^m \sim \frac{\partial^{3(m-2)}}{\partial^{2(m-3)}}$$

Double copy for conformal gravity?

at any loop order:

$$n_i \sim \partial^{(m+2L-2)}$$

$$\tilde{n}_i \sim \partial^m \sim \frac{\partial^{3(m+2L-2)}}{\partial^{2(m+3L-3)}}$$

The latter theory is marginal in D=6: $$\frac{\tilde{n}_i}{D_i} \sim \partial^{(6-6L-m)}$$

Guess for double copy: $$\frac{n_s\tilde{n}_s}{s} + \frac{n_t\tilde{n}_t}{t} + \frac{n_u\tilde{n}_u}{u}$$

$$\mathrm{CG} = \underbrace{(\mathrm{gauge\ th})}_{\text{marginal in } D=6} \otimes \underbrace{\mathrm{YM}}_{\text{marginal in } D=4}$$

(dimensional analysis)

HJ, Nohle

Dimension-six gauge theory

Two dim-6 operators: $$\underbrace{\frac{1}{2}(D_\mu F^{\mu\nu})^2}_{\text{correct } 1/k^4 \text{ propagator but trivial S-matrix}} - \frac{1}{3} g F^3$$

correct $1/k^4$ propagator
but trivial S-matrix

$$A_3 = \langle 1\,2\rangle\langle 2\,3\rangle\langle 3\,1\rangle$$

3pt double copy is promising: (Broedel, Dixon)

$$M_3 = \frac{\langle i\, j\rangle^4}{\langle 1\,2\rangle\langle 2\,3\rangle\langle 3\,1\rangle} \times \langle 1\,2\rangle\langle 2\,3\rangle\langle 3\,1\rangle = \langle i\, j\rangle^4 \sim \phi W^2$$

amplitude violates U(1) R-symmetry <-> S-matrix is non-trivial

(3-graviton amplitude vanish <-> Gauss-Bonnet term)

Dimension-six gauge theory

Candidate theory: $\frac{1}{2}(D_\mu F^{\mu\nu})^2 - \frac{1}{3} g F^3$

4pt ampl: $A_4(1^-, 2^-, 3^+, 4^+) = \frac{\langle 1\,2\rangle^2}{\langle 3\,4\rangle^2}(u-t)$

Check color-kinematics duality (BCJ relation):

$$0 \overset{?}{=} t A_4(1,2,3,4) - u A_4(2,1,3,4) = \frac{\langle 1\,2\rangle^2}{\langle 3\,4\rangle^2} s(t-u)$$

Missing contribution: $\Delta = \frac{\langle 1\,2\rangle^2 [3\,4]^2}{s} \quad \rightarrow \quad \varphi F^2$

new dim-6 operator!

Add scalar, new operators: $\left\{ (D_\mu \varphi)^2,\ \varphi F^2,\ \varphi^3 \right\}$

Ansatz for dimension-six theory

$$\mathcal{L} = \frac{1}{2}(D_\mu F^{a\,\mu\nu})^2 - \frac{1}{3} g F^3 + \frac{1}{2}(D_\mu \varphi^\alpha)^2 + \frac{1}{2} g\, C^{\alpha a b} \varphi^\alpha F^a_{\mu\nu} F^{b\,\mu\nu} + \frac{1}{3!} g\, d^{\alpha\beta\gamma} \varphi^\alpha \varphi^\beta \varphi^\gamma$$

scalar in some real representation of gauge group (not adjoint)

unknown Clebsh-Gordan coeff: $C^{\alpha a b}$, $d^{\alpha\beta\gamma}$ (symmetric)

Assume: diagrams with internal scalars reduce to $\sim f^{abc} f^{cde} \ldots$

4pt BCJ relation → $C^{\alpha a b} C^{\alpha c d} = f^{ace} f^{edb} + f^{ade} f^{ecb}$

6pt BCJ relation → $C^{\alpha a b} d^{\alpha\beta\gamma} = (T^a)^{\beta\alpha} (T^b)^{\alpha\gamma} + C^{\beta a c} C^{\gamma c b} + (a \leftrightarrow b)$

sufficient to compute any tree amplitude with external vectors!

Which representation for scalar ? "Bi-adjoint", "auxiliary" rep.

Construction works!

$$\mathcal{L} = \frac{1}{2}(D_\mu F^{a\,\mu\nu})^2 - \frac{1}{3}gF^3 + \frac{1}{2}(D_\mu\varphi^\alpha)^2 + \frac{1}{2}g\,C^{\alpha ab}\varphi^\alpha F^a_{\mu\nu}F^{b\,\mu\nu} + \frac{1}{3!}g\,d^{\alpha\beta\gamma}\varphi^\alpha\varphi^\beta\varphi^\gamma$$

Color-kinematics duality checked up to 8 pts !
(no new Feynman vertices beyond 6pt)

Double copy with YM agrees with conformal gravity: (Berkovits, Witten)

$$M^{\rm CG}(1^-,2^-,3^+,\ldots,n^+) = \langle 1\,2\rangle^4 \prod_{i=3}^{n}\sum_{\substack{j=1\\ j\neq i}}^{n} \frac{[i\,j]\,\langle j\,q\rangle^2}{\langle i\,j\rangle\,\langle i\,q\rangle^2}$$

All-plus amplitude is non-zero → no susy extension of $\mathcal{L}$

$$A(1^+,2^+,3^+,4^+) = u\frac{[1\,2]\,[3\,4]}{\langle 1\,2\rangle\,\langle 3\,4\rangle}$$

Supersymmetry of conformal supergravity sits on the YM side:

$$\mathrm{CSG} = (\text{dim-6 theory}) \otimes (\mathcal{N} = 1,2,4\ \mathrm{SYM})$$

Generalizations and deformations

Curiously no interacting scalars are obtained from dimensional reduction

Instead add regular scalars in adjoint...

$$\mathcal{L} = \frac{1}{2}(D_\mu F^{a\,\mu\nu})^2 - \frac{1}{3} g F^3 + \frac{1}{2}(D_\mu \varphi^\alpha)^2 + \frac{1}{2} g\, C^{\alpha ab} \varphi^\alpha F^a_{\mu\nu} F^{b\,\mu\nu} + \frac{1}{3!} g\, d^{\alpha\beta\gamma} \varphi^\alpha \varphi^\beta \varphi^\gamma$$

$$\underbrace{+(D_\mu \phi^{aA})^2 + \frac{1}{2} g C^{\alpha ab} \phi^{aA} \phi^{bA} \varphi^\alpha} \qquad \underbrace{+\frac{1}{3!} g \lambda f^{abc} \tilde{f}^{ABC} \phi^{aA} \phi^{bB} \phi^{cC}}$$

color-kinematics fixes interactions

Double copy: Maxwell-Weyl gravity:

$$\sqrt{-g} f(\phi)(W^2 + F^2 + \ldots)$$

Bi-adjoint ϕ^3

Double copy: Yang-Mills-Weyl

N=4 case: Witten's twistor string!

finally, deform with dim-4 operators: → Yang-Mills-Einstein-Weyl gravity

$$-\frac{1}{4} m^2 F^2 - \frac{1}{2} m^2 (\varphi^\alpha)^2 \qquad \rightarrow \qquad \sqrt{-g} f(\phi)(m^2 R + W^2 + F^2 + \ldots)$$

Summary

- Powerful framework for constructing scattering amplitudes in various gravitational theories – well suited for multi-loop UV calculations

- Color-kinematics duality and gauge symmetry underlies consistency of construction. (Kinematic Lie algebra ubiquitous in gauge theory.)

- Constructed new dim-6 theory using color-kinematics duality - theory has several unusual features.

- Checks: Explicitly up to 8pts tree level (loop level analysis remains…)

- First construction of conformal gravity as a double copy:
  - may simplify analysis of unresolved unitarity issues (if not yet resolved)
  - may be an interesting UV regulator of Einstein (super)gravity
    (div. $\mathcal{N}$=4 SG ←→ U(1) anomaly cf. Bern, Edison, Kosower, Parra-Martinez)

- An increasing number of gravitational theories exhibit double-copy structure (some in surprising ways) – more are likely to be found!