Spacetime, QM

\+

Positive Geometry

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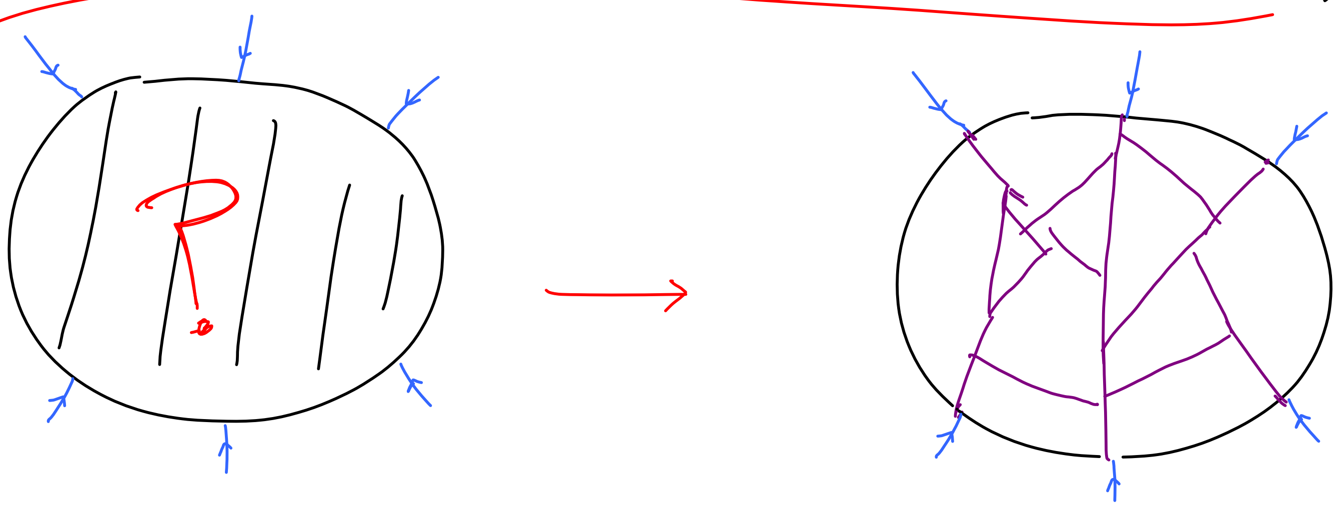
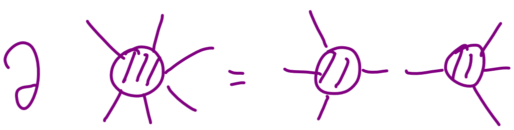
G.W. Yan, Y. Zhang

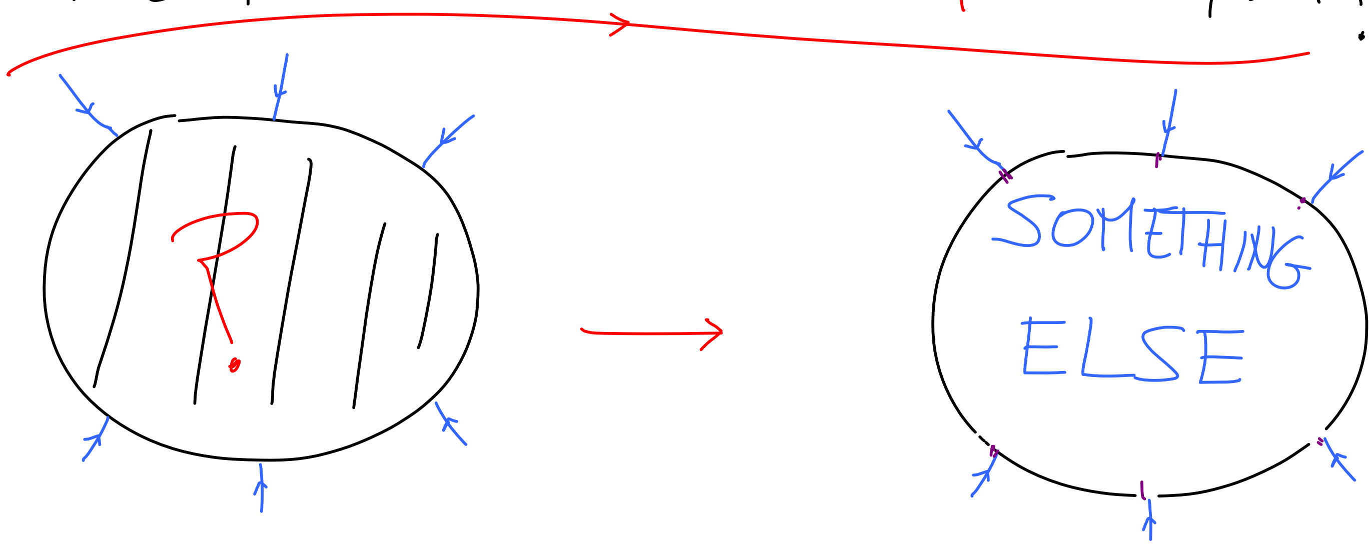
What is the Q to which A is the Answer?

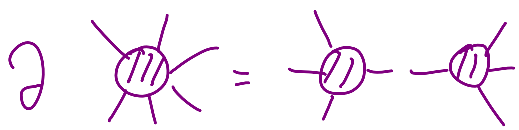




Local, Unitary Evolution
in Space time

What is the Q to which A is the Answer?





Deep Q left from 60's:

"How is Causality encoded in S-matrix?"

We ~~still~~ don't know the A, not even completely in pert. th! Not a technicality: TIME + DYNAMICS

(vs. non-p well defined EUCLIDEAN GAMES)

New Strategy: Look For

NEW PRINCIPLES, LAWS

from which CAUSAL, UNITARY

evolution — local Spacetime Physics + QM,

emerge together.

Unique Gauge Invariants → $\boxed{\text{Amplitudes as Canonical Forms of Positive Geometries}}$ ← Amplituhedron

Pert. String Amps

Scattering Eqns

Twistor String

$\boxed{\text{Worldsheets as Positive Geometries}}$

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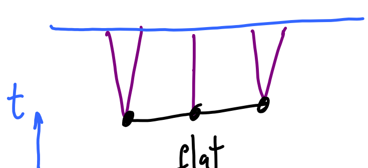
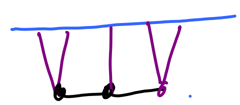
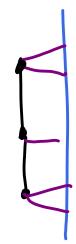


$\Psi^{flat}_{Vac}[\varphi]$



$\Psi^{dS}_{Vac}[\varphi]$



AdS Bndy Correlators ⟷ $\boxed{\text{Canonical Form of “Cosmological Polytopes”}}$

The Canvas

D-dim kinem. space

* Momenta
* Mandelstams
* $\lambda, \tilde{\lambda}$
* Twistors, Mom-twistors

⋮

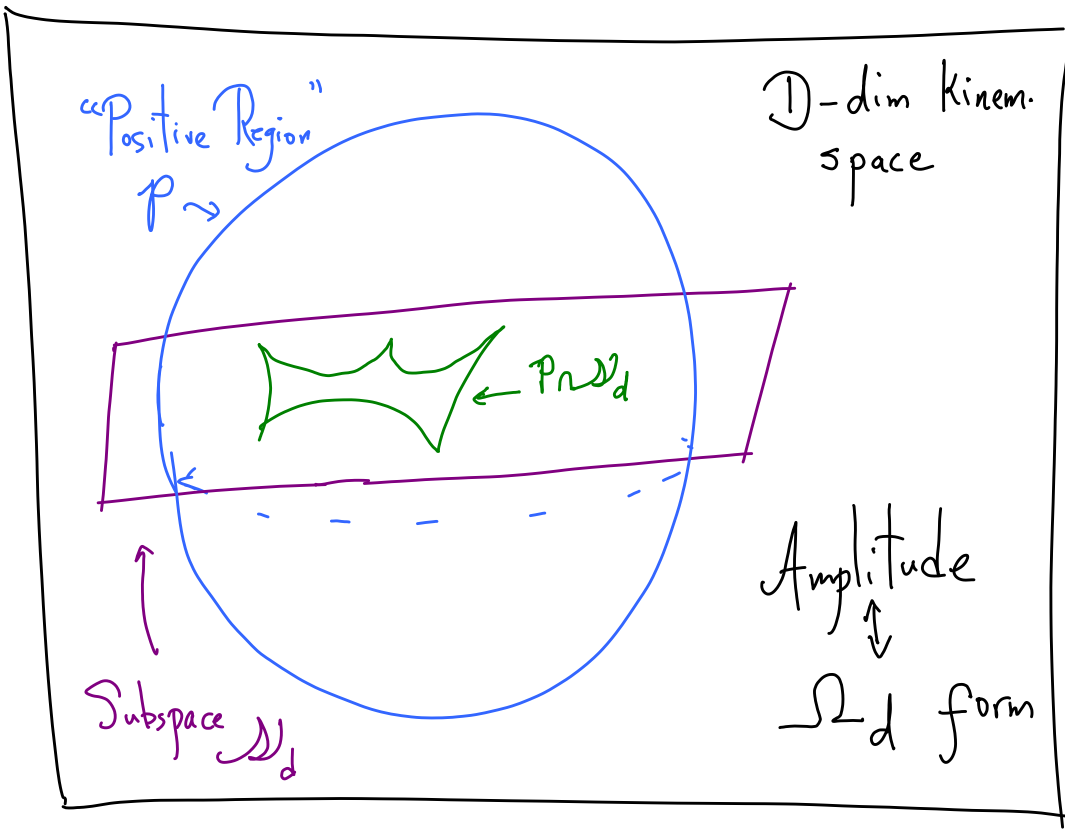
Note Unlike e.g AdS:

NO TIME

NO LOCALITY

WHAT IDEAS BREATHE PHYSICS-LIFE INTO THIS SPACE?

General Picture



Ω_d fixed thusly:

$\mathcal{S}_d$ intersects $\mathcal{P}$ in a POSITIVE GEOMETRY

Ω_d pulls back to CANONICAL FORM

Scattering Forms in Planar $\mathcal{N}=4$ SYM

Usually : $\mathcal{M}_{n,k}[Z_a^I, \eta_a^I]$ e.g. $\dfrac{(\langle Z_1Z_2Z_3Z_4\rangle \eta_5 + \cdots)^4}{\langle Z_1Z_2Z_3Z_4\rangle \cdots \langle Z_5Z_1Z_2Z_3\rangle}$

4 : Mom. Twistors

4: Grassmann η's

weight 4K

$\eta_a^I \to dZ_a^I$

Now : Just replace $\eta_a^I \longrightarrow dZ_a^I$

$$\mathcal{M}_{n,k} \longrightarrow \Omega_{4K}[Z_a]$$

$d\log \dfrac{\langle 2345\rangle}{\langle 1234\rangle} \wedge \cdots \wedge d\log \dfrac{\langle 5123\rangle}{\langle 1234\rangle}$

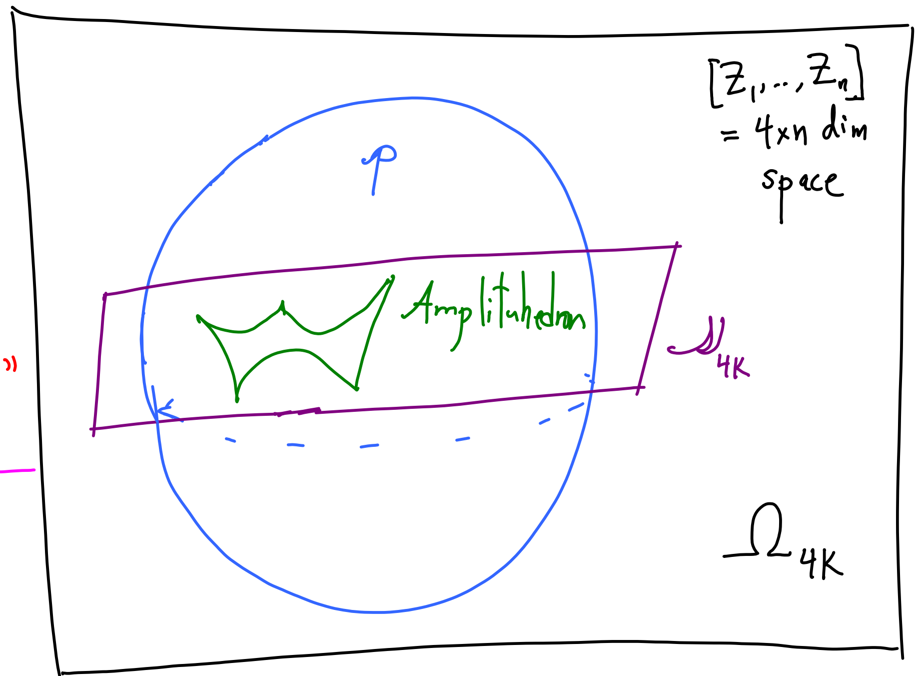
exposes underlying positive geometry!

{ Pushforward from Pos. Grassmannian, SUSY + ⋯ follows }

$\mathcal{P}$: Config. of $\{Z_1, \ldots, Z_n\}$ has fixed "binary code"

$\Rightarrow$ physical poles > 0

$+$ maximal "winding #"



In full:

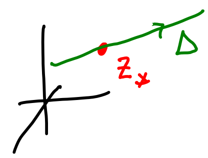
$\langle i\, i{+}1\; j\, j{+}1 \rangle > 0$

$\{\langle 1234\rangle, \ldots, \langle 123n\rangle\}$ has k sgn flips

$\langle AB_\alpha\, i\, i{+}1\rangle > 0$, $\langle AB_\alpha AB_\beta \rangle > 0$

$\{\langle AB_\alpha 12\rangle, \ldots, \langle AB_\alpha 1n\rangle\}$ has $k+2$ sgn flips

$\mathcal{S}_{4k}$: Affine subspace



$$Z^I_a = Z^I_{*a} + y^I_\alpha \Delta^\alpha_a$$

$$\begin{pmatrix} Z_* \\ \hline \Delta \end{pmatrix} \cap G_+(4+k, n)$$

Ex: MHV integrand for all n, L:

$$A_\alpha = Z_1 + x_\alpha Z_{i_\alpha} + y_\alpha Z_{i_\alpha+1}$$

$$B_\alpha = -Z_1 + z_\alpha \left(Z_{j_\alpha} + w_\alpha Z_{j_\alpha+1} \right)$$

$\langle (AB)_\alpha (AB)_\beta \rangle > 0$; that's it!

4L form .... 3L cut where $A_\alpha = Z_1 + x_\alpha Z_2$,

$$\Omega^{3L} = \sum_{\substack{N_i \\ N_3 + \cdots + N_L = L}} \prod (dx\,dy\,dz) \times$$

$$\frac{1}{x^{(3)}_1 (x^{(3)}_2 - x^{(3)}_1) \cdots (x^{(n)}_{N_n} - x^{(n)}_{N_n-1})} \left(\prod \frac{1}{z} \right) \prod_i \frac{1}{w^{(i)}_1 (w^{(i)}_2 - w^{(i)}_1) \cdots (w^{(i)}_{N_i} - w^{(i)}_{N_i-1})}$$

$$\boxed{\text{Trivial Computation of non-trivial } n \to \infty,\ L \to \infty \text{ information!}}$$

Scattering Forms in Momentum Space

$$\Omega = \mathcal{M}^{+}\, d^2\lambda + \mathcal{M}^{-}\, d^2\tilde{\lambda} + \cdots$$

[Can do even for non-SUSY QED, QCD !]

(not just planar) $\mathcal{N}=4$, Parity-symmetric on-shell superspace:

$$\mathcal{M}[\lambda_a, \tilde{\lambda}_a, \eta_a, \tilde{\eta}_a] \xrightarrow[\tilde{\eta}_a^{\dot\alpha} \to d\tilde{\lambda}_a^{\dot\alpha}]{\eta_a^{\alpha} \to d\lambda_a^{\alpha}} (Q)^4 \; \Omega_{(2n-4)}$$

$$Q \downarrow \quad \sum_a \lambda_a d\tilde{\lambda}_a = -\sum_a \tilde{\lambda}_a d\lambda_a$$

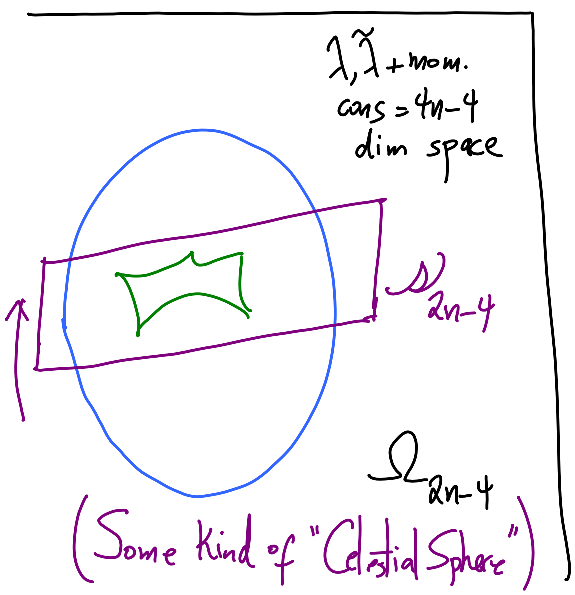
$\Omega_{(2n-4)}$: dlog form, e.g. $\Omega_{n=4} = d\log \frac{\langle 12 \rangle}{\langle 13 \rangle} \cdots d\log \frac{\langle 14 \rangle}{\langle 13 \rangle}$

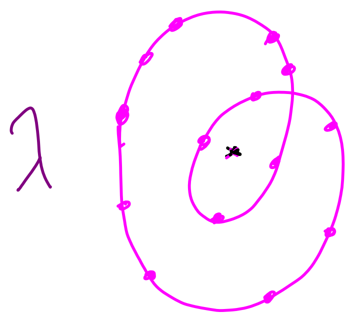
{ Again, ↑ follows from pushforward from Pos. Grassmannian }

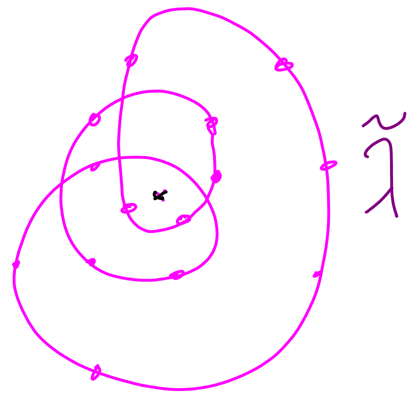
The Tree Amplituhedron Avatar in Mom. Space

$\{Z_1, \ldots, Z_n\}$ "correct binary code" $\longrightarrow$ Phys. Poles $(p_i + p_{i+1} + \cdots + p_j)^2 > 0$

+

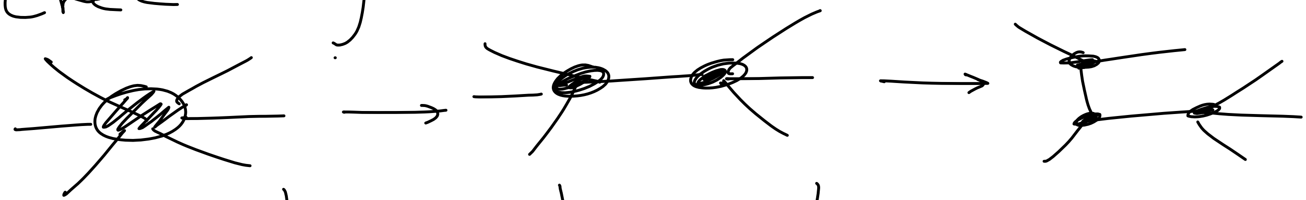






winding #	K		$K+2$
	$n-K-2$	or	$n-K-4$

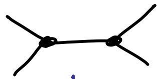
The most universal feature of tree amps is Factorization



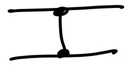
which always begged for a connection to the Associahedron itself a baby version of the simplest Positive Grassmannian $G_+(2,n)$ —

Yuntao's talk will finally realize the link!

Scattering Form



$\frac{ds}{s}$



$\frac{dt}{t}$

Well-defined on (s,t) proj space; inv. under

$[s,t] \to f(s,t)\,[s,t]$

$$\Longrightarrow \quad \Omega = \frac{ds}{s} - \frac{dt}{t} \quad \left(= d\log s/t \right)$$

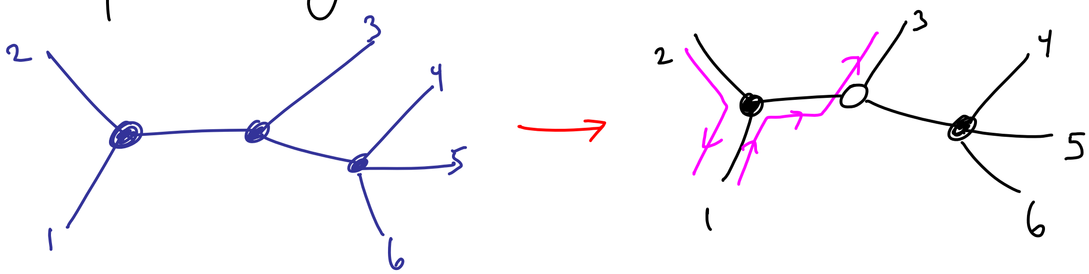
In general: $\Omega_{(n-3)}[1,\ldots,n] = \sum_{\substack{\text{planar}\\\text{cubic}\\\text{graphs}}} (\pm) \prod_{\text{prop}} \left(\frac{ds}{s}\right)$

* Yuntao's talk: Pulled back to $(n-3)$ dim subspace $s_{ij} = \text{const}_{ij}$ (non-adj), Ω is canonical form of an Associahedron in mom. space!

What's the theory? Bi Colored $(\phi_{aA})^3$

Double partial amps, $\{12\ldots n \,|\, \pi_1 \pi_2 \ldots \pi_n\}$

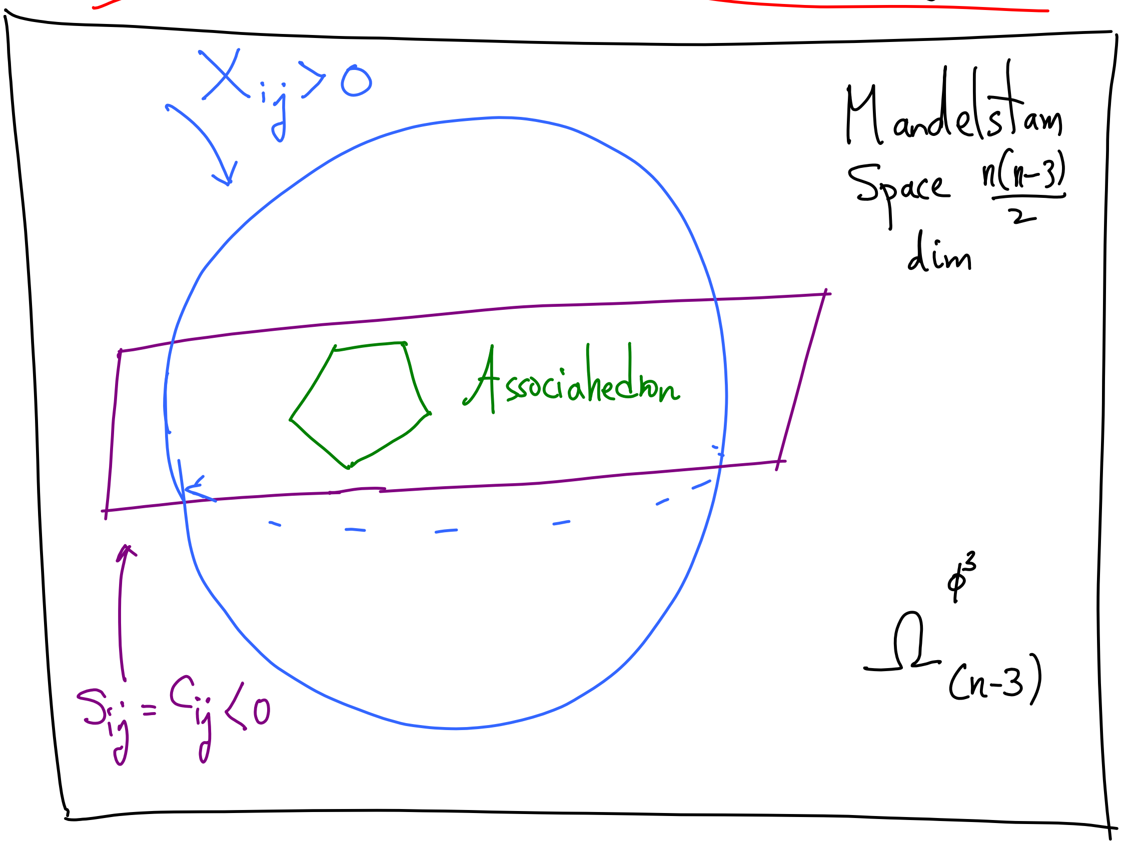
Captured by Facets of Associahedron:



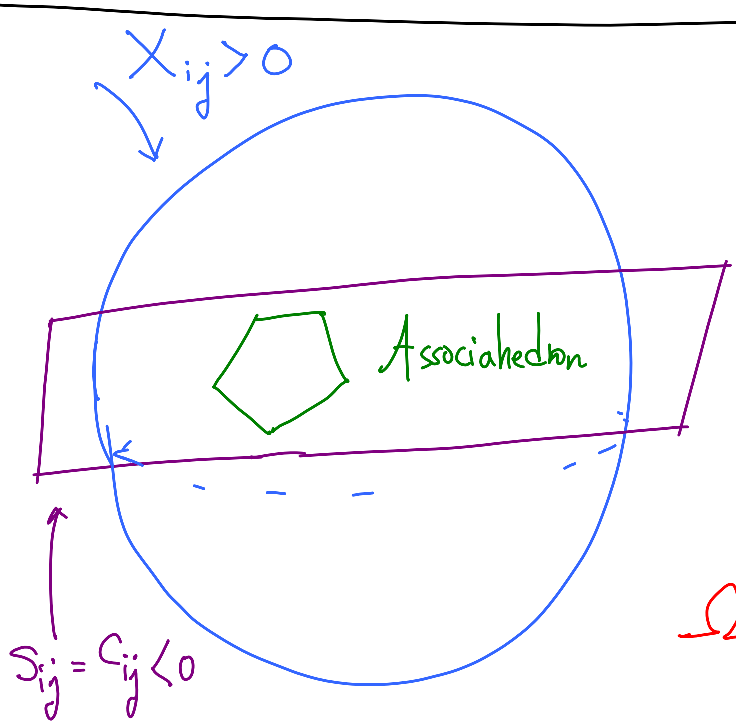
Left-Right Paths gives Ordering

Same $\Omega_{(n-3)}\Big|_{\text{zoom into facet}} = \{12\ldots n \,|\, \pi_1 \ldots \pi_n\}$

Bi-Colored $(\Phi_{aA})^3$ theory



Gluons



Mandelstam Space $\frac{n(n-3)}{2}$ dim

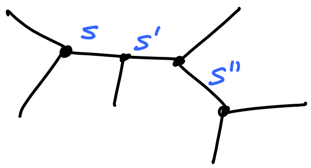
$\Omega^{YM}_{(n-3)}[s; \epsilon, p]$

* $(-1)^P$ under Perm.
* On-shell GI
* Minimal pole @ ∞

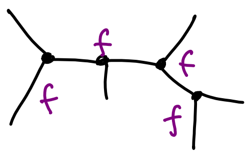
Ω^{YM} is Unique!

{non-trivial!}

Color IS Kinematics (Form)!

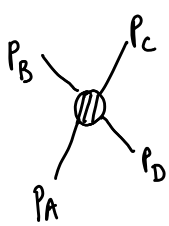


$\rho \equiv ds \wedge ds' \wedge ds''$



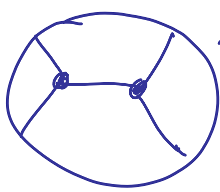
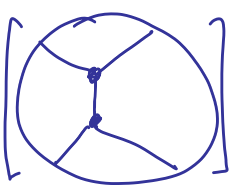
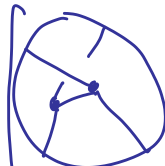
$f\,f\,f\,f$

Exactly the same Algebraic Relations

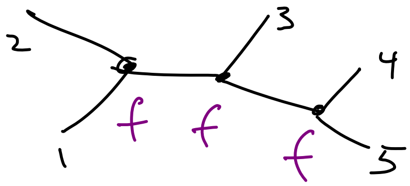
Why? 

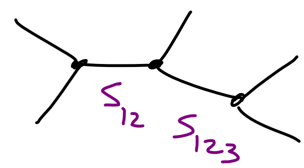
$$(p_A+p_B)^2 + (p_A+p_C)^2 + (p_A+p_D)^2 = p_A^2 + p_B^2 + p_C^2 + p_D^2$$

So $\left[d(p_A+p_B)^2 + d(p_A+p_C)^2 + d(p_A+p_D)^2\right] \wedge dp_A^2\, dp_B^2\, dp_C^2\, dp_D^2$

Hence $\rho\Big[$  $\Big] + \rho\Big[$  $\Big] + \rho\Big[$  $\Big] = 0$!

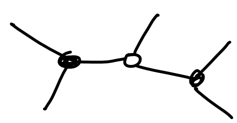
Trace Decomposition $\longleftrightarrow$ Pull back to Subspaces





$$=$$

$$\sum_{\substack{\text{colorings}\\ \text{B or W}}} \mathrm{Tr}\left(T^{\pi_1}\cdots T^{\pi_5}\right)$$



$$ds_{12}\wedge ds_{123}\Big|\quad s_{\pi_i\pi_j}=c_{ij}$$

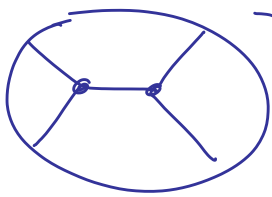
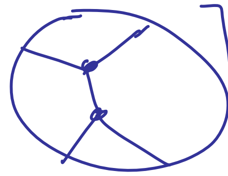
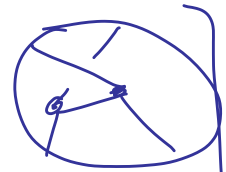
non-zero only for same orderings!

$$\Omega^{YM}_{(n-3)}\Big|_{s_{\pi_i\pi_j}=c_{ij}} = A^{YM}\left[\pi_1,\ldots,\pi_n\right]$$

Projective Avatar of Color-Kin

$$\Omega = \sum_{\substack{\text{all cubic} \\ \text{graphs}}} c_\Gamma \prod_{\text{prop}} \left(\frac{dS}{S}\right)_{\text{in } \Gamma}$$

Ω well-defined projectively $\Longrightarrow$

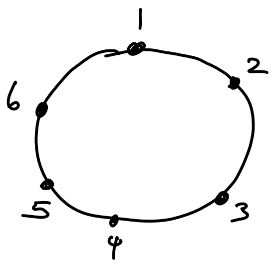
c [] + c [] + c [] $= 0$!

Also: $$\Omega^{\text{proj}} = \sum_{\text{orderings}} c(\pi)\,\Omega[\pi_1, \dots, \pi_n]$$

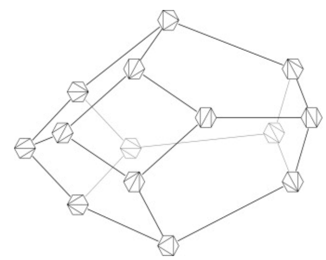
{ We have geometrized color + seen orderings emerge on subspaces.

Doors are opening to seeing Amplituhedron geometry for non-planar $\mathcal{N}=4$ SYM, for which "dlog structure" of integrand was already a strong impetus }

Worldsheet as Positive Geometry I : Associahedron



$\longleftrightarrow$



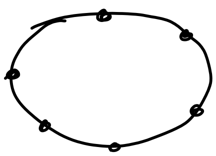
$$2\updownarrow \underbrace{\begin{pmatrix} \sigma_1 & \cdots & \sigma_n \end{pmatrix}}_{\longleftarrow\, n \,\longrightarrow} \Big/ \begin{matrix} GL(1)^n \\ \times SL(2) \end{matrix}$$

$$\langle \sigma_a \sigma_b \rangle > 0 \quad a<b$$

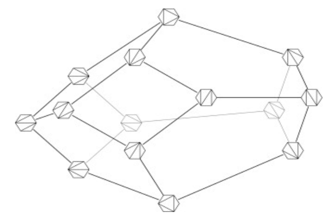
$$\Omega^{WS}_{(n-3)} = \frac{d^2\sigma_1 \,..\, d^2\sigma_n}{(12)(23)\cdots(n1)} \Big/ \begin{matrix} GL(1)^n \\ \times SL(2) \end{matrix}$$

Pushing Forward From Worldsheet [Yuntao]

WS



Mandelstam

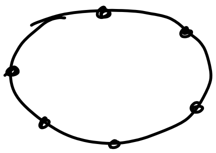


CHY Scattering Eqns!

$$\sum_{b\neq a} s_{ab} \frac{\sigma_a^{\alpha} \sigma_b^{\beta}}{(\sigma_a \sigma_b)} = 0$$

On $(n-3)$ mandelstam subspace $\rightarrow$ solve for s's in terms of σ's $\rightarrow$ Mandelstam associahedron is literally image of WS! Conversely: single real solution for σ's in WS assoc, iff s_{ab} live in some Mandelstam assoc.

Pushing Forward From Worldsheet [Yuntao]



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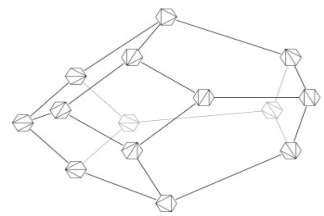
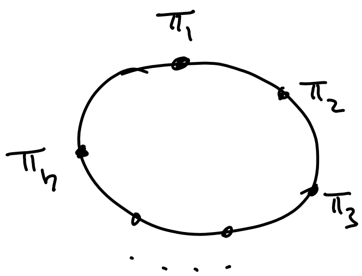


$$\Omega^{WS}_{(n-3)} \xrightarrow{\text{Push forward}} \Omega^{(\phi_{aA})^3}_{(n-3)}$$

= CHY formula

Gluons from Worldsheet

$$\tilde{\Omega}^{WS}_{(n-3)} = \sum_{\pi} \Omega^{WS}_{(n-3)}[\pi_1, \ldots, \pi_n]\, N(\epsilon, p)$$



Push forward on SE $\Big\downarrow$ Demand minimal # p + On-shell Gauge Inv.

$\tilde{\Omega}^{WS}_{(n-3)}$ is Unique! + Pushes forward to $\Omega^{YM}_{(n-3)}$

$\tilde{\Omega}^{WS} = \mathrm{Pf}[M]$ big + mysterious part of CHY magic $\longleftrightarrow$ But locked by G.I.

Assoc. is a baby version of simplest positive Grassmannian $G_+(2,n)$:

Assoc

$$(\sigma_1 \cdots \sigma_n)/GL(1)^n \times SL(2), \quad \underset{\text{dim}}{(n-3)}$$

$$(\sigma_a \, \sigma_b) > 0 \quad a < b$$

$$\Omega_{(n-3)} = \frac{d^2\sigma_1 \cdots d^2\sigma_n}{(12) \cdots (n1)} \Big/ GL(1)^n \times SL(2)$$

$G_+(2,n)$

$$(\sigma_1 \cdots \sigma_n)/GL(2), \quad \underset{\text{dim}}{(2n-4)}$$

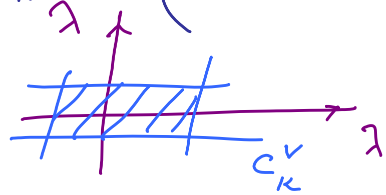
$$(\sigma_a \, \sigma_b) > 0 \quad a < b$$

$$\Omega_{(2n-4)} = \frac{d^2\sigma_1 \cdots d^2\sigma_n}{(12) \cdots (n1)} \Big/ GL(2)$$

Worldsheet as Pos. Geometry II : $G_+(2,n)$

In 4d, CHY + Twistor Strings become the same, WS becomes $G_+(2,n)$

$$\sigma = \begin{pmatrix} a \\ b \end{pmatrix} \xrightarrow{\text{Veronese}} \sigma_V = \begin{pmatrix} a^K \\ a^{K-1} b \\ \vdots \\ b^K \end{pmatrix} \; ; \quad C_K^{V} \cdot \tilde{\lambda} = \begin{pmatrix} \sigma_V^{(1)} \cdots & \sigma_V^{(n)} \end{pmatrix}$$



"RSV Eqns"

$$\Omega^{WS}_{(2n-4)} = \frac{d^2\sigma_1 \cdots d^2\sigma_n}{(12)\cdots(n1)} \Big/ GL(2) \xrightarrow{\text{Push Forward}} \Omega^{SYM}_{(2n-4)} [\lambda, \tilde{\lambda}]$$

Twistor String WS $\xrightarrow[\text{Pushforward}]{\text{RSV}}$ Mom. Space Amplituhedron!

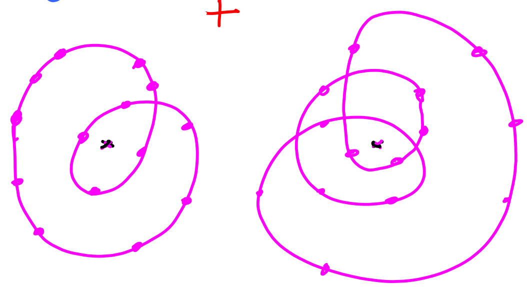
Strong Evidence:

RSV equations have single, positive soln in WS $G_+(2,n)$

iff

Phys. Poles $(p_i+p_{i+1}+\dots+p_j)^2>0$

+

λ  $\tilde{\lambda}$

winding #	k	$k+2$
		or
	$n-k-2$	$n-k-4$

Q: Why pushforward Ω^{WS}? It's a form, why not integrate it inside WS positive geometry?

A: It is logarithmically divergent!

Canonical way of dealing w/ this [c.f. Gelfand et. al. gen. hypergeometric functions]

$$\Omega^{WS}_{\alpha'} = \Omega^{WS} \times \underbrace{\prod_{(ab)} (\sigma_a \sigma_b)^{\alpha' s_{ab}}}_{\text{Koba-Nielsen}}$$

$\left\{ \sum_{b \neq a} s_{ab} = 0 \text{ for } SL(2) \text{ weight} \right\}$

So e.g. take $\tilde{\Omega}^{ws}$, whose "Pfaffian" structure was completely fixed by G.I. on support of Scatt Eqn.

Then

$$(\alpha')^{n-3} \int_{\substack{\text{Assoc} \\ (12..n)}} \tilde{\Omega}^{ws}_{\alpha'} = \text{Open Superstring Gluon Amp!}$$

Units

Unique Gauge Invariants

Amplitudes as Canonical Forms of Positive Geometries

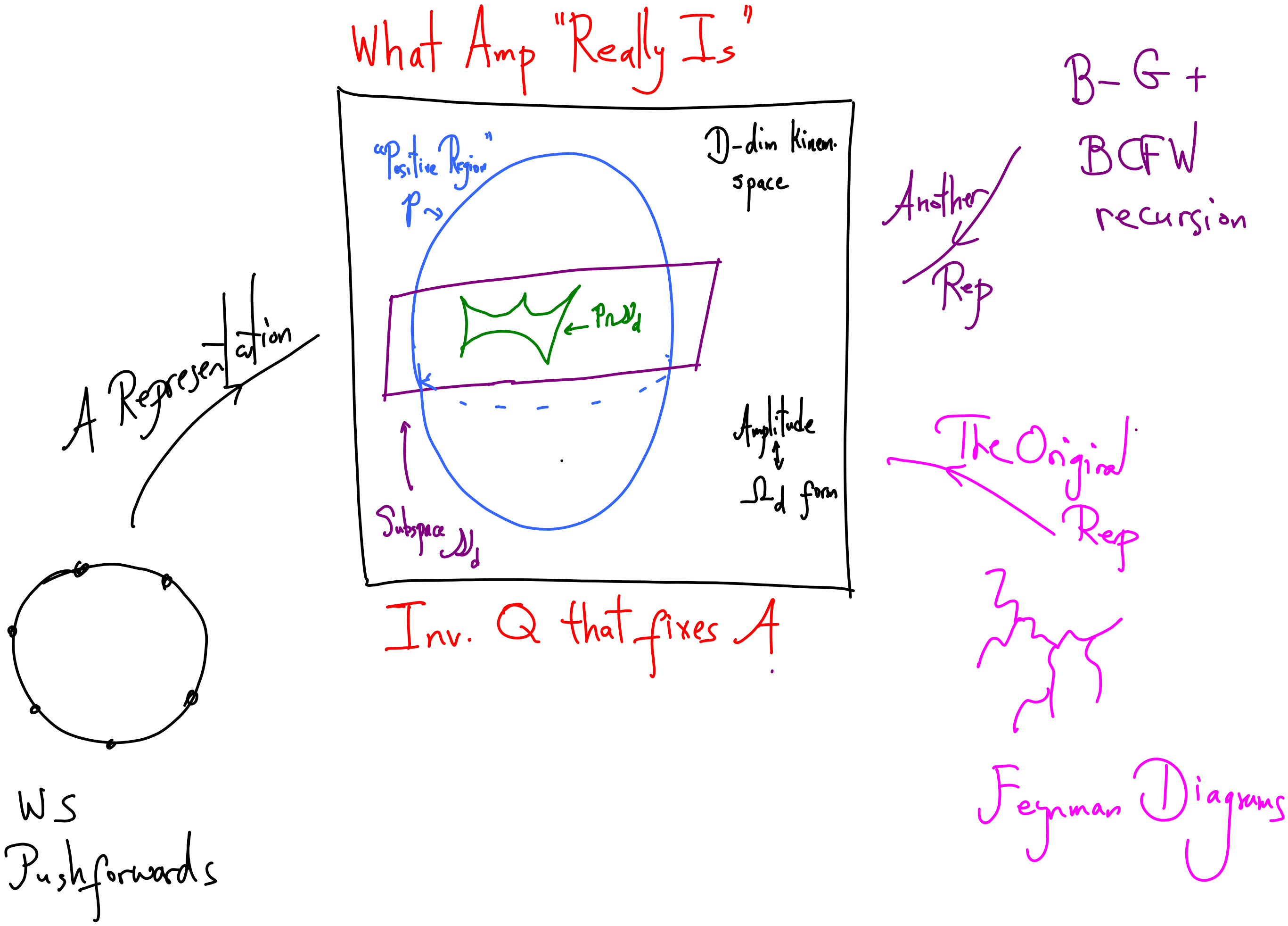
Amplituhedron

Pert. String Amps

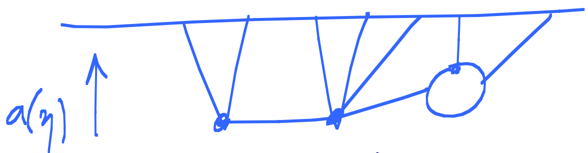
Scattering Eqns

Twistor String

Worldsheets as Positive Geometries


What Amp "Really Is"
"Positive Region"
P
D-dim kinem. space
$P \cap S_d$
Subspace S_d
Amplitude ↔ Ω_d form
Inv. Q that fixes A
A Representation
WS Pushforwards
Another Rep
B-G + BCFW recursion
The Original Rep
Feynman Diagrams

Toy Model for Cosm. WF of U

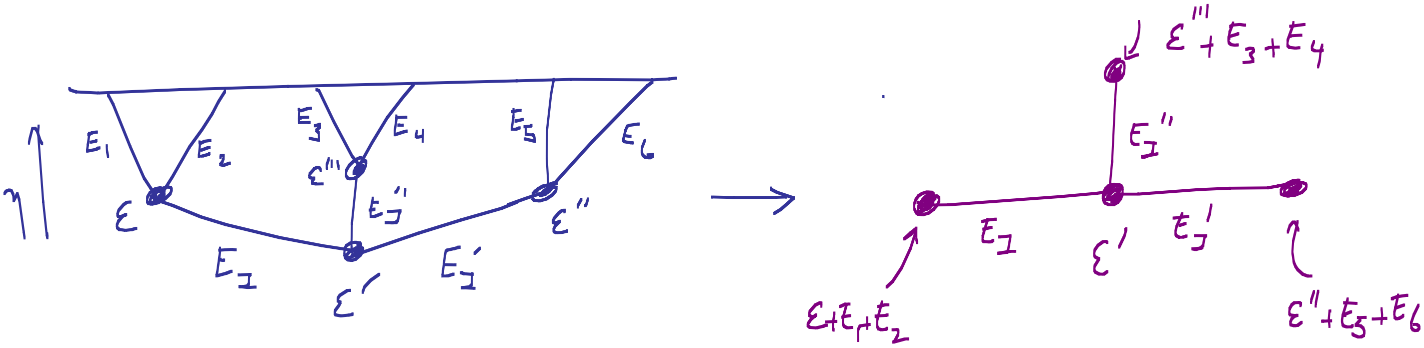


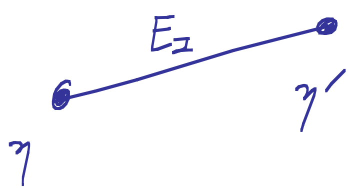
conf. coupled scalar, non-conf. polynomial interactions

$\Psi[\phi]$

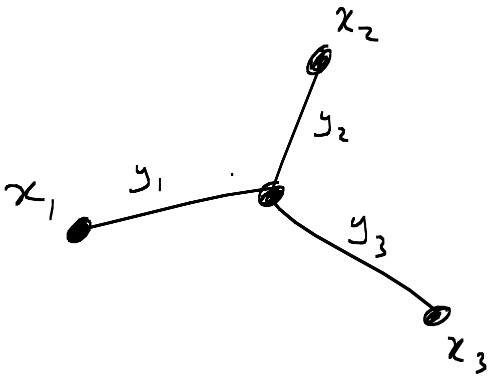
$$S = \int d\eta \, (\partial\phi)^2 + g_3(\eta)\,\phi^3 + g_4(\eta)\,\phi^4 + \cdots$$

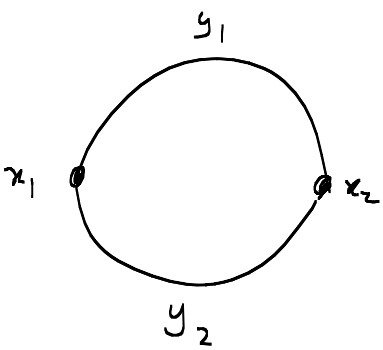
$$g_i(\eta) = \int d\varepsilon \; e^{i\varepsilon\eta}\,\tilde{g}(\varepsilon)$$





$$G_B[\eta,\eta'] = \frac{1}{E_I}\left[\underbrace{e^{iE_I(\eta'-\eta)}\,\theta(\eta-\eta') + (\eta\leftrightarrow\eta')}_{\text{Feynman Prop}} \underbrace{- e^{iE_I(\eta+\eta')}}_{\substack{\text{BC @ } \eta=0 \\ \text{to compute } \psi}} \right] \quad \text{3 terms}$$





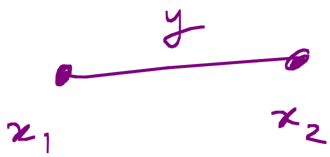
For any graph G:

$$\psi_G[x_i; y_I] = \int_{-\infty}^{0} d\eta_i \, e^{i\eta_i x_i} \times \prod_{\text{edges}} G_B[\eta_i, \eta_j, y_{ij}]$$

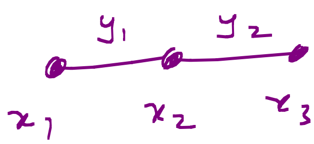
"Time evolution" representation

$\mathcal{O}(3^L)$ terms

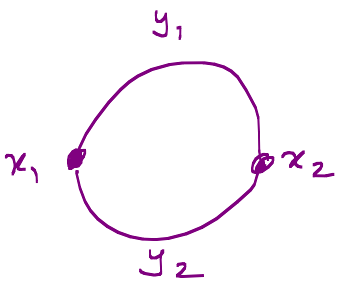
$$\frac{1}{x}$$



$$\frac{1}{y(x_1+y)(x_1+x_2)} + \frac{1}{y(x_2+y)(x_1+x_2)} - \frac{1}{y(x_1+y)(x_2+y)} = \frac{1}{(x_1+x_2)(x_1+y)(x_2+y)}$$



$$9 \text{ terms} = \frac{(x_1+x_3+2x_2+y_1+y_2)}{(x_1+y_1)(x_3+y_2)(x_2+y_1+y_2)(x_1+x_2+y_2)(x_3+x_2+y_1)}$$



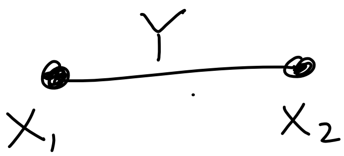
$$7 \text{ terms} = \frac{(x_1+x_2+y_1+y_2)}{(x_1+x_2+2y_1)(x_1+x_2+2y_2)(x_1+y_1+y_2)(x_2+y_1+y_2)}$$

* In Flat Space: the rational functions give $\Psi_{vac}[\varphi]$

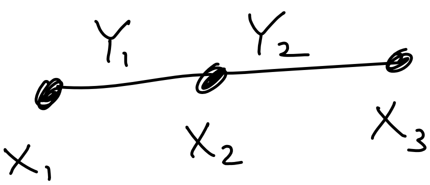
* In dS, further int. gives $\Psi_{dS}[\varphi]$.

(In AdS, ∂ correlators)

* We get "Cosmological POLYLOGS" even @ tree level!



$$S: \quad \frac{X_1+Y}{X_1-Y} \otimes \frac{X_2+Y}{X_2+X_1} + 1 \leftrightarrow 2$$



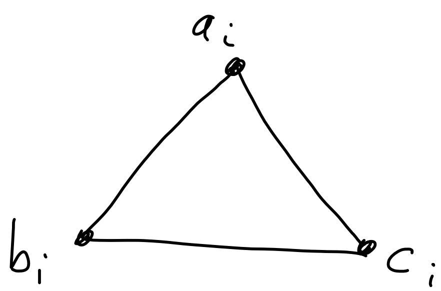
$S:$

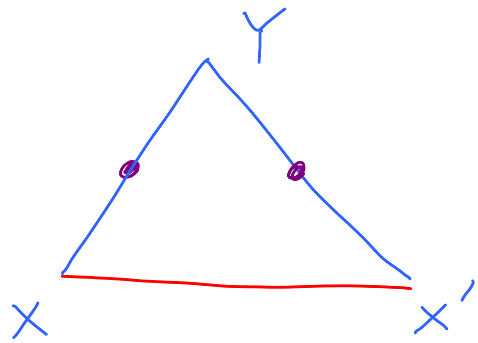
{{-1, X1 - Y1, X2 + Y1 + Y2, X2 + X3 + Y1}, {1, X1 - Y1, X1 + X2 + Y2, X1 + X2 + X3}, {1, X1 - Y1, X2 + Y1 - Y2, X2 + X3 + Y1}, {-1, X1 - Y1, X1 + X2 - Y2, X1 + X2 + X3}, {1, X1 - Y1, X1 + X2 - Y2, X3 + Y2}, {-1, X1 - Y1, X2 + Y1 - Y2, X3 + Y2}, {1, X1 - Y1, X2 + Y1 + Y2, X3 + Y2}, {-1, X1 - Y1, X1 + X2 + Y2, X3 + Y2}, {1, X1 + Y1, X1 + X2 - Y2, X1 + X2 + X3}, {-1, X1 + Y1, X1 + X2 + Y2, X1 + X2 + X3}, {-1, X1 + Y1, X1 + X2 - Y2, X3 + Y2}, {1, X1 + Y1, X1 + X2 + Y2, X3 + Y2}, {-1, X1 + Y1, X2 + Y1 - Y2, X2 + X3 + Y1}, {1, X1 + Y1, X2 + Y1 + Y2, X2 + X3 + Y1}, {1, X1 + Y1, X2 + Y1 - Y2, X3 + Y2}, {-1, X1 + Y1, X2 + Y1 + Y2, X3 + Y2}, {1, X1 - Y1, X3 - Y2, X2 + X3 + Y1}, {-1, X1 - Y1, X3 - Y2, X1 + X2 + X3}, {1, X1 - Y1, X3 - Y2, X1 + X2 + Y2}, {-1, X1 - Y1, X3 - Y2, X2 + Y1 + Y2}, {-1, X1 - Y1, X3 + Y2,

+ . . .

Already Pretty Complicated....

Cosmological Polytopes



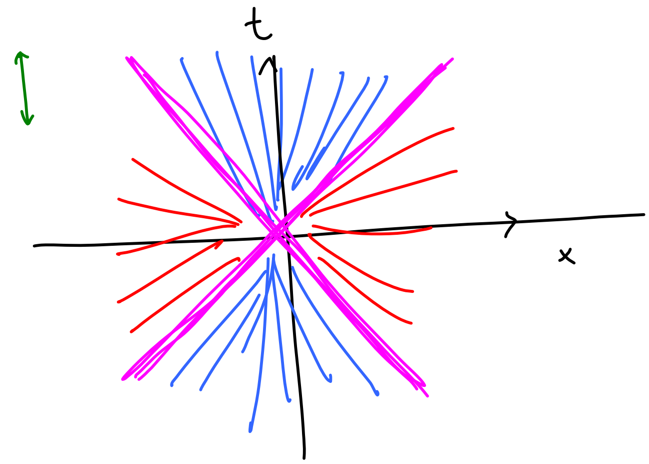


Collection of Triangles
Allowed to intersect
on 2 of 3 sides

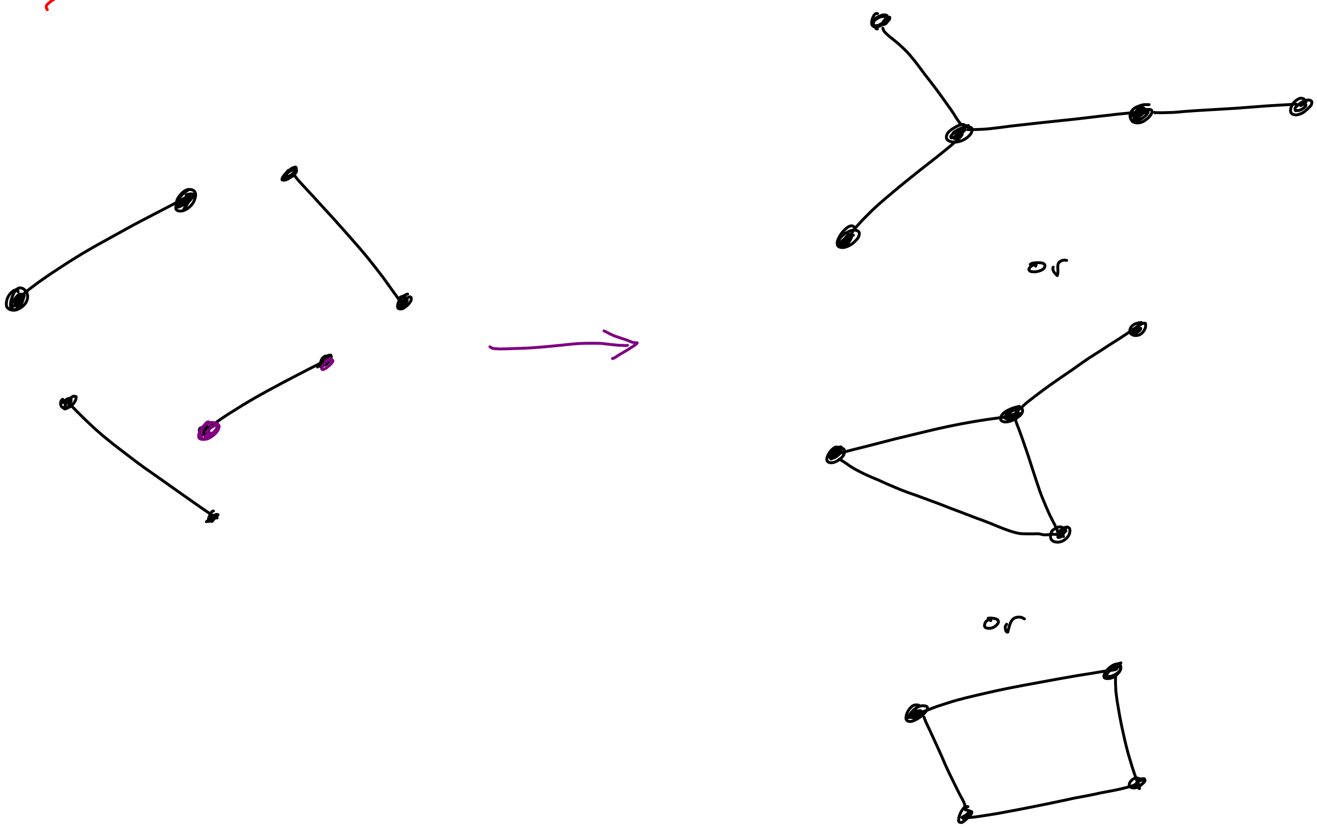
$$a_i + b_i = a_j + b_j$$

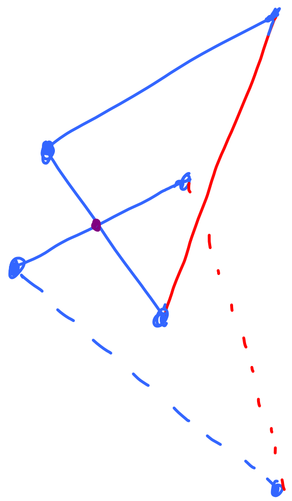
$\frac{\text{and}}{\text{or}}$ $a_i + c_i = a_j + c_j$

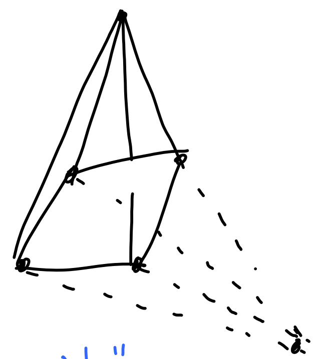
NOT $b_i + c_i = b_j + c_j$



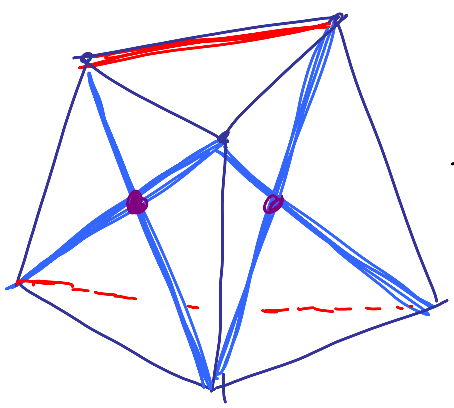
Polytopes Associated w/ Graphs

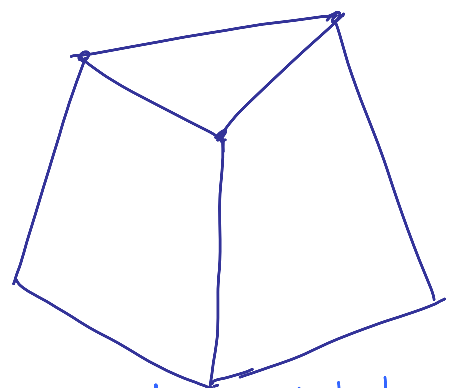




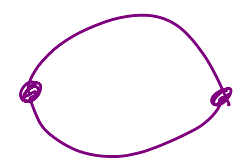


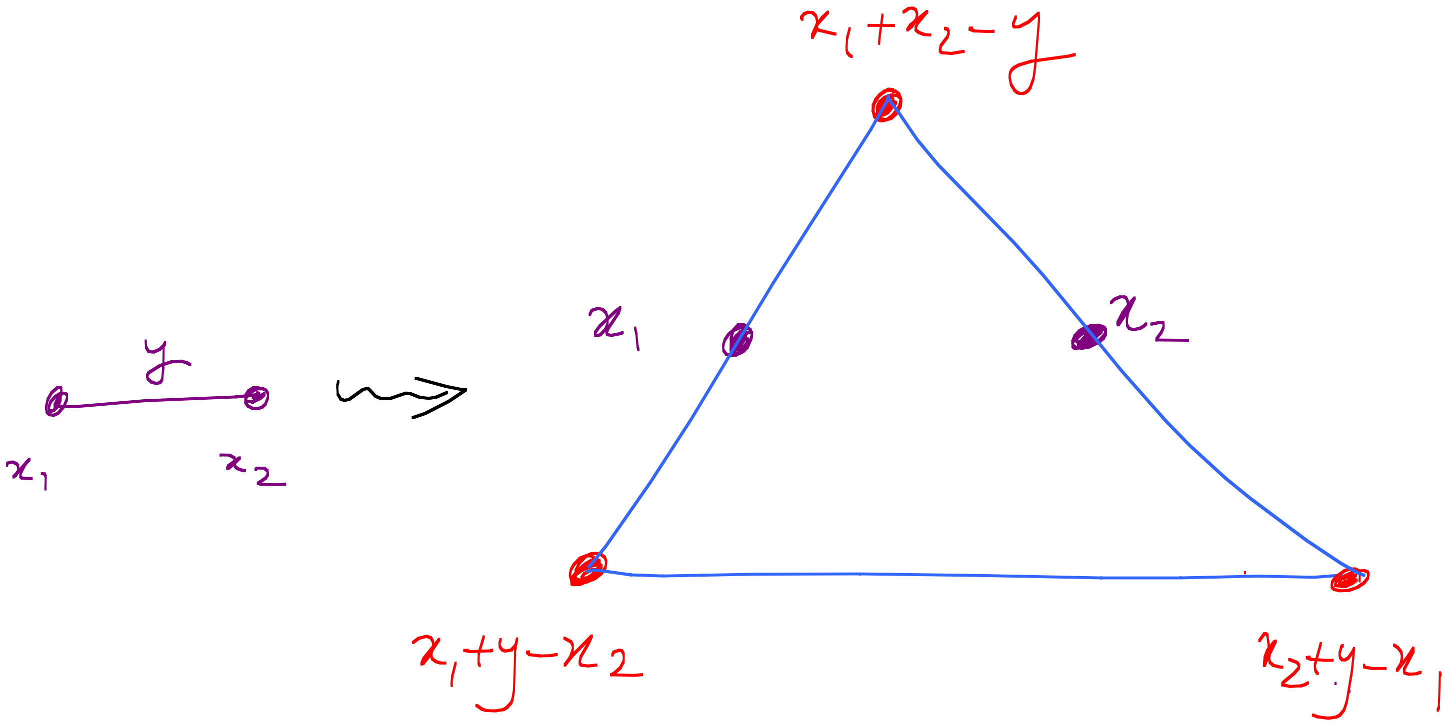
"Double" square pyramid



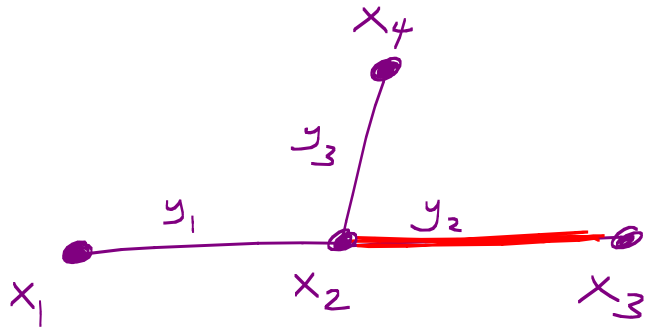


Truncated Tetrahedron



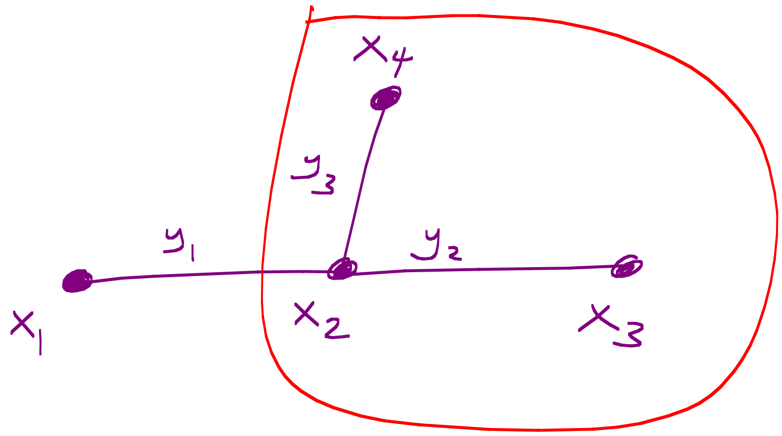


y
x_1
x_2
$x_1 + x_2 - y$
x_1
x_2
$x_1 + y - x_2$
$x_2 + y - x_1$



$$\left\{\begin{array}{l} x_2 + x_3 - y_2 \\ x_2 + y_2 - x_3 \\ x_3 + y_2 - x_2 \end{array}\right.$$

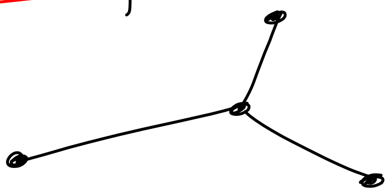
Vertices of P: Associated w/ Edges of G



$\tilde{x}_2 + \tilde{x}_3 + \tilde{x}_4 + \tilde{y}_1$

Faces of P : Associated with connected subgraphs of G

Wavefunction = Canonical Form!



$$X = \sum x X + y Y$$

$$\Omega[X, \text{Cosm. Polytope}[G]] = \left(\prod dx\, dy\right) \times \psi_G[x, y]$$

* Natural triangulations give → Time Int. Rep
  → Old-fash. pert. th.
* That ψ_G satisfies Sch. Eqn → Emergent from Polytope!
* [New triangulations → more efficient expressions for ψ_G]

Symbol of Cosmological Polylog =
Record of Projective Paths in Cosm. Polytope!

$\{ V_1, V_2, \dots, V_{V+E} \}$ s.t.

$$V_1 \longrightarrow V_2 \xrightarrow[\pi_{V_1}]{} V_3 \xrightarrow[\pi_{V_2}]{}$$

All connected by edges

$$\{ \tilde{W}_1, \dots, \tilde{W}_{V+E} ; V_1, \dots, V_{V+E} \}$$

$$= \text{Matrix}\left[\tilde{W}_I \cdot V_J \right]$$

$$S = \sum_{\substack{\{\tilde{x}_{i_1}, \dots, \tilde{x}_{i_V}\} \\ \{V_1, \dots, V_{V+E}\}}} (\pm) \prod \begin{array}{c} \begin{array}{cccccc} \tilde{y}_1 \cdots & \tilde{y}_E & \tilde{x}_{i_1} \cdots & \tilde{x}_{i_V} \end{array} \\ \left(\begin{array}{cccc} & & \cdots & \\ & & & \\ & & & \\ & & & \end{array} \right) \end{array} \begin{array}{c} V_1 \\ \vdots \\ V_{V+E} \end{array}$$

symbol = cons. nested minors

Huge # of open issues! And likely many twists + turns ahead.

But we are seeing concrete + meaningful Unification of different threads + seeing same general structure beyond planar $\mathcal{N}=4$ SYM!

The Canvas

D-dim kinem. space

⋮

* Momenta
* Mandelstams
* $\lambda, \tilde{\lambda}$
* Twistors, Mom-twistors

⋮

WHAT BREATHES PHYSICS LIFE INTO THIS SPACE?

In our examples:

Combinatorics $\leftrightarrow$ Pos Geometry $\leftrightarrow$ Can. Form