

$$\begin{aligned}
\mathcal{R}_{\text{BPS}}^{(2)} &:= \frac{3}{2} \text{Li}_4(u) - \frac{3}{4} \text{Li}_4\left(-\frac{uv}{w}\right) + \frac{3}{2} \log(w) \text{Li}_3\left(-\frac{u}{v}\right) - \frac{1}{16} \log^2(u) \log^2(v) \\
&\quad - \frac{\log^2(u)}{32} \left[\log^2(u) - 4 \log(v) \log(w) \right] - \frac{\zeta_2}{8} \log(u) [5 \log(u) - 2 \log(v)] \\
&\quad - \frac{\zeta_3}{2} \log(u) - \frac{7}{16} \zeta_4 + \text{perms}(u, v, w)
\end{aligned}$$