

$$\mathcal{S}_{\text{BP}_S}^{(2)} = u \otimes v \otimes \left[\frac{u}{w} \otimes_S \frac{v}{w} \right] + \frac{1}{2} u \otimes \frac{u}{(1-u)^3} \otimes \frac{v}{w} \otimes \frac{v}{w} + \text{perms}$$