Infrared Divergences in QED & Quantum Gravity

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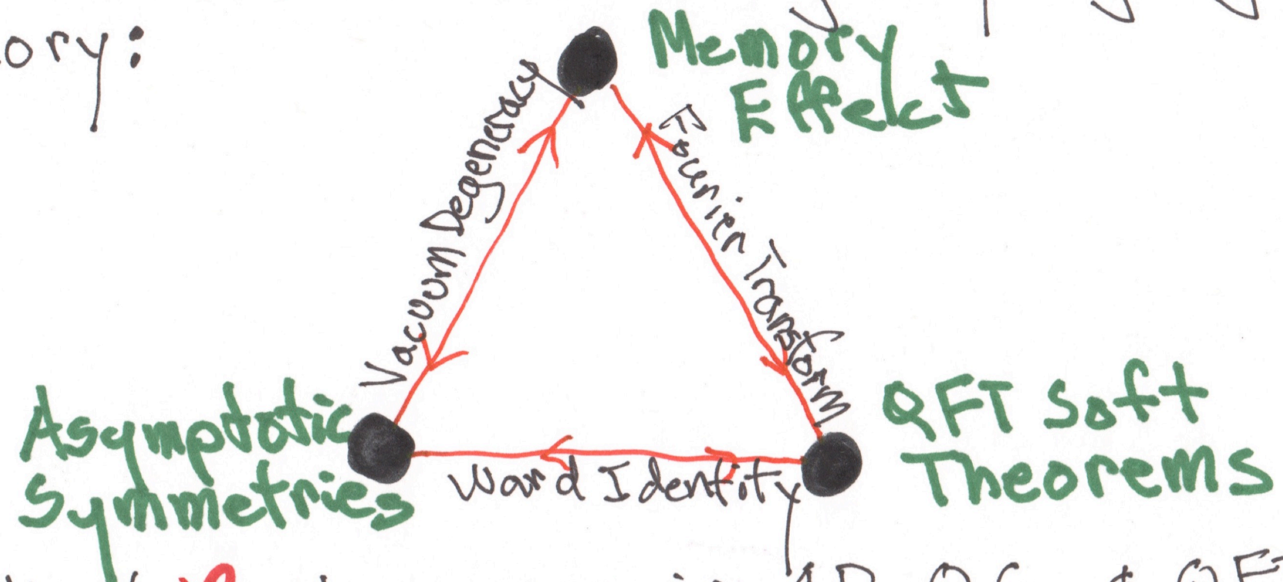
The last few years have seen new insights into the **IR** structure of gravity & gauge theory:



A salient **IR** phenomenon in 4D QG & QFT is the appearance of **IR divergences**. Do recent developments shed any light on their origin?

Yes!

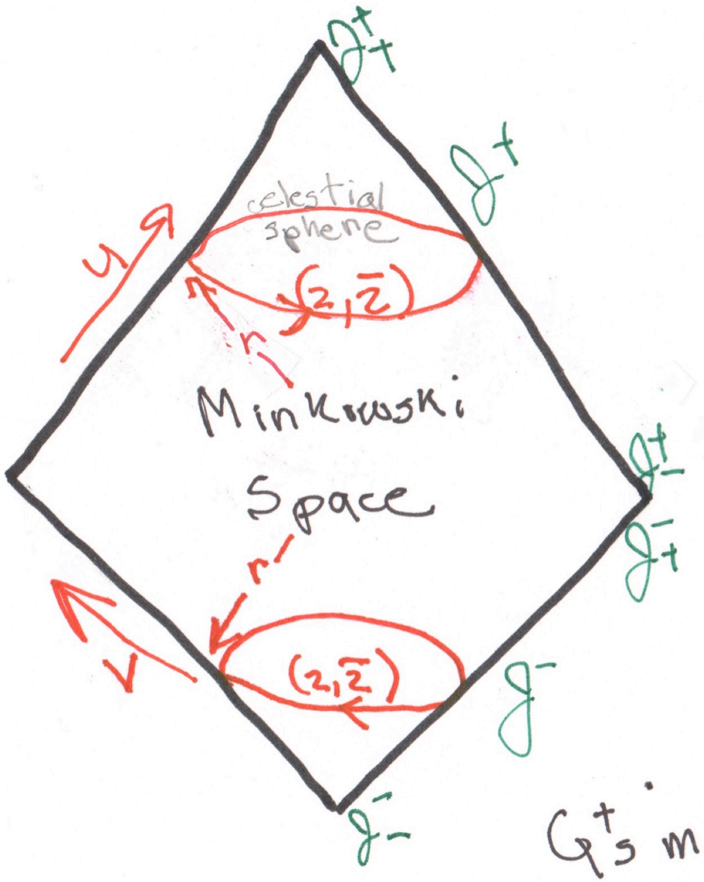
Kapec, Hawking, Perry, Baclariu, Zhiboedov, ..

Outline

1. Rederive 1970 Faddeev-Kulish (Chung-Kibble....) IR finite S-matrix from a modern perspective.

2. Use insight to motivate 3 conjectures in
   (i) QED
   (ii) Non-abelian gauge theory
   (iii) Black hole information

Lightning review: ∞ of conserved charges in QED



$$Q^+(z,\bar{z}) = F_{ru}(z,\bar{z})\Big|_{\mathscr{I}^+_-}$$

← $\frac{1}{r^2}$ component of radial electric field

antipodal

$$\doteq Q^-(z,\bar{z}) = F_{rv}(z,\bar{z})\Big|_{\mathscr{I}^-_+}$$

Integrating by parts

$$Q^+ = Q^+_S + Q^+_H$$

$$Q^+_S = \int_{-\infty}^{\infty} du \left(D^{z} F_{uz} + D^{\bar{z}} F_{u\bar{z}} \right)$$

← soft photon

$$Q^+_H = e^2 \int_{-\infty}^{\infty} du \, j_u$$

← charge matter current

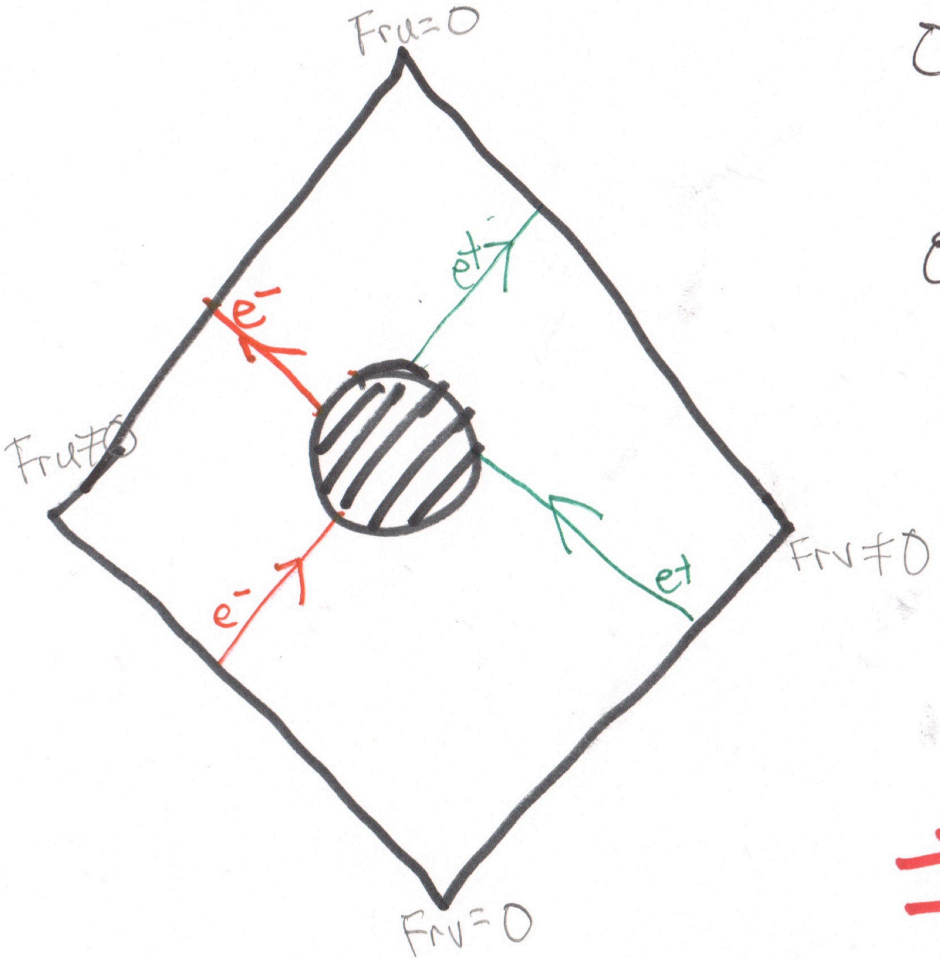
Q^+_S measures shift in flat connection
= shift between ∞-degenerate vacua

Ditto on $\mathscr{I}^-$.

$Q^+ = Q^-$ ⟺ soft photon theorem

Bhaba scattering $e^+e^- \to e^+e^-$

take $m_e \to 0$ for simplicity



Constraints

On $\mathcal{I}^+$

$$\partial_u F_{ru} + D^z F_{uz} + D^{\bar z} F_{u\bar z} = -e^2 j_u$$

($D^z F_{uz} \to 0$, $D^{\bar z} F_{u\bar z} \to 0$: no radiative modes)

On $\mathcal{I}^-$

$$\partial_v F_{rv} - D^z F_{vz} - D^{\bar z} F_{v\bar z} = -e^2 j_v$$

($D^z F_{vz} \to 0$, $D^{\bar z} F_{v\bar z} \to 0$)

$$\Rightarrow F_{ru}(z,\bar z)\big|_{\mathcal{I}^+_-} \neq F_{rv}(z,\bar z)\big|_{\mathcal{I}^-_+}$$

$$\Rightarrow a(e^+e^- \to e^+e^-) = 0$$

This is well known, and usually attributed to IR divergences:

$$= e^{-\infty} = 0.$$

Here we see the role of IR divergences as a clever trick by QFT to set to zero conservation-law-violating amplitudes.

No 'real' IR divergences.

But we need to compute something other than zero. Lets solve the constraints differently:

$$\cancel{\partial_u F_{ru}}^{\to 0} + D^z F_{uz} + D^{\bar z} F_{u\bar z} = -e^2 j_u = -e^2 \delta(u-u_0)\delta^2(z-z_0)$$

Radiative solution: $A_z = -\dfrac{\Theta(u-u_0)e^2}{4\pi(z-z_0)}$ has pole for $\omega \to 0$

Note $A_z|_{\mathcal{I}^+_+} - A_z|_{\mathcal{I}^+_-} \neq 0 \Rightarrow$ vacuum shift

Quantum state

$$|\psi\rangle_{dressed} = e^{-\frac{ie^2}{2\pi}\int \frac{d^2z}{(z-z_0)} A_{\bar z}(u_0,z,\bar z) + h.c.} |u_0, z_0\rangle$$

obeys $[D^z F_{uz} + D^{\bar z} F_{u\bar z} + e^2 j_u]|\psi\rangle_{dressed} = 0$ EXCEPT ZERO MODE ABSENT IN BAHABA

Coulomb field is 'shielded'. Dressing all particles $\Rightarrow F_{ru}|_{\mathcal{I}^+_-} = F_{rv}|_{\mathcal{I}^-_+} = 0$ & conservation laws trivially satisfied.

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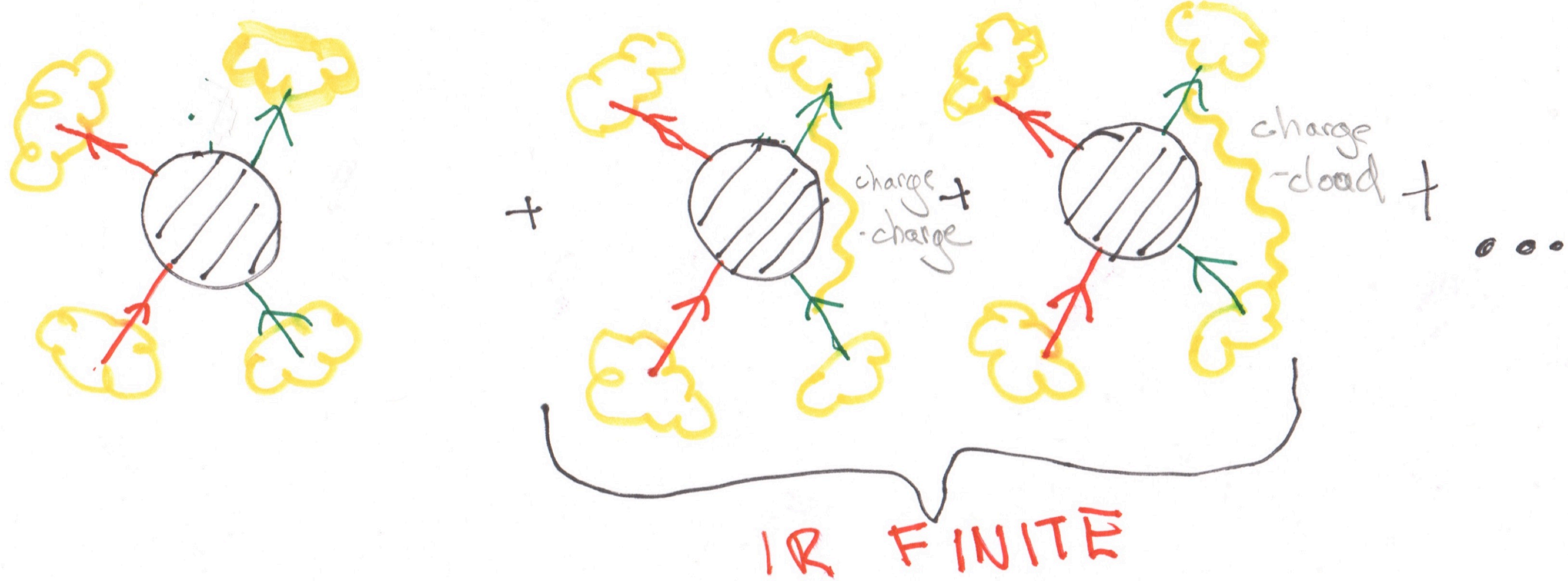
Scattering of these dressed state is IR finite!!! (Faddeev & Kulish 1970)



Reinterpretation of FK dressing: solves constraints radiatively, ensures conservation laws, implements required vacuum shift.

N.B. Soft clouds do not really 'surround' asymptotic hard particle.

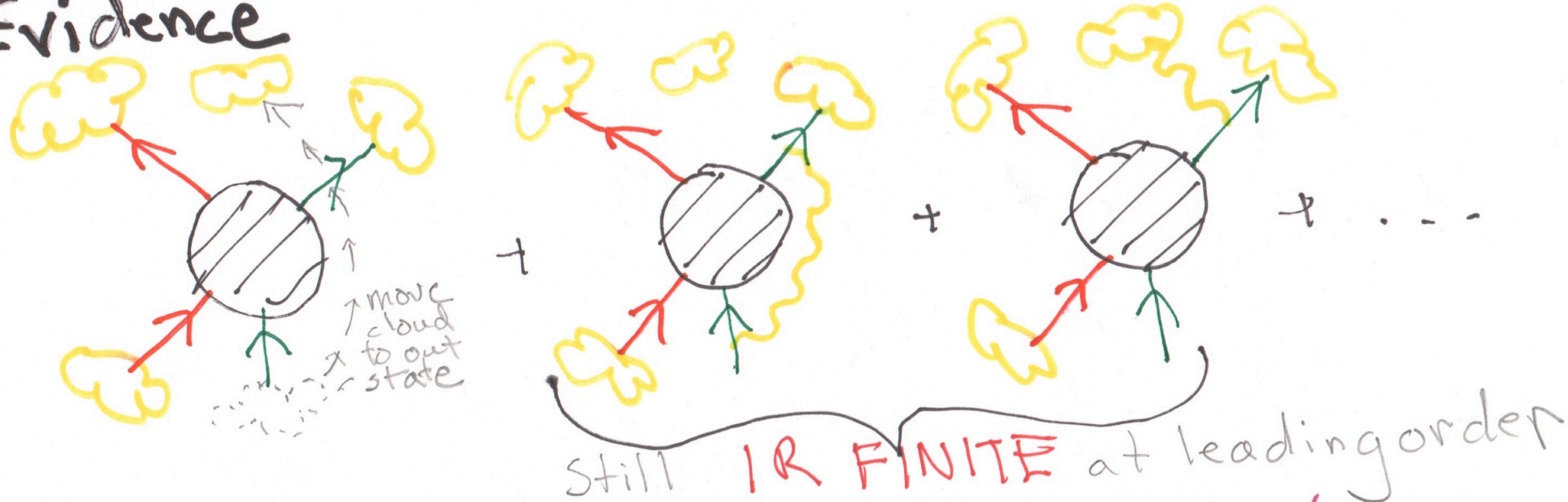
Now, on to the conjectures.

Conjecture for QED

All scattering amplitudes obeying ∞ of conservation laws are IR finite.

FK states are highly unphysical. We do not follow up LHC protons with finely-tuned charge-cancelling soft photon clouds!

Evidence



FK soft clouds are just the soft radiation produced in scattering as dictated by soft theorem = conservation laws.

Prove all-orders finite from crossing symmetry?

Conjecture for (unconfined & unhiggsed) nonabelian gauge theory:

Ditto.

Charge conservation ⇒ IR finite

Also ditto for gravity.

Recent generalization of FK to gravity: Ware, Saotome, Akhoury

Conjecture for Black Hole Information

BH evaporates at the Hawking temperature

$$T_H = \frac{1}{8\pi GM}$$

with energy distribution

$$N(\omega) \sim \frac{\omega^2}{e^{\omega/T_H} - 1} \xrightarrow{\omega \to 0} 0$$

so these are hard quanta.

Hawking (1975): $|\psi_{in}\rangle \to \sum_\alpha p_\alpha |H_\alpha\rangle\langle H_\alpha| \equiv \rho_{Hawking}$

Page (1980): $|\psi_{in}\rangle \to \sum_\alpha c_\alpha |H_\alpha\rangle$

Purity restored by late/early hard correlations. IMPOSSIBLE!

Alternative conjecture

$|\psi_{in}\rangle \to \sum_\alpha c_\alpha |H_\alpha\rangle|S_\alpha\rangle$, $\langle S_\alpha|S_\beta\rangle = \delta_{\alpha\beta}$

$\mathrm{tr}_{soft}|\psi_{out}\rangle\langle\psi_{out}| = \sum_\alpha |c_\alpha^2| |H_\alpha\rangle\langle H_\alpha| = \rho_{Hawking}$ for $c_\alpha = \sqrt{p_\alpha} e^{i\theta_\alpha}$

Exclusive detectors which cant measure soft see thermal spectrum.

Purity restored by hard/soft correlations.

Can $\langle S_\alpha | S_\beta \rangle = \delta_{\alpha\beta}$? Consider pure 4D gravity.

Supertranslation charge $Q^+(z,\bar{z})$, take

$$Q^+|\Psi_{in}\rangle = 0 = Q^+|\Psi_{out}\rangle = (Q^+_H + Q^+_S)\sum_\alpha c_\alpha |H_\alpha\rangle|S_\alpha\rangle$$

Diagonalize Q^+_H

$$Q^+_H(z,\bar{z})|H_\alpha\rangle = \Big[\sum_k E^\alpha_k \, \delta^2(z - z^\alpha_k)\Big]|H_\alpha\rangle$$

(E^α_k: hard graviton energy; z^α_k: angle)

$\Rightarrow$

$$Q^+_S(z,\bar{z})|S_\alpha\rangle = -\Big[\sum_k E^\alpha_k \, \delta^2(z - z_k)\Big]|H_\alpha\rangle$$

Ignoring spin, $[\ldots]$ uniquely determines hard radiation state up to probability-zero exactly collinear configurations. The spin degeneracy is lifted by a similar analysis employing the superrotation charge.

Conclusion $\langle S_\alpha | S_\beta \rangle = \delta_{\alpha\beta}$ for pure gravity. Tracing over soft quanta fully decoheres hard ones!

See also Carney, Chaurette, Neuenfeld & Semenoff.

No algorithm proposed here for phase in

$$c_\alpha = \sqrt{p_\alpha}\, e^{i\theta_\alpha}$$

Conclusion

Much remains to be understood about the soft structure of the world around us.