Advancements and Challenges in Multiloop Scattering Amplitudes for the LHC

Lorenzo Tancredi
TTP KIT Karlsruhe

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In collaboration with
A. Primo, A. von Manteuffel, E. Remiddi, J. Lindert, K. Melnikov, C. Wever

What are we interested in?
(Or what are we looking for?)

Beyond any doubts, there is still a lot we don't understand about **fundamental particle physics** and our **best chance** is the **LHC**

The **LHC** has discovered the **Higgs Boson** and has opened a window on the ***Electro Weak Symmetry Breaking mechanism***, about which we admittedly don't know much

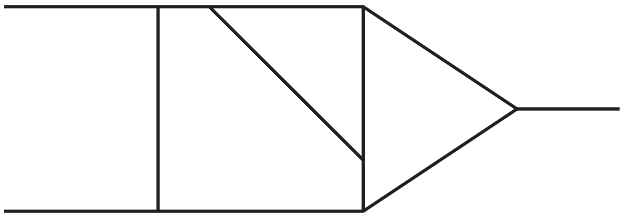
- Higgs width
- Higgs couplings to fermions
- Higgs self-couplings
- The Higgs potential
- … (not only Higgs !)

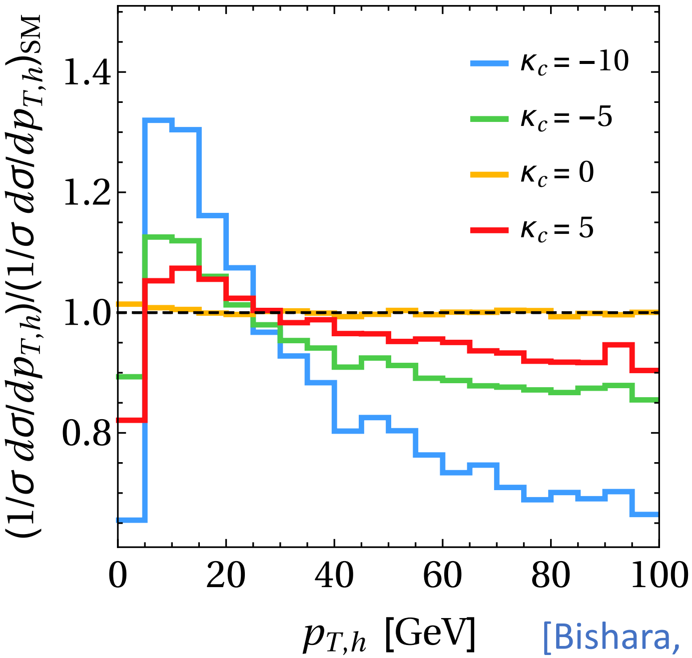
- **Higgs p_T distribution** can be used to constrain *couplings to light quarks*
- $\sigma_{gg \to ZZ}$ **at high energy** to constrain the *Higgs width*
- New observables will be crucial

The fascination of precision calculations

Precision is cool because it allows us to **pinpoint even small hints to new physics**, but (for many of us) this is not the only reason!

As theorists, precision requires **higher order calculations** and deep understanding of the physics that we are searching for





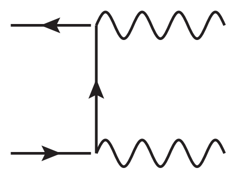
[Bishara, Haisch, Monni, Re '16]

- Going higher in perturbation theory allows us to expose **fascinating mathematical structures of QFT**
- And it can point us to the **limitations of our current approach**

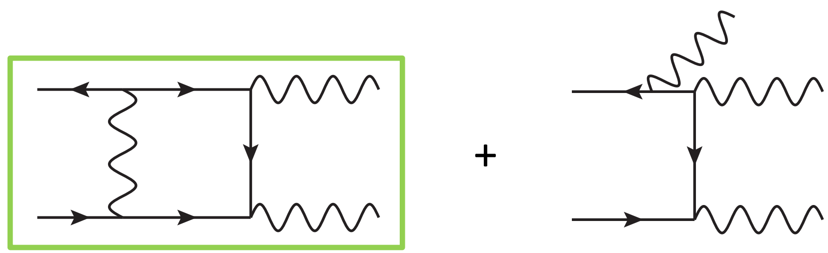
We still need to compute Feynman Diagrams!
(is a “revolution” finally due?)

Modulo revolutions, we still need to put together our physical quantities starting from not very well behaving building blocks…

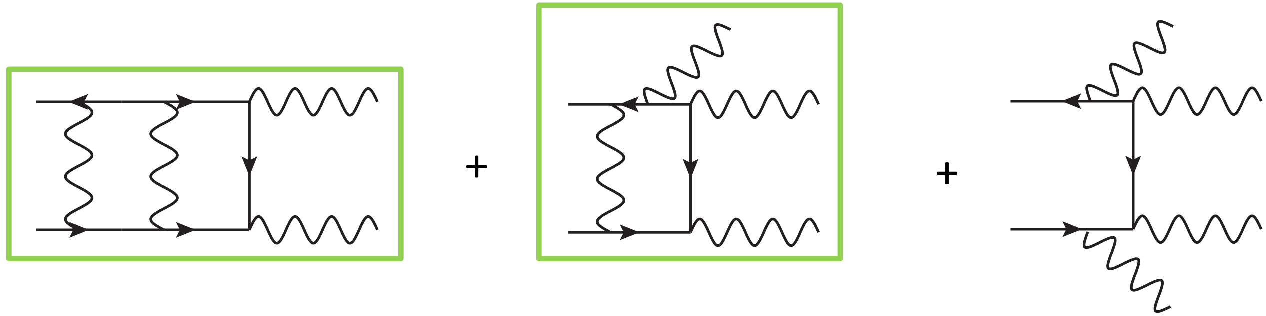
LO



NLO



NNLO



…

LHC energies give us the opportunity to study processes at **very high energy and transverse momentum**

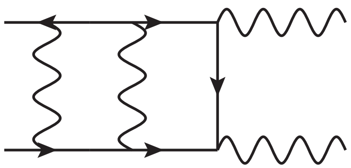
For realistic description, we **cannot neglect** heavy virtual particles in the loops!

In these regimes, we probe **high mass resonances** and in particular the **top-quark**, whose interactions are crucial to study the Higgs mechanism

We need a way to handle **(multi-)loop scattering amplitudes** which depend on **many scales** and, crucially, allow **massive internal states**!

How do we proceed (and can we do better?)

Any scattering amplitude is a collection of **scalar Feynman Integrals**



$$\int \prod_{j=1}^{l} \frac{d^d\, k_j}{(2\pi)^d}\ \frac{S_1^{\sigma_1} \ldots S_s^{\sigma_s}}{D_1^{\alpha_1} \ldots D_n^{\alpha_n}}\,, \qquad S_r = k_i \cdot p_j$$

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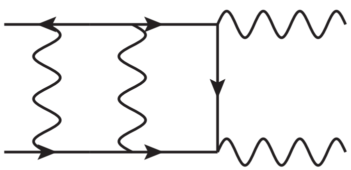
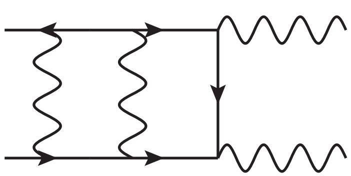


$$\int \prod_{j=1}^{l} \frac{d^d\, k_j}{(2\pi)^d} \, \frac{S_1^{\sigma_1} \dots S_s^{\sigma_s}}{D_1^{\alpha_1} \dots D_n^{\alpha_n}} \,, \qquad S_r = k_i \cdot p_j$$

Integrals are not all independent – there are **Integration-by-parts identities (IBPs)**

[Chetyrkin, Tkachov '81]

$$\int \prod_{j=1}^{l} \frac{d^d\, k_j}{(2\pi)^d} \left(\frac{\partial}{\partial k_j^\mu} v_\mu \, \frac{S_1^{\sigma_1} \dots S_s^{\sigma_s}}{D_1^{\alpha_1} \dots D_n^{\alpha_n}} \right) = 0 \qquad v^\mu = k_j^\mu, p_k^\mu$$

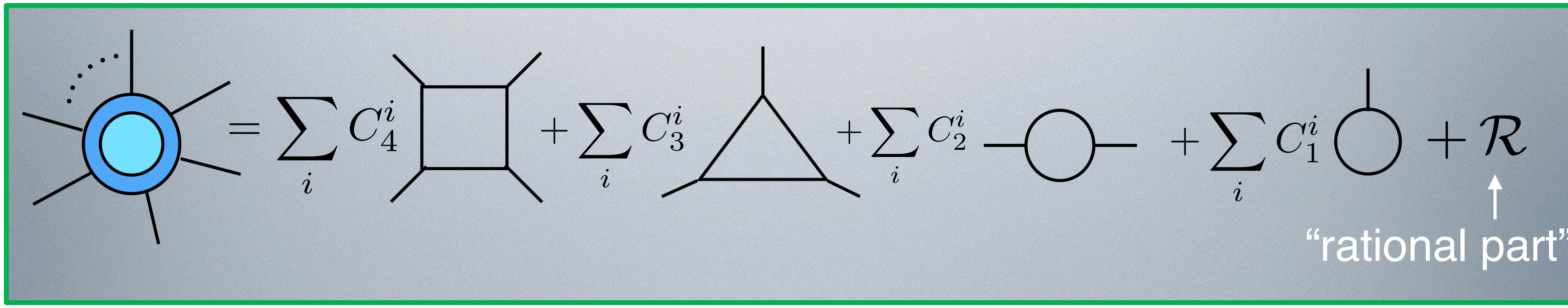


$$\sum_{j=1}^{N} C_j(d;\, x_k)\, I_j(d;\, x_k)$$

N Master Integrals

@ 1 loop everything is clear now

Reduction is understood, we are actually able to write amplitudes as combination of **4 Master Integrals** in terms of **on-shell quantities**



MIs are all known!

Space of functions is understood ➡ at every order in ε we only need **MPLs!**

$$G(0;x) = \ln(x)\,, \qquad G(a;x) = \ln\left(1 - \frac{x}{a}\right) \quad \text{for} \quad a \neq 0$$

$$G(\underbrace{0,\ldots,0}_{n};x) = \frac{1}{n!}\ln^n(x)\,, \qquad G(a,\vec{w};x) = \int_0^x \frac{dy}{y-a}\,G(\vec{w};y)\,.$$

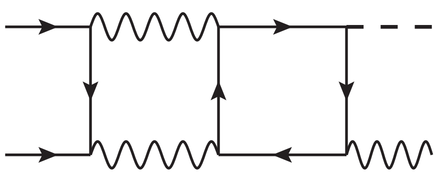
For finite piece in $d = 4$ only ***Li_2* functions**!

@ 2 loops and beyond
it is an entirely different story

We need realistic processes with **masses** and **many scales**

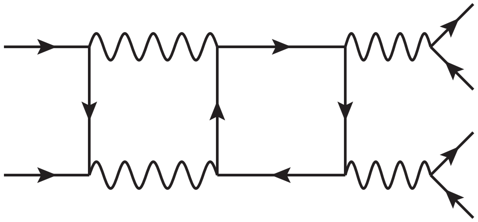
Classic examples of processes we need

- H+jet production **with a top quark**



4 scales, 3 ratios

- VV production **above top threshold**



5 scales, 4 ratios

No idea in general <u>what Mis are</u>, and <u>what is the space of functions needed</u>,
we only know that **MPLs are surely not enough**

We made a lot of progress in the last years

Most important development is probably the **differential equations method**

From the **IBPs** ⇨

$$\boxed{\frac{\partial}{\partial x_k} I_i(d;x_k) = \sum_{j=1}^{N} c_{ij}(d;x_k)\, I_j(d;x_k)}$$

[Kotikov '91; Remiddi '97; Gehrmann, Remiddi '00]

We want to compute the MIs **as Laurent series** in $\varepsilon = (4-d)/2$

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The reason why DEQs are so useful is because very they <u>often become triangular</u>
<u>In the limit $d \to 4$</u> (by choosing **wisely** basis of MIs)

Coefficients of Laurent series effectively satisfy **<u>First order linear DEQs</u>!**

They can expressed as well in terms of **MPLs**

$$\boxed{\frac{\partial}{\partial x} G(a,\vec{w};x) = \frac{1}{x-a}\, G(\vec{w};x)}$$

We discovered canonical bases

[Henn '13]

Beautiful systematization of the proceedure above, i.e. integrals expressed as **MPLs** or **iterated integrals over dlogs** ⟶ choose MIs with **unit leading singularities**

The concept of unit leading singularity is understood (?) for **integrals that fulfil this requirement** [Arkani-Hamed et al '10]

$$\boxed{d\vec{I}(d;x) = (d-4)\,A(x)\,\vec{I}(d;x)}$$

If such a basis exists, it must be possible to find it by transformations on DEQs only [Lee '14]

Differential equations take **very simple form**

A(x) is in **d-log form**

Results are trivially **Multiple Polylogarithms**

The **symbol** can be read off directly from A(x) !

Two other developments are *at least* as important...

We also have almost full control on **analytical** **and** **algebraic** **properties** of **Multiple Polylogarithms**

[Goncharov, Spradlin, Volovich '10]
[Duhr, Gangle, Rodes '11]
[Duhr '12]

We have **numerical routines** to evaluate **Multiple Polylogarithms** with **arbitrary precison**

[Vollinga, Weinzierl '05]

As long as we consider cases **in this subset**, we can do a lot!

We can compute **scattering amplitudes** **efficiently,** and get them in a form that is **useful to do real physics with them**!

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Beautiful Example (a bit biased):

$qq \rightarrow V_1 V_2$ @ 2 loop in QCD

without top-mass effects
but full V off-shellness effects

Use **all techniques above** to put amplitude in a usable form for pheno!

[Caola, Henn, Melnikov, Smirnov, Smirnov '14, '15]
[Gehrmann, von Manteuffel, Tancredi '15]

All this works so nicely without masses in the loops...

What changes otherwise? Multiple Polylogs are NOT enough to span all space of functions needed @ 2 loops! Elliptic Functions and possibly more...

In terms of **DEQs**, they cannot be decoupled anymore in $d = 4$
Laurent coefficients of MIs fulfill **irreducible higher order differential equations**

$$\frac{\partial}{\partial x_k} m_i(d; x_k) = \sum_{j=1}^{N} h_{ij}(d; x_k)\, m_j(d; x_k) + \sum_{j=1}^{M} nh_{ij}(d; x_k)\, sub_j(d; x_k)$$

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Concept of unit leading singularity unclear

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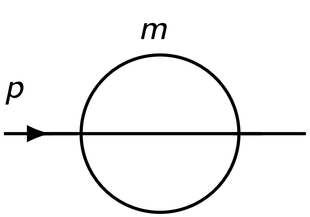
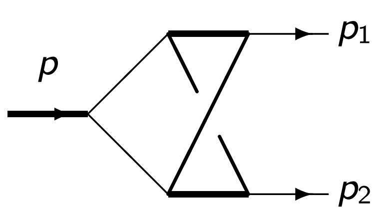
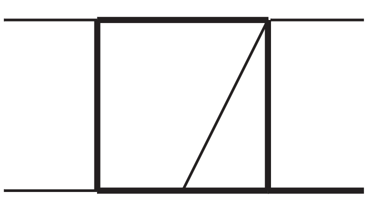
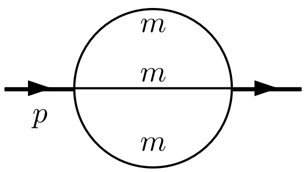






DEQs for 2-loop sunrise as an example



$$= S(d;u) = \int \frac{\mathfrak{D}^d k_1 \, \mathfrak{D}^d k_2}{[k_1^2 - m^2][k_2^2 - m^2][(k_1 - k_2 - p)^2 - m^2]}$$

Has two master integrals S_1 and S_2 plus one subtopology. $u = p^2/m^2$

$$\boxed{\frac{d}{du}\begin{pmatrix} S_1(u) \\ S_2(u) \end{pmatrix} = B(u)\begin{pmatrix} S_1(u) \\ S_2(u) \end{pmatrix} + \epsilon\, D(u)\begin{pmatrix} S_1(u) \\ S_2(u) \end{pmatrix} + \begin{pmatrix} N_1(u) \\ N_2(u) \end{pmatrix}}$$

$$B(u) = \frac{1}{6\,u(u-1)(u-9)}\begin{pmatrix} 3(3+14u-u^2) & -9 \\ (u+3)(3+75u-15u^2+u^3) & -3(3+14u-u^2) \end{pmatrix}$$

$$D(u) = \frac{1}{6\,u(u-9)(u-1)}\begin{pmatrix} 6\,u(u-1) & 0 \\ (u+3)(9+63u-9u^2+u^3) & 3(u+1)(u-9) \end{pmatrix}$$

Analytic solutions reloaded

A complete solution in series expansion in ε requires solving homogeneous equations in $\varepsilon = 0$, i.e. finding a matrix 2x2 such that

$$G(u) = \begin{pmatrix} I_1(u) & J_1(u) \\ I_2(u) & J_2(u) \end{pmatrix} \quad \Longrightarrow \quad \frac{d}{du}\begin{pmatrix} I_1(u) & J_1(u) \\ I_2(u) & J_2(u) \end{pmatrix} = B(u)\begin{pmatrix} I_1(u) & J_1(u) \\ I_2(u) & J_2(u) \end{pmatrix}$$

In general, given a <u>coupled system of equations</u>, no general method to determine all homogeneous solutions. **BUT** we can use **additional information from Maximal Cut**

$$\frac{\partial}{\partial x_k} m_i(d; x_k) = \sum_{j=1}^{N} h_{ij}(d; x_k)\, m_j(d; x_k) + \sum_{j=1}^{M} nh_{ij}(d; x_k)\, sub_j(d; x_k)$$

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$$\frac{\partial}{\partial x_k} \mathrm{Cut}\left(m_i(d; x_k)\right) = \sum_{j=1}^{N} h_{ij}(d; x_k)\, \mathrm{Cut}\left(m_j(d; x_k)\right)$$

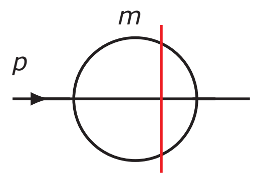
<u>**Maximal Cut provides solution of homogeneous equation**</u> [A. Primo, **L.Tancredi** ‘16]

Computed efficiently using **Baikov representation** [C. Papadopoulos, H. Frellesvig ‘17]

How to get all independent solutions

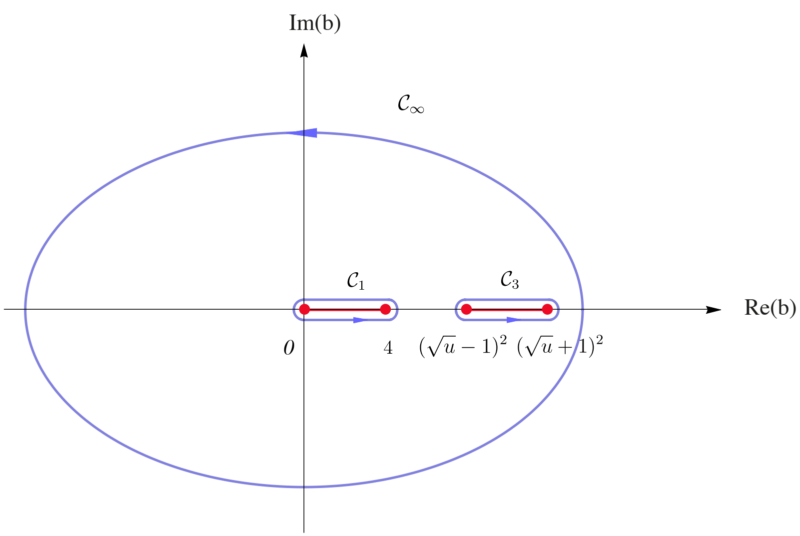
Compute max cut along all **independent contours**

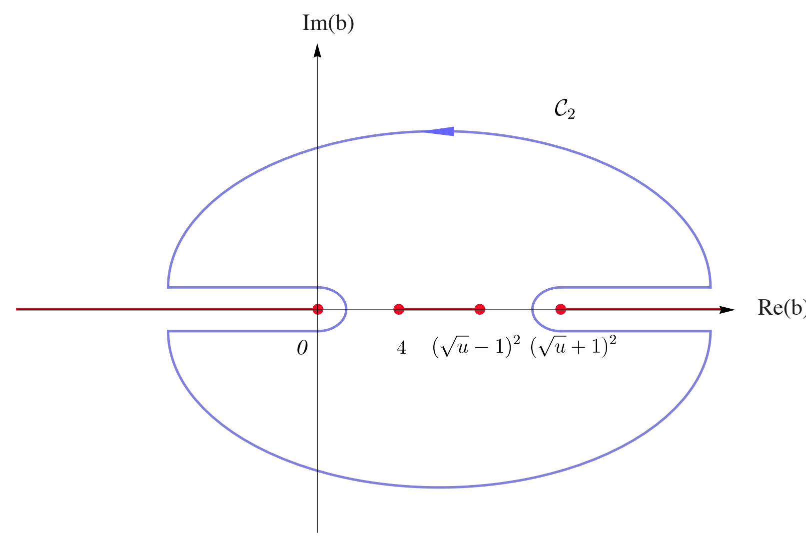
[Bosma, Sogaard, Zhang '17]
[A. Primo, **L. Tancredi** '17]
[Harley, Moriello, Schabinger '17]



$$= \oint_{\mathcal{C}} \frac{db}{\sqrt{\pm b\,(b-4)\,\left(b-(\sqrt{u}-1)^2\right)\,\left(b-(\sqrt{u}+1)^2\right)}}$$

$$= \oint_{\mathcal{C}} \frac{db}{\sqrt{\pm R_4(b,u)}}$$





$$J_1(u) \propto \oint_{\mathcal{C}_1} \frac{db}{\sqrt{R_4(b,u)}}$$

$$I_1(u) \propto \oint_{\mathcal{C}_2} \frac{db}{\sqrt{-R_4(b,u)}}$$

We obtain at once all solutions from the two master integrals

$$J_1(u) \propto \oint_{\mathcal{C}_1} \frac{db}{\sqrt{R_4(b,u)}} \qquad I_1(u) \propto \oint_{\mathcal{C}_2} \frac{db}{\sqrt{-R_4(b,u)}}$$

$$J_2(u) \propto \oint_{\mathcal{C}_1} \frac{db\ b^2}{\sqrt{R_4(b,u)}} \qquad I_2(u) \propto \oint_{\mathcal{C}_2} \frac{db\ b^2}{\sqrt{-R_4(b,u)}}$$

Cutting the second MI

And by construction we find

$$\frac{d}{du}\begin{pmatrix} I_1(u) & J_1(u) \\ I_2(u) & J_2(u) \end{pmatrix} = B(u)\begin{pmatrix} I_1(u) & J_1(u) \\ I_2(u) & J_2(u) \end{pmatrix}$$

It is the matrix of Maximal Cuts !

$$G(u) = \begin{pmatrix} I_1(u) & J_1(u) \\ I_2(u) & J_2(u) \end{pmatrix} = \begin{pmatrix} \mathrm{Cut}_{C_1}\left(S_1(u)\right) & \mathrm{Cut}_{C_2}\left(S_1(u)\right) \\ \mathrm{Cut}_{C_1}\left(S_2(u)\right) & \mathrm{Cut}_{C_2}\left(S_2(u)\right) \end{pmatrix}$$

Basis of unit leading singularity?

Rotate original basis
Using matrix G(u)

$$\begin{pmatrix} S_1(u) \\ S_2(u) \end{pmatrix} = G(u) \begin{pmatrix} m_1(u) \\ m_2(u) \end{pmatrix}$$

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$$\begin{pmatrix} m_1(u) \\ \\ m_2(u) \end{pmatrix} = G^{-1}(u) \begin{pmatrix} S_1(u) \\ \\ S_2(u) \end{pmatrix} = \frac{1}{W(u)} \begin{pmatrix} J_2(u)S_1(u) - J_1(u)S_2(u) \\ \\ -I_2(u)S_1(u) + I_1(u)S_2(u) \end{pmatrix}$$

$$\boxed{\begin{pmatrix} \mathrm{Cut}_{C_1}(m_1(u)) & \mathrm{Cut}_{C_2}(m_1(u)) \\ \mathrm{Cut}_{C_1}(m_2(u)) & \mathrm{Cut}_{C_2}(m_2(u)) \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}}$$

New basis' **max cuts** along two contours give **identity matrix**

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New basis' **max cuts** along two contours give **identity matrix**

For Sunrise, entries of matrix **G(u)** are proportional to **complete elliptic integrals**

Iterated integrals over new kernels

The new basis fulfils simple differential equations – homogeneous part factorized in ε

$$\frac{d}{du}\begin{pmatrix} m_1(u) \\ m_2(u) \end{pmatrix} = \epsilon \underbrace{G^{-1}(u)D(u)G(u)} \begin{pmatrix} m_1(u) \\ m_2(u) \end{pmatrix} + G^{-1}(u)\begin{pmatrix} N_1(u) \\ N_2(u) \end{pmatrix}$$

$$\Downarrow$$

Iterated integrals over products of **two** elliptic integrals and rational functions!

What are these functions (in the Sunrise case)?

- Elliptic Polylogarithms [Bloch, Vanhove '13]
- ELi functions [Adams, Bogner, Weinzierl '14,'15]
- Iterated integrals over **modular forms** [Adams, Weinzierl '17]
- …

Unclear how to extend them to other topologies with **more complicated kinematics**

The method is much more general!

We know a few examples at 2 loops (the number is increasing…)

Different kinematics (2,3,4-point functions), all reduced to **2x2 coupled system** whose solutions given by **complete elliptic integrals**

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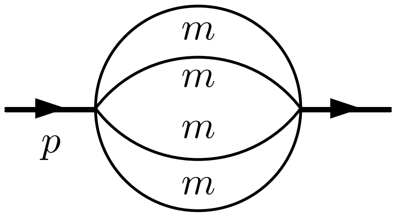
First 3x3 case known happens at 3 loops



Banana graph – 3 MIs, 3 coupled DEQs

[A. Primo, **L. Tancredi** '17]

$$\frac{d}{dx}\begin{pmatrix}\mathcal{I}_1(\epsilon;x)\\ \mathcal{I}_2(\epsilon;x)\\ \mathcal{I}_3(\epsilon;x)\end{pmatrix} = B(x)\begin{pmatrix}\mathcal{I}_1(\epsilon;x)\\ \mathcal{I}_2(\epsilon;x)\\ \mathcal{I}_3(\epsilon;x)\end{pmatrix} + \epsilon\, D(x)\begin{pmatrix}\mathcal{I}_1(\epsilon;x)\\ \mathcal{I}_2(\epsilon;x)\\ \mathcal{I}_3(\epsilon;x)\end{pmatrix} + \begin{pmatrix}0\\ 0\\ -\frac{1}{2(4x-1)}\end{pmatrix}$$

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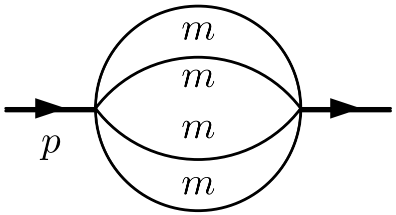

Banana graph – 3 MIs, 3 coupled DEQs

[A. Primo, **L. Tancredi** '17]

$$\boxed{\frac{d}{dx}\begin{pmatrix}\mathcal{I}_1(\epsilon;x)\\ \mathcal{I}_2(\epsilon;x)\\ \mathcal{I}_3(\epsilon;x)\end{pmatrix} = B(x)\begin{pmatrix}\mathcal{I}_1(\epsilon;x)\\ \mathcal{I}_2(\epsilon;x)\\ \mathcal{I}_3(\epsilon;x)\end{pmatrix}} + \epsilon\, D(x)\begin{pmatrix}\mathcal{I}_1(\epsilon;x)\\ \mathcal{I}_2(\epsilon;x)\\ \mathcal{I}_3(\epsilon;x)\end{pmatrix} + \begin{pmatrix}0\\ 0\\ -\frac{1}{2(4x-1)}\end{pmatrix}$$

3x3 coupled homogeneous system
Need a matrix of 3x3 independent solutions!

Same idea applied here

We study the max cut of the three loop banana graph along all independent contours bounded by branch cuts and find all independent solutions!

Generalization of complete elliptic integrals in 2x2 case

$$\mathcal{H}_1(x) = x\ \mathrm{K}\left(k_+^2\right)\mathrm{K}\left(k_-^2\right)\ ,$$
$$\mathcal{J}_1(x) = x\ \mathrm{K}\left(k_+^2\right)\mathrm{K}\left(1-k_-^2\right)\ ,$$
$$\mathcal{I}_1(x) = x\ \mathrm{K}\left(1-k_+^2\right)\mathrm{K}\left(k_-^2\right)\ ,$$

Homogeneous solutions are products of complete elliptic integrals!

Bessel moments
[Bailey, Borwein, Broadhurst '08]

$$k_\pm = \frac{\sqrt{(\gamma+\alpha)^2-\beta^2} \pm \sqrt{(\gamma-\alpha)^2-\beta^2}}{2\gamma} \qquad \text{with} \qquad k_- = \left(\frac{\alpha}{\gamma}\right)\frac{1}{k_+} = \frac{2\alpha}{k_+}$$

$$\alpha = \frac{\sqrt{x}+\sqrt{x(1-x)}}{2}\,, \qquad \beta = \frac{\sqrt{x}-\sqrt{x(1-x)}}{2}\,, \qquad \gamma = \frac{1}{2}$$

Already J. Joyce could solve this equation in 1973 in context of cubic lattice Green functions

More recently Bloch and Vanhove wrote the graph in d=2 as an Elliptic Trilog!

Is this the only approach worth trying?

Very promising developments: we have now a way to tackle complicated MIs by solving differential equations **even if they are coupled!**

We are making fast progress on the classifications of the **special functions involved**!

Still, a major issue remains. Calculations with many scales and internal masses generate typically huge algebraic complexity.

Complexity of amplitudes for realistic processes, even when written in terms of independent structures, **increases factorially**.

They become a problem **already for $2 \to 2$** even for **largest computers.** All our machinery breaks down!

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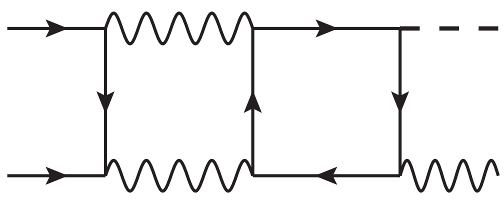
Maybe we need to **rethink entirely what we are doing?** →

Very promising **numerical approaches**:

- ttbar @ NNLO [Czakon et al. '13]
- HH @ NLO [Borowka et al. '16]

We should remember that we are physicists (mainly!) and that most of our calculations are performed in some sort of approximation…

Take for example **H+jet production** with a **massive bottom quark**



Important <u>source of uncertainty</u> :

Interference top-bottom @NLO in QCD

For large p_T of Higgs, large $\log\left(\frac{m_b}{p_T}\right), \log\left(\frac{m_h}{m_b}\right)$

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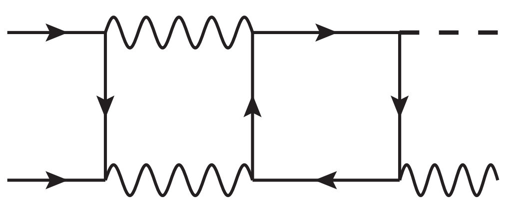

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Progress towards analytic computation of **planar MIs**

[Bonciani, Del Duca, Frellesvig, Henn, Moriello, Smirnov '16]

Impressive calculation, but results are <u>not really in a nice shape</u>, **not even the MPLs part!**

- Up to two-fold integral representations
- No analytic continuation
- ~ 200 MB large files and NPL integrals are still missing

In order to capture this effect, **no need of computing exact mass dependence**!
We can **expand all MIs** (and the whole amplitude) for **small bottom mass**!

[J. Lindert, K. Melnikov, **L. Tancredi**, C. Wever '16,'17]

$$\mathcal{I}_i^{MI}(m_b^2, s, t, m_h^2, \epsilon) = \sum_{ijkn} c_{ijkn}(s, t, m_h^2, \epsilon) \left(\frac{m_b^2}{m_h^2}\right)^{j-k\epsilon} \log^n\left(\frac{m_b^2}{m_h^2}\right)$$

Derive DEQs for coefficients
One less scale, no mass, much simpler!

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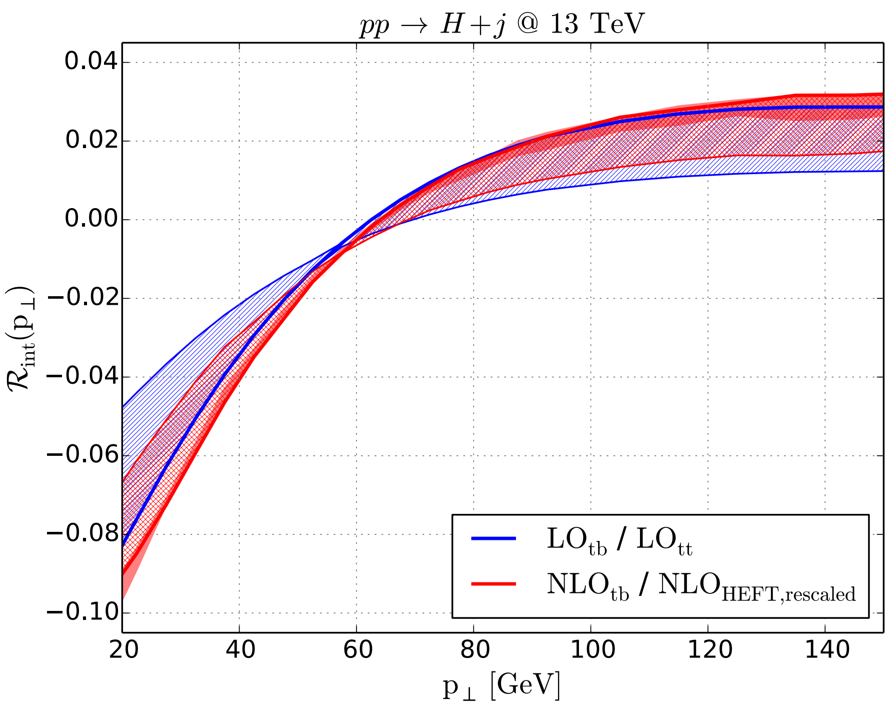

Message is:

Sometimes doing everything analytically can be **overkill**

If we find the **right approximation**, DEQs are very powerful also to get approximate results

Used already in similar context

[R. Mueller, G. Öztürk '16]

Conclusions

- **Scattering amplitudes** needed for realistic physical processes @ LHC are (appear to be?) **immensely complicated**
- Thanks to progress in theoretical understanding, a limited subset of these amplitudes is now under much better control (MPLs!)
- Also in more general cases, we start having an idea of how to proceed to tackle the problem (**Max cut**, **EPLs** and **Modular Forms**)
- Still, remains problem of **enormous algebraic complexity** (it's just simple combinatorics!)
- Given this complexity, it is unclear whether a purely analytical approach will be feasible in the near future.
- Hybrid Numerical/Analytical (series expansions?) might be the way to go…?

THANKS!