BERNHARD MISTLBERGER

QCD AT HIGH ORDERS FOR PRECISION PHENOMENOLOGY AT THE LHC

The LHC is currently our best tool to empirically test the inner workings of fundamental interactions at very high energies.

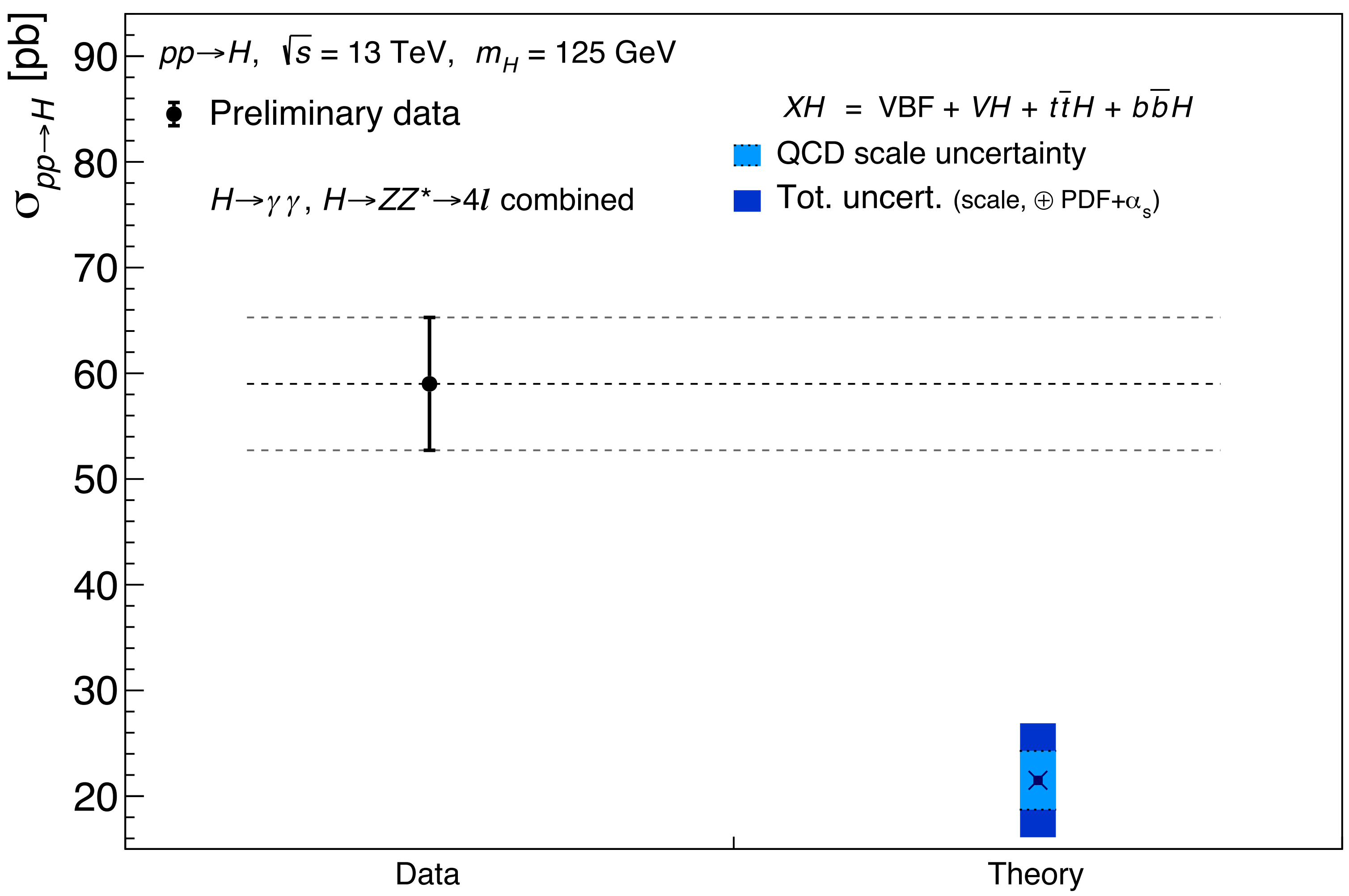
- Test QFT as a framework to describe high energy interactions.
- Compare specific models with actual physics realisation in nature.
- Measure fundamental parameters of these models.
  Example: Higgs boson mass:

$$m_h = 125.09\mathrm{GeV} \pm 0.2\%$$

The LHC is a precision machine!

- The motto: Predict and compare!

COMPARE DATA TO PREDICTION



- Precise measurement

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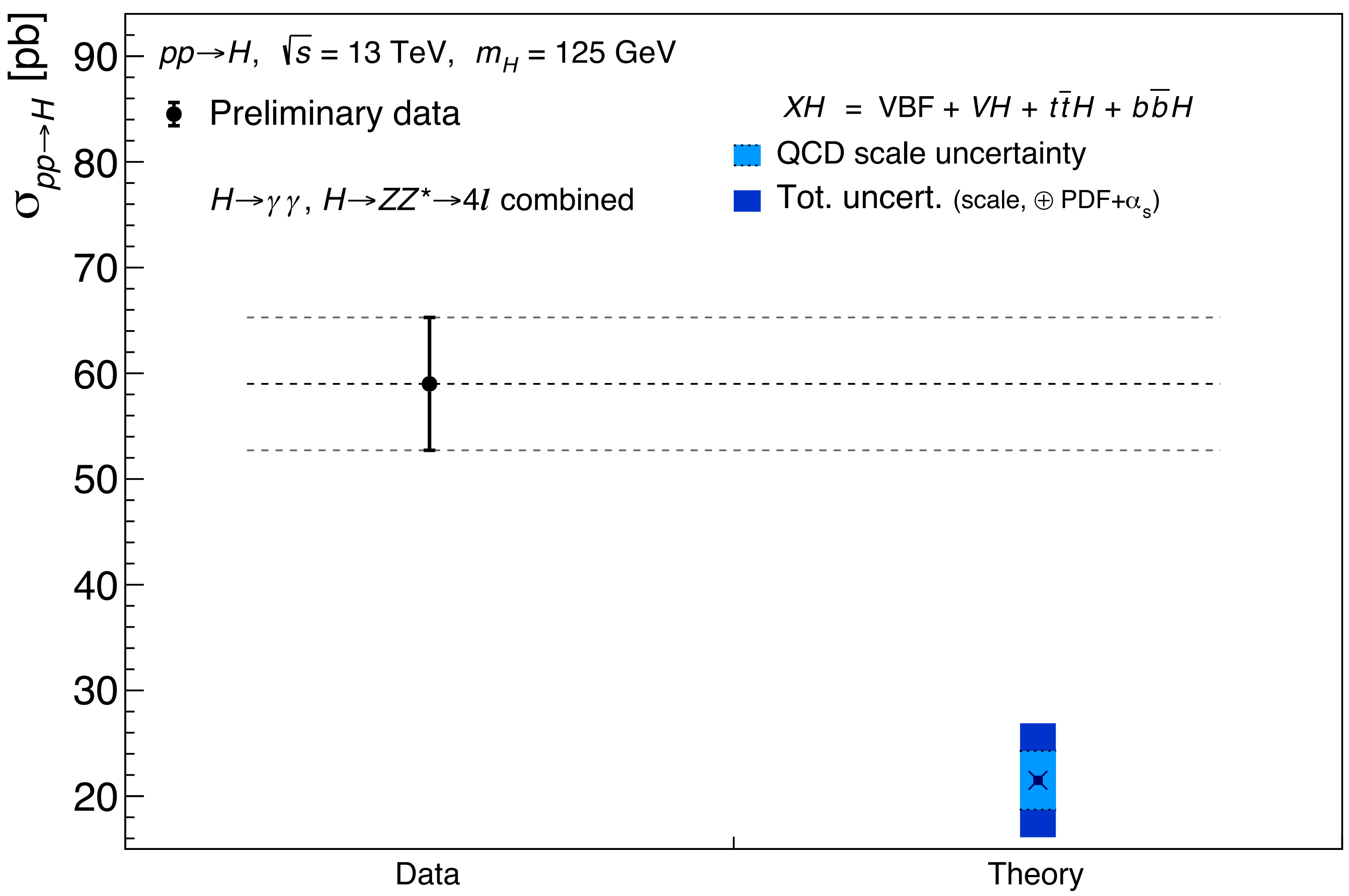

- Precise measurement
- 3.8 sigma deviation
- 1500 papers about new physics on the arXiv

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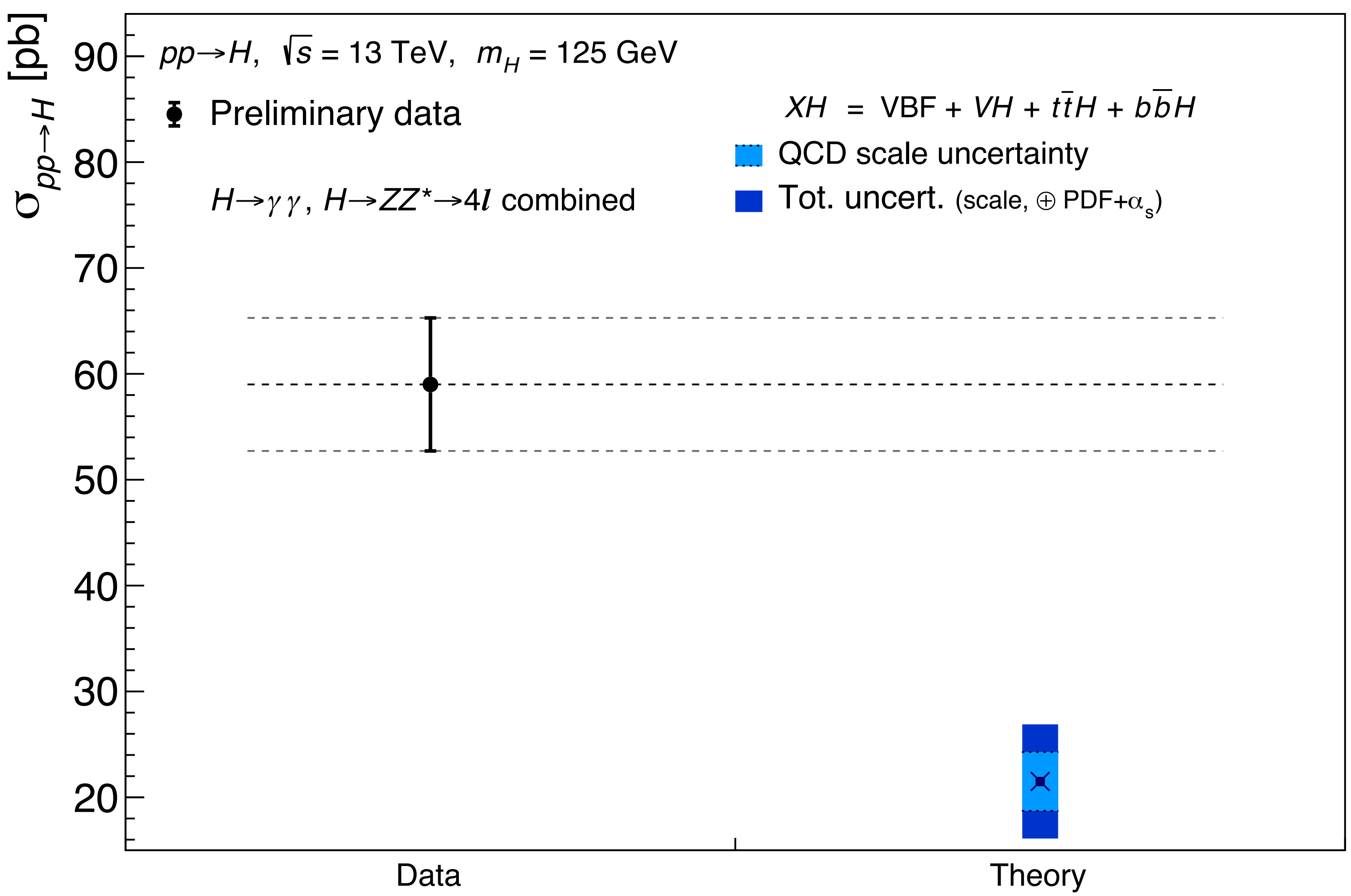
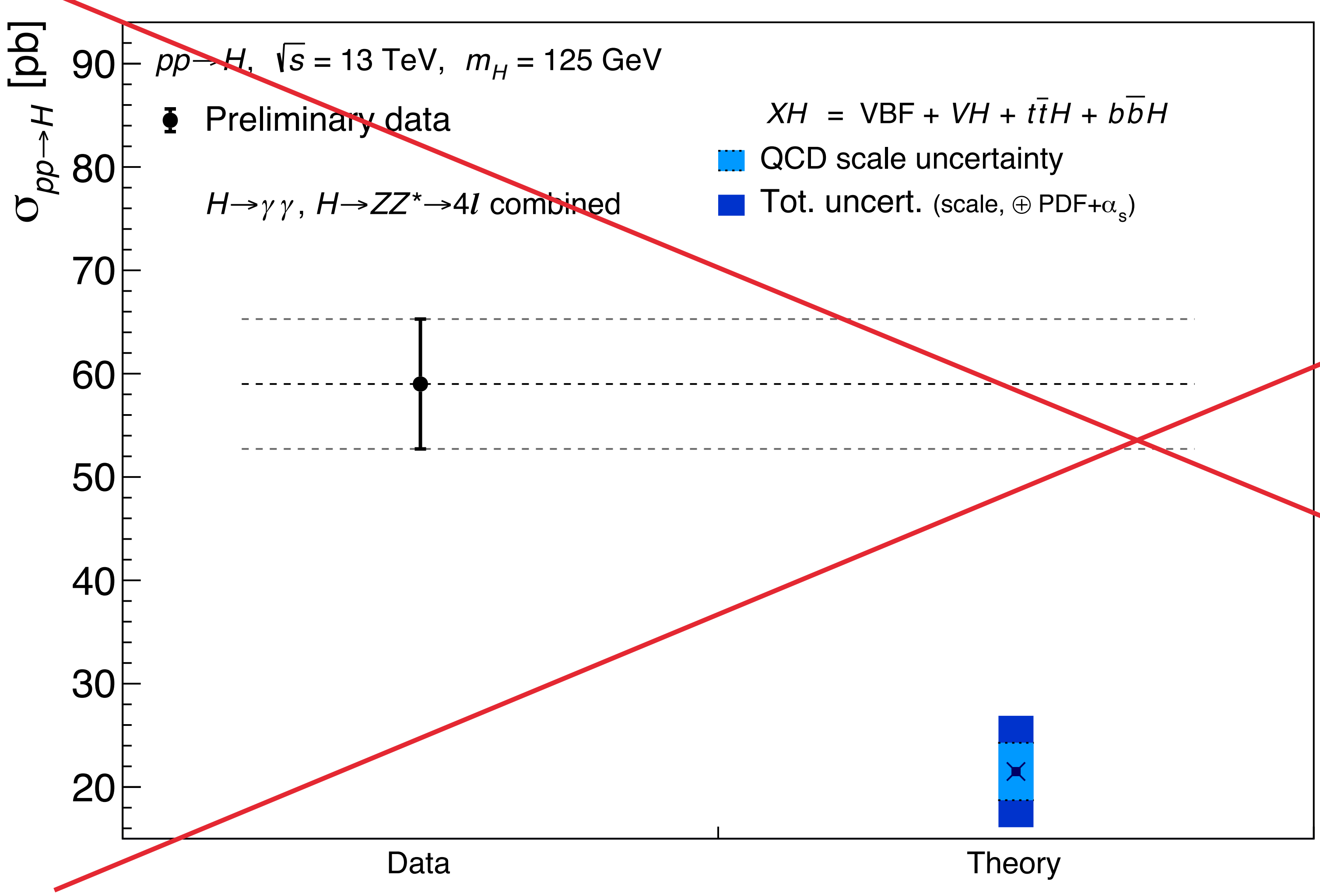


- Precise measurement
- 3.8 sigma deviation
- 1500 papers about new physics on the arXiv
- Standard Model fails

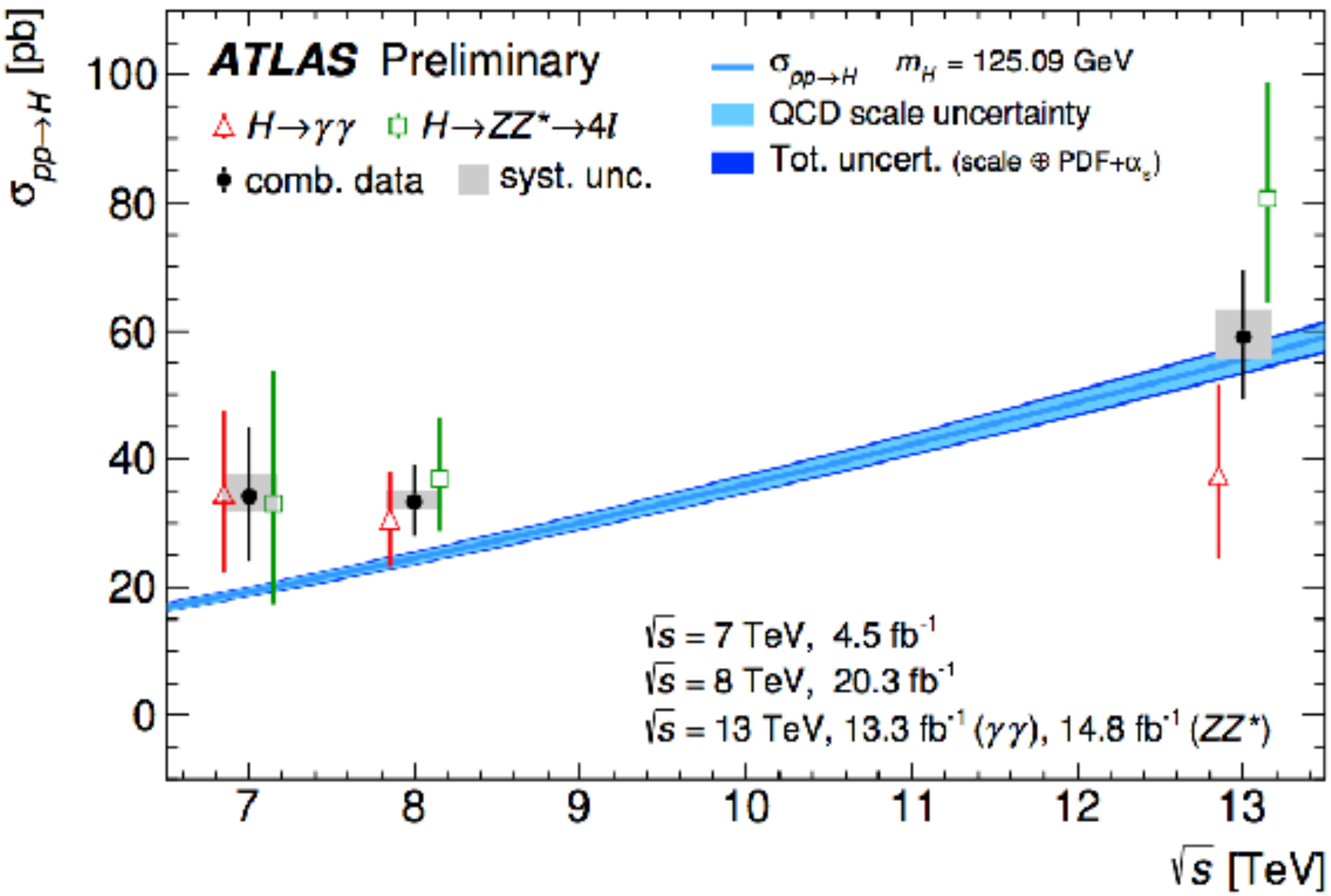
COMPARE DATA TO PREDICTION AT LEADING ORDER



- ~~Precise measurement~~
- ~~3.8 sigma deviation~~
- ~~1500 papers about new physics on the arXiv~~
- ~~Standard Model fails~~

PRECISION IS A MUST!

COMPARE DATA TO PREDICTION – ACTUALLY



Standard Model: Incredibly successful theory that leads to precise, quantitative predictions in agreement with experimental observation.

LHC – THE CURRENT STATUS

- Current precision level for inclusive cross sections:

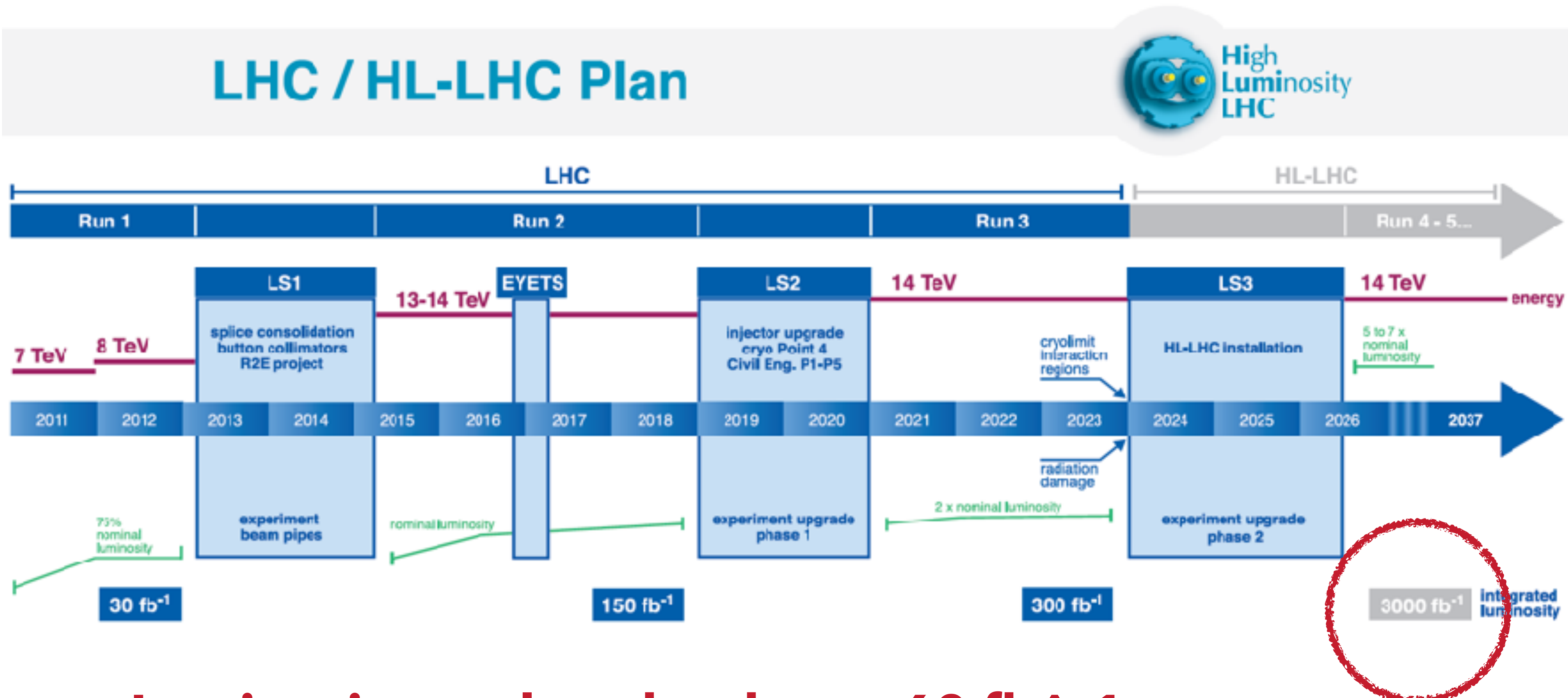
Higgs	$\sigma_{PP\to H} \sim 60pb \pm 16\%$
Top - Anti-Top	$\sigma_{PP\to t\bar{t}} \sim 818pb \pm 4\%$
Drell - Yan	$\sigma_{PP\to Z} \sim 1138pb \pm 1\%$

- Differential distributions typically of the order 10 - 20 %.

- New physics at 1 TeV?
  Modify Standard Model at $\mathcal{O}\left(\frac{Q^2}{(1TeV)^2}\right) \sim \%$?

LHC – THE CURRENT STATUS

- We are at the very beginning!



Luminosity analysed today: ~ 60 fb^-1

2 % of future LHC data.

PRECISION OBSERVABLES

- Opportunity to push the test of our understanding of nature at high energies to the percent level.
- Test: QFT / QCD, SM, parameters.
- Clean, high energy signatures required - Leptons / photons in the final state.
- Relatively low multiplicity of particles in final state.
- Excellent theoretical control required.

- H, H+J, H+JJ
- V, V+J, V+JJ
- t tbar
- JJ

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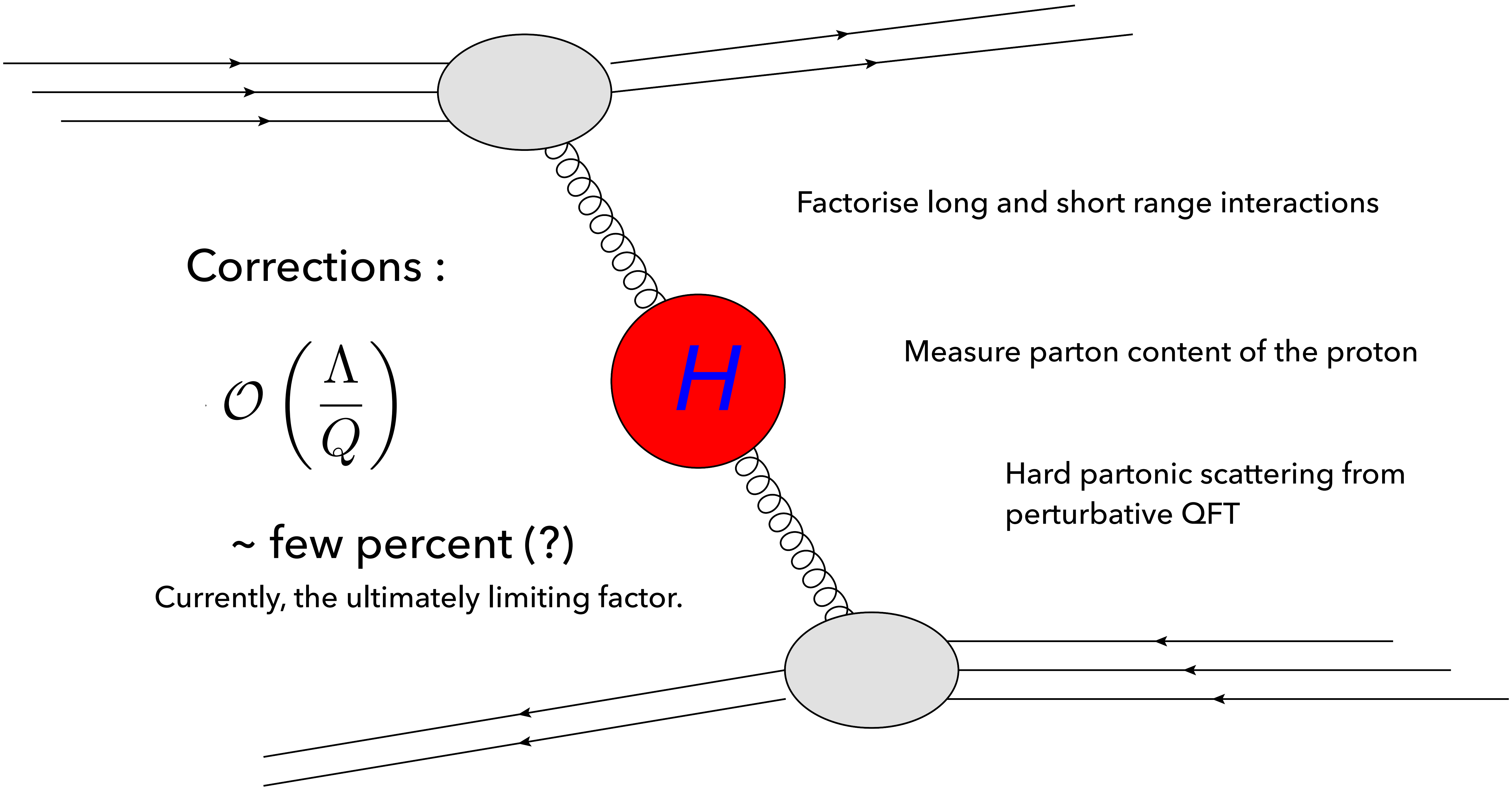
Percent level precision at the LHC?

FACTORISATION



Factorise long and short range interactions

Measure parton content of the proton

Hard partonic scattering from perturbative QFT

Corrections :

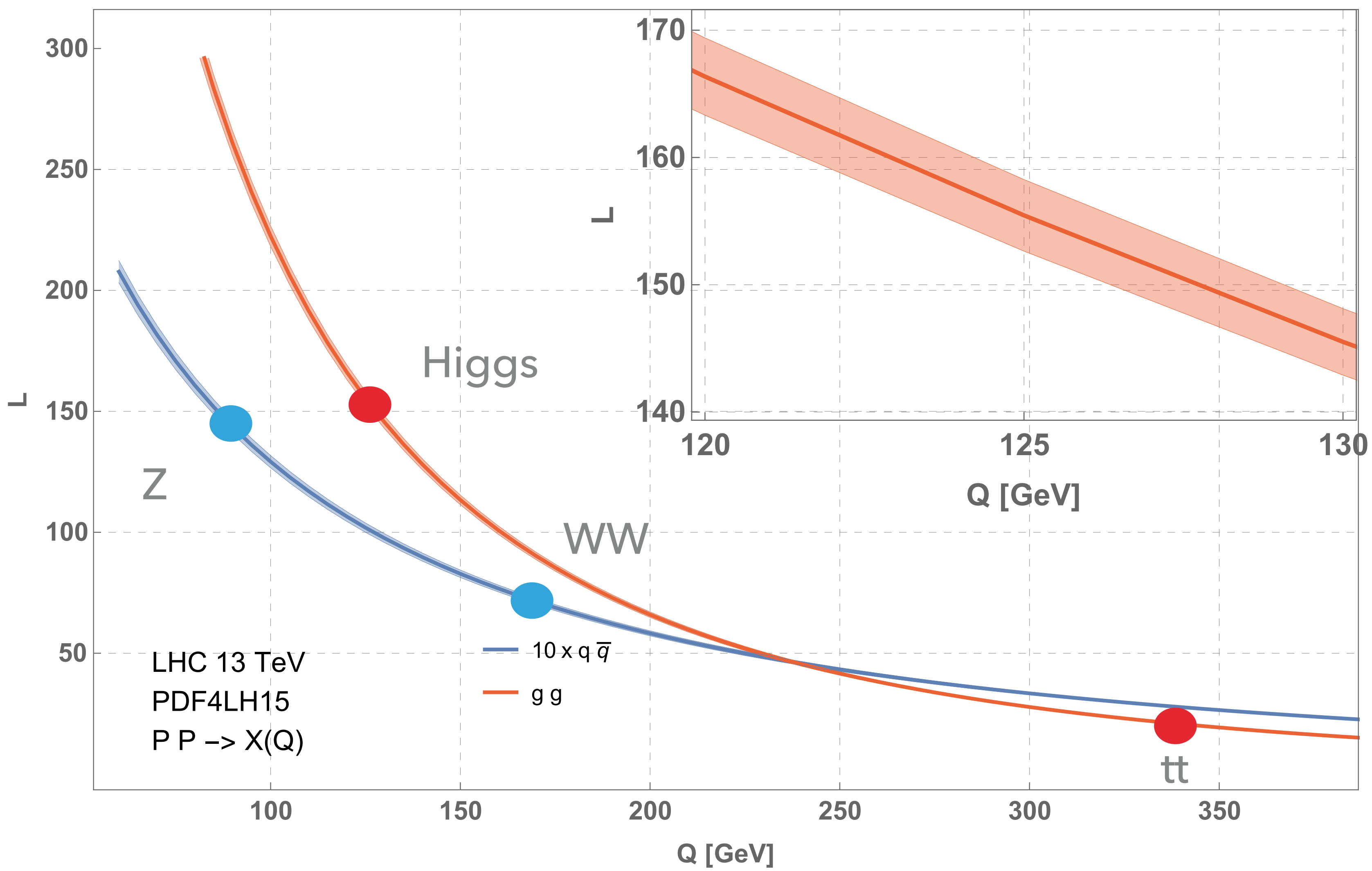
$$\mathcal{O}\left(\frac{\Lambda}{Q}\right)$$

~ few percent (?)

Currently, the ultimately limiting factor.

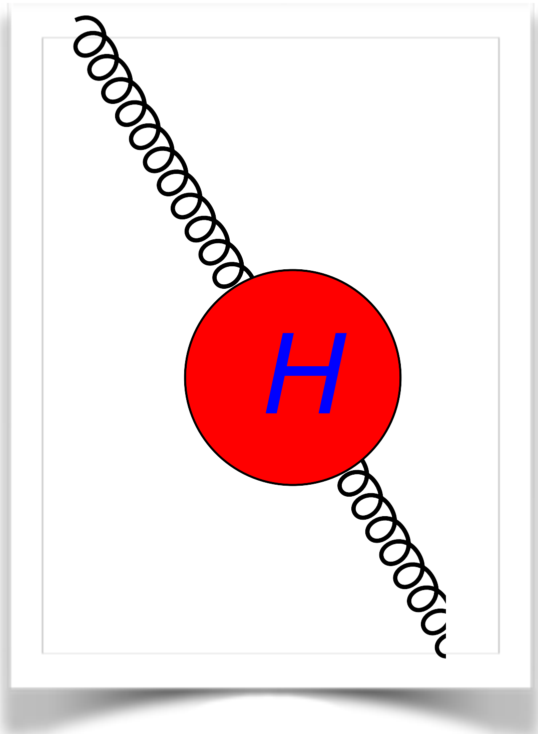
HOW WELL DO WE KNOW THE PROTON?

Probability to find parsons in the proton as a function of energy.



- Current knowledge of parton distribution functions is at the level of **2%** for LHC precision observables.
- Improvements due to new LHC data included in PDF fits.
- Ongoing developments on theoretical treatment of PDFs and their uncertainties. (Flavour thresholds, theoretic uncertainties, quark masses, etc.)

FIXED ORDER PERTURBATION THEORY



- Compute hard scattering process as fixed order perturbative expansion in coupling constant.
- Dominant at LHC: QCD corrections!

$$\alpha_S = 0.1181 \pm 1\%$$

$$\sigma = \sigma^{(0)} + \alpha_S \sigma^{(1)} + \alpha_S^2 \sigma^{(2)} + \alpha_S^3 \sigma^{(3)} + \mathcal{O}(\alpha_S^4)$$

Advantages of perturbation theory:

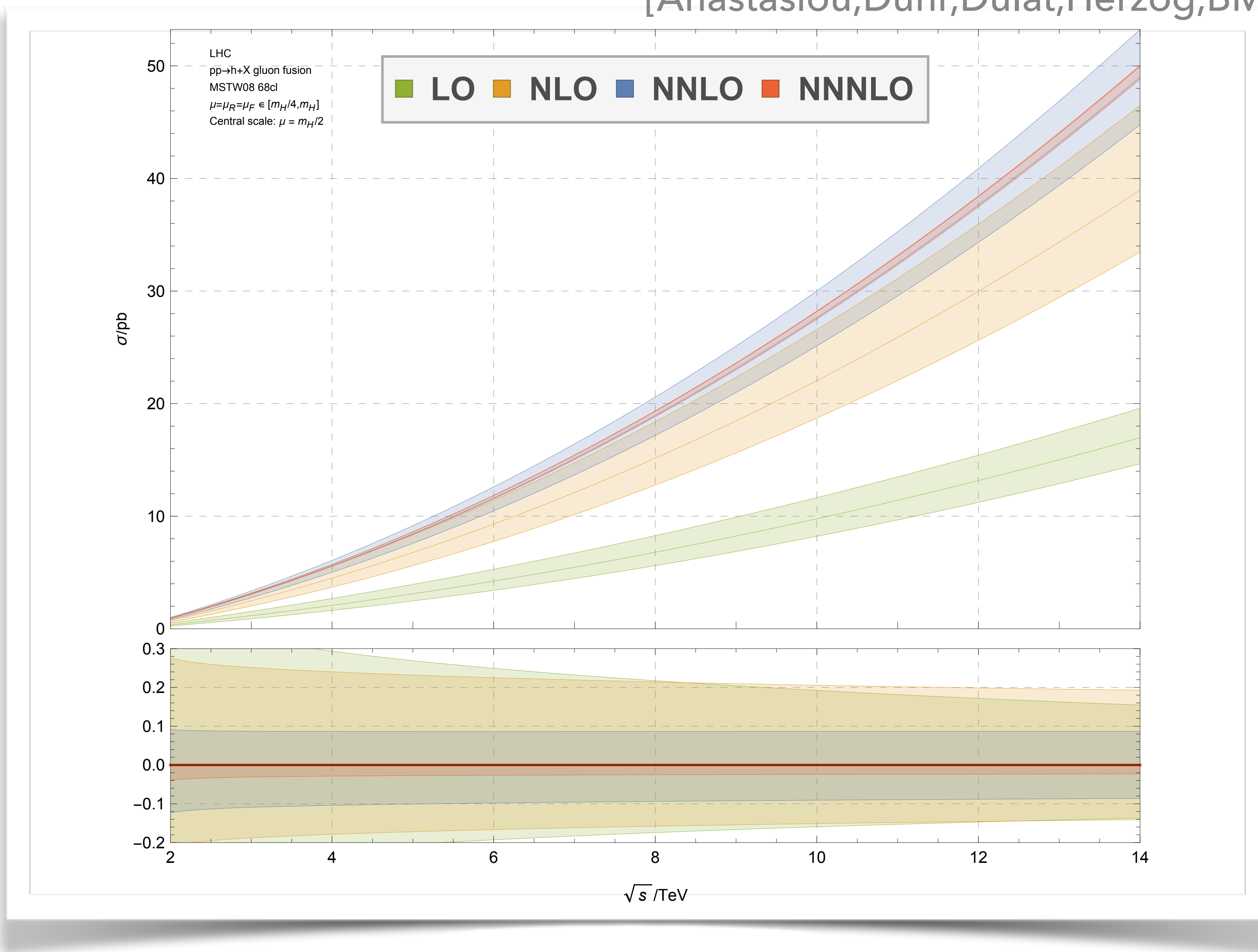
- Systematically improvable approximation.
  Precise and clean at high Q.
- Flexible to predict quantities directly related to experimental setup.
- High order process are multi-scale but at high Q all scales of similar size -> no large logarithms, perturbation theory is doing fine.

FIXED ORDER PERTURBATION THEORY

How well does it work?

- Higgs boson cross section: particularly bad perturbative behaviour.
- Stabilisation at N3LO.

Inclusive Higgs boson cross section as a function of LHC energy.
[Anastasiou,Duhr,Dulat,Herzog,BM]



FIXED ORDER PERTURBATION THEORY TODAY

NLO:

Automated tools, recursion relations / unitarity methods for amplitudes, large number of legs are no problem. "Solved", with notable exceptions.

NNLO :

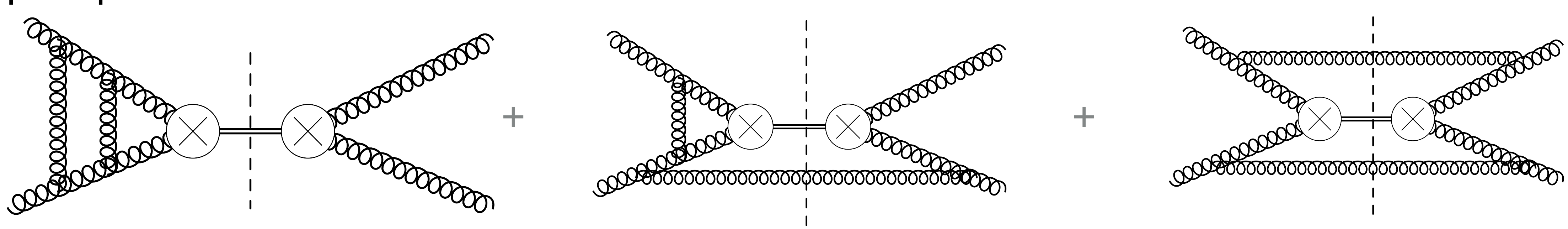
Current precision standard:

QCD corrections for most high precision **2 to 2** scattering processes.

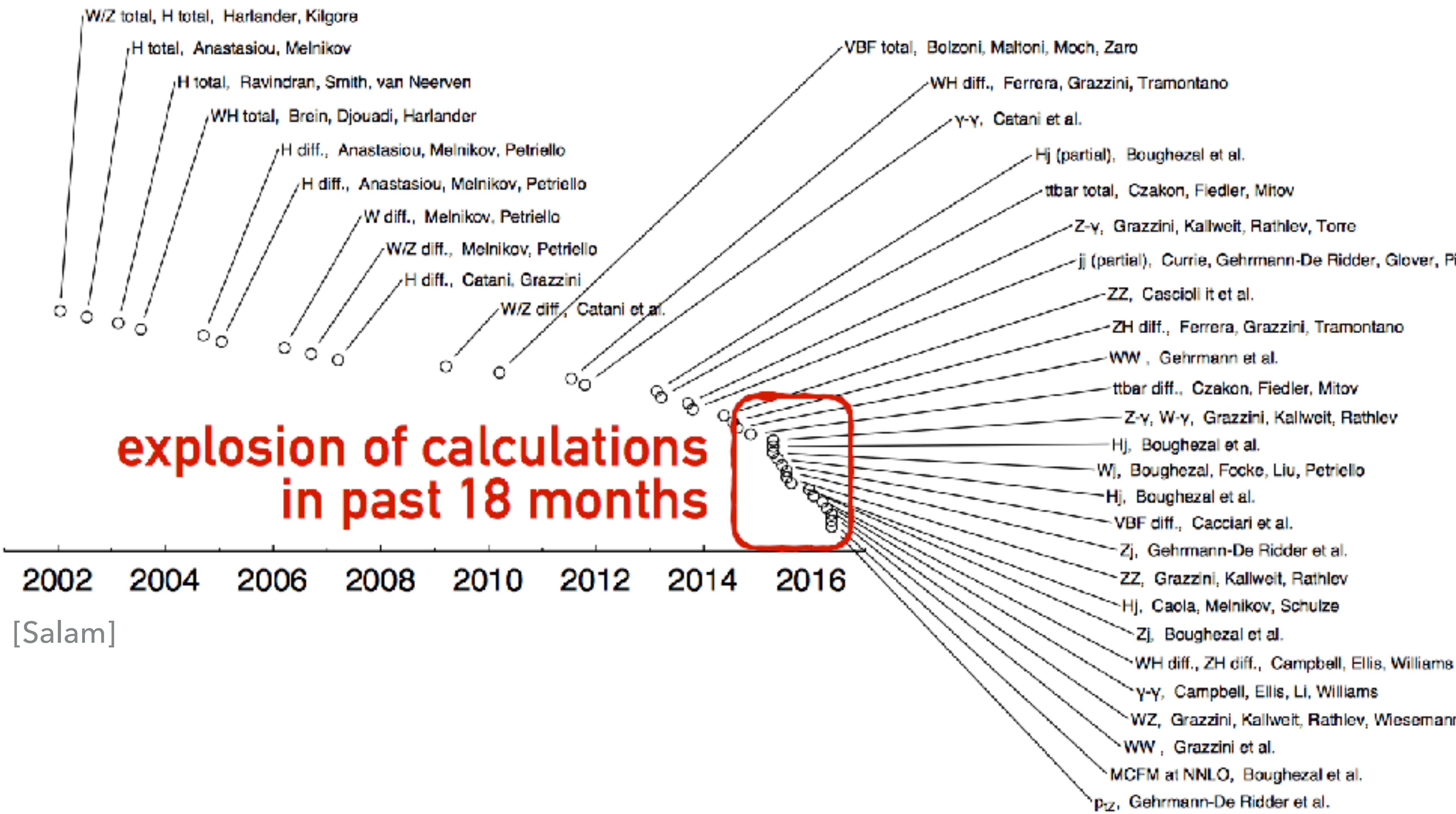
(+ VBF as a first approximate 2 to 3).

- Feynman diagram based.
  No reason to be scared of 10^5 diagrams.
- Split computation in separately divergent real and virtual contributions.
- Highly non-trivial problems to be solved for treatment loop amplitudes as well as real radiation.

NNLO:



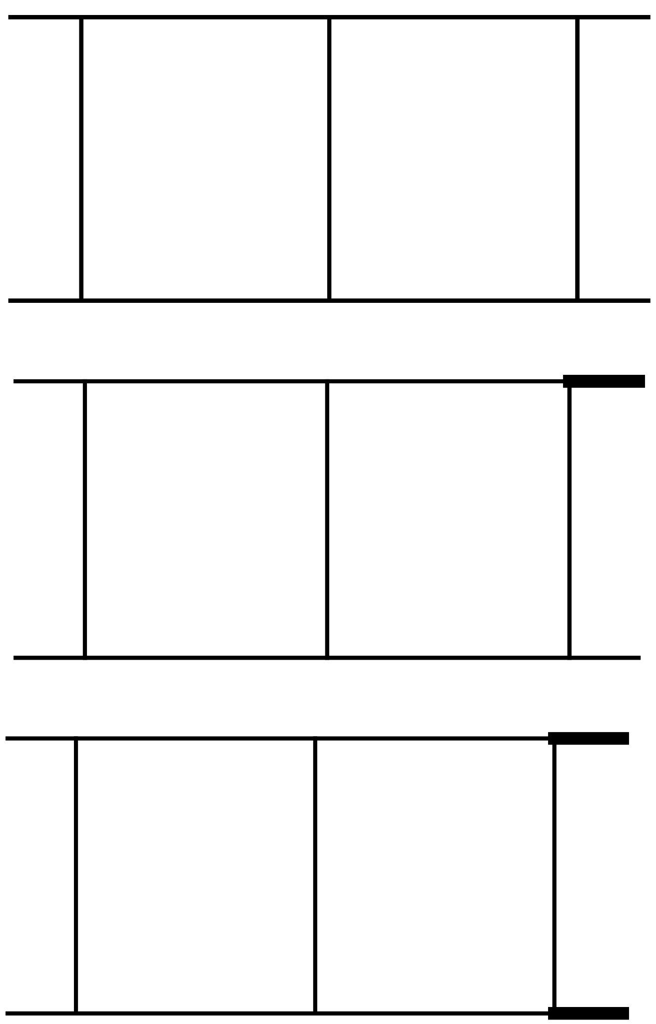
NNLO COMPUTATIONS AS A FUNCTION OF TIME

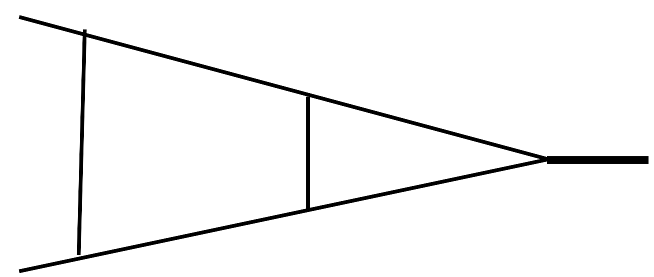


[Salam]

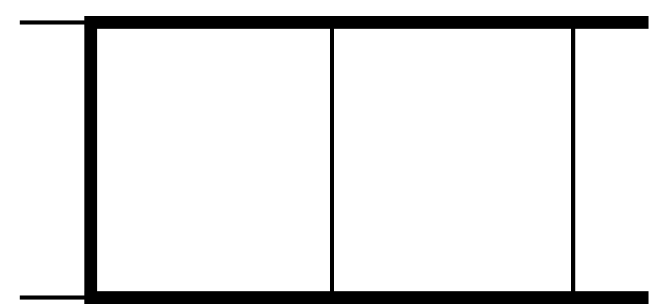
AMPLITUDE COMPUTATIONS

- 36 computations on the slide.
- Rely essentially on 5 different classes of two loop amplitudes.

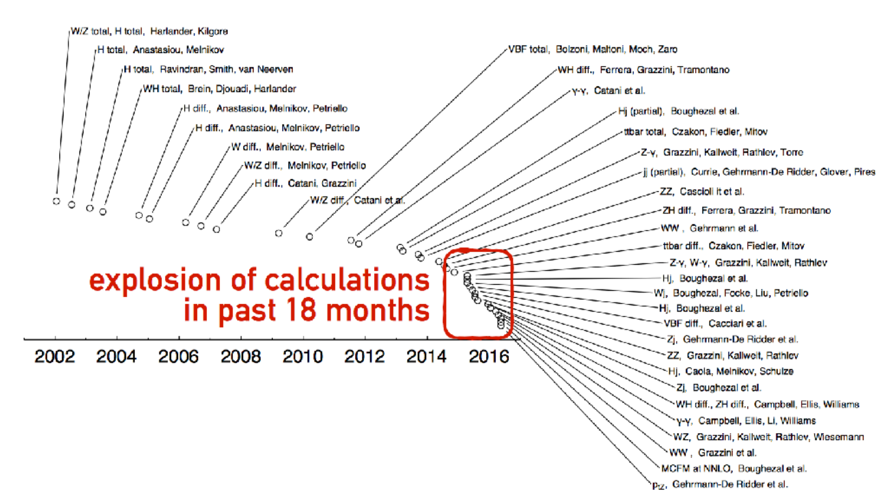




1983

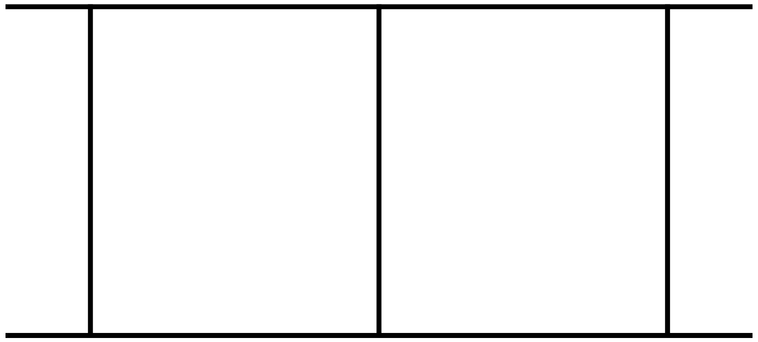


Numerical

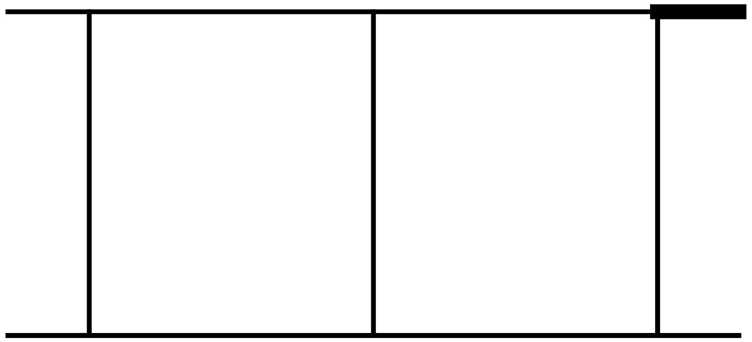


AMPLITUDE COMPUTATIONS

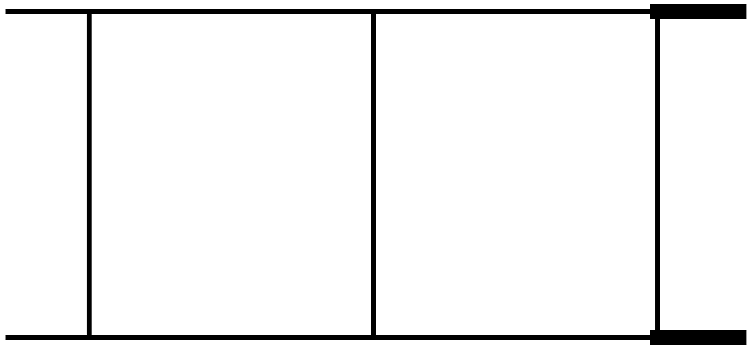
- Computing amplitudes for collider process happens hand-in-hand with technological development.



- Early 2 Loop unitarity techniques,
  Mellin-Barnes techniques,
  Integrand-expansions,
  Negative dimension approach,
  Direct computation, …



- Differential Equations,
  (2D) Harmonic Polylogarithms,
  Consistency for Boundary Conditions,…

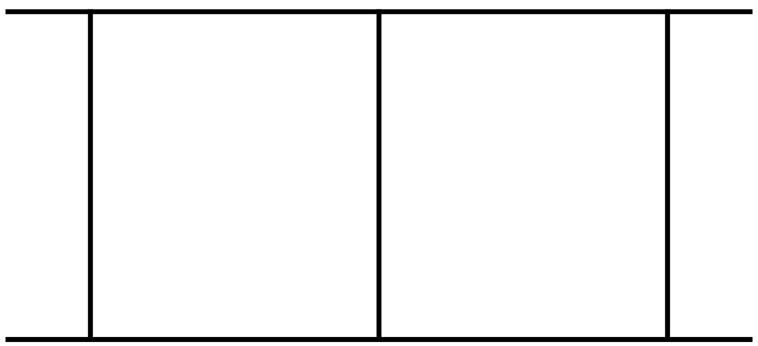


- Canonical Basis,
  Symbol Techniques,
  Stable and fast numerical implementations in physical region
  …

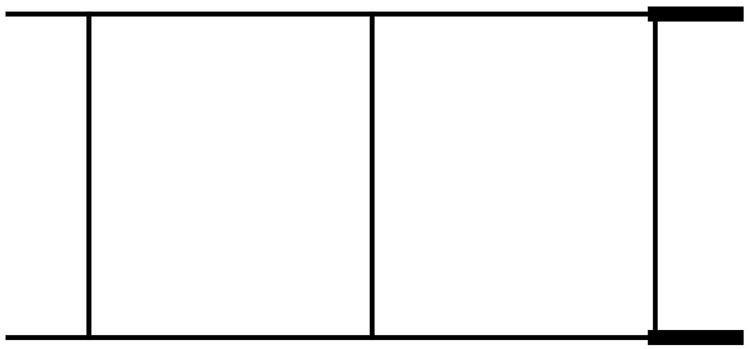
AMPLITUDE COMPUTATIONS

- Complexity reflected in the shear size of amplitudes

$gg \to \gamma\gamma$



$q\bar{q} \to WW$



$$
\begin{aligned}
G^{\rm L}_{-+-+} = \left(1-2\frac{x}{y^2}\right)\Big[& 4\mathrm{Li}_4(-y/x) + 4\mathrm{Li}_4(-y) - (3X - 2Y + i\pi)\mathrm{Li}_3(-y/x) \\
& -(X + 2Y + 3i\pi)\mathrm{Li}_3(-y) + ((X-Y)^2+\pi^2)\mathrm{Li}_2(-y/x) \\
& +((Y+i\pi)^2+\pi^2)\mathrm{Li}_2(-y) + \frac{1}{8}X^2(X-2Y)^2 \\
& -i\frac{\pi}{6}X\left((X+i\pi)^2 - 3XY\right)\Big] \\
-\frac{1}{2}\left(1+6\frac{x}{y^2}\right)\Big[& \mathrm{Li}_3(-x) - \zeta_3 - (X+i\pi)\left(\mathrm{Li}_2(-x) - \frac{\pi^2}{6}\right) \\
& -\frac{1}{6}X(X^2+4\pi^2) + \frac{1}{2}(X-2Y-i\pi)((X+i\pi)^2+\pi^2)\Big] \\
-\frac{1}{12}\left(5-2\frac{x}{y}\right)&(X+i\pi)((X+i\pi)^2-3\pi^2) + (X-Y)((X+i\pi)^2+\pi^2) \\
+\pi^2(X+i\pi) + \frac{1}{8}&\left(14\frac{x-1}{y} - 8y + 9y^2\right)((X-Y)^2+\pi^2) \\
+\frac{1}{8}\left(14\frac{1-x}{y}\right.&\left. - 8\frac{y}{x} + 9\frac{y^2}{x^2}\right)((Y+i\pi)^2+\pi^2) \\
+\frac{1}{8}\left(38\frac{x}{y^2}\right.&\left. - 13\right)((X+i\pi)^2+\pi^2) \\
-\frac{\pi^2}{6} - \frac{9}{4}\Big[&\left(y+2\frac{x}{y}\right)(X-Y) - \left(\frac{y}{x}+\frac{2}{y}\right)(Y+i\pi)\Big] + \frac{1}{2}\,, \qquad (5.6)
\end{aligned}
$$

[Bern,Dixon,Freitas]

`Aj_A-1.0.inc`	39 MB
`Aj_B-1.0.inc`	39 MB
`Aj_C-1.0.inc`	27 MB

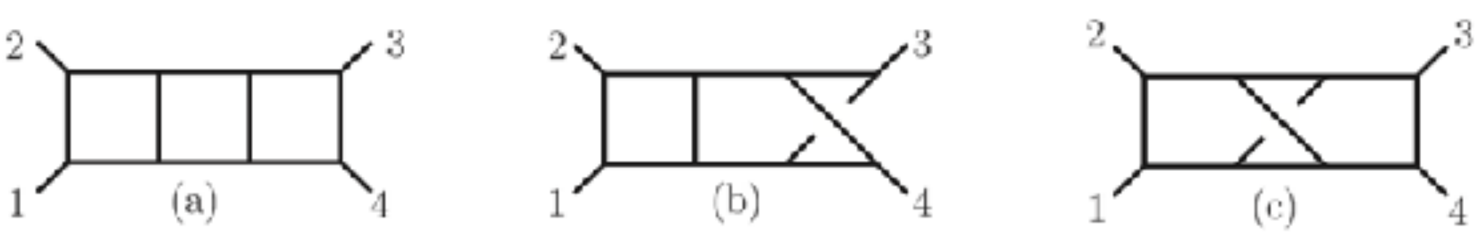
[Gehrmann, Manteuffel,Tancredi]

[Caola,Henn,Melnikov,Smirnov,Smirnov]

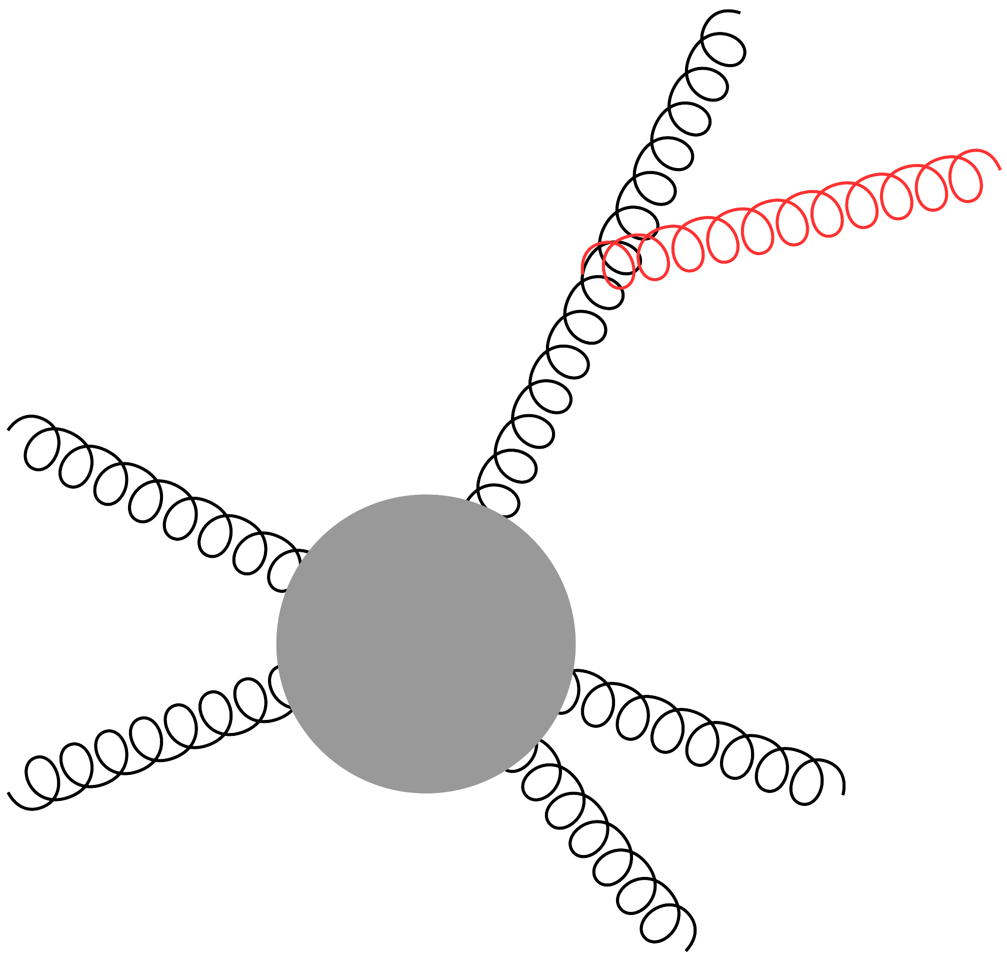
AMPLITUDE COMPUTATIONS - IMPRESSIVE PROGRESS

see talks by Andreas, Ben, Johannes, Lorenzo, Roman, Tiziano, …

- Numerical computation of Feynman integrals:
  SecDec. or numerical differential equations.
- Massive two-loop integrals: Progress in understanding extensions beyond generalised poly logarithms. (Application to ttbar, HH, H+J, Higgs possible ?)
- Two loop integrals at five points:
  Early / partial results appear! (5 partons, H+4 partons).
- Unitarity at two loops:
  (Generalised / d-dimensional / numerical /prescriptive, …)
- Spin - offs of technology to more formal places:
  4-point amplitudes in N=4 SYM



INTEGRATING OVER UNRESOLVED RADIATION



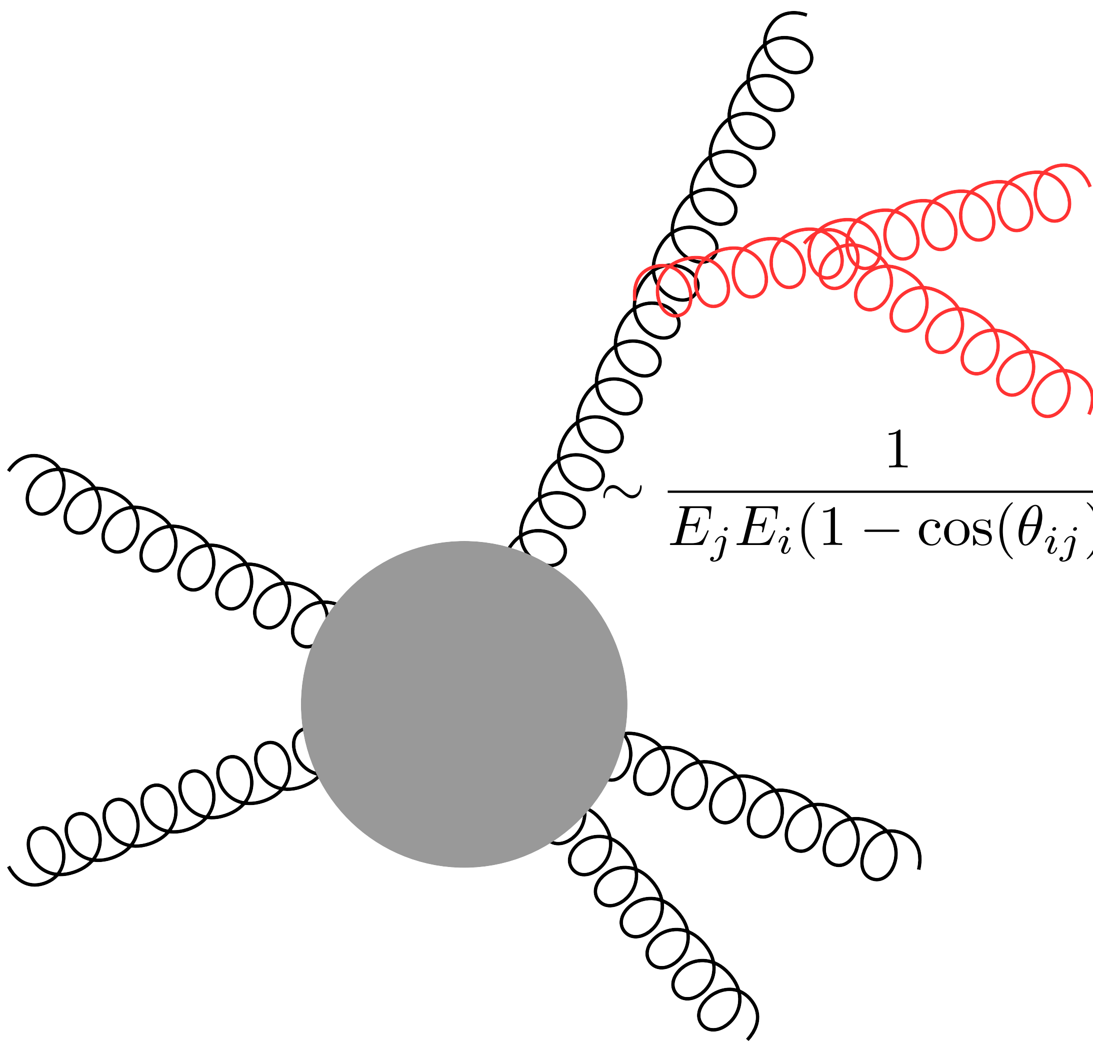
(morally)

$$\sim \frac{1}{E_j E_i (1 - \cos(\theta_{ij}))}$$

- Divergent as energies vanish or partons become collinear.
- 1 Parton subtraction:
  Essentially a solved issue.
- Still a challenge to be numerically stable in unresolved regions, especially if loops are involved.

$$\sigma_{\text{finite - j}} \sim \int dE_j dcos\theta_{ij} \left[|\mathcal{M}^{(1)}|^2 - \frac{1}{E_j E_i (1 - \cos(\theta_{ij}))} |\mathcal{M}^{(0)}|^2 \right]$$

INTEGRATING OVER UNRESOLVED RADIATION



- Non trivial structure of IR singularities. Subtraction becomes a major challenge!

$$\sim \frac{1}{E_j E_i(1-\cos(\theta_{ij}))}\,\frac{1}{E_i E_k(1-\cos\theta_{ik})+E_i E_j(1-\cos\theta_{ij})+E_j E_k(1-\cos\theta_{jk})}$$

- Overlap of singularities, increase of number of required subtractions.

INTEGRATING OVER UNRESOLVED RADIATION

- Past years have see several solutions to this problem.
  Some, in principle, fully general for NNLO.

- In practice, successful application to
  2 -> 2 processes.

- Life in practice:

- Sector decomposition
- Non-Linear Mappings
- qT
- FKS+
- N-Jettiness
- Antenna
- Colourful
- Projection-To-Born
- …

$2 \to 1$
Inclusive cross sections - analytic formulae: ~ seconds

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Differential cross sections for Higgs boson final states: 10 - 100 CPU hours

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- **Projection-To-Born**
- **…**

$2 \to 1$
Inclusive cross sections - analytic formulae: ~ seconds

$2 \to 1$
Differential cross sections for Higgs boson final states: 10 - 100 CPU hours

$2 \to 2$
Differential cross sections for Higgs + J boson final states: 100000+ CPU hours

REAL LIFE CROSS SECTIONS FOR THE LHC

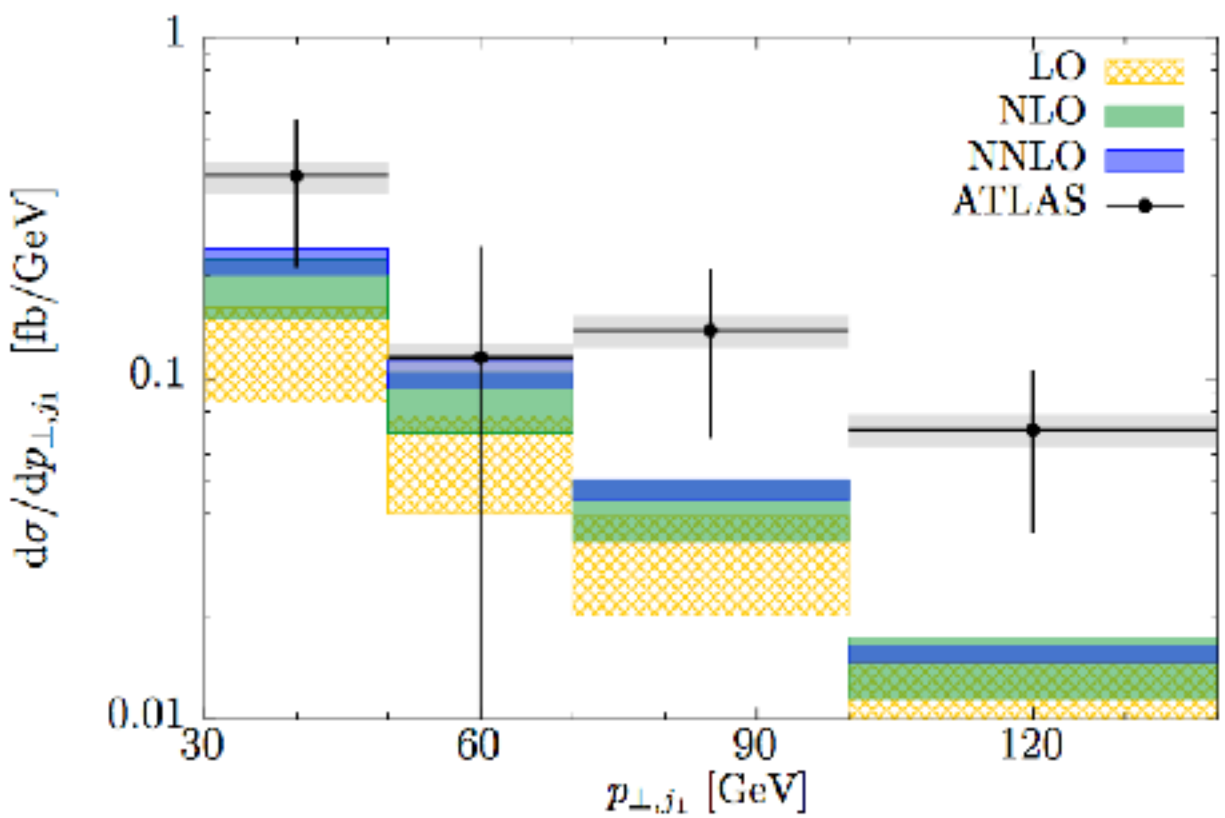
- Precision phenomenology is more than computing real and virtual amplitudes for QCD corrections.
- Implementation and numerics play a substantial role.
- Corrections due to masses, electro-weak physics, PDFs, resummation, hadronisation, underlying event, detector, etc.
- At high precision: many competing uncertainties of similar size.

Inclusive Higgs boson cross section:

$$
\begin{aligned}
48.58\,\mathrm{pb} = \; & 16.00\,\mathrm{pb} && (+32.9\%) && (\mathrm{LO,\ rEFT}) \\
& +20.84\,\mathrm{pb} && (+42.9\%) && (\mathrm{NLO,\ rEFT}) \\
& -\;\; 2.05\,\mathrm{pb} && (-4.2\%) && ((t,b,c),\ \mathrm{exact\ NLO}) \\
& +\;\; 9.56\,\mathrm{pb} && (+19.7\%) && (\mathrm{NNLO,\ rEFT}) \\
& +\;\; 0.34\,\mathrm{pb} && (+0.7\%) && (\mathrm{NNLO},\ 1/m_t) \\
& +\;\; 2.40\,\mathrm{pb} && (+4.9\%) && (\mathrm{EW,\ QCD\text{-}EW}) \\
& +\;\; 1.49\,\mathrm{pb} && (+3.1\%) && (\mathrm{N^3LO,\ rEFT})
\end{aligned}
$$

+ Strong coupling, PDFs, Missing mixed QCD -EWeak corrections etc.

NNLO PRECISION RESULTS

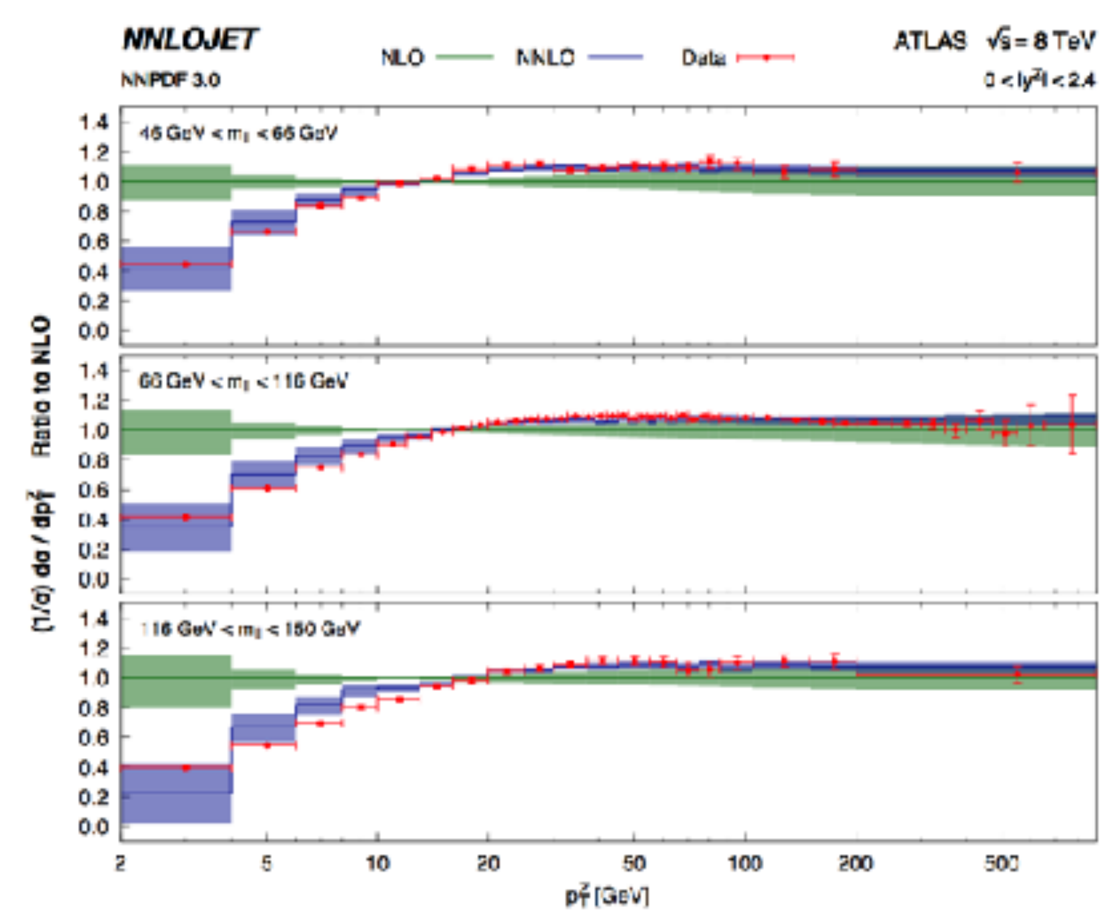


- Higgs + Jet:
  Still statistics limited.
  NNLO corrections reduce perturbative uncertainty.

[Boughezal et al., Chen et al., Caola et al.]

- Z - transverse momentum
  THE precision process.
  NNLO improves a lot agreement with data.
  Some puzzles to be investigated,
  (Luminosity uncertainties, PDFs, scales, low pt-effects,etc.)

[Boughezal et al., Gehrmann et al.]

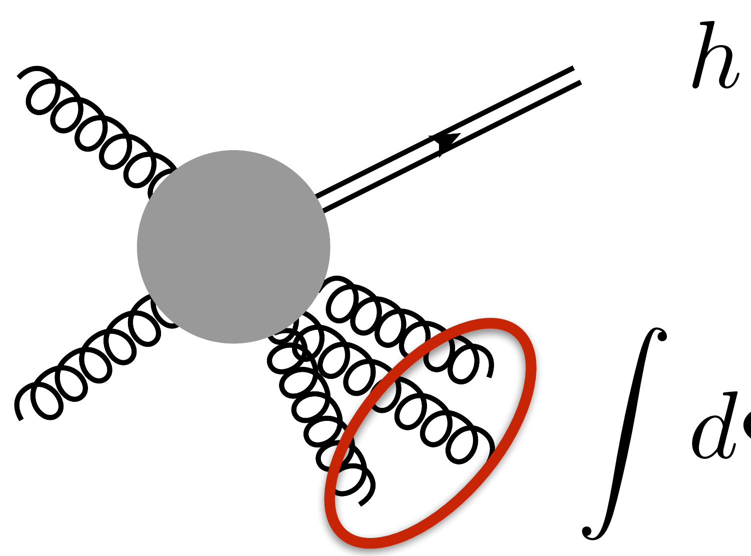


DIFFERENTIAL BEYOND NNLO 2->2 SCATTERING

- More legs:
    - Challenges for new subtraction schemes.
      Excellent phase space generation and numerical subtraction required.
      Fast and stable double real and real-virtual amplitudes
    - Virtual amplitudes become huge functions of many scales:
      Challenges at every step: Reduction, differential equations, analytic continuation, implementation.
      Numerical solution?
- (More) masses:
    - New analytic functions to be understood.
      Sunt hic leones?
- More loops
    - N3LO differential, even more complex subtraction required.

N3LO DIFFERENTIAL CROSS SECTIONS – HIGGS DIFFERENTIAL

- Analytic approach to N3LO differential cross sections.
- Ask specific questions about the final state that you are interested in:



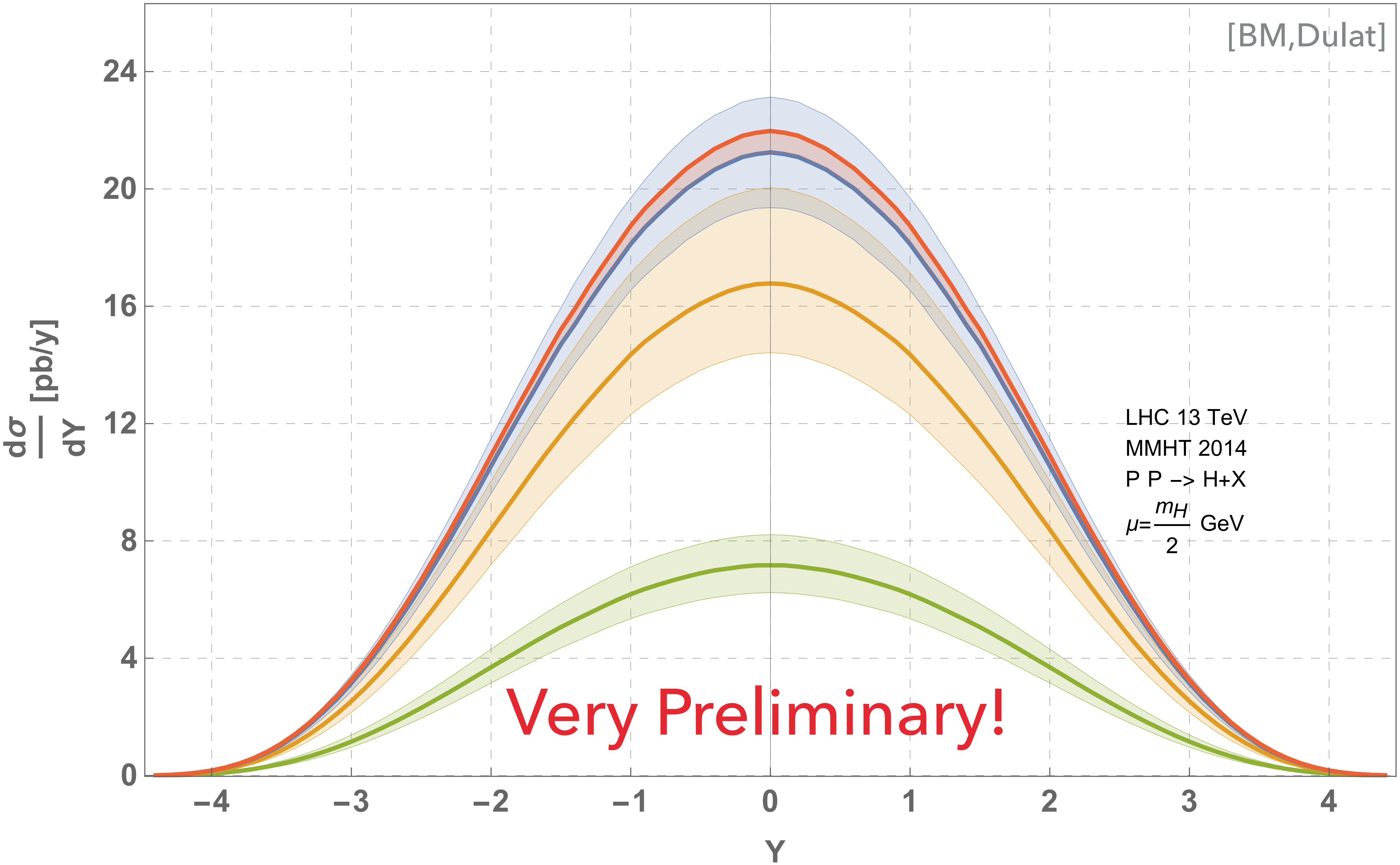
$$\int d\Phi_n$$

- Analytically integrate out all real radiation - takes care of subtraction.
- Maintain all differential information about the Higgs.
  Fiducial cross sections for interesting real-life final states.

$$P\ P \rightarrow H + X \rightarrow \gamma\gamma + X$$

$$P\ P \rightarrow H + X \rightarrow 4l + X$$

HIGGS DIFFERENTIAL – SOFT VIRTUAL N3LO CORRECTIONS



CONCLUSIONS

- High order precision predictions are inseparably intertwined with the success of LHC phenomenology.
- Remarkable progress in precision computation in past couple of years.
- Desire to predict better inspires to develop our understanding of QFT.
- High precision observables:
  A select class of LHC observables that deserve our special attention and that require NNLO precision and beyond.
  Percent level precision will be a reality.
  Many avenues for future developments!

Thank you!