Soft Gluon Resummation with (mostly very) Massive Quarks

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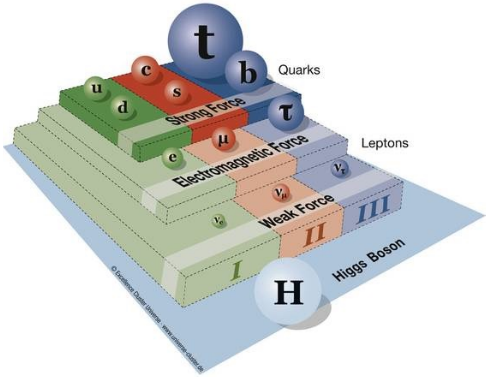
Heavy Flavours at High Pt Workshop, Edinburgh, 30.11.2023

Tops and Bottoms with A bit of Charm



- Realm of perturbative QCD, yet very different species
  - Different position of the mass scale in the hierarchy of scales for a given process or observable -> potentially large logarithms involving masses
  - Different lifetimes -> hadronize or not

- Resummation of soft gluon emission important in all cases
  - better description of physics: additional corrections to cross sections… especially applicable when higher fixed-order calculations out of reach
  - restoration of predictive power of perturbation theory
  - reduction of the theory error due to scale variation
  - cross-talk with parton shower techniques and tools

Factorization

- Factorization of soft and collinear emission is the underlying principle allowing resummation
- Intuitive: short distance incoherent with long distance dynamics, long-wavelength soft gluons "insensitive" to short wavelength physics
- Details of factorization / resulting expressions for cross section depend on the specific process and observable in question
  - $\alpha_s^n \log^m(\ldots)$ originating from soft/collinear emissions
    - threshold logs : $\alpha_s^n \log^m\left(1 - {}^{M^2}/_{s}\right)$ (absolute threshold, M= sum of final state particle masses), $\alpha_s^n \left(\frac{\log^m\left(1-{}^{Q^2}/_{s}\right)}{1-{}^{Q^2}/_{s}}\right)_+$ (invariant mass threshold), …
    - recoil logs $\alpha_s^n \log^m\left({}^{p_T^2}/_{Q^2}\right)$
    - mass logs $\alpha_s^n \log^m\left({}^{m_b^2}/_{Q^2}\right)$
    - …
- Various formalisms (direct QCD, SCET), shown to be equivalent

SOFT GLUON RESUMMATION

Systematic reorganization of perturbative series

$$\hat{\sigma} \sim c_{00} +$$

$$+\alpha_s \Big(c_{12} \log^2(\beta^2) + c_{11}\log(\beta^2) + c_{10} \Big) \leftarrow \text{NLO}$$

$$+\alpha_s^2 \Big(c_{24} \log^4(\beta^2) + c_{23}\log^3(\beta^2) + c_{22}\log^2(\beta^2) + \ldots \Big) \leftarrow \text{NNLO}$$

$$\ldots \qquad \ldots \qquad \ldots$$

$\alpha_s^n \log^{2n}(\beta^2)$ $\quad$ $\alpha_s^n \log^{2n-1}(\beta^2)$

$\log(\beta^2) \leftrightarrow \log(N) \equiv L$

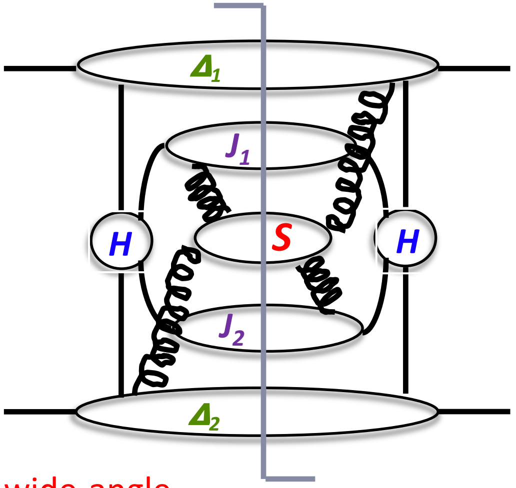
Factorization at threshold: space of Mellin moments *N*

$$\hat{\sigma}^{(N)} \sim \mathcal{C}(\alpha_s) \exp\left[L g_1(\alpha_s L) + g_2(\alpha_s L) + \alpha_s g_3(\alpha_s L) + \ldots\right]$$

sums up $\quad$ LL: $\alpha_s^n \log^{n+1}(N)$ $\quad$ NLL: $\alpha_s^n \log^{n}(N)$

Structure of the Resummed Cross Section

- Resummation ⟷ all-order factorization of soft and collinear emissions
- Exponentiation ⟷ solutions of RGEs
- For a generic process with n jets in the final state:



Hard function: hard off-shell radiation

Soft function: soft wide-angle

$$d\sigma^{\mathrm{res}}_{i\to f}(N,\mu_F,\mu_R,\ldots) = \mathrm{Tr}\,[\mathbf{H}_{i\to f}(\mu_F,\mu_R,\ldots)\mathbf{S}_{i\to f}(N,\mu_R,\ldots)]$$
$$\Delta_{i_1}(N,\mu_F,\mu_R,\ldots)\Delta_{i_2}(N,\mu_F,\mu_R,\ldots)\, J_{f_1}(N,\mu_R,\ldots)\ldots J_{f_n}(N,\mu_R,\ldots)$$

Jet functions: collinear emission: incoming & outgoing

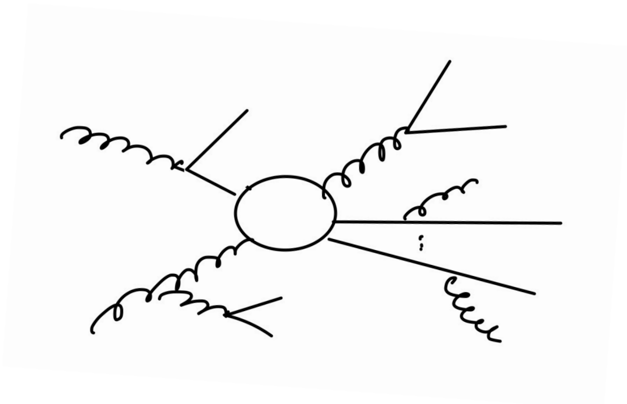
- If heavy particles in the final state (e.g. tops) no final state jets
- Processes with four or more legs carrying colour: hard and soft functions are matrices in colour space

Multi-Scale Problem

- Multi-scale problems: appearance of the logarithms of ratios of various scales
- Hierarchy of scales determines the importance of these terms, e.g. m_b , m_c << Q^2

4FS (tough)

5FS (easier)



DGLAP → resum logs of m_b/Q ← fragmentation

[Maltoni, Ridolfi, Ubiali'12] [Lim,Maltoni, Ridolfi, Ubiali'13] ...

[Cacciari, Greco, Nason'98] [Mele, Nason'91] [Ridolfi, Ubiali,Zaro'19] ...

see the talks yesterday!

+SOFT

- Combination of the massive fixed-order results with resummation of mass logarithms In the fragmentation approach*[Cacciari, Greco, Nason'98] [Cacciari, Frixione, Nason'98][Neubert'07][Forte, Laenen, Nason, Rojo'10],Forte, Napoletano, Ubiali'16, Bonvini, Papnastasiou, Tackmann'15, Pietrulewich et al.'17, Ridolfi, Ubiali, Zaro'19*

- Desirable to improve the predictions by resumming jointly with soft gluons, but different structure of soft logs in the two mass schemes (double log vs single log in the fully massive calculations) due to non-commuting massless and soft limits *[Cacciari, Corcella, Mitov'02][Corcella,Mitov'04][Aglietti et al.'07] [Gaggero, Ghira, Marzani, Ridolfi'22] [Aglietti, Ferrera'22]*

- New approach relies on modifying jet function through its calculation in the quasi-colinear limit *[Ghira, Marzani, Ridolfi'23] ,* recovers the resummation result for fragmentation *[Cacciari, Catani'01]* for mass scales smaller than soft scale and the usual soft gluon resummation for the reversed order

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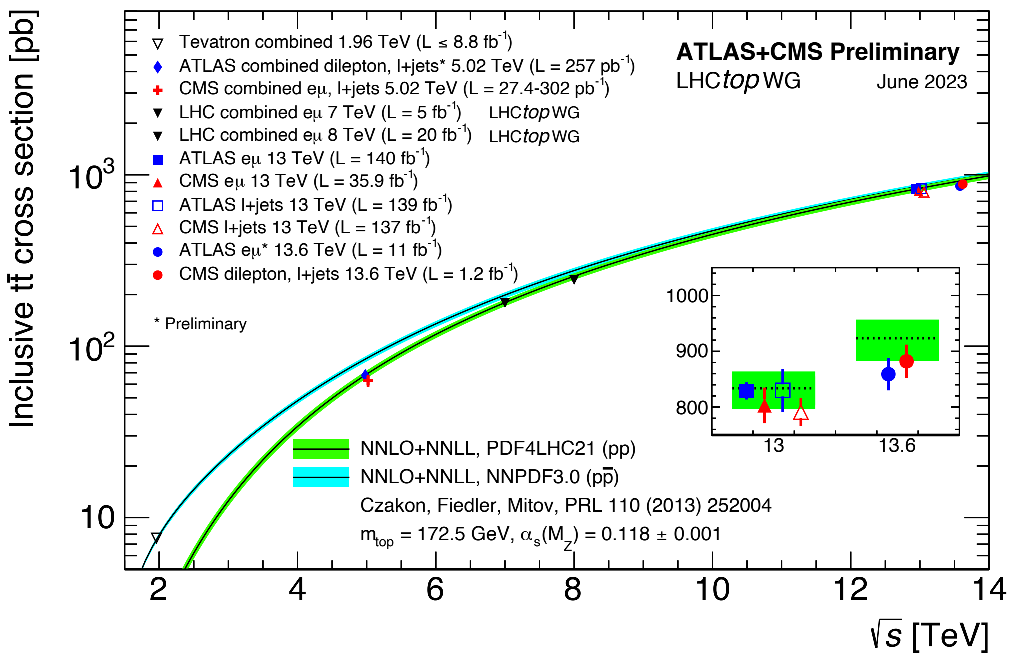
In this context, tops are much easier to handle

25+ Years of Resummation for Heavy Flavours

- Top-pair production
  - Pioneering analytical work of *[Kidonakis, Sterman'96 '97]* and *[Bonciani, Catani, Mangano, Nason,'98]* -> NLL resummation
  - Extension to NNLL and numerical results in *[Czakon, Mitov, Sterman'09] [Cacciari et al.'11]* (direct QCD) and *[Beneke et al. '09-'12] [Ahrens et al.'10-'11]* (SCET)



$$\log\left(1-\frac{4m_t^2}{\hat{s}}\right)$$

DIFFERENTIAL DISTRIBUTIONS tT

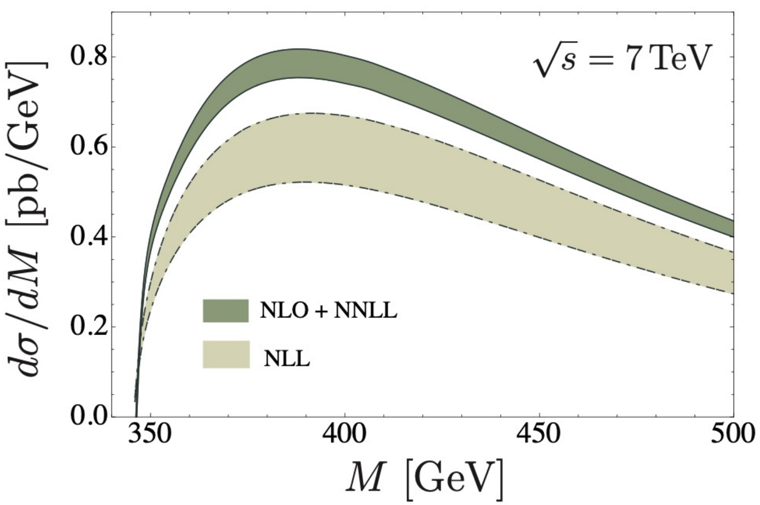
[Ahrens et al.'10-'11] (SCET)

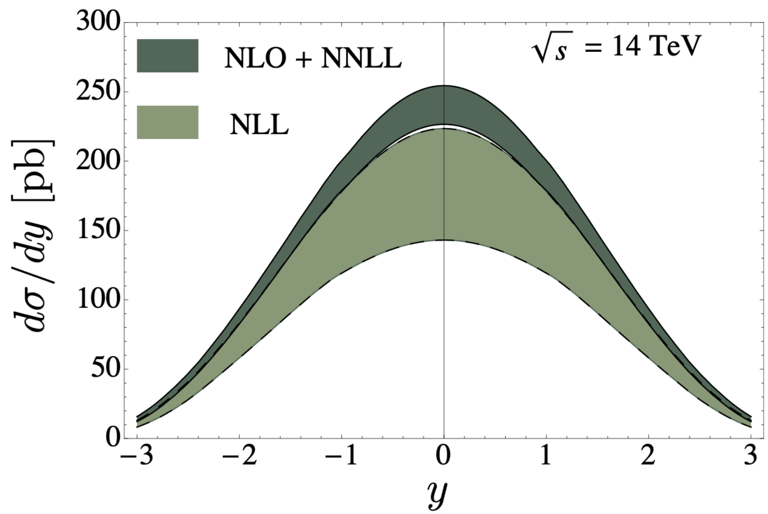
- Pair invariant-mass (PIM) kinematics

$$\log(1-z) \qquad z = \left(\frac{Q^2}{s}\right)$$

- 1-particle inclusive (1PI) kinematics

$$\log\left(\frac{s_4}{m_t^2}\right) \qquad s_4 = s + t + u$$





Boosted Tops

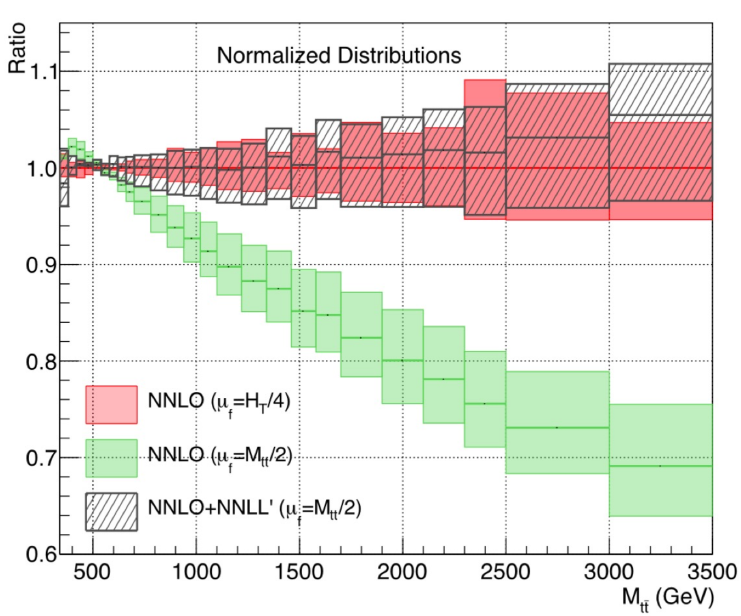
[Ferroglia,Pecjak, Yang'12-'13] [Ferroglia, Marzani, Pecjak,Yang'13] [Czakon et al.'19]

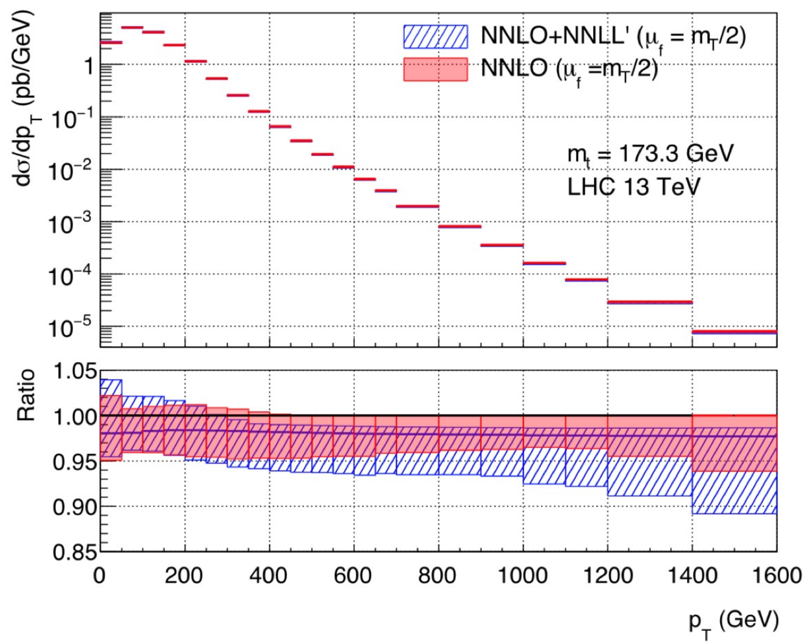
- Boosted soft limit $s \gg m_t^2 \gg s(1-z)^2$

$\log(1-z)$ $\qquad$ $\log\left(\frac{m_t^2}{Q^2}\right)$

$$d\hat{\sigma}_{ij} \sim \mathrm{Tr}[\mathbf{H}_{ij}(M,\mu)\mathbf{S}_{ij}(M(1-z),\mu)] \otimes C_D^2(m_t,\mu)S_D^2(m_t(1-z),\mu) + \mathcal{O}(1-z) + \mathcal{O}\left(\frac{m_t^2}{M^2}\right)$$

soft $\qquad$ quasi-collinear

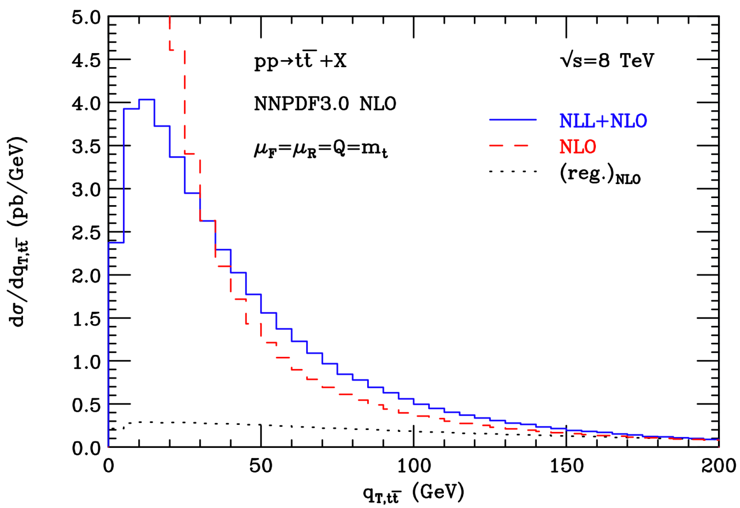




PT OF THE TOPS

$$\log\left(\frac{q^2_{t\bar{t},T}}{Q^2}\right)$$

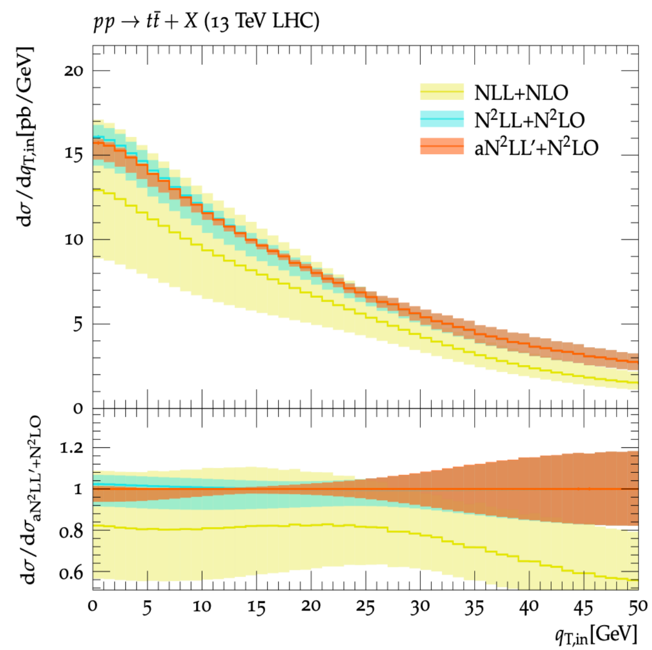
- Transverse momentum of the top pair



[Catani,Grazzini, Torre'14]
[Catani,Grazzini, Sargsyan'18]

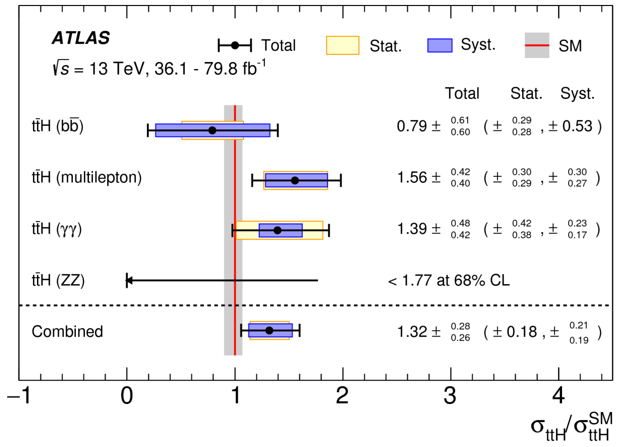
- Projected transverse momentum

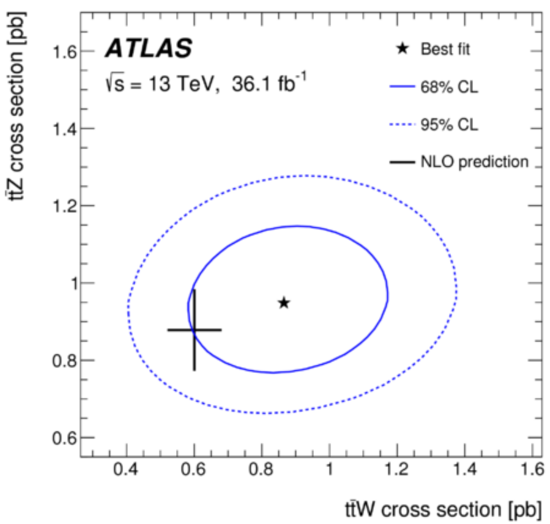
$$\frac{d\sigma}{dq_\tau} \qquad q_\tau = |\vec{q}_{t\bar{t},T}\ \vec{\tau}|$$



[Ju, Schönherr'23]

- Also available: 0-jettiness resummation *[Alioli, Broggio, Lim'21]*



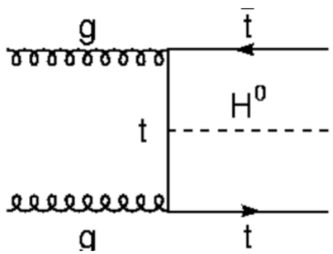


$t\bar{t}H,\ t\bar{t}W/Z,\ tH$

associated production

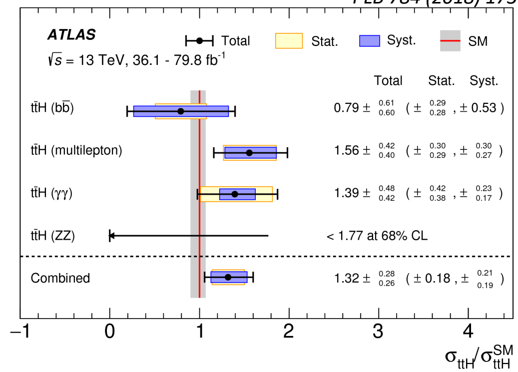
Associated Higgs Production with Top Quarks

$$pp \to t\bar{t}H$$



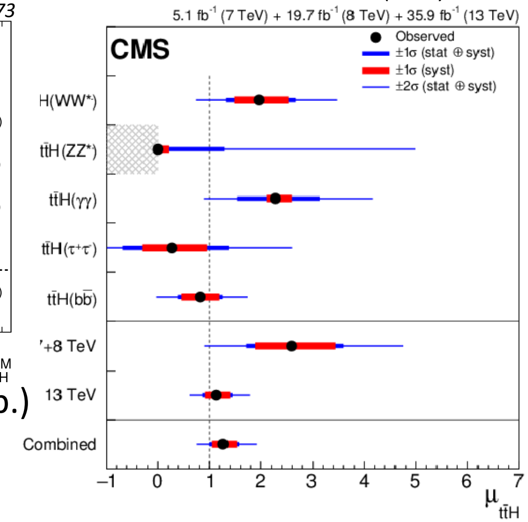
- Direct probe of the strength of the top-Yukawa coupling
- Crucial for understanding the Higgs sector and in searches for deviations from the SM
- Far-reaching consequences → stability of our Universe

PLB 784 (2018) 173

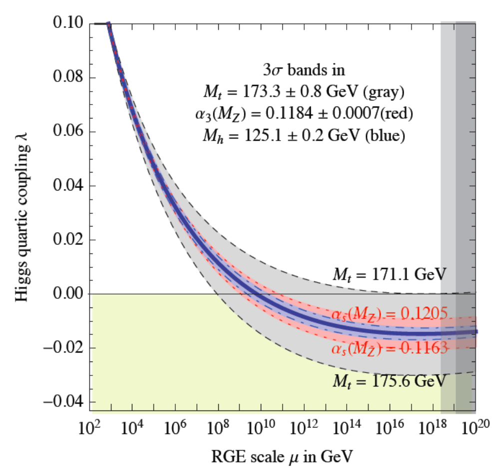


ATLAS, Run1 + Run2: **6.3σ** (5.1σ exp.)

PRL 120 (2018) 231801



CMS, Run1 + Run2: **5.2σ** (4.9σ exp.)



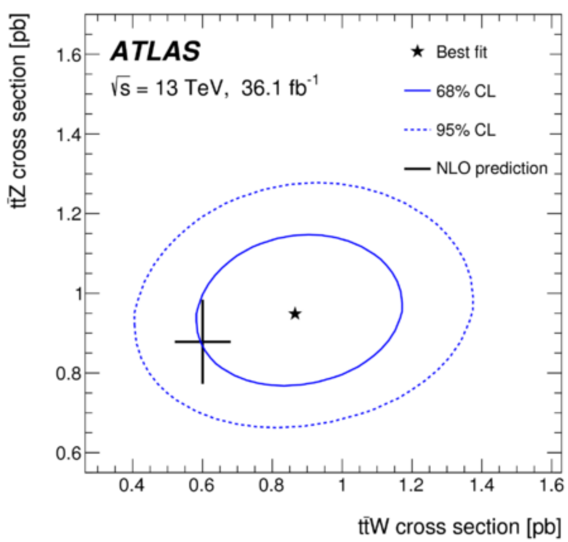
$$4\pi^2 \frac{d\lambda}{d\ln\mu^2} \cong -3y_t^4 + 6\lambda y_t^2 + 12\lambda^2 + \ldots$$

[Buttazzo et al.'13]

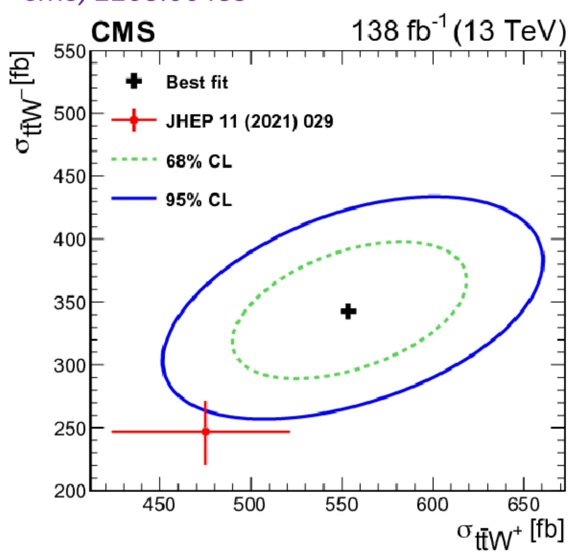
TTBAR+W,Z

- Probes of top-quark couplings
- Sensitive to BSM contributions
- Dominant backgrounds to searches and SM precision measurements (*ttH* included)

ATLAS, PRD 99 (2019) 072009



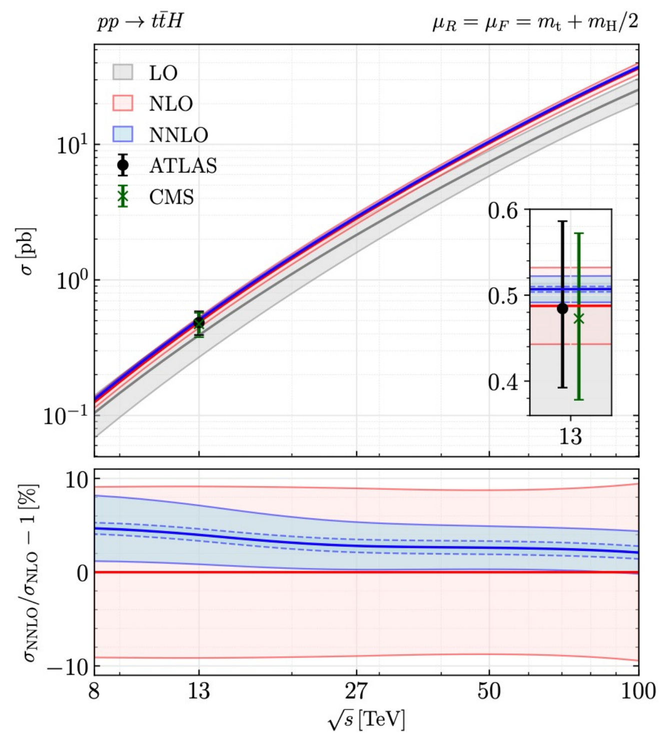
CMS, 2208.06485



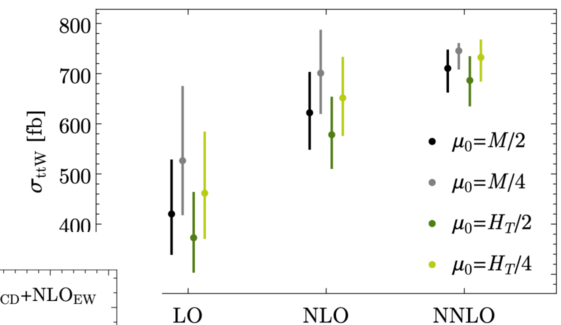
- Measurements of differential distributions of the Z boson produced in association with ttbar

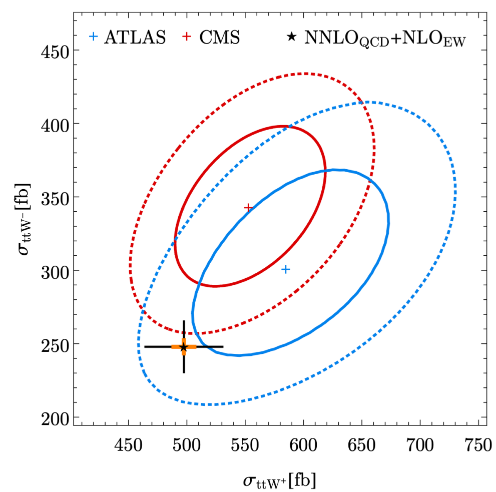
TTBARH AND TTBARW AT NNLO

- soft Higgs approximation for the 2-loop amplitudes *[Catani et al. '22]*



- soft approximation to W emission as well as massification procedure to W+4 parton amplitudes *[Buonocore et al.'23]*





Resummation for ttbar+B

- Threshold limit in the invariant mass kinematics: $\hat{s} \to Q^2 = (p_t + p_{\bar{t}} + p_B)^2$
- Large logarithmic contributions of the form $\alpha_s^n \left(\frac{\log^m(1-\hat{\rho})}{1-\hat{\rho}}\right)_+$; $\hat{\rho} = Q^2/\hat{s}$
- Direct QCD → N space:

$$\frac{d\tilde{\hat{\sigma}}_{ij\to klB}}{dQ^2}(N, Q^2, \{m^2\}, \mu_F^2, \mu_R^2) = \int_0^1 d\hat{\rho}\, \hat{\rho}^{N-1} \frac{d\hat{\sigma}_{ij\to klB}}{dQ^2}(\hat{\rho}, Q^2, \{m^2\}, \mu_F^2, \mu_R^2)$$

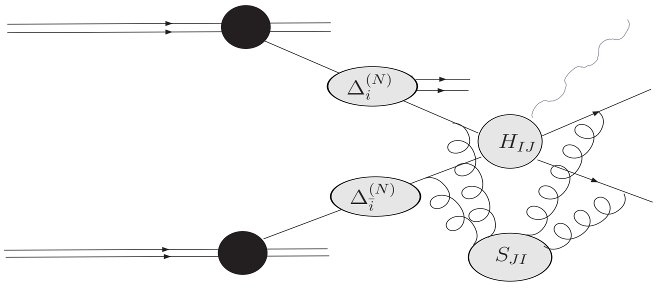
- In Mellin space, logarithms of $1 - \hat{\rho}$ become logarithms of Mellin moments N
- Resummation

$$\hat{\sigma}^{(N)} \sim \mathcal{C}(\alpha_s) \exp\left[L g_1(\alpha_s L) + g_2(\alpha_s L) + \alpha_s g_3(\alpha_s L) + \ldots\right]$$

LL: $\alpha_s^n \log^{n+1}(N)$ NLL: $\alpha_s^n \log^{n}(N)$ NNLL: $\alpha_s^n \log^{n-1}(N)$

RESUMMATION FOR TTBAR+B

- Resummation ⟷ all-order factorization of soft and collinear emissions
- Exponentiation ⟷ solutions of RGEs



$$\frac{d\tilde{\hat{\sigma}}^{(\mathrm{res})}_{ij\to klB}}{dQ^2}(N, Q^2, \{m^2\}, \mu_F^2, \mu_R^2) =$$

$$= \mathrm{Tr}\left[\mathbf{H}_{ij\to klB}(Q^2, \{m^2\}, \mu_F^2, \mu_R^2)\mathbf{S}_{ij\to klB}(N+1, Q^2, \{m^2\}, \mu_R^2)\right]\Delta^i(N+1, Q^2, \mu_F^2, \mu_R^2)\Delta^j(N+1, Q^2, \mu_F^2, \mu_R^2)$$

Hard off-shell — Soft wide-angle: Process-dependent

Collinear: Universal, known ✓

[Kodaira, Trentadue'82-'83][Catani, d'Emilio, Trentadue'88][Sterman'87][Moch, Vermaseren, Vogt'04]

Processes with four or more legs carrying colour: hard and soft functions are matrices in colour space

NNLL FOR TTBAR+B

- Soft function from the solution of the RG equation

$$\mathbf{S}_{ij\to klB}(N,Q^2,\{m^2\},\mu_{\mathrm{R}}^2) = \bar{\mathbf{U}}_{ij\to klB}(N,Q^2,\{m^2\},\mu_{\mathrm{R}}^2)\ \tilde{\mathbf{S}}_{ij\to klB}(\alpha_{\mathrm{s}}(Q^2/\bar{N}^2))\mathbf{U}_{ij\to klB}(N,Q^2,\{m^2\},\mu_{\mathrm{R}}^2)$$

$$\mathbf{U}_{ij\to klB}(N) = \mathrm{P}\exp\left[\int_{\mu}^{Q/\bar{N}} \frac{dq}{q}\mathbf{\Gamma}_{ij\to klB}\left(\alpha_{\mathrm{s}}\left(q^2\right)\right)\right] \qquad \mathbf{\Gamma}_{ij\to klB} = \left[\left(\frac{\alpha_s}{\pi}\right)\mathbf{\Gamma}^{(1)} + \left(\frac{\alpha_s}{\pi}\right)^2\mathbf{\Gamma}^{(2)} + \dots\right]$$

  - Soft anomalous dimensions $\mathbf{\Gamma}$ known at two loops for any number of legs *[Mert-Aybat, Dixon, Sterman'06] [Becher, Neubert'09] [Mitov, Sterman, Sung'09-'10] [Ferroglia, Neubert, Pecjak, Yang'09] [Beneke, Falgari, Schwinn'09], [Czakon, Mitov, Sterman'09] [Kidonakis'10]* ✓
  - To calculate, eliminate path-ordered exponentials:
    - use basis that diagonalize $\mathbf{\Gamma}^{(1)}$ $\rightarrow$ enough for NLL accuracy
    - For NNLL, treat matrices **U** perturbatively *[Buras'80][Buchalla, Buras, Lautenbacher'96]] [Ahrens, Ferroglia, Neubert, Pecjak, Yang'10]*

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- Hard function $\mathbf{H}_{ij\to klB} = \mathbf{H}^{(0)}_{ij\to klB} + \frac{\alpha_s}{\pi}\mathbf{H}^{(1)}_{ij\to klB} + \ldots$
  - $\mathbf{H}^{(1)}$ extracted from aMC@NLO *[Frederix, Frixione, Hirschi, Maltoni, Pittau, Torrielli'11]* (*ttH, ttW, ttZ*) and PowHel (HELAC-NLO *[Bevilacqua et al.'11]* +POWHEG-Box) package *[Garzelli, Kardos, Papadopoulos,Trocksanyi'11]* (*ttH*)

Resummation-improved NNLL+N(N)LO total cross section

- NNLL resummed expression has to be matched with the full N(N)LO QCD(+EW) result

$$
\begin{aligned}
\sigma^{(\mathrm{match})}_{h_1 h_2 \to kl}(\rho, \{m^2\}, \mu^2) \;=\; & \sum_{i,j=q,\bar{q},g} \int_{C_{\mathrm{MP}}-i\infty}^{C_{\mathrm{MP}}+i\infty} \frac{dN}{2\pi i}\, \rho^{-N}\, f^{(N+1)}_{i/h_1}(\mu^2)\, f^{(N+1)}_{j/h_2}(\mu^2) \\
& \times \left[\hat{\sigma}^{(\mathrm{res},N)}_{ij\to kl}(\{m^2\}, \mu^2) - \hat{\sigma}^{(\mathrm{res},N)}_{ij\to kl}(\{m^2\}, \mu^2)\,|_{N(N)LO\ QCD} \right] \\
& + \sigma^{N(N)LO\ QCD(+EW)}_{h_1 h_2 \to kl}(\rho, \{m^2\}, \mu^2)\,,
\end{aligned}
$$

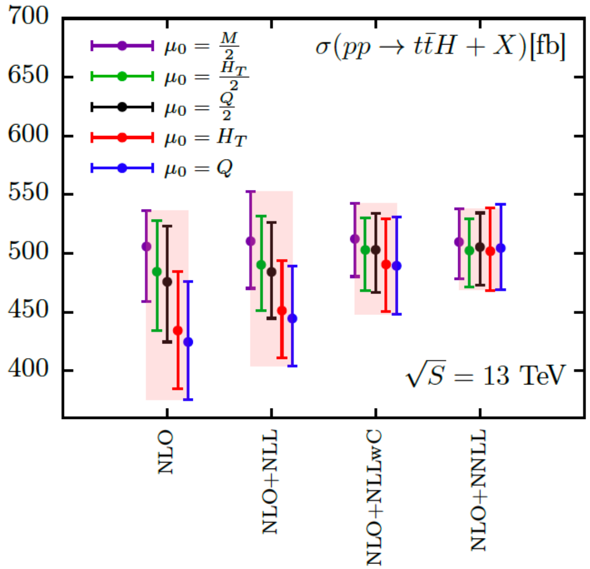
- Inverse Mellin transform evaluated using a contour in the complex *N* space according to 'Minimal Prescription' *[Catani, Mangano, Nason Trentadue'96]*

NLO+NNLL FOR TTBARH

[AK, Motyka, Stebel, Theeuwes'17] [AK, Motyka, Schwartläder, Stebel, Theeuwes'20]

μ_0	NLO[fb]	NLO+NLL[fb]	NLO+NLLwC[fb]	NLO+NNLL[fb]	K_{NNLL}
Q	$425^{+12.1\%}_{-11.6\%}$	$445^{+10.0\%}_{-9.2\%}$	$489^{+8.4\%}_{-8.5\%}$	$505^{+7.5\%}_{-7.0\%}$	1.19
H_T	$434^{+11.6\%}_{-11.4\%}$	$451^{+9.5\%}_{-8.9\%}$	$491^{+7.9\%}_{-8.2\%}$	$502^{+7.3\%}_{-6.7\%}$	1.16
$Q/2$	$476^{+9.9\%}_{-10.8\%}$	$484^{+8.7\%}_{-8.2\%}$	$503^{+6.2\%}_{-7.3\%}$	$505^{+5.7\%}_{-6.4\%}$	1.06
$H_T/2$	$484^{+8.9\%}_{-10.4\%}$	$490^{+8.4\%}_{-8\%}$	$503^{+5.5\%}_{-6.8\%}$	$502^{+5.4\%}_{-6.1\%}$	1.04
$M/2$	$506^{+6\%}_{-9.3\%}$	$510^{+8.2\%}_{-7.8\%}$	$512^{+5.9\%}_{-6.2\%}$	$510^{+5.6\%}_{-6.1\%}$	1.01

$K_{NNLL} = \sigma_{NLO+NNLL} / \sigma_{NLO}$



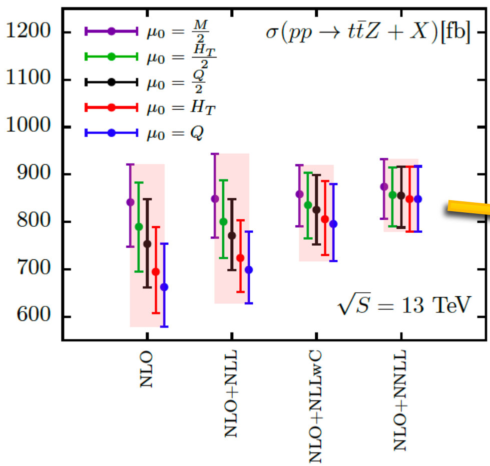
- Compared to NLO, remarkable stability of NLO+NNLL
- Stability improves with increasing accuracy of resummation
- Reduction of the theory scale error
- "Best" (envelope) NNLL+NLO prediction in agreement with NLO QCD+EW at $\mu_0 = M/2$

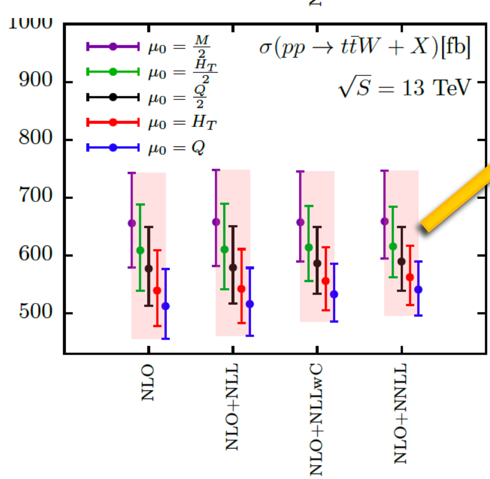
$$\sigma_{t\bar{t}H}^{\text{NLO+NNLL}} = 504^{+7.6\%+2.4\%}_{-7.1\%-2.4\%} \text{ fb}$$

(± scale) (± pdf)

TTBAR+W,Z AT NLO+NNLL

[AK, Motyka, Schwartländer, Stebel, Theeuwes'18 '20]





Envelope over results for different central scales

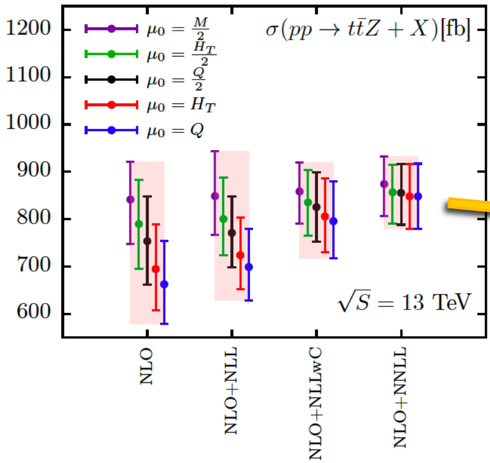
NLO(QCD+EW) +NNLL (± scale) (± pdf)

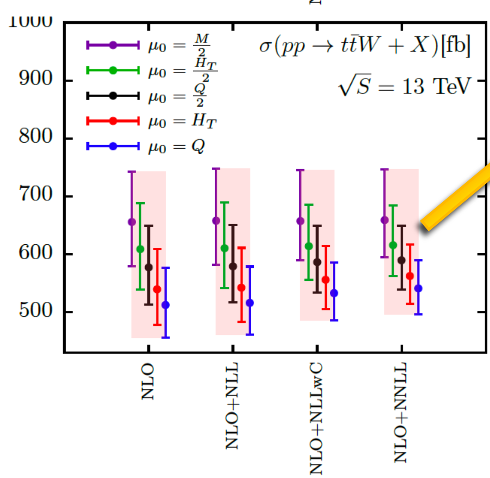
$$\sigma^{\text{NLO+NNLL}}_{t\bar{t}Z} = 859^{+8.6\%+2.3\%}_{-9.5\%-2.3\%}\ \text{fb}\,,$$
$$\sigma^{\text{NLO+NNLL}}_{t\bar{t}W} = 592^{+26.1\%+2.1\%}_{-16.2\%-2.1\%}\ \text{fb}$$

- Significant reduction (up to by half) in the scale dependence of the *ttZ* total cross section
- Depending on the central scale, correction up to ~30% for the *ttZ* cross section
- Smaller impact on the ttW predictions: only $q\bar{q}$ initial channel at LO

TTBAR+W,Z AT NLO+NNLL

[AK, Motyka, Schwartländer, Stebel, Theeuwes'18 '20]





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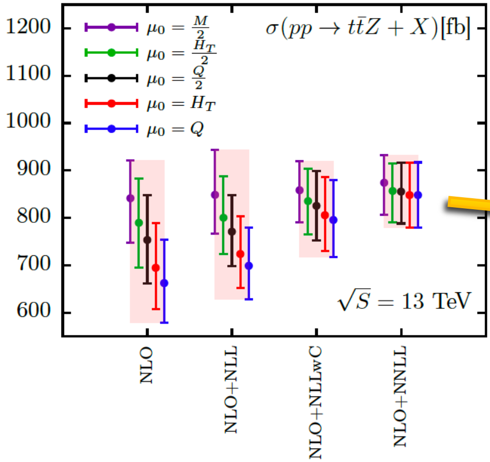
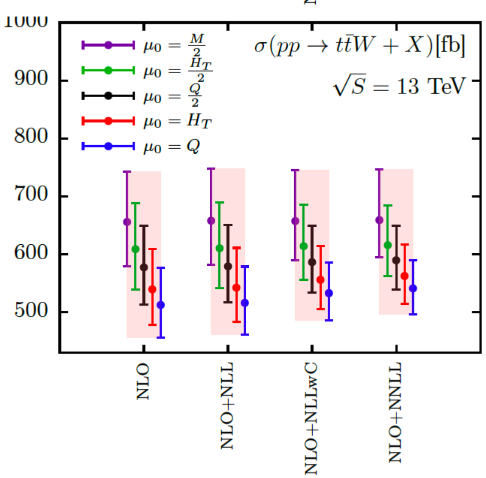
$$\sigma_{t\bar{t}W}^{\text{NLO+NNLL}} = 592^{+26.1\%+2.1\%}_{-16.2\%-2.1\%}\ \text{fb}$$

- NLO+NNLL results for ttbar+W,Z,H also obtained in the SCET formalism *[Broggio, Ferroglia, Pecjak, Signer, Yang'15] [Broggio, Ferroglia, Ossola, Pecjak'16] [Broggio, Ferroglia, Pecjak, Yang'16][Broggio, Ferroglia, Ossola, Pecjak, Samoshima'17] [Broggio et al. 19]*

TTBAR+W,Z AT NLO+NNLL

[AK, Motyka, Schwartländer, Stebel, Theeuwes'18 '20]





Envelope over results for different central scales

NLO(QCD+EW) +NNLL (± scale) (± pdf)

$$\sigma_{t\bar{t}Z}^{\text{NLO+NNLL}} = 859^{+8.6\%+2.3\%}_{-9.5\%-2.3\%}\ \text{fb} = 0.859^{+0.073+0.028}_{-0.085-0.028}\ \text{pb}$$

ATLAS, EPJC 81 (2021) 737:

$$\sigma_{pp\to t\bar{t}Z}^{\text{ATLAS,2021}} = 0.99 \pm 0.05(\text{stat.}) \pm 0.08(\text{syst.})$$

CMS, JHEP 03 (2020) 056:

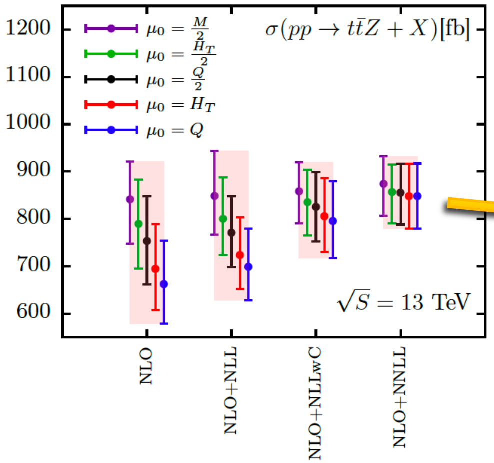
$$\sigma_{pp\to t\bar{t}Z}^{\text{CMS}} = 0.95 \pm 0.05(\text{stat.}) \pm 0.06(\text{syst.})$$

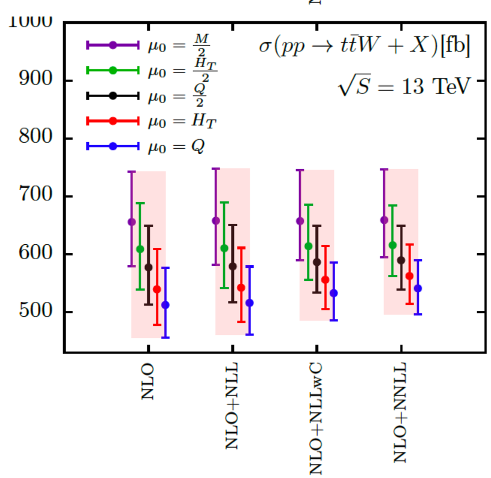
Accuracy of the NLO+NNLL result comparable with experimental precision!

Central value brought closer to the measured cross section

TTBAR+W,Z AT NLO+NNLL

[AK, Motyka, Schwartländer, Stebel, Theeuwes'18 '20]





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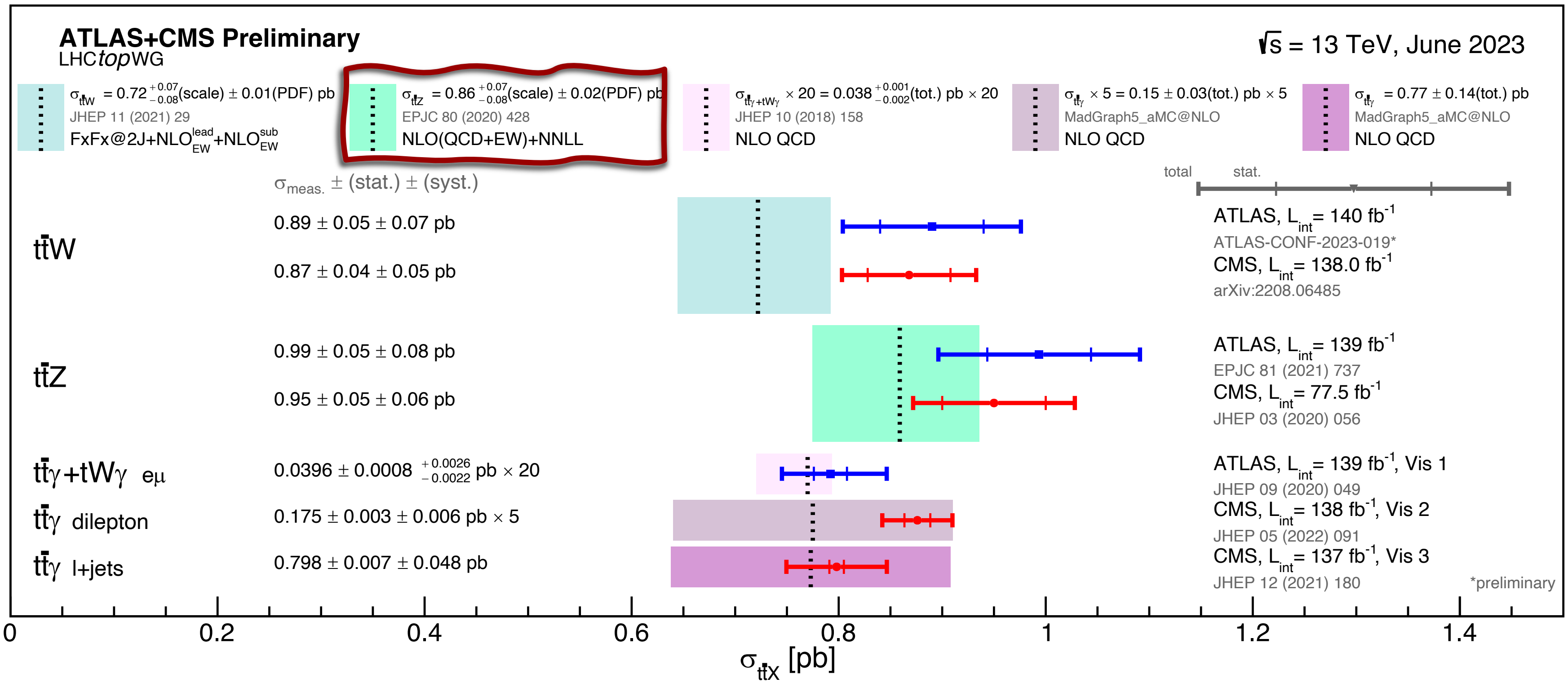
ATLAS-CONF-2023-065

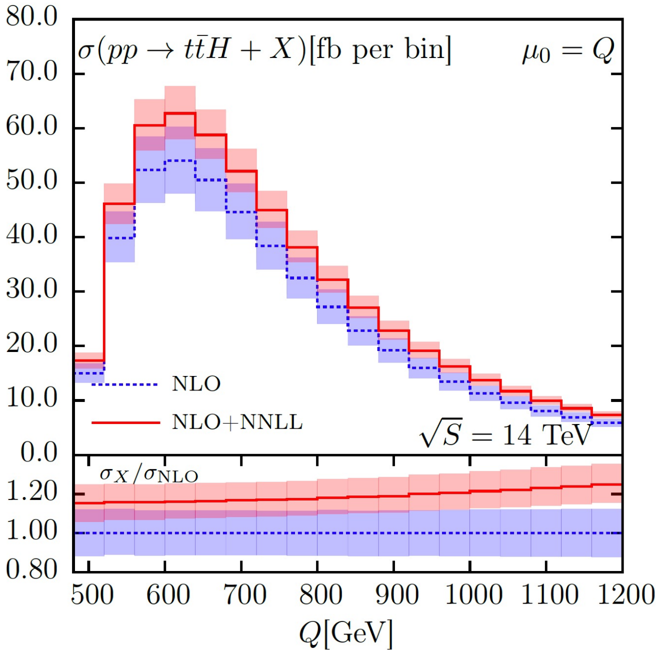
$$\sigma_{pp\to t\bar{t}Z}^{\mathrm{ATLAS,2023}} = 0.86 \pm 0.04(\mathrm{stat.}) \pm 0.04(\mathrm{syst.})$$

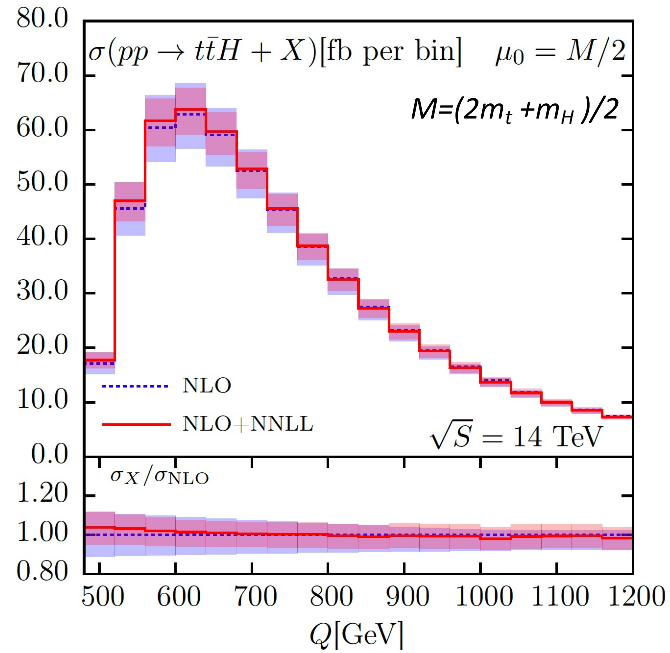
TTBAR+V: TOTAL X SECTION



Invariant Mass Distribution for TTbarH

[AK, Motyka, Stebel, Theeuwes'17 '20]





- NNLL+NLO distributions for the two considered scale choices very close, NLO results differ visibly → $K_{\mathrm{NNLL}} = \sigma_{\mathrm{NLO+NNLL}} / \sigma_{\mathrm{NLO}}$ factors also different
- NNLL+NLO error band slightly narrower than NLO (7-point method)

Invariant Mass Distribution for TTbarH

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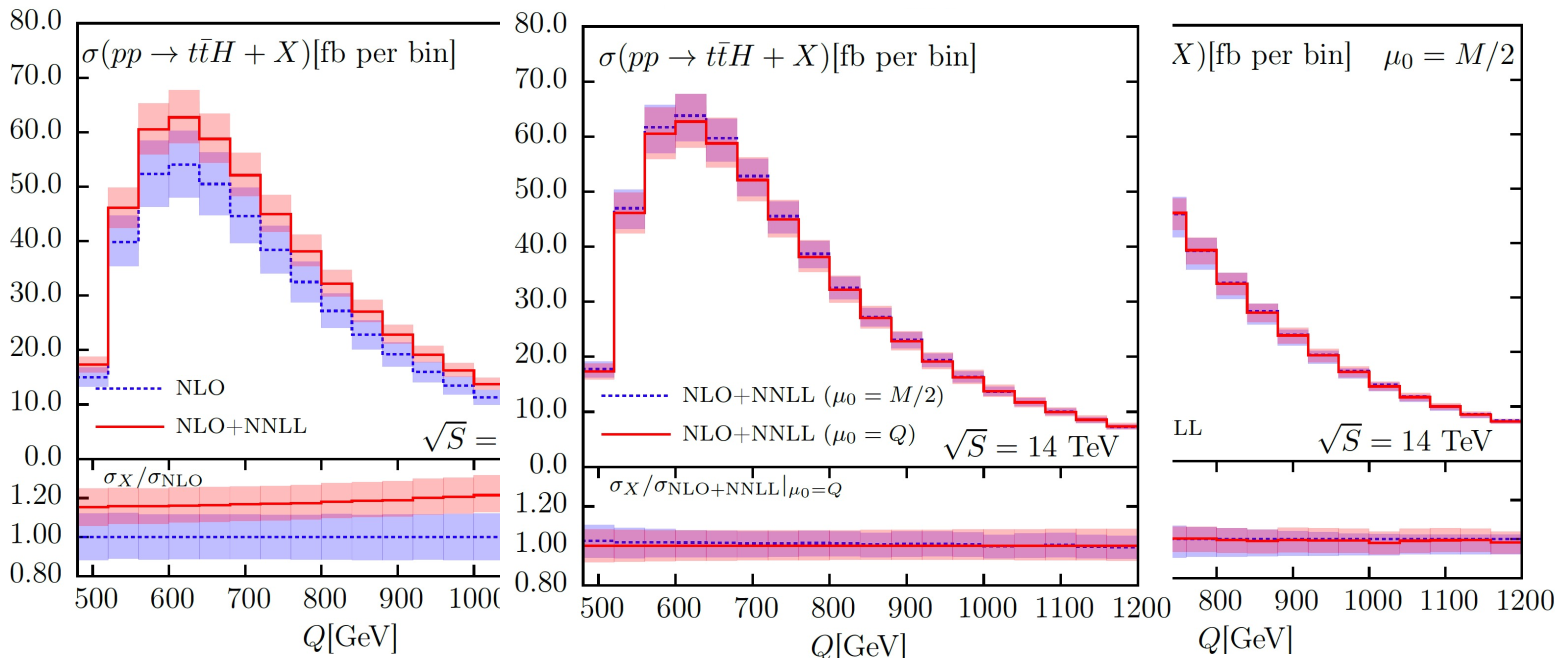

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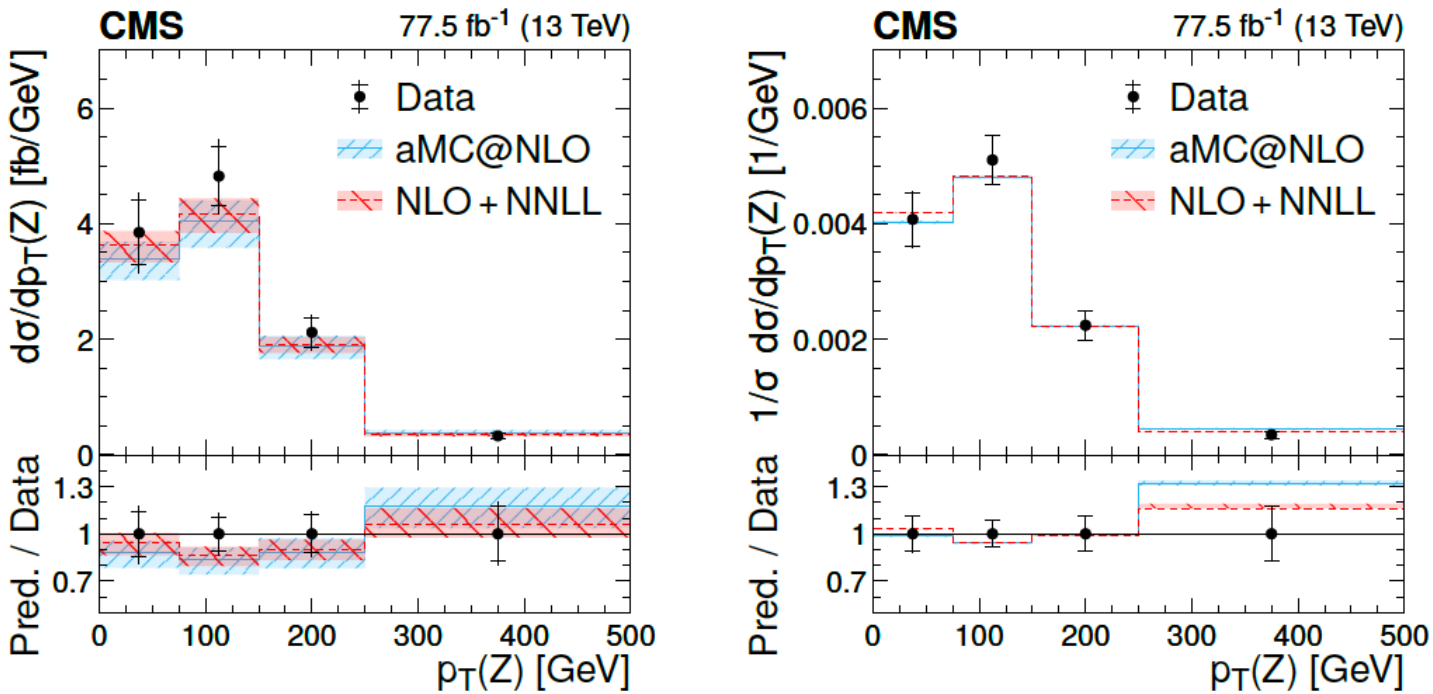
TTbarZ: Z's Transverse Momentum Distribution at NLO(QCD+EW)+NNLL

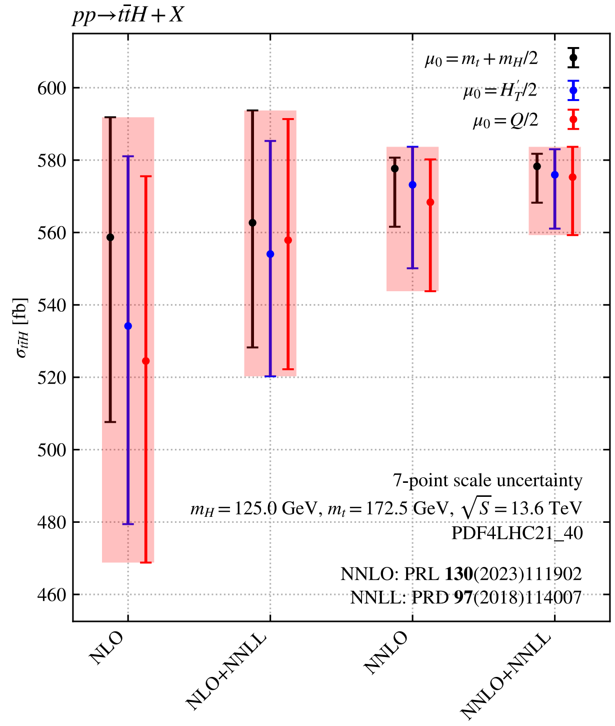
[AK, Motyka, Schwartländer, Stebel, Theeuwes'19 '20]



CMS, JHEP 03 (2020) 056

NNLO+NNLL

- Matching to the new NNLO results for tTH *[Catani et al.'22],* undertaken within framework of the LHCHWG



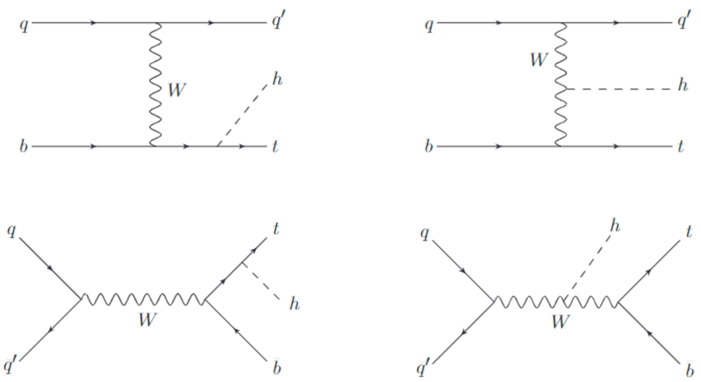
- Ongoing comparison with results based on the approach of *[Broggio et al.'19]*

ASSOCIATED SINGLE TOP AND HIGGS PRODUCTION

- Measured together with ttbarH *[ATLAS Collaboration, PRL125.061802 (2015)] [CMS Collaboration PRD.99.092005 (2019)]*
- Sensitive to the top Yukawa coupling and HWW coupling
- At LO negative interference between the diagrams involving the two couplings in the dominant t-channel → sensitivity to BSM

t-channel*

s-channel



- NLO QCD for t- and s-channel *[Demartin et al.'15]* and NLO(QCD+EW) for combined *[Pagani, Tsinikos, Vryonidou'20]*
- Approximation of NNLO based on the expansion of the resummed formula *[Forslund, Kidonakis'20]*

ThJet Resummation

$$d\sigma^{\rm res}_{ij\to tHk}(N,\mu_F,\mu_R,\ldots) = {\rm Tr}\left[\mathbf{H}_{ij\to tHk}(\mu_F,\mu_R,\ldots)\mathbf{S}_{ij\to tHk}(N,\mu_R,\ldots)\right]$$
$$\Delta_i(N,\mu_F,\mu_R,\dot{)}\Delta_j(N,\mu_F,\mu_R,\ldots)\; J_k(N,\mu_R,\ldots)$$

- Exploratory study for s-channel (t-channel suffers from flavour scheme dependence)
- Resummation of logarithms that are large in the invariant mass threshold limit $s \to Q^2$

$$Q^2 = (p_t + p_H + p_j)^2$$

triple invariant mass kinematics

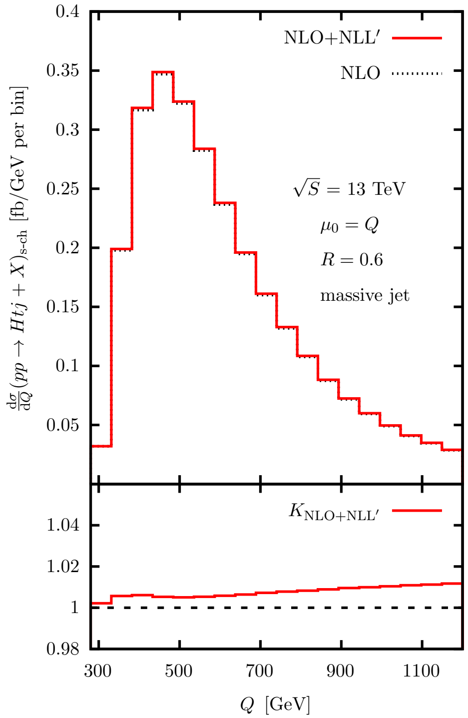
- NLL accuracy augmented with NLO contributions (NLLwC, also called NLL')
  - calculations of 1-loop soft anomalous dimension (NLL), NLO non-logarithmic contributions to **S**, **H** and ***J*** functions
- New aspect: final state jet
  - can consider jets that at threshold are either massive (non-zero p_j^2) or massless ($p_j^2 = 0$)

tHJET AT THE LHC

Massive jets at the threshold:

[Moreno Valero, AK, Theeuwes'21]

- Invariant mass distribution

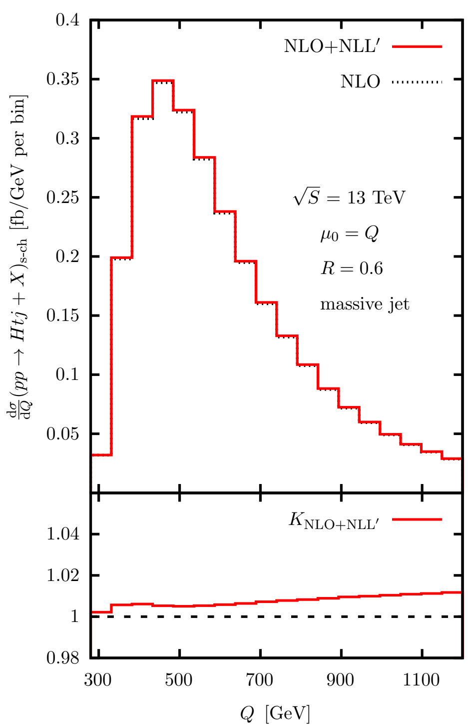


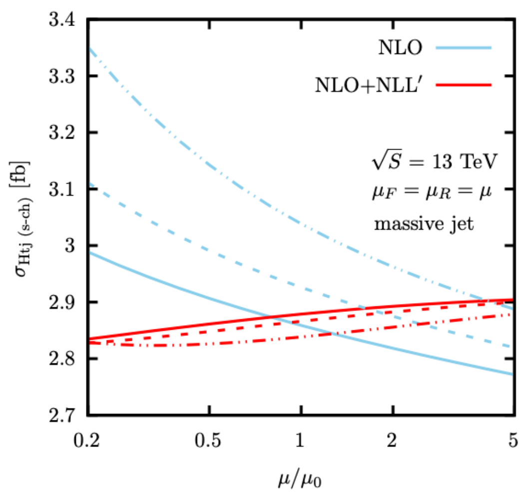
tHJET AT THE LHC

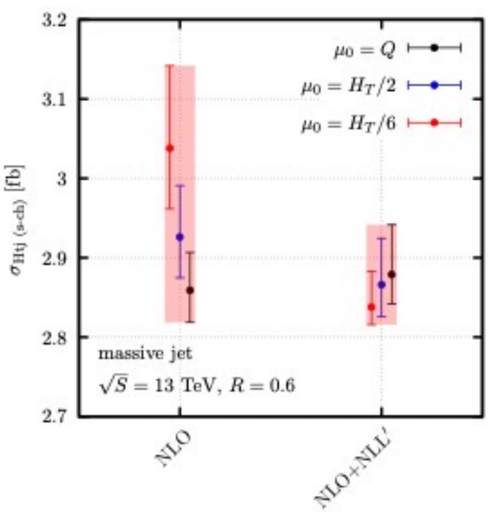
Massive jets at the threshold:

[Moreno Valero, AK, Theeuwes'21]

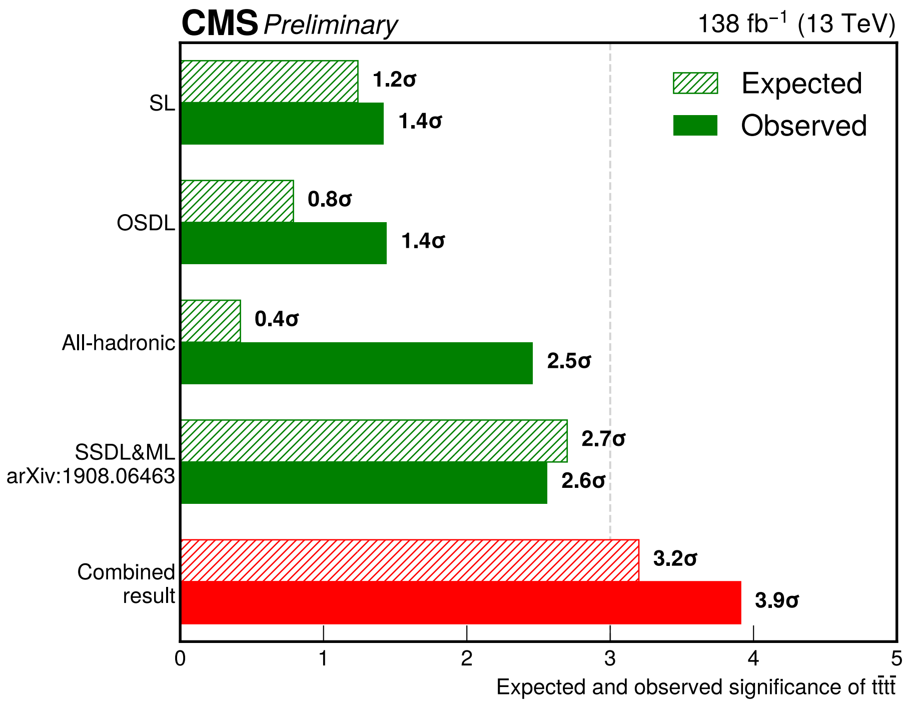
- Invariant mass distribution
- Total cross section





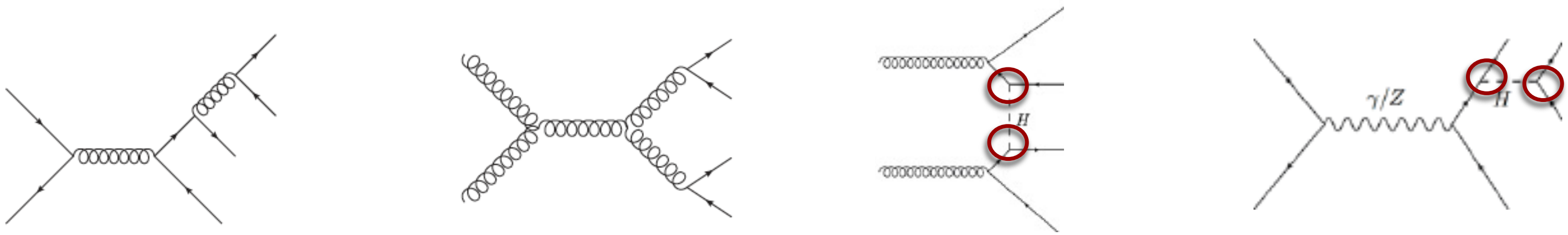


- Promising, but more studies needed: relation to massless jets, t-channel contributions..

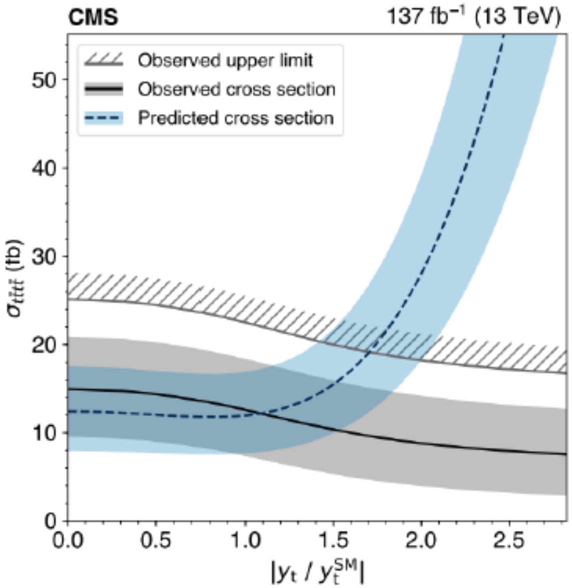


$t\bar{t}t\bar{t}$ production

4-TOP

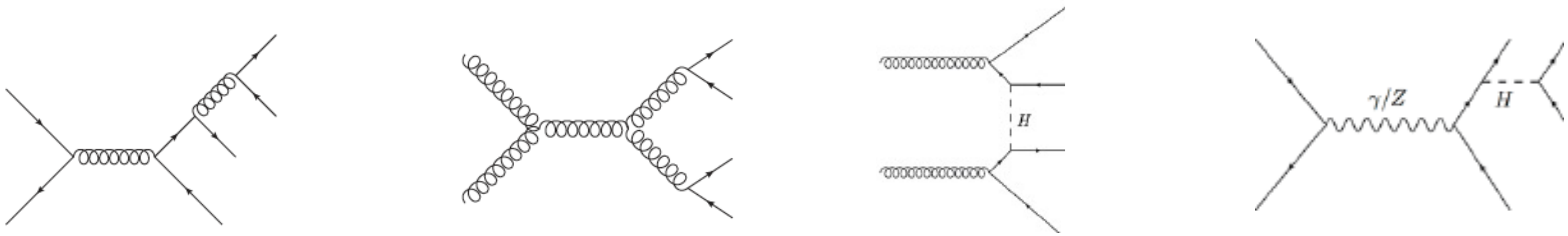


- Sensitive to the Yukawa coupling



[CMS Collaboration , JHEP 11 (2021) 118]

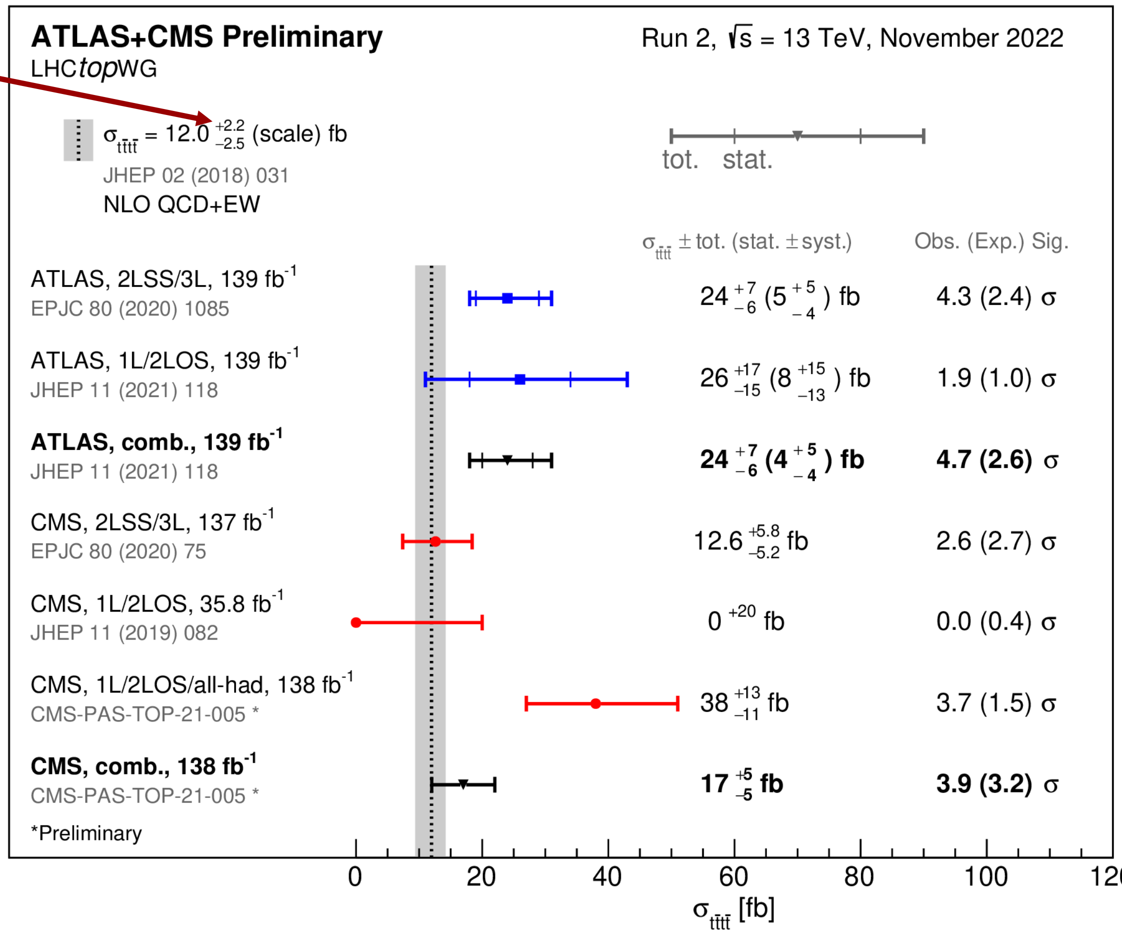
4-TOP



- Sensitive to the Yukawa coupling
- Can receive significant contributions in BSM scenarios: SUSY, 2HDM, simplified DM models…
- Used to constrain four-fermion operators in SMEFT
- Known in the SM at NLO (QCD+EW), matched to parton showers *[Bevilacqua, Worek'12], [Maltoni, Pagani, Tsinikos'16][Frederix, Pagani, Zaro'17], [Jezo, Kraus'21]*

4-TOP AT THE LHC

20% scale error



Colour Structure

$$d\sigma^{\text{res}}_{ij\to t\bar{t}t\bar{t}}(N,\mu_F,\mu_R) = \text{Tr}\left[\mathbf{H}_{ij\to t\bar{t}t\bar{t}}(\mu_F,\mu_R)\mathbf{S}_{ij\to t\bar{t}t\bar{t}}(N,\mu_R)\right]\Delta_i(N,\mu_F,\mu_R)\Delta_j(N,\mu_F,\mu_R)$$

- Processes with four or more legs carrying colour: hard and soft functions are matrices in colour space ↔ soft radiation sensitive to the overall colour structure of the Born process

$q\bar{q}$ channel:

$$\mathbf{3}\otimes\bar{\mathbf{3}} = \mathbf{3}\otimes\bar{\mathbf{3}}\otimes\mathbf{3}\otimes\bar{\mathbf{3}}$$

$$\mathbf{1}\oplus\mathbf{8} = (2\times\mathbf{1})\oplus(2\times\mathbf{8})\oplus\mathbf{8}_S\oplus\mathbf{8}_A\oplus\mathbf{10}\oplus\overline{\mathbf{10}}\oplus\mathbf{27}$$

6-dimensional colour space

gg channel:

$$\mathbf{8}\otimes\mathbf{8} = \mathbf{3}\otimes\bar{\mathbf{3}}\otimes\mathbf{3}\otimes\bar{\mathbf{3}}$$

$$\mathbf{0}\oplus\mathbf{1}\oplus\mathbf{8}_S\oplus\mathbf{8}_A\oplus\mathbf{10}\oplus\overline{\mathbf{10}}\oplus\mathbf{27} = \mathbf{0}\oplus(2\times\mathbf{1})\oplus(2\times\mathbf{8})\oplus\mathbf{8}_S\oplus\mathbf{8}_A\oplus\mathbf{10}\oplus\overline{\mathbf{10}}\oplus\mathbf{27}$$

14-dimensional colour space

- Construction of the colour basis as in *[Keppeler, Sjodahl'12]*, with modification for the *gg* channel

Resummation for 4 -tops

- Resummation performed at the production threshold limit $s \to 4m_t^2 = M^2$
- Logarithms of N arise from Mellin transform of logarithms of $1-4m_t^2/s$
- The soft function can be written as an exponential if $\mathbf{\Gamma}^{(1)}$ diagonal

$$\mathbf{S}_{ij\to t\bar{t}t\bar{t},R} = \exp\left[\frac{\mathrm{Re}\left(\mathbf{\Gamma}^{(1)}_{\mathbf{ij\to t\bar{t}t\bar{t},R}}\right)}{b_0\pi}\log(1-2\lambda)\right]$$

- Construction of the colour basis as in *[Keppeler, Sjodahl'12],* with modification for the *gg* channel
- The calculated $\mathbf{\Gamma}^{(1)}_{\mathbf{ij},R}$ become in the threshold limit

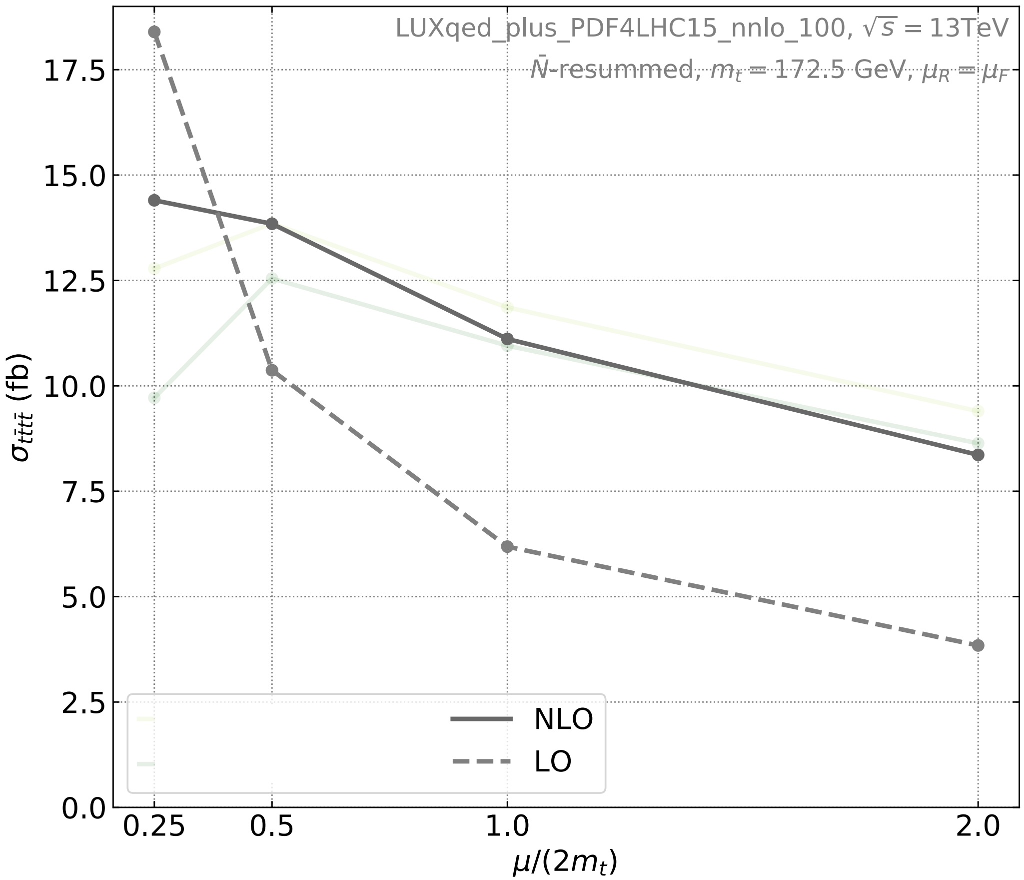
$$2\mathrm{Re}[\overline{\mathbf{\Gamma}}_{\mathbf{q\bar{q}},R}] = \mathrm{diag}\,(0,0,-3,-3,-3,-3)$$

$$2\mathrm{Re}[\overline{\mathbf{\Gamma}}_{\mathbf{gg},R}] = \mathrm{diag}(-8,-6,-6,-4,-3,-3,\ -3,-3,-3,-3,-3,-3,0,0)$$

negative values of the quadratic Casimir operators for the corresponding SU(3) irreducible representations

SCALE VARIATION AT NLO

[van Beekveld, Moreno Valero, AK'22]



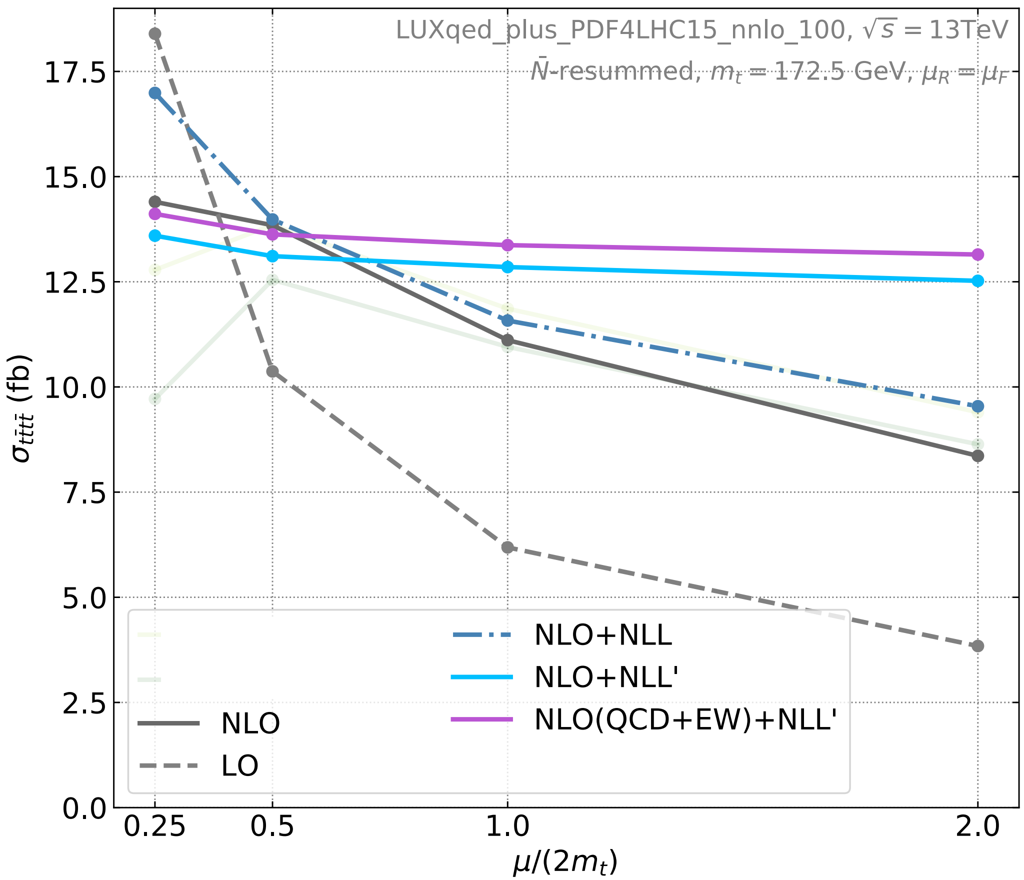
$\mu = \mu_R = \mu_F$

QCD only

Fixed-order results obtained with aMC@NLO *[Alwall et. al.'14][Frederix'18]*

SCALE VARIATION AT NLO+NLL'

[van Beekveld, Moreno Valero, AK'22]



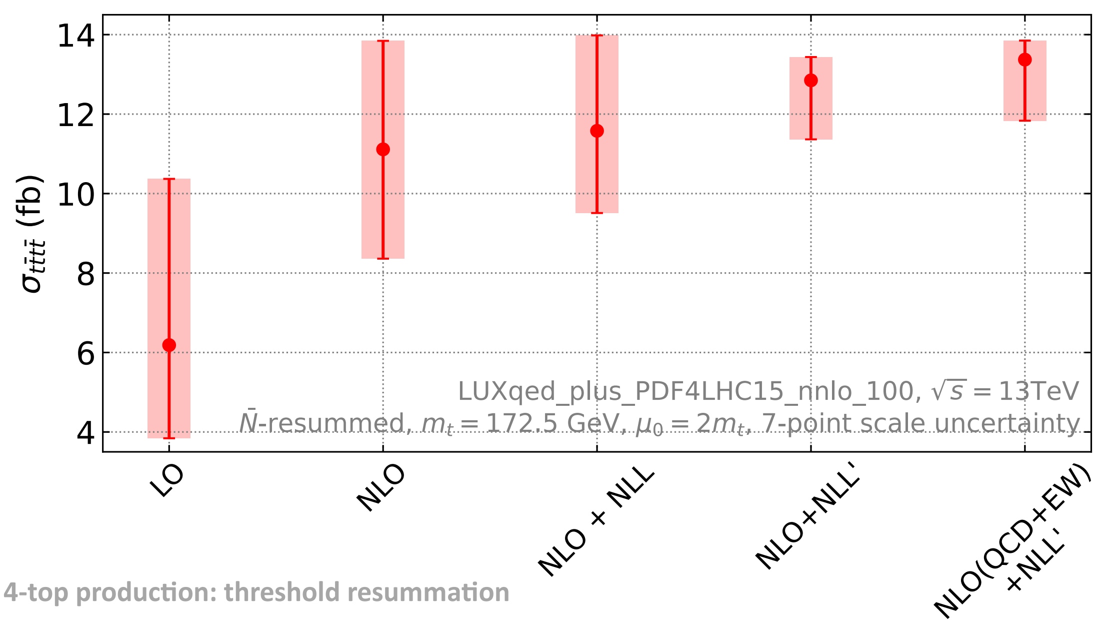
$$\mu = \mu_R = \mu_F$$

Fixed-order results obtained with aMC@NLO *[Alwall et. al.'14][Frederix'18]*

4-TOP AT 13 TEV

[van Beekveld, AK, Moreno Valero'22]

$\sqrt{s}$ (TeV)	NLO	NLO+NLL	NLO+NLL'	$K_{NLL'}$	
13	$11.00(2)^{+25.2\%}_{-24.5\%}$ fb	$11.46(2)^{+21.3\%}_{-17.7\%}$ fb	$12.73(2)^{+4.1\%}_{-11.8\%}$ fb	1.16	QCD only
$\sqrt{s}$ (TeV)	NLO(QCD+EW)	NLO(QCD+EW)+NLL	NLO(QCD+EW)+NLL'	$K_{NLL'}$	
13	$11.64(2)^{+23.2\%}_{-22.8\%}$ fb	$12.10(2)^{+19.5\%}_{-16.3\%}$ fb	$13.37(2)^{+3.6\%}_{-11.4\%}$ fb	1.15	



- 7-point scale variation error
- Additional pdf error : +/- 6.7 %

4-TOP AT 13 TEV

[van Beekveld, AK, Moreno Valero'22]

$\sqrt{s}$ (TeV)	NLO	NLO+NLL	NLO+NLL'	$K_{NLL'}$	
13	$11.00(2)^{+25.2\%}_{-24.5\%}$ fb	$11.46(2)^{+21.3\%}_{-17.7\%}$ fb	$12.73(2)^{+4.1\%}_{-11.8\%}$ fb	1.16	QCD only

$\sqrt{s}$ (TeV)	NLO(QCD+EW)	NLO(QCD+EW)+NLL	NLO(QCD+EW)+NLL'	$K_{NLL'}$
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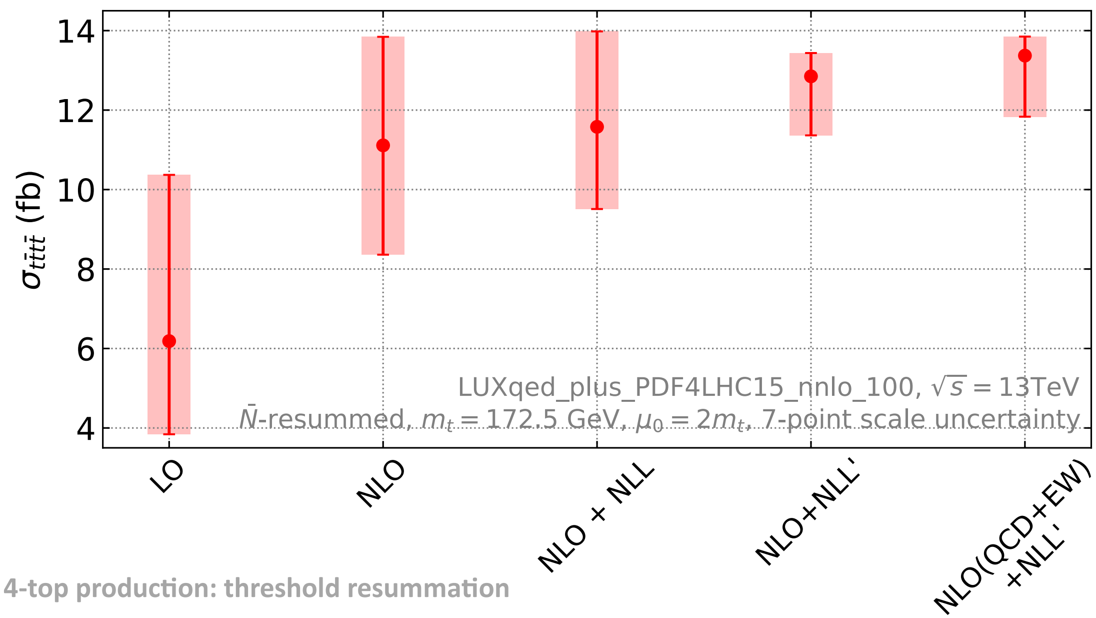

- 15 % correction to the NLO (QCD+EW) prediction due to NLL' resummation

4-TOP AT 13 TEV

[van Beekveld, AK, Moreno Valero'22]

$\sqrt{s}$ (TeV)	NLO	NLO+NLL	NLO+NLL′	$K_{NLL'}$	
13	$11.00(2)^{+25.2\%}_{-24.5\%}$ fb	$11.46(2)^{+21.3\%}_{-17.7\%}$ fb	$12.73(2)^{+4.1\%}_{-11.8\%}$ fb	1.16	QCD only
$\sqrt{s}$ (TeV)	NLO(QCD+EW)	NLO(QCD+EW)+NLL	NLO(QCD+EW)+NLL′	$K_{NLL'}$	
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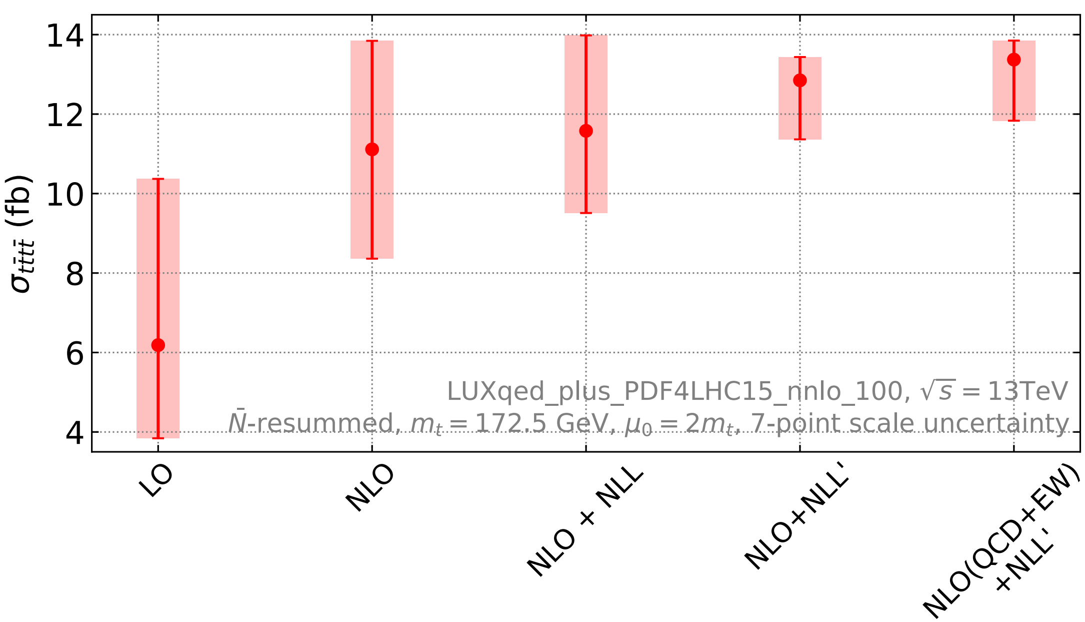
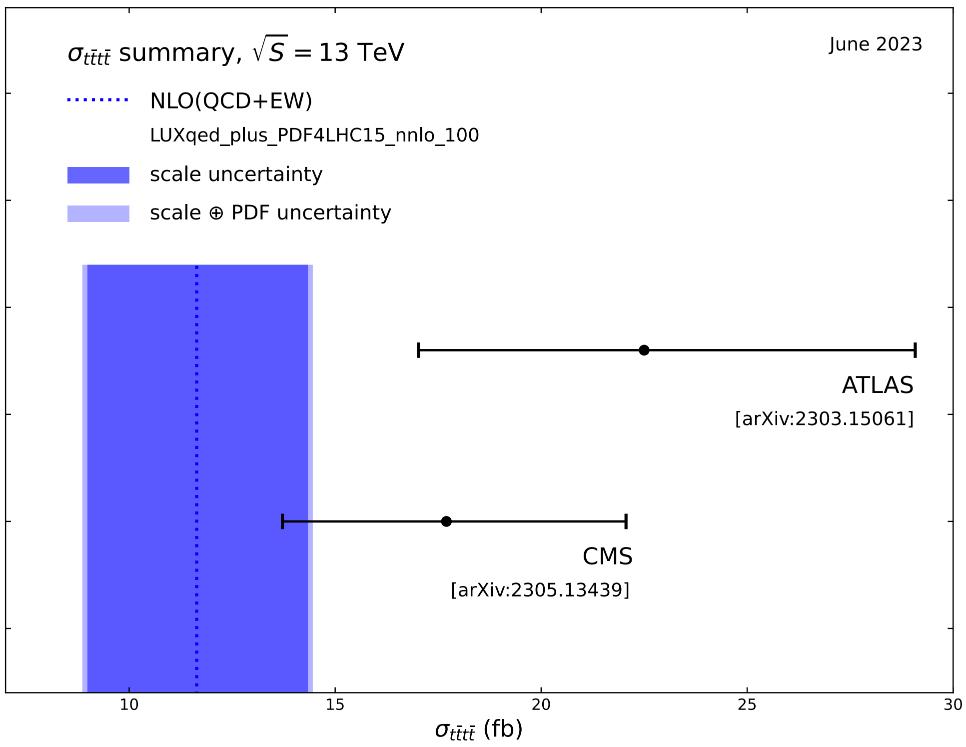


- Reduction of the scale error by more than a factor of 2

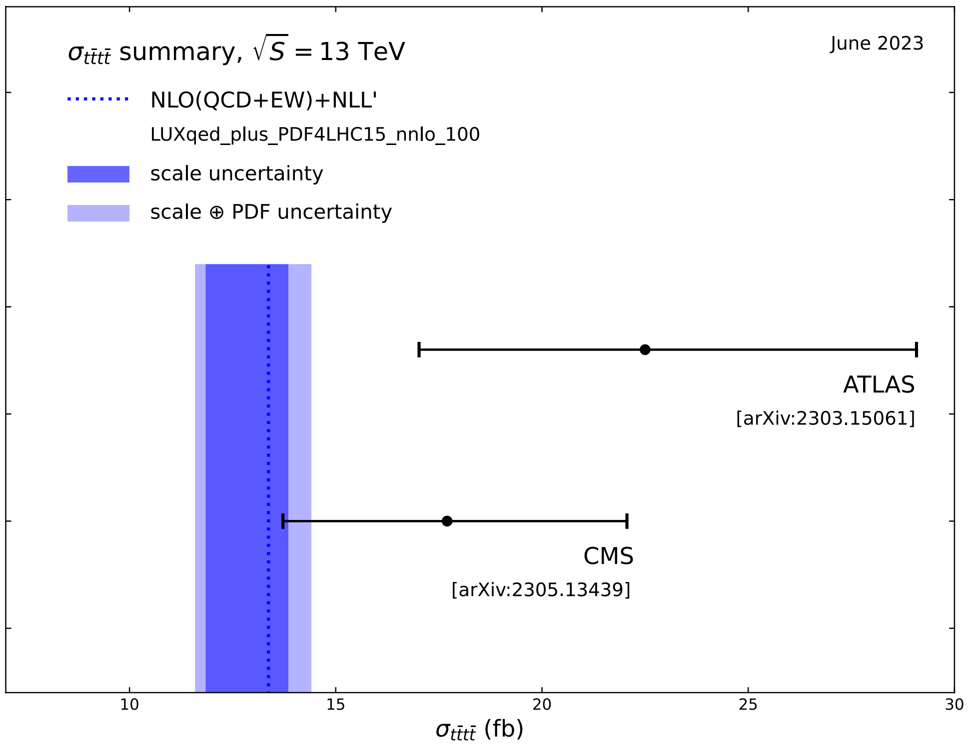
Comparison with Measurements

[van Beekveld, Moreno Valero, AK'22]



Comparison with Measurements

[van Beekveld, Moreno Valero, AK'22]



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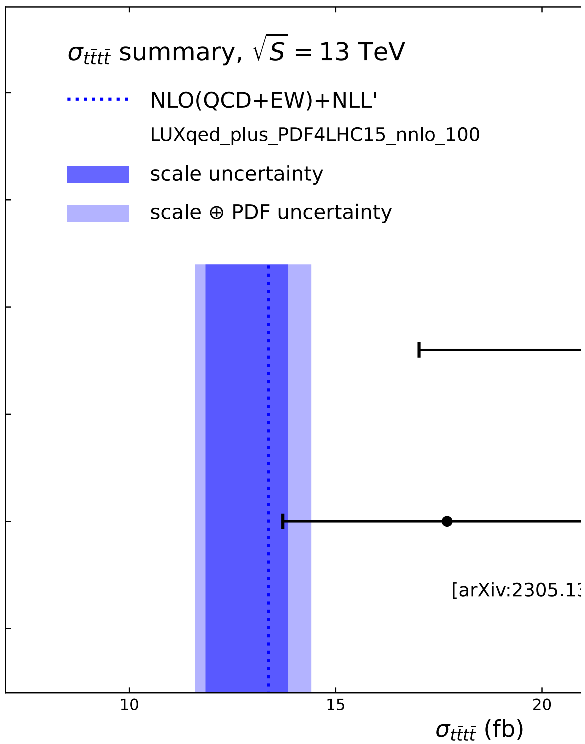

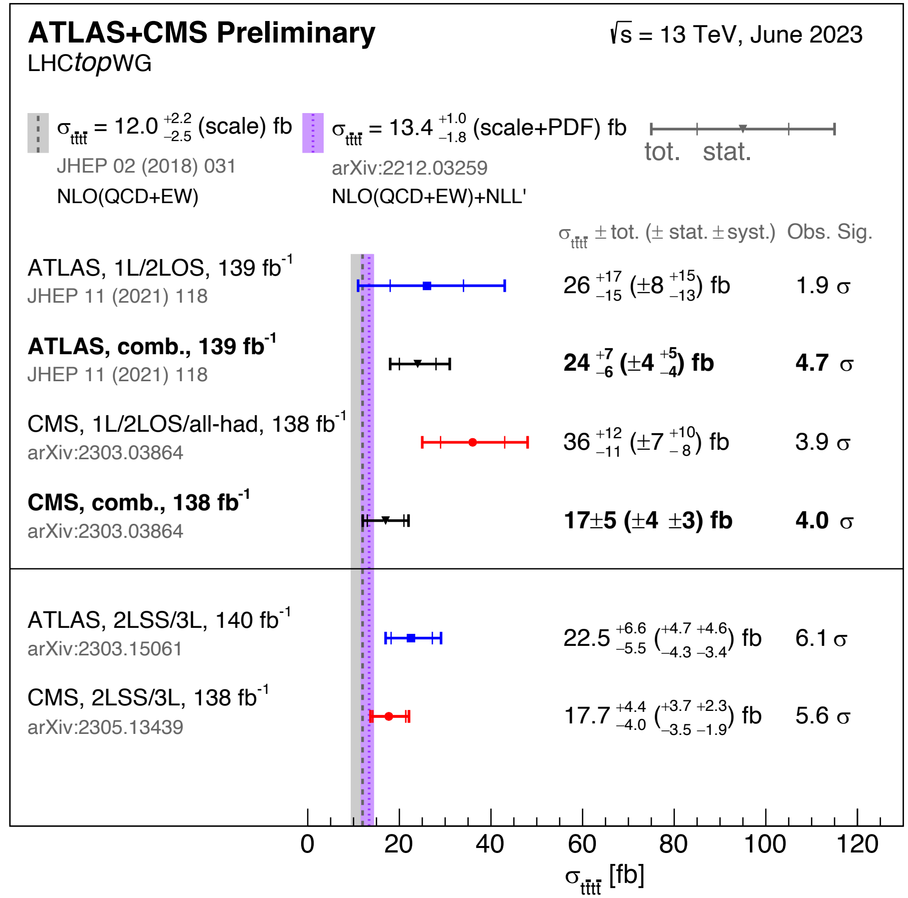

SUMMARY

- 25+ years of soft gluon resummation for heavy flavours
- Soft gluon resummation for processes involving b quarks brings new challenges due to the gap between bottom mass and the typical scale of the process but progress is made
- In the context of soft gluons, ground zero are the studies for top quarks
- For tops, new developments for top-pair production and first application of threshold resummation to a class of 2 → 3 and 2 → 4 processes reaching N(N)LL+N (N)LO (QCD+EW) accuracy
  - Remarkable stability of the total and differential and cross sections w.r.t. scale variation
  - Reduction of the theory error due to scale variation with the increasing logarithmic order of the calculations