Heavy flavours at small-x

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Heavy Flavours at High pT

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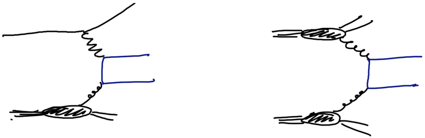
Scales of heavy quark production

Consider heavy quark production in either DIS or pp collisions



There are (at least) four scales in the game

- m: mass of the heavy quark
- Q: hard scale (virtuality of the photon, invariant mass of the final state)
- $\sqrt{s}$: parton-level centre-of-mass energy
- $\sqrt{S}$: hadron-level centre-of-mass energy

Depending on the hierarchy, there may be large logs $\to$ need to resum them

Dimensionless ratios:

$$\frac{m^2}{Q^2}, \qquad \tau = \frac{Q^2}{S}, \qquad x = \frac{Q^2}{s}, \qquad \tau \leq x \leq 1$$

Focus on the high-energy limit $\tau \ll 1$: large $\log x$ appear and must be resummed.
If m^2/Q^2 is also small, also $\log \frac{m^2}{Q^2}$ must be resummed

small-x resummation

[Catani,Ciafaloni,Colferai,Hautmann,Salam,Stasto]
[Thorne,White]
[Altarelli,Ball,Forte]
[Neill,Pathak,Rothstein,Stewart]

Small-x logarithms in the context of collinear factorization

Collinear factorization: $$\sigma(x,Q^2)=\int_x^1\frac{dz}{z}\,C_i(z,\alpha_s(Q^2))\,f_i\left(\frac{x}{z},Q^2\right)$$

DGLAP evolution: $$\mu^2\frac{d}{d\mu^2}f_i(x,\mu^2)=\int_x^1\frac{dz}{z}\,P_{ij}(z,\alpha_s(\mu^2))\,f_j\left(\frac{x}{z},\mu^2\right)$$

Heavy-quark matching: $$f_i^{[n_f+1]}(x,\mu^2)=\int_x^1\frac{dz}{z}\,A_{ij}(z,\alpha_s(\mu^2))\,f_j^{[n_f]}\left(\frac{x}{z},\mu^2\right)$$

Any object with a perturbative expansion can exhibit a logarithmic enhancement:

- observable: coefficient functions $C(x,\alpha_s)$
- evolution: splitting functions $P(x,\alpha_s)$ and matching conditions $A(x,\alpha_s)$

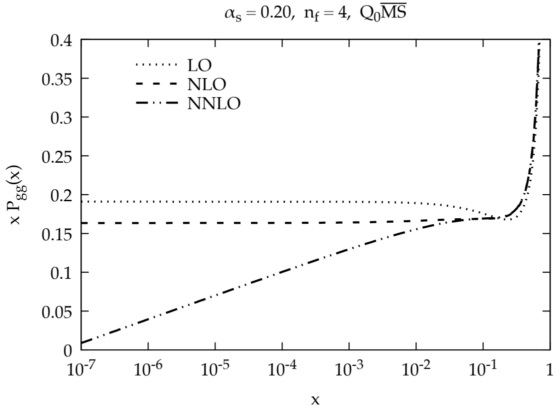
Small-x logarithms: single logs $\alpha_s^n\frac{1}{x}\log^k\frac{1}{x}\quad(0\le k\le n-1)$

When $\alpha_s\log\frac{1}{x}\sim 1$ perturbativity is spoiled $\to$ **all-order resummation needed**

In $\overline{\mathrm{MS}}$ and related schemes, both coefficient $C(x,\alpha_s)$ and splitting $P(x,\alpha_s)$ functions, and also matching conditions $A(x,\alpha_s)$, are logarithmically enhanced at small x (in the singlet sector)

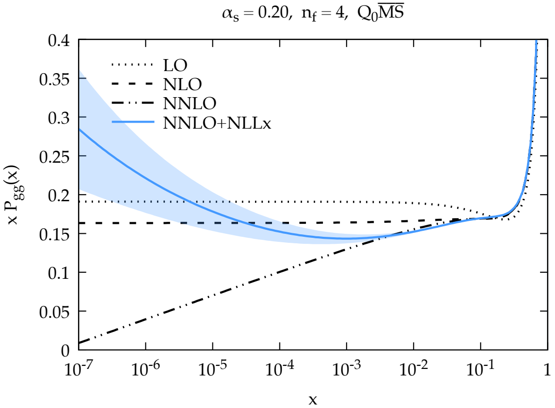
Small-x logarithms in gluon-gluon splitting function

$P_{gg}(x, \alpha_s)$ splitting function at fixed order



Logarithms start to grow for $x \lesssim 10^{-2}$ → **perturbative instability** for $x \lesssim 10^{-3}$ (for $Q \sim 5$GeV)

$P_{gg}(x, \alpha_s)$ splitting function at fixed order



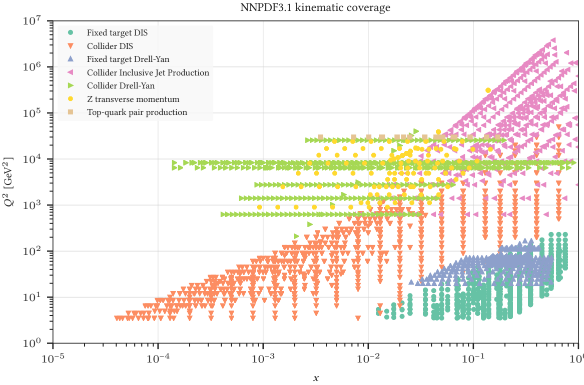
Logarithms start to grow for $x \lesssim 10^{-2}$ → **perturbative instability** for $x \lesssim 10^{-3}$ (for $Q \sim 5$GeV)

Resummation obtained with the `HELL` public code

[MB,Marzani,Peraro 1607.02153] [MB,Marzani,Muselli 1708.07510] [MB,Marzani 1805.06460]

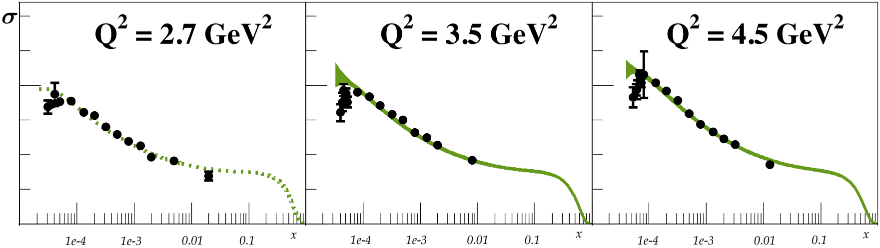
Do we experience the need for small-x resummation?

Hint: look at PDF fits...

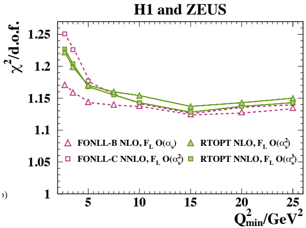


Deep-inelastic scattering (DIS) data from HERA extend down to $x \sim 3 \times 10^{-5}$ in the "perturbative region" $Q^2 > 2\,\mathrm{GeV}^2$

Tension between HERA data at low Q^2 and low x with fixed-order theory



Also leads to a deterioration of the χ^2 of PDF fits when including low-Q^2 data



The first PDF fits with small-x resummation

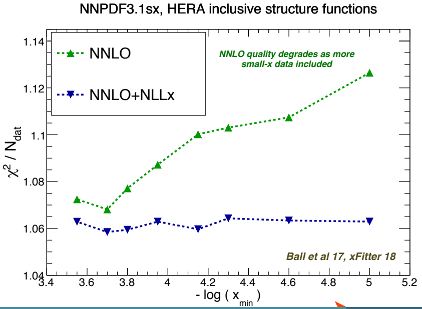
HELL $\rightarrow$ makes possible a PDF fit with small-x resummation

NNPDF3.1sx [1710.05935]	xFitter [1802.00064, see also 1902.11125]
NeuralNet parametrization of PDFs	polynomial paramterization
MonteCarlo uncertainty	Hessian uncertainty
charm PDF is fitted	charm PDF perturbatively generated
DIS+tevatron+LHC ($\sim$ **4000** datapoints)	only HERA data ($\sim$ **1200** datapoints)
NLO, NLO+NLLx, NNLO, NNLO+NLLx	NNLO, NNLO+NLLx

The quality of the fit improves substantially including small-x resummation

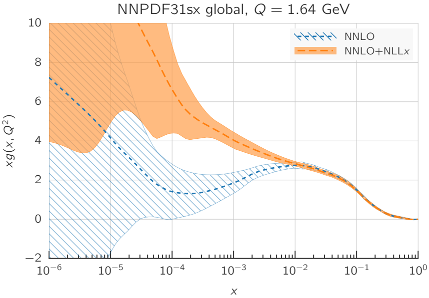
$\chi^2/N_{\rm dat}$	NNLO	NNLO+NLLx
xFitter	**1.23**	**1.17**
NNPDF3.1sx	**1.130**	**1.100**
		smaller!

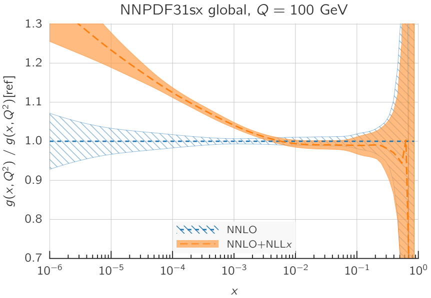
Stable upon inclusion of low-x data $\rightarrow$



Impact of small-x resummation on PDFs: the gluon

Small-x resummation mostly affects the gluon PDF (and the total quark singlet)





Dramatic effect of resummation on the gluon PDF at $x \lesssim 10^{-3}$

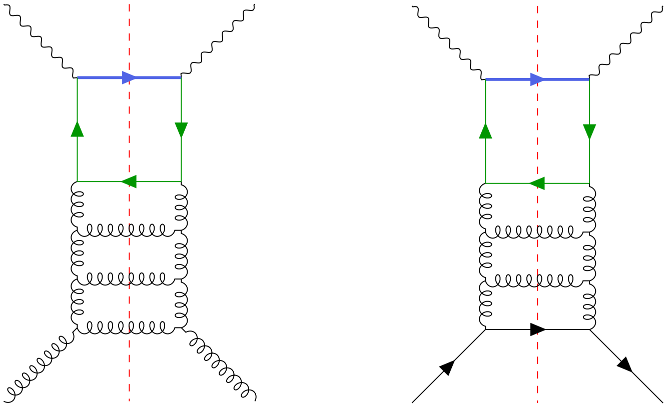
Persists at higher energy scales $\Rightarrow$ impact for LHC and FCC-hh phenomenology

heavy quark production in DIS

[Catani,Ciafaloni,Hautmann NPB 366 (1991) 135]
[Ball,Ellis 0101199]
[MB,Marzani,Muselli 1708.07510]

Heavy quark production in DIS at small x

Heavy quark production in neutral or charged current

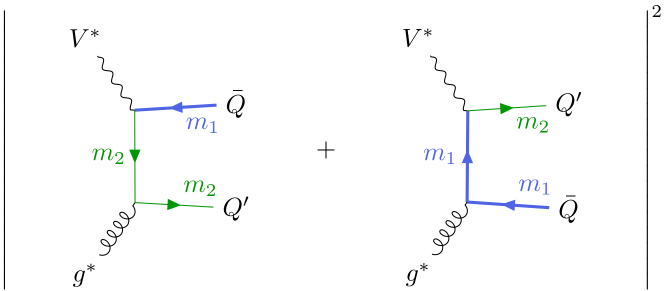


Leading small-x logarithms produced by chains of gluon emissions (in axial gauge)

Factorization into a universal ladder and a process dependent *off-shell* coefficient function

Off-shell coefficient function

We computed the off-shell coefficients, projected onto $\boldsymbol{F_2}$, $\boldsymbol{F_L}$ and $\boldsymbol{F_3}$ structure functions [MB,Marzani,Muselli 1708.07510]



In the charged current case, the flavour changes $\rightarrow \boldsymbol{m_1 \neq m_2}$

Ignoring the top quark, and considering CKM suppression in $\boldsymbol{V_{cb}}$, we can assume one of the quarks to be massless ($\boldsymbol{m_2 = 0}$).

Note: massless limit is finite as long as gluon is off-shell!

Variable flavour number scheme (1)

Consider the charm quark, with mass m. Collinear emissions are regulated by the mass, so no need to factorize them → 3FS (3 light active flavours)

$$F_2(Q^2, m^2) = \sum_i^3 C_i^{[3]}(Q^2, m^2) \otimes f_i^{[3]}(Q^2)$$

$C_i^{[3]}(Q^2, m^2)$ contains logs of m^2/Q^2, which may get large for $Q^2 \gg m^2$ → resummation → 4FS (the charm is considered massless)

$$= \sum_k^4 C_k^{[4]}(Q^2, 0) \otimes f_k^{[4]}(Q^2) + \mathcal{O}\left(\frac{m^2}{Q^2}\right)$$

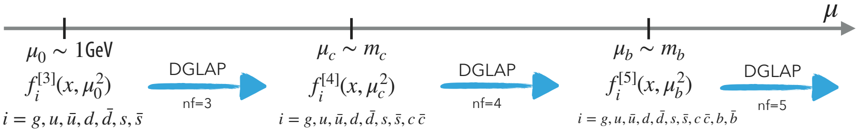
collinear emissions are factorized in the 4FS PDF, massless limit finite, power corrections lost.

Power corrections can be restored from the 3FS computation, after removing power counting

Linear relation between $f_i^{[3]}(Q^2)$ and $f_k^{[4]}(Q^2)$ (factorization scheme change)

Variable flavour number scheme (2)

Need the relation between PDFs in the two schemes



$$f_k^{[4]}(Q^2) = \sum_p^4 \sum_{i,j}^3 U_{kp}^{[4]}(Q^2,\mu_c^2) \otimes A_{pj}^{[4\leftarrow 3]}(\mu_c^2, m^2) \otimes U_{ji}^{[3]}(\mu_c^2,\mu_0^2) \otimes f_i^{[3]}(\mu_0^2)$$

where $A_{ki}^{[4\leftarrow 3]}(\mu_c^2, m^2)$ depends on m through logarithms of m^2/μ_c^2

- Choosing $\mu_c \sim m$ the logs of m^2/Q^2 are resummed through DGLAP evolution from μ_c^2 to Q^2
- Choosing $\mu_c = Q$ resummation is switched off and we get

$$f_k^{[4]}(Q^2) = \sum_i^3 A_{ki}^{[4\leftarrow 3]}(Q^2, m^2) \otimes f_i^{[3]}(Q^2)$$

Variable flavour number scheme (3)

We can thus write

$$\begin{aligned} F_2(Q^2, m^2) &= \sum_i^3 C_i^{[3]}(Q^2, m^2) \otimes f_i^{[3]}(Q^2) \\ &= \sum_k^4 C_k^{[4]}(Q^2, 0) \otimes f_k^{[4]}(Q^2) + \mathcal{O}\left(\frac{m^2}{Q^2}\right) \\ &= \sum_i^3 \sum_k^4 C_k^{[4]}(Q^2, 0) \otimes A_{ki}^{[4\leftarrow 3]}(Q^2, m^2) \otimes f_i^{[3]}(Q^2) + \mathcal{O}\left(\frac{m^2}{Q^2}\right) \end{aligned}$$

from which we get

$$\lim_{m^2 \ll Q^2} C_i^{[3]}(Q^2, m^2) = \sum_k^4 C_k^{[4]}(Q^2, 0) \otimes A_{ki}^{[4\leftarrow 3]}(Q^2, m^2)$$

out of which we can compute the matching functions $A_{ki}^{[4\leftarrow 3]}(Q^2, m^2)$ and the power corrections

$$\begin{aligned} \mathcal{O}\left(\frac{m^2}{Q^2}\right) &= \sum_i^3 \left[C_i^{[3]}(Q^2, m^2) - \sum_k^4 C_k^{[4]}(Q^2, 0) \otimes A_{ki}^{[4\leftarrow 3]}(Q^2, m^2) \right] \otimes f_i^{[3]}(Q^2) \\ &= \sum_k^4 \Delta C_k^{[4]}(Q^2, m^2) \otimes f_k^{[4]}(Q^2) \end{aligned}$$

Matching conditions at small x

Expanding this equation to NLL at small x

$$\lim_{m\ll Q} C_i^{[3]}(Q^2,m^2) = \sum_k^4 C_k^{[4]}(Q^2,0) \otimes A_{ki}^{[4\leftarrow 3]}(Q^2,m^2)$$

we get (in Mellin space) at LL [MB,Marzani,Muselli 1708.07510]

$$A_{ki}^{[4\leftarrow 3],\mathrm{LL}}(Q^2,m^2) = \delta_{ki}$$

and at NLL

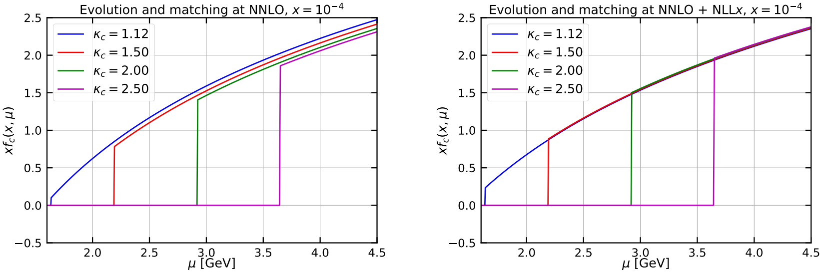
$$A_{cg}^{[4\leftarrow 3],\mathrm{NLL}}(Q^2,m^2) = \lim_{m\ll Q} C_g^{[3],\mathrm{NLL}}(Q^2,m^2) - C_g^{[4],\mathrm{NLL}}(Q^2,0)$$

$$A_{cq}^{[4\leftarrow 3],\mathrm{NLL}}(Q^2,m^2) = \frac{C_F}{C_A} A_{cg}^{[4\leftarrow 3],\mathrm{NLL}}(Q^2,m^2)$$

$$A_{ki}^{[4\leftarrow 3],\mathrm{NLL}}(Q^2,m^2) = 0 \quad \text{if } k \text{ is light } (g,u,d,s)$$

Perturbative stability of charm matching conditions

$$f_c^{[4]}(Q^2) = \sum_p^4 \sum_{i,j}^3 U_{cp}^{[4]}(Q^2, \mu_c^2) \otimes A_{pj}^{[4\leftarrow 3]}(\mu_c^2, m^2) \otimes U_{ji}^{[3]}(\mu_c^2, \mu_0^2) \otimes f_i^{[3]}(\mu_0^2)$$



$\kappa_c = \mu_c/m$, μ_c = charm matching scale (threshold)

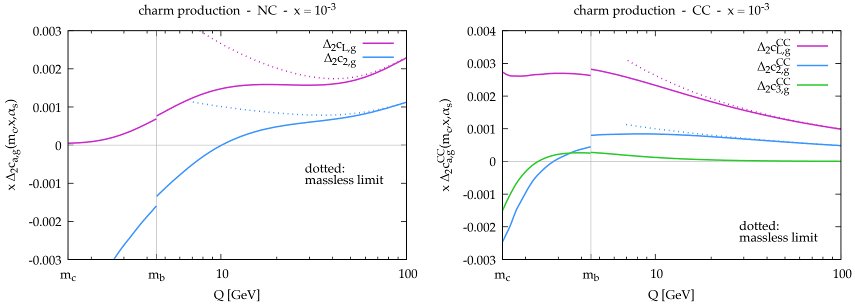
[xFitter 1802.00064]

The perturbatively generated charm PDF is much less dependent on the (unphysical) matching scale when small-x resummation is included!

Resummed result with power corrections included:

$$F_2(Q^2,m^2) = \sum_k^4 \left[C_k^{[4]}(Q^2,0) + \Delta C_k^{[4]}(Q^2,m^2) \right] \otimes f_k^{[4]}(Q^2)$$

Resummed contribution to $F_{2,g}$, $F_{L,g}$ and $F_{3,g}$ DIS coefficient functions for the charm with mass effects



heavy quark production at LHC

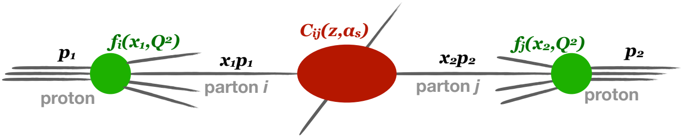
[Catani,Ciafaloni,Hautmann NPB 366 (1991) 135]
[Ball,Ellis 0101199]
[Chachamis,Deák,Hentschinski,Rodrigo,Sabio Vera 1507.05778]
[Guiot 1812.02156]
[Bolognino,Celiberto,Fucilla,Ivanov,Mohammed,Papa 1909.03068, 2109.11875]
[MB,Silvetti 2211.10142]

Theoretical predictions with hadrons in the initial state

QCD collinear factorization:

$y = Y - \frac{1}{2}\log\frac{x_1}{x_2}$

$$\frac{d\sigma}{dQ^2\,dY\,dp_t\ldots} = \sum_{i,j=g,q}\int_\tau^1 dx_1 \int_\tau^1 dx_2\, f_i(x_1,Q^2)\, f_j(x_2,Q^2)\, C_{ij}\left(\frac{\tau}{x_1 x_2}, y, p_t, \ldots, \alpha_s\right)$$

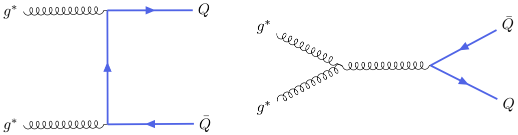


$x_1, x_2, \frac{\tau}{x_1 x_2}$ can get as small as $\tau = \frac{Q^2}{S}$ (note: typical values $x_1, x_2 \sim \sqrt{\tau}$)

τ	Higgs	low mass DY	$b\bar{b}$	$c\bar{c}$
LHC (13 TeV)	10^{-4}	$\sim 10^{-6}$	$\sim 10^{-6}$	$\sim 10^{-7}$
FCC-hh (100 TeV)	10^{-6}	$\sim 10^{-8}$	$\sim 10^{-8}$	$\sim 10^{-9}$

Heavy-quark production at pp colliders probes rather small x!!

Fully differential heavy-quark pair production [MB,Silvetti 2211.10142]



$$g^*(k_1) + g^*(k_2) \to Q(p) + \bar{Q}(\bar{p}) \qquad \text{(pair with momentum } q = p + \bar{p}\text{)}$$

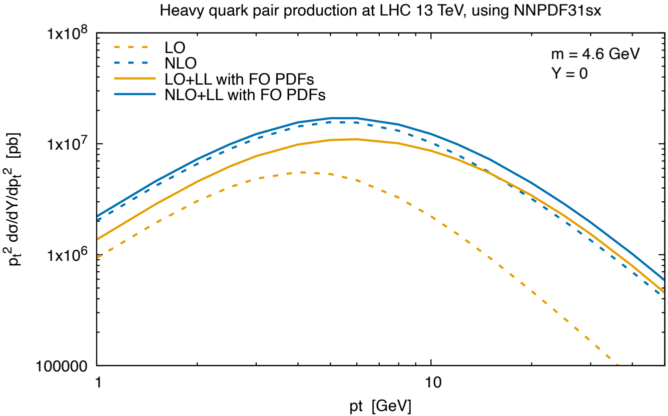
We have resummed the cross section for different kinematics:

- single heavy quark $\dfrac{d\sigma}{dY\, dp_t}$ → e.g. D, B meson production
- heavy-quark pair $\dfrac{d\sigma}{dQ^2\, dY\, dq_t}$ → e.g. J/ψ, Υ production

Small-x resummation crucial for charm and bottom production

- sensitive to very small x → constrain the PDFs [Gauld, Rojo 1610.09373]
- key process at a forward physics facility (FPF) [Feng et al 2203.05090]

Heavy-quark pair production at LHC: single-quark kinematics

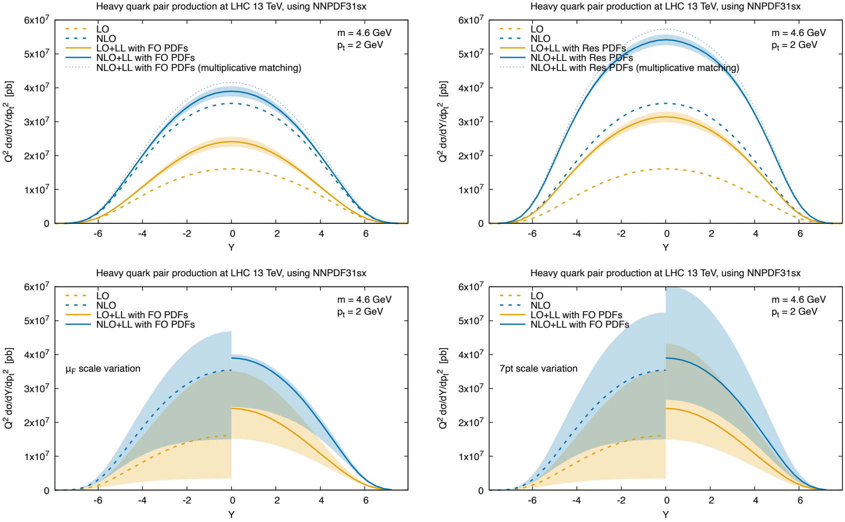


At large p_t a larger perturbative instability, likely due to small-x logs, as it is cured by resummation

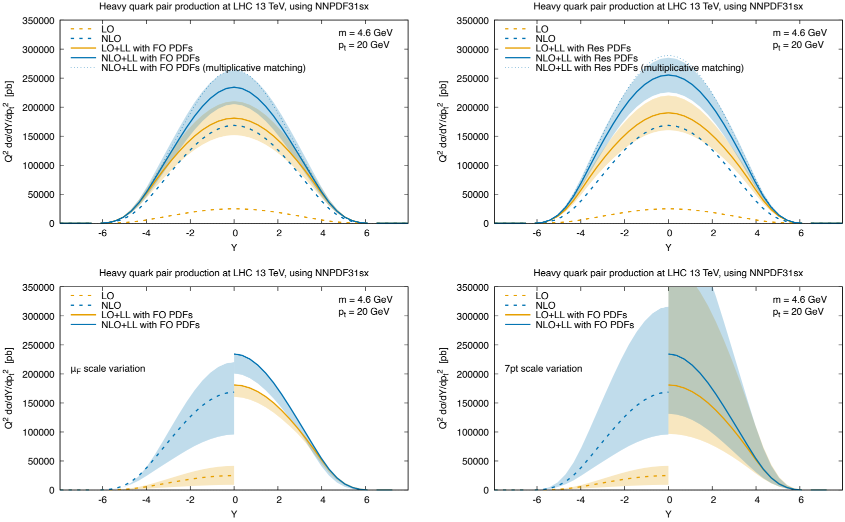
Induced by kinematic constraint $\quad x\, e^{2|y|}\left(1+\frac{p_t^2}{m^2}\right) \leq 1$

Heavy-quark pair production at LHC: single-quark kinematics

Low $p_t = 2$ GeV



High $p_t = 20$ GeV



conclusions

Conclusions

Why small x resummation:

- stabilises perturbative expansion at small $x \lesssim 10^{-3}$
- has important effects in PDFs at small x, especially the gluon
- impact on LHC and future high-energy colliders

Heavy quark production in DIS:

- crucial for DIS description and so for PDF determination
- small-x resummation at lowest non-trivial order known (available from HELL)
- variable flavour number scheme at small x fully ready (from HELL)

Heavy quark production at pp colliders:

- can probe very small x
- fully differential resummed results available (from HELL)
- single-quark and quark-pair kinematics both considered
- potential for improving knowledge of PDFs at small x

backup slides

The role of the longitudinal structure function

The HERA data are reduced cross sections, given by

$$\sigma_{r,\mathrm{NC}} = F_2(x, Q^2) - \frac{y^2}{1 + (1-y)^2} F_L(x, Q^2) \qquad\qquad y = \frac{Q^2}{x\, s}$$

in terms of the structure functions F_2, F_L

The turnover can be explained by a larger F_L, contributing mostly at small x

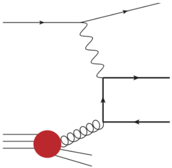
The other option, a turnover in F_2, seems unlikely (requires peculiar PDF shape)

Note that $F_L = \mathcal{O}(\alpha_s)$, and it is gluon dominated

It plays a key role in DIS at small x

$\Rightarrow$ having good measurements of F_L is very important!

Future ep colliders (LHeC, FCC-eh) could provide precise F_L measurements!!

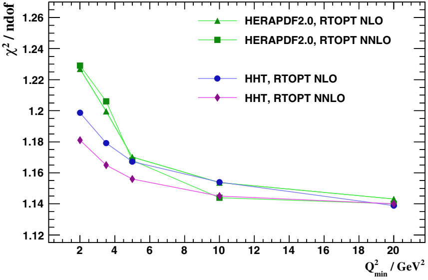


Higher twist explanation of HERA low-x data

$$F_L \to F_L \times \left(1 + \frac{A_L}{Q^2}\right)$$

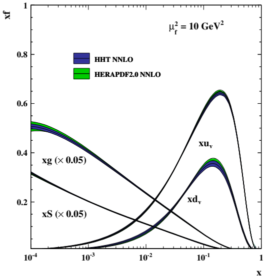
with A_L fitted from data

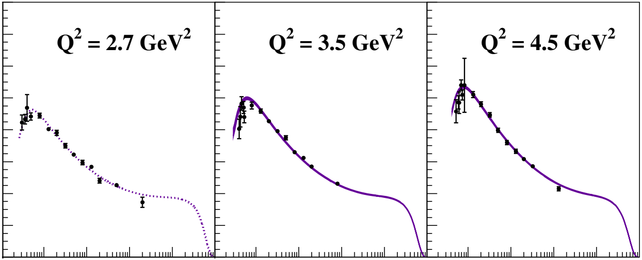
Improved description, but χ^2 still grows



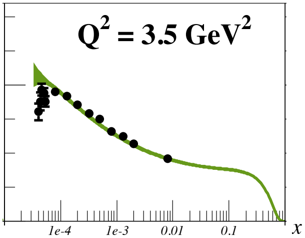
PDFs unaffected

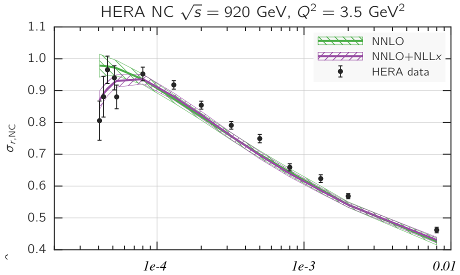
↓ ↓ ↓ ↓ ↓





[Abt, Cooper-Sarkar, Foster, Myronenko, Wichmann, Wing 1604.02299]

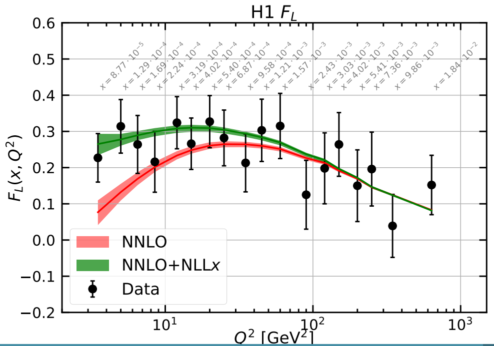




Improved description of the data, turnover well reproduced

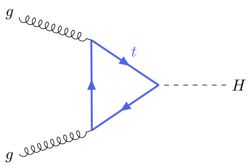
The better description mostly comes from a larger resummed F_L

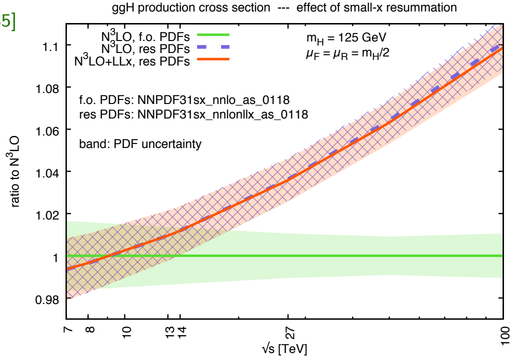
Note: no extra parameters in the fit, just improved theory



$gg \to H$ inclusive cross section

[MB,Marzani 1802.07758] [MB 1805.08785]





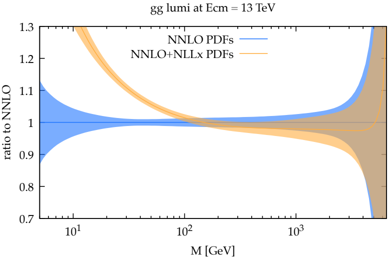
ggH cross section at FCC-hh $\sim \mathbf{10\%}$ larger than expected!

At LHC $+1\%$ effect; larger effect expected at differential level

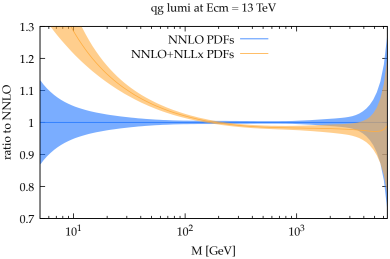
Other recent works on Higgs production [Hentschinski,Kutak,vanHameren 2011.03193] [Celiberto,Ivanov,Mohammed,Papa 2008.00501]

Parton luminosities at LHC

gg:

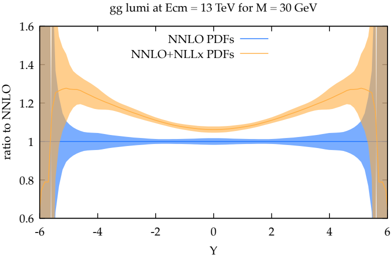


qg:

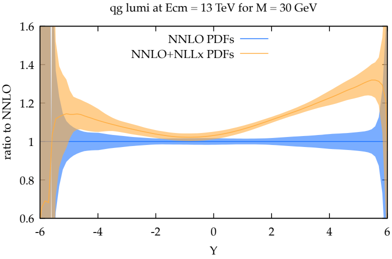


Difference more pronounced in differential distributions at large rapidity

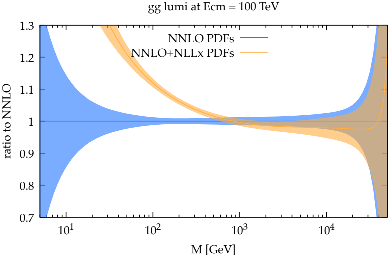
gg:



qg:



Parton luminosities at FCC-hh

gg:


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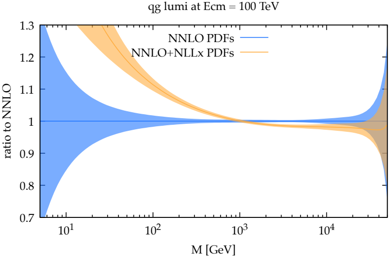
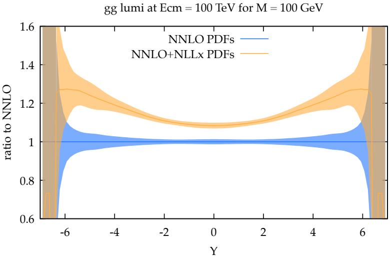


Large effects also at the EW scale, especially at large rapidities

gg:


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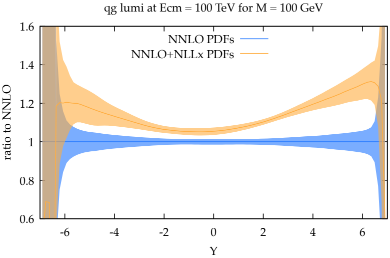
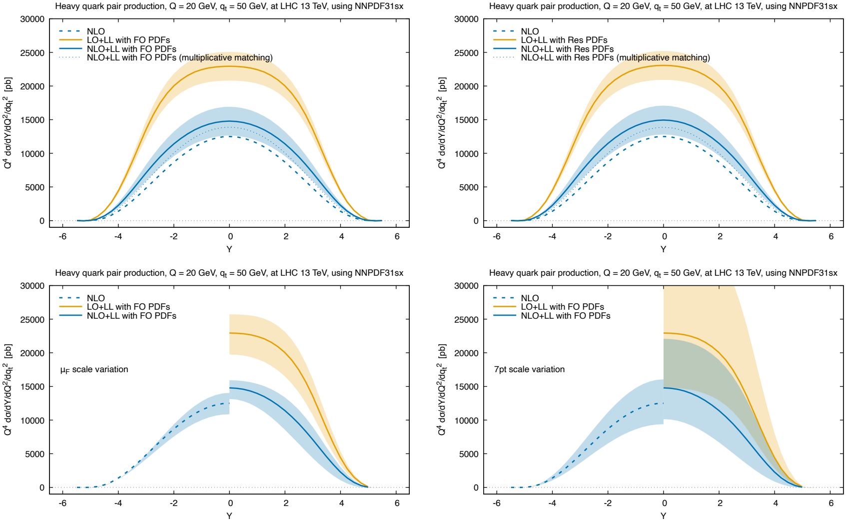
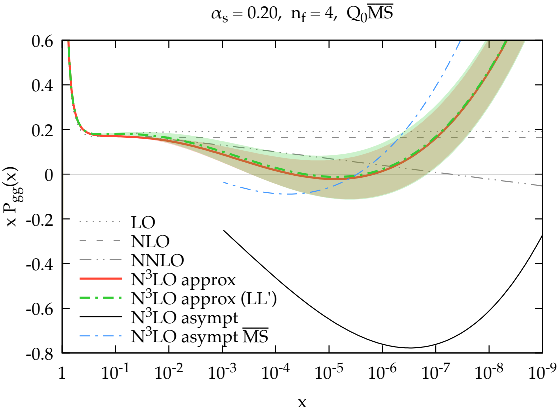


Heavy-quark pair production at LHC: quark-pair kinematics



Approximate N^{3}LO splitting functions from resummation [MB,Marzani 1805.06460]



Large uncertainties from subleading logs

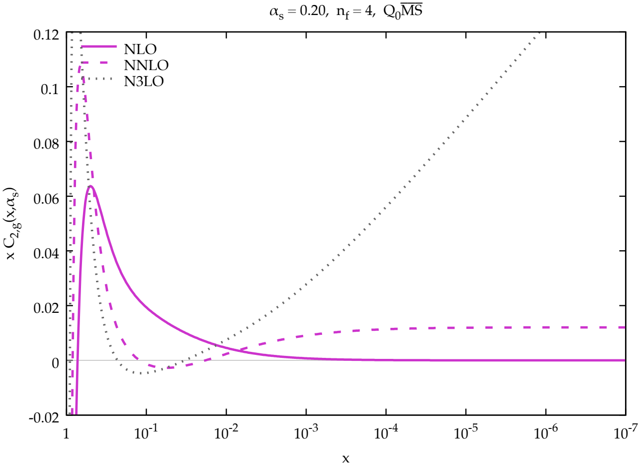
$$xP_{gg}^{(3)}(x) = a_{\rm LL}^{\rm known} \log^3 \frac{1}{x} + a_{\rm NLL}^{\rm known} \log^2 \frac{1}{x} + a_{\rm NNLL}^{\rm not\ known} \log \frac{1}{x} + \ldots$$

Could be constrained with the recent impressive progress (Mellin moments)

[Davies,Vogt,Ruijl,Ueda,Vermaseren 1610.07477] [Moch,Ruijl,Ueda,Vermaseren,Vogt 1707.08315]

Small-x logarithms in DIS coefficient functions

Only **singlet sector** affected: $\boldsymbol{C_{a,g}}$, $C^{\rm S}_{a,q}$, $a = \mathbf{2}, L, 3$



fixed-order $xC_{2,g}(x, \alpha_s)$ splitting function at small x:

LO:
0

NLO:
$\alpha_s \times 0$

NNLO:
$\alpha_s^2 \times \text{const}$

N^{3}LO:
$\alpha_s^3 \left(\ln \frac{1}{x} + \text{const}\right)$

DIS coefficient functions are NLL quantities

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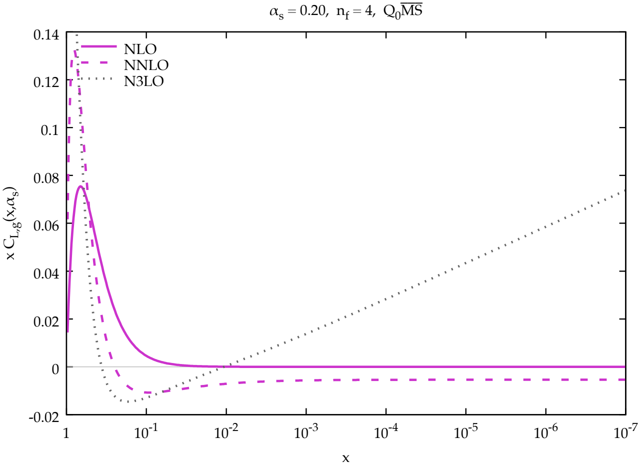

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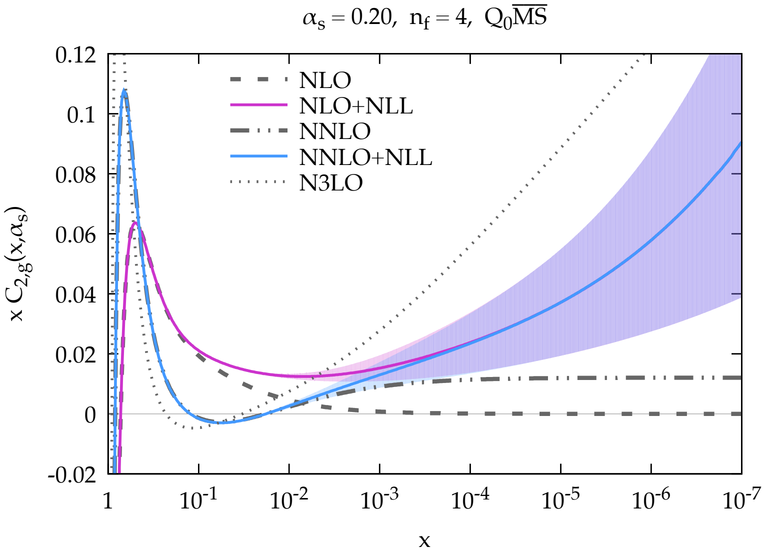
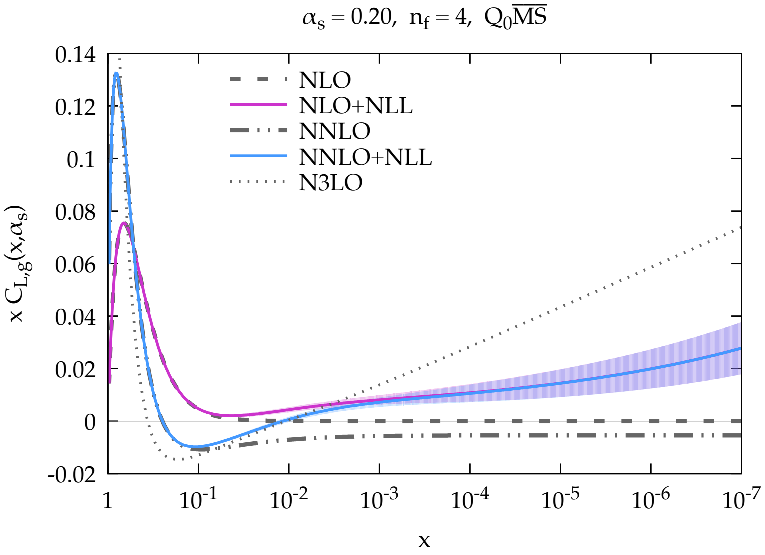
DIS coefficient functions are NLL quantities

Some representative HELL results: DIS coefficient functions

$F_{2,g}$ and $F_{L,g}$ massless DIS coefficient functions



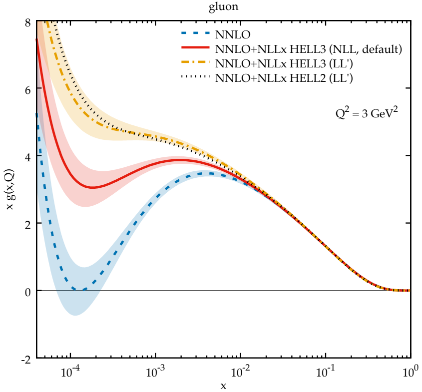
$F_{2,g}$ and $F_{L,g}$ massless DIS coefficient functions

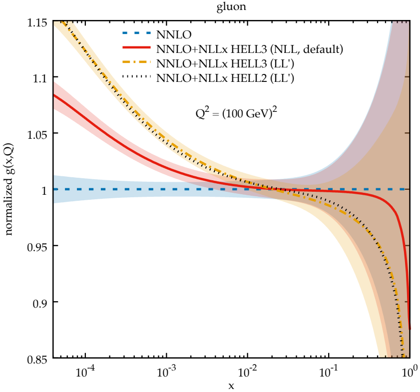


Impact of subleading logs (with xFitter)

First fit with `HELL 3.0` [MB,Giuli 1902.11125]

Red and yellow curves differ by subleading logs





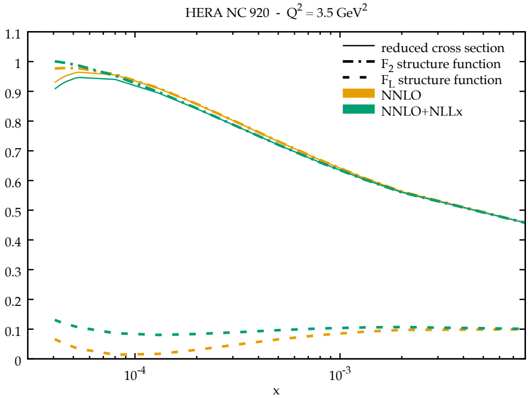
Achieved with a new parametrization, more flexible at small x

$$xf(x,\mu_0^2) = A\,x^B(1-x)^C\left[1 + Dx + Ex^2 + F\log x + G\log^2 x + H\log^3 x\right]$$

Improved description of the low x data even at fixed order

Impact of subleading logs (with xFitter)

The good agreement obtained at fixed order with the low x HERA data is achieved in a different way with respect to the resummed case [MB,Giuli 1902.11125]



At resummed level, both F_L and F_2 grow

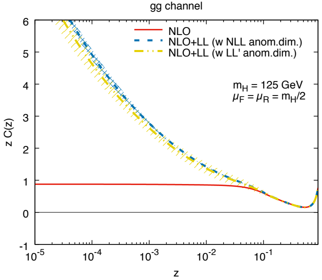
At fixed order, F_L grows below $x \sim 10^{-4}$ and F_2 decreases, due to the sudden growth of the gluon PDF

Why is the effect of resummation mostly driven by the PDFs?

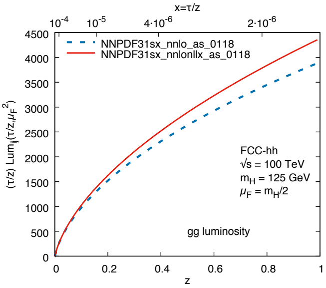
$$\frac{d\sigma}{dQ^2 dY...} = \int_\tau^1 \frac{dz}{z} \int d\hat{y}\, f_i\left(\sqrt{\frac{\tau}{z}} e^{\hat{y}}, Q^2\right) f_j\left(\sqrt{\frac{\tau}{z}} e^{-\hat{y}}, Q^2\right) C_{ij}(z, Y - \hat{y}, ..., \alpha_s)$$

The small z integration region, where logs in C are large, is weighted by the PDFs at large momentum fractions $x = \sqrt{\frac{\tau}{z}} e^{\pm\hat{y}}$
Since PDFs die fast at large x, especially the gluon, the small-z region is suppressed!

Rather, the large z region is enhanced by the gluon-gluon luminosity
In that region, the difference between fixed-order and resummed PDFs is large



gg partonic cross section



gg luminosity