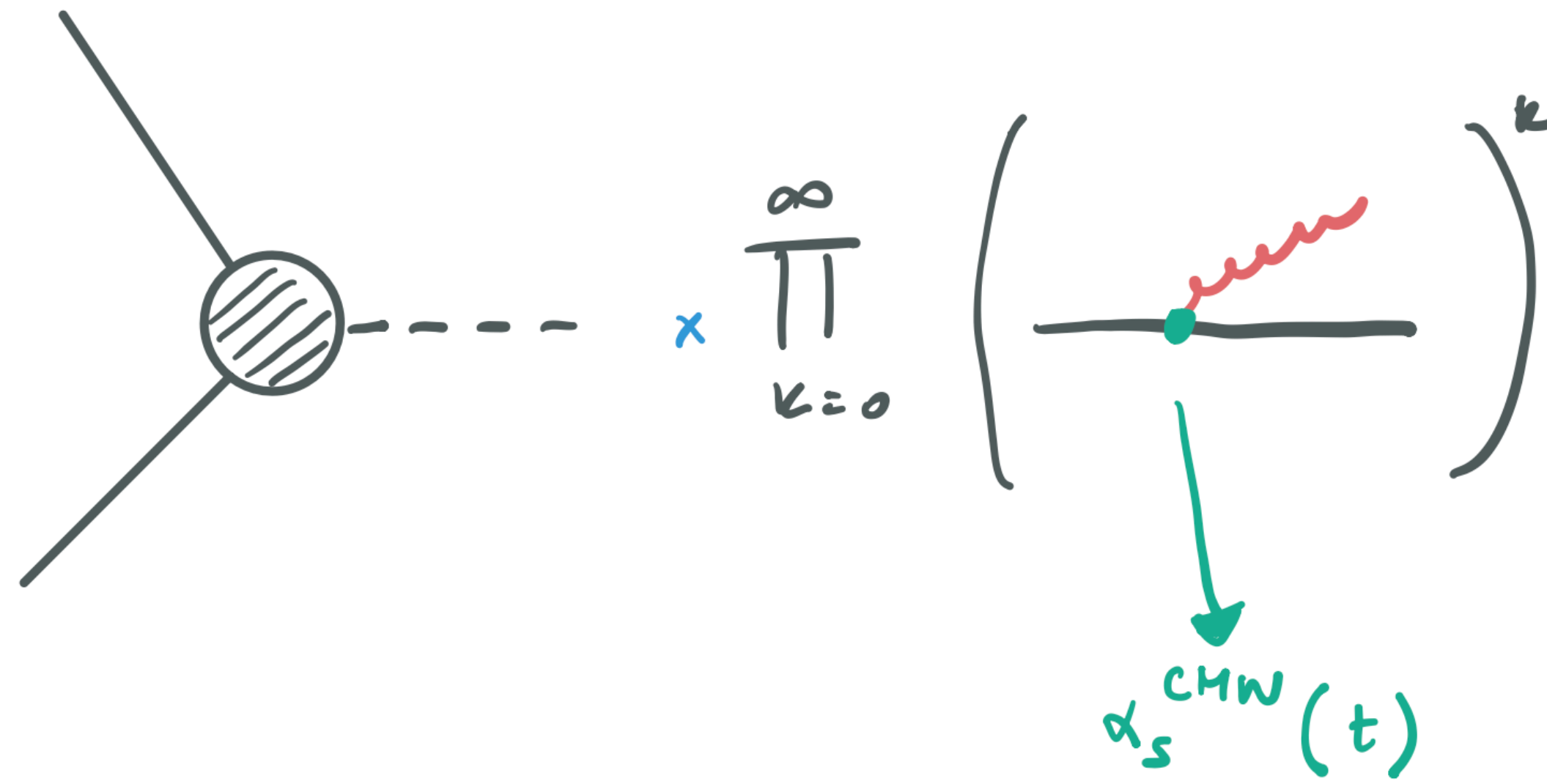


Parton Showers and Massive Quarks

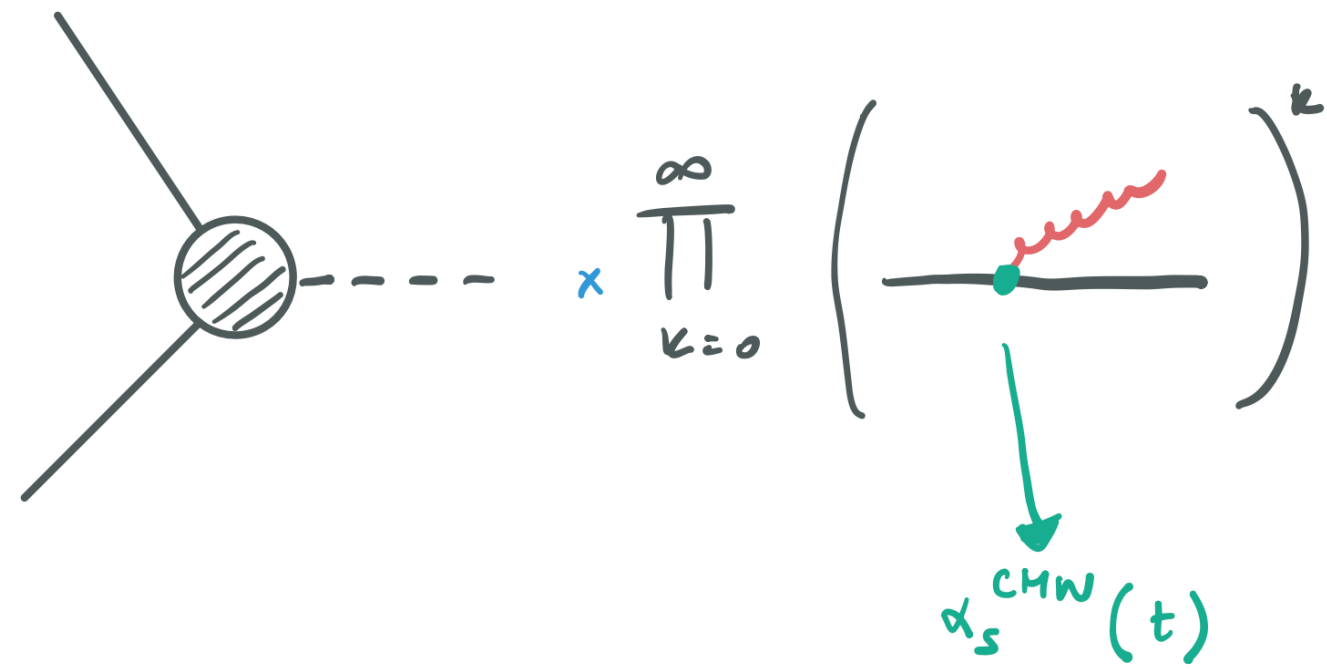
Heavy Flavours @ High p_T , Edinburgh, 30/11/2023



Parton Showers



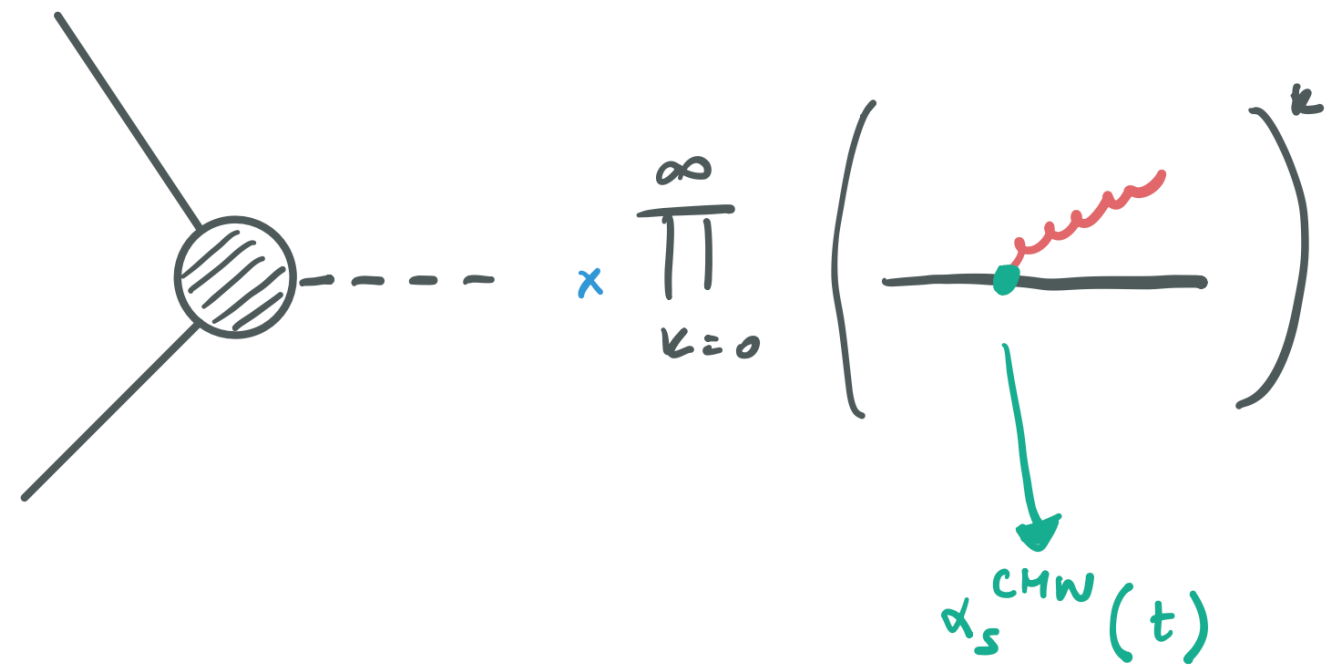
Parton Showers



$$d\sigma = d\sigma_0 \times \exp \left\{ - \int_{z_0}^{z_1} dz \int_{t_f}^{t_i} \frac{dt}{t} \frac{\alpha_s(t)}{2\pi} \mathbf{V}(z, t) \right\}$$

Evolution Scale

Parton Showers



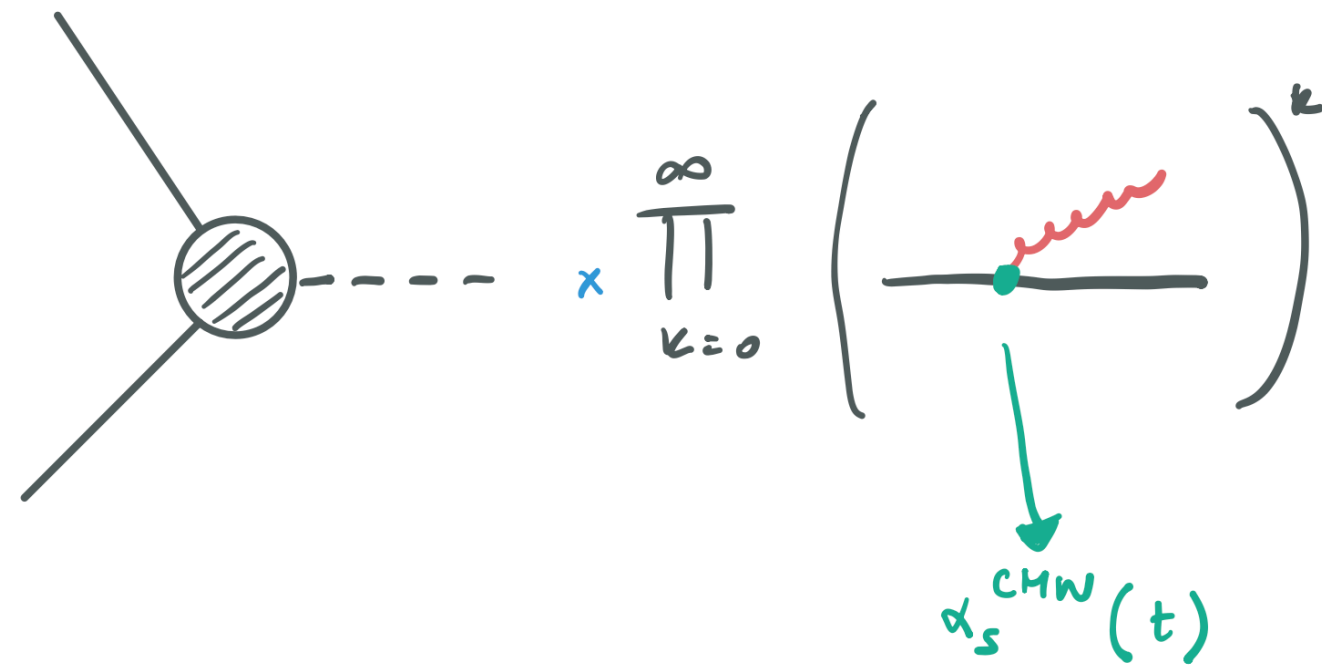
$$d\sigma = d\sigma_0 \times \exp \left\{ - \int_{z_0}^{z_1} dz \int_{t_f}^{t_i} \frac{dt}{t} \frac{\alpha_s(t)}{2\pi} \mathbf{V}(z, t) \right\}$$

Evolution Scale

$\alpha_s^{CHW}(t)$



Parton Showers



$$d\sigma = d\sigma_0 \times \exp \left\{ - \int_{z_0}^{z_1} dz \int_{t_f}^{t_i} \frac{dt}{t} \frac{\alpha_s(t)}{2\pi} \mathbf{V}(z, t) \right\}$$

Evolution Scale

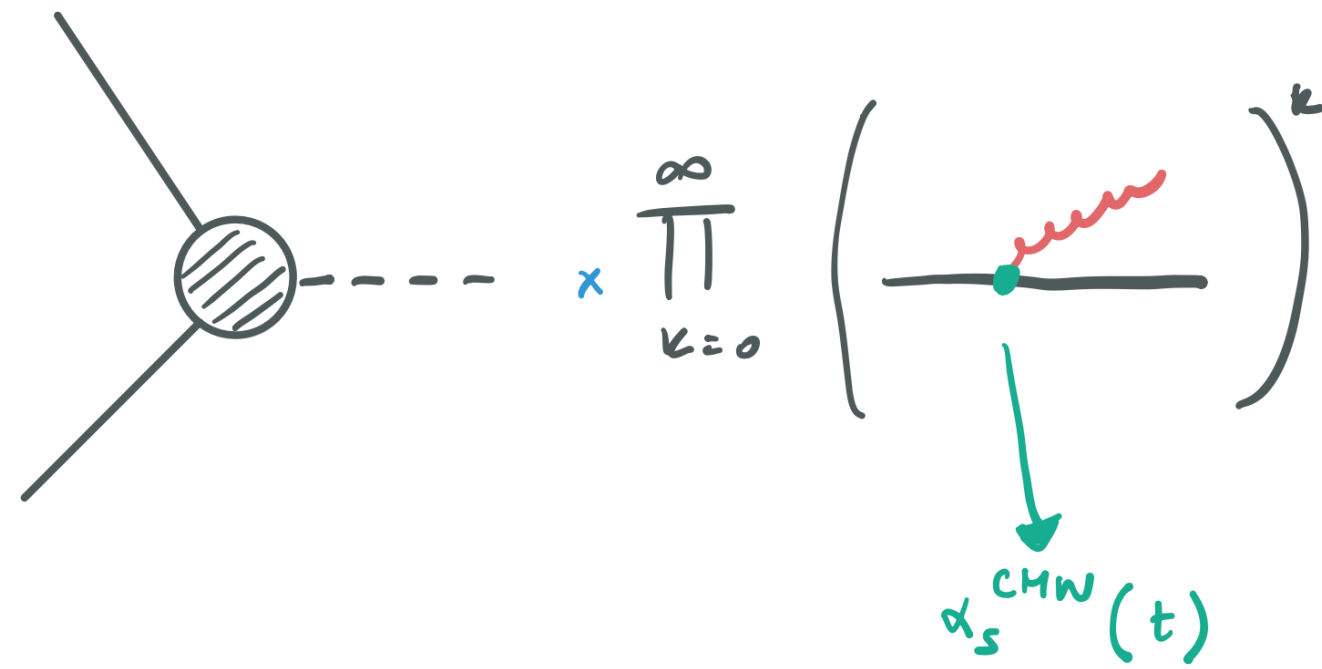
$\alpha_s^{CHW}(t)$



KINEMATICS...

(See for example JHEP 04 (2020) 019)

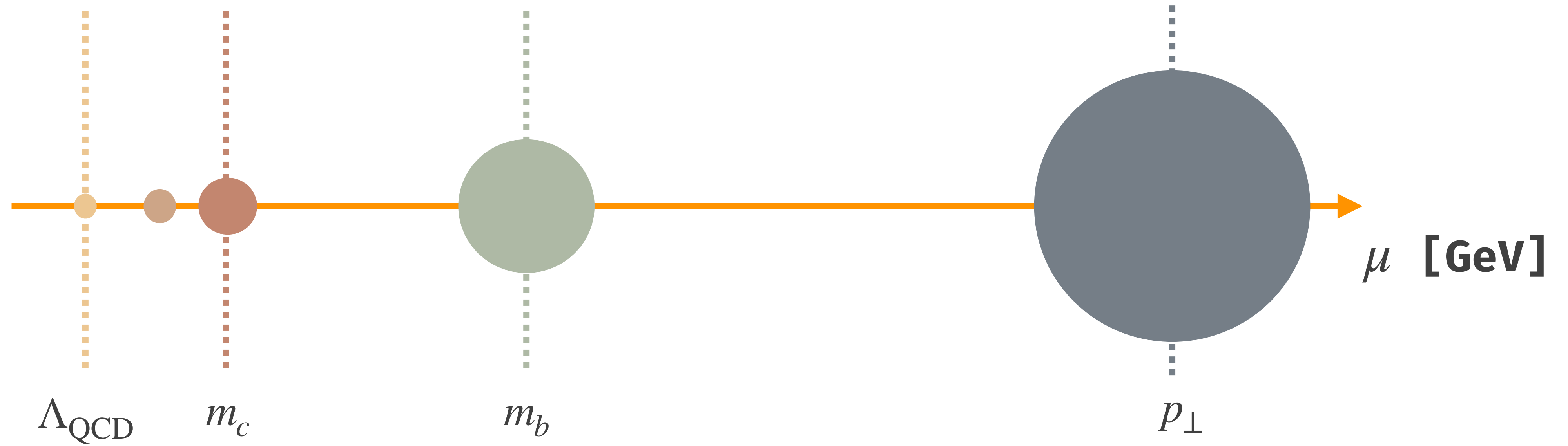
Parton Showers



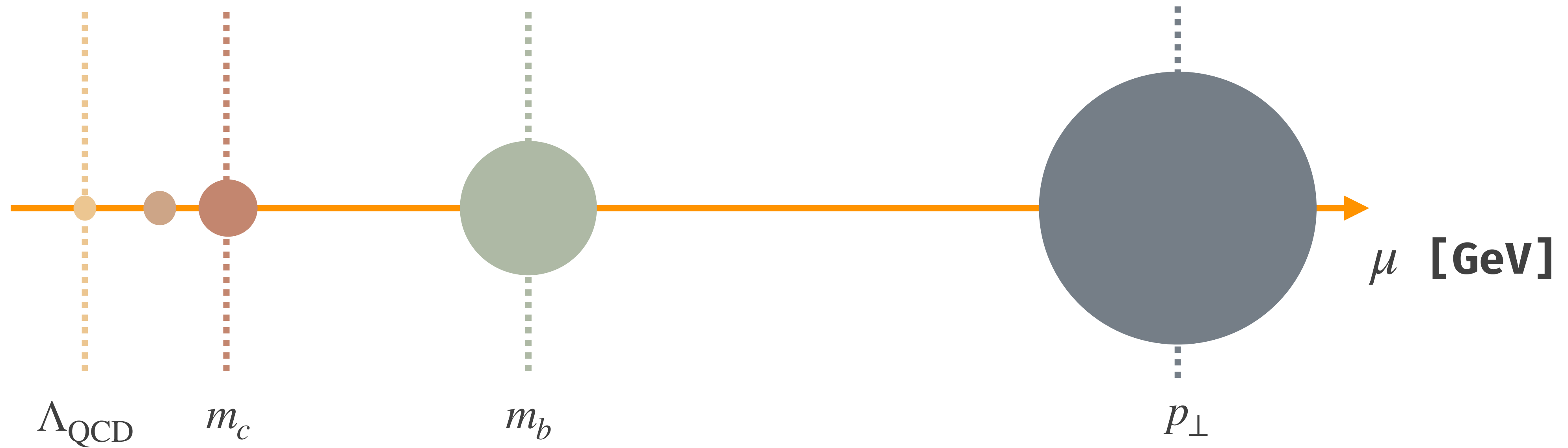
$$d\sigma = d\sigma_0 \times \exp \left\{ - \int_{z_0}^{z_1} dz \int_{t_f}^{t_i} \frac{dt}{t} \frac{\alpha_s(t)}{2\pi} \mathbf{V}(z, t) \right\}$$

- **Accuracy of the PS either comparing to resummation**
- **Or comparing to Fixed-Order predictions**

Mass Effects



Mass Effects



- $p_{\perp} \sim m_Q \Rightarrow$ **Retain mass effects**
- $p_{\perp} \gg m_Q \Rightarrow$ **Neglect mass effects**

Combining the two

- PS cannot ignore quark masses, necessary for hadronisation

Minimal Approach

- Massless Evolution
- At the end, shuffle momenta

Fully Massive Approach

- Mass included in kinematics and splittings
- Evolution same as massless

Combining the two

- PS cannot ignore quark masses, necessary for hadronisation

Minimal Approach

- Massless Evolution
- At the end, shuffle momenta

Fully Massive Approach

- Mass included in kinematics and splittings
- Evolution same as massless

$$t = \mathbf{k}_\perp^2 = 2 p_i \cdot p_j z (1 - z) - m_i^2 (1 - z)^2 - m_j^2 z^2$$

• Evolution scale/Argument of α_S (massless)

Figure 1 consists of two parts, (a) and (b), showing Feynman diagrams. Part (a) shows the Dyson equation for the inverse propagator: a horizontal line with a central dot is equal to a horizontal line with a shaded oval labeled 'NS' containing a loop with a dot. Part (b) shows the skeleton expansion of the flavour non-singlet kernel: a shaded oval labeled 'NS' is equal to a sum of four diagrams. The first is a horizontal line with a central dot. The second is a horizontal line with a loop. The third is a horizontal line with a loop and a wavy line. The fourth is a horizontal line with a loop and a wavy line with a dot. The expansion ends with '+ ...'.

Fig. 1. Dyson equation for the inverse propagator and skeleton expansion of the flavour non-singlet kernel.

$k'^2 = 0$ in the arguments of $\Gamma_{\text{qqg}}^2 d_g$ and $k'^2 = k^2 z$ (i.e., its kinematical limit) in d_q . We then find that vertices and propagators appear in the combination

$$\frac{g^2}{4\pi} \Gamma_{\text{qqg}}^2(k^2, 0, -k^2(1-z); k_{i||}) d_g(-k^2(1-z)) d_q(-k^2 z) = d_q^{-1}(-k^2) G^{\text{coll}}|_{\xi=0}. \quad (2.16)$$

We will now argue that it is legitimate to replace $G^{\text{coll}}|_{\xi=0}$ in eq. (2.16) by $\alpha_s(k^2(1-z))$. Let us first notice that for fixed z the kernel depicted in fig. 1b is the

Approaches: Evolution Scale

- Evolution scale/Argument of α_s (massive)
- Everything seems fine for Q-Qg splitting, what about g->QQ?

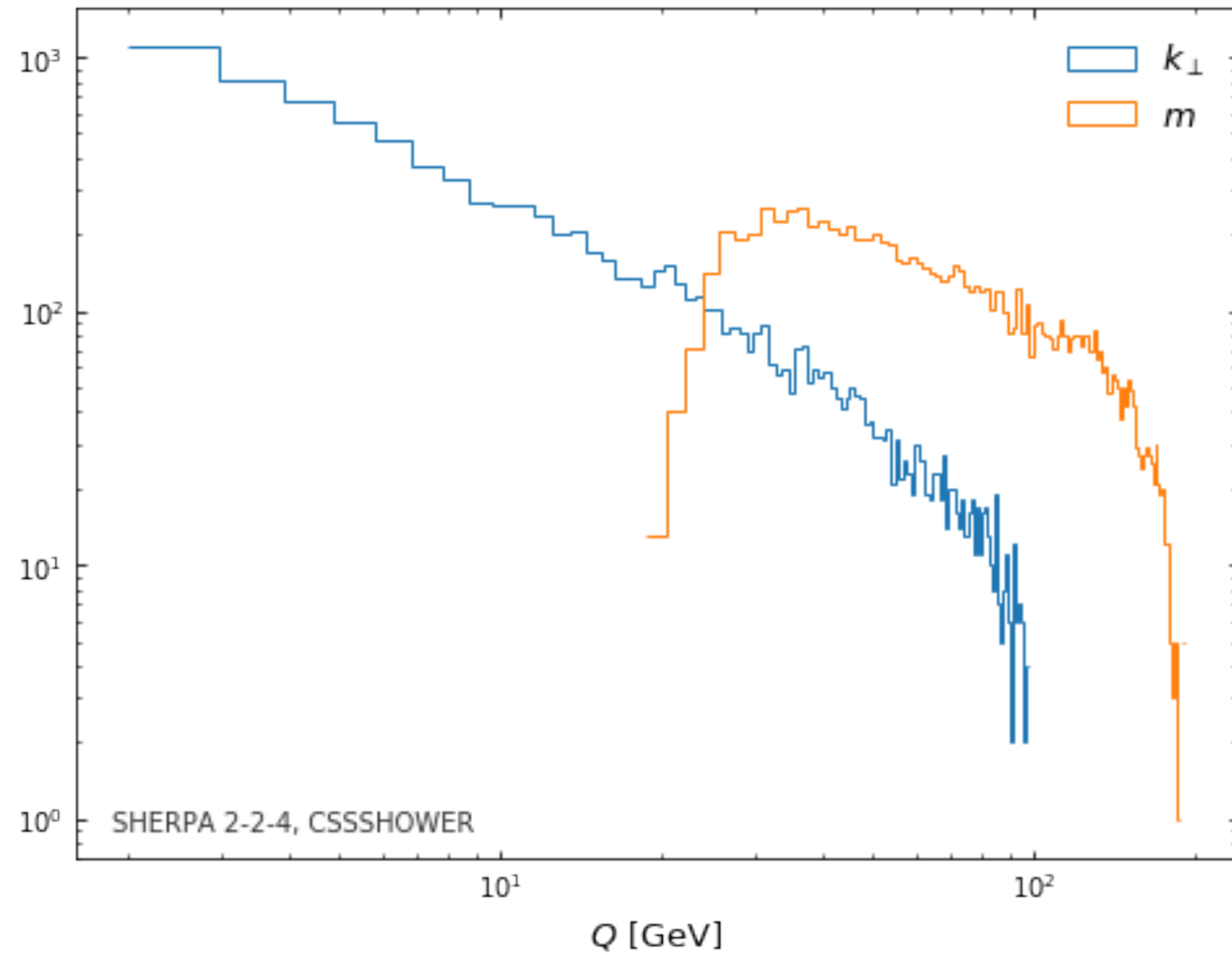


A Feynman diagram showing a gluon (represented by a red wavy line) splitting into a quark-antiquark pair (represented by two black lines meeting at a green vertex). The diagram is positioned to the left of the mathematical expression.

$$\alpha_s (m_{QQ}^2, t, m_{QQ}^2 + t, \sqrt{m_{QQ}^2 \cdot t}, \dots)$$

Approaches: Evolution Scale, $g \rightarrow QQ$

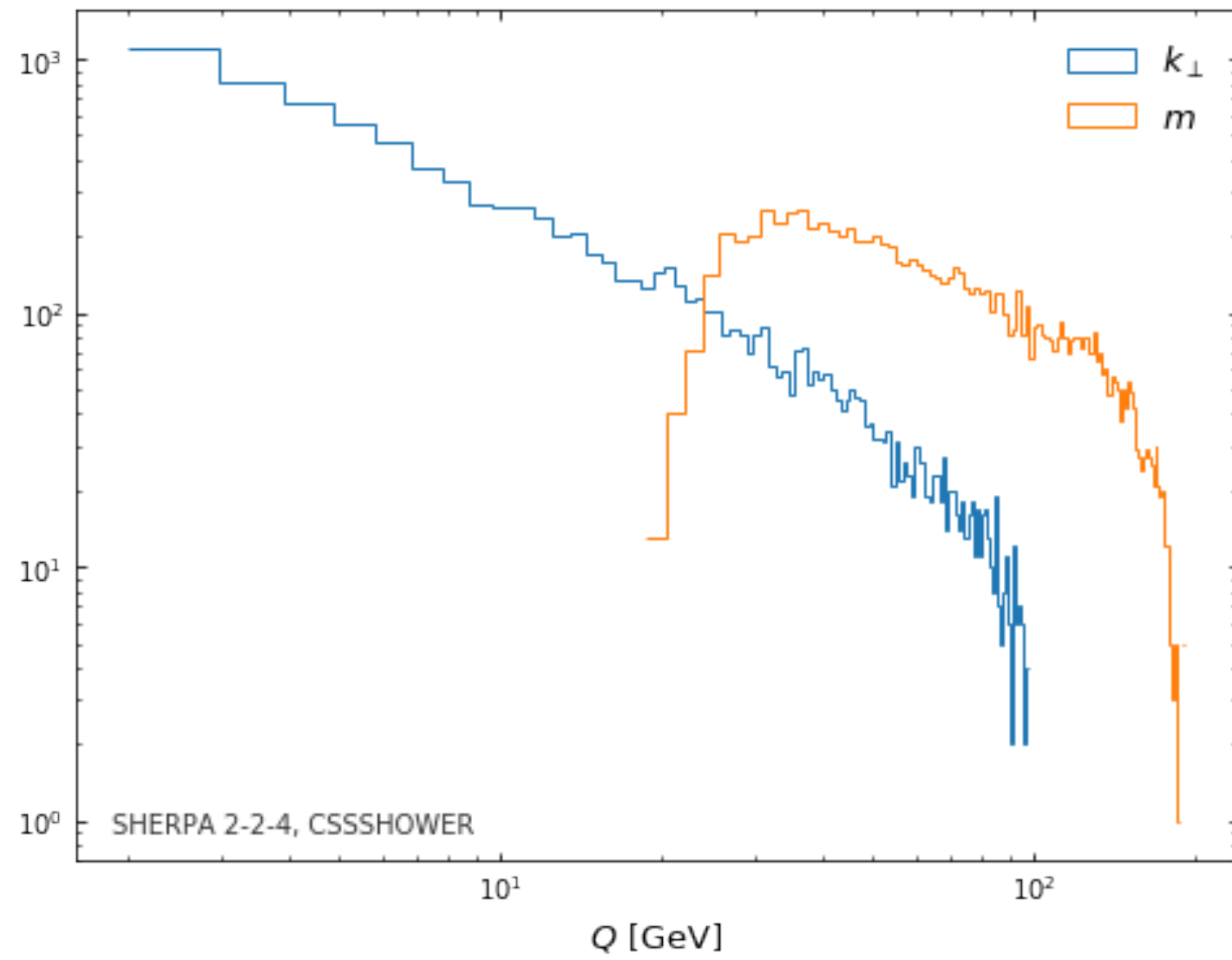
[Nucl.Phys. B436 (1995) 163-183]



$$\tau_g \sim \frac{E}{t^2} \ll \tau_{QQ\bar{Q}} \sim \frac{E'}{t'^2} + \frac{1}{m} \quad k_{\perp} \gg m$$

Approaches: Evolution Scale, g->QQ

[Nucl.Phys. B436 (1995) 163-183]



$$\tau_g \sim \frac{E}{t^2} \ll \tau_{Q\bar{Q}} \sim \frac{E'}{t'^2} + \frac{1}{m} \quad k_{\perp} \gg m$$

- **Evolution of heavy quarks, take $\gamma^* \rightarrow qqQQ$, the matrix element reads**

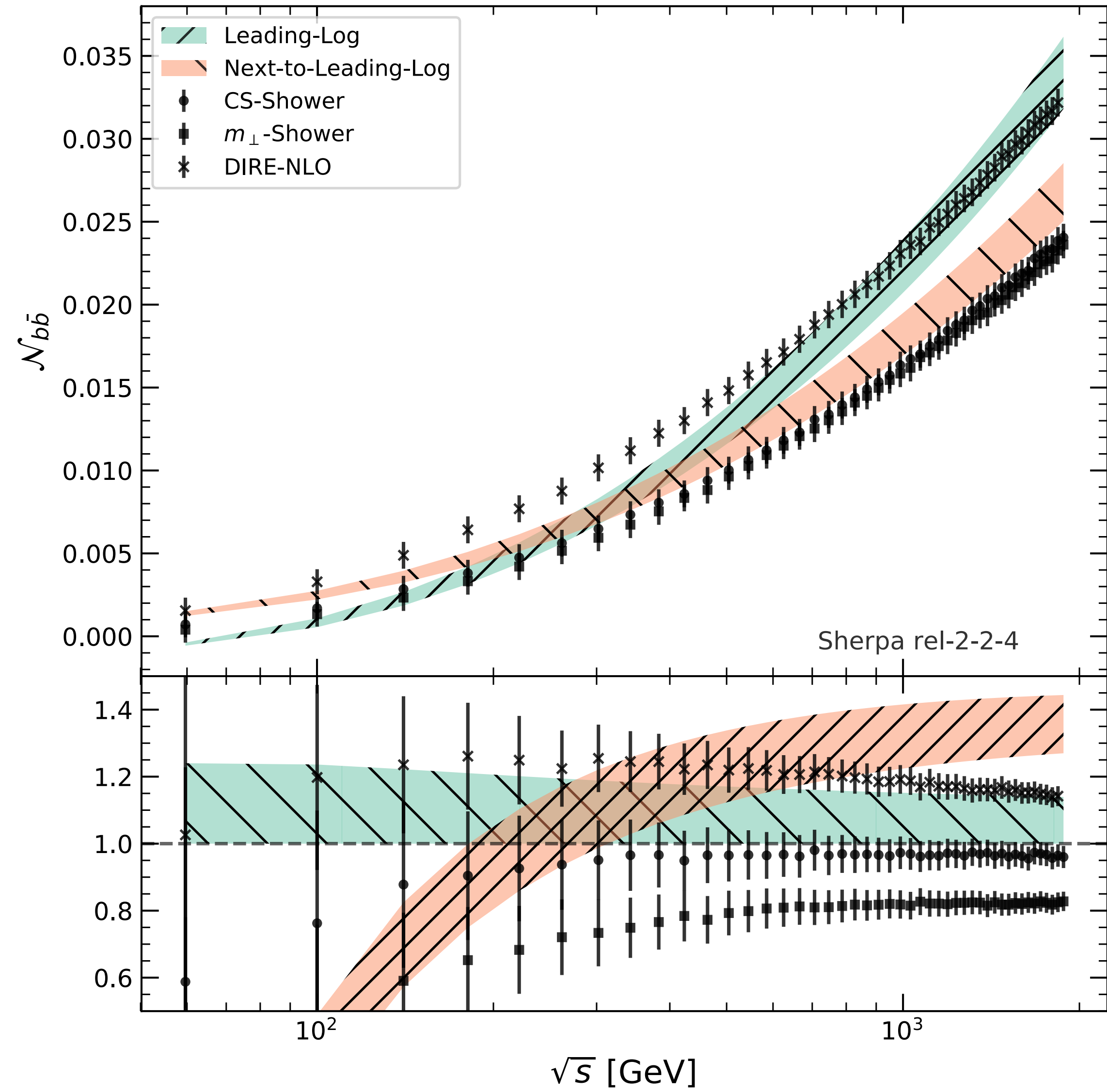
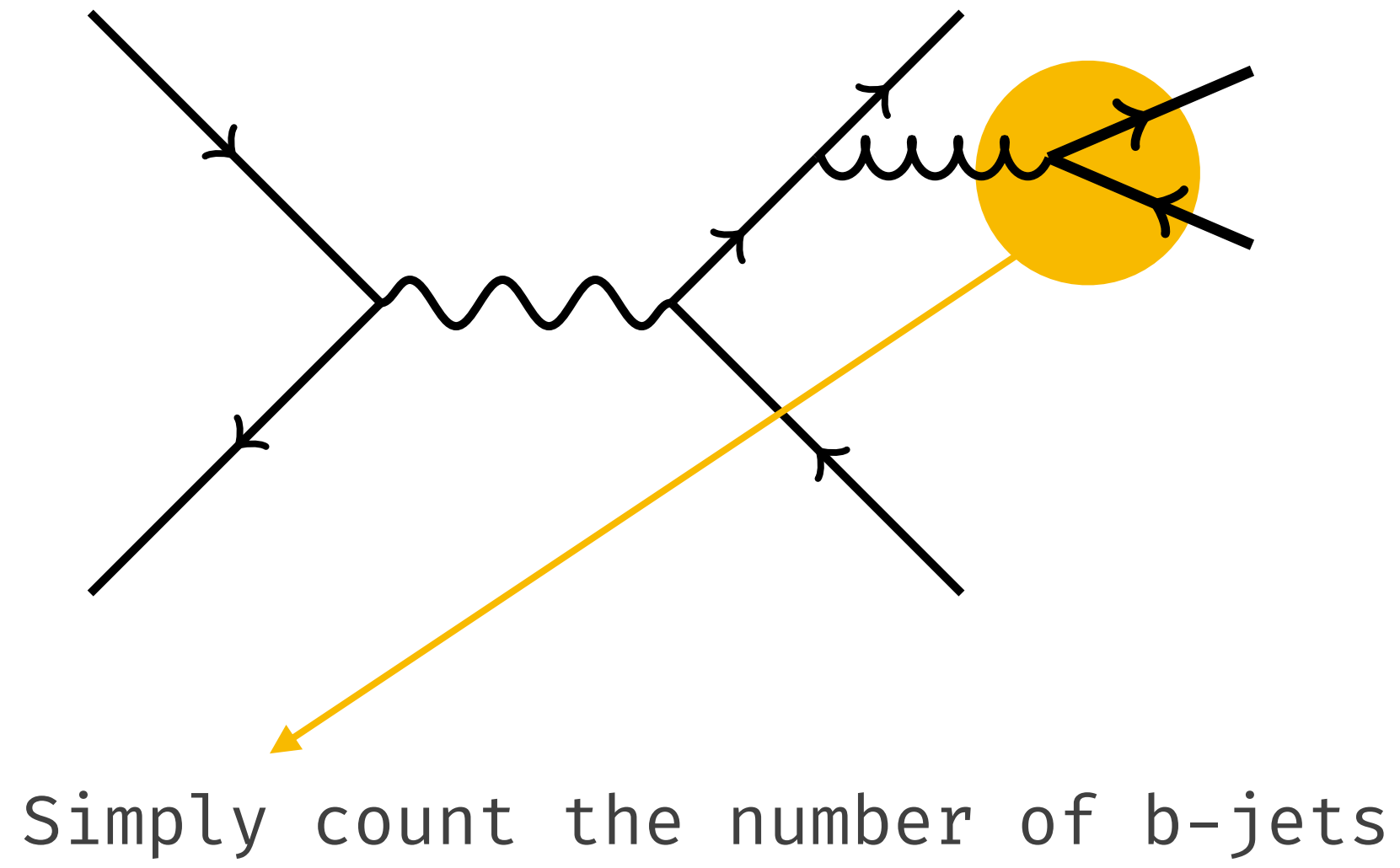
$$\frac{d\sigma}{\sigma_0} \propto \frac{dK_{\perp}^2}{K_{\perp}^2} \dots$$

- **If we take the QQ as if emitted from an off shell gluon**

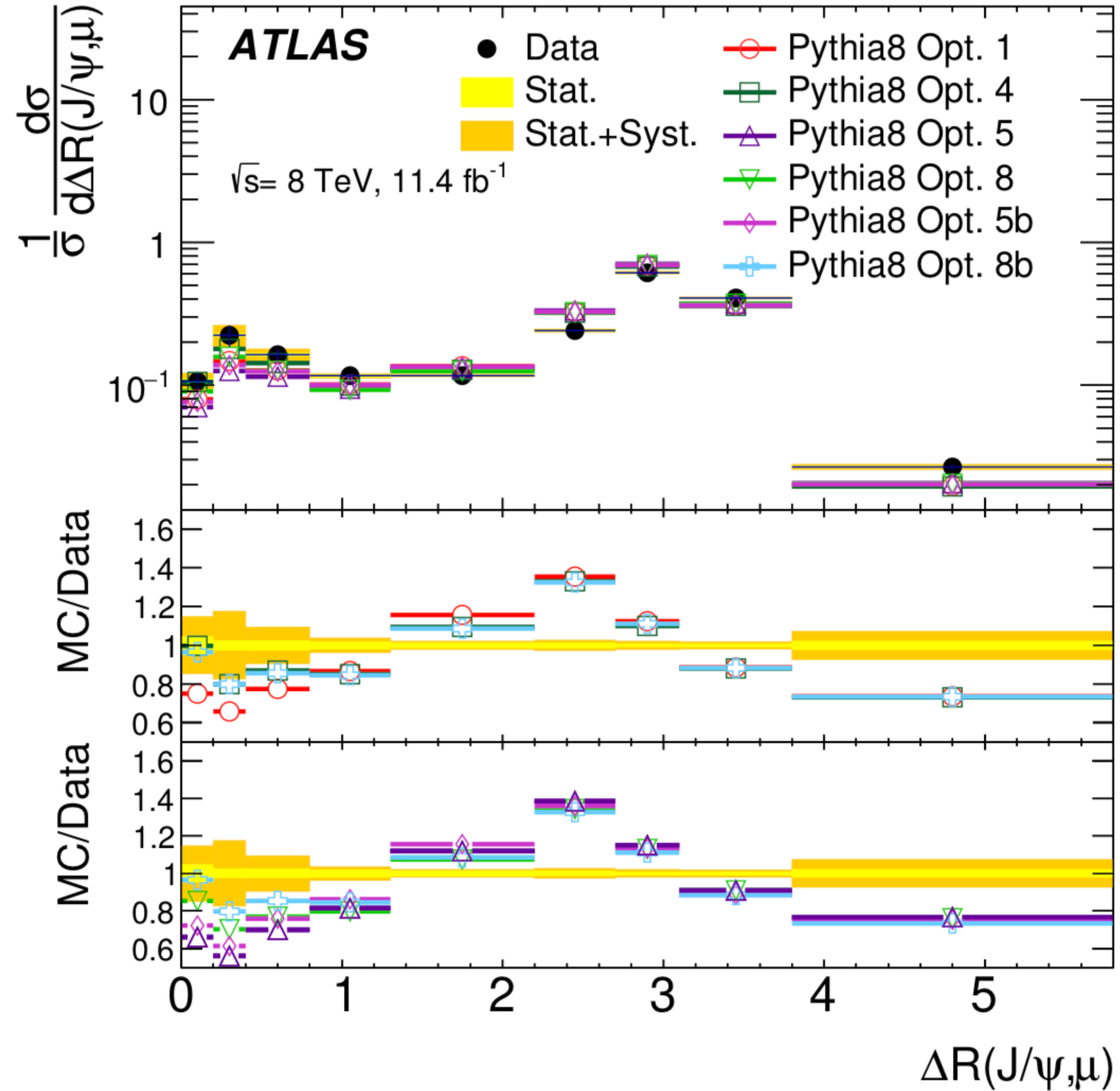
$$\frac{d\sigma}{\sigma_0} \propto \frac{dK_{\perp}^2}{K_{\perp}^2 - m_g^2} \dots$$

=> Unphysical overestimates for $K_T \sim m_g$

Approaches: Evolution Scale, g->QQ

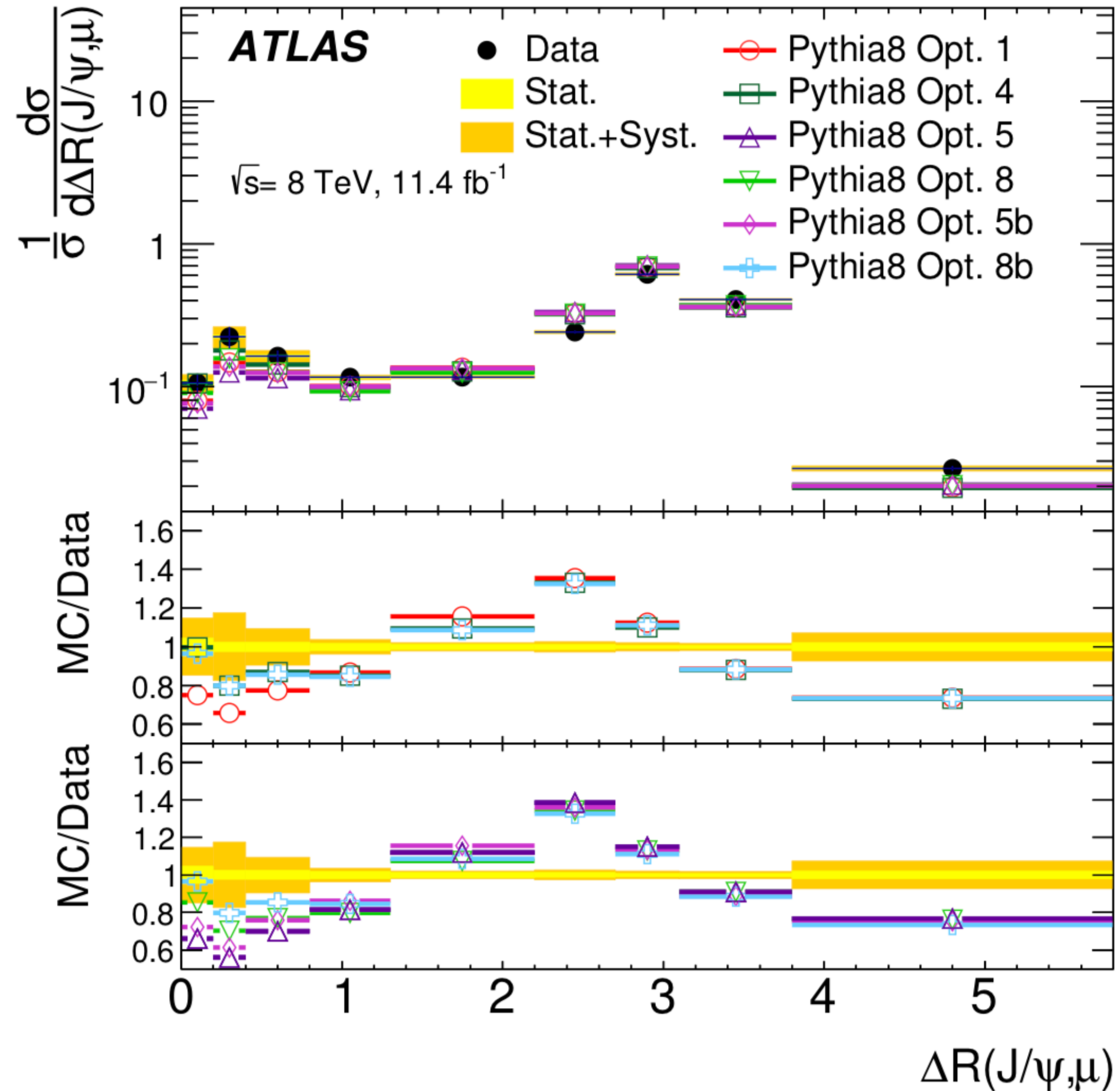


Approaches: Splitting Functions



$$\frac{d\mathcal{P}}{d \log \mu^2} \propto \frac{\alpha_s(X(t))}{2\pi} \left[P_{qg} + A \frac{m^2}{Y} + C \right]$$

Approaches: Splitting Functions



$$\frac{d\mathcal{P}}{d \log \mu^2} \propto \frac{\alpha_s(X(t))}{2\pi} \left[P_{qg} + A \frac{m^2}{Y} + C \right]$$

- (At LL all of these options are equally good!)