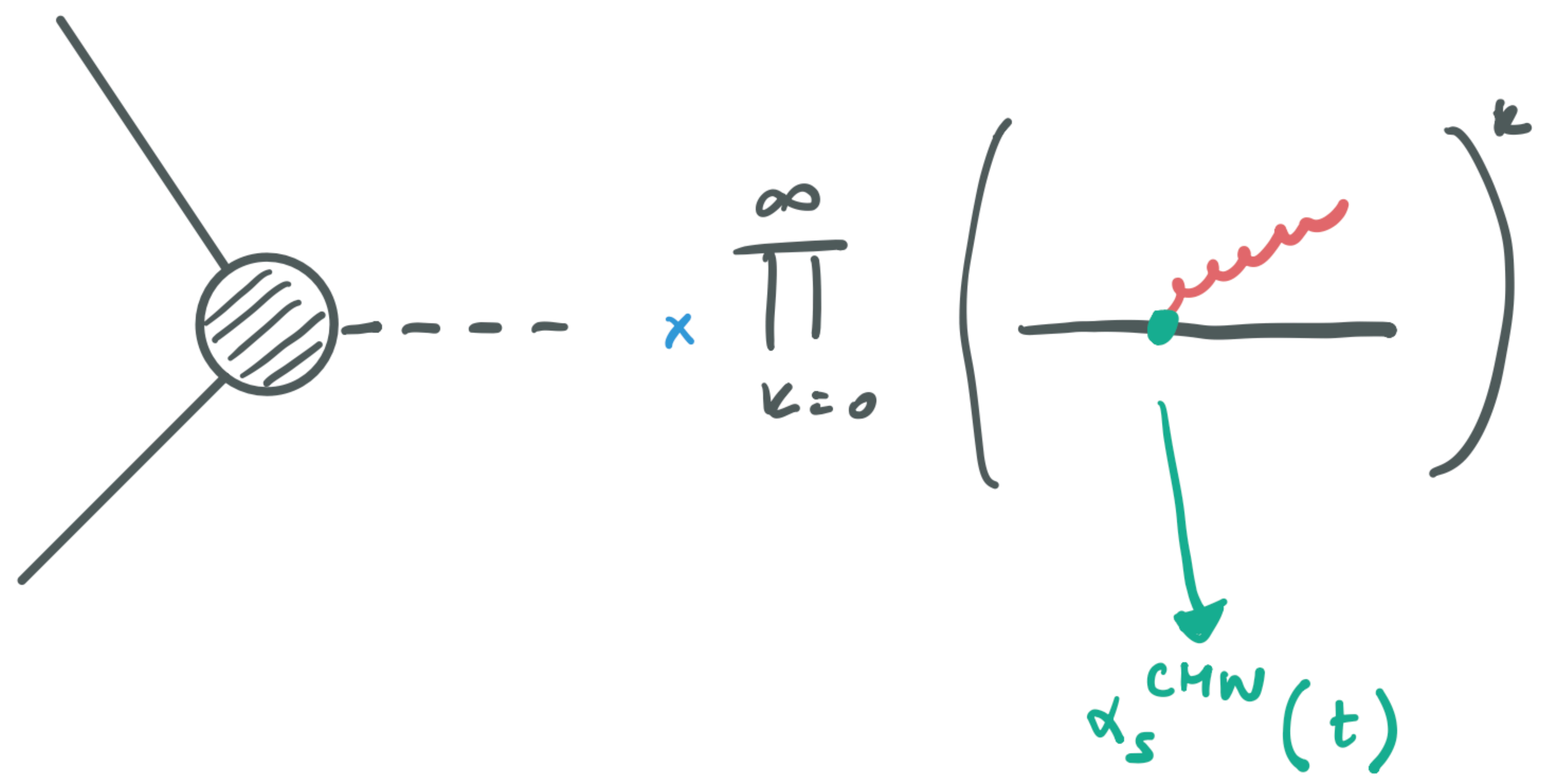
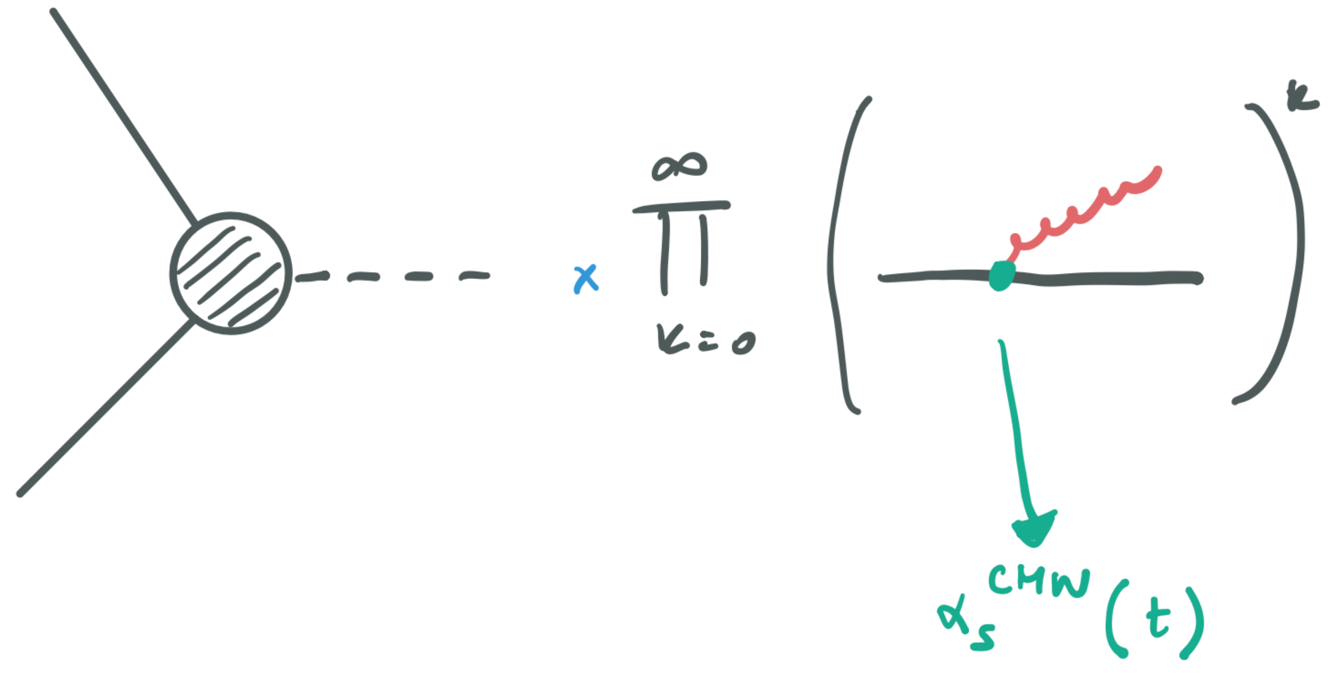
Parton Showers and Massive Quarks

Heavy Flavours @ High pT, Edinburgh, 30/11/2023

Parton Showers



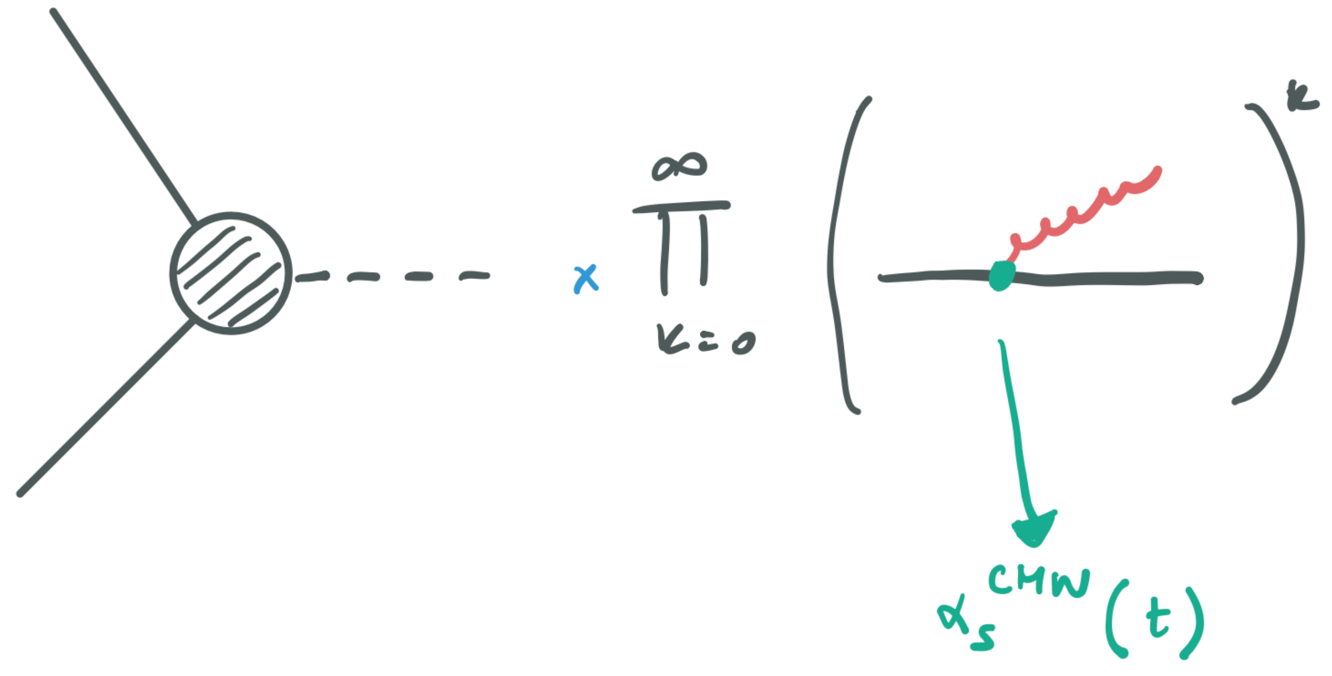
Parton Showers



$$d\sigma = d\sigma_0 \times \exp\left\{-\int_{z_0}^{z_1} dz \int_{t_f}^{t_i} \frac{dt}{t} \frac{\alpha_s(t)}{2\pi} \mathbf{V}(z,t)\right\}$$

Evolution Scale

Parton Showers



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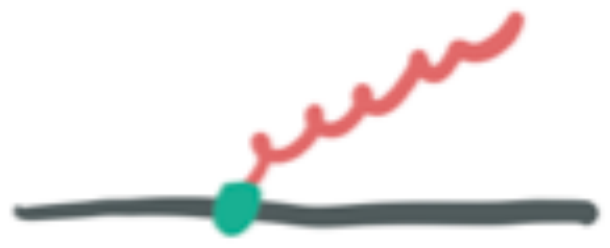
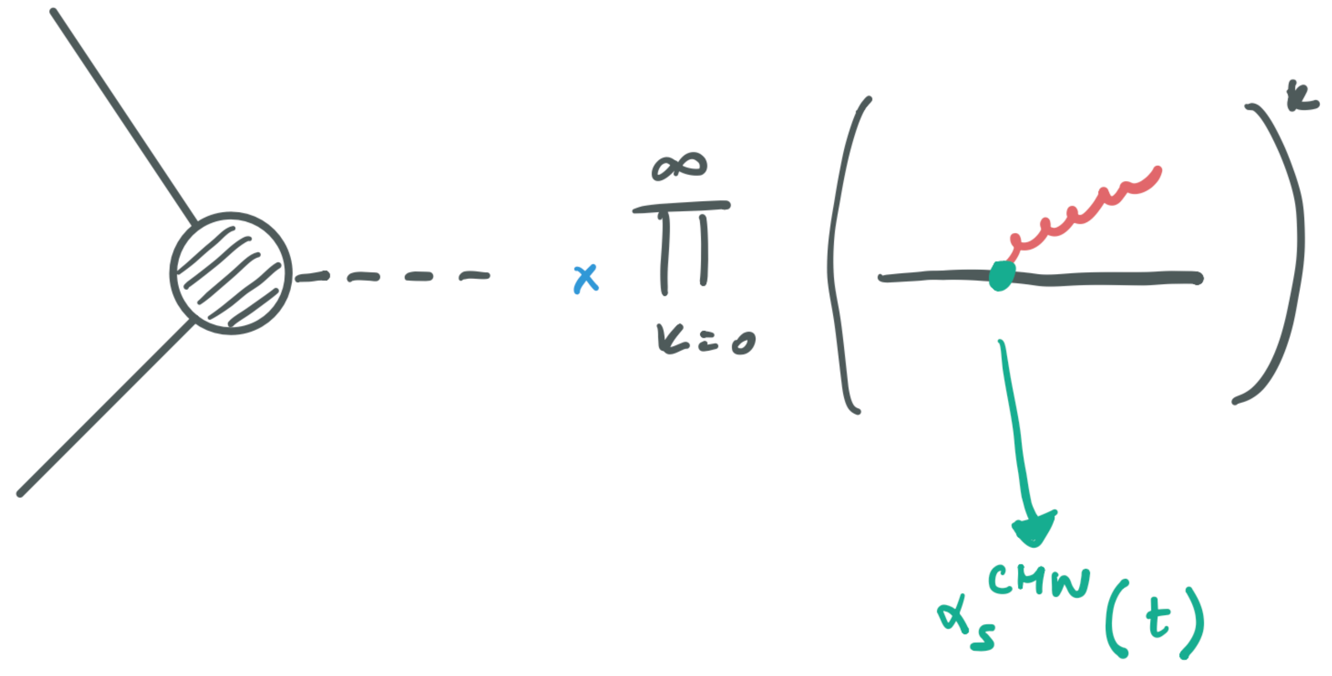
Evolution Scale

$\alpha_s^{CMW}(t)$



Parton Showers



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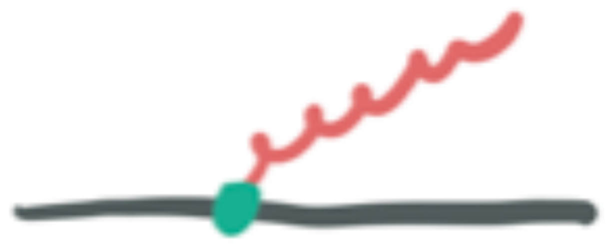
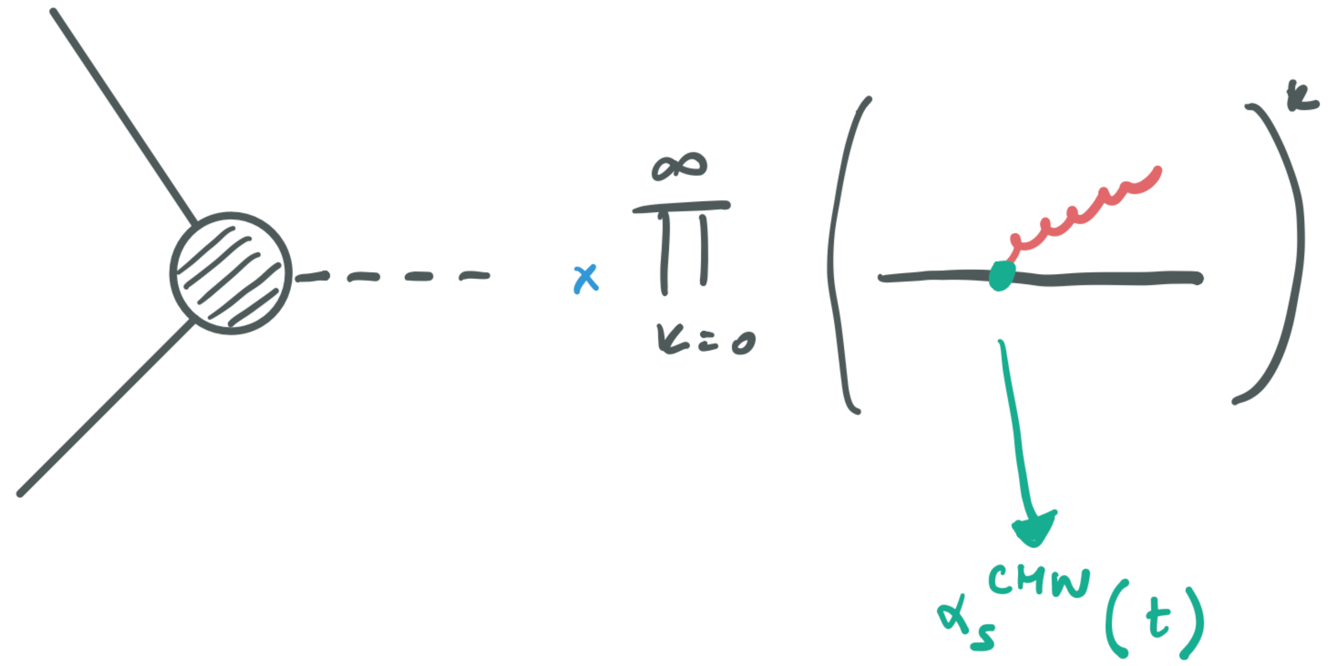


KINEMATICS...

(See for example JHEP 04 (2020) 019)

Parton Showers



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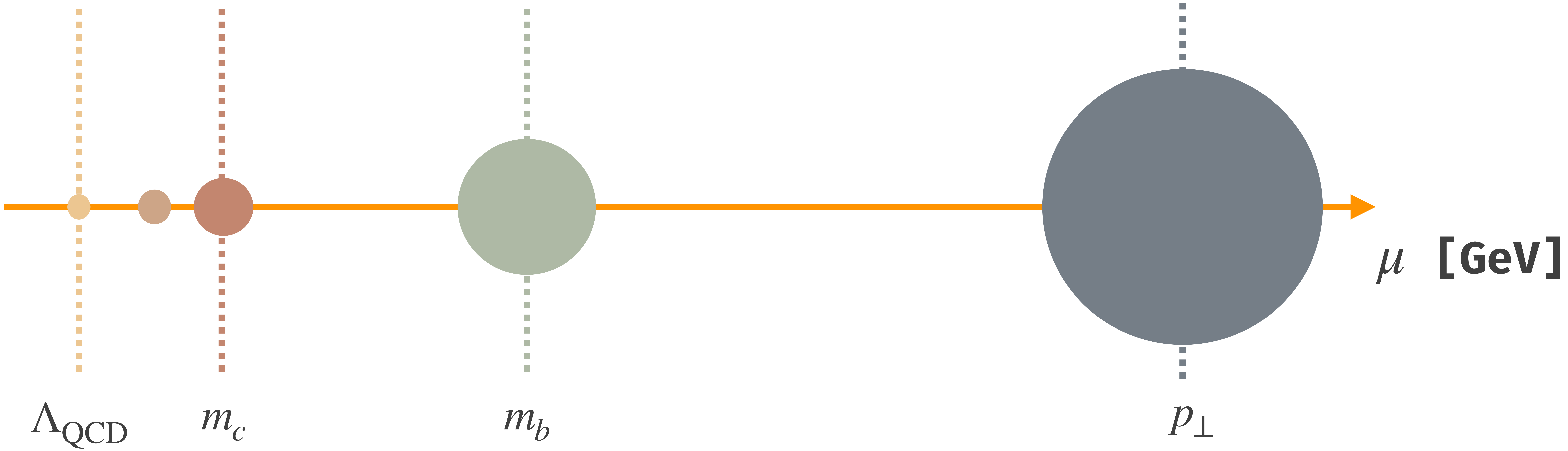
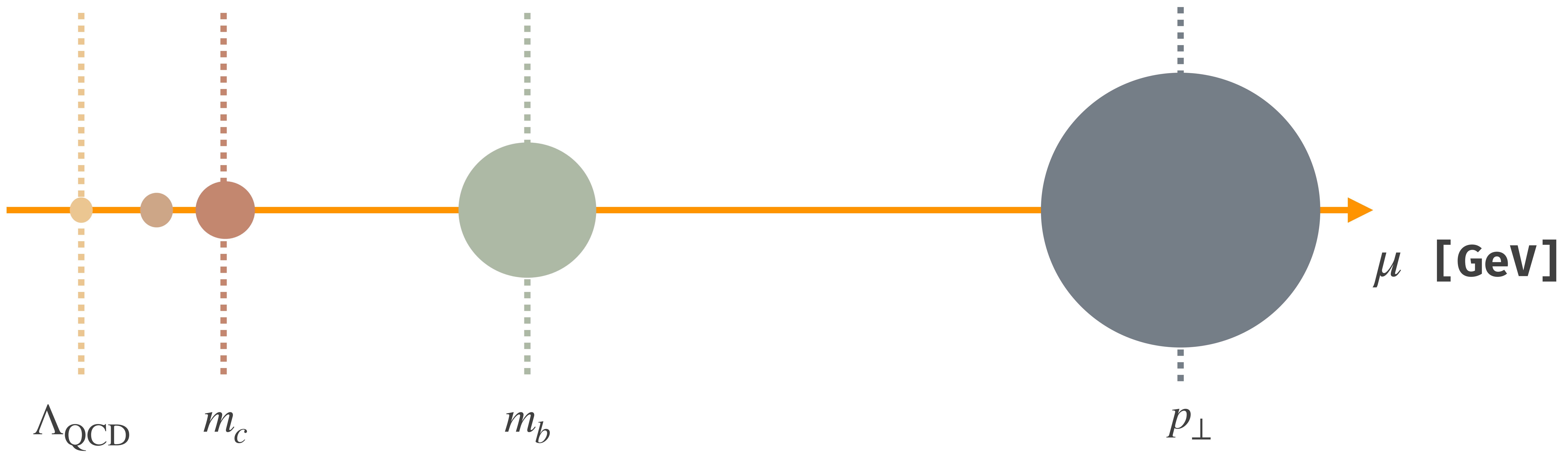
- Accuracy of the PS either comparing to resummation
- Or comparing to Fixed-Order predictions

Mass Effects



Mass Effects



- $p_\perp \sim m_Q$ => **Retain mass effects**
- $p_\perp \gg m_Q$ => **Neglect mass effects**

Combining the two

- **PS cannot ignore quark masses, necessary for hadronisation**

Minimal Approach

- Massless Evolution
- At the end, shuffle momenta

Fully Massive Approach

- Mass included in kinematics and splittings
- Evolution same as massless

Combining the two

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$$t = \mathbf{k}_\perp^2 = 2\, p_i \cdot p_j \, z\,(1 - z) \; - \; m_i^2 \; (1 - z)^2 \; - \; m_j^2 \, z^2$$

Approaches: Evolution Scale

[Nucl.Phys. B173 (1980) 429-455]

- **Evolution scale/Argument of alphaS (massless)**

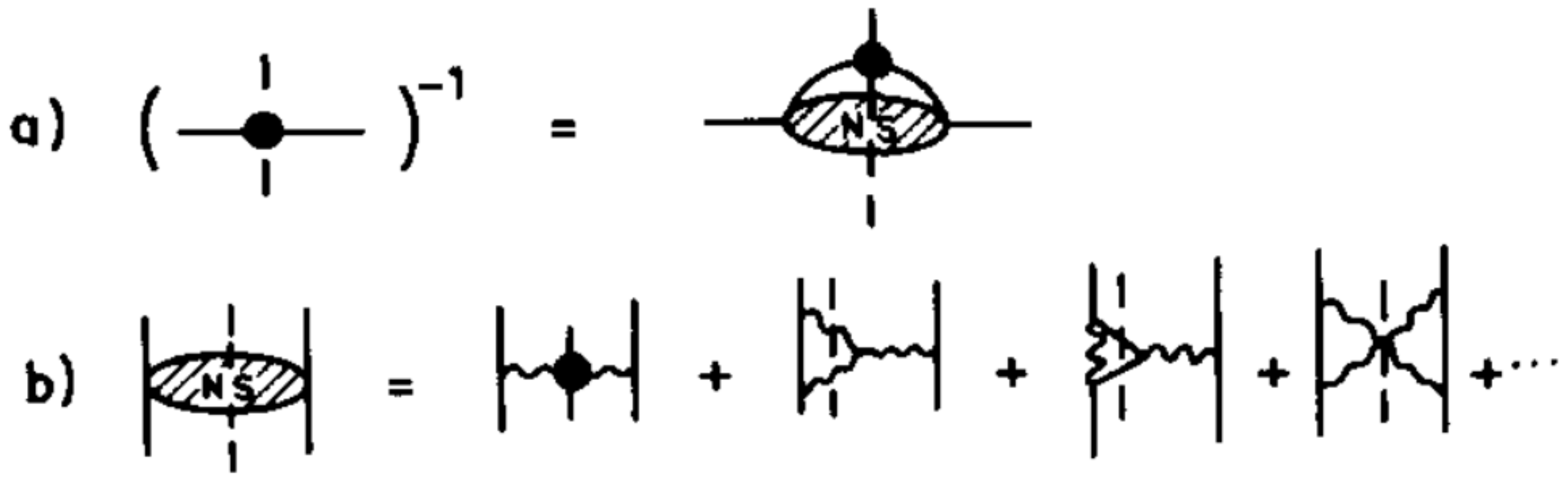


Fig. 1. Dyson equation for the inverse propagator and skeleton expansion of the flavour non-singlet kernel.

$k'^2 = 0$ in the arguments of $\Gamma^2_{\mathrm{qqg}} d_{\mathrm{g}}$ and $k'^2 = k^2 z$ (i.e., its kinematical limit) in d_{q}. We then find that vertices and propagators appear in the combination

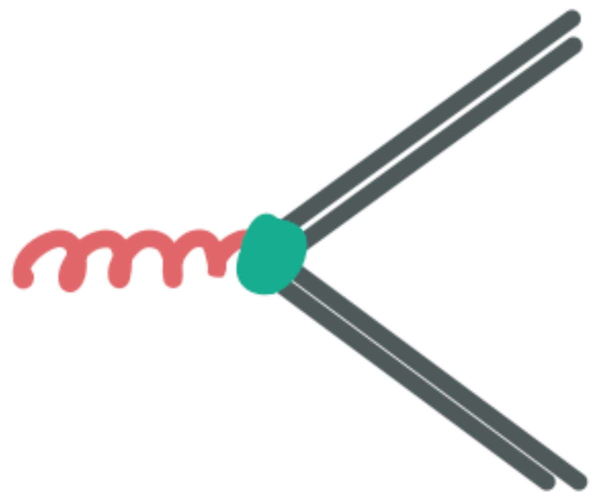
$$\frac{g^2}{4\pi}\Gamma^2_{\mathrm{qqg}}(k^2, 0, -k^2(1-z); k_{i\|})d_{\mathrm{g}}(-k^2(1-z))d_{\mathrm{q}}(-k^2 z) = d_{\mathrm{q}}^{-1}(-k^2)G^{\mathrm{coll}}|_{\xi=0}\,. \quad (2.16)$$

We will now argue that it is legitimate to replace $G^{\mathrm{coll}}|_{\xi=0}$ in eq. (2.16) by $\alpha_{\mathrm{s}}(k^2(1-z))$. Let us first notice that for fixed z the kernel depicted in fig. 1b is the

Approaches: Evolution Scale

- Evolution scale/Argument of alphaS (massive)

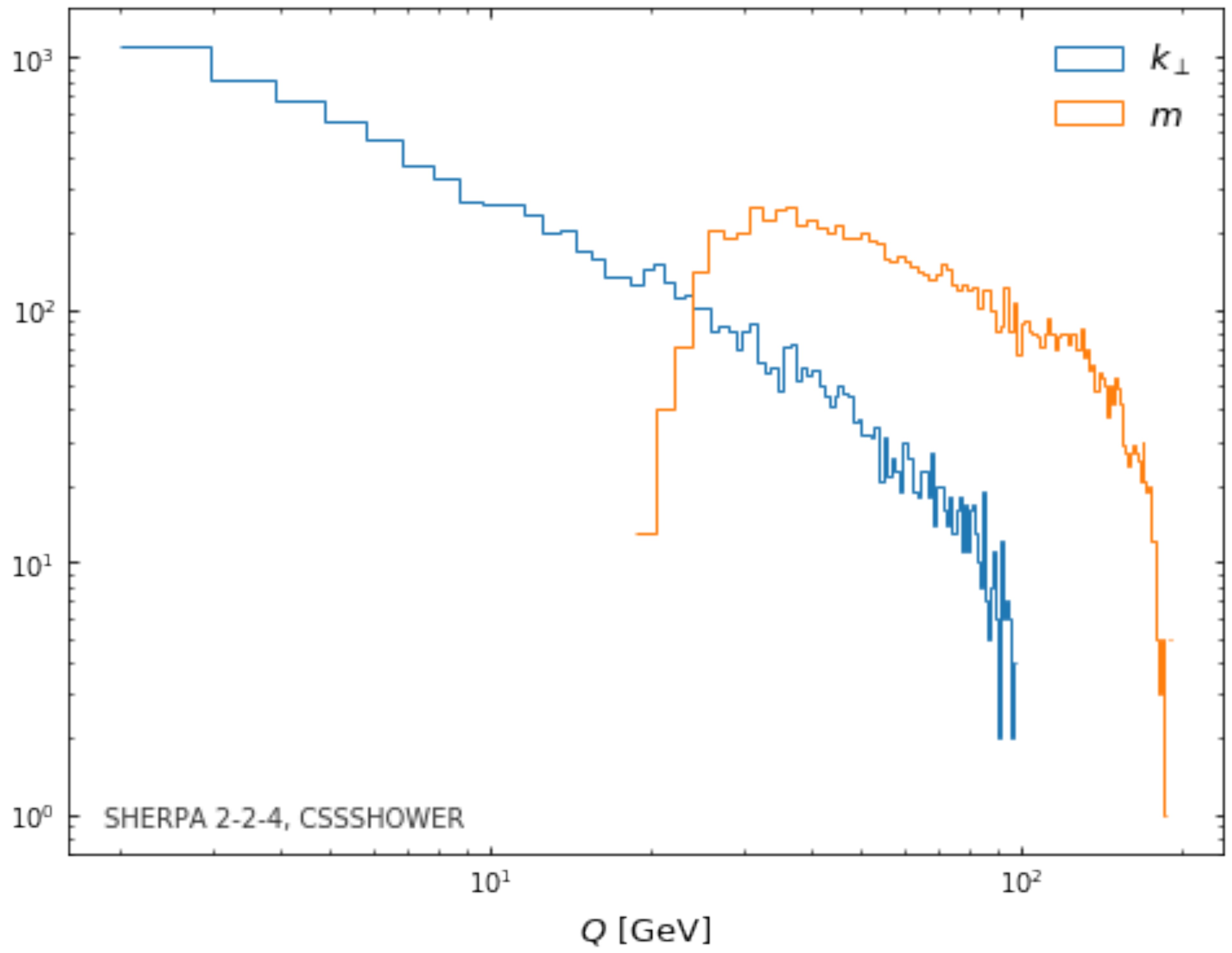
- Everything seems fine for Q-Qg splitting, what about g->QQ?



$$\alpha_s\left(m_{QQ}^2,\ t,\ m_{QQ}^2 + t,\ \sqrt{m_{QQ}^2 \cdot t},\ \dots\right)$$

Approaches: Evolution Scale, g->QQ

[Nucl.Phys. B436 (1995) 163-183]



$$\tau_g \sim \frac{E}{t^2} \ll \tau_{Q\bar{Q}} \sim \frac{E'}{t'^2} + \frac{1}{m} \quad k_\perp \gg m$$

Approaches: Evolution Scale, g->QQ

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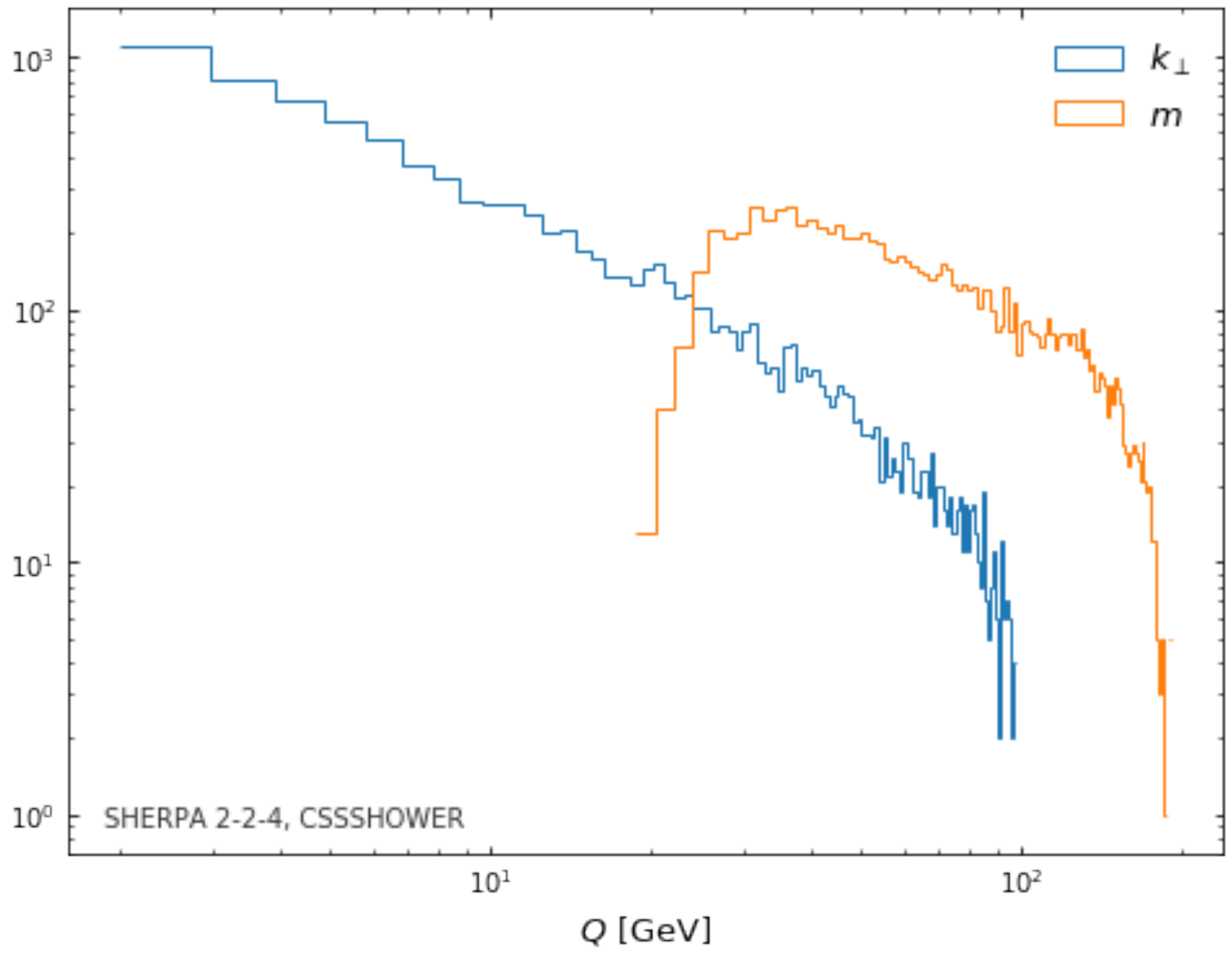

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Approaches: Evolution Scale, g->QQ

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- Evolution of heavy quarks, take $\gamma^* -> qqQQ$, the matrix element reads

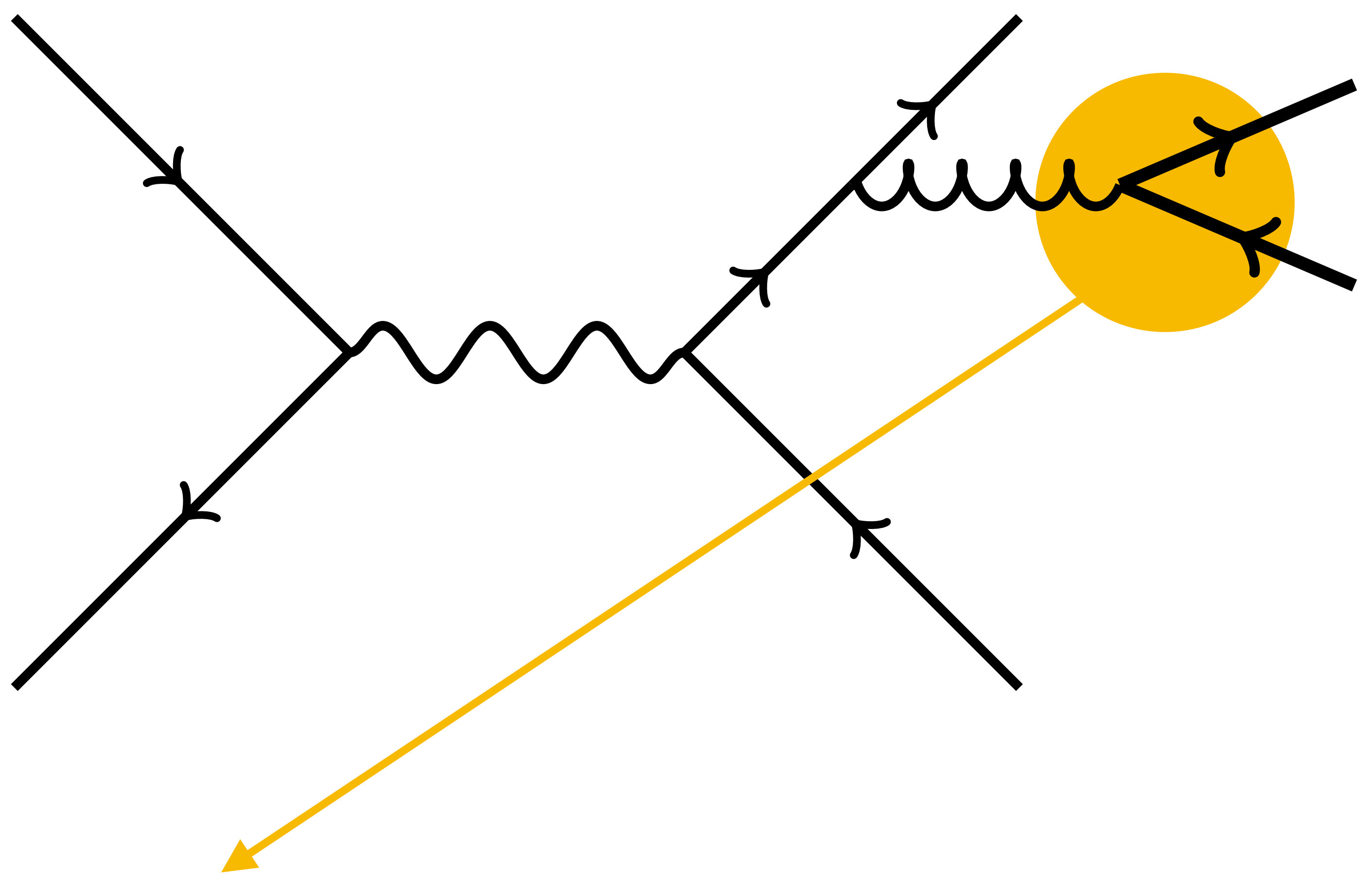
$$\frac{\mathrm{d}\sigma}{\sigma_0} \propto \frac{\mathrm{d}K_\perp^2}{K_\perp^2} \cdots$$

- If we take the QQ as if emitted from an off shell gluon

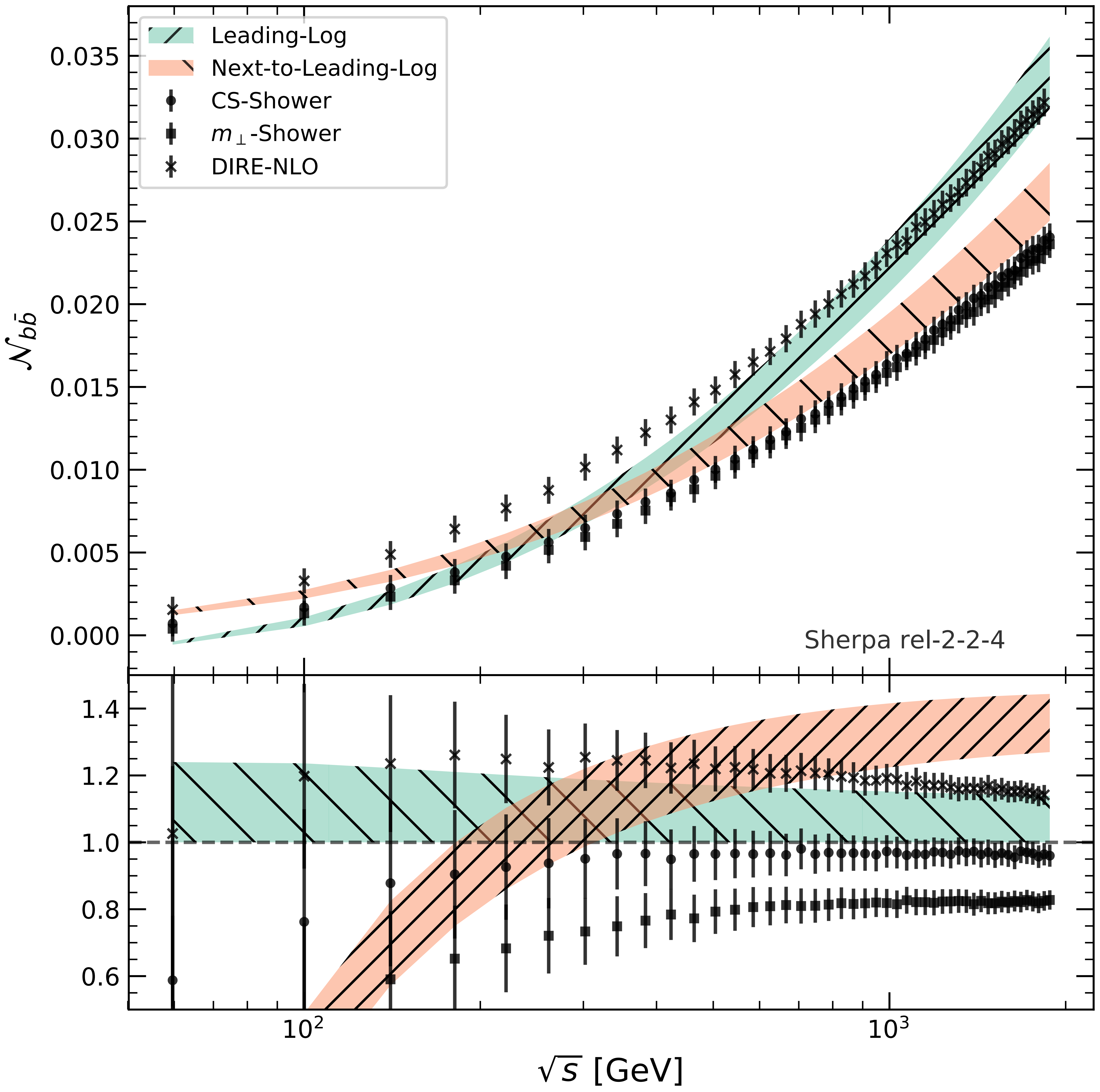
$$\frac{\mathrm{d}\sigma}{\sigma_0} \propto \frac{\mathrm{d}K_\perp^2}{K_\perp^2 - m_g^2} \cdots$$

=> Unphysical overestimates for KT ~ mg

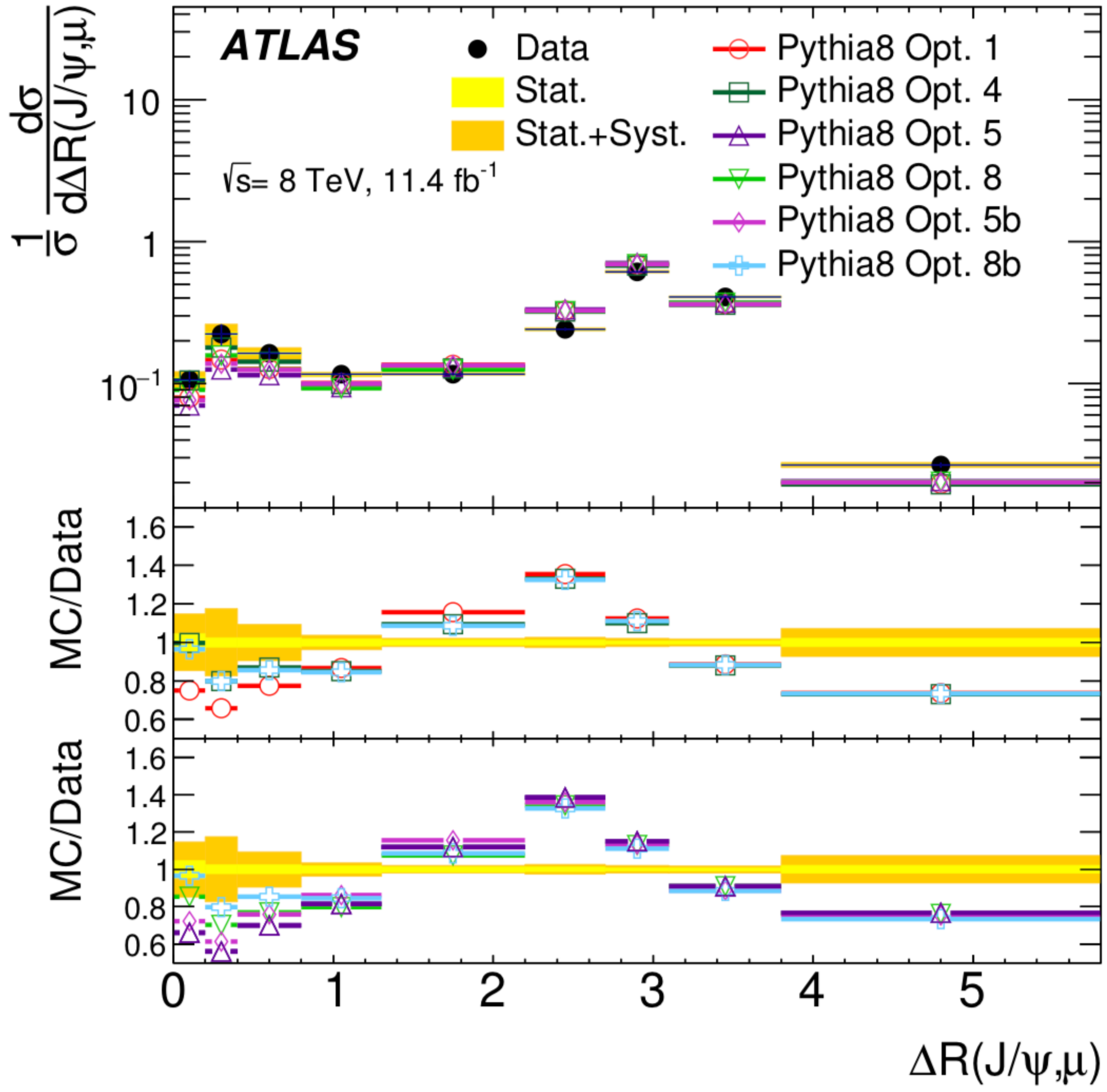
Approaches: Evolution Scale, g->QQ



Simply count the number of b-jets

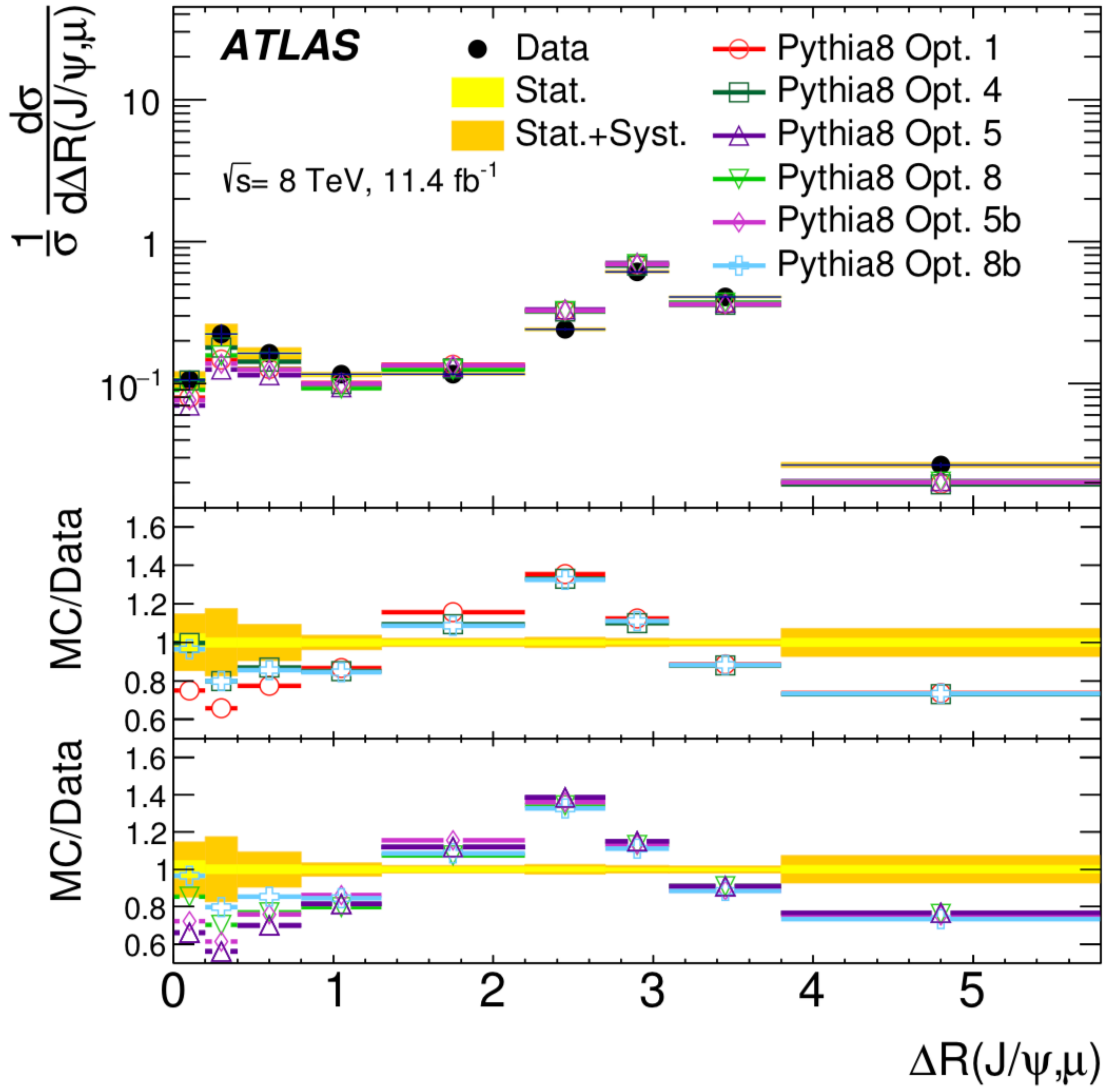


Approaches: Splitting Functions



$$\frac{\mathrm{d}\mathcal{P}}{\mathrm{d}\log\mu^2} \propto \frac{\alpha_s(X(t))}{2\pi}\left[P_{qg} + A\,\frac{m^2}{Y} + C\right]$$

Approaches: Splitting Functions



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- (At LL all of these options are equally good!)