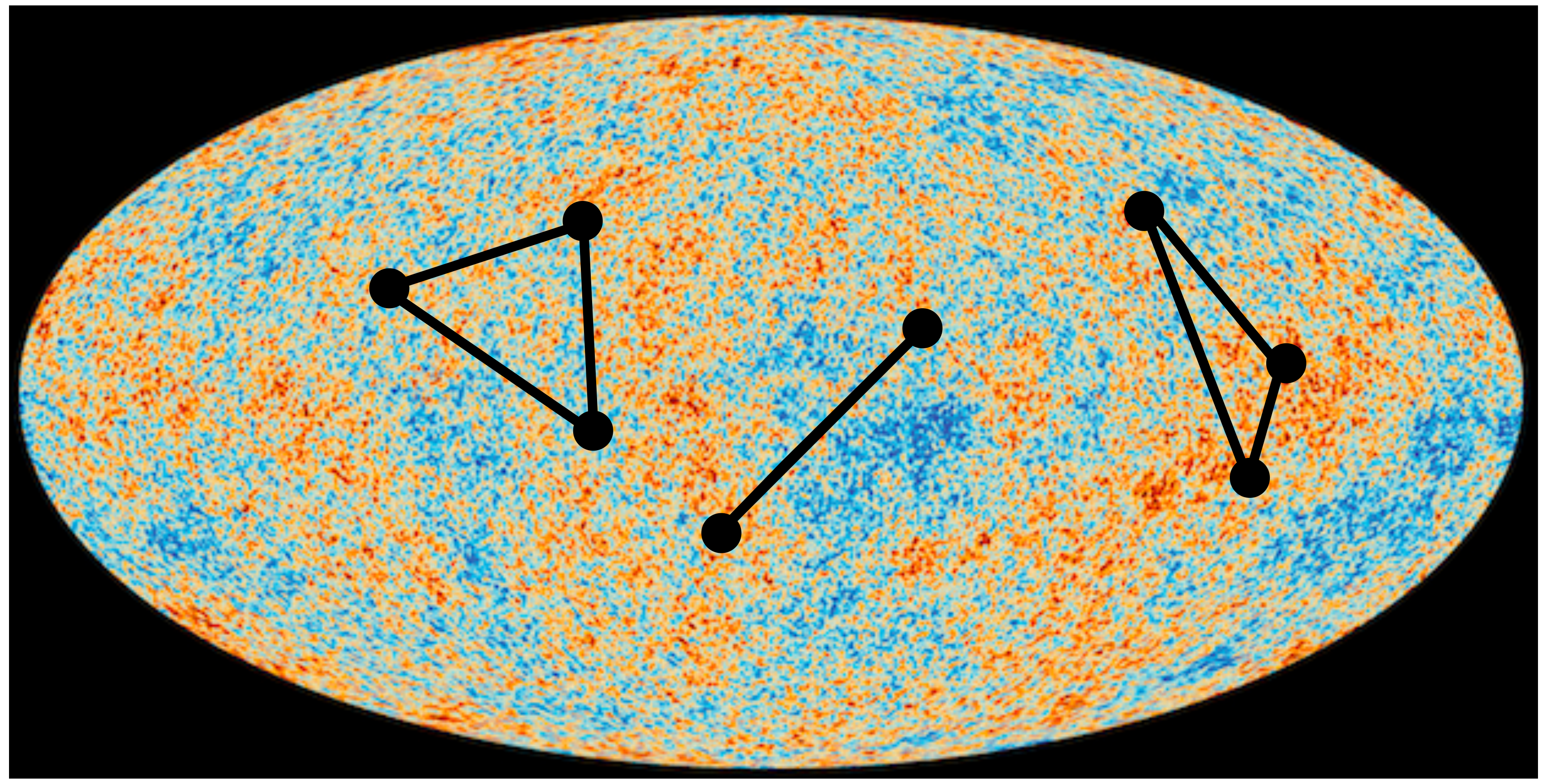
Probing *charming* and beautiful dynamics with energy correlators

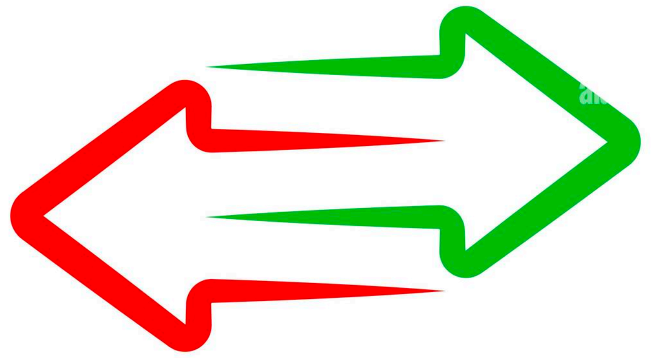
Kyle Lee
CTP, MIT

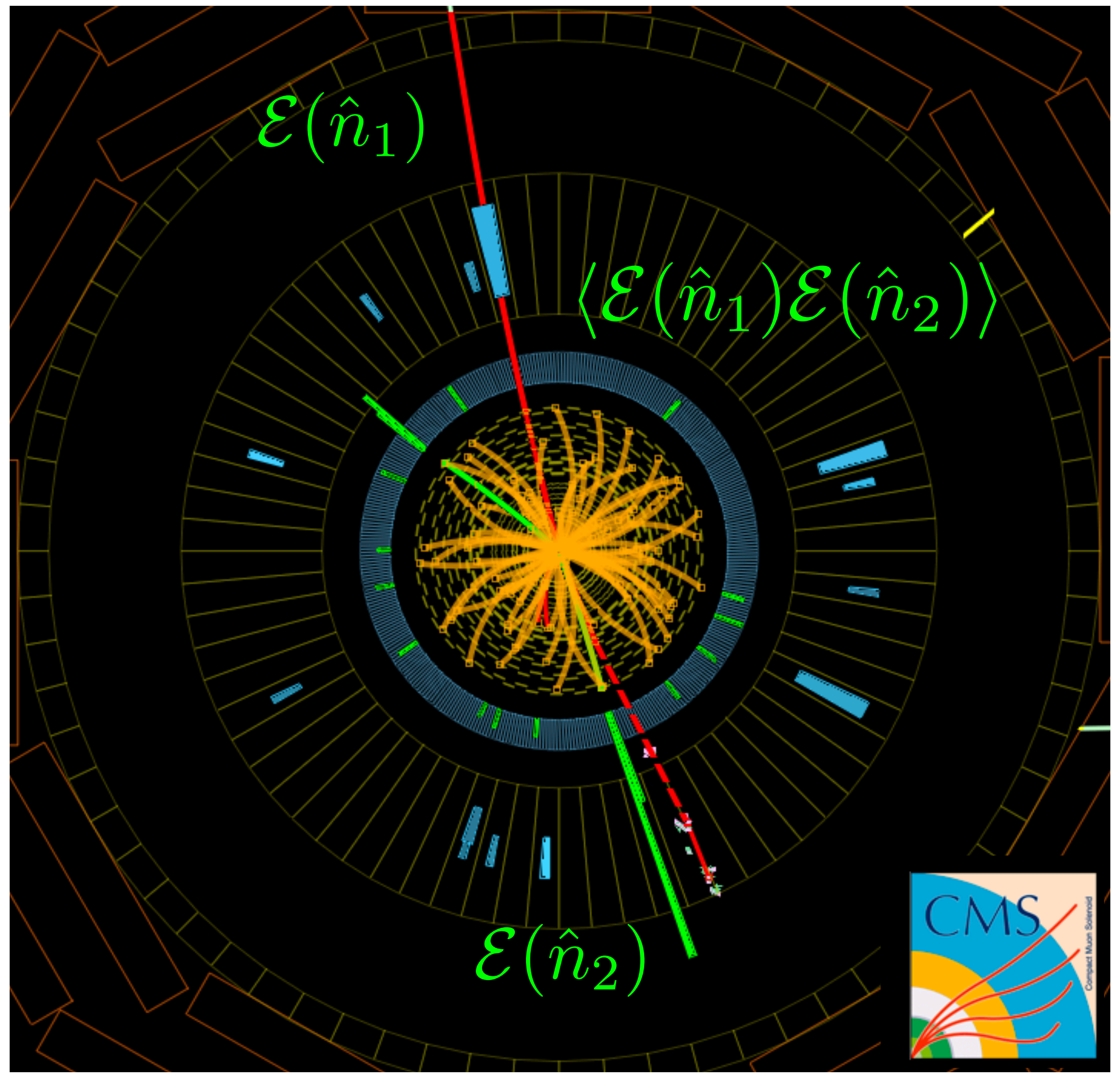
Higgs Centre - Heavy Flavours at high pT, 2023

ENERGY CORRELATORS

- In many fields, **correlation functions** are considered to be fundamental objects which encode the dynamics of the underlying theory.



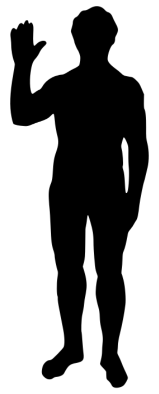




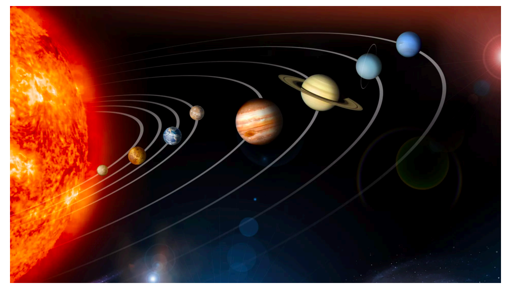
- Much like cosmology, we observe **asymptotic energy flux** at the detectors that are placed at **cosmic scale away** from where the events originated.

(Collision events happen at $\delta x \sim \frac{\hbar}{10\text{TeV}} \sim 2\times 10^{-20}$ meters, and observed at ~ 10 meters away)

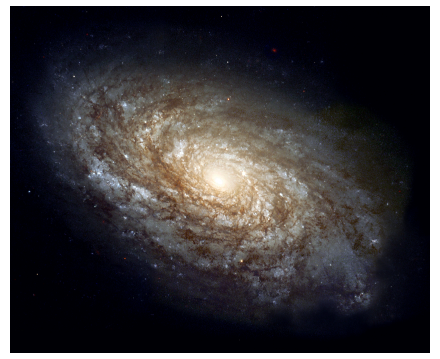
10^{21} orders in distance!



$\mathcal{O}(1)$ meters



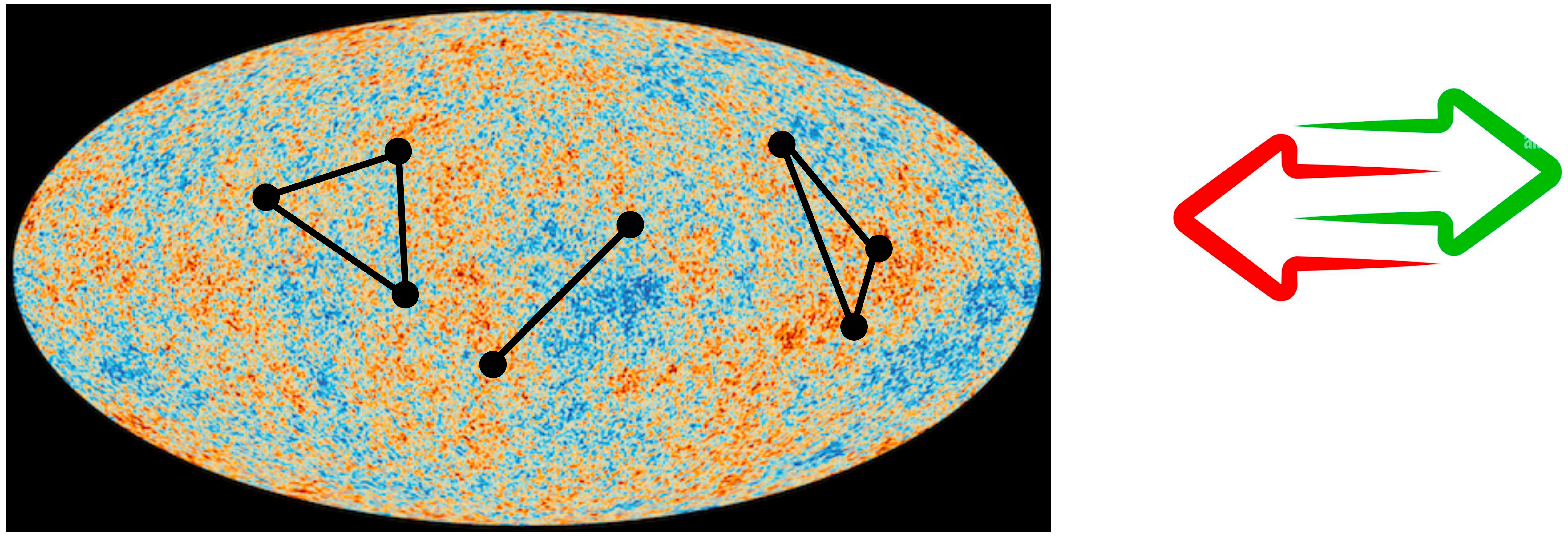
$\mathcal{O}(10^{15})$ meters

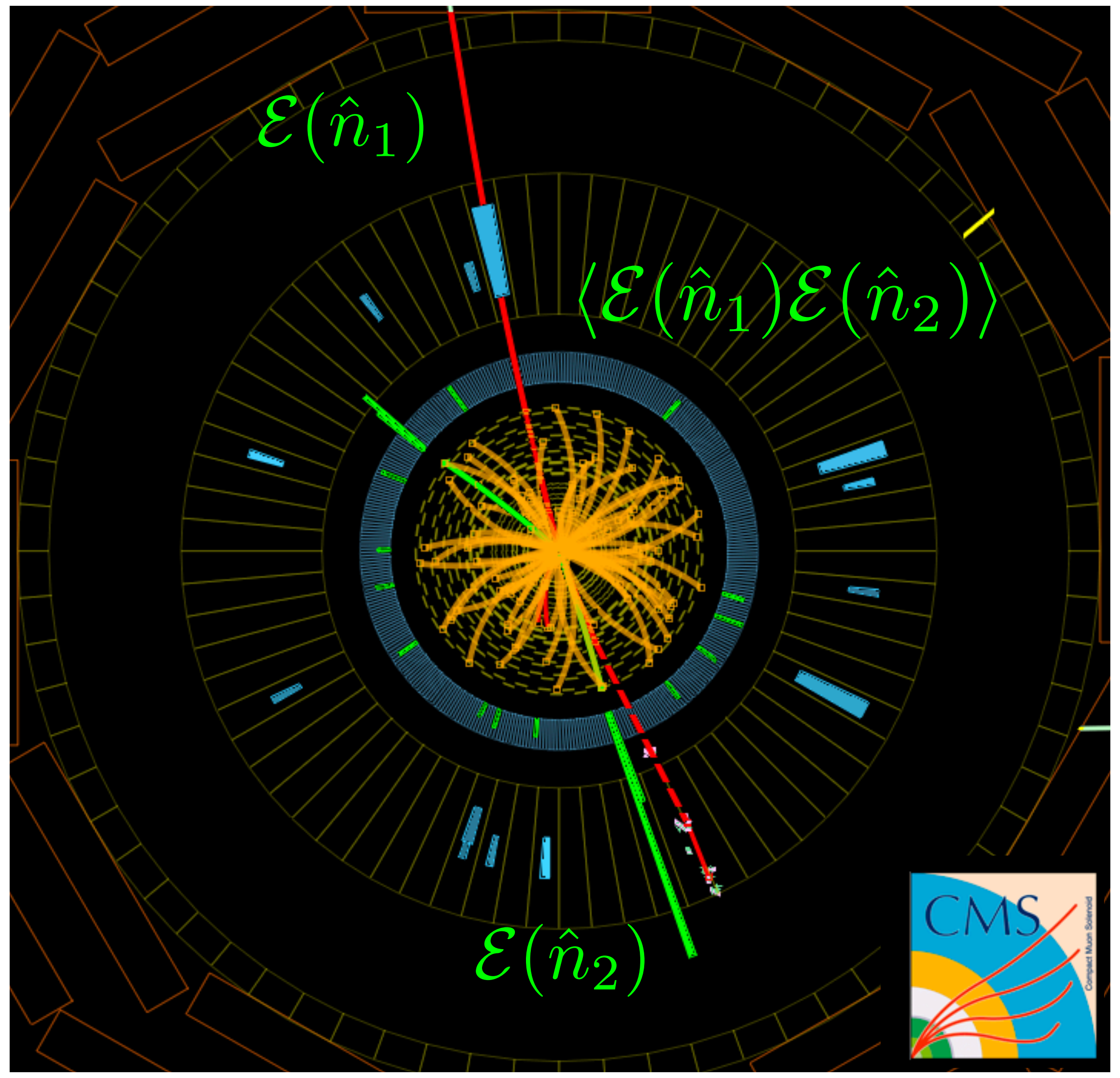


$\mathcal{O}(10^{21})$ meters

ENERGY CORRELATORS

- **In many fields, correlation functions are considered to be fundamental objects which encode the dynamics of the underlying theory.**

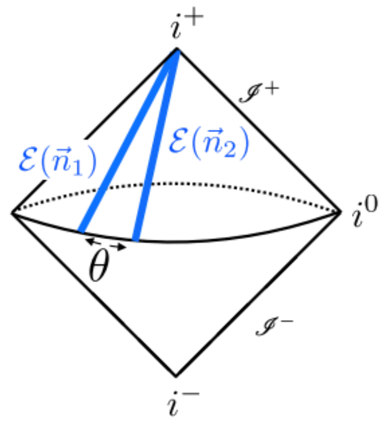




- **Much like cosmology, we observe asymptotic energy flux at the detectors that are placed at cosmic scale away from where the events originated.**

Energy Flow Operators (Light Ray Operators)

$$\mathcal{E}(\hat{n}) = \int_0^\infty dt \lim_{r\to\infty} r^2 n^i T_{0i}(t, r\hat{n})$$



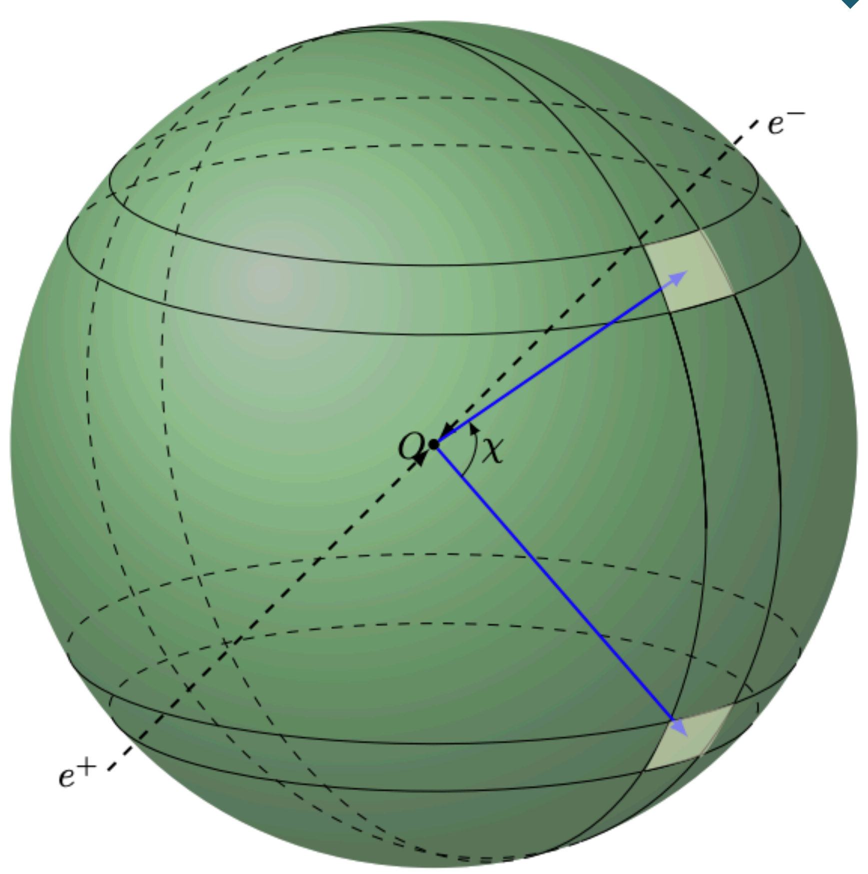
$$\mathcal{E}(\hat{n})|X\rangle = \sum_a E_a \delta^{(2)}\left(\Omega_{\vec{p}_a} - \Omega_{\hat{n}}\right)|X\rangle$$

Basham, Brown, Ellis, Love, `78-79
Sveshnikov, Tkachov, `95
Korchemsky, Sterman, `01

ENERGY CORRELATORS

Indeed, energy-energy correlators are one of the very first studied event shape (or correlations) observables in QCD

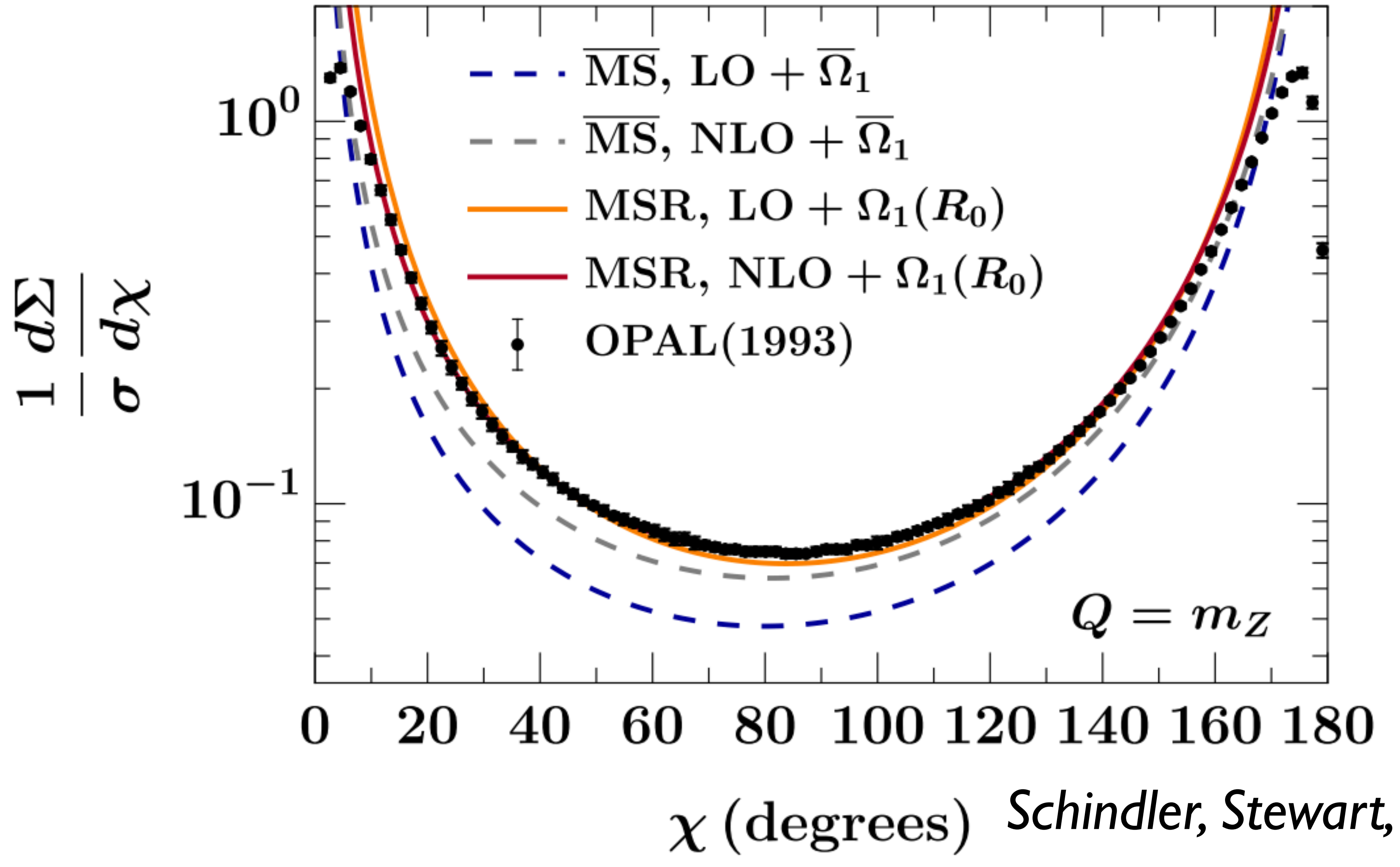
Basham, Brown, Ellis, Love, `78-79



$$\frac{\mathrm{d}\sigma}{\mathrm{d}\theta} \sim \langle \Psi | \mathcal{E}(\hat{n}_1)\, \mathcal{E}(\hat{n}_2) | \Psi \rangle$$

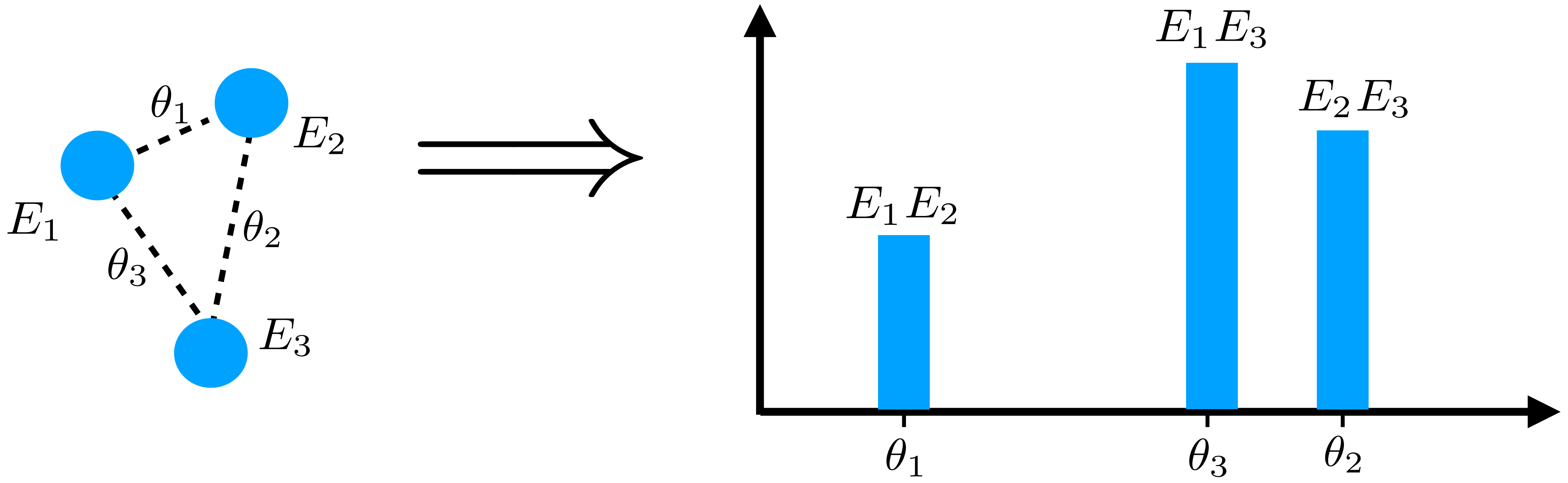
Many precise calculations!

Belitsky, Hohenegger, Korchemsky, Sokatchev, Zhiboedov `13
Dixon, Luo, Shtabovenko, Yang, Zhu `18
Luo, Shtabovenko, Yang, Zhu `19
Henn, Sokatchev, Yan, Zhiboedov `19



Schindler, Stewart, Sun `23

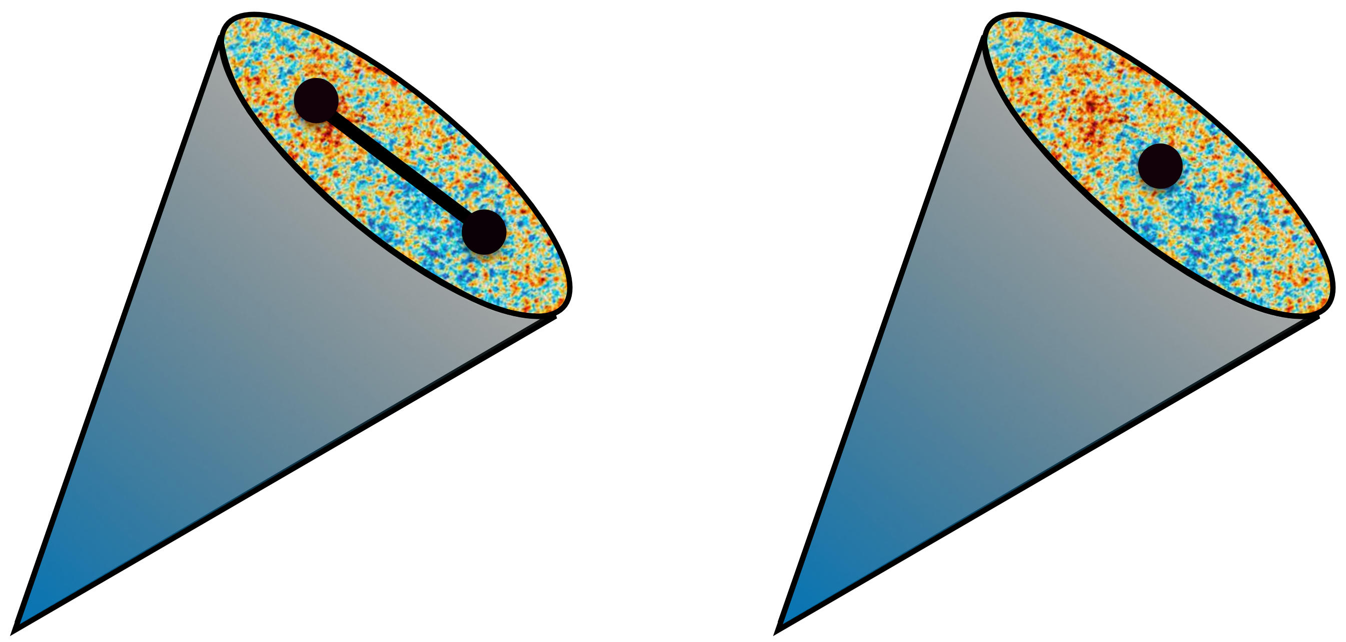
Impressive agreements from recent calculation, without any fits!



$$\frac{\mathrm{d}\sigma}{\mathrm{d}\theta} = \sum_{i,j} \int d\sigma \frac{E_i E_j}{Q^2} \delta(\theta - \theta_{ij}) \sim \langle \Psi | \mathcal{E}(\hat{n}_1)\, \mathcal{E}(\hat{n}_2) | \Psi \rangle$$

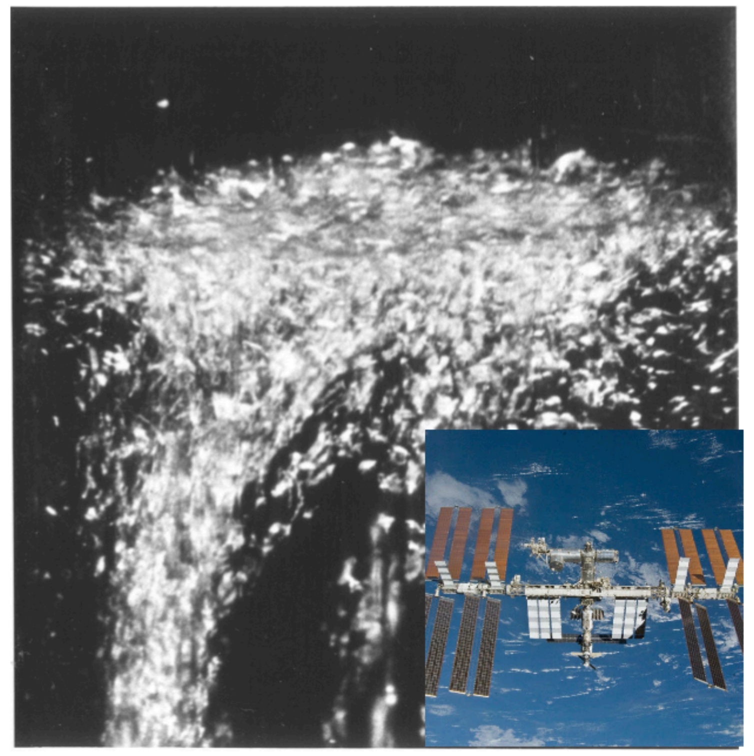
Weighted cross-section, or, ensemble averaged observable

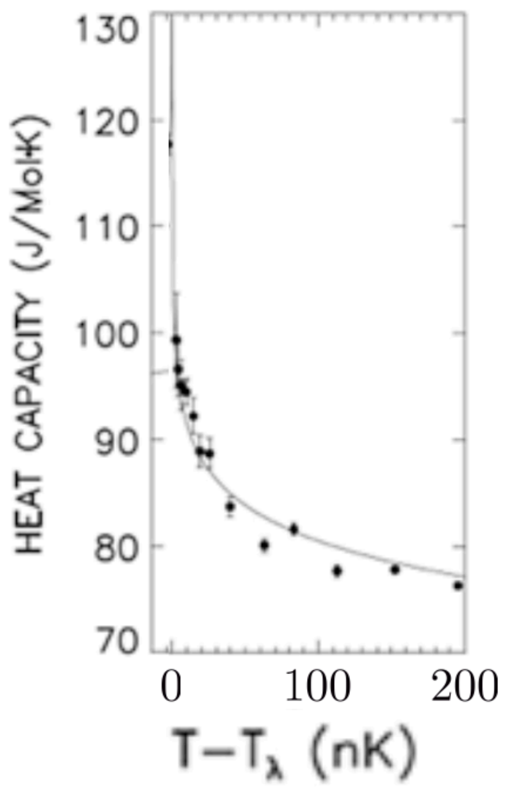
JET SUBSTRUCTURE AS CORRELATION FUNCTIONS



- **Jet limit corresponds to the collinear limit (OPE limit) of the correlation functions of the Energy Flow Operators**

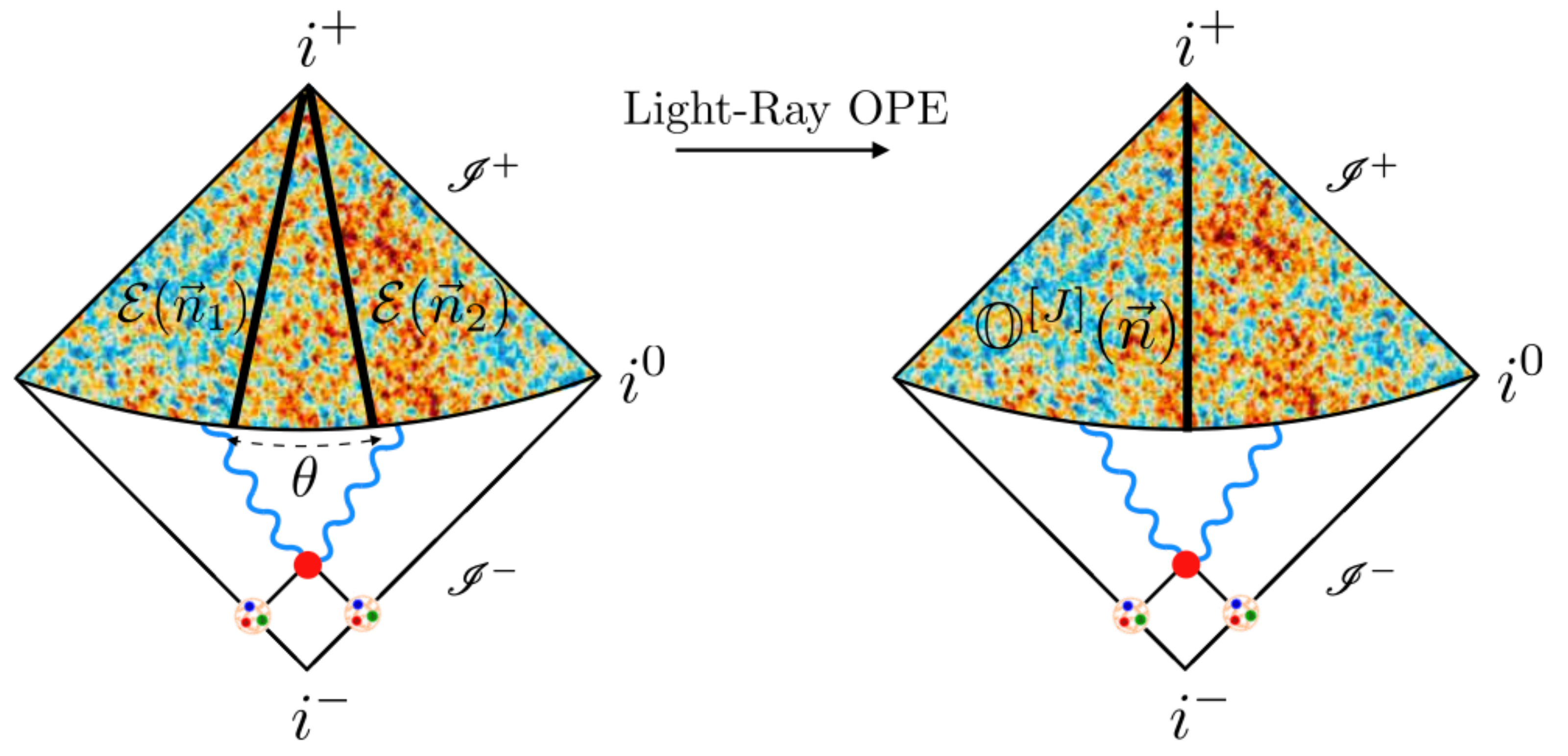
- **Field theory often predicts universal scaling as operators approach each other**





$$\mathcal{O}(x)\mathcal{O}(0) = \sum x^{\gamma_i} c_i \mathcal{O}_i$$

$$\Longleftrightarrow$$



$$\mathcal{E}(\hat{n}_1)\,\mathcal{E}(\hat{n}_2) \sim \sum \theta^{\tau_i - 4}\,\mathbb{O}_i(\hat{n}_1)$$

Hofman, Maldacena, `08

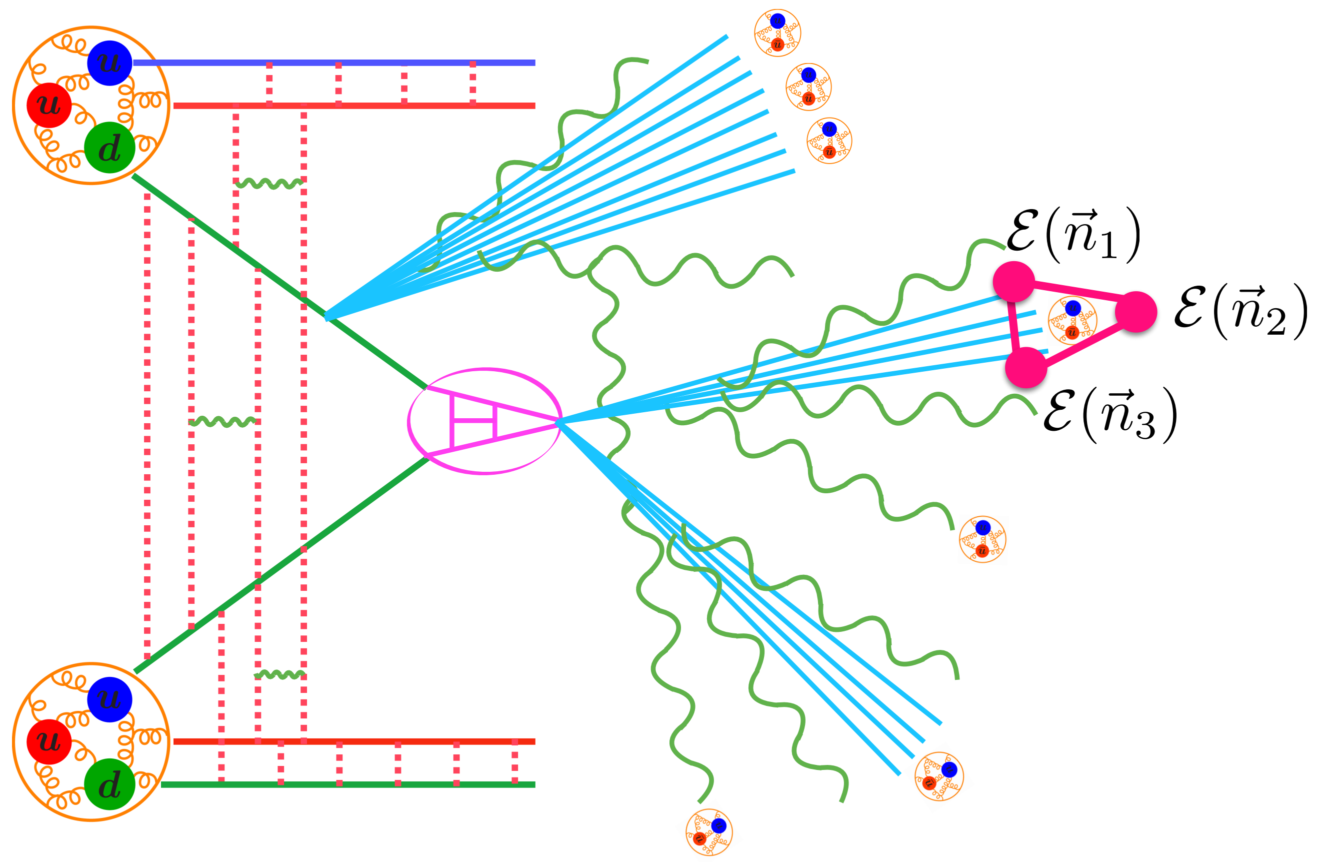
Much interests from the formal theory:

Kravchuk, Simmons Duffin, `18
Henn, Sokatchev, Yan, Zhiboedov, `19
Korchemsky, `19
Belin, Hofman, Mathys, `19
Belitsky, Hohenegger, Korchemsky, Sokatchev, Zhiboedov, `13
Kologlu, Kravchuk, Simmons Duffin, Zhiboedov, `19
Chang, Kologlu, Kravchuk, Simmons-Duffin, `20
Caron-Huot, Kologlu, Kravchuk, Meltzer, Simmons-Duffin, `22

CAN THIS UNIVERSAL SCALING OF THE FIELD THEORY BE OBSERVED IN JETS AT THE LHC???

CONFORMAL COLLIDERS MEET THE LHC

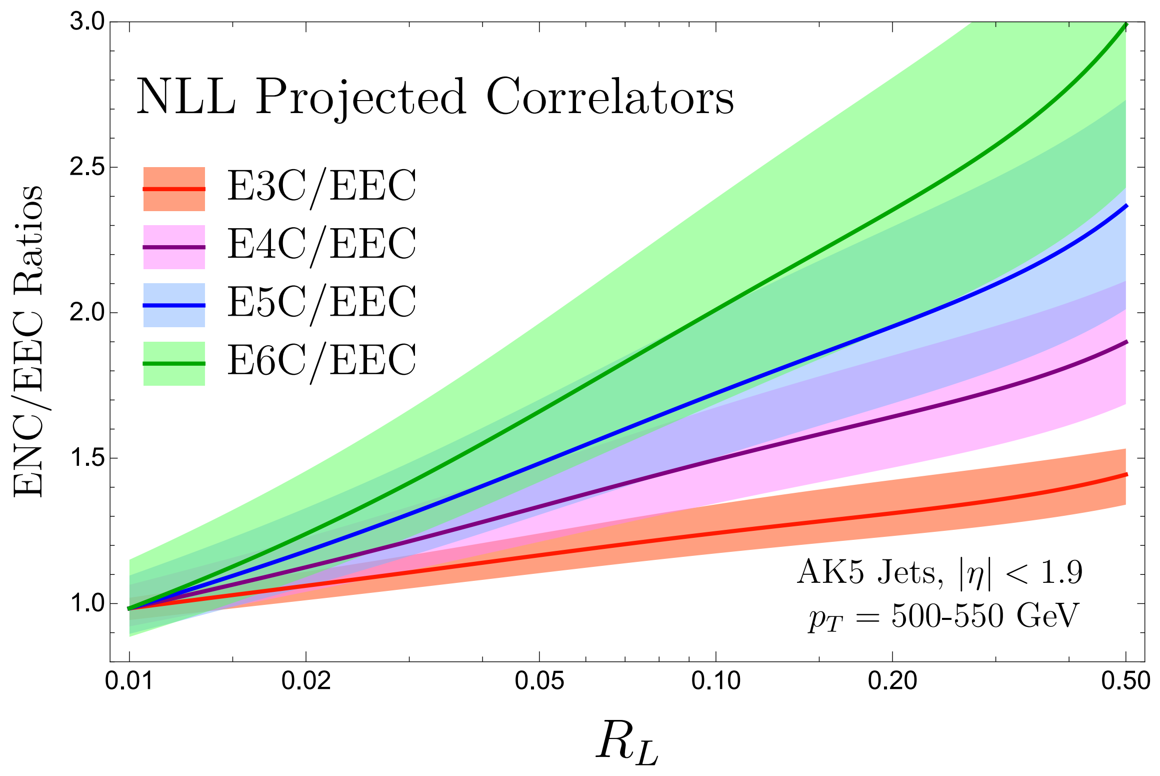
2019/2020

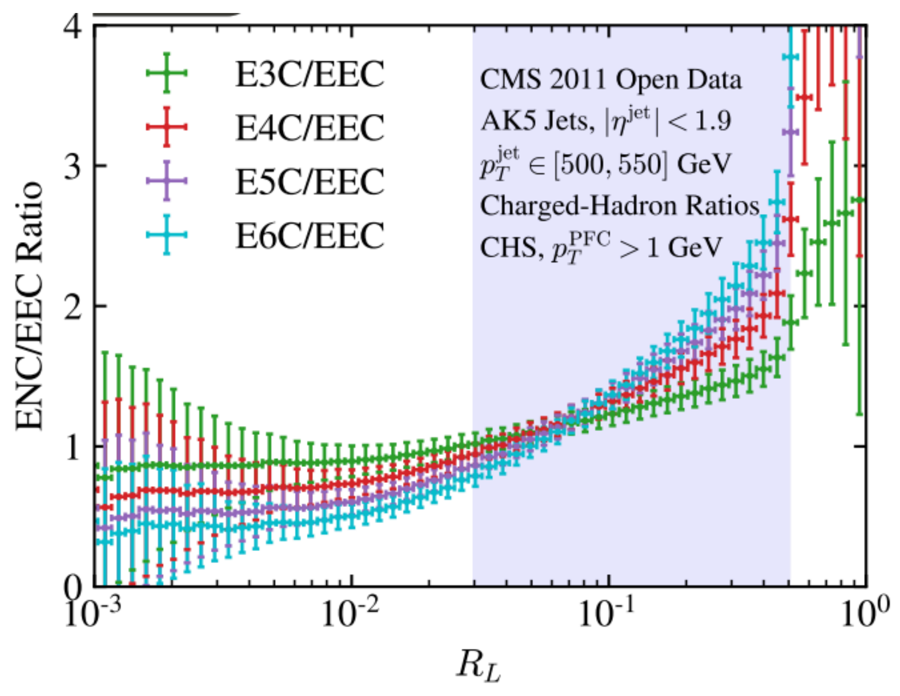


Rethinking Jets with Energy Correlators

Chen, Moult, Zhang, Zhu `20
Dixon, Moult, Zhu `19

2022





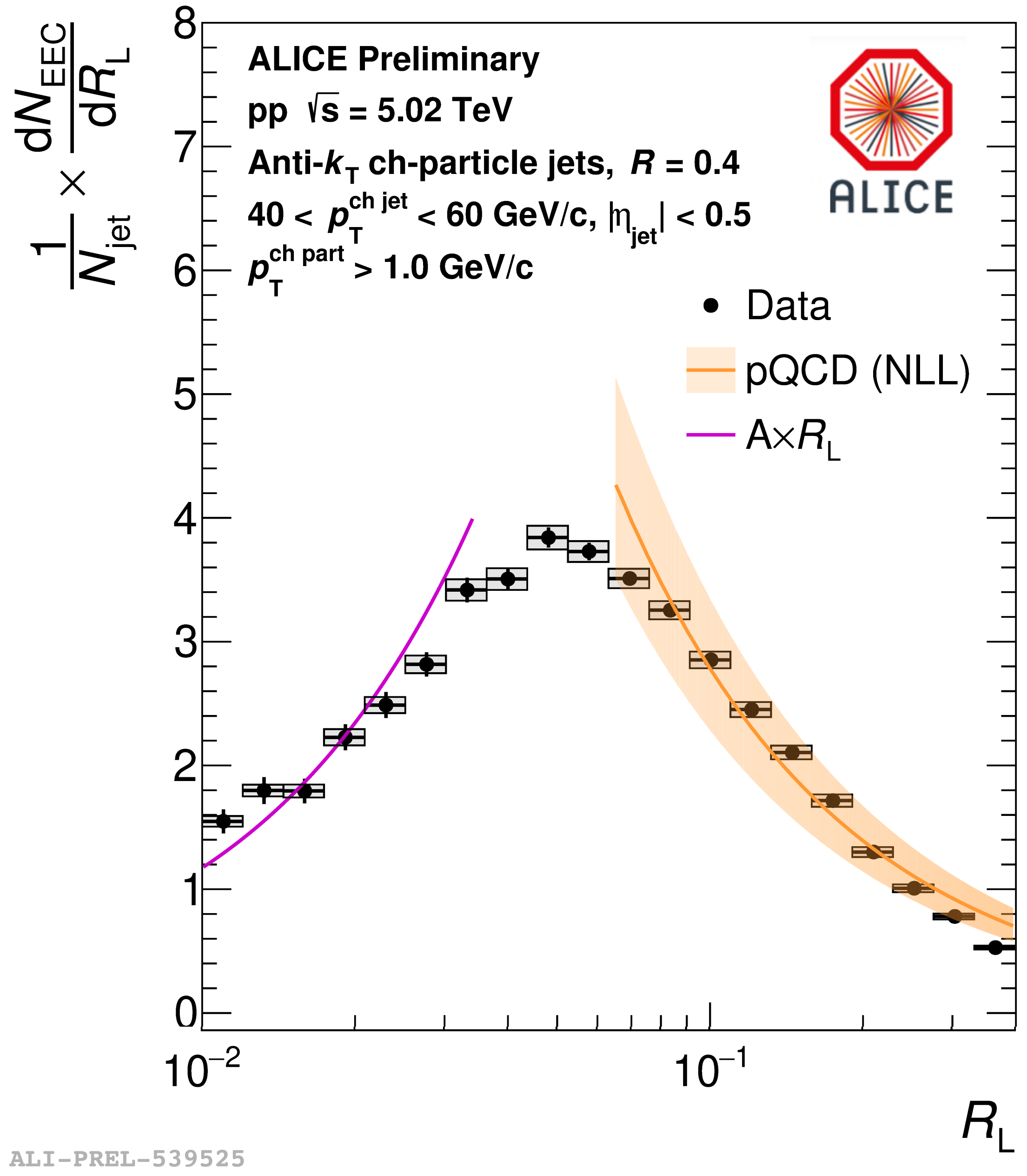
Open data analyses and QCD Factorization

KL, Meçaj, Moult `22
Komiske, Moult, Thaler, Zhu `22

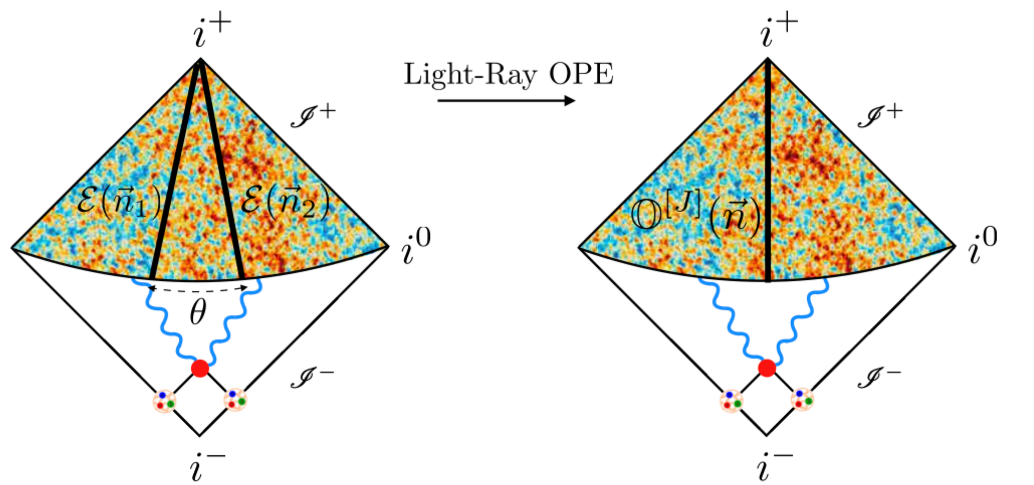
$$\mathcal{E}(\hat{n}_1)\,\mathcal{E}(\hat{n}_2) \sim \sum \theta^{\tau_i - 4}\,\mathbb{O}_i(\hat{n}_1)$$

2023

ALICE `23



Real Data analyses at the LHC

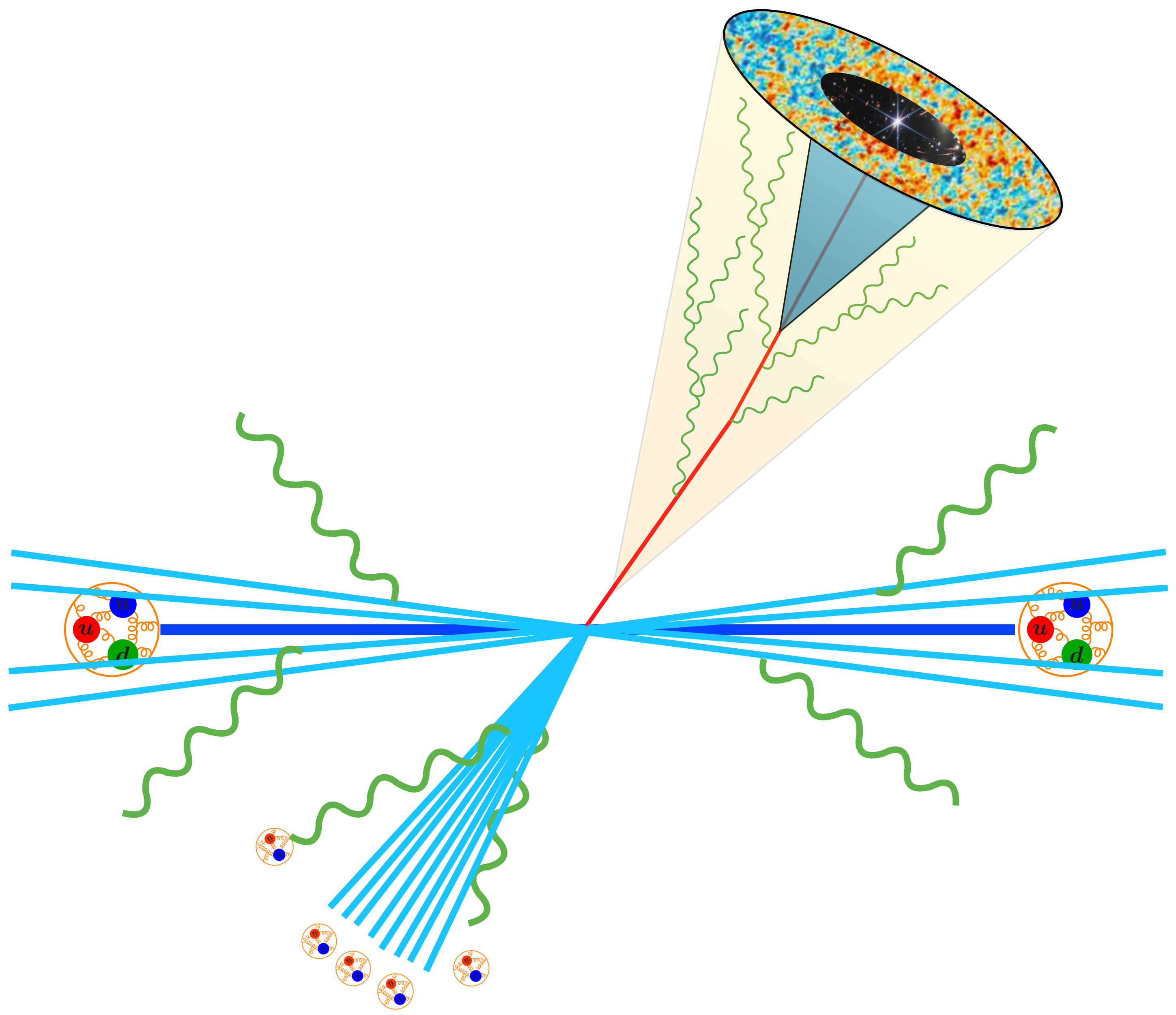


Observation of the universal of QCD predicted at the operator levels from the light-ray operator product expansion!

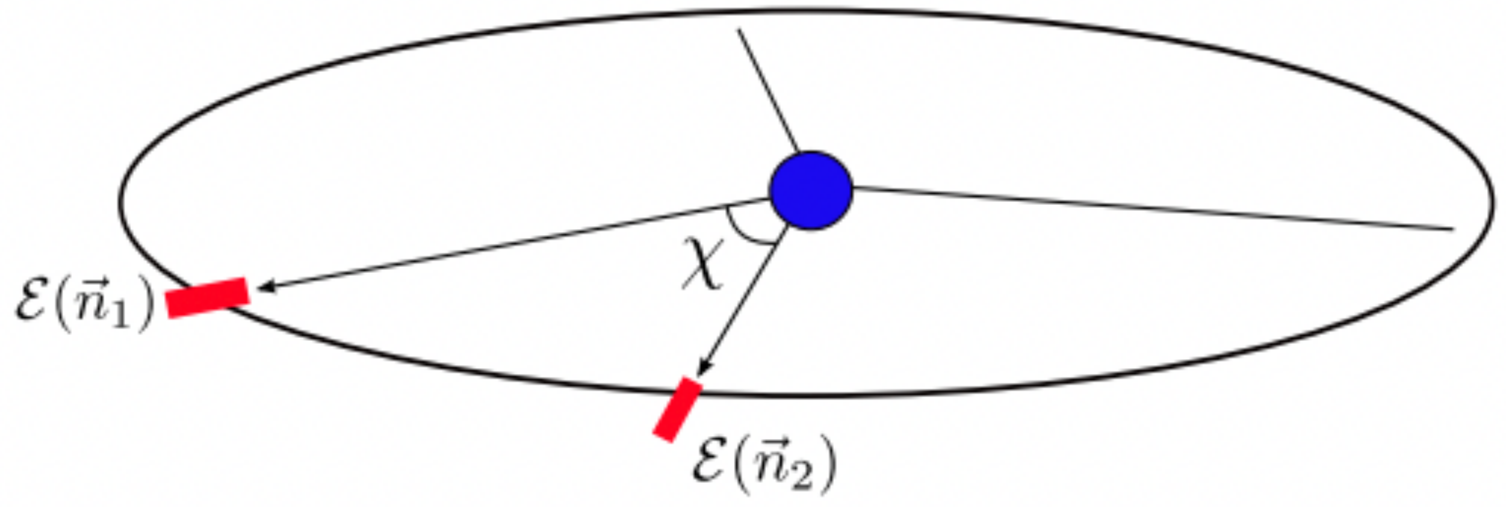
ENERGY CORRELATORS MAPPING HIGH ENERGY COLLIDER EVENTS

Collinear limit

$\chi \to 0$

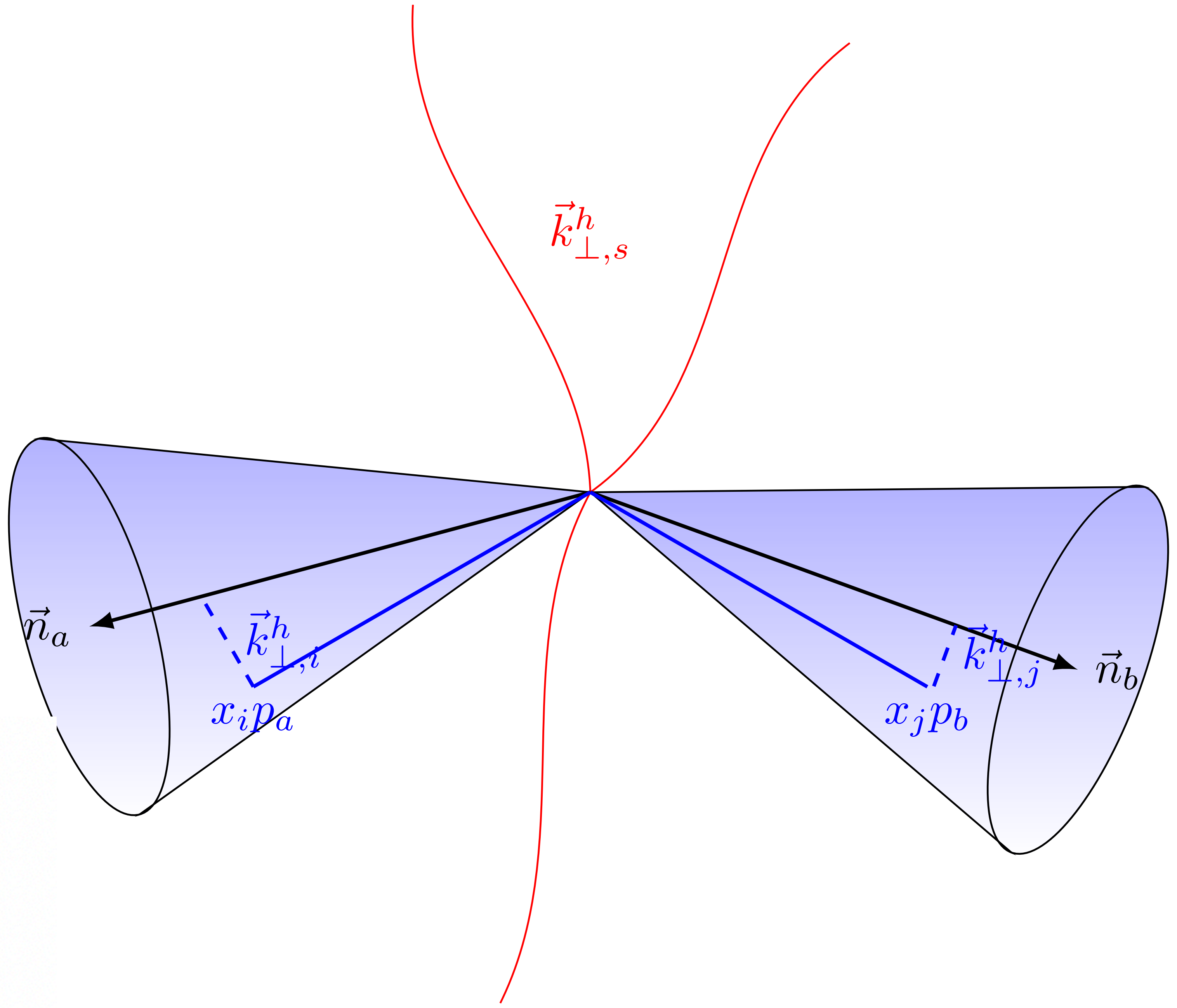


General angle region

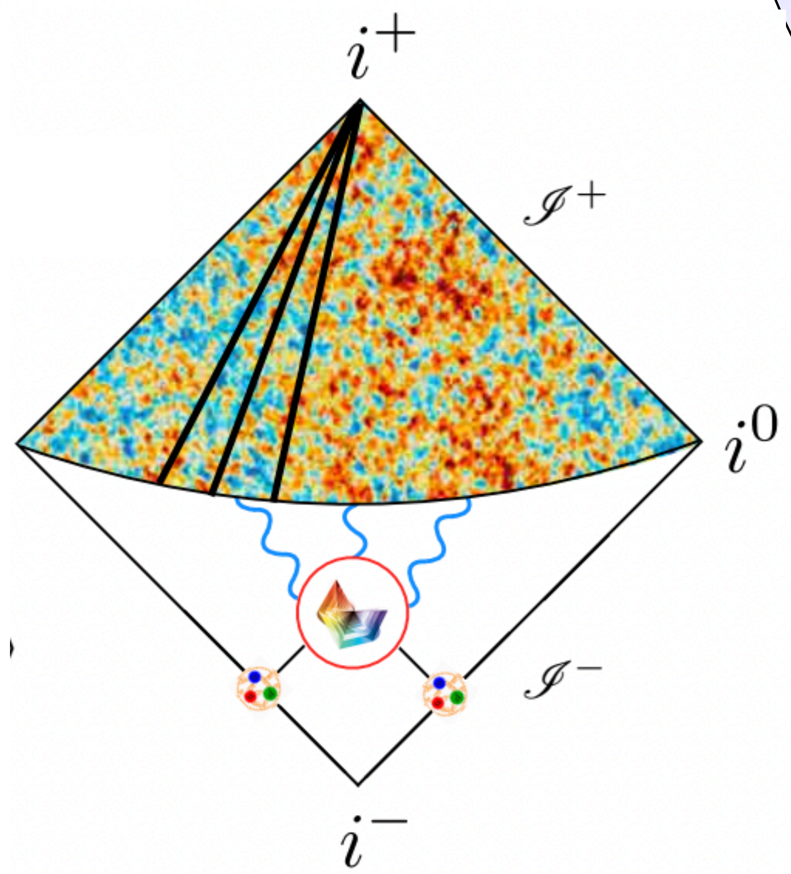


Back-to-back region

$\chi \to \pi$



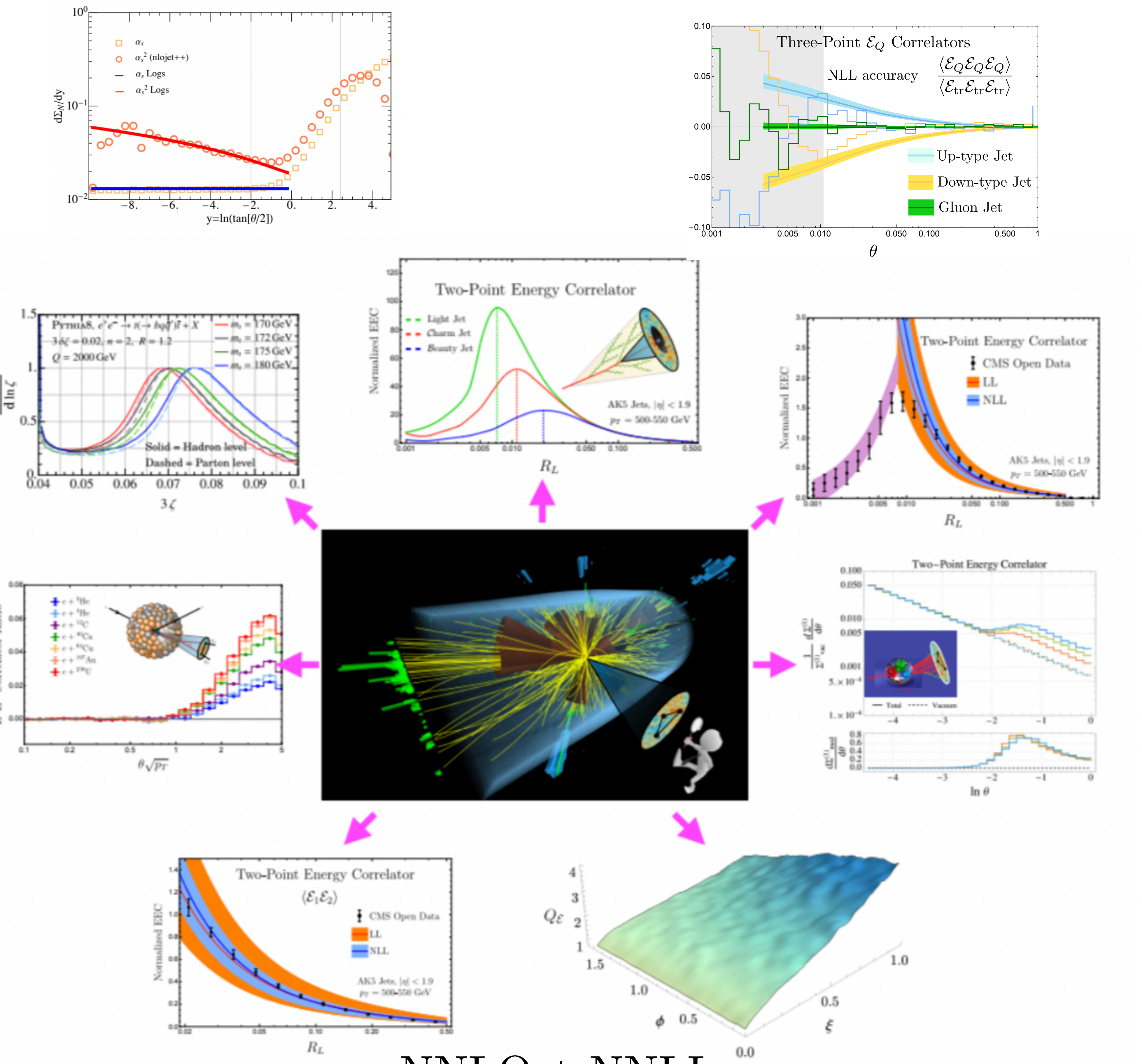
$$\mathcal{E}(\hat{n}) = \int_0^\infty dt \lim_{r\to\infty} r^2 n^i T_{0i}(t, r\hat{n}) \Longleftrightarrow$$



ENERGY CORRELATORS MAPPING HIGH ENERGY COLLIDER EVENTS

Collinear limit

$\chi \to 0$

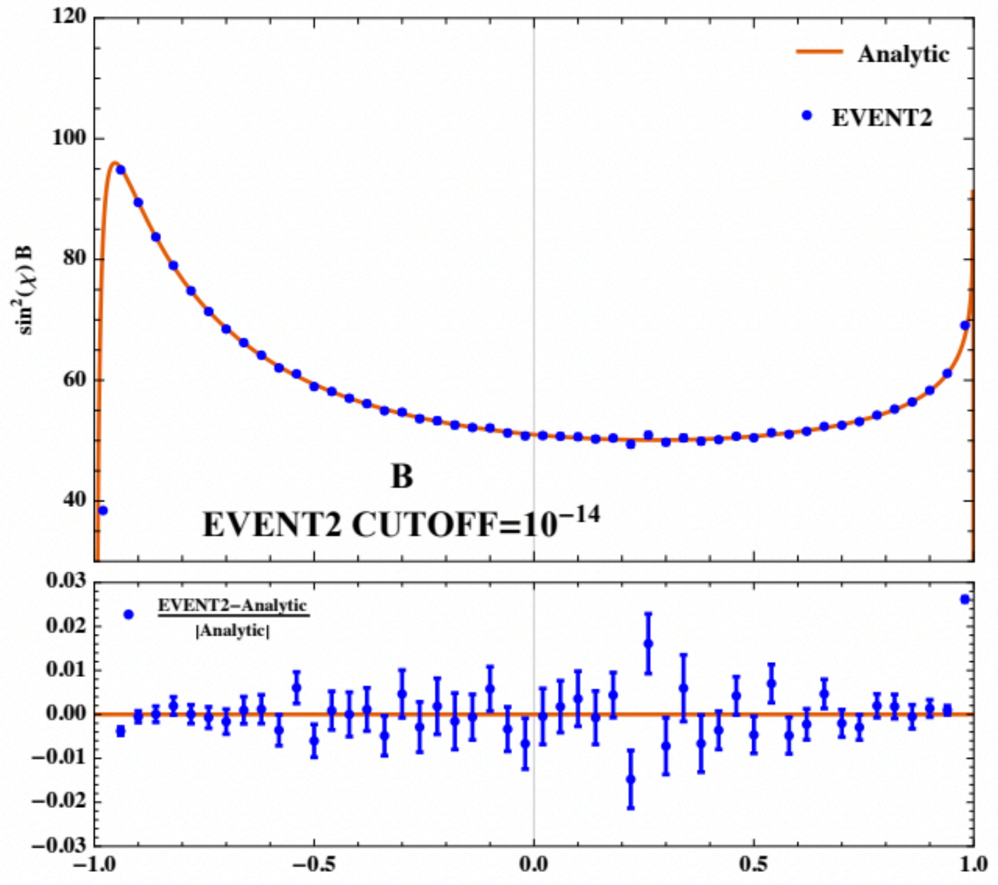


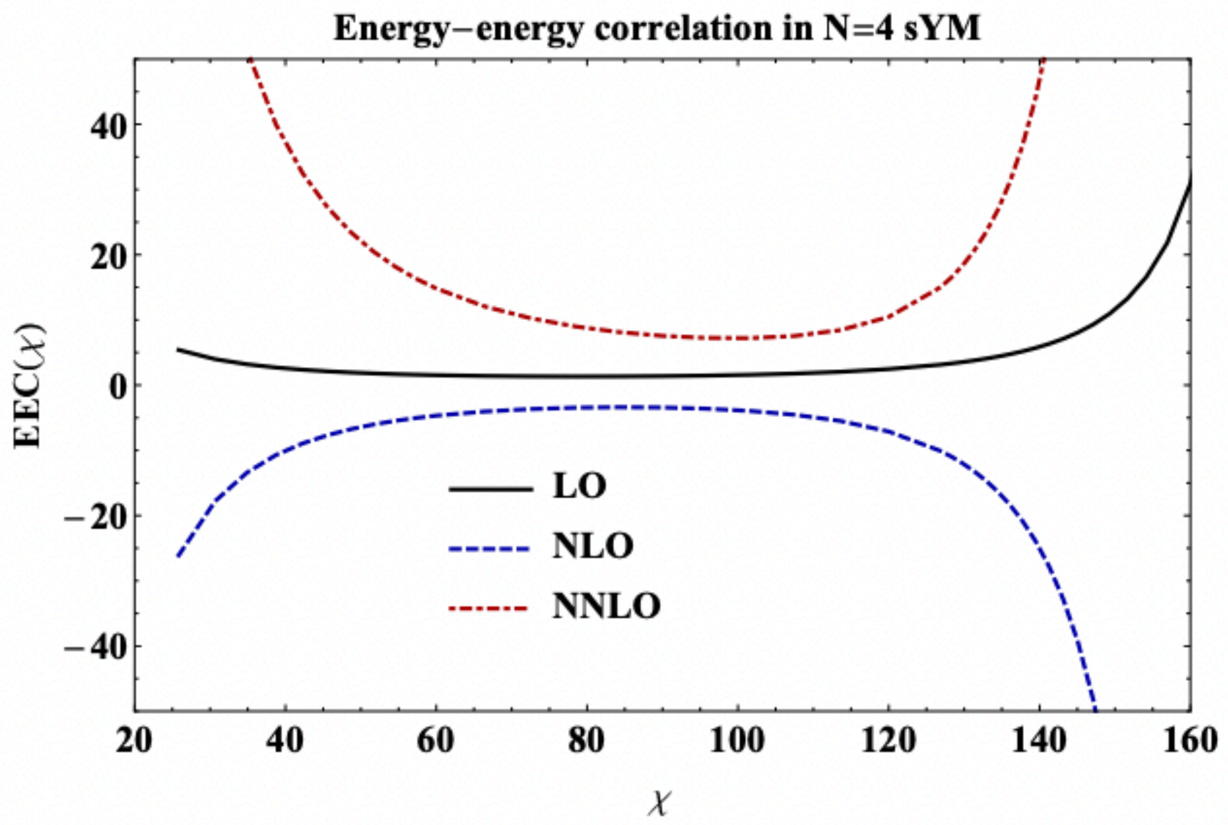
NNLO + NNLL

Dixon, Moult, Zhu, `19

General angle region

Dixon, Luo, Shtabovenko, Yang, Zhu, `18

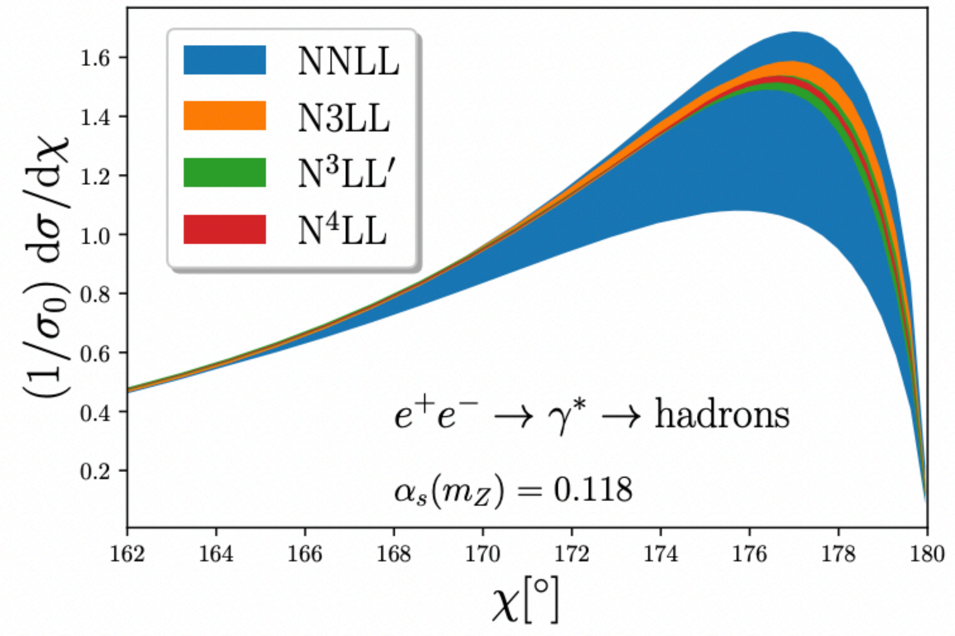




Henn, Sokatchev, Yan, Zhiboedov, `19

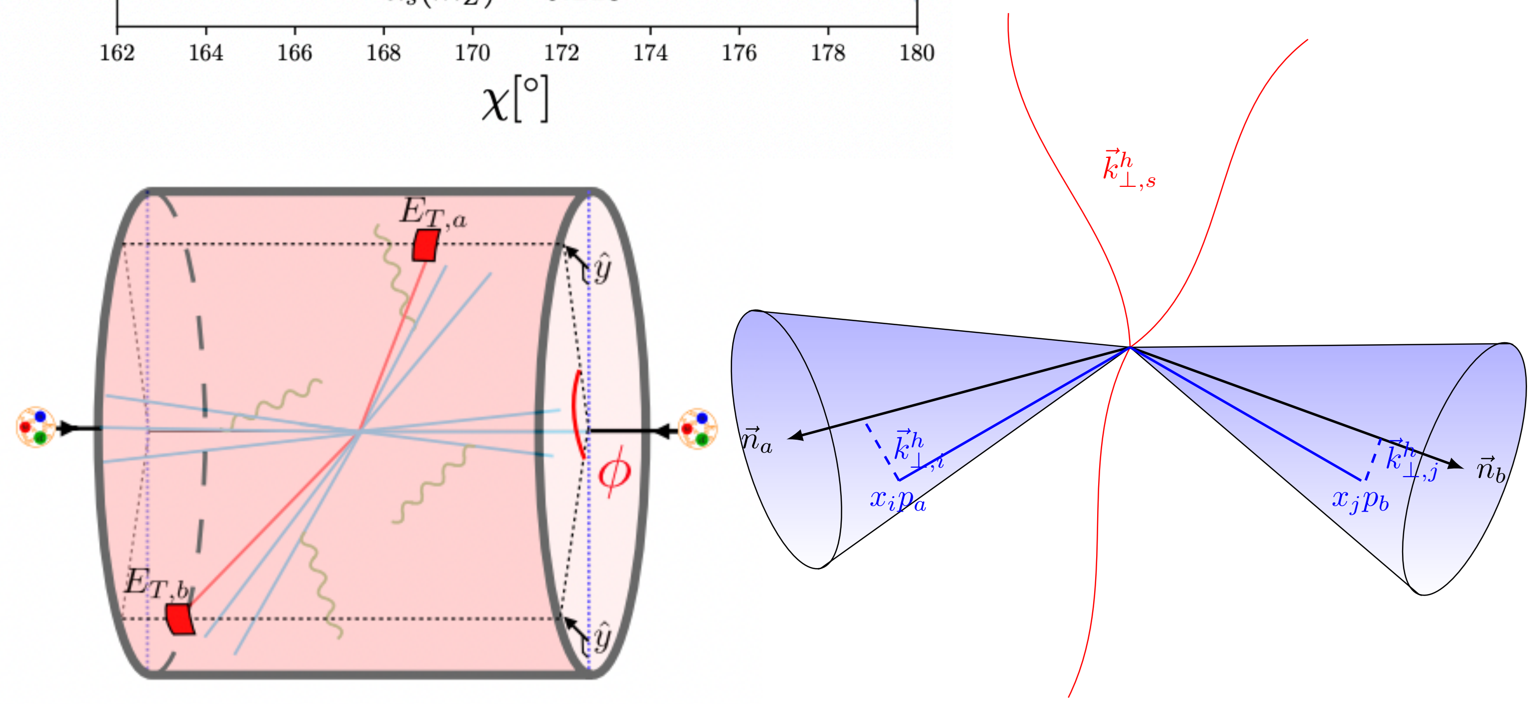
Back-to-back region

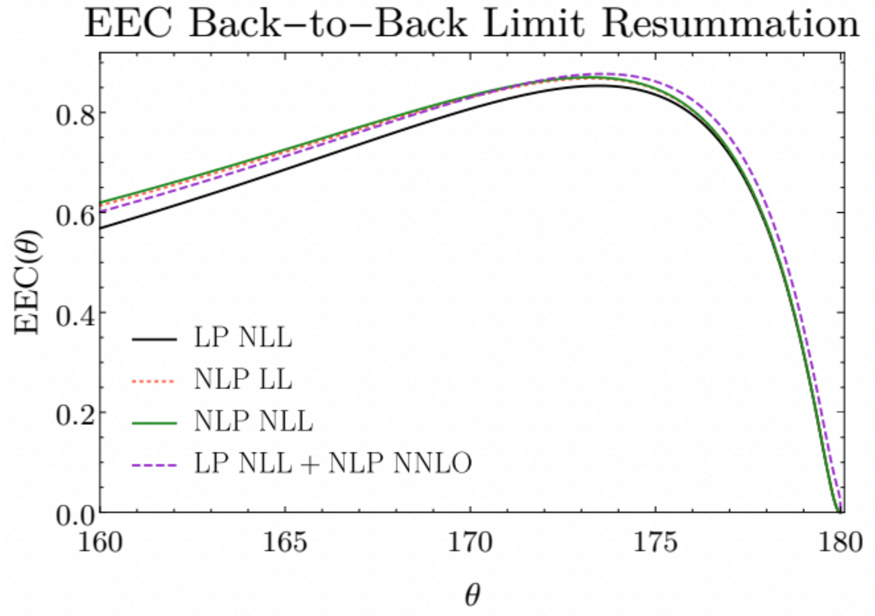
$\chi \to \pi$



$\mathrm{N}^3\mathrm{LO} + \mathrm{N}^4\mathrm{LL}$

Duhr, Mistlberger, Vita, `22





ENERGY CORRELATORS MAPPING HIGH ENERGY COLLIDER EVENTS

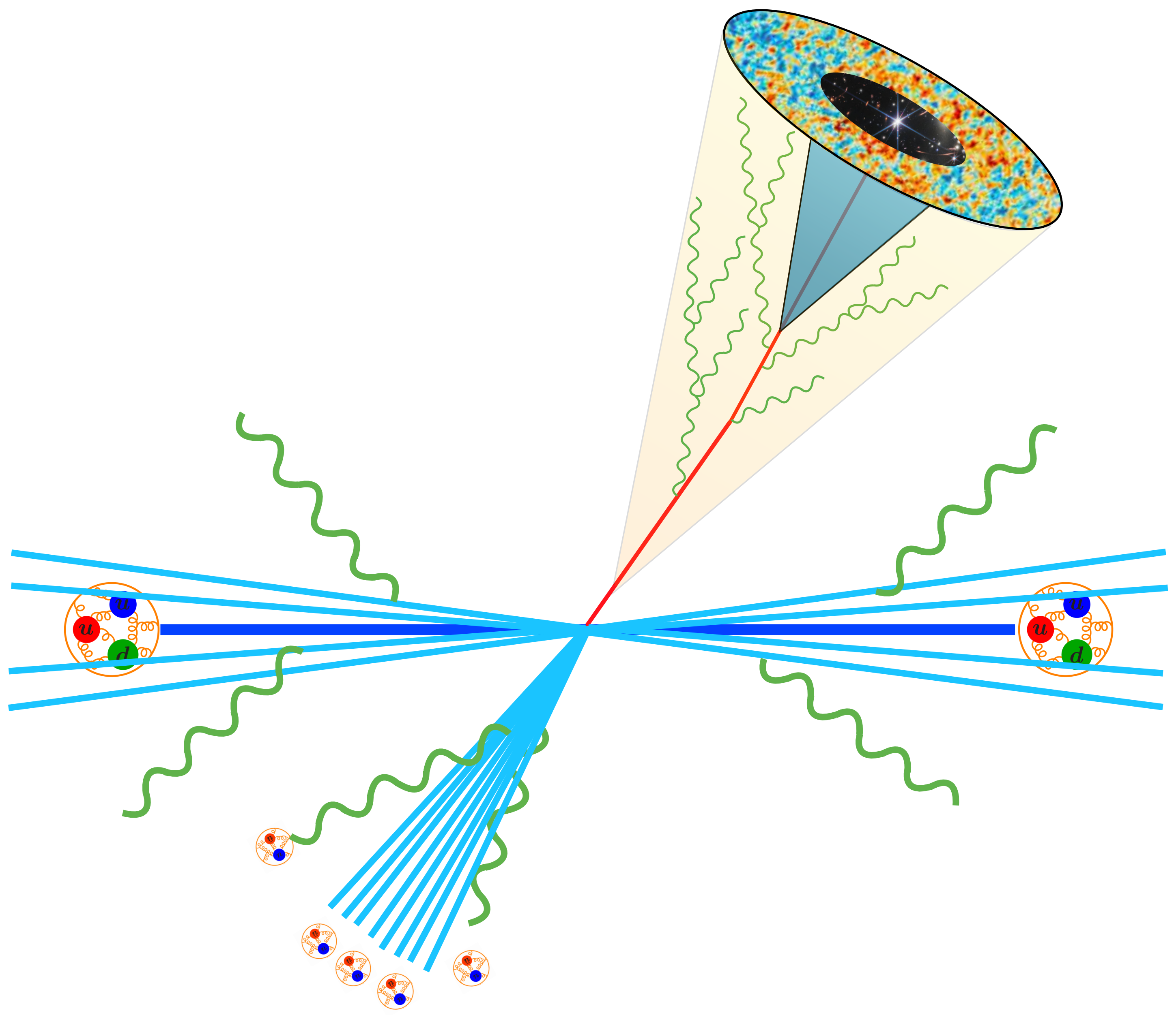
Collinear limit

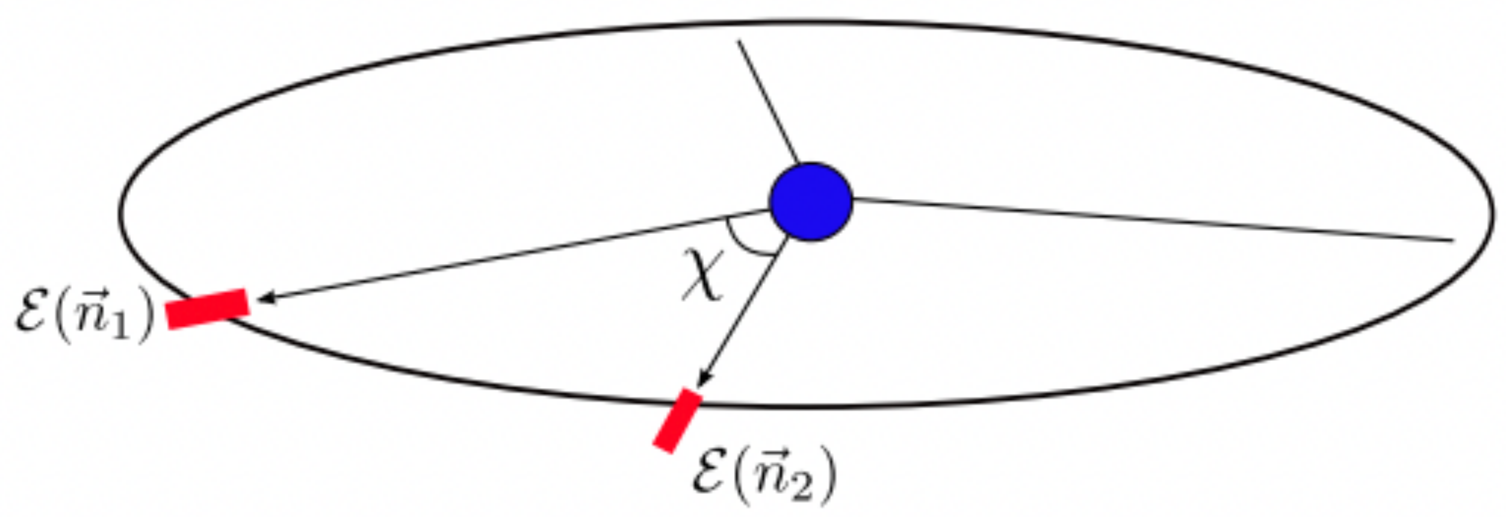
$\chi \to 0$

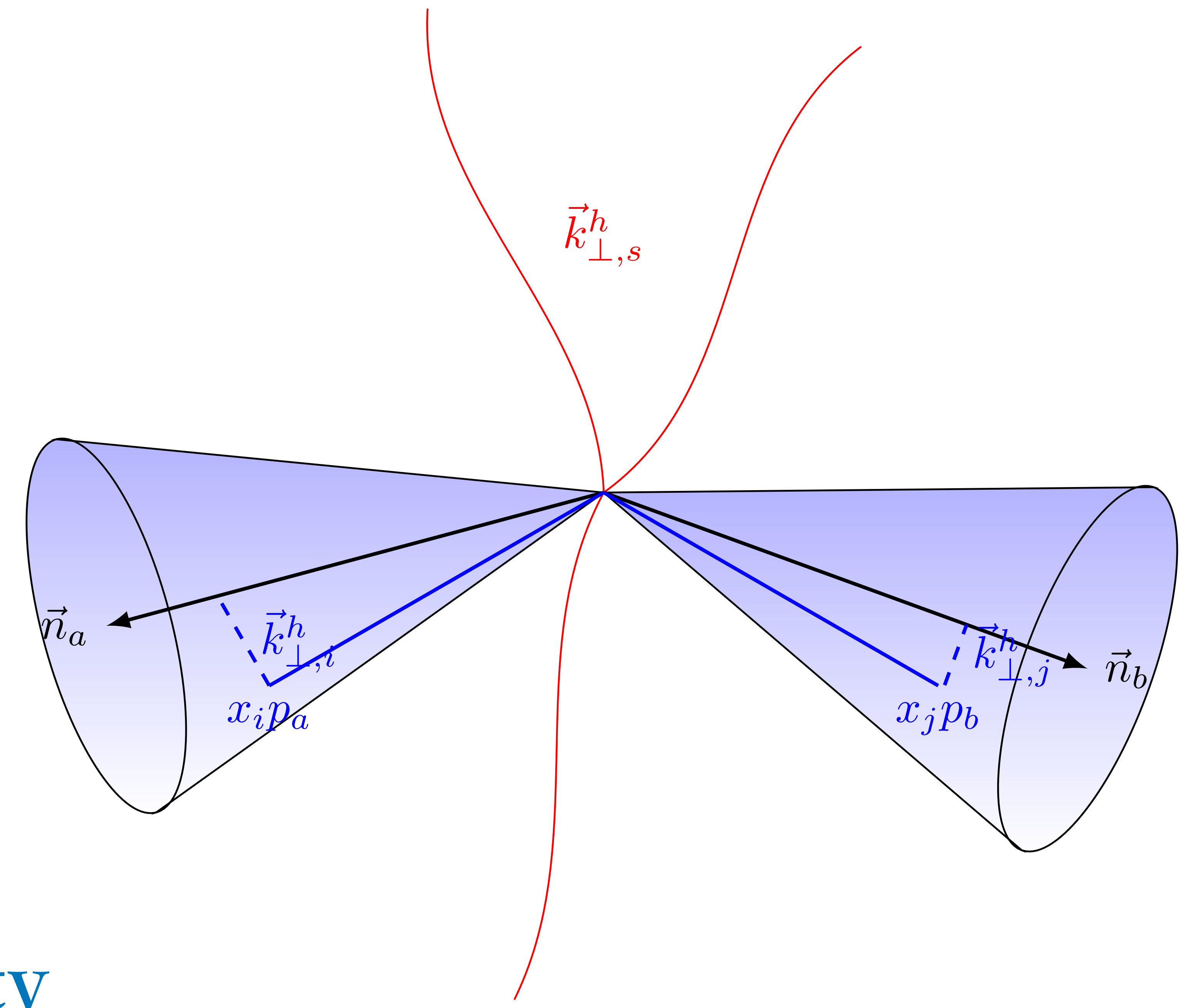
General angle region

Back-to-back region

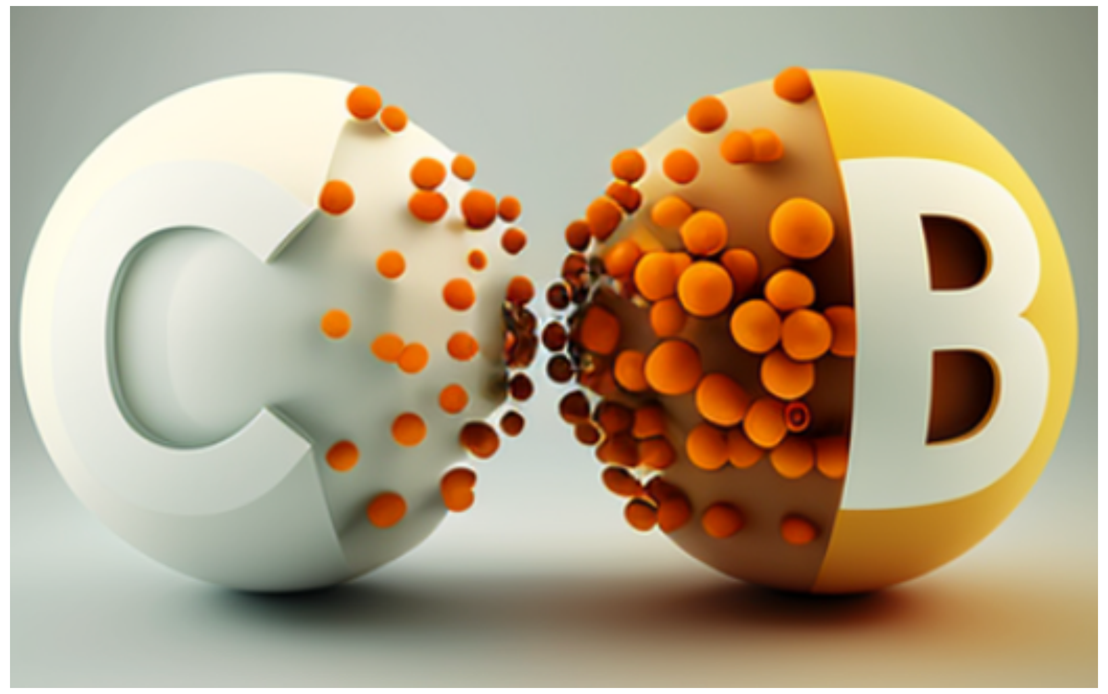
$\chi \to \pi$







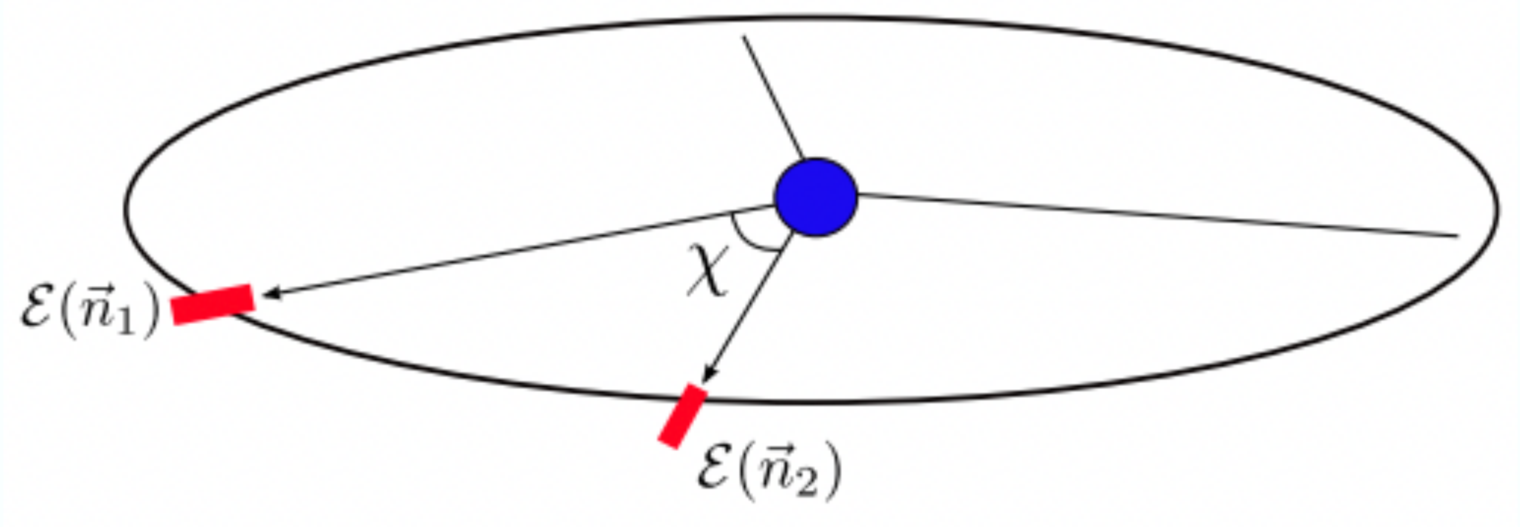
Charm



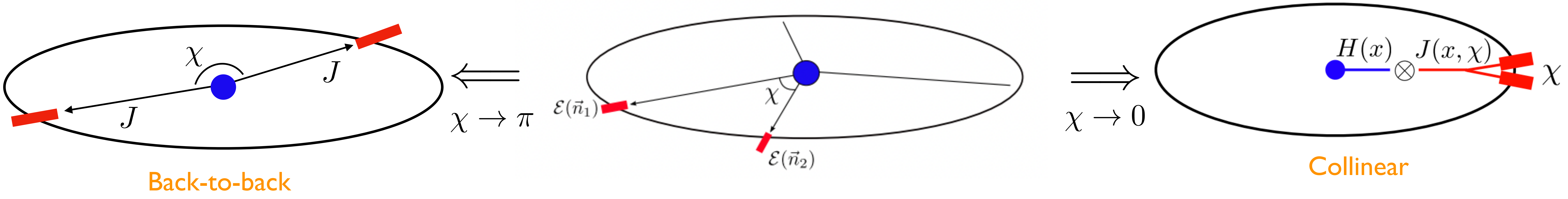
Beauty

How does intrinsic **heavy quark mass** affect each of these regions of particle collisions?

ENERGY-ENERGY CORRELATORS AT GENERAL ANGLE WITH MASS



ENERGY-ENERGY CORRELATORS AT GENERAL ANGLE WITH MASS

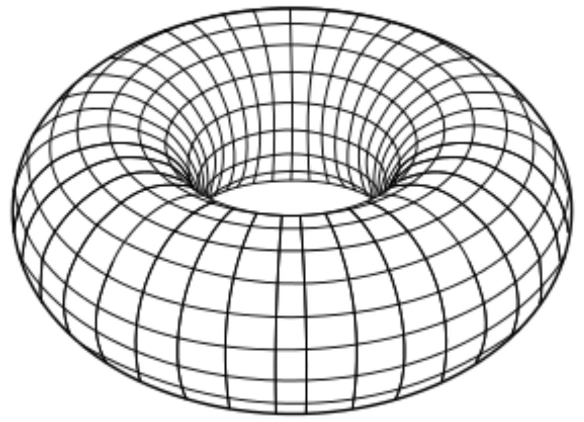


- **Sum rule connects different singular regions**

$$\int_0^{\pi} d\chi \frac{d\Sigma}{d\chi} = \sigma_{\mathrm{tot}}$$

- **Provides the mean to study singular regions**
  - **useful for developing factorization formalism**

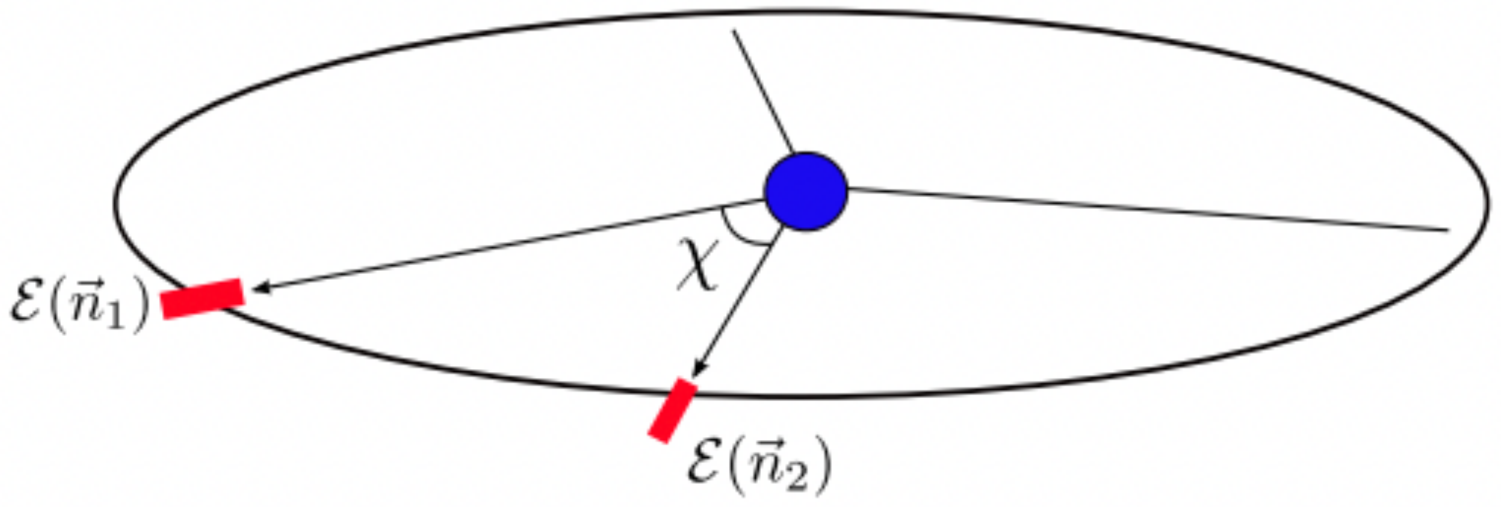
- **Opportunity to explore the function space of the observable**



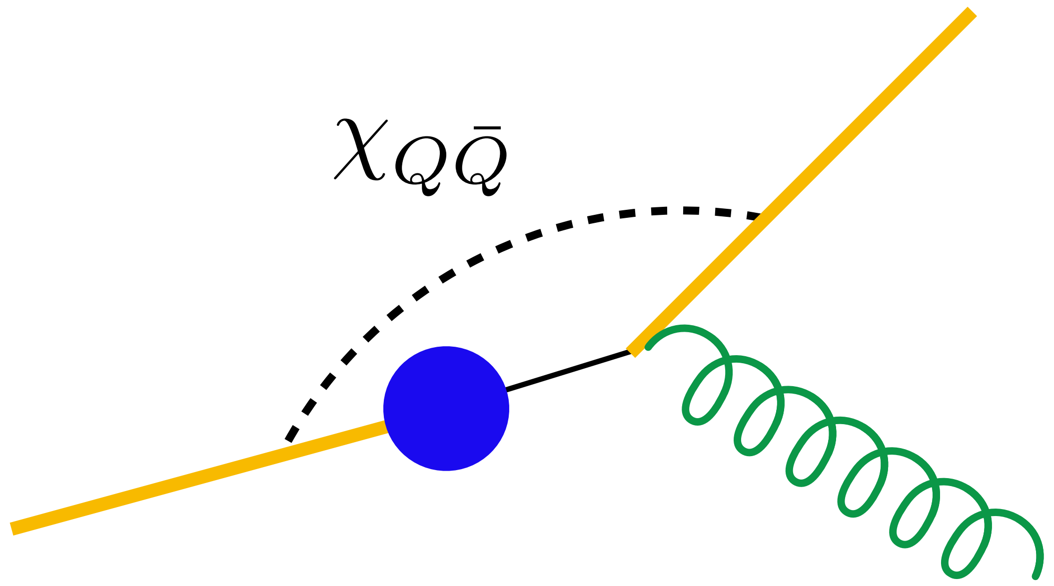
ENERGY-ENERGY CORRELATORS AT GENERAL ANGLE WITH MASS

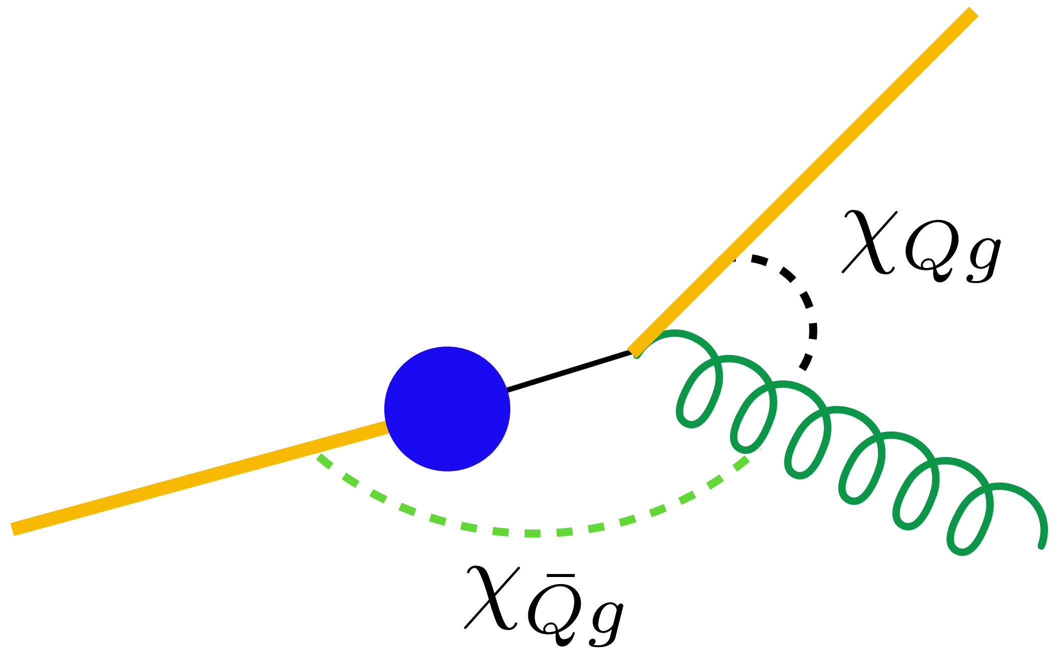
I II III III

$$\left.\frac{d\Sigma_M^{(1)}}{d\chi}\right|_{e^+e^-} \sim \frac{\alpha_s C_F}{2\pi}\int dx_Q dx_{\bar{Q}} H\left(x_Q, x_{\bar{Q}}, y\right)\left[2x_Q x_{\bar{Q}}\delta\left(\chi_{Q\bar{Q}} - f_{Q\bar{Q}}\left(x_Q, x_{\bar{Q}}, y\right)\right) + 2x_Q x_g \delta\left(\chi_{Qg} - f_{Qg}\left(x_Q, x_{\bar{Q}}, y\right)\right) + \bar{Q}g\text{case}\right]$$

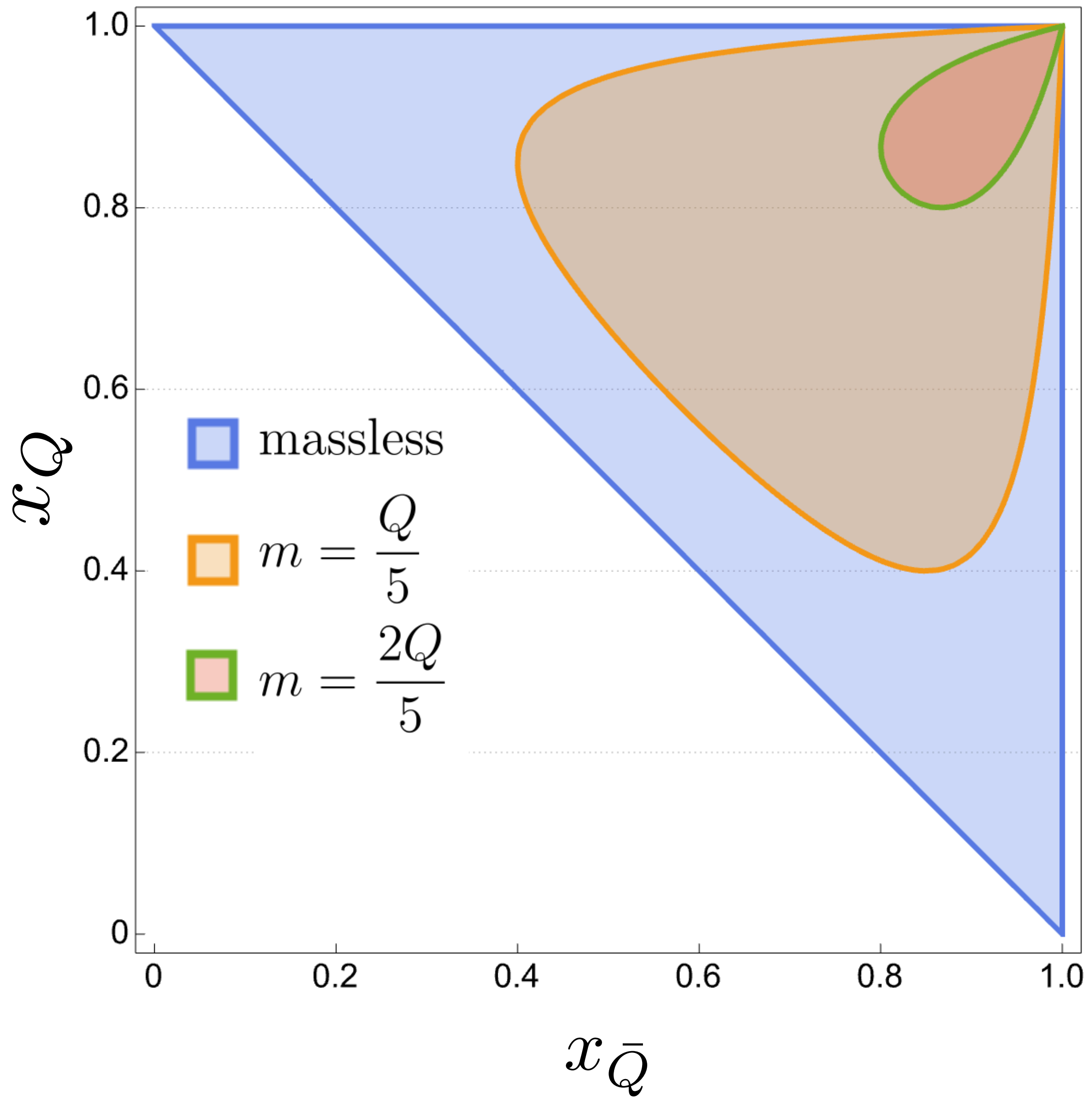


$$0 < y = \frac{2m}{Q} < 1$$





I Phase space

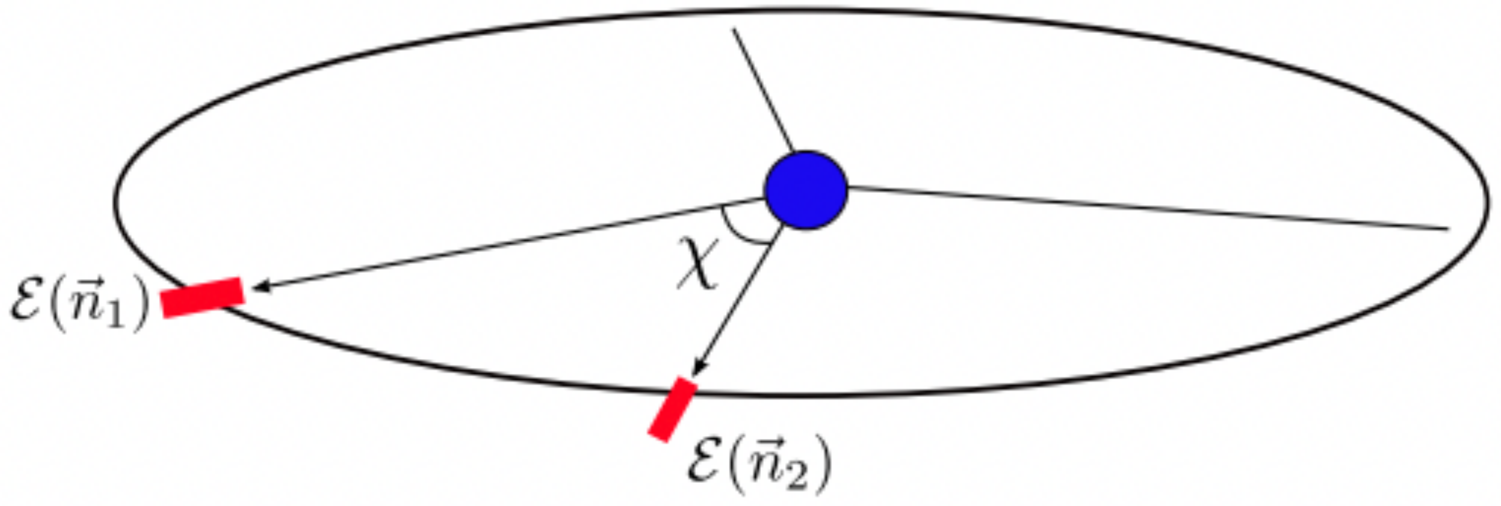


- Phase space suppression for massive final states

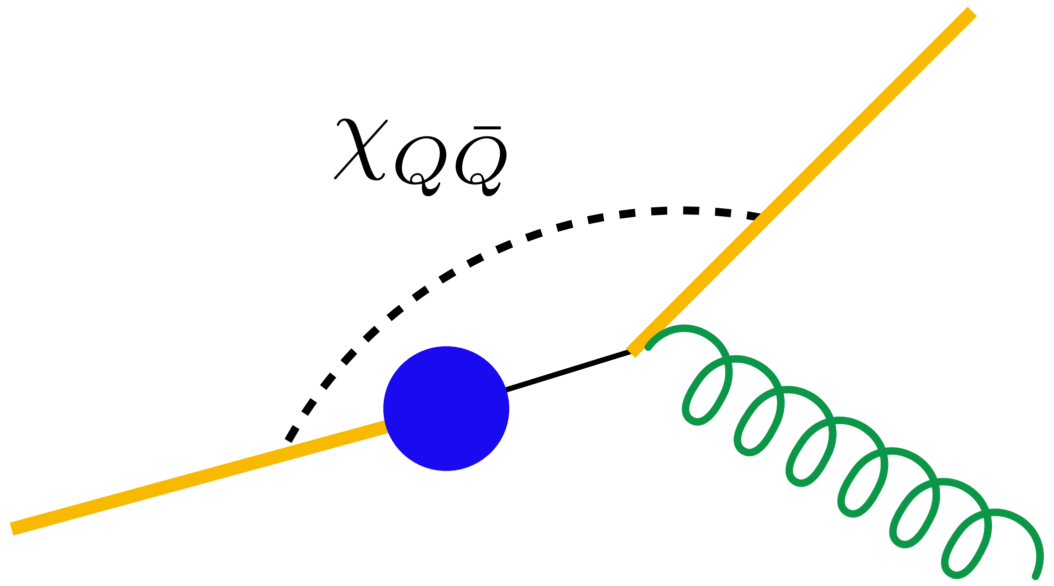
ENERGY-ENERGY CORRELATORS AT GENERAL ANGLE WITH MASS

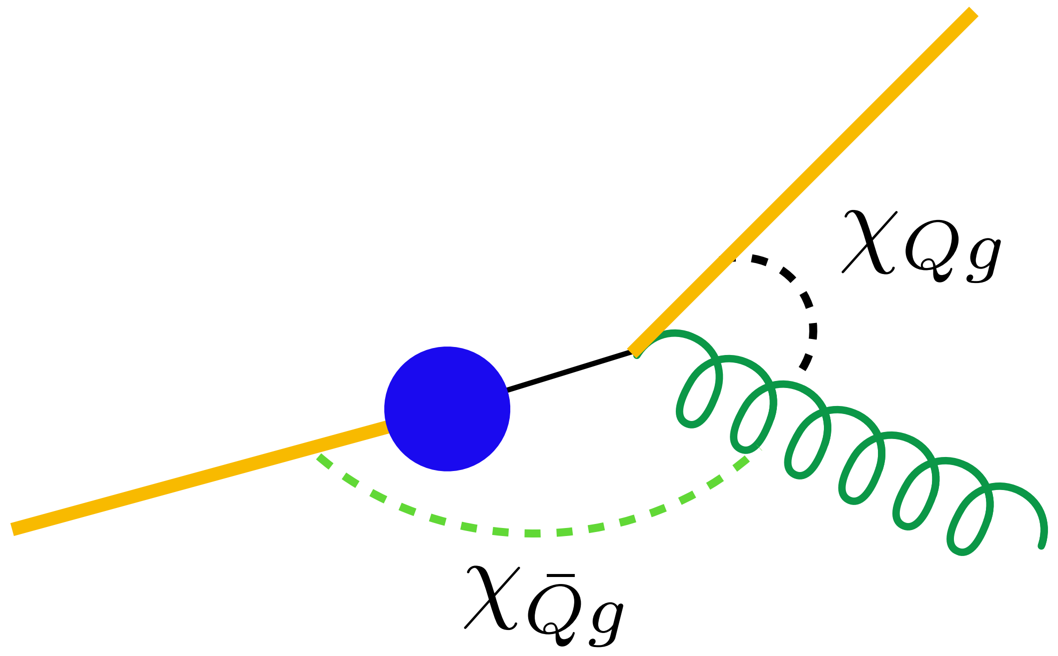
I II III III

$$\left.\frac{d\Sigma_M^{(1)}}{d\chi}\right|_{e^+e^-} \sim \frac{\alpha_s C_F}{2\pi}\int dx_Q dx_{\bar{Q}} H\left(x_Q, x_{\bar{Q}}, y\right)\left[2x_Q x_{\bar{Q}}\delta\left(\chi_{Q\bar{Q}} - f_{Q\bar{Q}}\left(x_Q, x_{\bar{Q}}, y\right)\right) + 2x_Q x_g \delta\left(\chi_{Qg} - f_{Qg}\left(x_Q, x_{\bar{Q}}, y\right)\right) + \bar{Q}g\text{case}\right]$$



$$0 < y = \frac{2m}{Q} < 1$$





II Matrix Elements

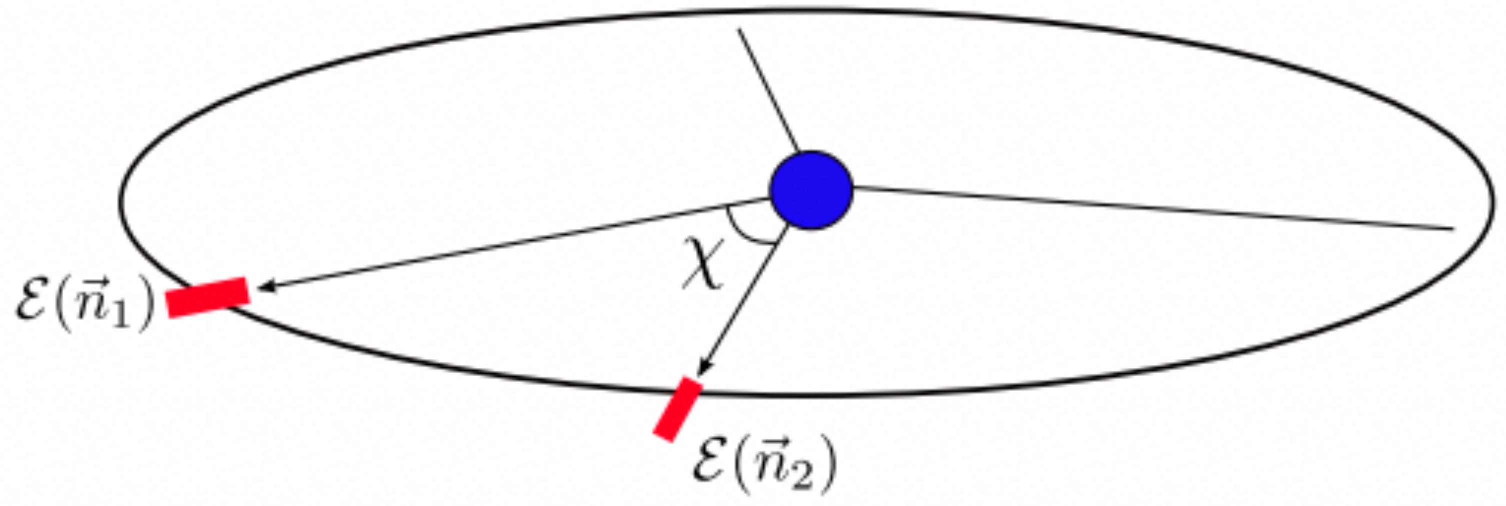
$$H(x_Q, x_{\bar{Q}}, y) = H_0(x_Q, x_{\bar{Q}}) + y^2 H_2(x_Q, x_{\bar{Q}}) + y^4 H_4(x_Q, x_{\bar{Q}})$$

The usual massless one $H_0(x_Q, x_{\bar{Q}}) \sim \dfrac{x_Q^2 + x_{\bar{Q}}^2}{(1-x_Q)(1-x_{\bar{Q}})}$

ENERGY-ENERGY CORRELATORS AT GENERAL ANGLE WITH MASS

I **II** **III** **III**

$$\left.\frac{d\Sigma_M^{(1)}}{d\chi}\right|_{e^+e^-} \sim \frac{\alpha_s C_F}{2\pi}\int dx_Q dx_{\bar{Q}} H\left(x_Q, x_{\bar{Q}}, y\right)\left[2x_Q x_{\bar{Q}}\delta\left(\chi_{Q\bar{Q}} - f_{Q\bar{Q}}\left(x_Q, x_{\bar{Q}}, y\right)\right) + 2x_Q x_g \delta\left(\chi_{Qg} - f_{Qg}\left(x_Q, x_{\bar{Q}}, y\right)\right) + \bar{Q}g\text{case}\right]$$



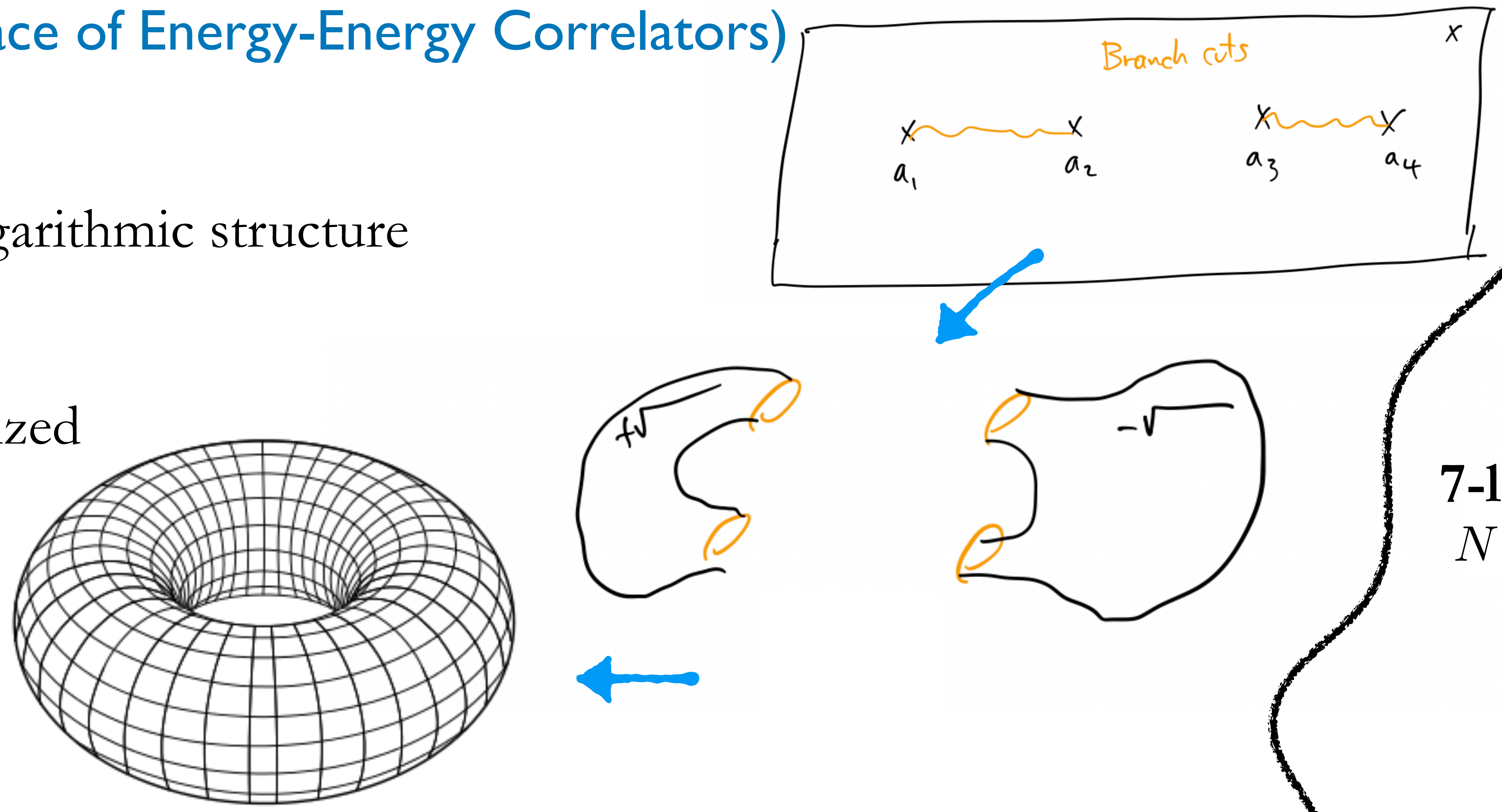
$$0 < y = \frac{2m}{Q} < 1$$

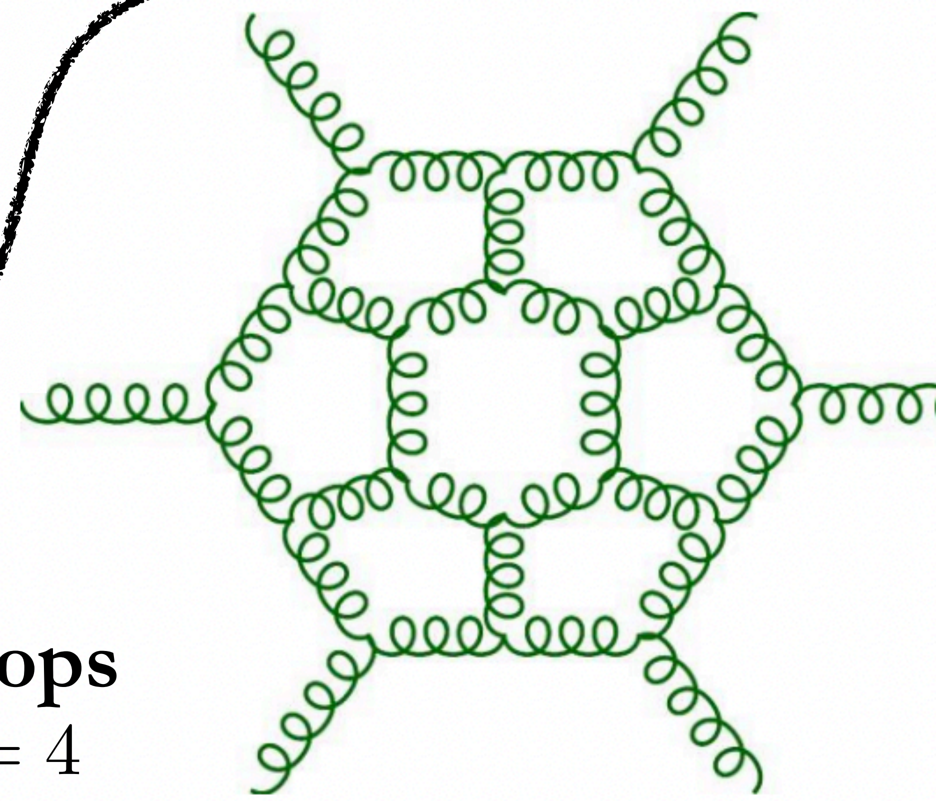
III Measurements (Study of function space of Energy-Energy Correlators)

$$f_{Qg} \sim \frac{1}{x_g\sqrt{x_Q^2 - y^2}}$$ can be rationalized, usual logarithmic structure

$$f_{Q\bar{Q}} \sim \frac{1}{\sqrt{x_Q^2 - y^2}\sqrt{x_{\bar{Q}}^2 - y^2}}$$ cannot be rationalized

$$\Longrightarrow \int dx^* \frac{1}{\sqrt{P_4(x^*)}}$$ **Elliptic integrals**

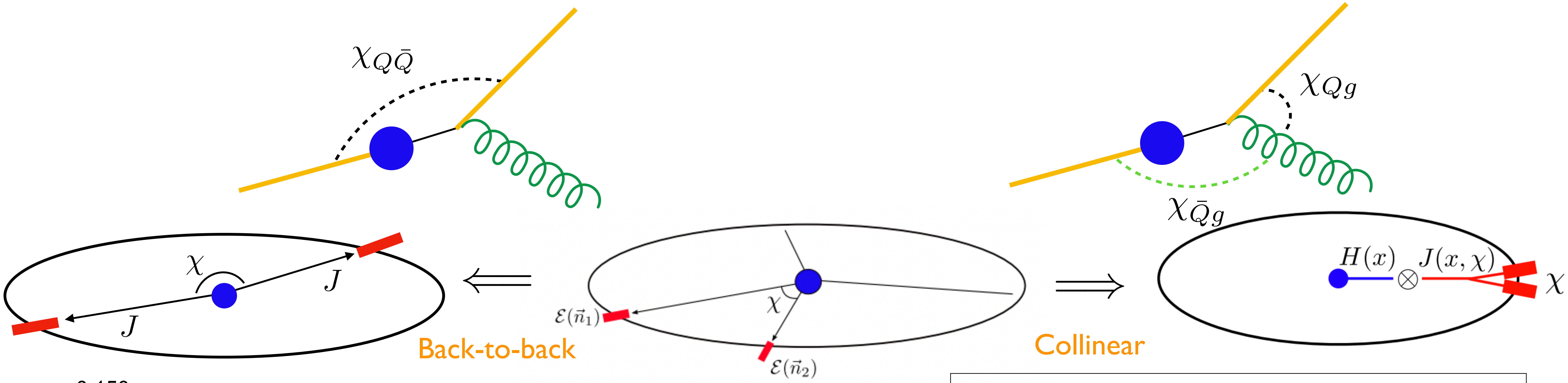


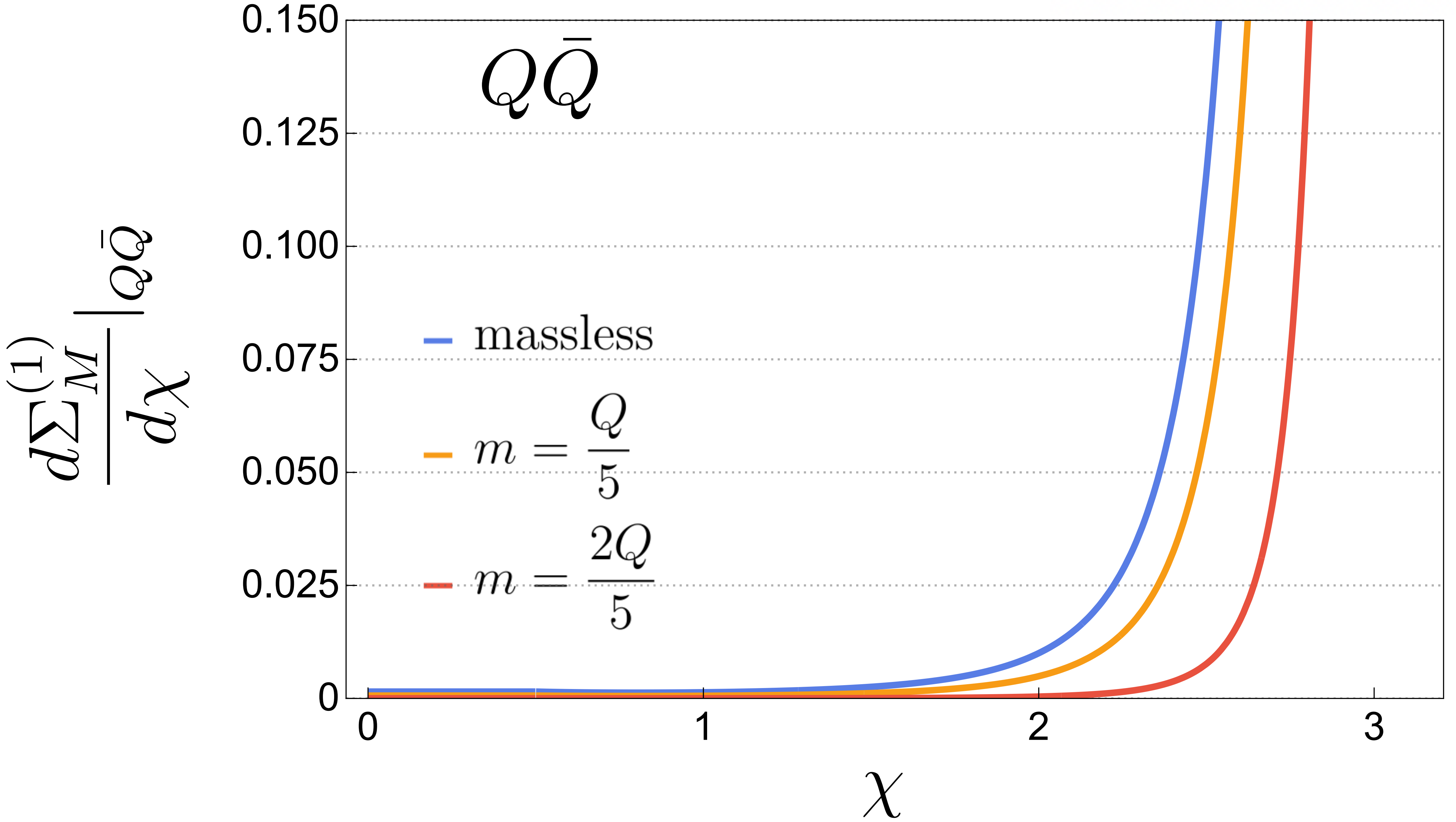


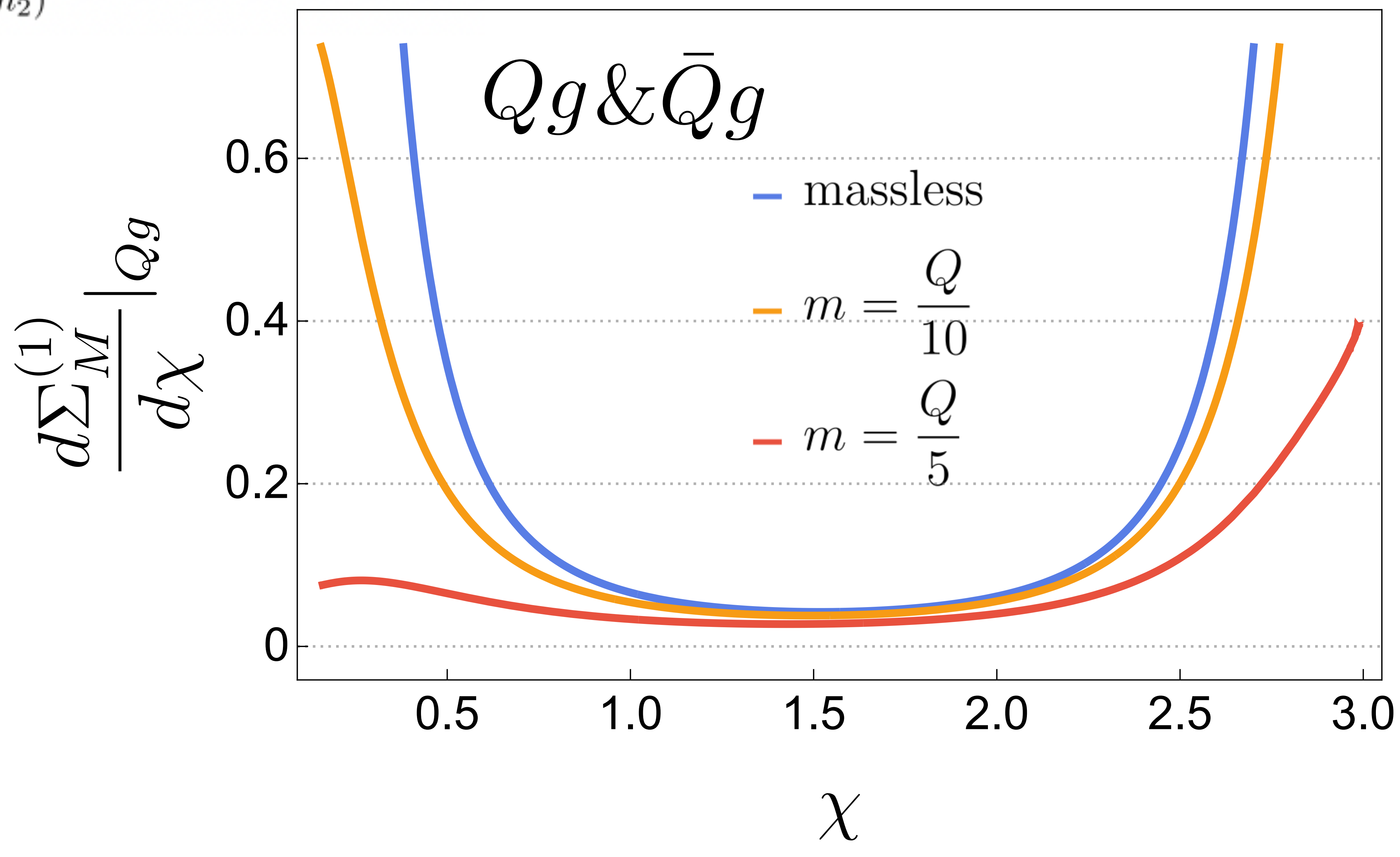
7-loops
$N = 4$

Amplitude bootstrap
Dixon et al, Henn et al, …

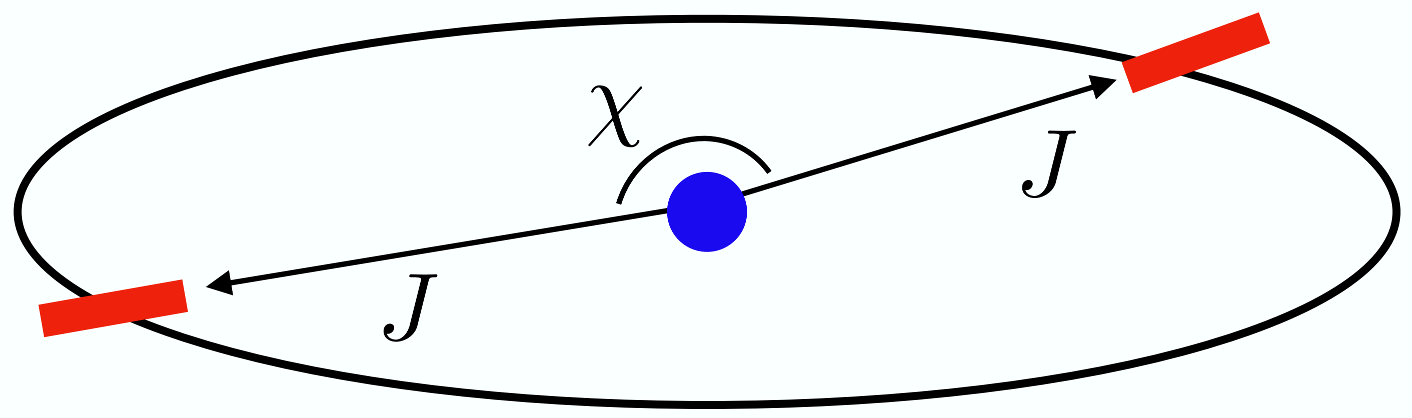
ENERGY-ENERGY CORRELATORS AT GENERAL ANGLE WITH MASS



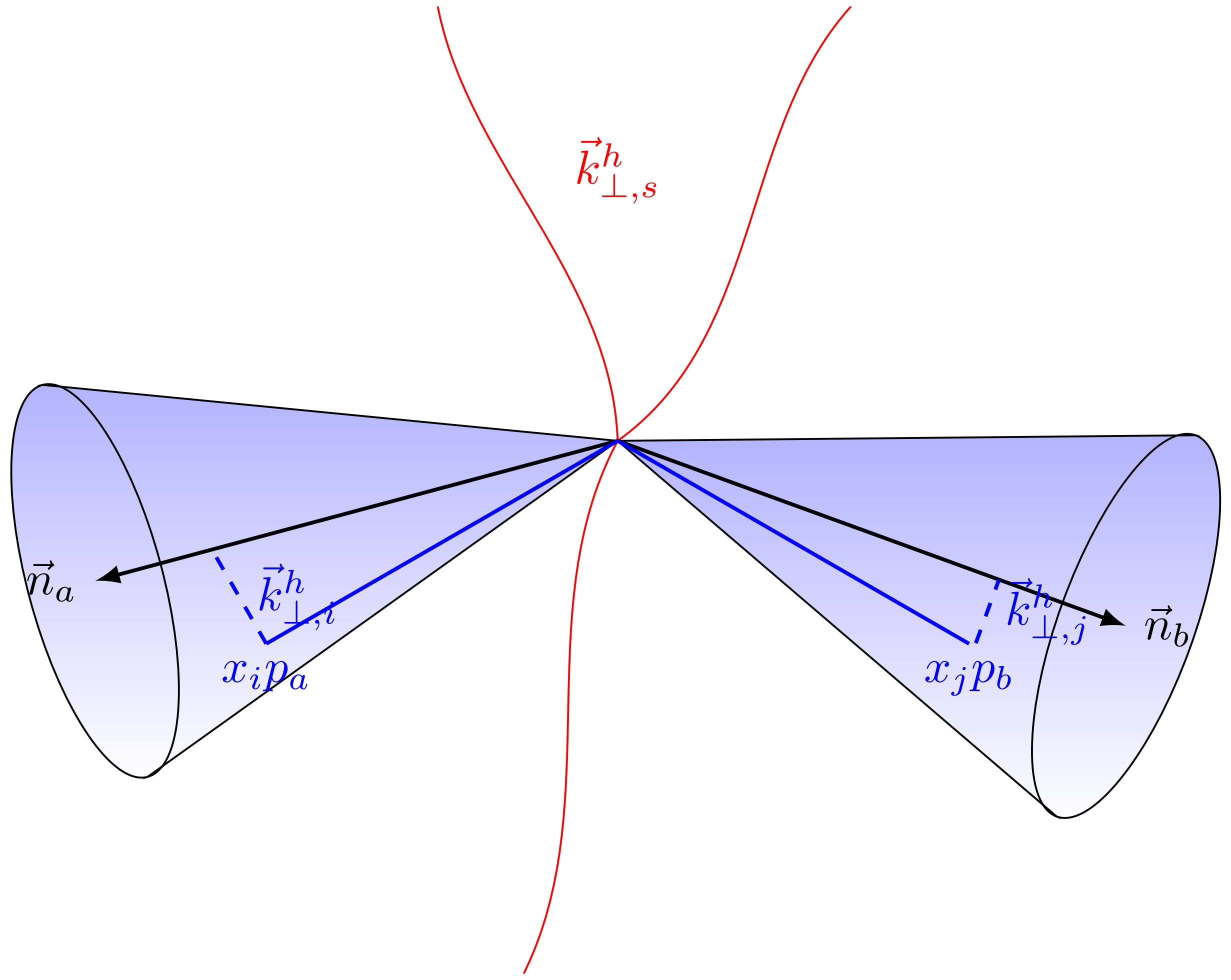




BACK-TO-BACK ENERGY-ENERGY CORRELATORS WITH MASS



BACK-TO-BACK ENERGY-ENERGY CORRELATORS WITH MASS



$$\frac{d\Sigma_M}{dz} = \frac{1}{2}\int d^2\vec{k}_\perp \int \frac{d^2\vec{b}_\perp}{(2\pi)^2} e^{-i\vec{b}_\perp\cdot\vec{k}_\perp} H(Q,\mu) J_n^M\left(\vec{b}_\perp, m,\mu,\nu\right) J_{\bar{n}}^M\left(\vec{b}_\perp, m,\mu,\nu\right) S(\vec{b}_\perp,\mu,\nu)\,\delta\left(1-z-\frac{\vec{k}_\perp^2}{Q^2}\right)$$

where $z = \dfrac{1+\cos\chi}{2}$

Assuming $\Lambda^2_{\rm QCD} \ll Q^2(1-z) \sim m^2$, [See **Rebecca**'s talk for NP matching]

von Kuk, Michel, Sun `23

$$J_n^M(b_\perp, m,\mu,\nu) = \sum_h \int dx\, x\, D_{h/Q}(x, b_\perp, m,\mu,\nu) = \int dx\, x\, d_{Q/Q}(x, b_\perp, m,\mu,\nu) \boxed{\sum_H \chi_H} = 1$$

Favored contribution

$$+\int dx\, x\, d_{\bar{Q}/Q}(x, b_\perp, m,\mu,\nu) \boxed{\sum_{\bar{H}} \chi_{\bar{H}}} = 1$$

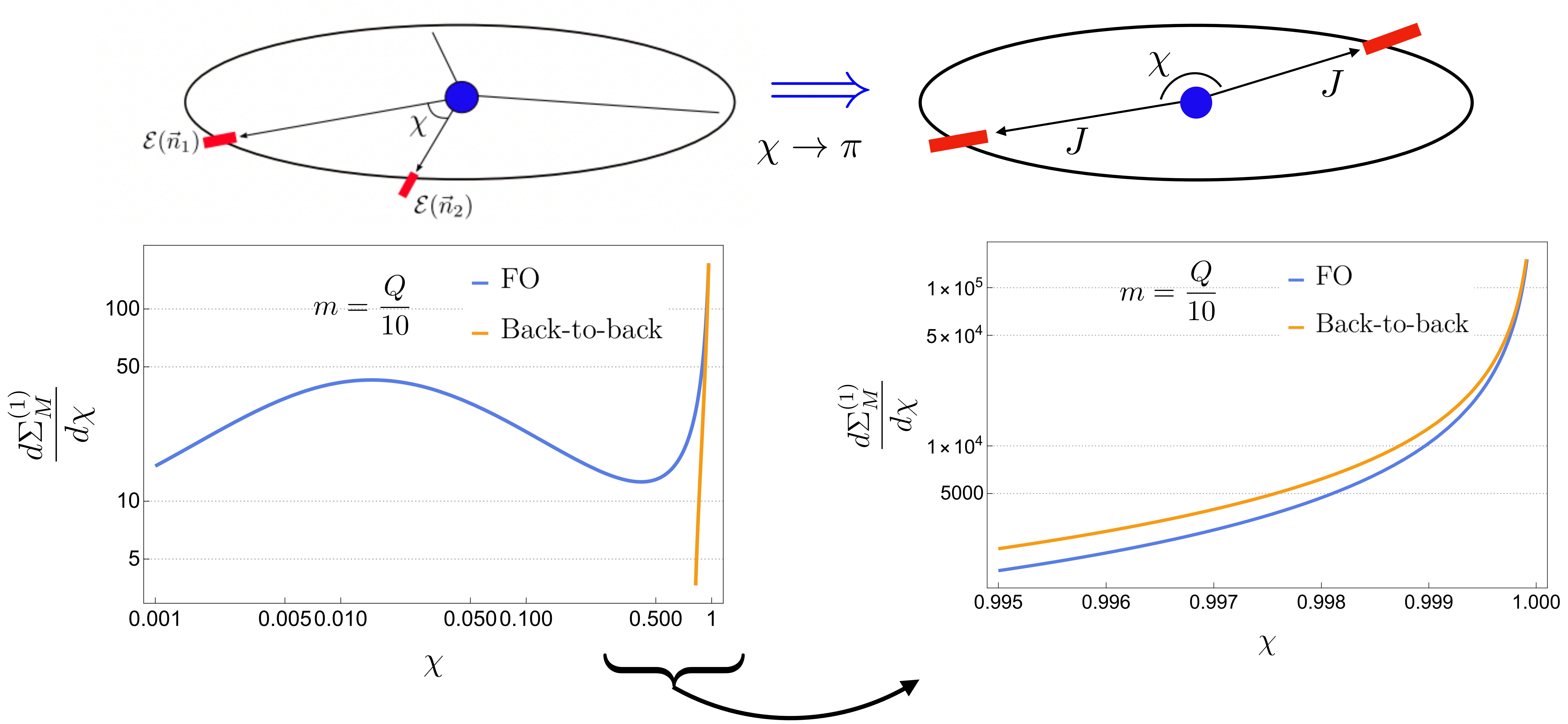
Disfavored contribution

$$+\int d\tau\, \tau \sum_j \mathcal{I}_{Qj}(\tau, b_\perp, m,\mu,\nu) \boxed{\sum_h \int dz\, z\, D_{j\to h}(z,\mu)} = 1$$

Light hadron contribution

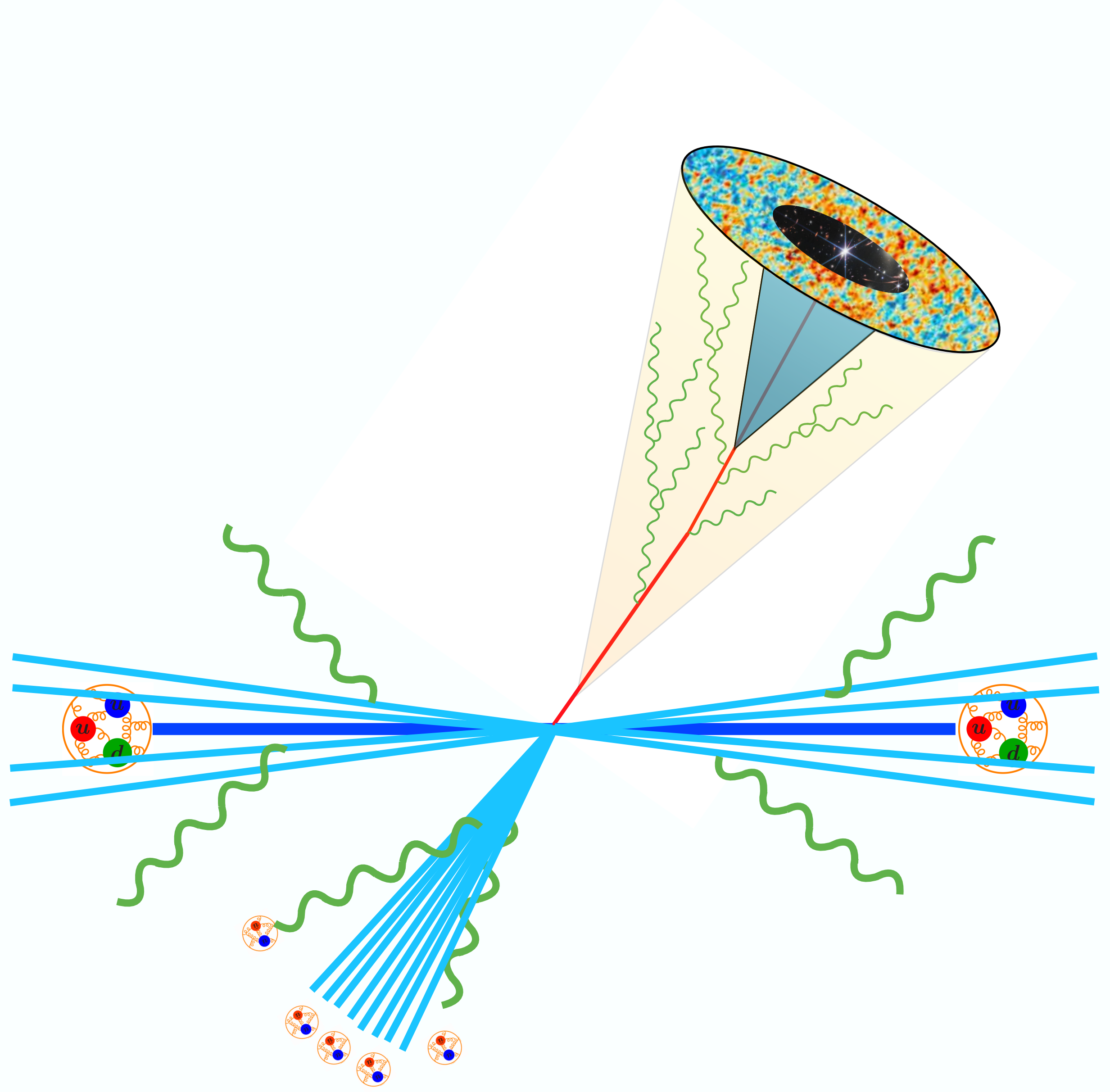
$$+\,\mathcal{O}\left(\frac{\Lambda_{\rm QCD}}{m}\right) + \mathcal{O}\left(\frac{\Lambda_{\rm QCD}}{Q\sqrt{1-z}}\right)$$

BACK-TO-BACK ENERGY-ENERGY CORRELATORS WITH MASS

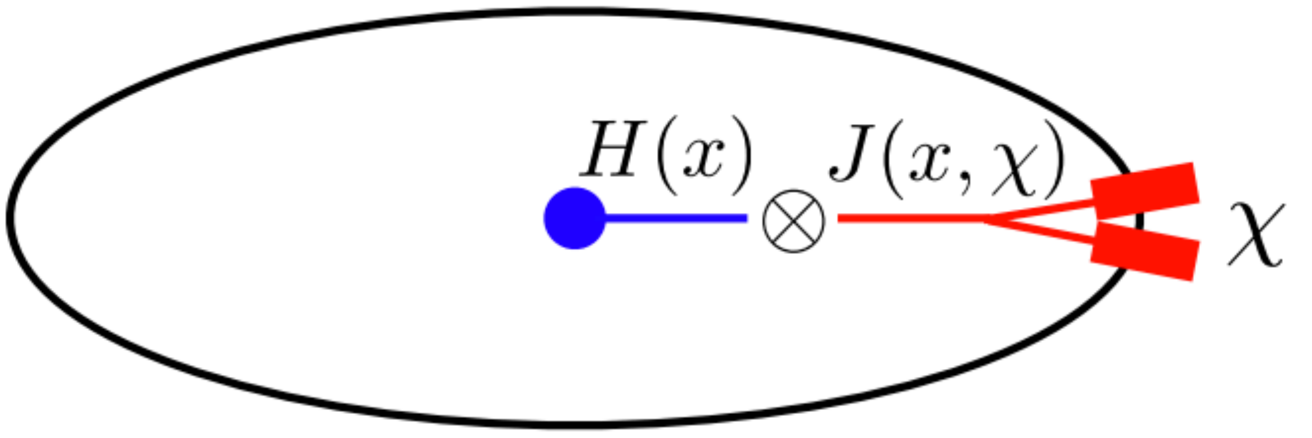


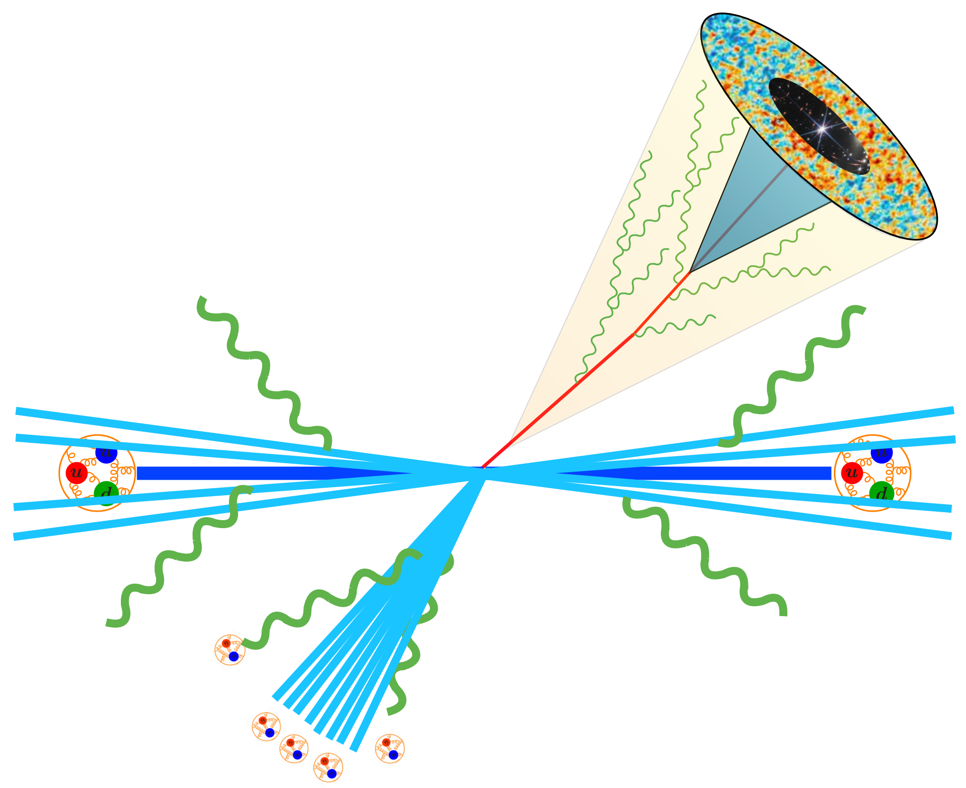
- **One can see that the fixed order and factorization results with finite mass agrees in the back-to-back region!**

COLLINEAR ENERGY-ENERGY CORRELATORS WITH MASS



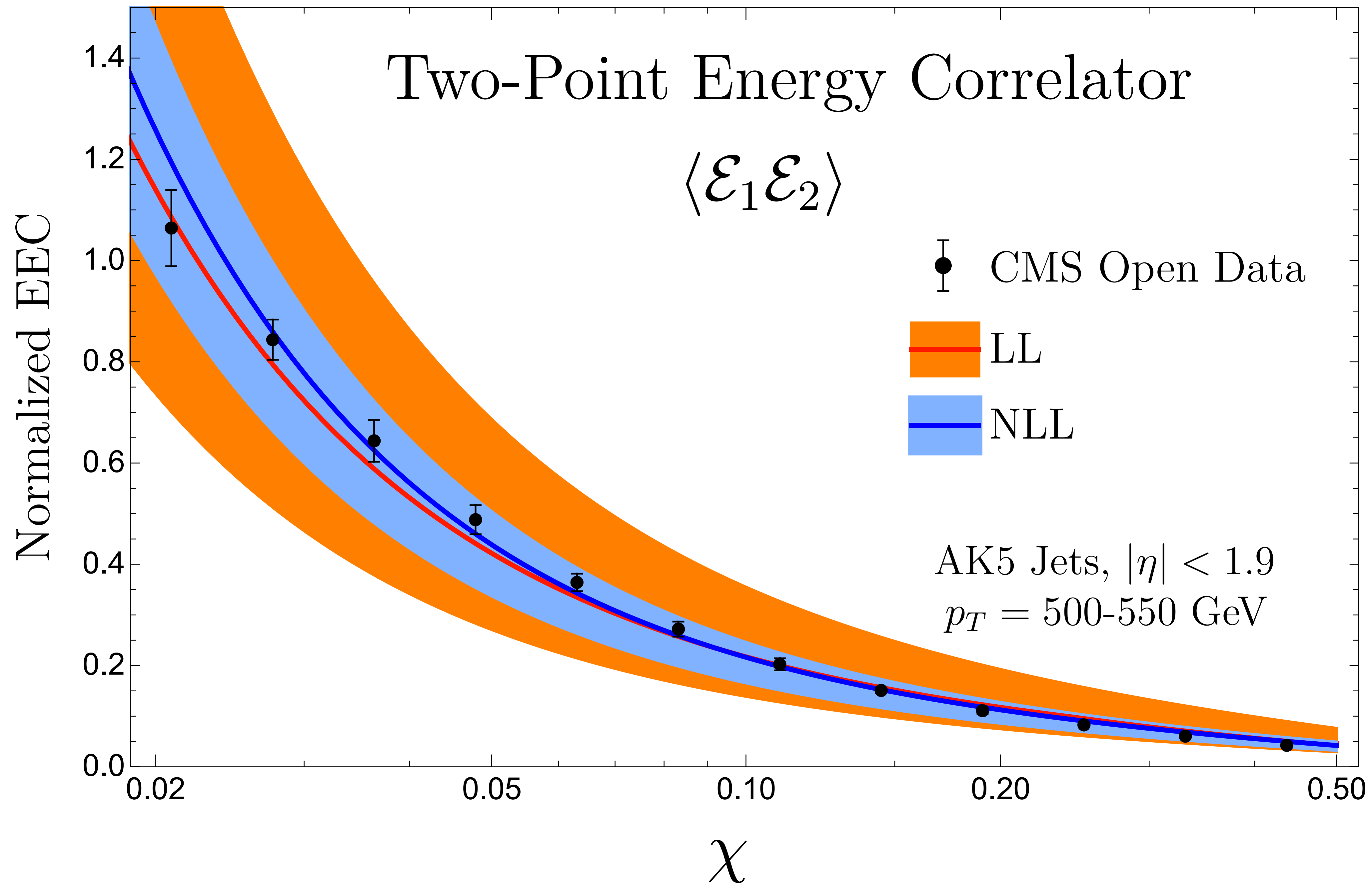
COLLINEAR ENERGY-ENERGY CORRELATORS WITH MASS



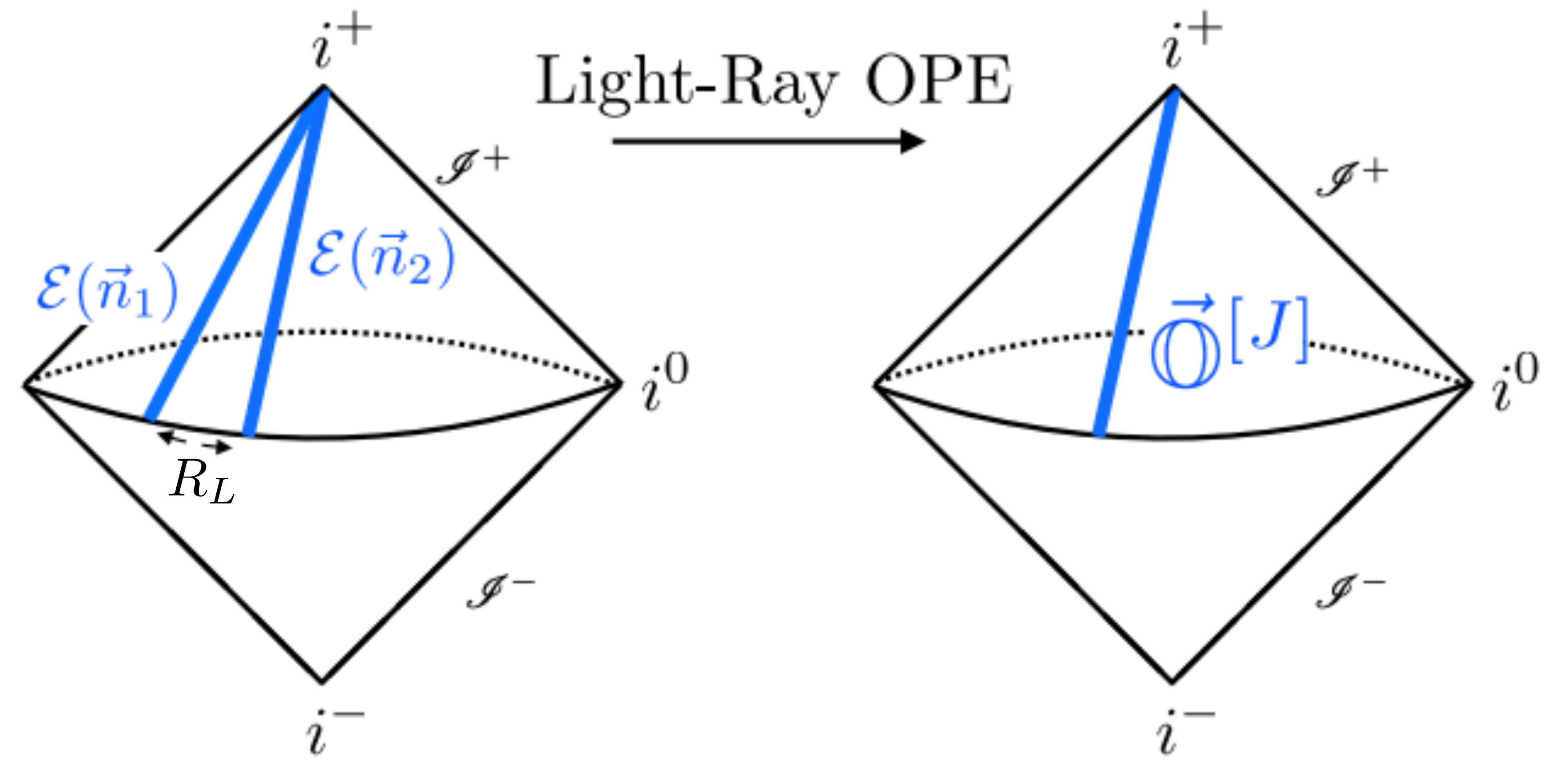


$$\Sigma\left(\chi, p_T^2, m_Q, \mu\right) = \int_0^1 dx\, x^2 \vec{J}\left(\chi, x, m_Q, \mu\right) \cdot \vec{H}\left(x, p_T^2, \mu\right)$$

Collinear dynamics factorize identically for different collider environment



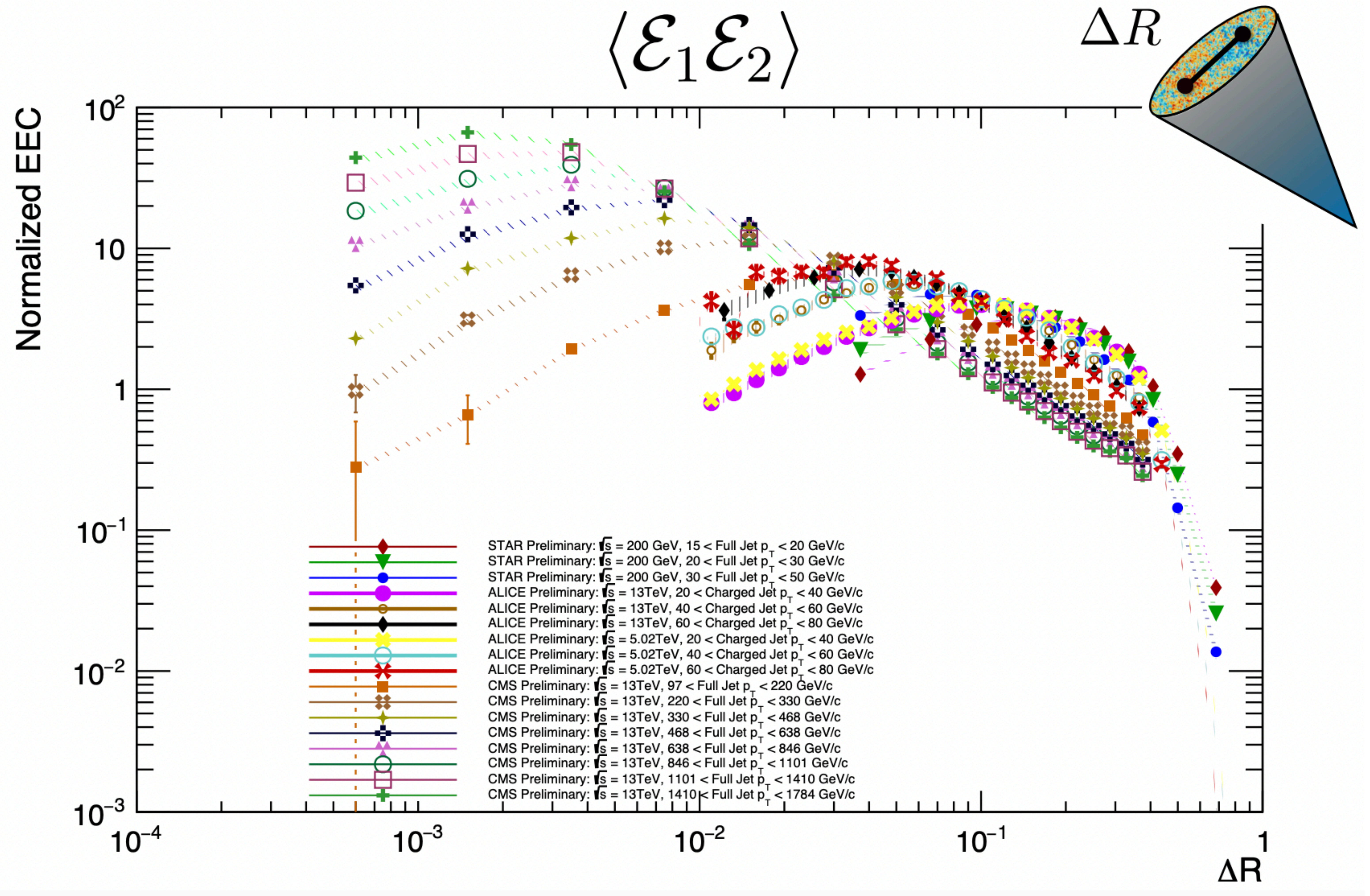
$\Longleftrightarrow$



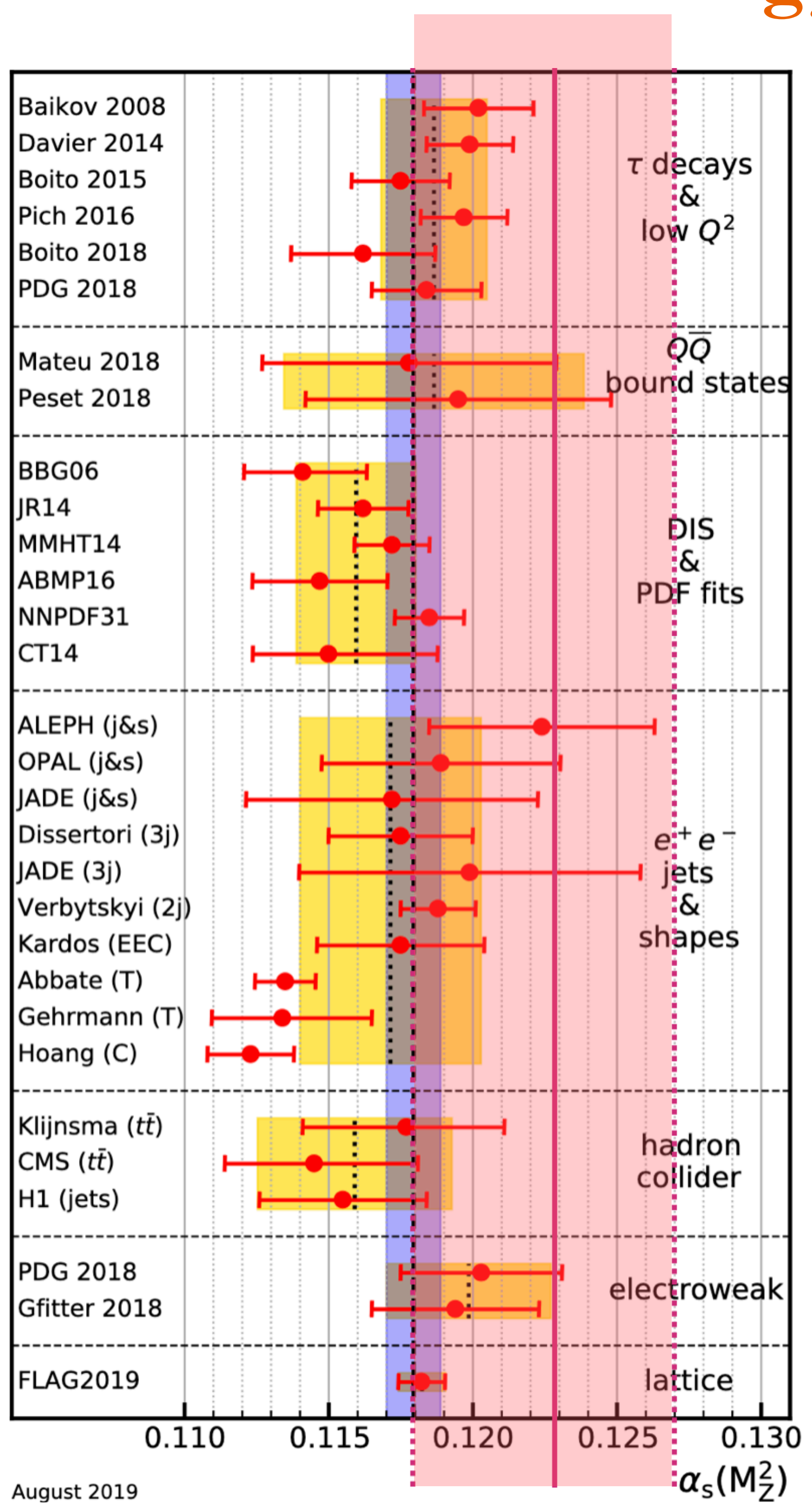
$$\mathcal{E}\left(\hat{n}_1\right) \mathcal{E}\left(\hat{n}_2\right) = \frac{1}{\theta^{2(1-\gamma_3)}} \sum \vec{\mathbb{O}}_i^{J=3}\left(\hat{n}_1\right)$$

- The universal scaling behavior is given by the Light-Ray OPE in the conformal region.

COLLINEAR ENERGY-ENERGY CORRELATORS WITH MASS



Energy Correlators in Jet



CMS Collaboration `23

$$\alpha_s(m_Z) = 0.1229^{+0.0040}_{-0.0050} = 0.1229^{+0.0014(stat.)+0.0030(theo.)+0.0023(exp.)}_{-0.0012(stat.)-0.0033(theo.)-0.0036(exp.)}$$

major source

Covariance matrix

QCD scale of NNLL$_{\text{approx}}$

Neutral hadron energy scale

Uncertainty ~ 4%, most precise from jet-substructure measurement to date

- **Universal scaling measured in real data from ALICE, CMS, and STAR from 15 GeV to 1784 GeV!**

COLLINEAR ENERGY-ENERGY CORRELATORS WITH MASS

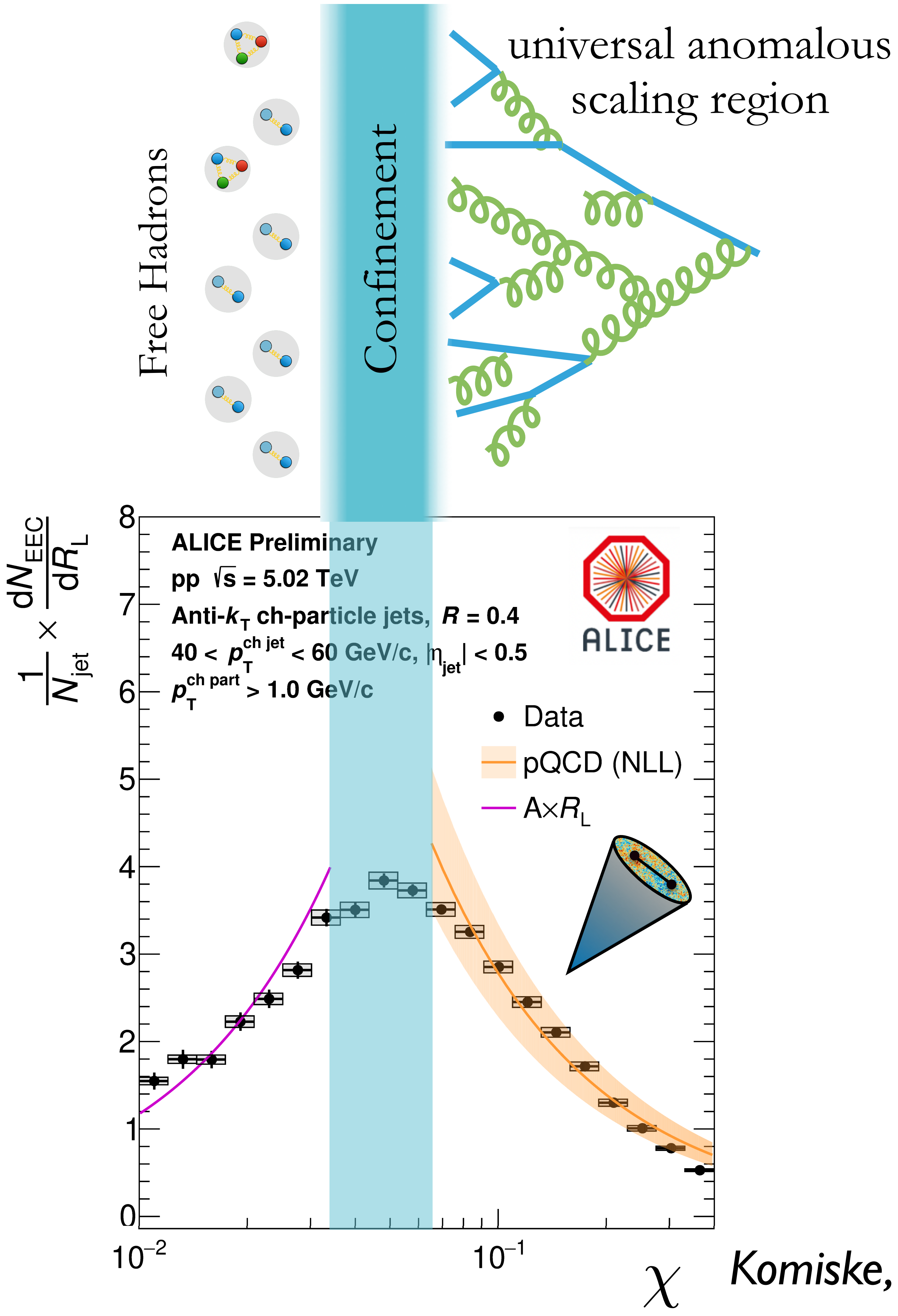
- **Energy correlators allow the hadronization process to be directly imaged inside high energy jets: transition from interacting quarks and gluons and free hadrons is clearly visible!**

Free hadrons

$$\frac{d\sigma}{d\theta^2} = \text{const}$$

$$\frac{d\sigma}{d\theta} = \text{const} \times 2\theta$$

EEC gives angular scale $\mu \sim p_T \theta_{ij}$



Interacting quarks and gluons

$$\mathcal{E}(\hat{n}_1)\,\mathcal{E}(\hat{n}_2) \sim \sum \theta^{\tau_i - 4} \mathbb{O}_i(\hat{n}_1)$$

Hofman, Maldacena, `08

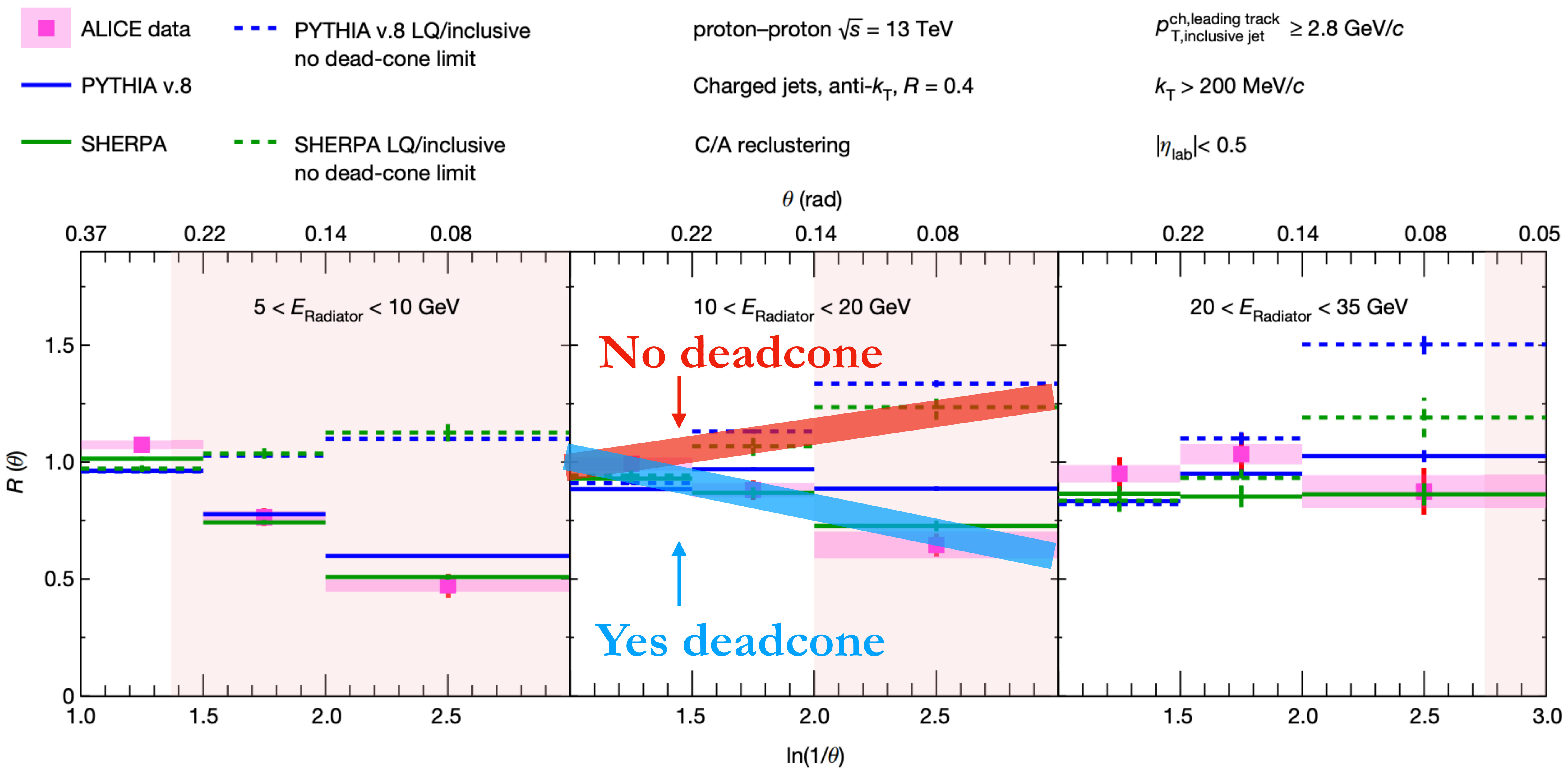
KL, Meçaj, Moult `22
Komiske, Moult, Thaler, Zhu `22

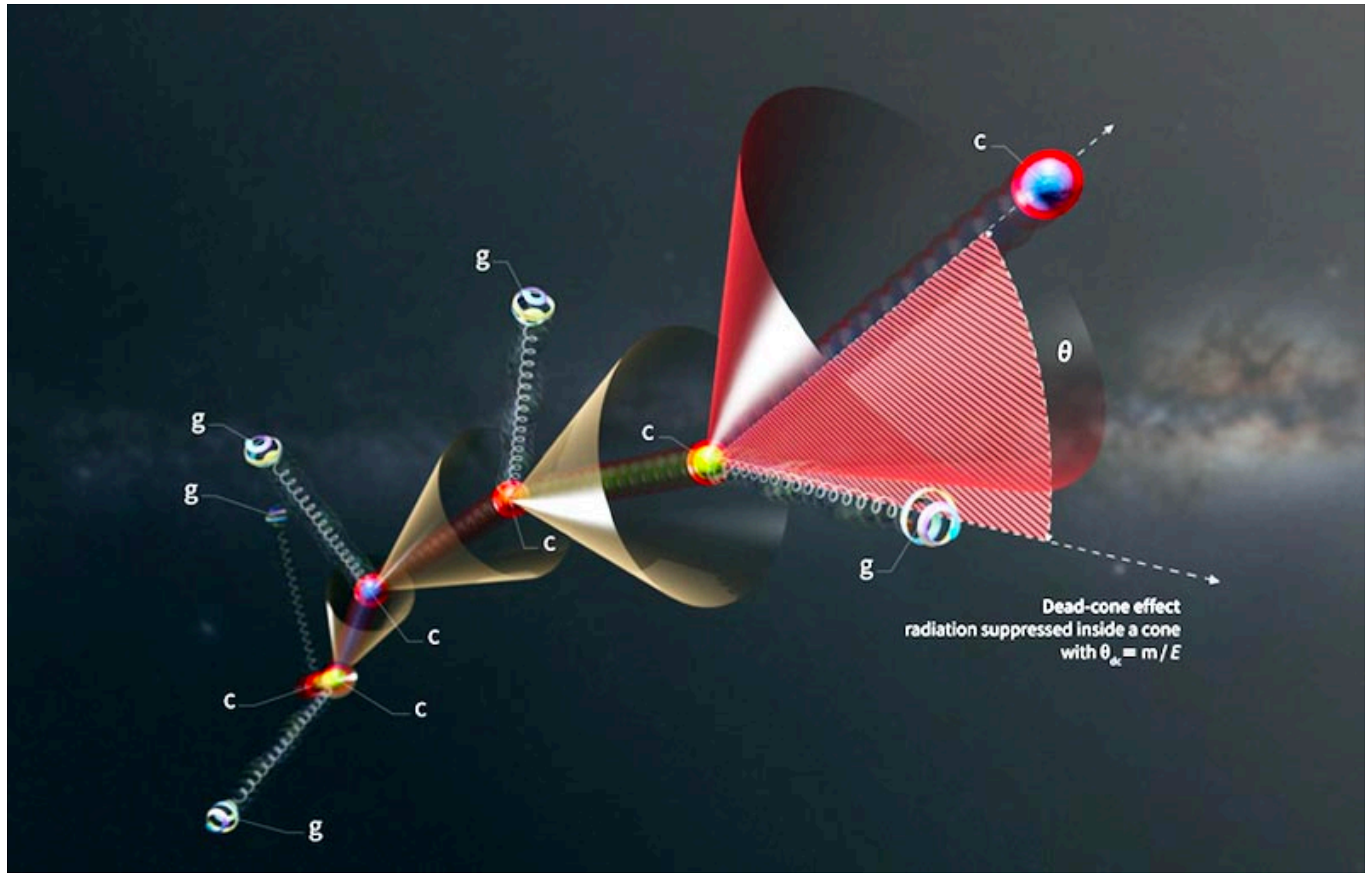
VERY FIRST DIRECT DETECTION OF DEADCONE

- **Fundamental predictions of our gauge theory— directly observed for the very first time last year!**

[See **Nima**'s talk]
[See also **Ibrahim**'s talk]

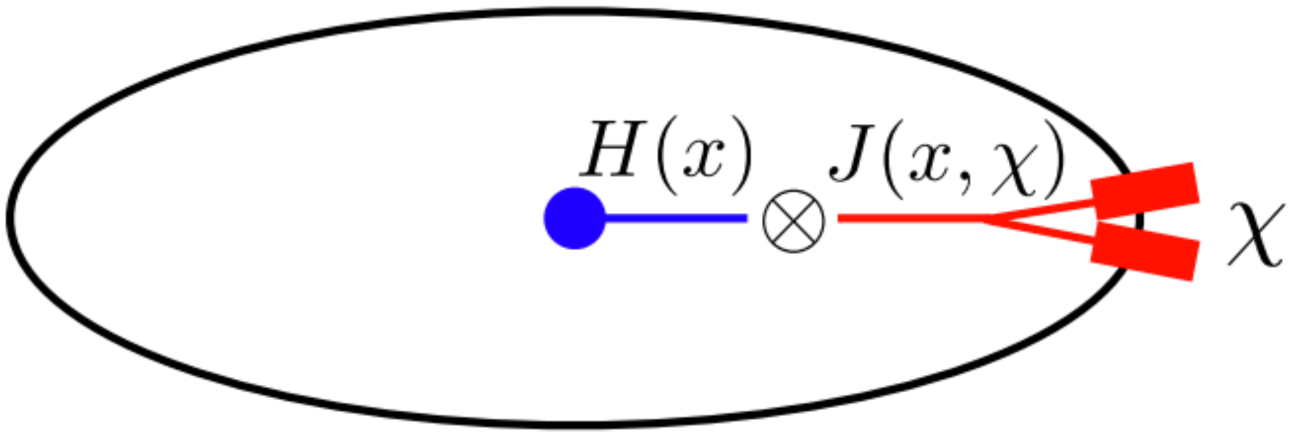
nature

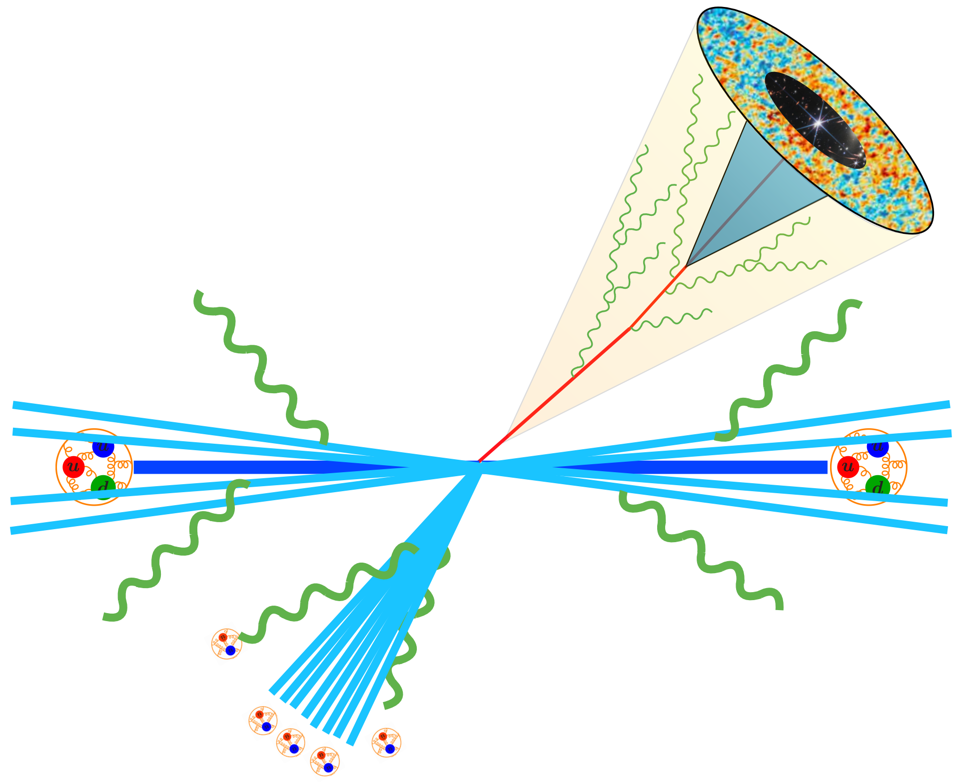




[ALICE 2022]

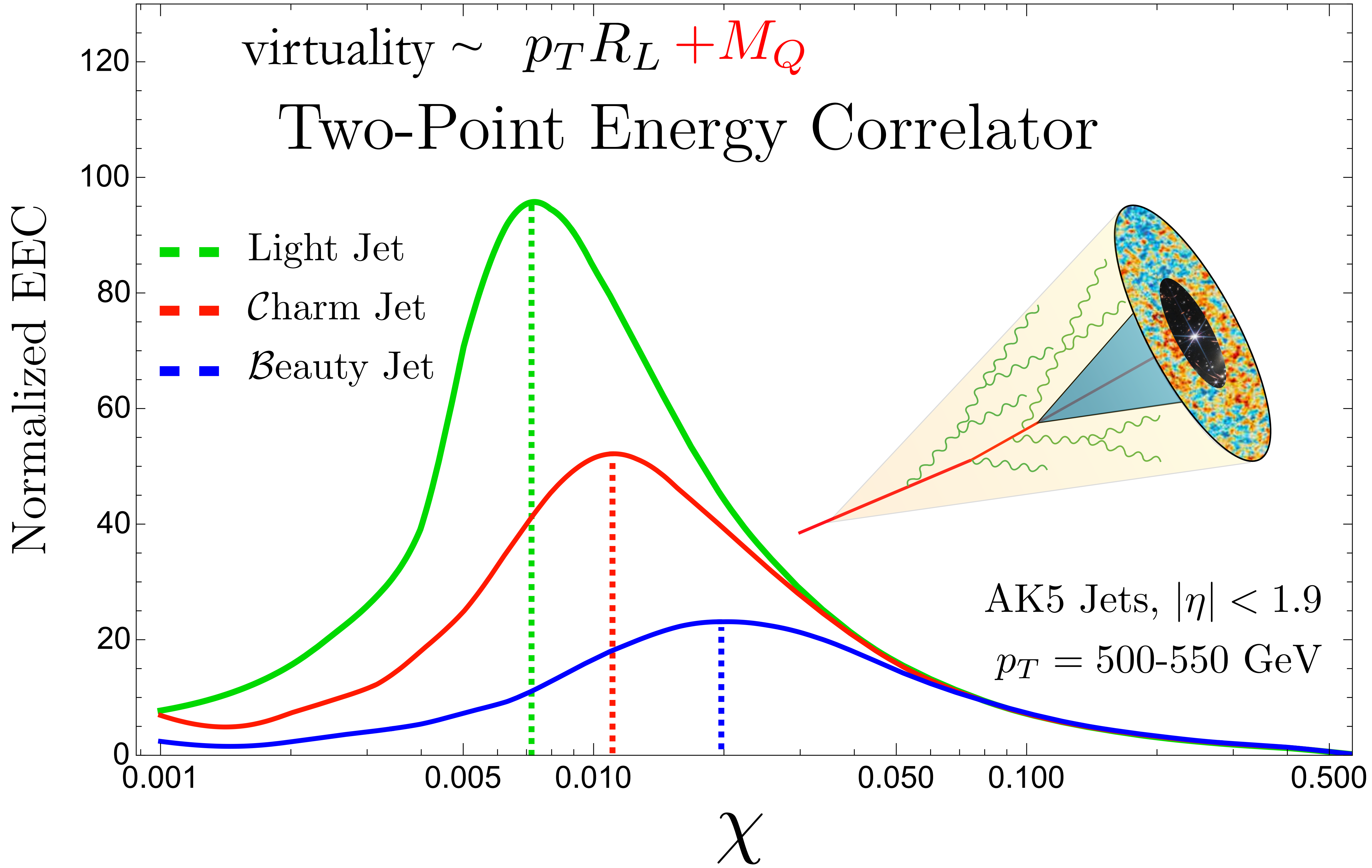
COLLINEAR ENERGY-ENERGY CORRELATORS WITH MASS

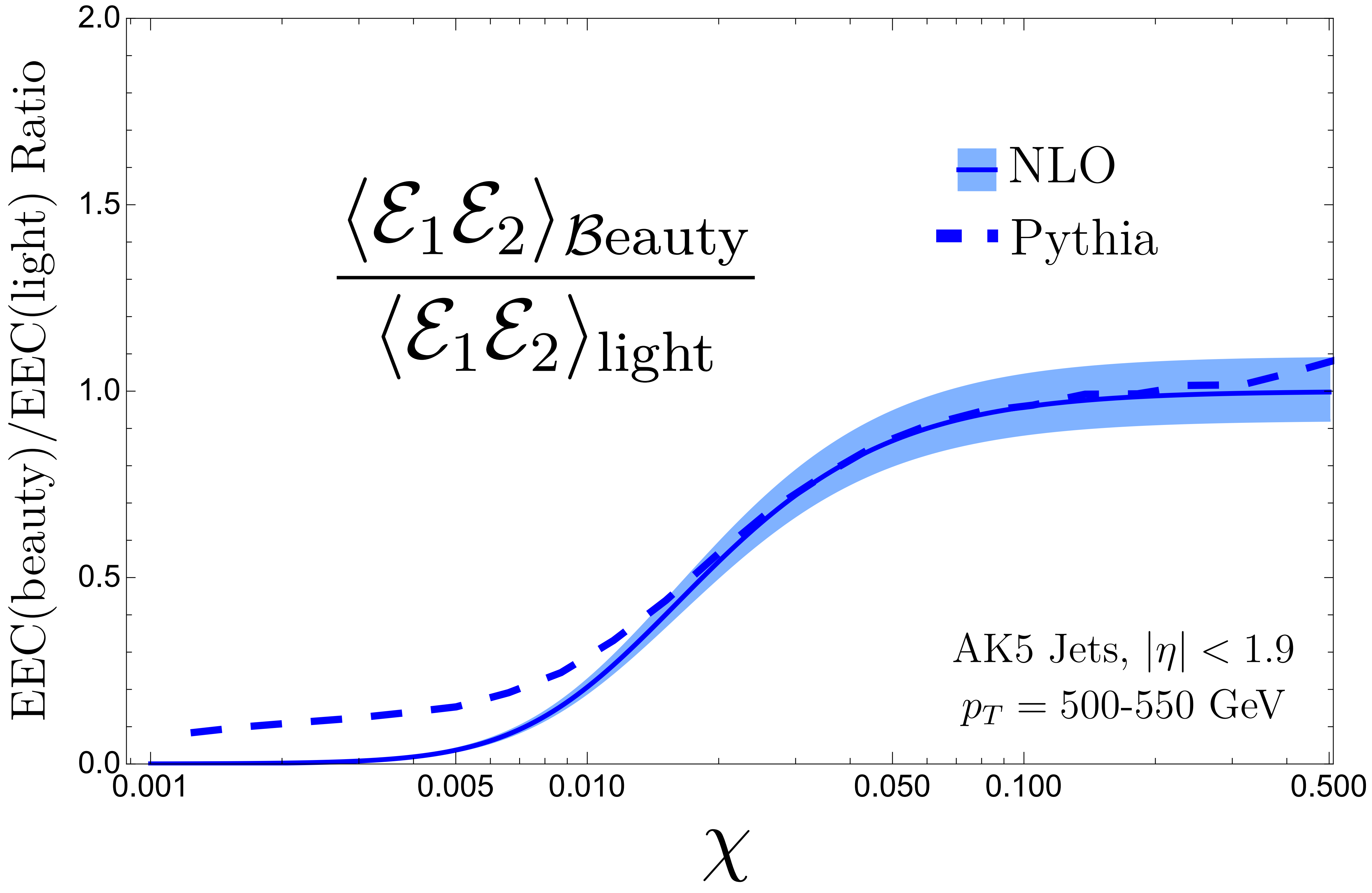




$$\Sigma\left(\chi, p_T^2, m_Q, \mu\right) = \int_0^1 dx\, x^2 \vec{J}\left(\chi, x, m_Q, \mu\right) \cdot \vec{H}\left(x, p_T^2, \mu\right)$$

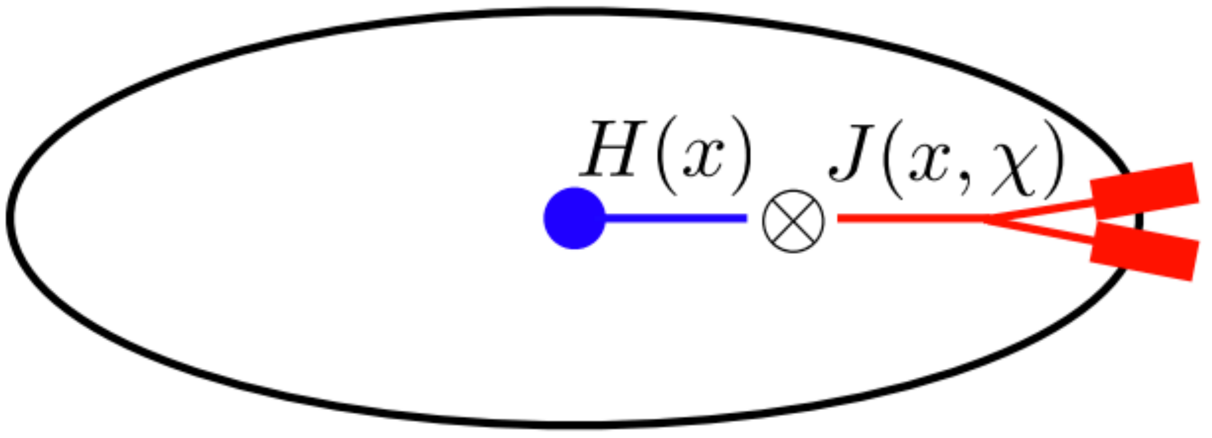
Collinear dynamics factorize identically for different collider environment

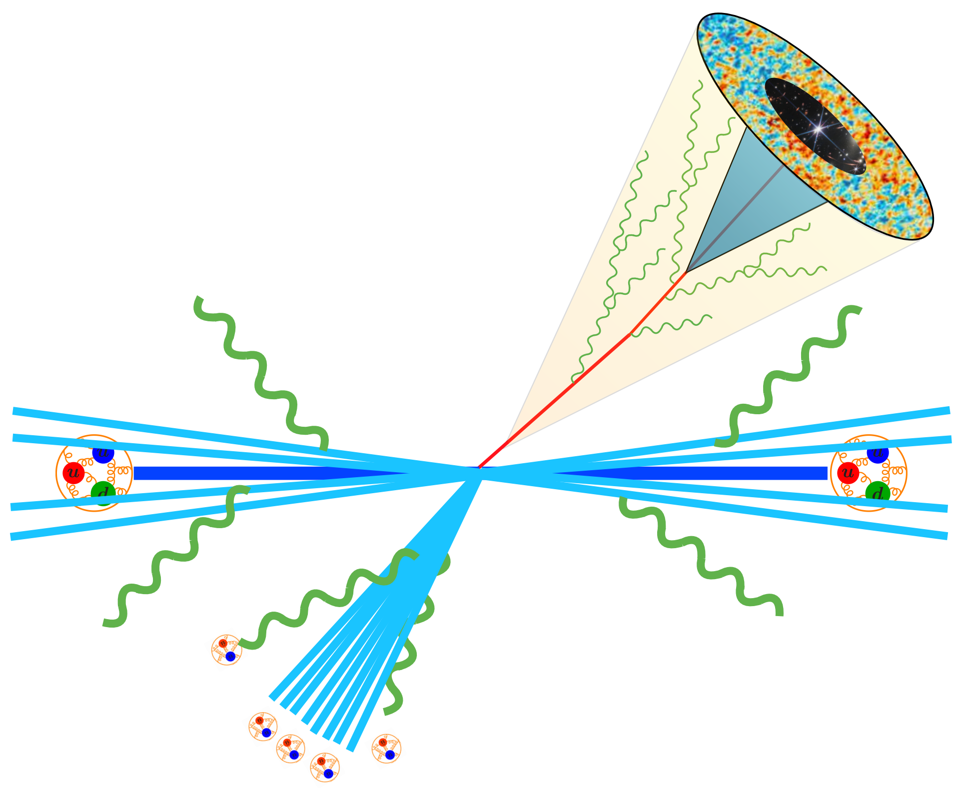




- Presence of the mass forces heavy flavor hadrons to be formed at an earlier time. This is clearly probed via energy correlators!

COLLINEAR ENERGY-ENERGY CORRELATORS WITH MASS

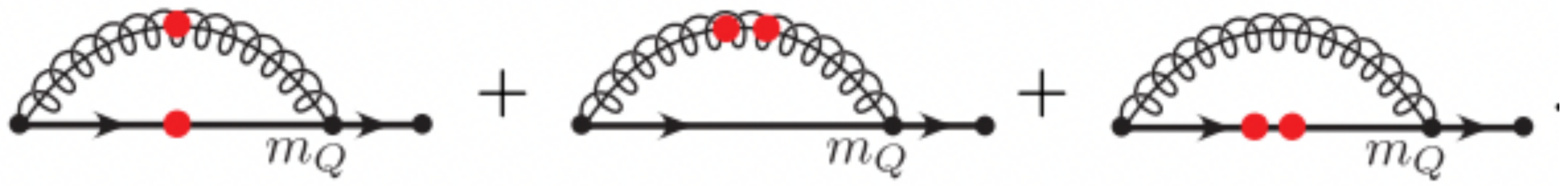




$$\Sigma\left(\chi, p_T^2, m_Q, \mu\right) = \int_0^1 dx\, x^2 \vec{J}\left(\chi, x, m_Q, \mu\right) \cdot \vec{H}\left(x, p_T^2, \mu\right)$$

Collinear dynamics factorize identically for different collider environment

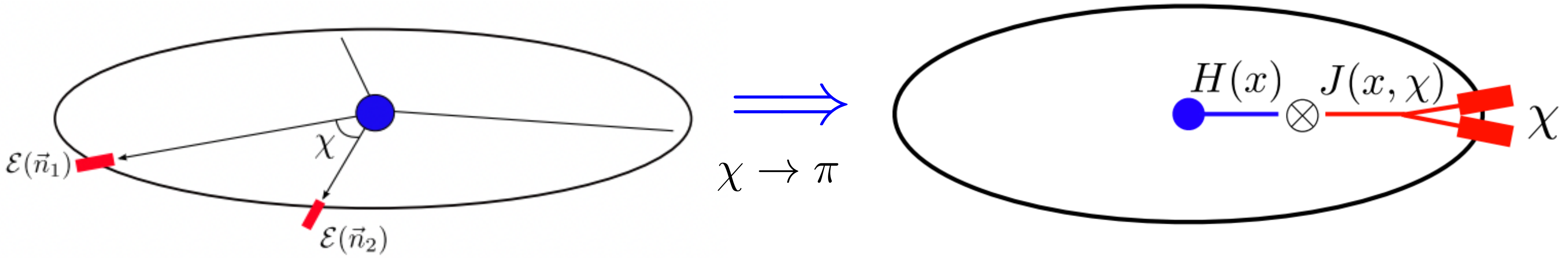
At NLO, $J_Q\left(\chi, m_Q\right)$

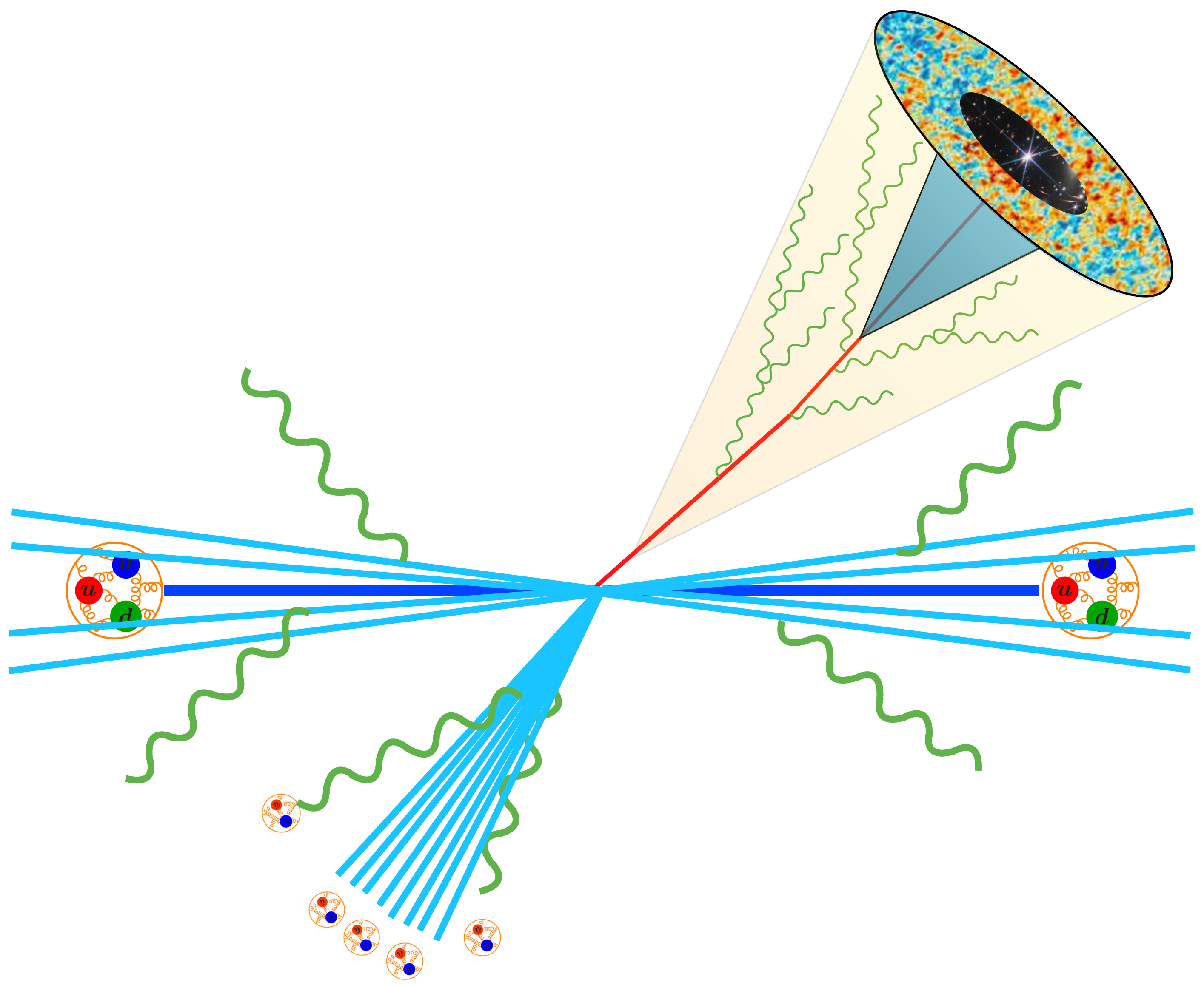


$$J_Q|_{\chi\neq 0}\left(\chi, m\right) = \frac{\alpha_s C_F}{4\pi}\left\{\left[\delta^4 + 4i\delta^3 - 2\delta^2 - 3\right] \ln\left(\frac{i\delta}{1+i\delta}\right) + \frac{1}{2}\left(9\delta^2 - \frac{31}{6}\right)\right\} + c.c$$

where $\delta = \dfrac{m}{p_T \chi}$

COLLINEAR ENERGY-ENERGY CORRELATORS WITH MASS





Making contact with the general angle result:

$$\left.\frac{d\sigma}{dz}\right|_{Qg+\bar{Q}g}^{z\to 0} = \mathrm{LP} + \mathrm{NLP} + \mathrm{NNLP} + \cdots$$

$$\mathrm{LP} \propto \frac{1}{2z}\left\{\left[-4(i\delta)^4 + 12(i\delta)^3 - 4(i\delta)^2\right]\ln\left(\frac{i\delta}{1+i\delta}\right) + \left[-(i\delta)^4 + 4(i\delta)^3 - 2(i\delta)^2 + 3\right]\frac{1}{(1+i\delta)} + 9(i\delta)^2\right\} + c.c,$$

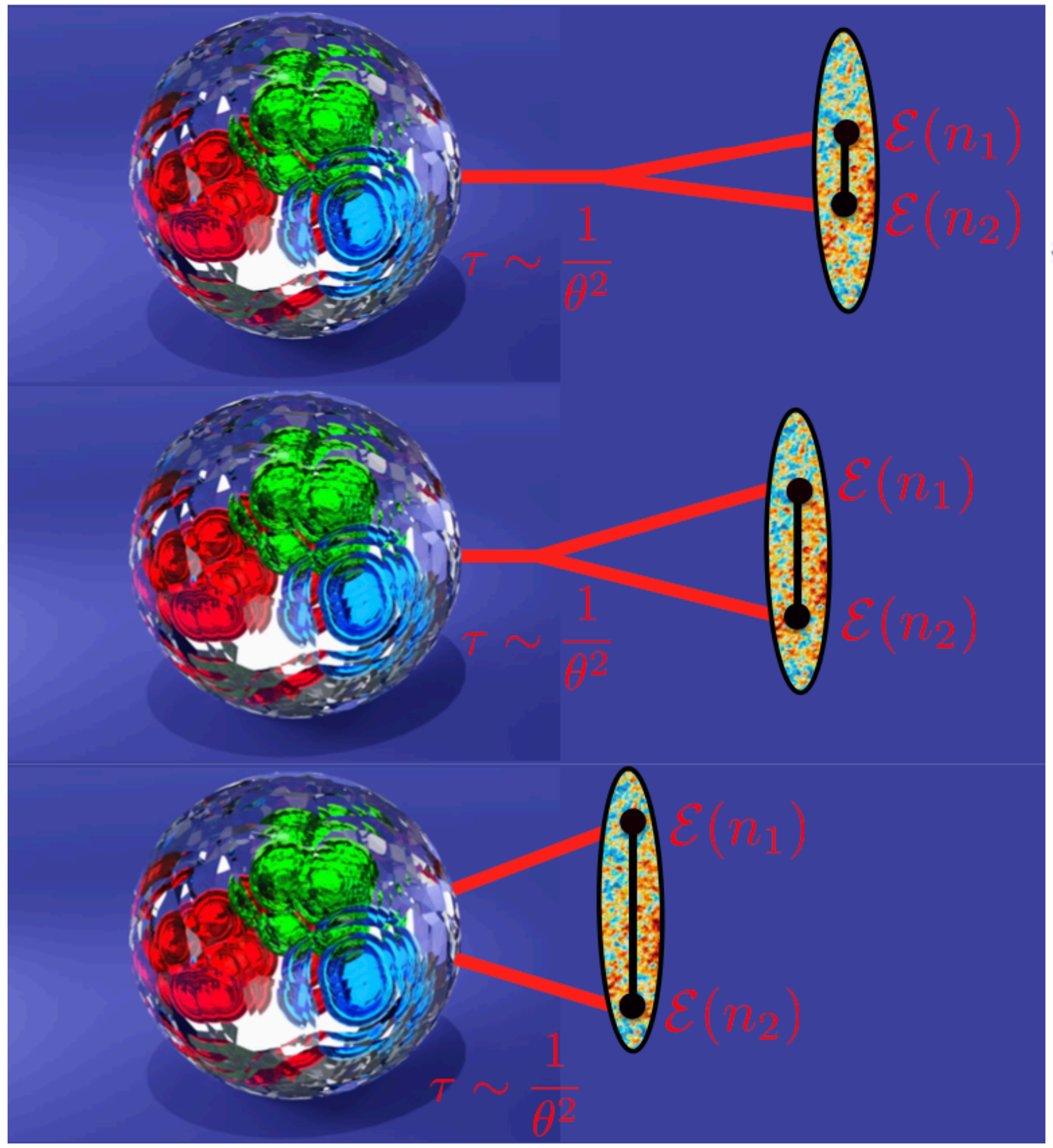
$$\mathrm{NLP} \propto \frac{-2\left(300\delta^8 + 530\delta^6 + 208\delta^4 + 41\delta^2 - 27\right)}{15\left(\delta^2+1\right)^2} + \frac{15\delta^3\left(\delta^2+1\right)^2\left(6\delta\log\left(\frac{\delta^2+1}{\delta^2}\right) + i\left(1-20\delta^2\right)\log\left(\frac{\delta+i}{\delta-i}\right)\right)}{15\left(\delta^2+1\right)^2}$$

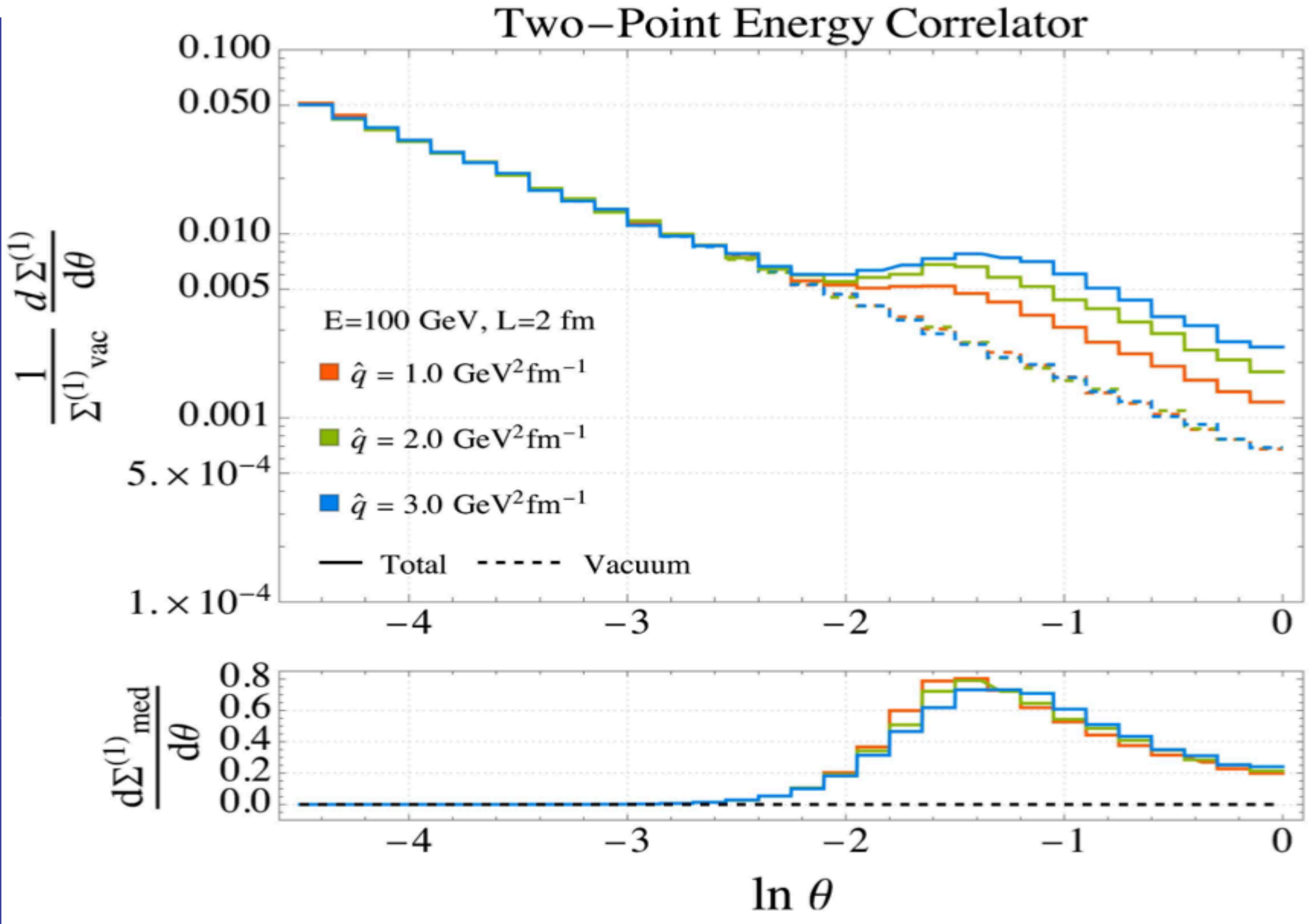
Leading power agrees and one can obtain higher power results from expanding the general angle result!

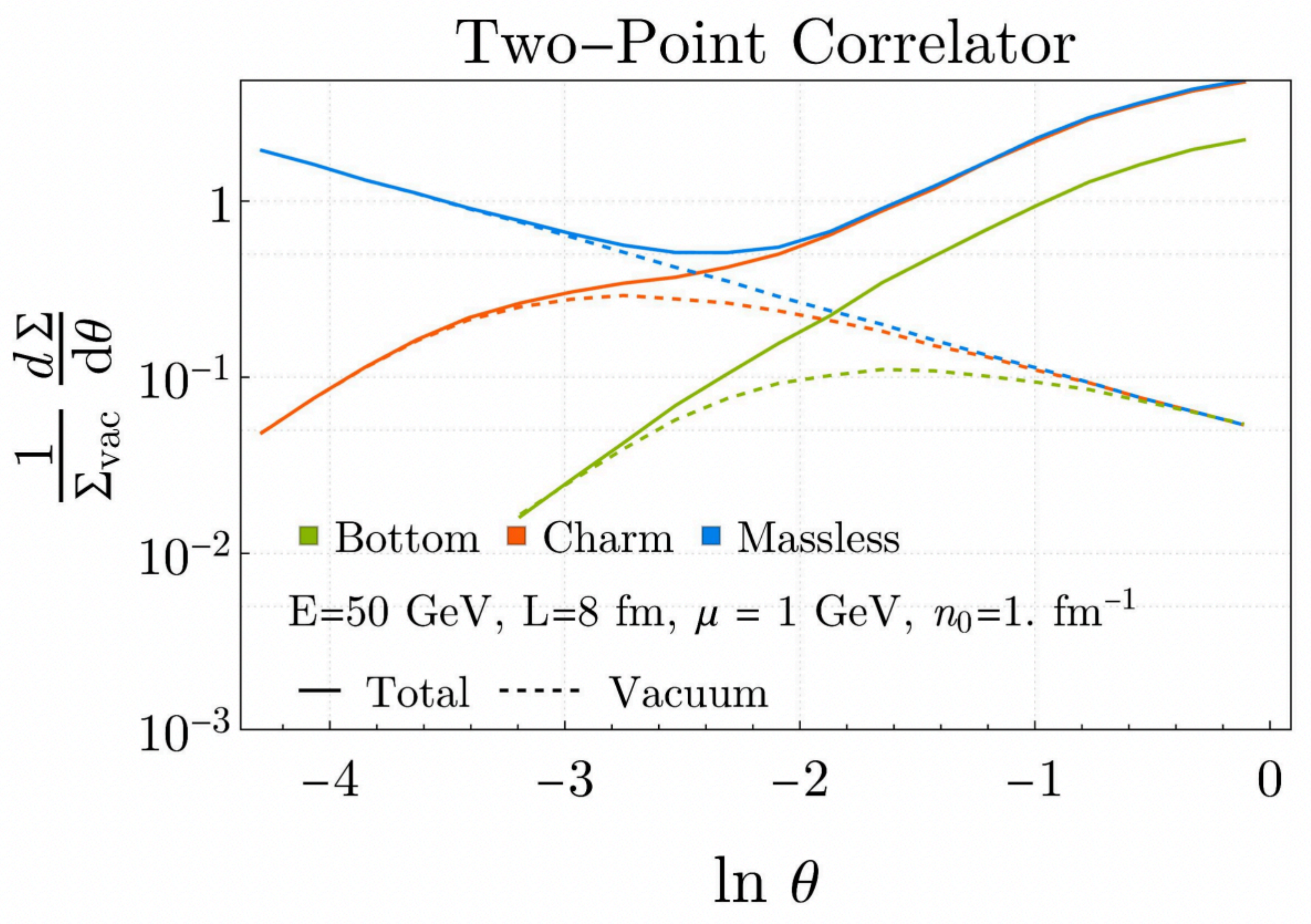
RESOLVING THE QGP USING ENERGY CORRELATORS

- The standard energy loss corresponds to the measurement of the one-point energy correlator
- Two-point energy correlators clearly identify the scale at which the energy loss occurs, and gives robust prediction across different models!

EEC gives angular scale $\mu \sim p_T\theta_{ij}$







Andres, Dominguez, Holguin, Kunnawalkam Elayavalli, Marquet, Moult `22
Andres, Dominguez, Holguin, Marquet, Moult `23
Barata, Mehtar-Tani `23

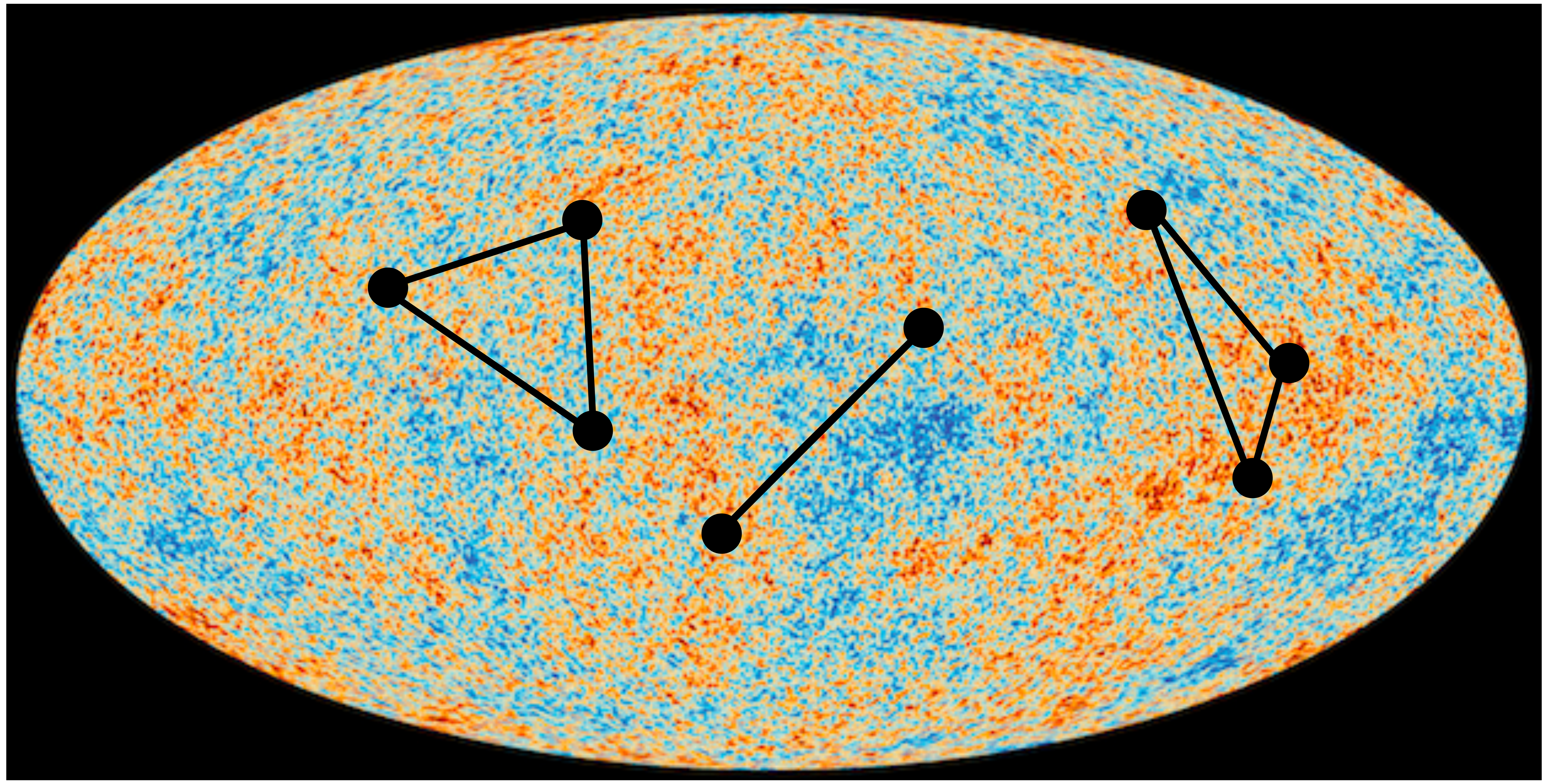
PROBING HIGHER POINT STRUCTURE

- Higher-point correlators probe **more detailed** aspects of interactions.

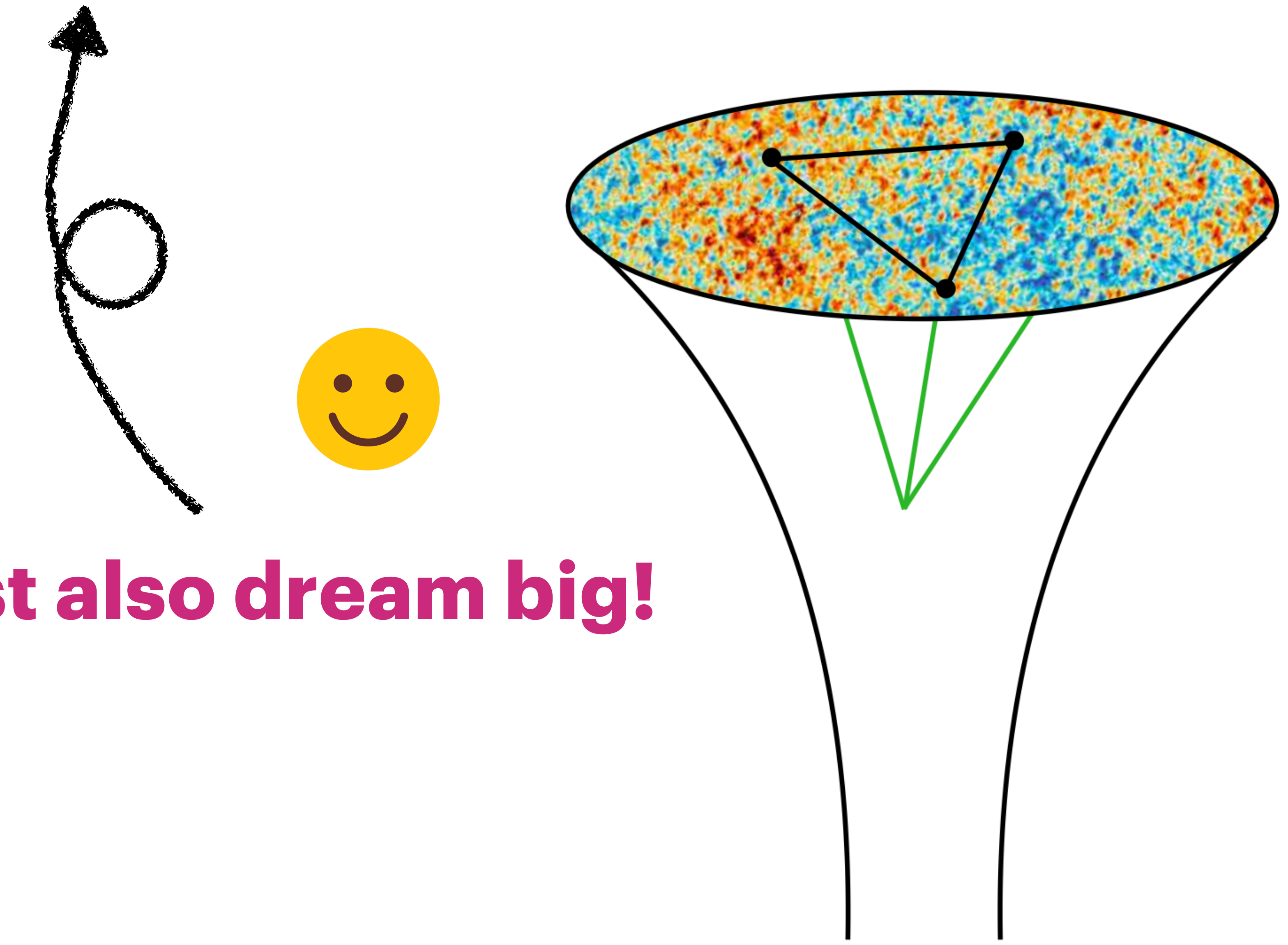
- Hunting for non-gaussianities to distinguish the models of inflation.

  Maldacena `02, Komatsu `10
  Cabass, Pajer, Stefanyszyn, Supel `21,…

  Extremely interesting physics detail hiding under the 1 part in 100000 non-gaussianity in CMB!



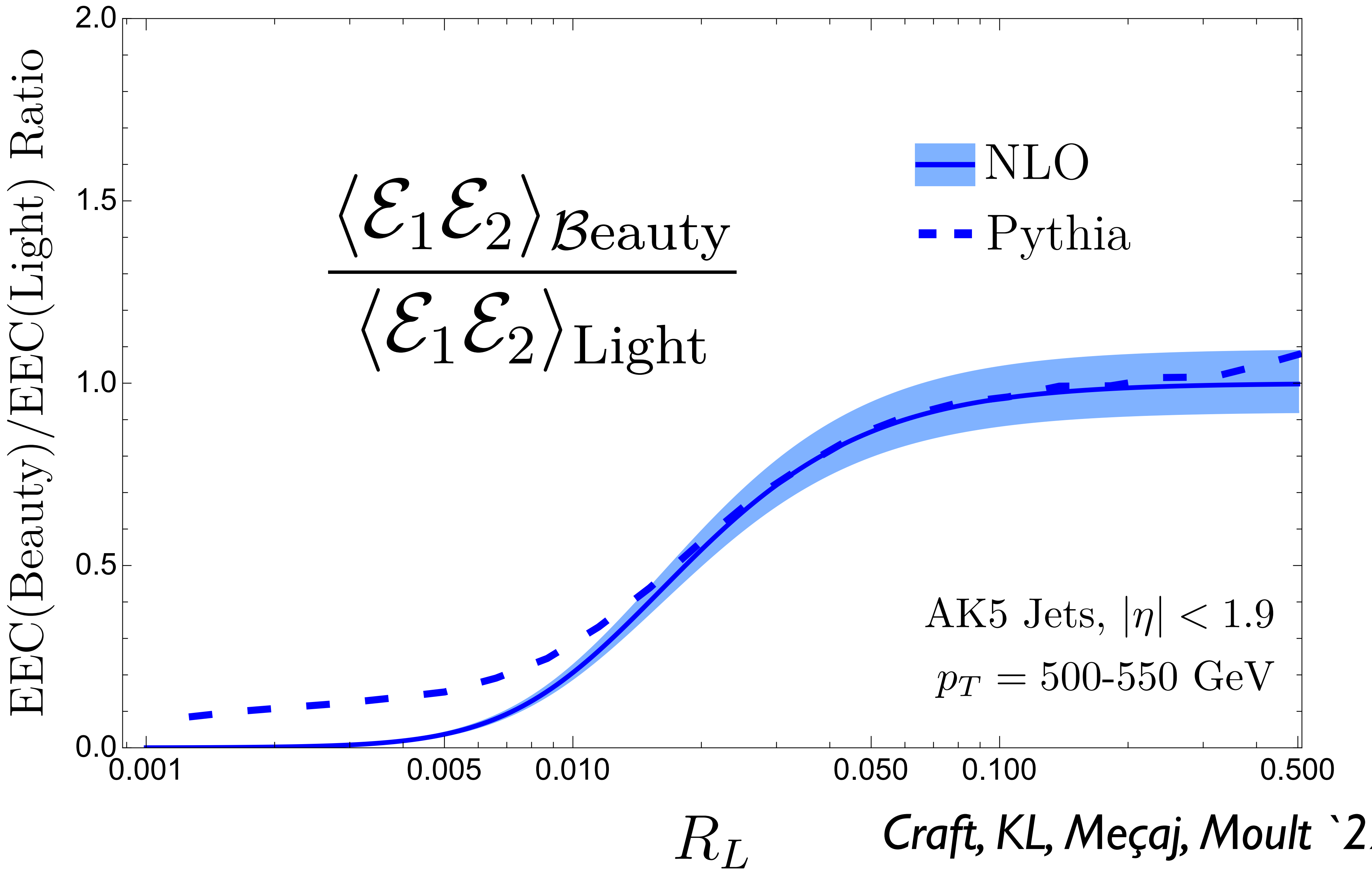
We must also dream big!



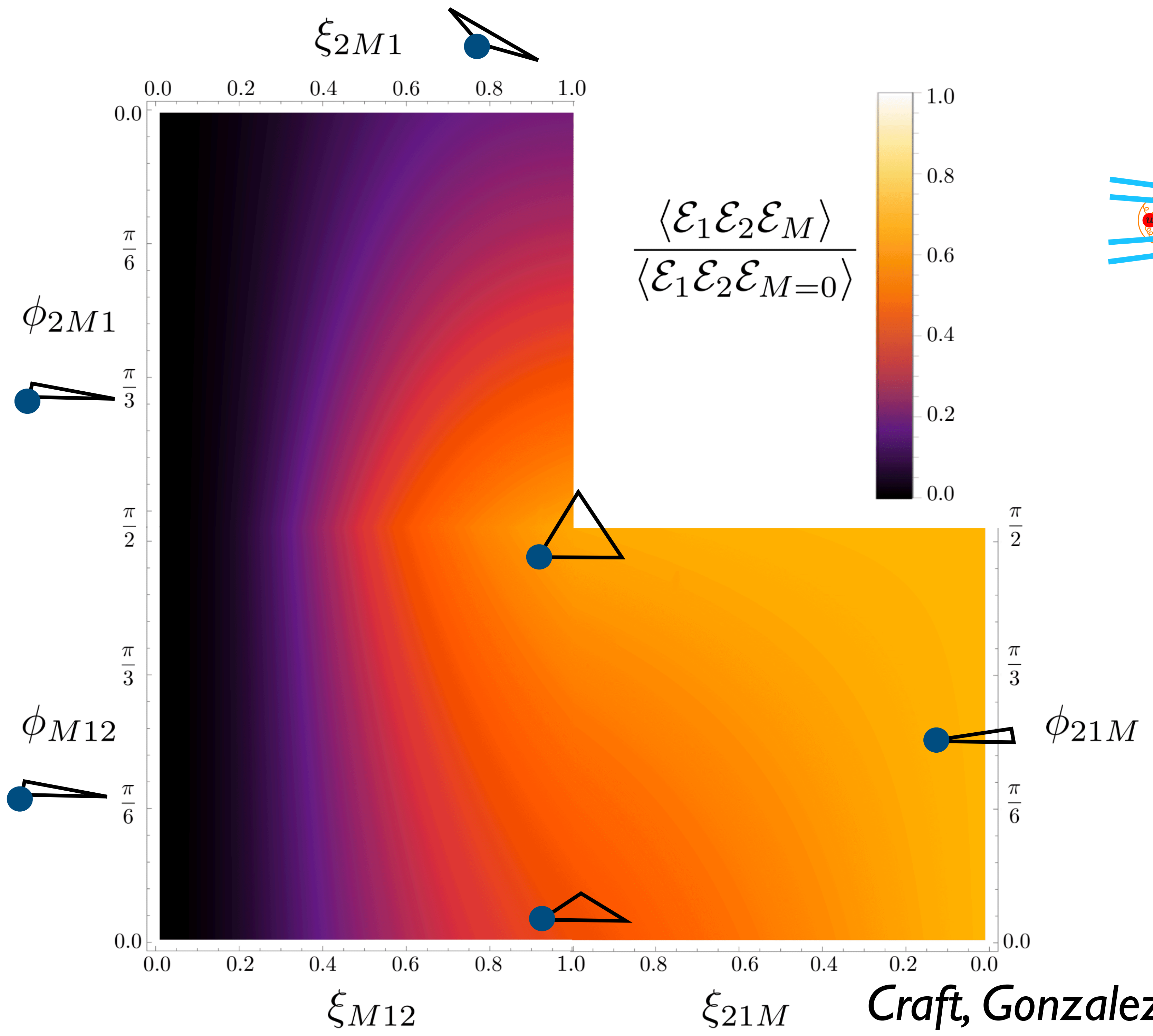
BEAUTIFUL AND CHARMING ENERGY CORRELATORS

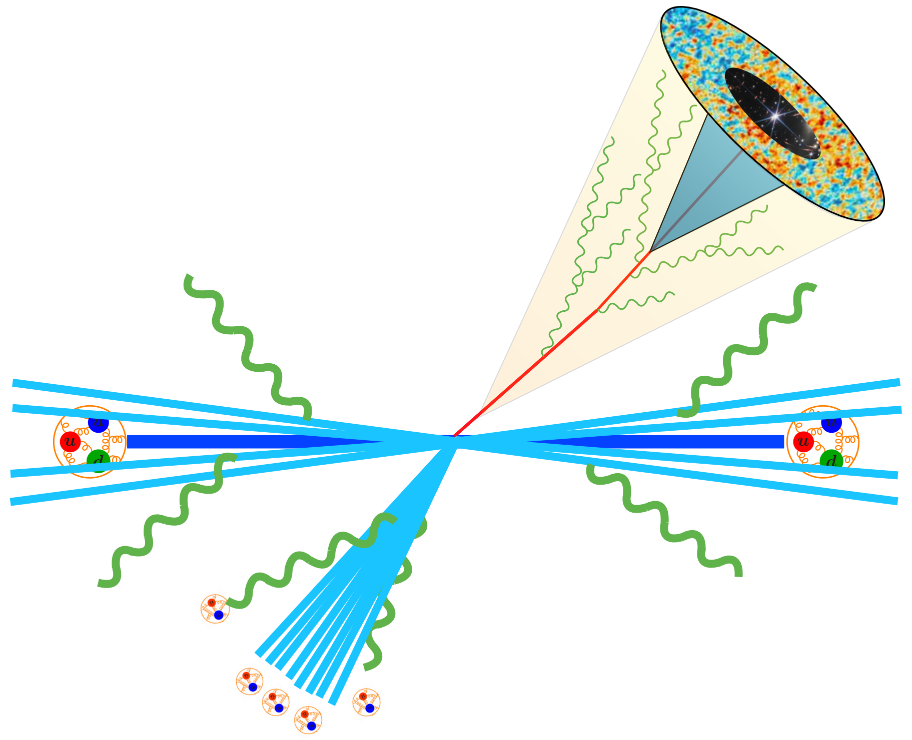
- **One can statistically measure the gluon suppression (dead cone) within the heavy jets as compared to the light jets by taking the ratio of energy correlators.**

scale knob  **EEC gives angular scale** $\mu \sim p_T \theta_{ij}$



Craft, KL, Meçaj, Moult `22





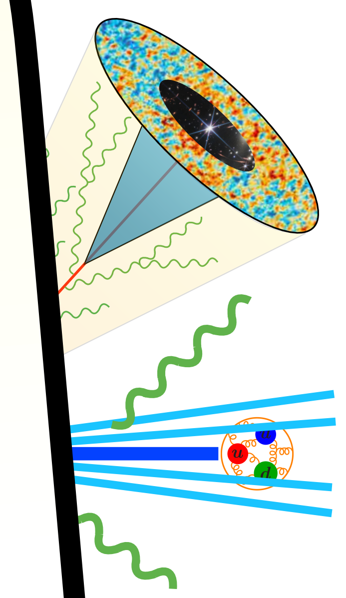
Craft, Gonzalez, KL, Meçaj, Moult `23

- **Higher-point structure provides nontrivial shape information**

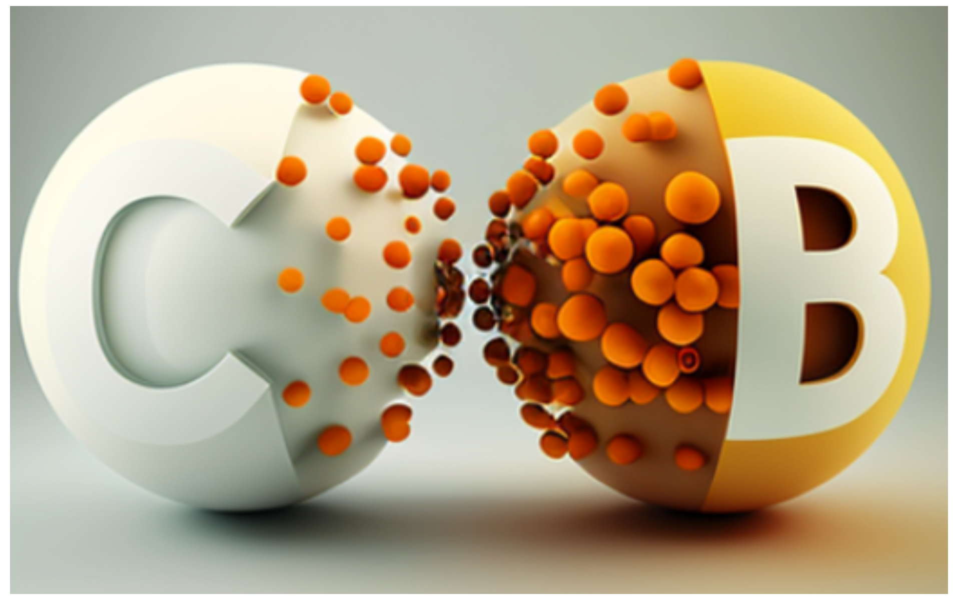
BEAUTIFUL AND CHARMING ENERGY CORRELATORS

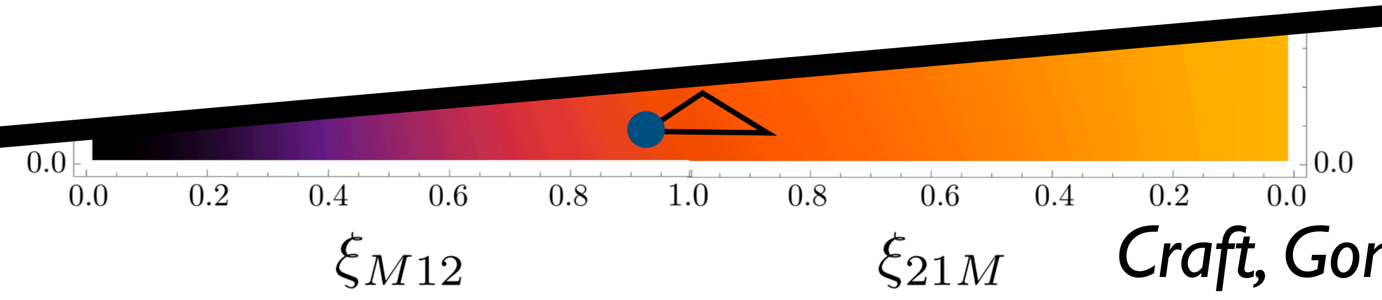
- **One can statistically measure the gluon suppression (dead cone) within the heavy jet as compared to the light jets by taking the ratio of**

scale kno



Beautiful and Charming future ahead with Energy Correlators!





Craft, KL, Meçaj, Moult `22

Craft, Gonzalez, KL, Meçaj, Moult `23

- **Higher-point structure provides nontrivial shape information**