Transverse Momentum Distributions of Heavy Hadrons and Polarized Heavy Quarks.

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Outline.

Motivation and introduction

- very short introduction to **t**ransverse **m**omentum **d**ependent distributions (TMDs)
- why are heavy quarks interesting?

Heavy TMD fragmentation functions (FFs)

- discuss heavy TMD FFs in two regimes: $\Lambda_{\rm QCD} \sim k_T \ll m$ and $\Lambda_{\rm QCD} \ll k_T \sim m$

Towards phenomenology

- $e^+e^- \to HHX$
- Semi-inclusive deep inelastic scattering (SIDIS)

Outlook

- heavy TMD FFs within Jets

Intro.

Transverse Momentum Dependent Distributions.

Transverse momentum dependent (TMD) factorization formalism [Collins '11]

- collinear factorization [Collins, Soper, Sterman '89]: describes longitudinal momentum distribution (1D)
- TMDs allow for extraction of **3D** structure of the hadronization cascade
- TMDs are universal across processes [Collins, Metz '04]
- natural power-counting with transverse momentum $k_T \ll Q$

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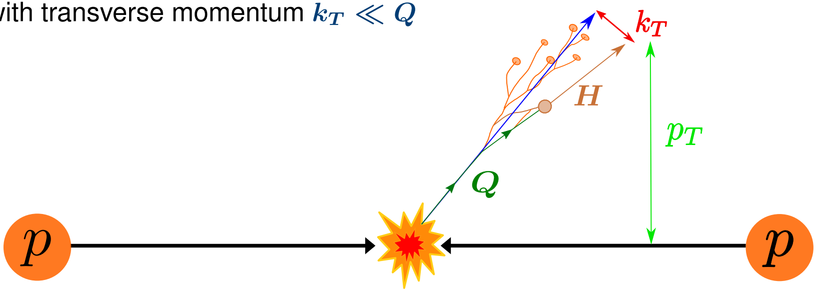
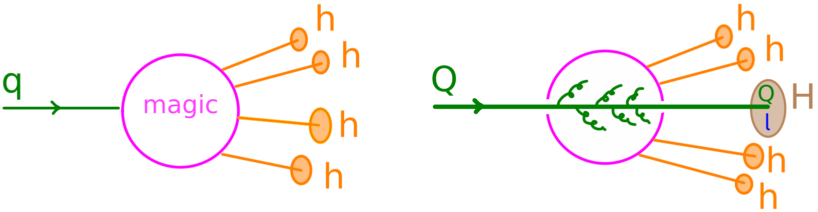


Heavy quarks.

Why are heavy quarks interesting?

- bottom and charm quarks have $m_b, m_c \gg \Lambda_{\text{QCD}}$
  - provides perturbative scale on otherwise non-perturbative dynamics of hadronization
- serve as static color source coupling to light degrees of freedom
- model independent prediction to tune shower to improve heavy flavor modeling
- **Ideal to study hadronization process**



Heavy TMD FFs.

Basics of TMD FFs.

- TMD quark-quark correlator describing fragmentation

$$\Delta_{H/Q}(z_H, b_\perp) = \frac{1}{2 z_H N_c} \int \frac{\mathrm{d}b^+}{4\pi} e^{ib^+(P_H^-/z_H)/2} \, \mathrm{Tr} \sum_X \langle 0 | W^\dagger(b) \, \psi_Q(b) | H X \rangle \langle H X | \bar{\psi}_Q \, W | 0 \rangle \, ,$$

- unpolarized TMD FF:

$$D_{1\,H/Q}(z_H, b_T) = \mathrm{tr}\Big[\frac{\not{\bar{n}}}{2} \Delta_{H/Q}(z_H, b_\perp) \Big]$$

- Collins TMD FF:

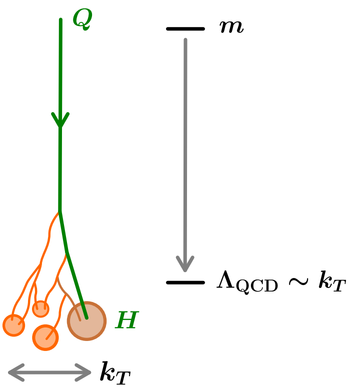
$$H_{1\,H/Q}^{\perp(1)}(z_H, b_T) = \mathrm{tr}\Big[\frac{\not{\bar{n}}}{2} \frac{\not{b}_\perp}{M_H b_T^2} \Delta_{H/Q}(z_H, b_\perp) \Big]$$

- in general: more TMD FFs if we allow polarized hadrons
- **today: focus on unpolarized TMD FF!**

Notation: momenta in terms of n^μ, $\bar{n}^\mu$ with $n^2 = \bar{n}^2 = 0$ and $n \cdot \bar{n} = 2$:
$p^\mu = \bar{n} \cdot p \, \frac{n^\mu}{2} + n \cdot p \, \frac{\bar{n}^\mu}{2} + p_\perp^\mu \equiv (p^-, p^+, p_\perp)$, with $p_\perp^2 \equiv p_\perp \cdot p_\perp < 0$, and $p_T = \sqrt{-p_\perp^2}$

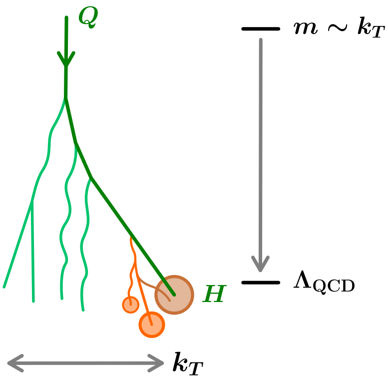
Two different regimes

Regime 1:



k_T set by soft emissions

Regime 2:



k_T set by perturbative emissions

Regime 1.

Regime 1 $\Lambda_{\mathrm{QCD}} \sim k_T \ll m$

- use bHQET:

$$\Delta_{H/Q}(z_H, b_\perp) = \frac{\delta(1-z_H)}{\bar{n}\cdot v} C_m(m) \underbrace{\frac{1}{2N_c} \mathrm{Tr} \sum_X \langle 0 | W^\dagger(b_\perp)\, h_v(b_\perp) | H_v X \rangle \langle H_v X | \bar{h}_v\, W | 0 \rangle}_{F_H(b_\perp)}$$

- project out unpol. TMD FF

$$D_{1\,H/Q}(z_H, b_T, \mu, \zeta) = \delta(1-z_H)\, C_m(m, \mu, \zeta)\, \chi_{1,H}\left(b_T, \mu, \frac{\sqrt{\zeta}}{m}\right) + \mathcal{O}\left(\frac{1}{m}\right)$$

[C_m calculated by Hoang, Pathak, Pertrulewicz, Stewart '15]

- new scalar bHQET TMD fragmentation factors $\chi_{1,H}(b_T) = \frac{1}{2}\, \mathrm{tr}\, F_H(b_\perp)$

bHQET: Use bHQET to describe dynamics at non-perturbative scale
$\mathcal{L} = \bar{h}_v(iv\cdot D)h_v + \mathcal{L}_{\mathrm{light}} + \mathcal{O}\left(\frac{1}{m}\right), \quad \psi_Q(x) = e^{-imv\cdot x} h_v(x)$

Regime 1: $\Lambda_{\text{QCD}} \sim k_T \ll m$

- Now: decouple spin d.o.f. from light dynamics [Korchemsky and Radyushkin '92, Bauer, Pirjol and Stewart '02]

$$F_H(b_\perp) = \frac{1}{2} \sum_{\substack{h_H, h_Q,\\ h'_Q, h_\ell, h'_\ell}} u(v, h_Q)\, \bar{u}(v, h'_Q) \overbrace{\langle s_Q, h_Q; s_\ell, h_\ell | s_H, h_H \rangle \langle s_H, h_H | s_Q, h'_Q; s_\ell, h'_\ell \rangle}^{\text{Clebsch-Gordan-Coefficients}} \rho_{\ell, h_\ell h'_\ell}(b_\perp)$$

  - $\rho_{\ell, h_\ell h'_\ell}(b_\perp)$: light spin density matrix, encodes all non-perturbative physics

Results for the unpolarized TMD FF

- performing trace sets $h_Q = h'_Q$ and $h_\ell = h'_\ell$

$$D_{1\,H/Q}(z_H, b_T, \mu, \zeta) \propto \chi_{1,H}(b_T) = \frac{1}{2} \sum_{h_H} \sum_{h_Q} \sum_{h_\ell} |\underbrace{\langle s_Q, h_Q; s_\ell, h_\ell | s_H, h_H \rangle}_{\text{Clebsch-Gordan-Coefficients}}|^2 \rho_{\ell, h_\ell h_\ell}(b_\perp)$$

bHQET: $h_v(x) = Y_v(x)\, h_v^{(0)}(x)$, $Y_v(x) = P\Big[\exp\Big(ig \int_0^\infty ds\, v \cdot A(x + vs)\Big)\Big]$

$h_v(x)\, |s_Q, h_Q; s_\ell, h_\ell, f_\ell; X\rangle = u(v, h_Q)\, Y_v(x)\, |s_\ell, h_\ell, f_\ell; X\rangle$, where $u(v, h_Q) = u(mv, h_Q)/\sqrt{m}$

Results for the unpolarized TMD FF

- example: D ($s_\ell = 1/2\,,\ s_H = 0$) vs. D^* meson ($s_\ell = 1/2\,,\ s_H = 1$)

$$\chi_{1,D}(b_T) = \frac{1}{4}\big[\rho_{\ell,++}(b_\perp) + \rho_{\ell,--}(b_\perp)\big] \quad \text{vs.} \quad \chi_{1,D^*}(b_T) = \frac{3}{4}\big[\rho_{\ell,++}(b_\perp) + \rho_{\ell,--}(b_\perp)\big]$$

- $\chi_{1,H}(b_T)$: conditional probability of Q to fragment into H given k_T

$$D_{1\,D/Q} = \frac{1}{3} D_{1\,D^*/Q}$$

- **three times as likely to produce D^* than D!**
- **for the first time: show that relations hold point by point in k_T**
  [proven for inclusive fragmentation Falk and Peskin '94, Manohar and Wise '00]

Unpolarized for $\Lambda_{\mathrm{QCD}} \ll k_T \sim m$

- match onto bHQET at $\mu \sim k_T \sim m$ [Nadolsky, Kidonakis, Olness and Yuan ’03]

$$D_{1\,H/Q}(z_H, b_T, \mu, \zeta) = \boxed{d_{1\,Q/Q}(z_H, b_T, \mu, \zeta)}\,\boxed{\chi_H} + \mathcal{O}\Big(\frac{\Lambda_{\mathrm{QCD}}}{m}\Big) + \mathcal{O}(\Lambda_{\mathrm{QCD}} b_T)$$

- ▶ new perturbative matching coefficient:

$$\boxed{d_{1\,Q/Q}(z_H, b_T, \mu, \zeta)} = \mathrm{tr}\Big[\frac{\not{n}}{2}\,\Delta_{Q/Q}(z_H, b_\perp)\Big] = \delta(1 - z_H) + \mathcal{O}(\alpha_s)$$

- $\boxed{\chi_H}$ total probability of Q to fragment into H

Unpolarized for $\Lambda_{\rm QCD} \ll k_T \sim m$: consistency between regimes

- dependence on the hadronic final state is purely encoded in χ_H
- still valid for $m \ll k_T$, but need to resum logs at $\mu \sim m$ → refactorization

$$d_{1\,Q/Q}(z_H, b_T, \mu, \zeta) = \frac{1}{z_H^2} \sum_i \int \frac{\mathrm{d}z}{z} \mathcal{J}_{i/Q}(z, b_T, \mu, \zeta)\, d_{Q/i}\Big(\frac{z_H}{z}, \mu\Big) + \mathcal{O}(m^2 b_T^2)$$

with collinear FFs $d_{j/i}$ [Mele, Nason '91] and coefficients $\mathcal{J}_{i/k}$ [Collins '11, Echevarria, Idilbi, Sciememi '14]

- for $k_T \ll m$

$$d_{1\,Q/Q}(z, b_T, \mu, \zeta) = \delta(1-z)\, C_m(m, \mu, \zeta)\, C_1\Big(b_T, \mu, \frac{\sqrt{\zeta}}{m}\Big) + \mathcal{O}\Big(\frac{1}{b_T m}\Big)$$

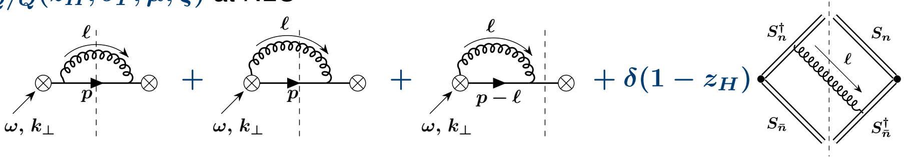
▶ **symmetry relations hold for all values of k_T and to all orders in α_s!**

$$D_{1\,H/Q} = \frac{1}{3} D_{1\,H^*/Q}$$

Unpolarized TMD FF: work in progress

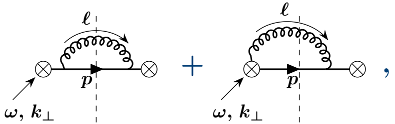
- calculate $d_{1\,Q/Q}(z_H, b_T, \mu, \zeta)$ at NLO

$d_{1Q/Q} =$

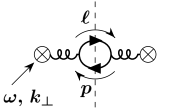


- also consider $d_{1\,g/Q}(z_H, b_T, \mu, \zeta)$ and $d_{1\,Q/g}(z_H, b_T, \mu, \zeta)$

$d_{1g/Q} =$



, $d_{1Q/g} =$



- **recall light quark limit:** use and $d_{g/i}$ [Mele, Nason '91] and $d_{Q/i}$ [Cacciari, Nason, Oleari '05] at $\mathcal{O}(\alpha_s)$

Heavy TMD FFs.

numerical model

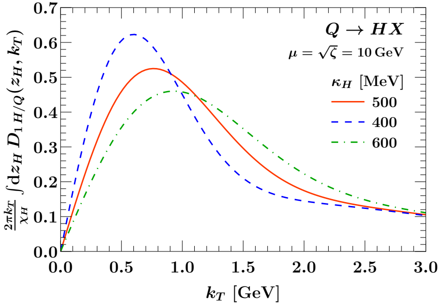
- at LL: unpol. TMD FF completely specified by $\chi_{1,H}$ and TMD evolution

$$\int_{z_{\rm cut}} {\rm d}z_H \, D_{1\,H/Q}(z_H, b_T, \mu, \zeta) = \chi_{1,H}(b_T, \mu_0, \rho_0)\, U_q(\mu_0, \zeta_0, \mu, \zeta)$$

- assume simple Gaussian model:

$$\chi_{1,H}(b_T, \mu_0, \rho_0) = \chi_H \exp\left(-\kappa_H^2 b_T^2\right)$$

- plot identical for both quark flavors $(b\,,c)$:
  - exactly equal for $k_T \ll m$
  - $k_T \sim m$: independent of m up to $\mathcal{O}(\alpha_s)$

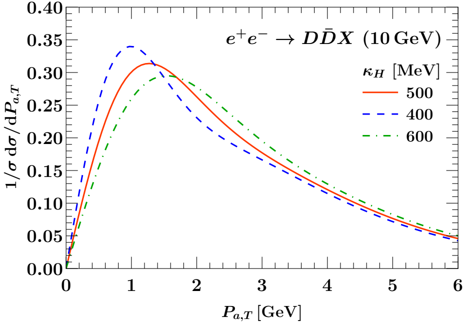


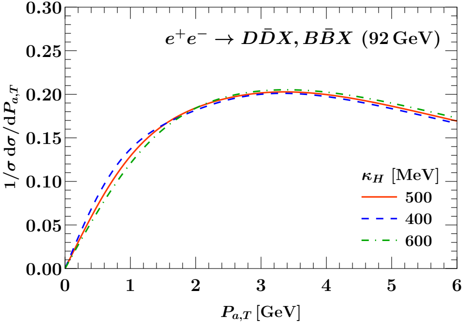
Towards Phenomenology.

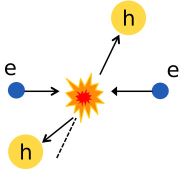
Towards Phenomenology: $e^+e^- \to HHX$.

Charm continuum production

- cross section depends on transverse momentum of the hadron $P_{a,T}$ and partonic transverse momentum $q_T = P_{a,T}/z_a$
- again: plots identical for b and c quarks

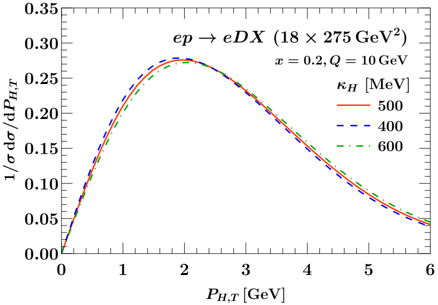


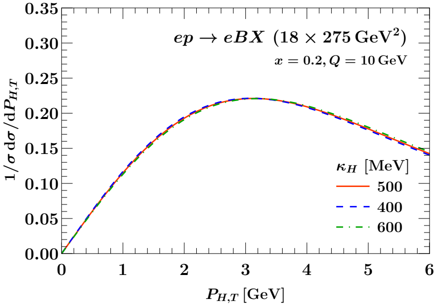




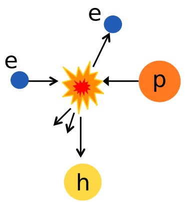
Towards Phenomenology: SIDIS at future EIC.

cross section





- SIDIS: interesting for spin correlations
- broader peak for B because of different size of phase space



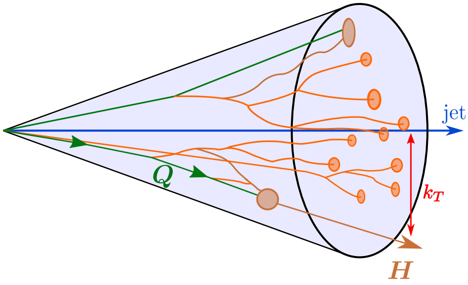
Outlook.

TMD FFs within jets

- consider heavy-quark TMD fragmentation within jets
- **straightforward extension: our results are independent of the factorization theorem!**
- NLO factorization theorem for TMD FFs within jets in pp collisions for $Q \sim R\, p_T^{\text{jet}} \gg k_T,\, m$ w.r.t. standard jet axis [Kang, Liu, Ringer, Xing '17]

Non-global logs

- starting from NNLO effects from NGLs [Dasgupta, Salam '01, Banfi, Marchesini, Smye '02]
- studying k_T w.r.t to groomed jet axis could mitigate NGL effects [Makris, Neill, Vaidya '17, Makris, Vaidya '18]



Outlook.

Polarized Hadrons

- so far: considered TMD FFs only for unpol. hadrons
  - gives us access to two TMD FFs (unpolarized D_1 and Collins $H_1^\perp$)
- now: also consider polarized hadrons
  - study all eight TMD FFs for heavy quarks
- relevant for LHC: polarized hadrons can be reconstructed from angular distribution of decay products

Outlook and Summary.

Summary

- heavy transverse momentum dependent fragmentation functions:
  - $\Lambda_{\mathrm{QCD}} \sim k_T \ll m$: unpol. TMD FF matches onto TMD fragmentation factor $\chi_{1,H}$
  - $\Lambda_{\mathrm{QCD}} \ll m \sim k_T$: used light-quark twist expansion and combined it matching collinear FFs onto bHQET to identify relevant bHQET matrix elements
  - use HQ symmetry to prove relations between TMD FFs within in the same spin multiplet $\to$ hold point by point in k_T
- towards phenomenology
  - expect TMD-sensitive samples at current $e^+e^- \to HHX$ and SIDIS
- Outlook: numerous follow up projects $\to$ **Stay tuned!**

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Back up.

Notation and Conventions.

lightcone momenta

- write momenta in terms of lightlike vectors n^μ, $\bar{n}^\mu$ with $n^2 = \bar{n}^2 = 0$ and $n \cdot \bar{n} = 2$:

$$p^\mu = \bar{n} \cdot p \, \frac{n^\mu}{2} + n \cdot p \, \frac{\bar{n}^\mu}{2} + p_\perp^\mu \equiv p^- \frac{n^\mu}{2} + p^+ \frac{\bar{n}^\mu}{2} + p_\perp^\mu \equiv (p^-, p^+, p_\perp)$$

- always take $p_\perp^2 \equiv p_\perp \cdot p_\perp < 0$, and denote their magnitude by $p_T = \sqrt{-p_\perp^2}$

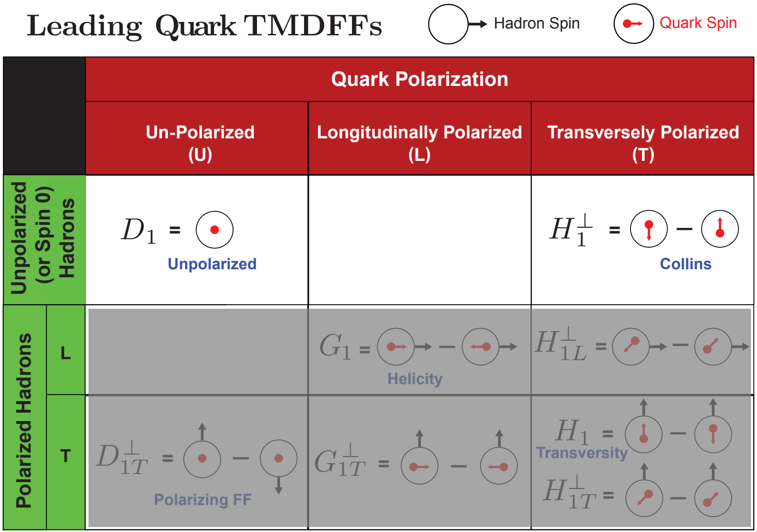
Important quantities

- heavy quark Q fragmenting into heavy hadron H that contains Q and carries momentum

$$P_H^\mu = P_H^- \frac{n^\mu}{2} + \frac{M_H^2}{P_H^-} \frac{\bar{n}^\mu}{2}, \quad \text{where} \quad P_H^- \gg P_H^+ = M_H^2 / P_H^-$$

- $k_\perp$: transverse momentum of the heavy quark
- $b = (0, b^+, b_\perp)$: spatial separation between quark fields
- z_H: fraction of quarks lightcone momentum retained by hadron

Heavy TMD FFs.



[Figure taken from TMD Handbook '23]

Regime 1.

Results for the Collins TMD FF

$$H^{(1),\perp}_{1,H/Q}(z_H, b_T) \propto \chi^{\perp}_{1,H}(b_T) = \frac{1}{2}\,\mathrm{tr}\left[\frac{\not{b}_\perp}{b_T}\not{z}\, F_H(b_\perp)\right]$$

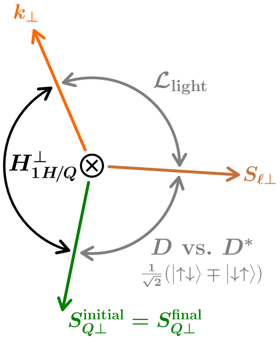
- encodes correlation between $S^{\text{initial}}_{q\perp}$ and $k_\perp$
- naive expectation from HQ symmetry: Collins TMD FF should be suppressed by $1/m$

Light quarks:

- correlation directly from non-perturbative dynamics of $\mathcal{L}_{\mathrm{QCD}}$

Heavy quarks:

- the angle between $S_{Q\perp}$ and $S_{\ell\perp}$ determines which hadron in the spin symmetry multiplet is produced
- reconstructing this information induces a correlation between $S_{Q\perp}$ and $S_{\ell\perp}$
- $S_{\ell\perp}$ and $k_\perp$ are correlated by nonperturbative dynamics of $\mathcal{L}_{\mathrm{QCD}}$



Results for the Collins TMD FF

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- encodes correlation between $S^{\text{initial}}_{q\perp}$ and $k_\perp$
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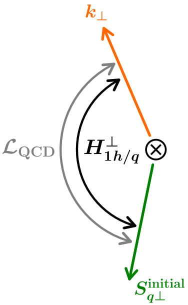
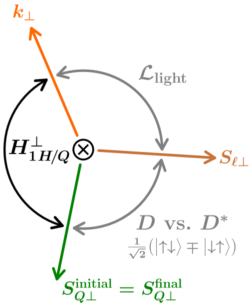




- **Collins effect is not suppressed by $1/m$!**

Regime 1.

Results for the Collins TMD FF

$$H^{(1),\perp}_{1,H/Q}(z_H, b_T) \propto \chi^{\perp}_{1,H}(b_T) = \frac{1}{2}\,\mathrm{tr}\Big[\frac{\not{b}_\perp}{b_T}\not{z}\,F_H(b_\perp)\Big]$$

- **Collins TMD FF is not suppressed by mass!**
- example $s_\ell = 1/2\,,\ s_H = 0$

$$\chi^{\perp}_{1,H}(b_T) = \frac{1}{4}\big[\rho_{\ell,-+}(b_\perp) - \rho_{\ell,+-}(b_\perp)\big]$$

- sum over all hadrons within same spin multiplet M_ℓ (identical light spin and falvor state ℓ)

$$\sum_{H\in M_\ell} \chi^{\perp}_{1,H}(b_T) = 0 \quad \to \quad H^{\perp}_{1,\Lambda_c/c} = 0, \quad H^{\perp}_{1,D/c} = -H^{\perp}_{1,D^*/c}, \quad \dots$$

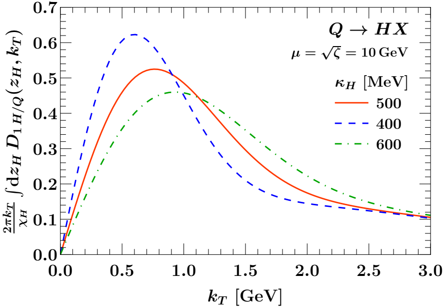
▶ **Heavy quark limit lets us sum rule point by point in q_T!**

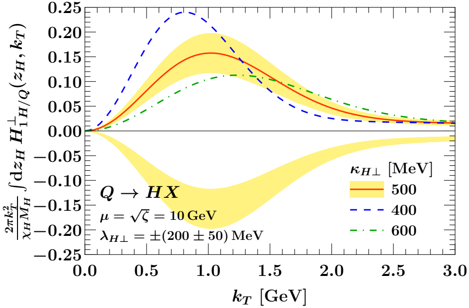
[Previously shown in bare case. Requires sum over all hadrons and integration of z_H and k_T. Schäfer and Teryaev '00, Meissner, Metz and Pitonyak '10]

numerical model

- at LL: TMD FFs completely specified by $\chi_{1,H}$, $\chi^{\perp}_{1,H}$ and TMD evolution
- assume simple Gaussian models

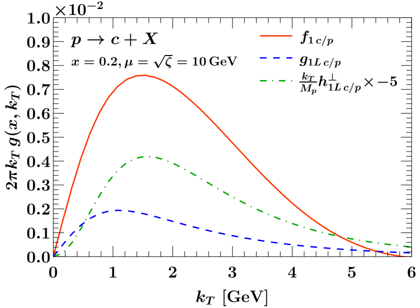
$$\chi_{1,H}(b_T, \mu_0, \rho_0) = \chi_H \exp\left(-\kappa_H^2 b_T^2\right)$$

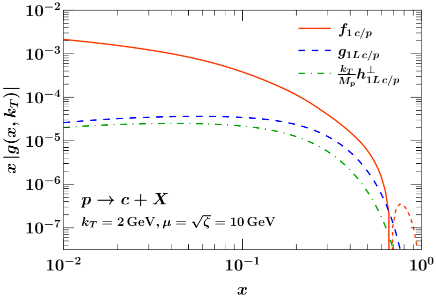




Heavy TMD PDFs.

numerical model

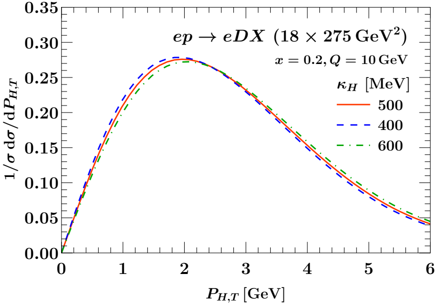


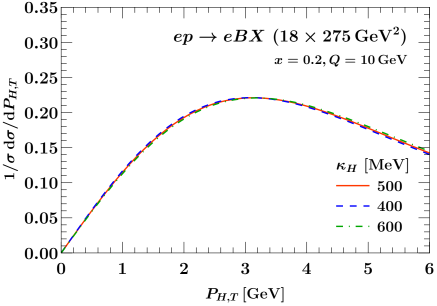


- unpolarized and helicity are linear in small k_T region, worm-gear L quadratic (after including Jacobian)
- unpolarized rises much more rapidly for small x
  - expected from smaller gluon polarized fraction at small x
- need for resummation of subleading-power threshold logs of $1 - x$

Towards Phenomenology: SIDIS at future EIC.

cross section





- compare cross sections D and B mesons
- broader peak for B because of different size of phase space
- very small dependence on parameter κ_H

Towards Phenomenology: $e^+e^- \to HHX$.

Charm continuum production

- absolute sign of Collins effect lost in Collins effect strength:

$$R_{\cos(2\phi_0)}(Q^2, q_T) = \frac{H_1^{\perp(1)} \otimes H_1^{\perp(1)}}{D_1 \otimes D_1}$$

- **but**: can extract relative factor between D and D^* Collins function

$$R^{D\bar{D}}_{\cos(2\phi_0)} = -\frac{1}{3} R^{D^*\bar{D}}_{\cos(2\phi_0)} = -\frac{1}{3} R^{D\bar{D}^*}_{\cos(2\phi_0)} = +\frac{1}{9} R^{D^*\bar{D}^*}_{\cos(2\phi_0)}$$

- for parameters $(\kappa_H, \kappa_{H,\perp}, \lambda_{H,\perp})$ of $\mathcal{O}(\Lambda_{\text{QCD}})$: expect Collins effect of several percent

Towards Phenomenology: SIDIS at future EIC.

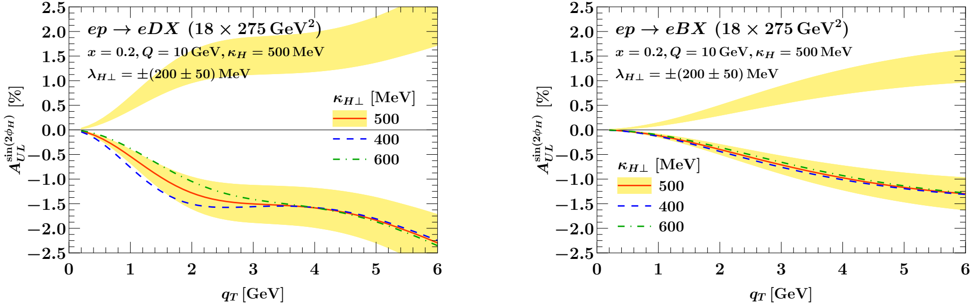
SIDIS kinematics

- electron-nucleon collisions $e^-(\ell) + N(P) \to e^-(\ell') + H(P_H) + X$
- scattering mediated by an off-shell photon with $q = \ell - \ell'$ and $Q^2 \equiv -q^2 > 0$
- SIDIS cross section depends on: $x = Q^2/(2P \cdot q)$, $y = (P \cdot q)/(P \cdot \ell)$, $z_H = (P \cdot P_H)/(P \cdot q)$
- estimate sample at EIC:

$\sigma(eN \to eHX)$ [pb]	c, $x > 0.01$	c, $x > 0.1$	b, $x > 0.01$	b, $x > 0.1$
$q_T < 2\,\mathrm{GeV}, Q > 4\,\mathrm{GeV}$	84	3.47	18	0.65
$q_T < 4\,\mathrm{GeV}, Q > 10\,\mathrm{GeV}$	16	1.45	4.9	0.42

acceptance cuts: $0.01 < y < 0.95\,, \quad W^2 = \left(\frac{1}{x} - 1\right)Q^2 > 100\,\mathrm{GeV}^2$

Collins effect



- get access to the sign of the Collins via spin asymmetry

$$A_{UL}^{\sin(2\phi_H)}(Q^2, x, q_T) = h_{1L}^{\perp} \otimes H_1^{\perp} / f_1 \otimes D_1$$

- resolving the sign should be possible within expected statistics at EIC!