Factorisation breaking for heavy-quark induced processes

Based on FC , K. Melnikov, D. Napoletano & L. Tancredi, PRD103 (2021)

Heavy Flavours at High pT, Edinburgh, Dec 1st 2023

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An old result…

Can we do consistent calculations with intrinsic massive quarks?

COUNTER-EXAMPLE TO NON-ABELIAN BLOCH–NORDSIECK CONJECTURE

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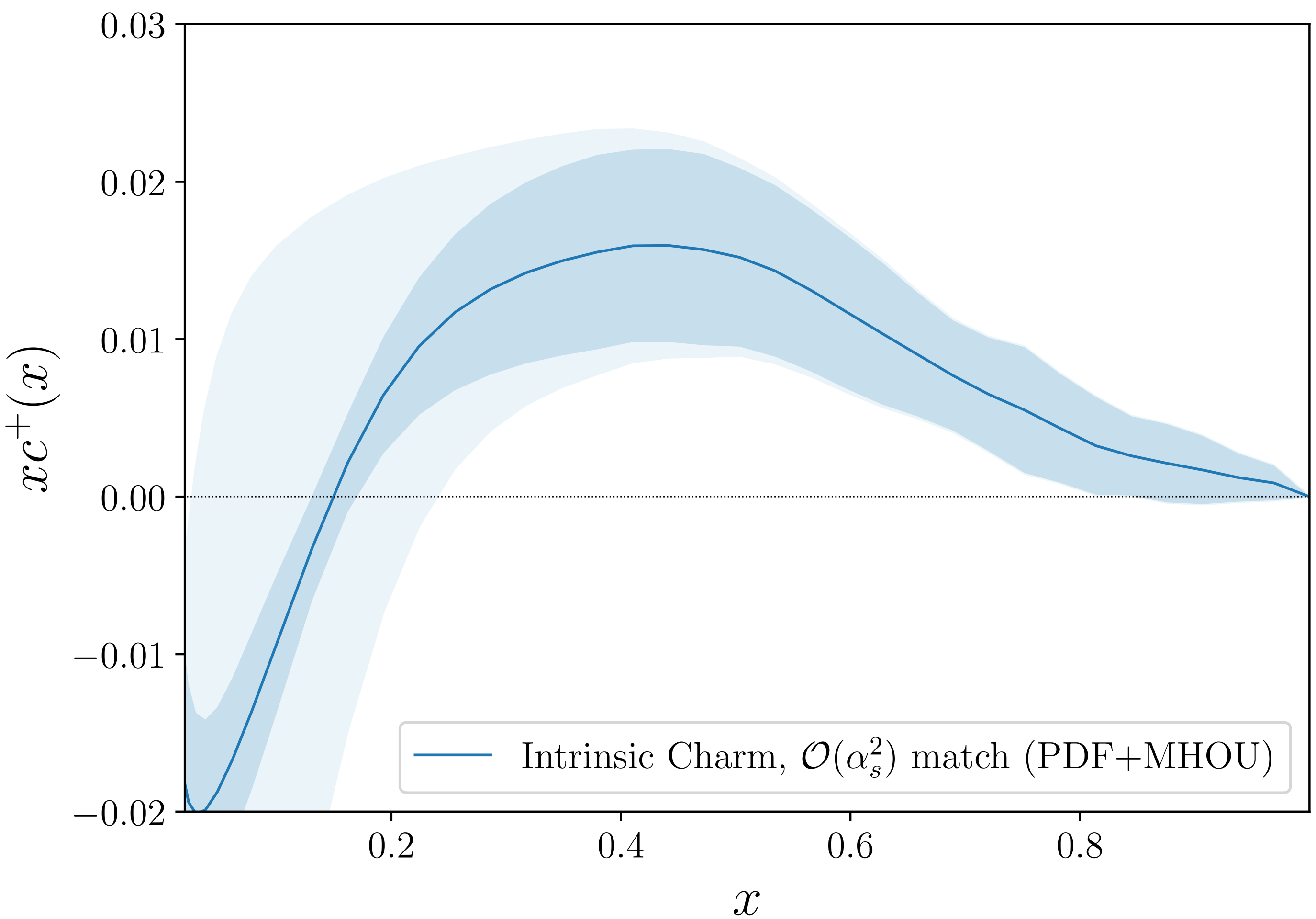
An example is given of a reaction, with two quarks (with non-zero mass, and colour averaged) in the initial state, in which non-leading infrared divergences coming from two soft gluons do not cancel between real and virtual diagrams.

… which may no longer be purely academic

INTRINSIC CHARM

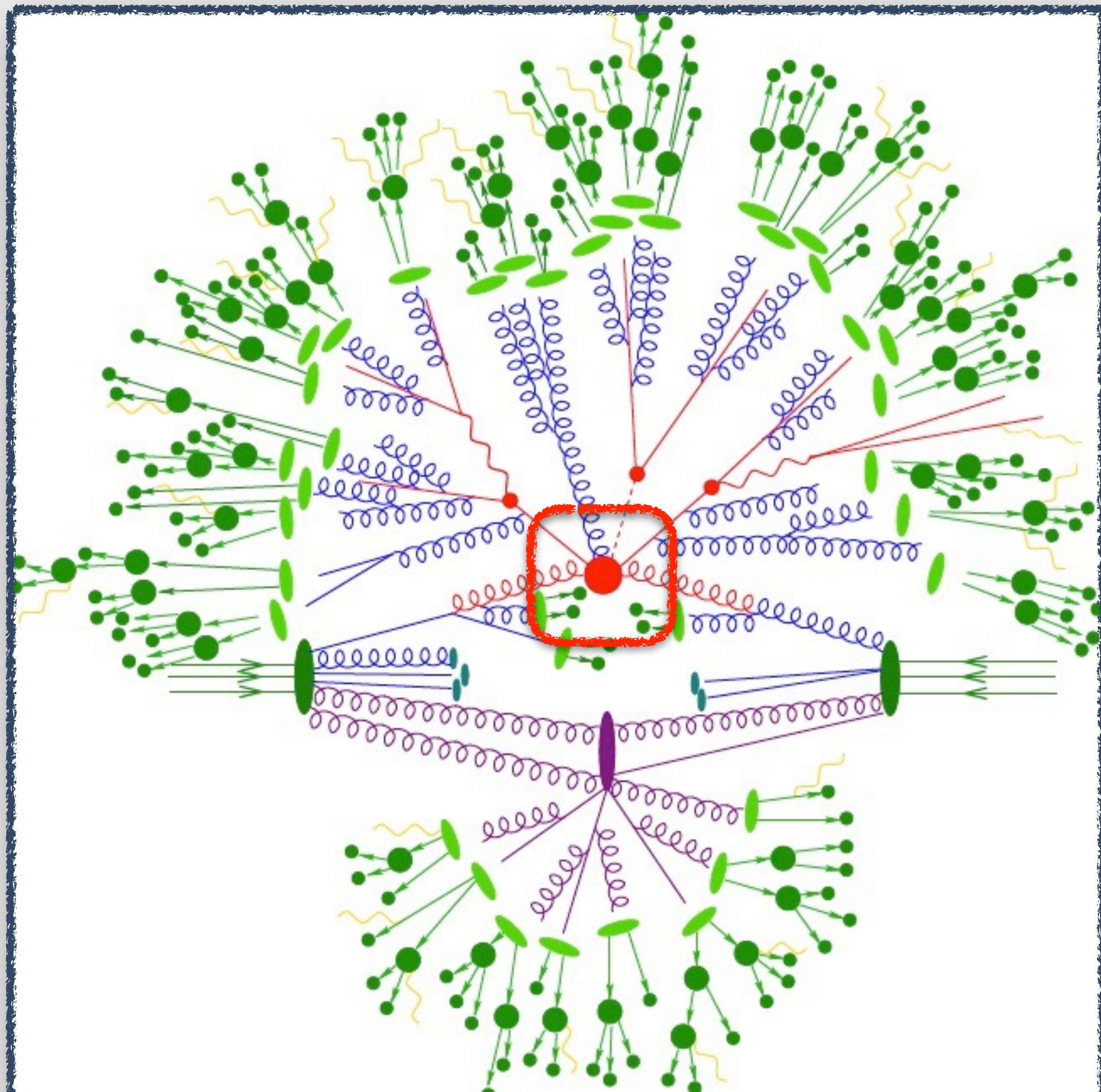
- MHOU ESTIMATED FROM N^{3}LO-NNLO MATCHING DIFFERENCE
  - LARGE UNCERTAINTY AT SMALL x
  - NEGLIGIBLE UNCERTAINTY IN VALENCE REGION
- COMPATIBLE WITH ZERO AT SMALL x
- CLEAR EVIDENCE FOR INTRINSIC VALENCE PEAK

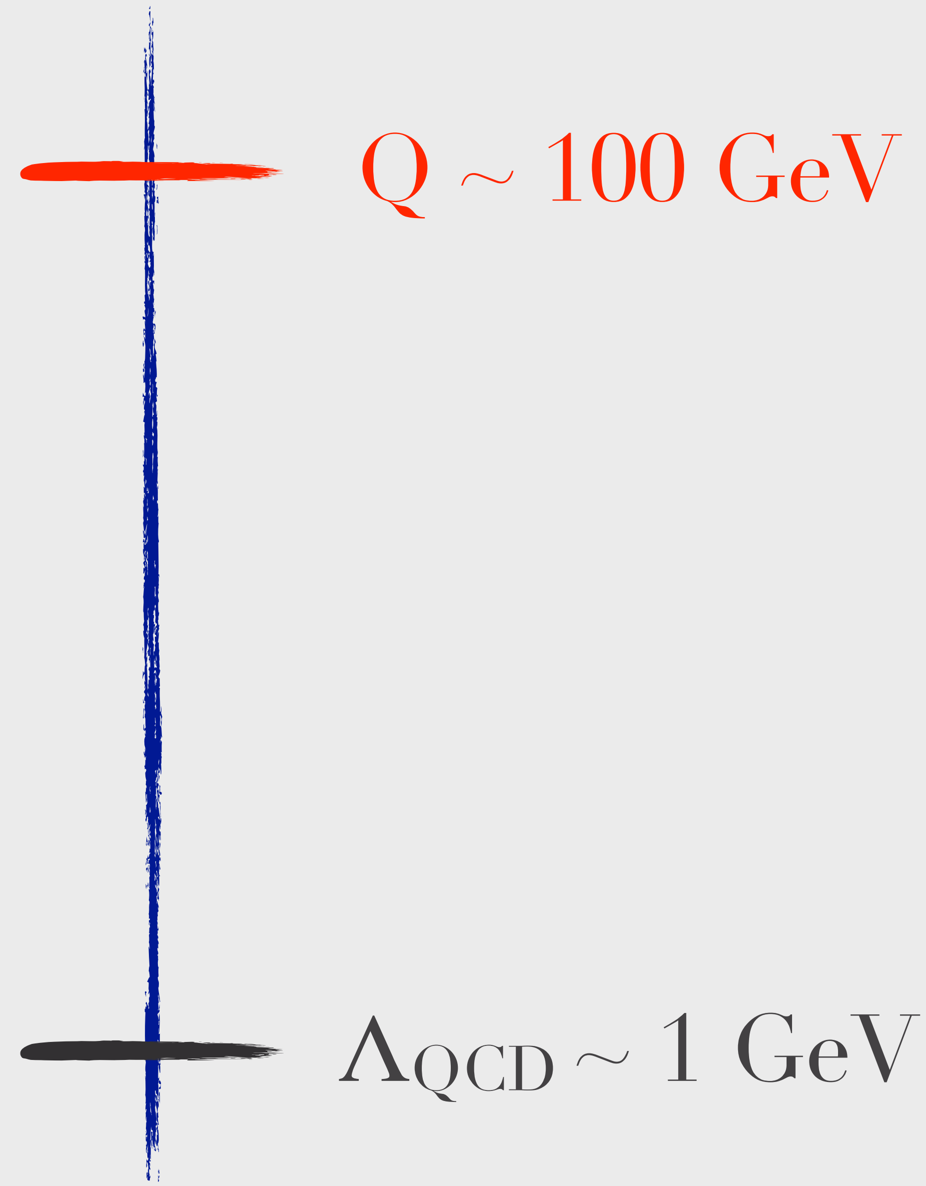
3FNS



Stefano's slide from yesterday

Back to the basics: (collinear) factorisation



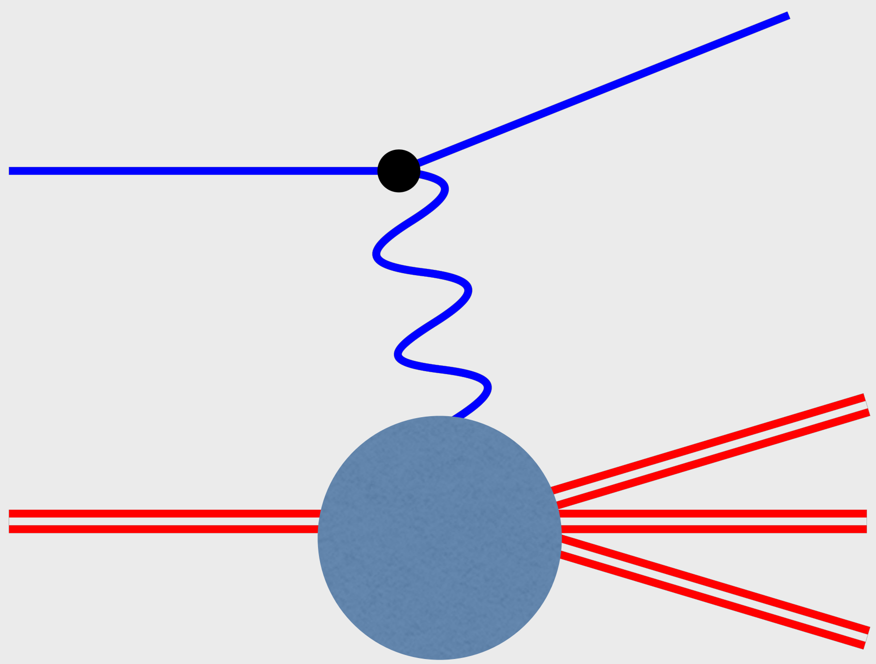


$$d\sigma = \int dx_1 dx_2 f(x_1) f(x_2) d\sigma_{\rm part}(x_1, x_2) F_J (1 + \mathcal{O}(\Lambda_{\rm QCD}/Q))$$

↓

logarithmically insensitive to $\Lambda_{\rm QCD}$

A solid framework: DIS



$$\longrightarrow \int e^{iq\cdot x}\langle P|J^{\dagger,\mu}(0)J^{\nu}(x)|P\rangle dx$$

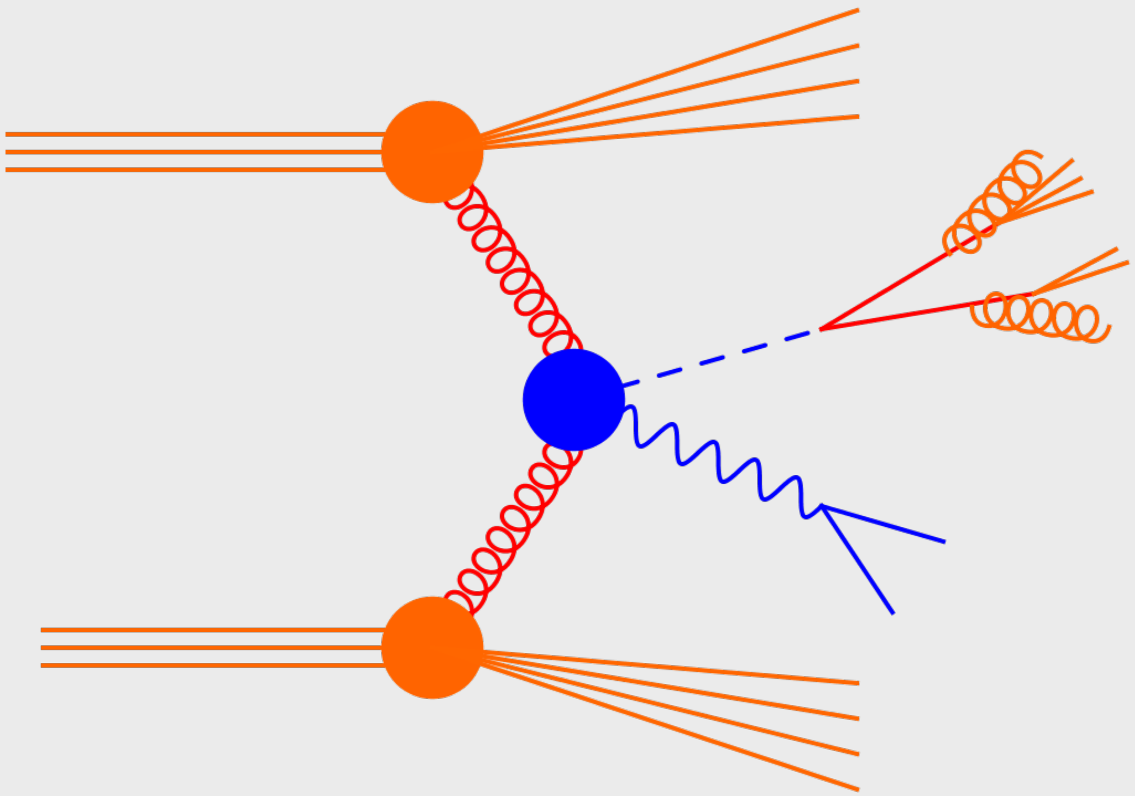
OPE: $$\langle P|J^{\dagger,\mu}(0)J^{\nu}(x)|P\rangle \sim \sum_i c_i(x)\langle P|\mathcal{O}_i|P\rangle$$

twist-2

$$\sigma = \int \mathrm{d}x\,\sigma_{\text{part}}(x)f(x)(1+\mathcal{O}(\Lambda^2_{\text{QCD}}/Q^2))$$

higher-twist

Hadronic collisions: more complex



A potential problem: non-trivial (long-distance) interactions among protons / coloured objects

"Standard" collinear factorisation

- Works at all order for simple processes, e.g. Drell-Yan [Collins, Soper, Sterman]
- Works up to NNLO for any process and (IR-safe) observables
- There may be issues at N^{3}LO for complex-enough processes [Beneke, Ruiz-Femenia; Catani, de Florian, Rodrigo; Forshaw, Seymour, Siodmok...]

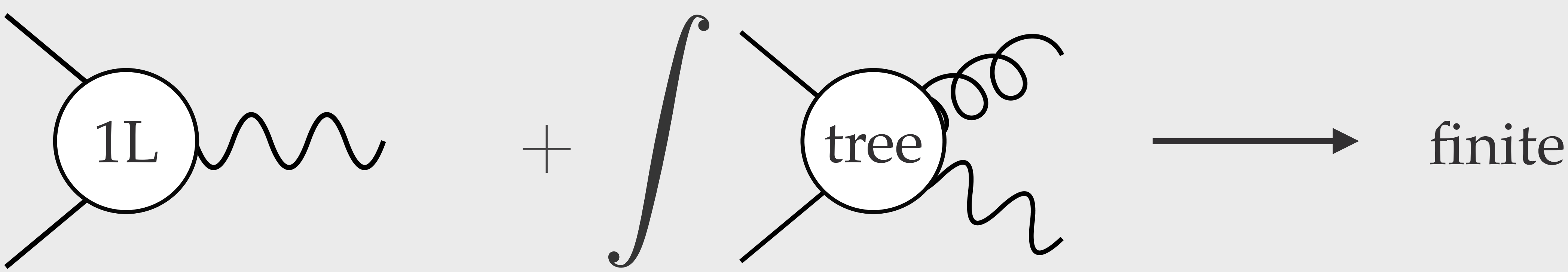
Initial-state heavy quarks: more delicate...

The simplest set-up: Drell-Yan

Is the total partonic cross-section for heavy-quark induced Drell-Yan IR finite?

The “standard” Bloch/Nordsiek mechanism

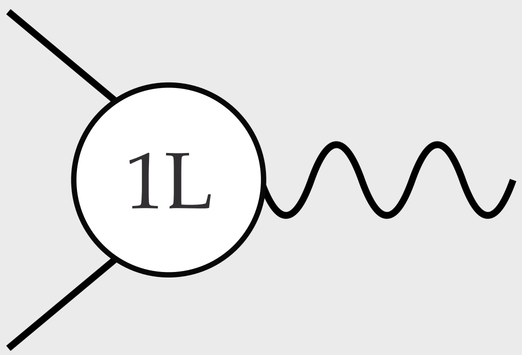
$$\sigma_{\text{virt}} + \sigma_{\text{real}} = \sigma$$



-1/ε from loop integration

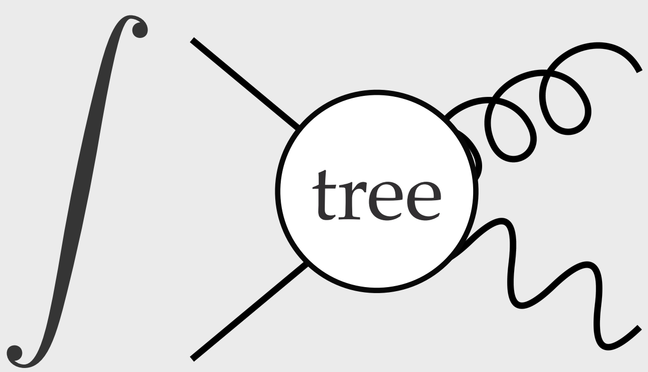
1/ε from integrating over unresolved parton phase-space

Warm-up: massive DY at NLO



$$d\sigma_V = \frac{\alpha_s(\mu)}{2\pi}\left\{-\frac{2C_F}{\epsilon}\left[\frac{1}{2v}\ln\left(\frac{1-v}{1+v}\right)+1\right]\right\}d\sigma_{LO} + d\sigma_{V,\mathrm{fin}}$$

$$v = \sqrt{1-m^4/(p_1\cdot p_2)^2}$$

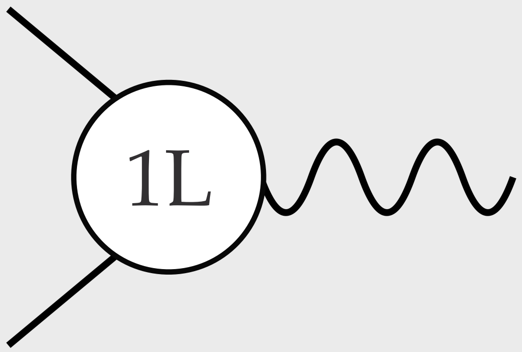


Singularities only from the soft $E_g \sim 0$ region

$$d\sigma_R = \int_0^{E_{max}} \frac{dE_g}{E_g^{1+2\epsilon}}\frac{d\Omega_g^{(3)}}{16\pi^3}\lim_{E_g\to 0}\left[F_g^{(4)}(p_1,p_2,p_V;p_g)\right] + d\sigma_R^{\mathrm{fin}}$$

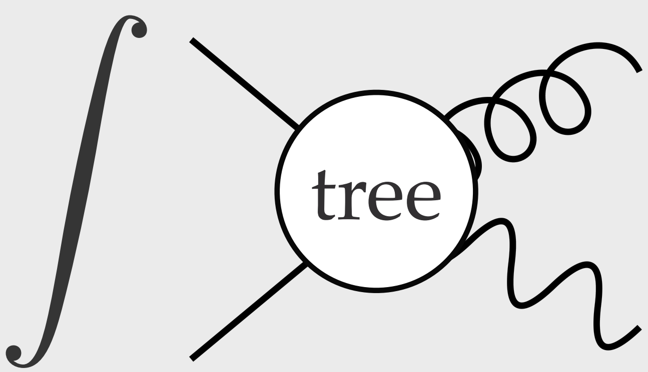
$$-\frac{1}{2\epsilon}$$

Warm-up: massive DY at NLO



$$d\sigma_{\mathrm{V}} = \frac{\alpha_s(\mu)}{2\pi}\left\{-\frac{2C_F}{\epsilon}\left[\frac{1}{2v}\ln\left(\frac{1-v}{1+v}\right)+1\right]\right\}d\sigma_{\mathrm{LO}} + d\sigma_{\mathrm{V,fin}}$$

$$v = \sqrt{1 - m^4/(p_1\cdot p_2)^2}$$



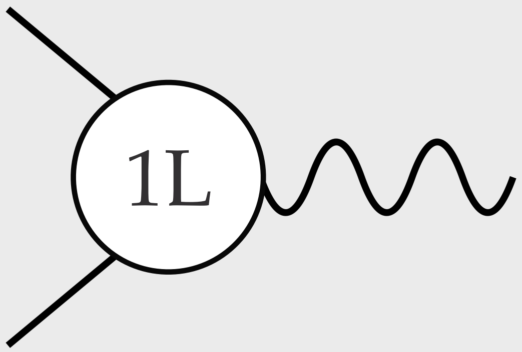
Singularities only from the soft $E_g \sim 0$ region

Soft region: eikonal approximation

$$\mathcal{M}_0(p_1, p_2; p_V, p_{g^a}) \approx g_s^2 \varepsilon^\mu J_\mu^{a,(0)}(p_1, p_2; p_g)\mathcal{M}_0(p_1, p_2; p_V),$$

$$J_\mu^{a,(0)}(p_1, p_2; p_g) = \sum_{i=1}^{2} T_i^a \frac{p_{i,\mu}}{p_i \cdot p_g},$$

Warm-up: massive DY at NLO



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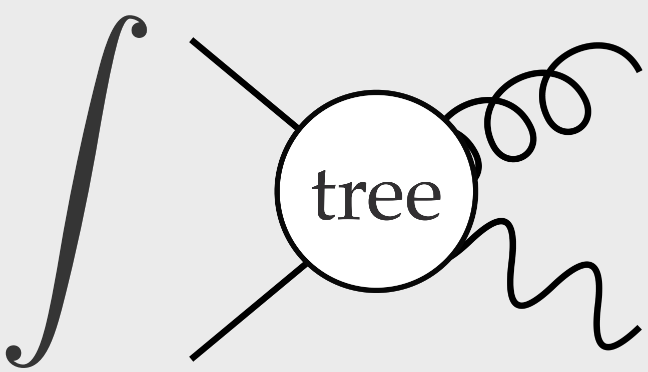

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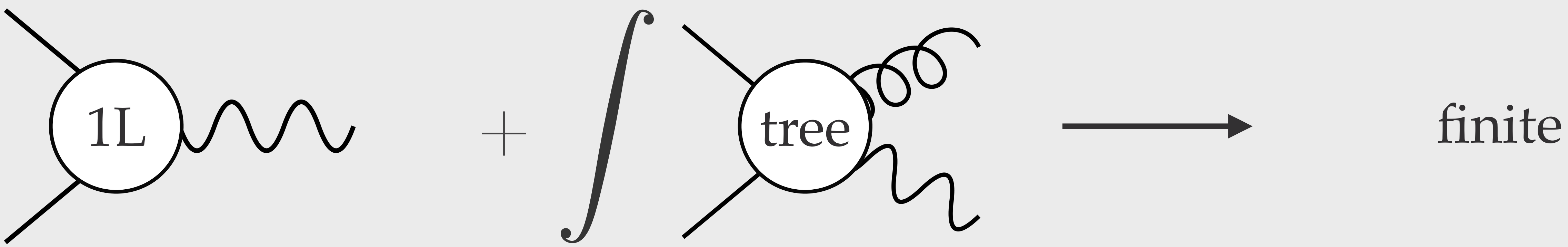
$d\sigma_R^{\mathrm{div}} = \mathrm{Eik}_0(p_1,p_2)\times d\sigma_{LO}$, universal factor, cancels the pole in σ_V

Warm-up: massive DY at NLO



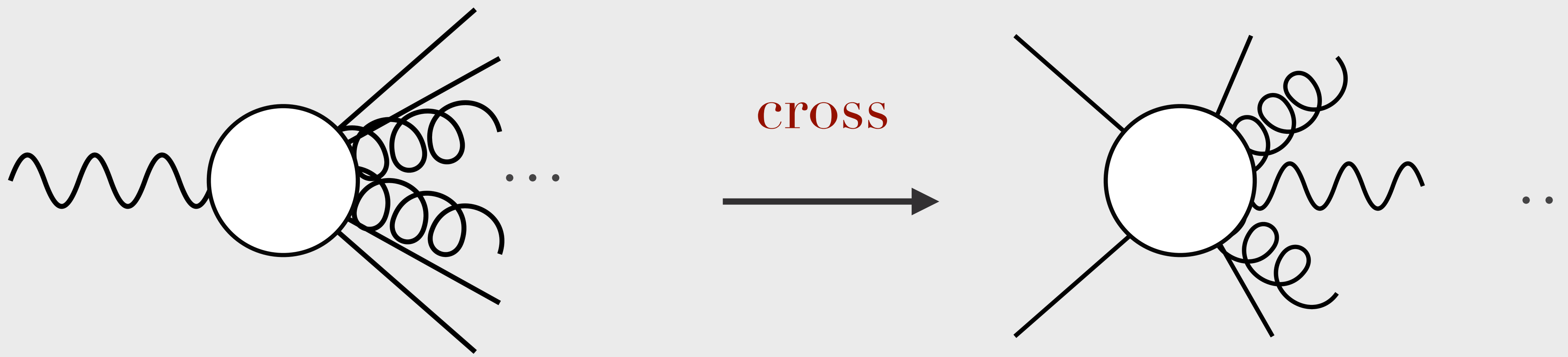
Drell-Yan at NLO

- IR sensitivity cancels among reals and virtuals, as in the standard case
- Only relevant region: soft gluon
- No need for PDFs
- Full calculation not needed to see the cancellation
- Still, going beyond NLO non-trivial

Is this result obvious? Yes

Massive DY@NLO: a different approach

- Consider now Z decay, related to DY by crossing



Massive DY@NLO: a different approach

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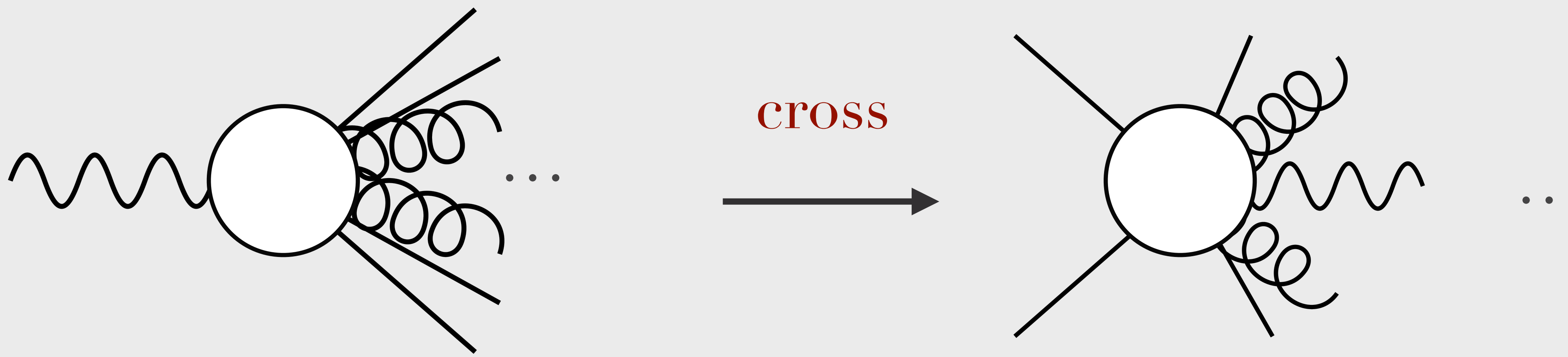
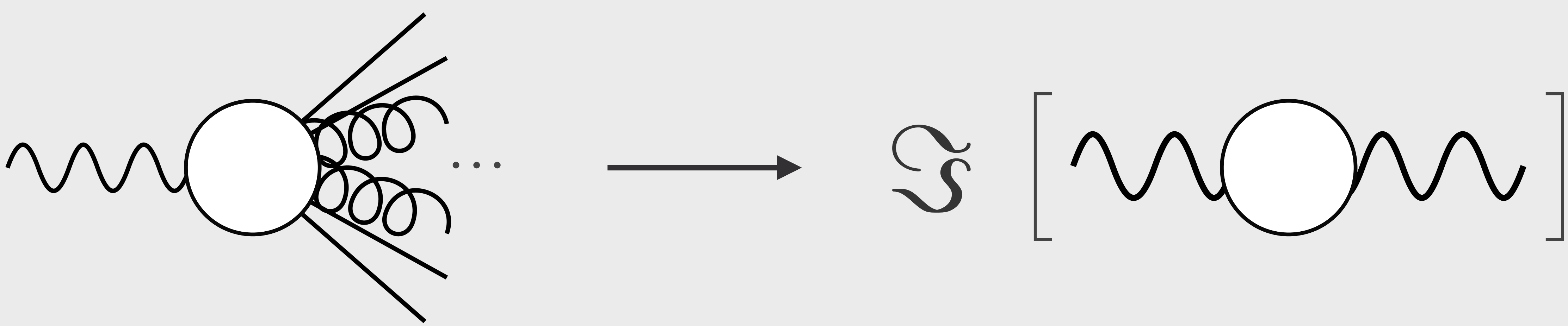


- Optical theorem



Massive DY@NLO: a different approach

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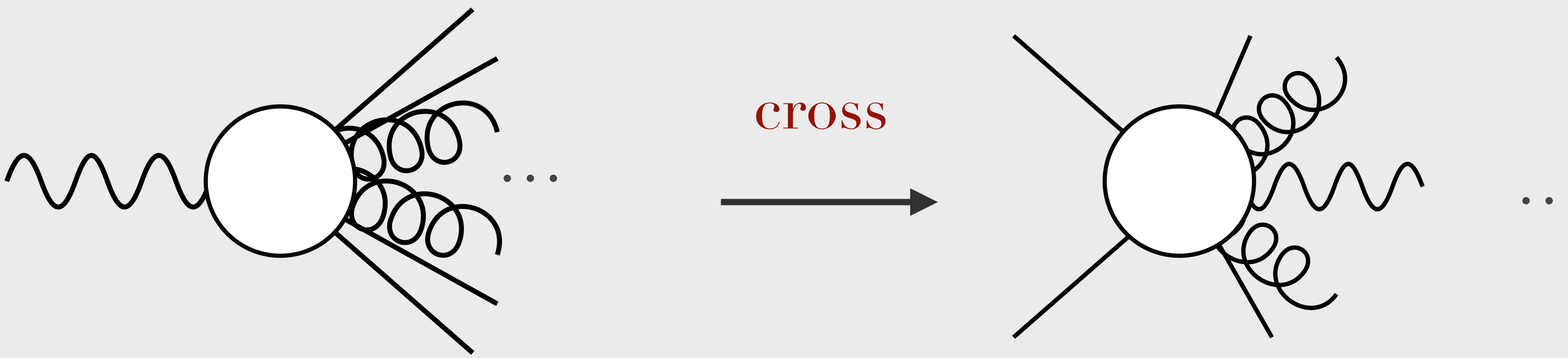
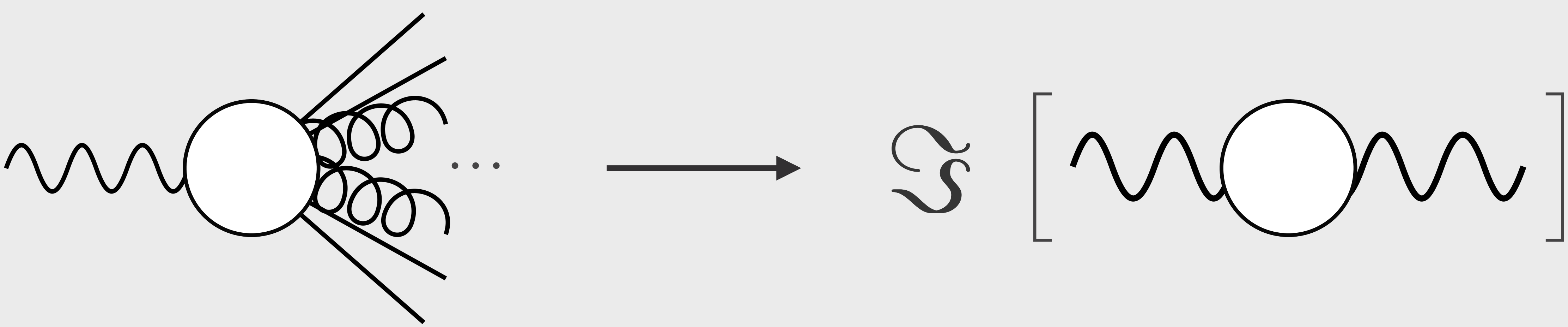
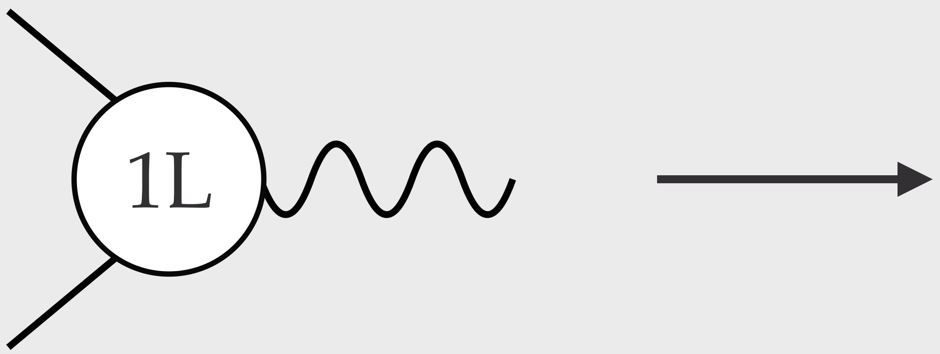


- Optical theorem

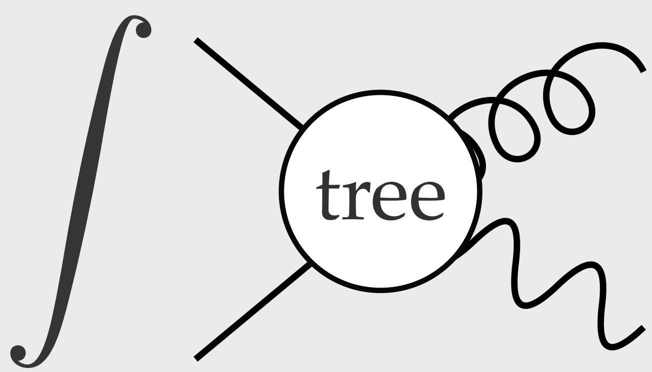


- Off-shell correlator: finite → any IR sensitivity in DY must come from non-trivial behaviour under crossing

Back to NLO



trivial under crossing



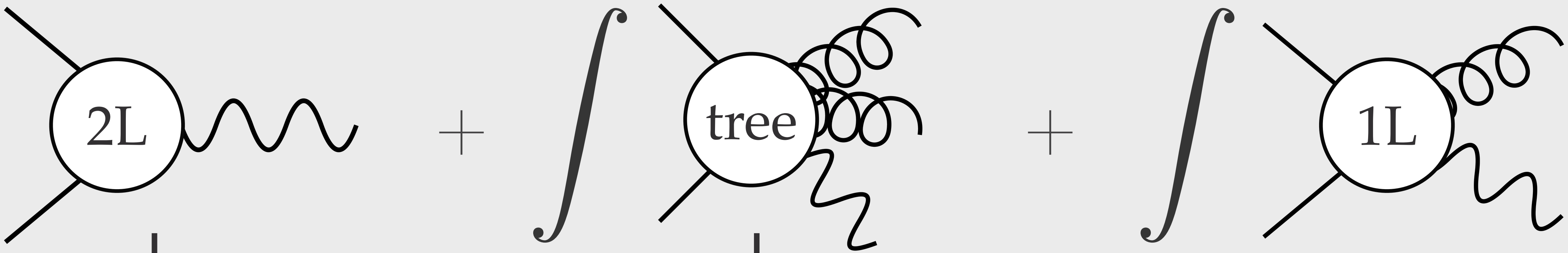
Soft region: eikonal approximation

$$J_\mu^{a,(0)}(p_1, p_2; p_g) = \sum_{i=1}^{2} T_i^a \frac{p_{i,\mu}}{p_i \cdot p_g},$$

Eikonal current invariant under $p_i \to -p_i$

At NLO, heavy-quark induced DY is trivially IR-insensitive

Now at NNLO

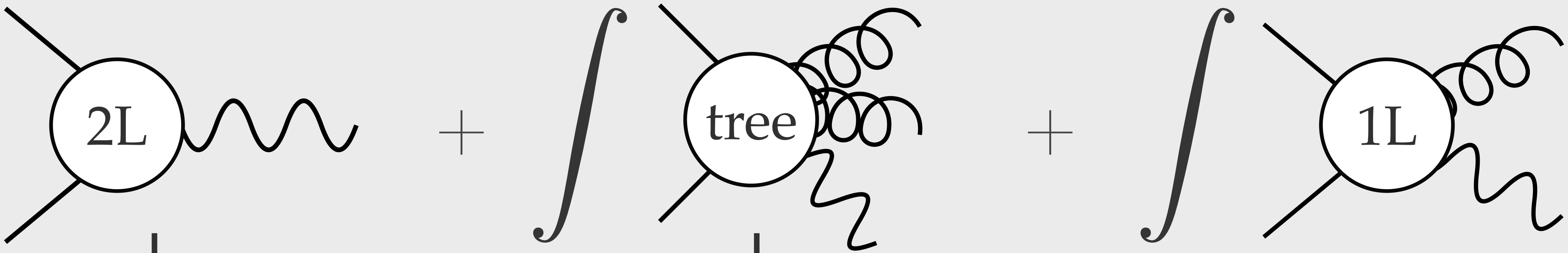


VV: trivial under crossing

RR: in the soft limit, everything controlled by soft currents which are homogeneous in hard momenta → same IR structure of decay process

RV: loop → non-trivial under crossing.
When gluon is soft: potential source for problems

Now at NNLO



VV: trivial under crossing

RR: in the soft limit, everything controlled by soft currents which are homogeneous in hard momenta → same IR structure of decay process

RV: loop → non-trivial under crossing.
When gluon is soft: potential source for problems

Note: in QED, the soft current does not receive corrections → NNLO QED DY is finite

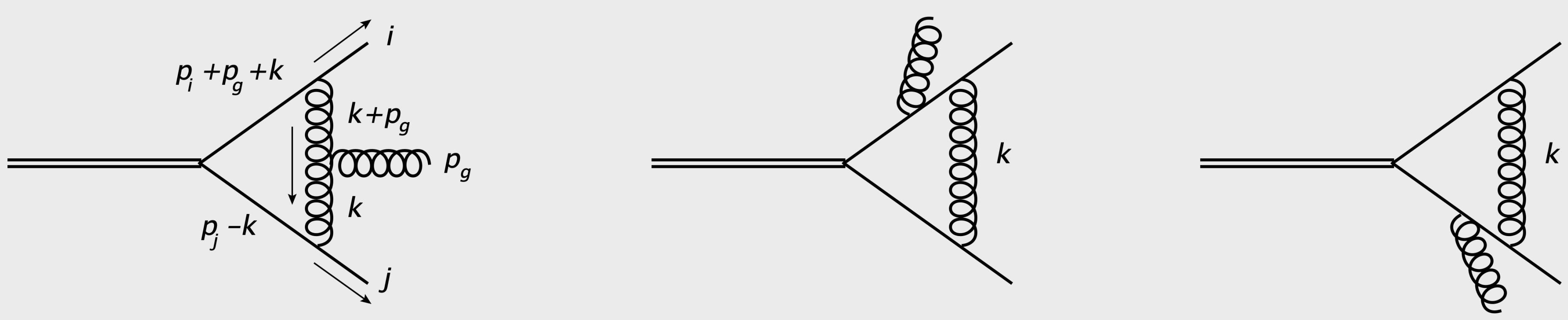
The soft current at 1L

$$\mathcal{M}_1(p_V; p_1, p_2, p_g) \approx g_s^2 \varepsilon^\mu \Big[J_\mu^{a,(0)}(p_1, p_2; p_g) \mathcal{M}_1(p_V; p_1, p_2) + g_s^2 J_\mu^{a,(1)}(p_1, p_2; p_g) \mathcal{M}_0(p_V; p_1, p_2) \Big].$$

$$J^{a,(1),\mu}(p_1, p_2; p_g) = i f_{abc} \sum_{\substack{i,j=1 \\ i \neq j}}^{2} T_i^b T_j^c \left(\frac{p_i^\mu}{p_i \cdot p_g} - \frac{p_j^\mu}{p_j \cdot p_g} \right) g_{ij}^{(1)}(\epsilon, p_g; p_i, p_j)$$

$$= g_{12}^{(1)}(\epsilon, p_g; p_1, p_2)\, C_A\, J^{a,(0),\mu}(p_1, p_2; p_g).$$

NLO current



p_g, k soft

[Explicit expressions: Catani, Grazzini (2000) + Bierenbaum, Czakon, Mitov (2012)]

The soft current at 1L: massless case

$$J^{a,(1),\mu}(p_1,p_2;p_g) = i f_{abc} \sum_{\substack{i,j=1 \\ i\neq j}}^{2} T_i^b T_j^c \left(\frac{p_i^\mu}{p_i\cdot p_g} - \frac{p_j^\mu}{p_j\cdot p_g}\right) g_{ij}^{(1)}(\epsilon, p_g; p_i, p_j)$$

$$= g_{12}^{(1)}(\epsilon, p_g; p_1, p_2)\; C_A\; J^{a,(0),\mu}(p_1,p_2;p_g).$$

Massless case:

$$g_{12}^{(1)}(\epsilon, p_g; p_1, p_2) = -\frac{1}{16\pi^2}\frac{1}{\epsilon^2}\frac{\Gamma^3(1-\epsilon)\Gamma^2(1+\epsilon)}{\Gamma(1-2\epsilon)}\left[\frac{(-s_{12}-i\delta)}{(-s_{1g}-i\delta)(-s_{2g}-i\delta)}\right]^\epsilon$$

- Under crossing → simple phase
- $\sigma_{RV} = 2\,\mathrm{Re}[A^0\, A^{1,*}]$ → drops out
- Standard cancellation of soft singularities applies

The soft current at 1L: massive case

Massive case more complicated. Under crossing:

$$g_{12}^{(1)}(\epsilon, p_g; -p_1, -p_2) = e^{-2i\epsilon\pi} g_{12}^{(1)}(\epsilon, p_g; p_1, p_2).$$

$$g_{12}^{(1)}(p_1, p_2) = \frac{\alpha_s}{2\pi} E_g^{-2\epsilon} \left[-\frac{1}{2\epsilon} - \frac{i\pi}{2\epsilon} + \frac{i\pi}{2\epsilon} \frac{(1-v)}{v} + \ldots \right]$$

$$\int \frac{dE_g}{E_g^{1+4\epsilon}} \to -\frac{1}{4\epsilon}$$

Universal soft pole

standard "ln(-s-iδ)"

Non-trivial massive cut → extra phase

The soft current at 1L: massive case

Massive case more complicated. Under crossing:

$$g_{12}^{(1)}(\epsilon, p_g; -p_1, -p_2) = e^{-2i\epsilon\pi} g_{12}^{(1)}(\epsilon, p_g; p_1, p_2).$$

$$g_{12}^{(1)}(p_1, p_2) = \frac{\alpha_s}{2\pi} E_g^{-2\epsilon} \left[-\frac{1}{2\epsilon} - \frac{i\pi}{2\epsilon} + \boxed{\frac{i\pi}{2\epsilon} \frac{(1-v)}{v}} + \ldots \right]$$

$$\Re\left[g_{12}^{(1)}(\epsilon, p_g; -p_1, -p_2) \right] = \Re\left[g_{12}^{(1)}(\epsilon, p_g; p_1, p_2) \right] + \boxed{\frac{\alpha_s}{2\pi} E_g^{-2\epsilon} \left[\left(\frac{1-v}{v} \right) \pi^2} + \mathcal{O}(\epsilon) \right].$$

↓

Unmatched IR contribution, leftover from non-cancelling Coulomb phase

Massive DY@NNLO: summing it up

$$\Re\left[g_{12}^{(1)}(\epsilon, p_g; -p_1, -p_2)\right] = \Re\left[g_{12}^{(1)}(\epsilon, p_g; p_1, p_2)\right] + \frac{\alpha_s}{2\pi} E_g^{-2\epsilon}\left[\left(\frac{1-v}{v}\right)\pi^2 + \mathcal{O}(\epsilon)\right].$$

$$\sigma_{\mathrm{DY,NNLO}} = \sigma_{\mathrm{DY,VV}} + \sigma_{\mathrm{DY,RR}} + \sigma_{\mathrm{DY,RV}} =$$

$$\sigma_{\mathrm{Zdec,VV}} + \sigma_{\mathrm{Zdec,RR}} + \left[\sigma_{\mathrm{Zdec,RV}} + \Delta\sigma_{\mathrm{RV}}\right] =$$

$$\mathrm{finite} + \Delta\sigma_{\mathrm{RV}}$$

$$\mathrm{d}\sigma_{\mathrm{NNLO}} = \Delta[\mathrm{d}\sigma_{\mathrm{RV}}^{\mathrm{div}}] + \cdots =$$

$$\left[\frac{\alpha_s(\mu)}{2\pi}\right]^2 \frac{2C_A C_F\,\pi^2}{\epsilon}\left[\frac{1}{2v}\ln\left(\frac{1-v}{1+v}\right) + 1\right]\left(\frac{1-v}{v}\right)\mathrm{d}\sigma_{\mathrm{LO}} + \cdots$$

Massive DY@NNLO: remarks

$$\mathrm{d}\sigma_{\mathrm{NNLO}} = \Delta[\mathrm{d}\sigma_{\mathrm{RV}}^{\mathrm{div}}] + \cdots =$$
$$\left[\frac{\alpha_s(\mu)}{2\pi}\right]^2 \frac{2C_A C_F\ \pi^2}{\epsilon}\left[\frac{1}{2v}\ln\left(\frac{1-v}{1+v}\right)+1\right]\left(\frac{1-v}{v}\right)\mathrm{d}\sigma_{\mathrm{LO}} + \cdots$$

- Derivation based on general properties of IR factorisation →
  trivial to generalise $(C_F \to -T^1 \cdot T^2)$

- For different masses: $v \to \sqrt{1 - m_1^2 m_2^2/(p_1 \cdot p_2)^2} \to$
  requires two massive incoming partons

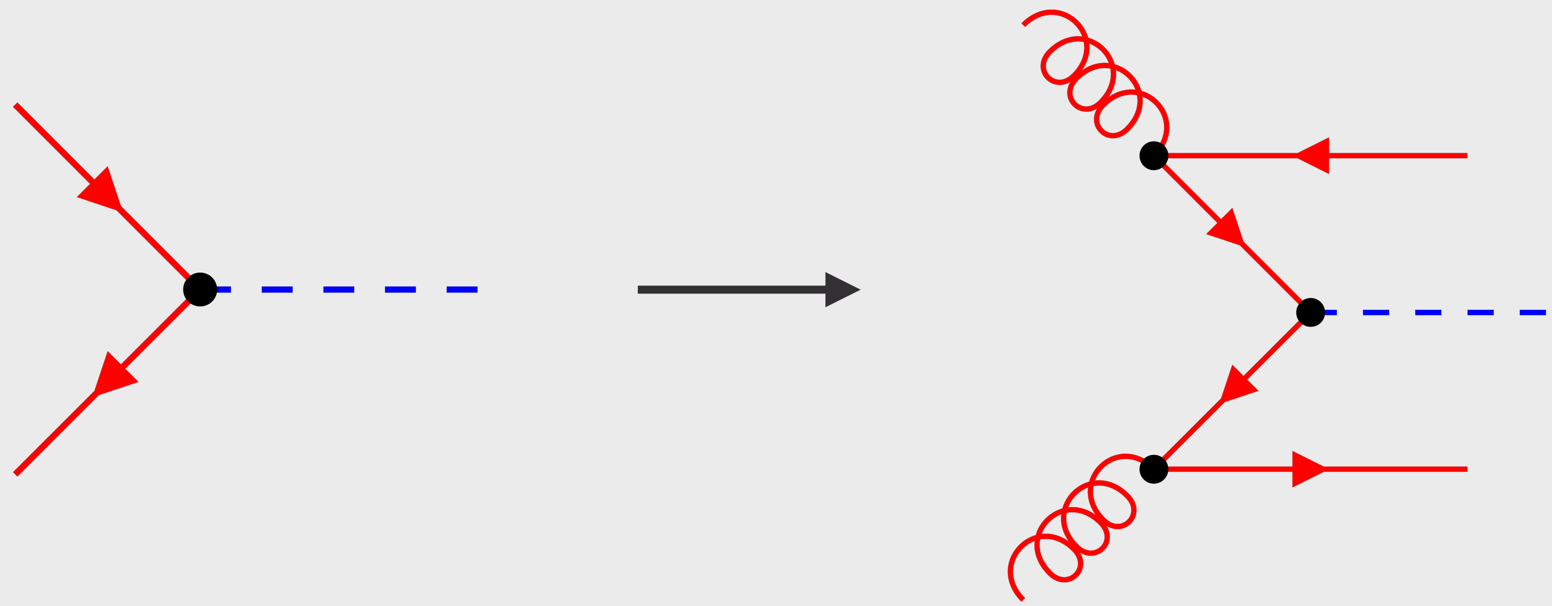
- $\Delta\sigma \sim m_q^4/m_V^4 \to$ "higher twist", and consistent with standard factorisation arguments.
  imagine target hadron H_t at the origin, and hadron H moving in the z direction.
  Field experienced by H_t: F$\sim \dfrac{e\gamma(\beta t - z)}{\sqrt{x^2+y^2+\gamma^2(\beta t - z)^2}} \sim \dfrac{1}{\gamma^2} \sim \dfrac{m^4}{s^2}$

Massive DY@NNLO: remarks

$$\mathrm{d}\sigma_{\mathrm{NNLO}} = \Delta[\mathrm{d}\sigma_{\mathrm{RV}}^{\mathrm{div}}] + \cdots =$$

$$\left[\frac{\alpha_s(\mu)}{2\pi}\right]^2 \frac{2C_A C_F\,\pi^2}{\epsilon} \left[\frac{1}{2v}\ln\left(\frac{1-v}{1+v}\right)+1\right]\left(\frac{1-v}{v}\right)\mathrm{d}\sigma_{\mathrm{LO}} + \cdots$$

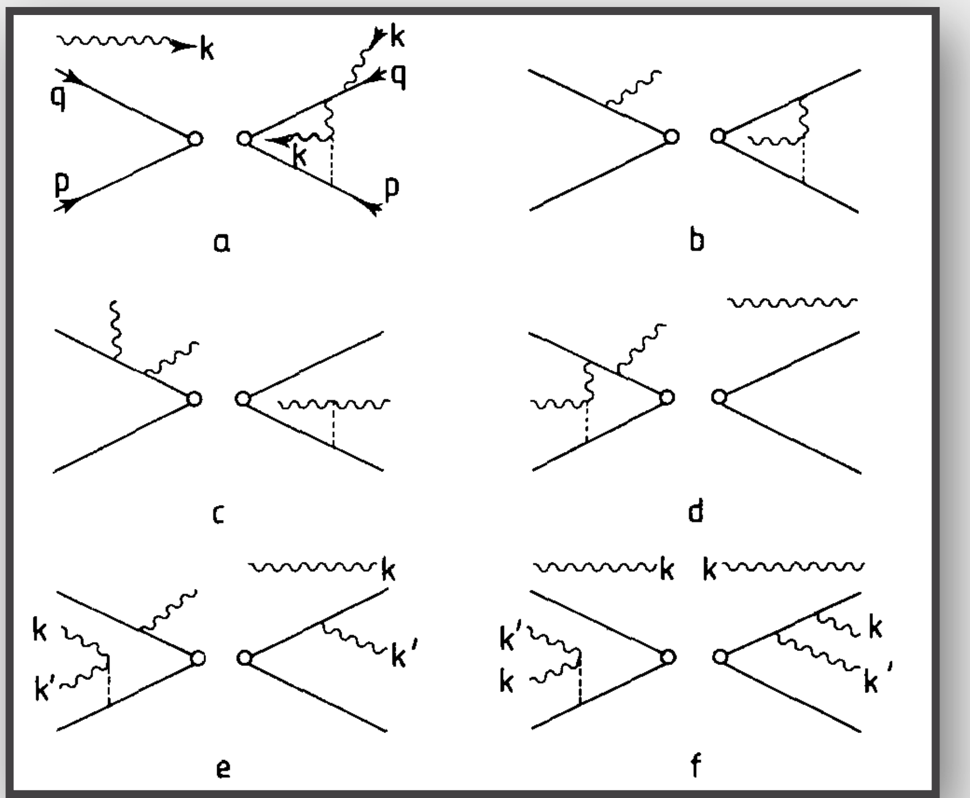
This IR sensitivity is only an issue for intrinsic heavy quarks



Perturbative HF: off-shellness acts as cut-off (though it may require tweaking of e.g. FONLL…)

Massive DY@NNLO: remarks

$$\mathrm{d}\sigma_{\mathrm{NNLO}} = \Delta[\mathrm{d}\sigma_{\mathrm{RV}}^{\mathrm{div}}] + \cdots =$$
$$\left[\frac{\alpha_s(\mu)}{2\pi}\right]^2 \frac{2C_A C_F\,\pi^2}{\epsilon}\left[\frac{1}{2v}\ln\left(\frac{1-v}{1+v}\right)+1\right]\left(\frac{1-v}{v}\right)\mathrm{d}\sigma_{\mathrm{LO}} + \cdots$$



This is *not* a violation of KLN.
Solution within KLN well-known
(disconnected gluons, spectators,
coherent states…)

Issue: interplay with collinear factorisation

Conclusions

- Even for the simplest processes, challenges for standard collinear factorisation if two heavy quarks are present in the initial state
- Origin very simple: non trivial Coulomb phase that does not cancel (contrary to QED and massless QCD)
- To some extent, expected. Light quark mass probes IR physics, power-like "higher twist" behaviour m^4/Q^4 recovered
- Not a problem for "proxy" massive 4FNS calculations, but would require tweaking → more work needed
- If m/Q small but finite, could still look at first terms in the expansion and learn something interesting. E.g. $p_t \sim m_q << Q$ (see Davide's talk yesterday) → more work needed
- Simplest example factorisation breaking. Something more severe could happen in the massless sector beyond NNLO… → more work needed

Thank you very much for your attention