Higher Derivative Theories

John F. Donoghue

Present case for higher derivative theories

With detours into:

Arrow of Time
Emergent Causality
Ostrogradsky
Quadratic gravity
Starobinsky

Work with Gabriel Menezes
- also new with Buccio, Menezes, Percacci
- and upcoming with Bazavov, D. Lee, Mandlecha, Menezes

Edinburgh
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AMHERST CENTER FOR FUNDAMENTAL INTERACTIONS
Physics at the interface: Energy, Intensity, and Cosmic frontiers
University of Massachusetts Amherst

Outline:

1) Arrow of Causality (arrow of time)

2) Higher derivatives and “Merlin modes”

3) Stability and Ostrogradsky

4) Unitarity - briefly

5) Quadratic gravity and Starobinsky inflation

6) Higher derivatives as a “new” paradigm

1) Arrow of Causality (arrow of time)

A) **"Laws of Physics" are not invariant under Time Reversal**

- but **"covariant"**

Recall that *T* is **anti-unitary**

Neglecting *T*-violating phases, Lagrangian/Action is **invariant**

$$T^\dagger \mathcal{L} T = \mathcal{L} \quad , \qquad T^\dagger S T = S$$

But the "Laws of Physics" are more than the Lagrangian
- also need to include quantization rules

The path integral (or canonical quantization) is **not** invariant

$$T^\dagger Z_+ T = T^\dagger \int [d\phi]\ e^{+iS}\ T \rightarrow Z_- = \int [d\phi] e^{-iS}$$

We will see that this distinction is meaningful

B) Requirements for Causality

i) Operators commute at spacelike separations
ii) All fields share a common definition of past and future lightcones

The second is less commonly stated, but it implied
- past lightcone can propagate influences, future lightcone cannot

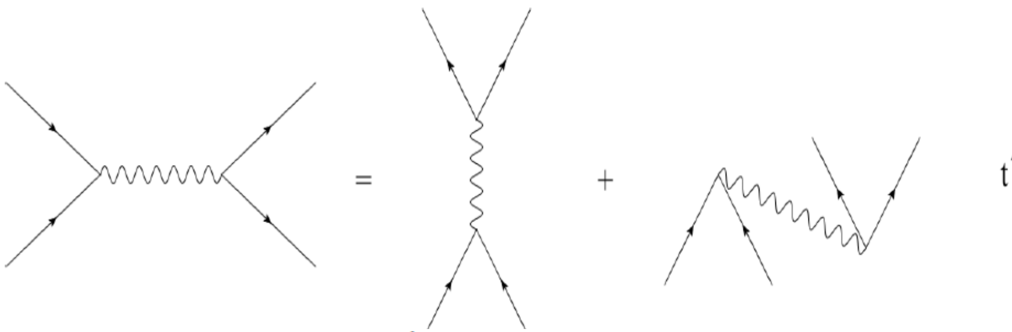
This is enforced by the iε prescription. Standard choice is +iε

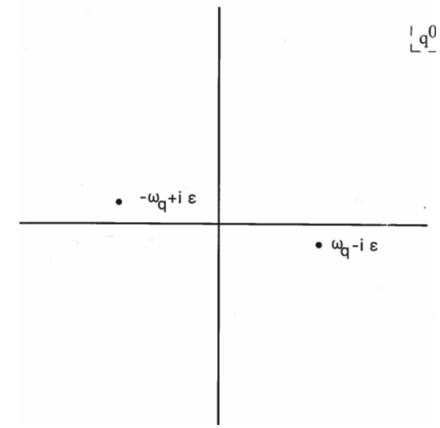
All field share a common +iε prescription in propagators

If not, causality violation.
Calculations due to Lee &Wick; Coleman;
and Grinstein, O'Connell, Wise

C) The $i\varepsilon$ prescription defines a time direction

- The forward light cone





$$iD_F(q) = \frac{i}{q^2 - m^2 + i\epsilon}$$

Decompose into time orderings:

$$iD_F(x) = D_F^{\mathrm{for}}(x)\theta(t) + D_F^{\mathrm{back}}(x)\theta(-t)$$

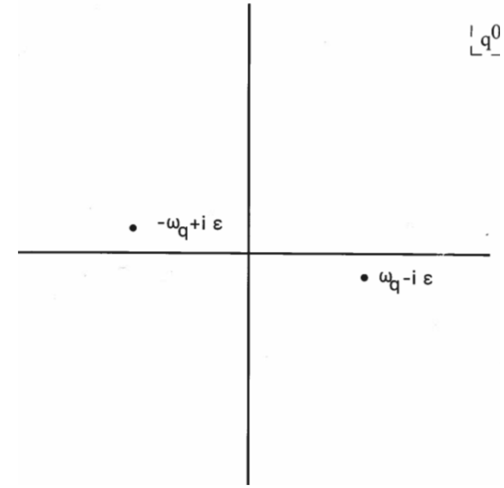
Positive energies propagate forward in time

$$D_F^{\mathrm{for}}(x) = \int \frac{d^3q}{(2\pi)^3 2E_q} e^{-i(E_q t - \vec{q}\cdot\vec{x})}$$

$$D_F^{\mathrm{back}}(x) = (D_F^{\mathrm{for}}(x))^*$$

This is the arrow of causality

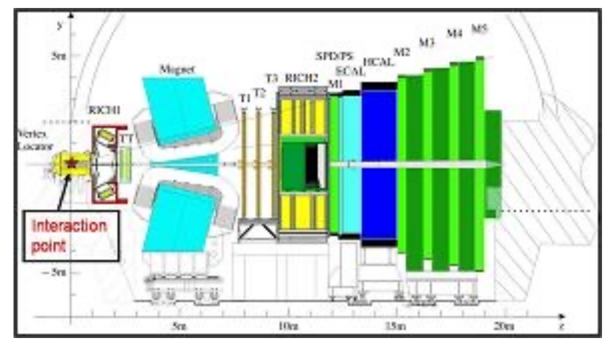
Enforced by analyticity properties



$$iD_F(q) = \frac{i}{q^2 - m^2 + i\epsilon}$$

Example: long-lived resonance production

- production A+B → R
- decay R → C+D
- decay always happens later
  - this is the arrow of causality



LHCb

Note: Time reversal relates A+B → C+D and C+D → A+B
- but experiment runs both reactions forward in time

Recall:

"Cause before effect" is not enough
- leads to effects outside light cone

Causality also requires "effect before cause"
- negative energy / antiparticles

D) The $e^{\pm iS}$ and $\pm i\varepsilon$ choices are connected

Consider generating functions:

$$Z_\pm[J] = \int [d\phi] e^{\pm i S(\phi, J)}$$
$$= \int [d\phi] e^{\pm i \int d^4x [\frac{1}{2}(\partial_\mu \phi \partial^\mu \phi - m^2 \phi^2) + J\phi]} .$$

Need to make this better defined – add

$$e^{-\epsilon \int d^4x \phi^2 / 2}$$

Solved by completing the square:

$$Z_\pm[J] = Z[0] \exp\left\{ -\frac{1}{2} \int d^4x d^4y J(x)\, iD_{\pm F}(x-y) J(y) \right\}$$

Yield propagator with specific analyticity structure

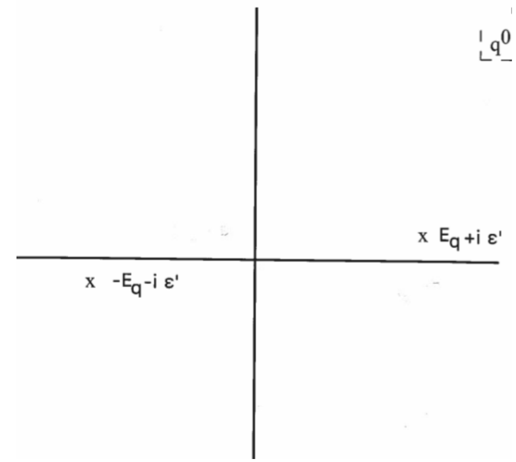
$$iD_{\pm F}(x-y) = \int \frac{d^4q}{(2\pi)^4} e^{-iq\cdot(x-y)} \frac{\pm i}{q^2 - m^2 \pm i\epsilon}$$

Using e^{-iS} results in time-reversed propagator

$$iD_{-F}(x) = D^{\rm for}_{-F}(x)\theta(t) + D^{\rm back}_{-F}(x)\theta(-t)$$

Positive energy propagates backwards in time

$$D^{\rm back}_{-F}(x) = \int \frac{d^3q}{(2\pi)^3 2E_q} e^{-i(E_q t - \vec{q}\cdot\vec{x})}$$



Use of this generating functional yields **time reversed scattering processes**

Opposite arrow of causality

Overall

Under T, laws of physics are not invariant, but covariant
- transform into similar laws with opposite flow of time

But also, quantum physics carries a **single** preferred direction
- positive energy reactions propagate in this direction
- arrow of causality - analyticity
- determined by factors of *i* in the quantization condition

Quantum physics is unidirectional
- classical physics is bidirectional

Arrow of causality → arrow of thermodynamics

2) Spectrum of higher derivative theories

Higher derivative scalar

$$\mathcal{L}_{hd} = \frac{1}{2}\left[\partial_\mu\phi\partial^\mu\phi - m^2\phi\phi - \frac{1}{M^2}\Box\phi\Box\phi\right]$$

Interacting with normal matter

$$\mathcal{L} = \mathcal{L}_\chi + \mathcal{L}_\phi - g\phi\chi^\dagger\chi$$

$$\mathcal{L}_\chi = \partial_\mu\chi^\dagger\partial^\mu\chi - m_\chi^2\chi^\dagger\chi - \lambda(\chi^\dagger\chi)^2$$

For our purposes, consider M to be very large

I will pretend that renormalized $m \to 0$

Propagator

$$iD(q) = \frac{i}{q^2 - \frac{q^4}{M^2} + i\epsilon}$$

1) Avoid spacelike poles (tachyons)
   - requires $\frac{1}{M^2} > 0$

2) Poles at $q^2 = 0 \; - i\epsilon$ and $q^2 = M^2 + i\epsilon$

$$iD(q) = \frac{i}{q^2 + i\epsilon} - \frac{i}{q^2 - M^2 - i\epsilon}$$

Massive pole has opposite arrow of causality

Massive mode decays to light particles

- $M \rightarrow \chi\bar{\chi}$ - positive energy resonance
- this is important – massive mode not an asymptotic state

$$iD(q^2) = \frac{i}{q^2 - m^2 + i\epsilon - \frac{q^4}{M^2} + \Sigma(q)} \quad .$$

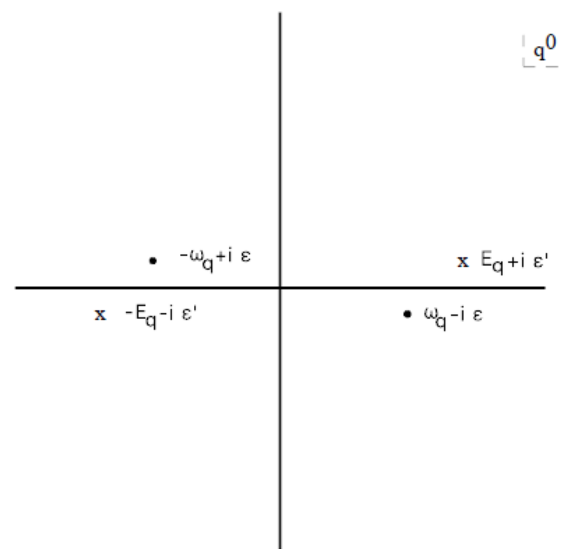
Has a positive imaginary component

$$\mathrm{Im}\Sigma \sim \frac{g^2}{32\pi} \equiv \gamma$$

Leads to exponential decay (not growth)

$$D^{\rm for}(t,\vec{x}) = \int \frac{d^3q}{(2\pi)^3}\left[\frac{e^{-i(\omega_q t - \vec{q}\cdot\vec{x})}}{2\omega_q} - \frac{e^{i(E_q t - \vec{q}\cdot\vec{x})}}{2(E_q + i\frac{\gamma}{2E_q})}e^{-\frac{\gamma t}{2E_q}}\right]$$

$$D^{\rm back}(t,\vec{x}) = \int \frac{d^3q}{(2\pi)^3}\left[\frac{e^{i(\omega_q t - \vec{q}\cdot\vec{x})}}{2\omega_q} - \frac{e^{-i(E_q t - \vec{q}\cdot\vec{x})}}{2(E_q + i\frac{\gamma}{2E_q})}e^{-\frac{\gamma|t|}{2E_q}}\right]$$



Merlin modes:

-Merlin (the wizard in the tales of King Arthur) ages backwards

"Now ordinary people are born forwards in Time, if you understand what I mean, and nearly everything in the world goes forward too. (...) But I unfortunately was born at the wrong end of time, and I have to live backwards from in front, while surrounded by a lot of people living forwards from behind."

T. H. White *Once and Future King*

Note, there is a key distinction with usual nomenclature "ghosts"

- ghost is anything with a minus sign in the numerator
- these Merlin modes refer to crucial sign $-i\gamma$ in denominator in addition

3) Stability at low energy

$$Z_\phi[\chi] = \int [d\phi] e^{i\int d^4x\left[\frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2M^2}\Box\phi\Box\phi - g\phi\chi^\dagger\chi\right]}$$

Rewrite exactly using auxiliary field to remove higher derivative

$$Z_\phi[\chi] = \int [d\phi][d\eta] e^{i\int d^4x[\mathcal{L}(\phi,\eta)]}$$

$$\mathcal{L}(\phi,\eta) = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \eta\Box\phi + \frac{1}{2}M^2\eta^2 - g\phi\chi^\dagger\chi$$

Redefine field variables using $\phi(x) = a(x) - \eta(x)$

$$\begin{aligned} Z_\phi[\chi] &= \int [da] e^{i\int d^4x\left[\frac{1}{2}\partial_\mu a\partial^\mu a - ga\chi^\dagger\chi\right]} \\ &\times \int [d\eta] e^{-i\int d^4x\left[\frac{1}{2}\partial_\mu\eta\partial^\mu\eta - \frac{1}{2}M^2\eta^2 - g\eta\chi^\dagger\chi\right]} \quad \longleftarrow \quad ** \\ &= Z_a \times Z_\eta \end{aligned}$$

The three forms are exactly equivalent

Recognize the decomposition of modes and e^{-iS} .

For low energy, integrate out the heavy η

Usual gaussian integral – seen above

$$\eta'(x) = \eta(x) - \int d^4x iD_{-F}(x-y)\ \chi^\dagger(y)\chi(y)$$

$$iD_{-F}(x-y) = \int \frac{d^4k}{(2\pi)^4} \frac{-i}{k^2 - M^2 - i\epsilon}$$

This result in:

$$Z_\eta = N e^{\int d^4x d^4y \frac{1}{2} g\chi^\dagger(x)\chi(x)\ iD_{-F}(x-y)\ g\chi^\dagger(y)\chi(y)}$$

At low energy, this becomes a contact interaction

$$Z_\eta = N e^{i\int d^4x \frac{g^2}{2M^2}[\chi^\dagger(x)\chi(x)]^2}$$

The result is just a shift in λ in the χ interaction

$$\lambda \to \lambda' = \lambda - \frac{g^2}{2M^2}$$

Low energy limit is a normal theory

The field χ has a shifted value of λ

Normal massless field a

- classical physics is the wave equation

No sign of Ostrogradsky instability

Note: really not $\hbar \to 0$ because $\hbar$ is a constant.

- Classical "limit" is kinematics where $\hbar$ is not important

Ostrogradsky

Mikhail Ostrogradsky 1801- 1862

The Ostrogradsky instability (1850)

- theories with higher time derivatives
- requires extra canonical coordinates and canonical momenta
- Hamiltonian chosen to reproduce Hamilton's equations
- result is not positive definite – even at low energy

The instability is often used to rule out higher derivative theories

What would Ostrogradsky say?

Extra canonical coordinates and momenta

$$\phi_1 = \phi \qquad \pi_1 = \frac{\partial \mathcal{L}}{\partial \dot{\phi}} - \frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \ddot{\phi}} = \left(\frac{\Box + M^2}{M^2}\right)\dot{\phi}$$

$$\phi_2 = \dot{\phi} \qquad \pi_2 = \frac{\partial \mathcal{L}}{\partial \ddot{\phi}} = -\frac{\Box}{M^2}\phi \ .$$

Hamiltonian

$$\mathcal{H}(\phi_1, \phi_2, \pi_1, \pi_2) = \pi_1 \dot{\phi}_1 + \pi_2 \dot{\phi}_2 - \mathcal{L}$$

But we have to eliminate $\ddot{\phi}$ in favor of the coordinates and momenta
This leads to the final Hamiltonian

$$\mathcal{H} = \pi_1 \phi_2 + \pi_2 \left(\nabla^2 \phi - M^2 \pi_2\right) - \mathcal{L}(\phi_1, \phi_2, \nabla^2 \phi - M^2 \pi_2)$$

The first term is the Ostrogradsky instability
- π_1 and ϕ_2 can have either sign
- this is the only place where π_1 enters the Hamiltonian

Ostrogradsky construction is not the classical version of the QFT

Canonical quantization also does not follow Ostrogradsky path

-Indefinite metric quantization (**Lee-Wick** and others)

$$\mathcal{L}(a,\eta) = \left[\frac{1}{2}\partial_\mu a \partial^\mu a - g a \chi^\dagger \chi\right] - \left[\frac{1}{2}\partial_\mu \eta \partial^\mu \eta - \frac{1}{2}M^2\eta^2 - g\eta\chi^\dagger\chi\right]$$

Roughly:

$$\pi_\eta = \frac{\partial \mathcal{L}}{\partial \dot{\eta}} = -\dot{\eta}$$

$$[\eta(x,t), \pi_\eta(x',t)] = i\hbar\delta^3(x-x') \quad \rightarrow \quad [\eta(x,t), \dot{\eta}(x',t)] = -i\hbar\delta^3(x-x')$$

Solved by:

$$[a(p), a^\dagger(p')] = -\delta^3(p-p')$$

such that

$$H_\eta = -\int \frac{d^3p}{(2\pi)^3}\sqrt{p^2+M^2}\; a^\dagger(p)a(p)$$

yields positive energy when acting on states (!)

Nonperturbative Stability? - Lattice studies

Jansen, Kuti, Liu 1992-94

- third order HD operator – makes theory finite
- study of heavy Higgs and triviality

$$\mathcal{L} = -\frac{1}{2}\Phi_\alpha(x)\left(m^2 + \Box + M^{-4}\Box^3\right)\Phi_\alpha(x) - \lambda_0\left(\Phi_\alpha(x)\Phi_\alpha(x)\right)^2,$$

Bazavov, JFD, Dean Lee, Mandlecha, Menezes (ongoing)

- second order operator
- goals: renormalizable non-linear sigma models

JKL: Various results:

- main issue for this talk is **non-perturbative stability**

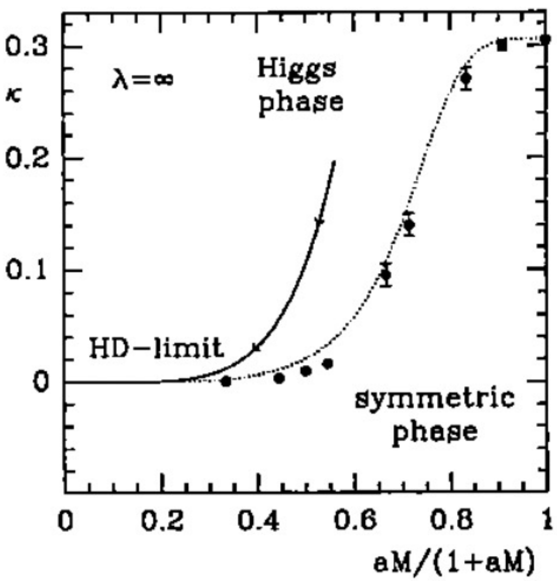


Figure 1. The phase diagram of the lattice model at infinite bare coupling. The dotted line is calculated in the large-N expansion. The solid line displays the fixed M_R/m_H ratio towards the continuum limit of the higher derivative theory.

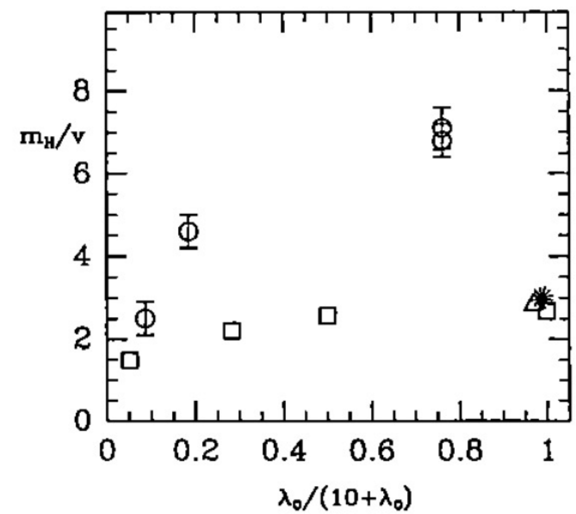


Figure 2. The circles are from our simulation results. They are compared with the simple $O(4)$ model on a hypercubic lattice [2] (squares), with Symanzik improved action on a hypercubic lattice [5] (star), and with dimension six interaction terms added on F_4 lattices [6] (triangle).

Exploring physics of a heavy Higgs

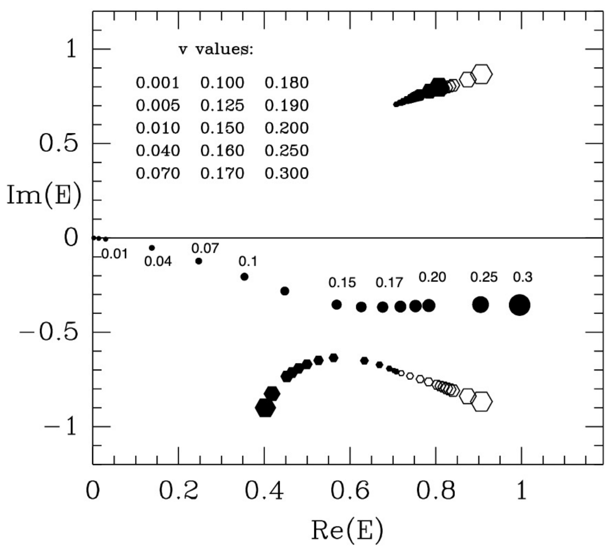


Figure 3.9: The complex poles of the large N Higgs propagator is shown on the first and the second Riemann sheets. The bare coupling constant is set to infinity in this figure. The open hexagonal points represent the ghost pair poles on the first Riemann sheet. The filled hexagonal points are the 'image' of the ghost on the second Riemann sheet. The filled circles are the Higgs poles on the second sheet. The size of the points reflects the different v values.

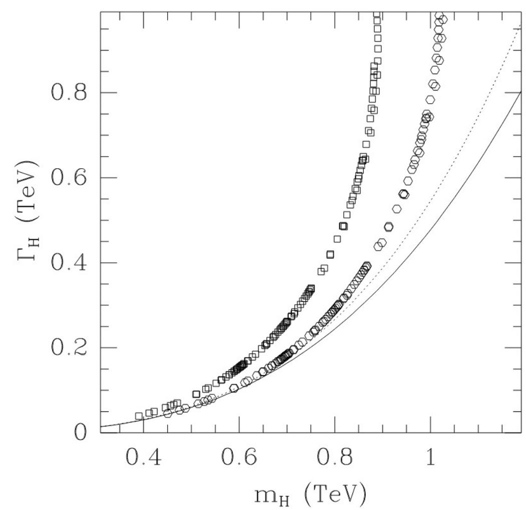


Figure 3.11: The large N result for the width of the Higgs particle as a function of the Higgs mass is shown in the Pauli-Villars higher derivative $O(N)$ theory. The open squares are the naive large N prediction at $N = 4$. The open hexagons are the large N results after the number of decay channels has been corrected. The solid line is the leading order perturbation result and the dashed line is the perturbation result up to the second order. The corrected large N width agrees with the perturbative prediction very well in the weakly interacting regime as it should. The naive large N result overshoots by about 30 to 40 percent.

4) Unitarity:

$$\langle f|T|i\rangle - \langle f|T^\dagger|i\rangle = i\sum_j \langle f|T^\dagger|j\rangle\langle j|T|i\rangle$$

Who counts in unitarity relation?

- Veltman 1963
- **only stable particles count**
- they form asymptotic Hilbert space
- **do not** make any cuts on unstable resonances

UNITARITY AND CAUSALITY IN A RENORMALIZABLE FIELD THEORY WITH UNSTABLE PARTICLES

M. VELTMAN *)

In HD theory, massive Merlin mode is not asymptotic state

- decay to light states

Veltman proof of unitarity goes through here also

- if only cuts are on the stable particles

Unitarity, stability, and loops of unstable ghosts

John F. Donoghue and Gabriel Menezes

Phys. Rev. D **100**, 105006 – Published 12 November 2019

However, practical issues remain:

Recall causality ~ analyticity

Veltman-style proof does not use Wick rotation
 - but loop calculations do

Unitarity at one loop uses “Lee-Wick” contour

Higher loops get complicated
 - Cutkosky-Landshoff-Olive-Polkinghorne
 - Anselmi working to sort this out

Here be dragons - not fully understood

This is one reason lattice work is useful
 - albeit Euclidean

5) Quadratic gravity:

$$S = \int d^4x\sqrt{-g}\left[-\Lambda_{vac} + \frac{1}{16\pi G}R + \frac{1}{6f_0^2}R^2 - \frac{1}{2f_2^2}C_{\mu\nu\alpha\beta}C^{\mu\nu\alpha\beta}\right]$$

Renormalizable QFT for quantum gravity Stelle

- New but technical – can be tachyon free and asymptotically free

(Buccio, JFD, Menezes, Percacci arXiv:2403.02397)

This is a HD theory $R \sim \partial^2 g$ $R^2 \sim \partial^2 g \partial^2 g$

The C^2 term leads to a spin 2 Merlin mode (partner with graviton)

$$m_2^2 = f_2^2 M_P^2$$

The R^2 term is spin 0 and ghost free (Ghost is gauge artifact)

$$m_0^2 = f_0^2 M_P^2$$

Mixed causal structure due to spin-2 Merlin

- near $m_2^2 = f_2^2 M_P^2$
- do we even expect usual causality in QG near Planck scale?

A Quadratic Gravity layer of reality?

If inflation occurs and is Starobinsky style:
- requires R^2 to be dynamically active
- not a small EFT perturbation
- then C^2 also expected to be dynamical

Both couplings are required with matter loops
- mixed under RG flow

$$\beta_{f_2^2} = -\frac{1}{(4\pi)^2}\frac{(539 f_2^2 + 20 f_0^2) f_2^2}{30} ,$$
$$\beta_{f_0^2} = \frac{1}{(4\pi)^2}\frac{6 f_0^4 + 36 f_2^2 f_0^2 - 420 f_2^4}{36} ,$$

This implies a layer of reality with active Quadratic Gravity
- need not be ultimate theory
- but at least temporary renormalizable theory

Can we utilize and/or observe lack of microcausality?

Possibly homogeneous initial conditions for Starobinsky inflation?
- recall acausal homogeneity arguments
- issues resurfaces in initial conditions for inflation

Backwards-in-time propagation can spread uniformity
- effective outside the light cone

Perhaps even a possible alternative to inflation?

I would be interested in discussing this during this conference!

Related:

Higher-Order Gravity, Finite Action, and a Safe Beginning for the Universe

Jean-Luc Lehners, K.S. Stelle (Dec 21, 2023)

e-Print: 2312.14048 [hep-th]

Can HD become a "new" paradigm?

Actually goes back to Lee-Wick and Coleman in the 1960s
- makes renormalizable theories finite
- makes non-renormalizable theories renormalizable

Also, recall Wilsonian RG methods
- **operators of all dimensions generated by RG flow**

Is causality/analyticity emergent?
- physics is experimental science
- causality only tested on macroscopic scales
- fully consistent with HD theories

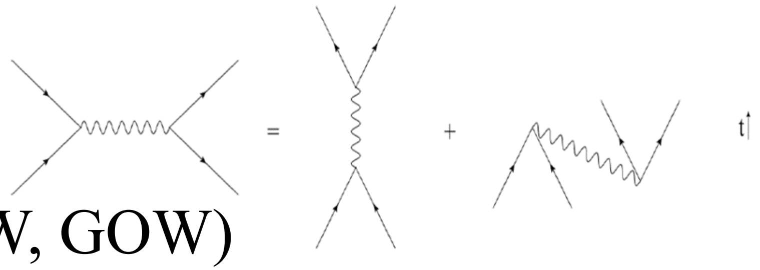
Causality as an emergent macroscopic phenomenon: The Lee-Wick $O(N)$ model

Benjamín Grinstein, Donal O'Connell, and Mark B. Wise
Phys. Rev. D **79**, 105019 – Published 21 May 2009

<u>Testing causlity:</u>

Lee, Wick
Coleman
Grinstein, O'Connell, Wise
Alvarez, Da Roid, Schat, Szynkman

Vertex displacements: (ADSS)
- look for final state emergence
- before beam collision

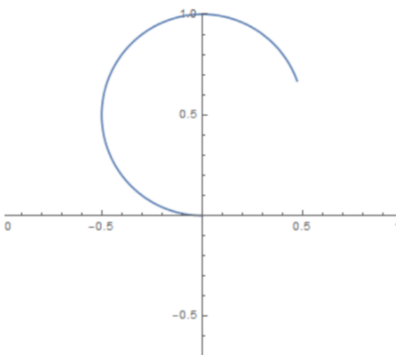


Form wavepackets – early arrival (LW, GOW)
- wavepacket description of scattering process
- some components arrive at detector early

Resonance Wigner time delay reversal
- normal resonaces counterclockwise on Argand diagram

$$\Delta t \sim \frac{\partial \delta}{\partial E} \sim> 0$$

- Merlin modes are clockwise resonance



Swampland EFT coefficients
- causality/analyticity constraints

Summary

Main focus here was causality:

1) Arrow of causality tied to factors of i in quantization

2) Higher derivative theories have mixed causal structure

3) Such theories appear stable and unitary

4) Quadratic gravity is potential application (+Starobinsky)

5) More generally, can macrocausality be emergent?