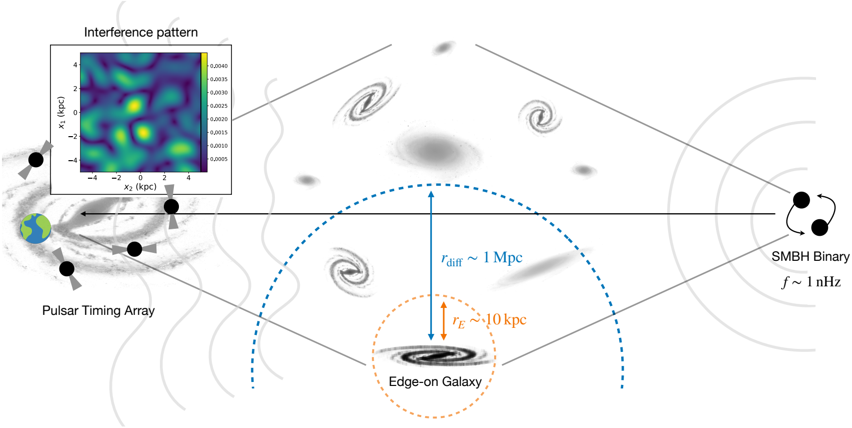
Resolving the Hubble tension through diffraction of gravitational waves

Ue-Li Pen, Dylan Jow

April 19, 2024

Diffractive/Evanescent Gravitational Wave Lensing



Jow+ in prep

Gravitational Waves – Wave optics

- Coherent, distant source of radiation
- Interference effects under multi-path propagation
- Kirchoff-Fresnel path integral
- semi-classical concepts: stationary phase, Eikonal limit
- Witten 2010: generalized by Picard-Lefschetz theory
- Morse index, evanescent/complex images: measure time delays in weak lensing

Pulsar Timing Arrays (PTAs)

- initial GW evidence in 2023 (nanoGrav++)
- currently from a few dozen nearby PSRs
- full map of galaxy with SKA, orders of magnitude improvement in sensitivity, resolution
- Precision distances with Scintillometry
- Coherent imaging GW telescope with arcsecond resolution
- no “stochastic” regime: small number of SMBH near merger
- counterparts of supermassive BH binaries, precise redshifts

Diffractive (evanescent) imaging

- most galaxies too weak to form real gravitational lens images
- always form evanescent images through wave optics
- analogous to quantum mechanical tunelling
- diffractive angle $\theta \sim \lambda/D$
- $\lambda \sim$ pc, $D \sim 500$pc $\longrightarrow \theta \sim 0.1^o$
- dominated by edge-on spirals!

Time delay H_0

- stack diffractive GW flux from all edge-on galaxies
- distance from angle-delay relation $\tau \sim \theta^2 L$
- Shapiro delay neglible off-axis
- sensitivity in SKA/DSA era

Waves, Particles, Geodesics

- Everything is a wave
- Short wavelength limit has classical particle interpretation: Eikonal
- Picard-Lefschetz: no waves, everything is a particle!
- evanescent lensing: complex eikonal lensing
- New tool for QM Cosmology, FRBs, pulsars
- alternative picture for quantization: quantum gravity?

Fermat's Principle

- light takes shortest path. Why?
- Huygen's principle: light takes all paths
- Kirchoff/Feynmann path integral
- stationary phase dominates
- classical equation of motion: extremal paths, independent of wavelength

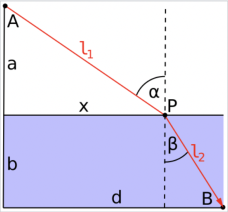


Fig. 1: Fermat's principle in the case of refraction of light at a flat surface between (say) air and water. Given an object-point *A* in the air, and an observation point *B* in the water, the refraction point *P* is that which

Huygen's Principle: Path Integral

- Kirchoff integral $A(\mu) = \int e^{iS(\theta,\mu)} d\theta$
- $S = \nu[(\theta - \mu)^2 + \Psi(\theta)]$
- Highly oscillatory integral, even for $\Psi = 0$
- Stationary phase points: $\partial_\theta S = 0$ leads to (complex) Eikonal images θ_i.
- flux/phase through curvature expansion (known as *steepest descent*): exact as $\nu \to \infty$
- Geometric limit considers only *Real* solutions θ_i and gives up phase information (length of trajectory)
- Geometric optics applicable at short wavelengths for extended sources (e.g. optical gravitational lensing of finite size sources, stars)

Optics: Geometric, Eikonal, Wave, P-L

- Consider 1-D lens
- lensing potential $\Psi(\theta)$
- deflection Ψ'
- simplify for $D_{\mathrm{ds}} = \infty$

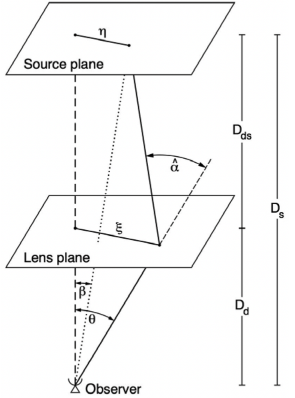
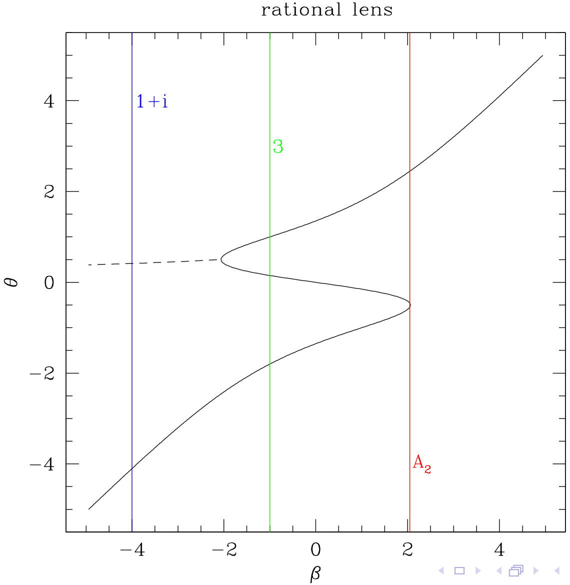


Figure 1. Geometry of a lensing event (reproduced from Schneider et al. 1992). See text for details.

Evanescent Images

- consider "rational lens" potential $\psi(\theta) = \alpha/(1+\theta^2)$
- Geometric/eikonal images at $\psi' = \theta$
- 5 roots. 1 or 3 real roots, rest imaginary
- P-L: at most one imaginary image contributes!
- Resurgence theory to classify?
- Evanescent (imaginary) image can be brighter than unlensed real image

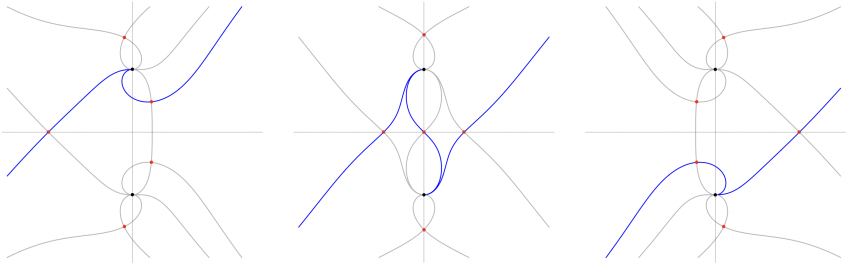
Rational 1-D lens



Picard-Lefschetz Theory

- descend integral along real line along Morse function Im(S)
- contour deforms into finite number of Thimbles of constant phase with maximum at saddle point (extrema $dS = 0$)
- correctly identifies relevant saddle points
- resolves numerical challenges of oscillatory integral
- complex analysis works in multiple variables
- elevates concept of “image” deep into wave optics
- multiple public implementations (Feldbrugge+, Jow+)

Picard-Lefschetz Theory



(a) $\alpha = 2, \mu < -\mu_c$ (b) $\alpha = 2, -\mu_c < \mu < \mu_c$ (c) $\alpha = 2, \mu > \mu_c$

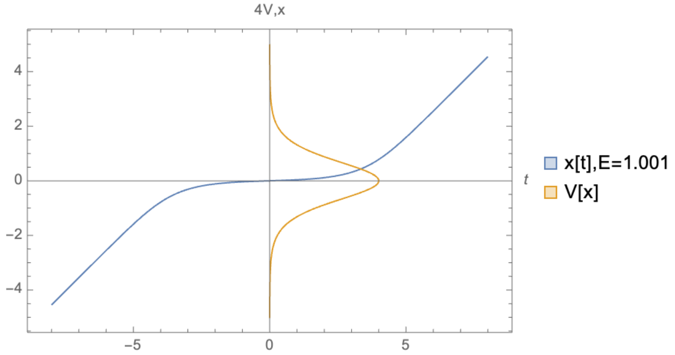
Feldbrugge+2019

Quantum Mechanics

- Feynmann path integral saddle points are similarly complex
- interprets tunneling as complex position path

Rosen-Morse Potential

- $V(x) = \mathrm{sech}^2(x)$
- exactly solvable: $\sinh[x(t)] = \sqrt{\frac{1-E}{E}} \cosh[\sqrt{E}(t + t_0)]$
- for $\lim_{E \to 1+\epsilon^2} = \sinh^{-1}[\epsilon \sinh(t)]$



Reinterpretation of Quantum Mechanics: 2309.12420 w/Feldbrugge, Jow

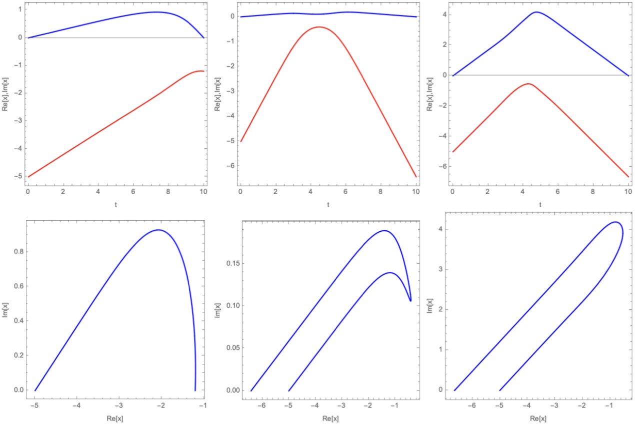


Figure 10: The complex paths as a function of time (top) and in the complex plane (bottom). *Left:* a classical path on the left blue arc. *Centre:* a classical path on the right blue arc. *Right:* a classical path on the green arc.

Discussion

- Eikonal effects applicable to compact radio sources, e.g. FRBs, pulsars
- full wave effect dominates for long wavelengths as Fresnel scale is bigger then Einstein radius
- microlensing down to planet size
- gravitational waves: LIGO, LISA, PTA
- with SKA PTA parameters and favourable source geometries, H_0 measurement at 0.1% possible.

Conclusions

- wave optics changes nature of astrophysical observables: Coherent FRB/pulsar/GW radiation one of the potentially most precise measurements in physics
- PTA weak diffractive lensing may give new tool for Hubble Constant tension
- evanescent lensing/instantons
- evanescent lensing images understood through Picard-Lefschetz theory and complex/evanescent images
- at long wavelength, evanescent lensed images are unsuppressed