

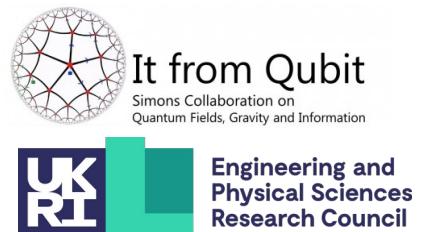
# A postquantum theory of classical gravity?



WITH  
Andrzej Grudka, Muhammad Sajjad, Andrea  
Russo, Zach Weller-Davies

Phys. Rev. X 13, 041040 (2023)  
Nature Comms 14, 7910 (2023)  
arXiv:2302.07283  
arXiv:2402.19459  
arXiv:2402.17844

Jonathan Oppenheim  
@postquantum  
Edinburgh  
April 19, 2024



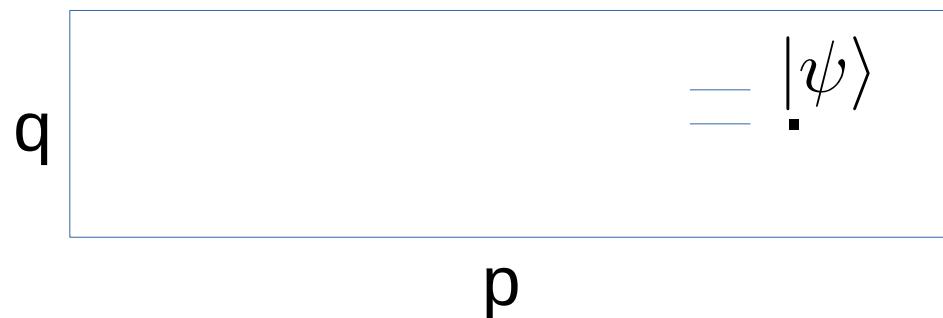
# Frameworks

## Quantum Mechanics

$$\frac{\partial \hat{\sigma}}{\partial t} = -i[\hat{H}, \hat{\sigma}]$$

## Classical Mechanics

$$\frac{\partial \rho(q,p)}{\partial t} = \{H(q,p), \rho(q,p)\}$$



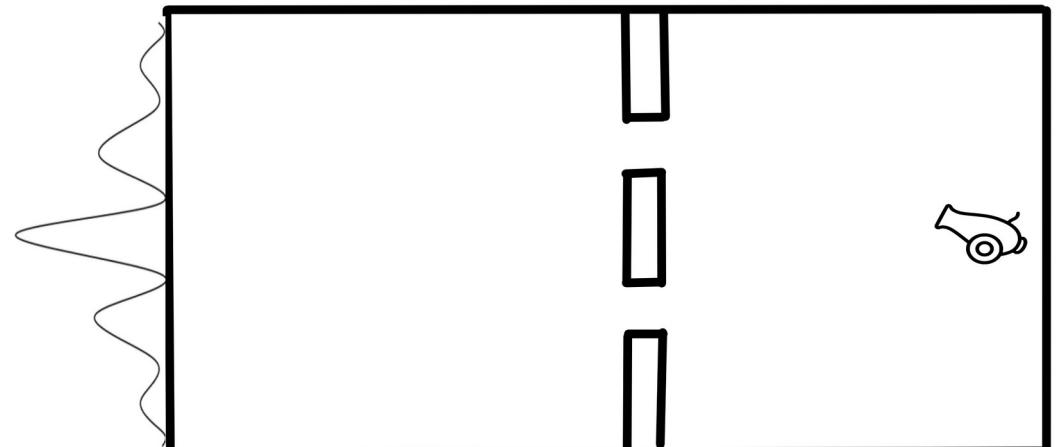
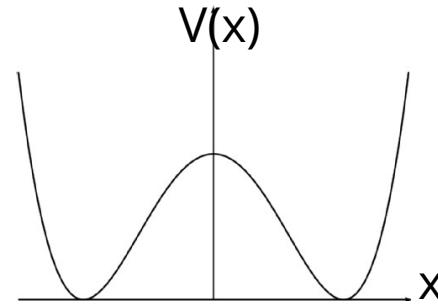
# Frameworks

## Quantum Mechanics

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## Classical Mechanics

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**Back-reaction**

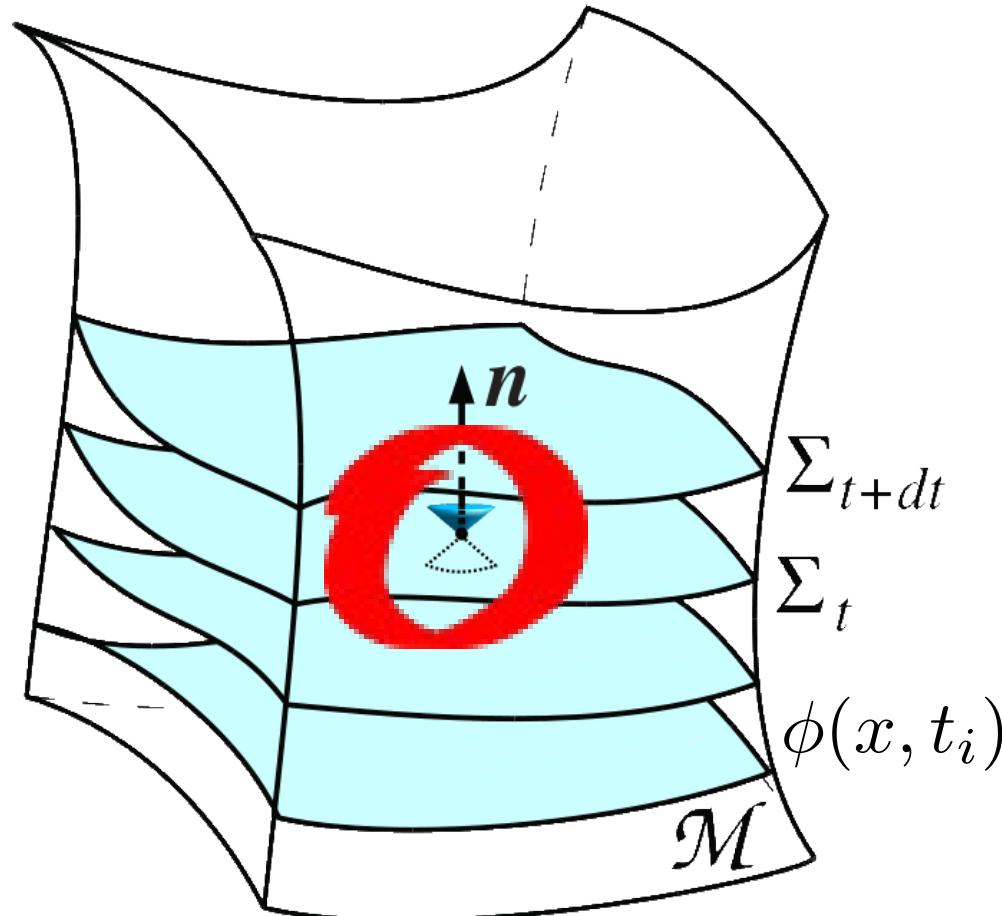
# Motivation (fundamental or effective)

$$\psi(t_f)$$

$$[\phi(x), \phi(y)] = 0$$

$$\mathcal{H}(x)|\psi\rangle = 0$$

$$\psi(t_i)$$



# Classical-quantum dynamics

## Debate

---

No

Feynman (1957)  
DeWitt (1962)  
Unruh (1984)  
Aharonov (~1986)  
Eppley & Hannah (1977)  
Unruh (1984)  
Caro & Salcedo (1999)  
Terno (2004)  
Carlip (2008)  
Marletto Vedral (2017)  
Galley, Giacomini, Selby (2022)

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Maybe

Sherry & Sudarshan (1978)  
Boucher & Traschen (1988)  
Kapral (1999)  
Peres & Terno (2001)  
Hall & Reginatto (2005)  
Mattingly (2006)  
Albers, Kiefer & Reginatto (2008)  
Kent (2018)

# Classical-quantum dynamics

## History

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Semi-classical Einstein  
(pathological when  
fluctuations are large)

Page & Geilker (1981);  
Gisin (1989)

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### Simple examples

Blanchard & Jadczyk (1994);  
Diosi (1995);  
Poulin (2017);

Kafri, Taylor, Milburn (2014);  
Diosi, Tilloy (2016)

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Quantum chemistry  
(negative probabilities)

Kapral review (2006);  
Koopman-von Neumann (1931-32)

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### Experiments!

Kafri & Taylor (2013);  
Bose et. al. (2017);  
Marletto et. al. (2017)  
Lami, Pedarnals, Plenio (2022)  
Carney (2108.06320)

# Classical-quantum dynamics

## Results

---

Most general form

---

Born rule & No need for  
Measurement postulate

---

**Renormalisable  
without Ghosts**

---

Decoherence vs  
Diffusion trade-off

---

Small  $\Lambda$   
dark matter?

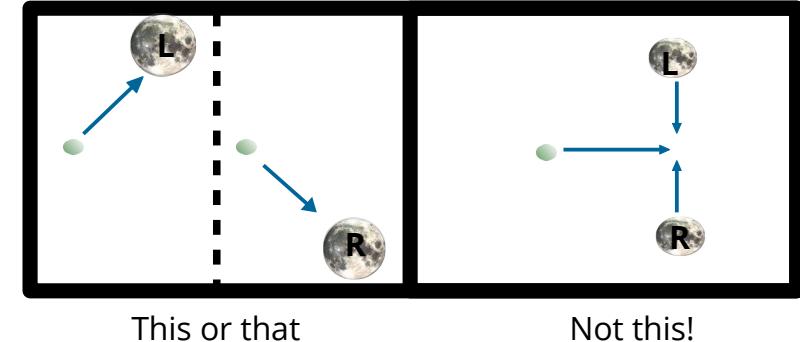
EXPERIMENT

EXPERIMENT

# Semi-classical equations are pathological

$$G_{\mu\nu} = \frac{8\pi G}{c^4} \langle \hat{T}_{\mu\nu} \rangle$$

$$\rho^{(gm)} = \frac{1}{2}\rho_L^{(g)}\rho_L^{(m)} + \frac{1}{2}\rho_R^{(g)}\rho_R^{(m)}$$



**Consistent Dynamics: linear and preserve state-space (completely positive and norm preserving)**

**Requires decoherence and diffusion**

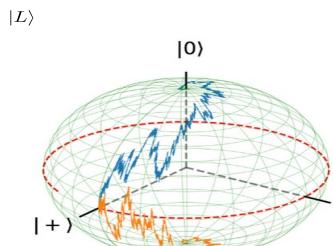
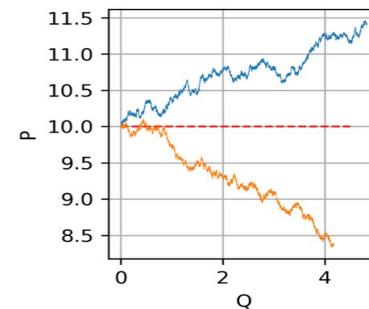
# CQ Dynamics

## Path Integral

$$\rho(q, p, \phi^\pm, t_f) = \int \mathcal{D}q \mathcal{D}p \mathcal{D}\phi^\pm e^{iS_C[q,p] + iS[\phi^+] - iS[\phi^-] + iS_{FV}[\phi^\pm] + iS_{CQ}[q,p,\phi^\pm]} \delta(\dot{q} - \frac{p}{m}) \rho(q, p, \phi^\pm, t_i)$$

JO, Zach Weller-Davies

## Trajectories



JO, I. Layton, Z. Weller-Davies

## Master Eqn

$$\frac{\partial \hat{\rho}}{\partial t} \approx \{H^{(grav)}, \hat{\rho}\} - i[\hat{H}^{(m)}, \hat{\rho}] + \frac{1}{2}\{\hat{H}^{(m)}, \hat{\rho}\} - \frac{1}{2}\{\hat{\rho}, \hat{H}^{(m)}\}$$

$$+ \int dxdx' \frac{\delta^2}{\delta\pi_\Phi(x)\delta\pi_\Phi(x')} (D_2(x, x')\hat{\rho}) + \frac{1}{2} \int dxdx' D_0(x, x') ([\hat{m}(x), [\hat{\rho}, \hat{m}(x')]])$$

CPTP MAP

JO, Sparaciari, Soda, Weller-Davies

# A post-quantum theory of classical gravity?

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Renormalisable  
without Ghosts!

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Anomalous  
contribution to the  
metric (dark matter,  
dark energy?)

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What is the most  
general form of  
CQ dynamics?

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Decoherence vs  
Diffusion: testing  
quantum gravity

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# Path Integrals

## Quantum Mechanics

$$\langle q_f, t_f | q_i, t_i \rangle = \int_{q_i}^{q_f} \mathcal{D}q \ e^{i \int dt (\frac{1}{2} m \dot{q}^2 - V(q))}$$



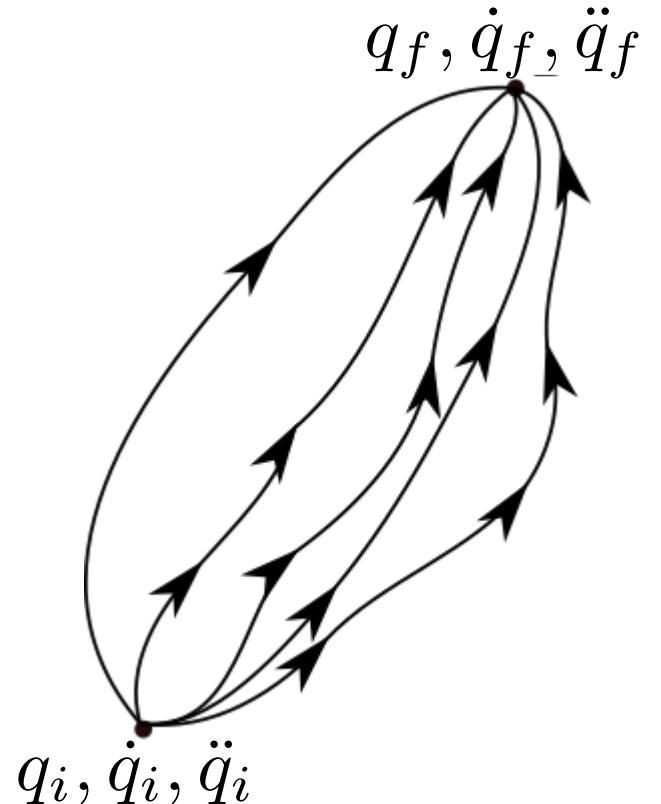
# Brownian motion

## Classical Mechanics (stochastic)

$$\ddot{q} = \frac{F}{m} + j(t)$$

$$\langle j(t) \rangle = 0, \quad \langle j(t)j(s) \rangle = D_2 \delta(s, t)$$

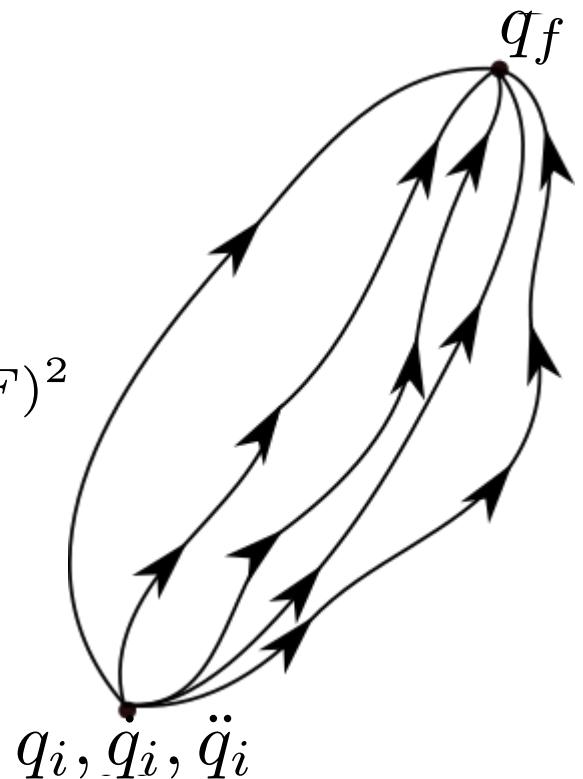
Langevin Eqn (or Fokker-Planck, or Ito)



# Path Integral for Brownian motion

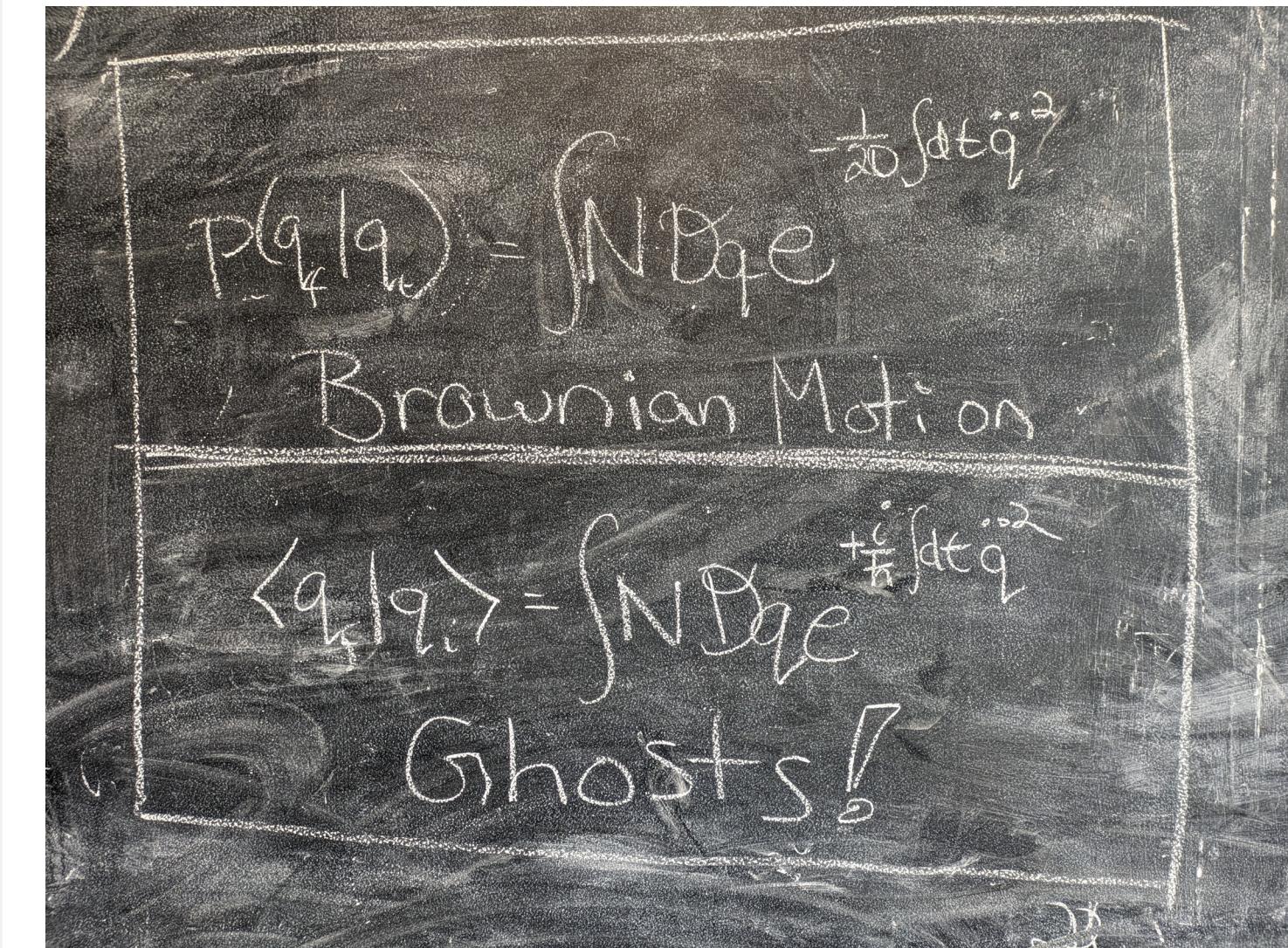
## Classical Mechanics (stochastic)

$$\rho(q_f, t_f | q_i, \dot{q}_i, \ddot{q}_i, t_i) = \mathcal{N} \int_{q_i, \dot{q}_i, \ddot{q}_i}^{q_f} \mathcal{D}q e^{-\frac{1}{2D_2} \int dt (m\ddot{q} - F)^2}$$



Onsager-Machlup

# Main Message!



# No Ostrogradsky Instability

$$\mathcal{I}_{OM} = -\frac{1}{2D_2} \int dt (m\ddot{q} - F)^2$$

$$\mathcal{I}_{QM} = \frac{i}{\hbar} \int dt (m\ddot{q} - F)^2$$

$$H = \frac{p^2}{2m} + V(q) + j(t)$$

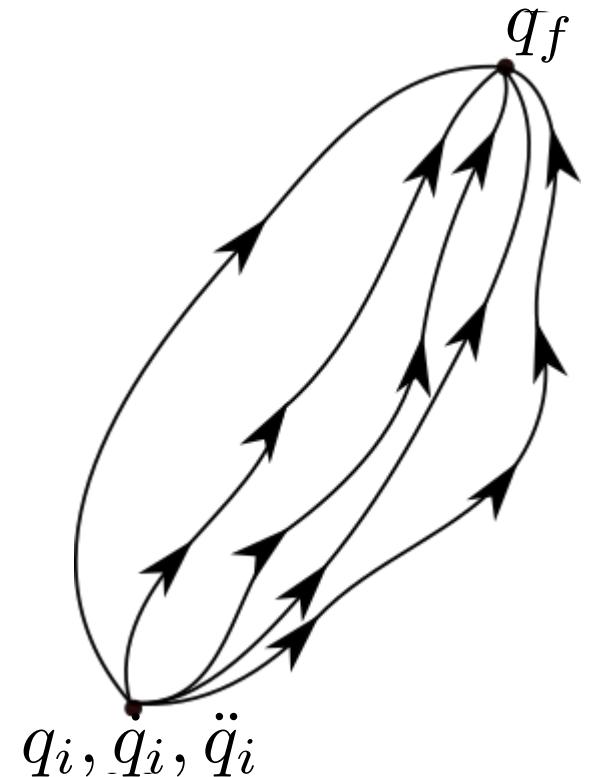
$$H = \frac{D_2 P_1^2}{2m^2} + P_1 X_2 + \frac{F P_2}{m}$$

# Path Integral for Stochastic Dynamics

**“Equation of Motion Squared”**

$$\mathcal{I} = -\frac{1}{2D_2} \int dt (m\ddot{q} - F)^2$$

$$\mathcal{I} = -\frac{1}{2D_2} \int dt \left( \frac{d}{dt} \frac{\partial \mathcal{L}}{\partial \dot{q}} - \frac{\partial \mathcal{L}}{\partial q} \right)^2$$



Onsager-Machlup

# Path Integral for Stochastic GR

**“Equation of Motion Squared”**

$$\mathcal{I} = -\frac{1}{2D_2} \int dt \left( \frac{d}{dt} \frac{\partial \mathcal{L}}{\partial \dot{q}} - \frac{\partial \mathcal{L}}{\partial q} \right)^2$$

$$\mathcal{I} = -\frac{1}{2D_2} \int dt \left( R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R \right) \left( R^{\mu\nu} - \frac{1}{2} g^{\mu\nu} R \right)$$

# Path Integral for Stochastic GR

## Stochastic GR vs Quadratic Gravity

$$\mathcal{I} = -\frac{1}{2D_2} \int dt \left( R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R \right) \left( R^{\mu\nu} - \frac{1}{2} g^{\mu\nu} R \right)$$

$$\mathcal{I} = -\frac{1}{2D_2} \int dt \left( R_{\mu\nu} R^{\mu\nu} + |\beta| R^2 \right)$$

Grudka, JO, Sajjad, Russo,  
2402.17844

# Path Integral for Stochastic GR

## Stochastic GR vs Quadratic Gravity

$$\mathcal{I}^{(gr)} = - \int d^4x \left( \frac{1}{2D_2} R^2 + \frac{1}{2D_w} C^2 + \frac{1}{\alpha} \mathcal{G} \right)$$

$$\mathcal{I}^{(m)} = - \frac{G_N}{16\pi D_2} \int dx \sqrt{-g} \left[ -2\bar{T}^{\mu\nu}R_{\mu\nu} + (1 - 2\beta)\bar{T}R - 8\pi G_N \beta \bar{T}^2 + 8\pi G_N \bar{T}^{\mu\nu} \bar{T}_{\mu\nu} \right]$$

Grudka, JO, Sajjad, Russo,  
2402.17844

# A post-quantum theory of classical gravity?

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Renormalisable  
without Ghosts!

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contribution to the  
metric (dark energy,  
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What is the most  
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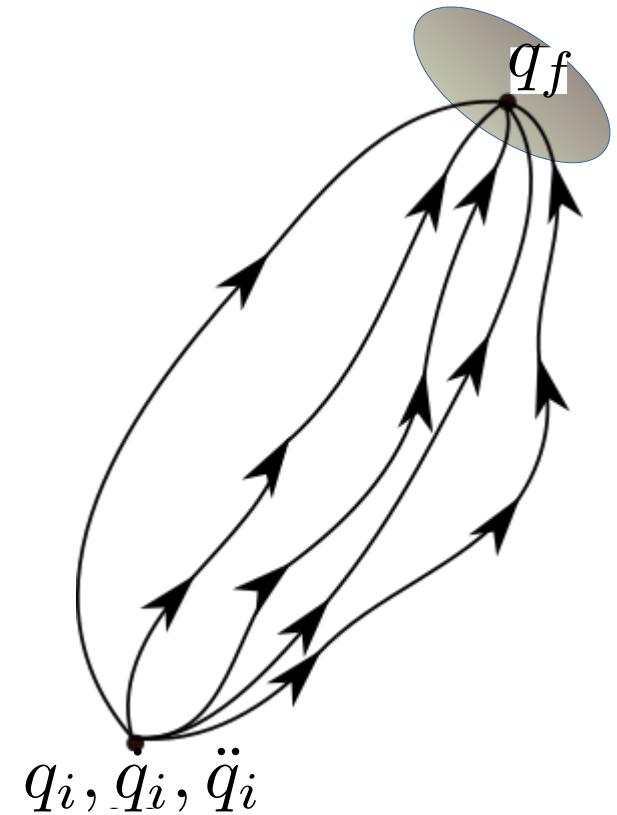
# Anomalous contribution (Brownian motion)

**The most probable path (MPP)**

$$\mathcal{I} = -\frac{1}{2D_2} \int dt (m\ddot{q})^2$$

$$\frac{d^4 q}{dt^4} = 0$$

$$q_{\text{MPP}} = \kappa_0 + \kappa_1 t + \frac{1}{2}\kappa_2 t^2 + \frac{1}{6}\kappa_3 t^3$$



Onsager-Machlup

# Anomalous contribution (Brownian motion)

## The most probably path (MPP)

$$\mathcal{I} = -\frac{1}{2D_2} \int dt (m\ddot{q})^2$$

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$$Z = \int \mathcal{D}q e^{-\frac{1}{2D_2} \int dt (m\ddot{q})^2}$$

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# Anomalous contribution (Brownian motion)

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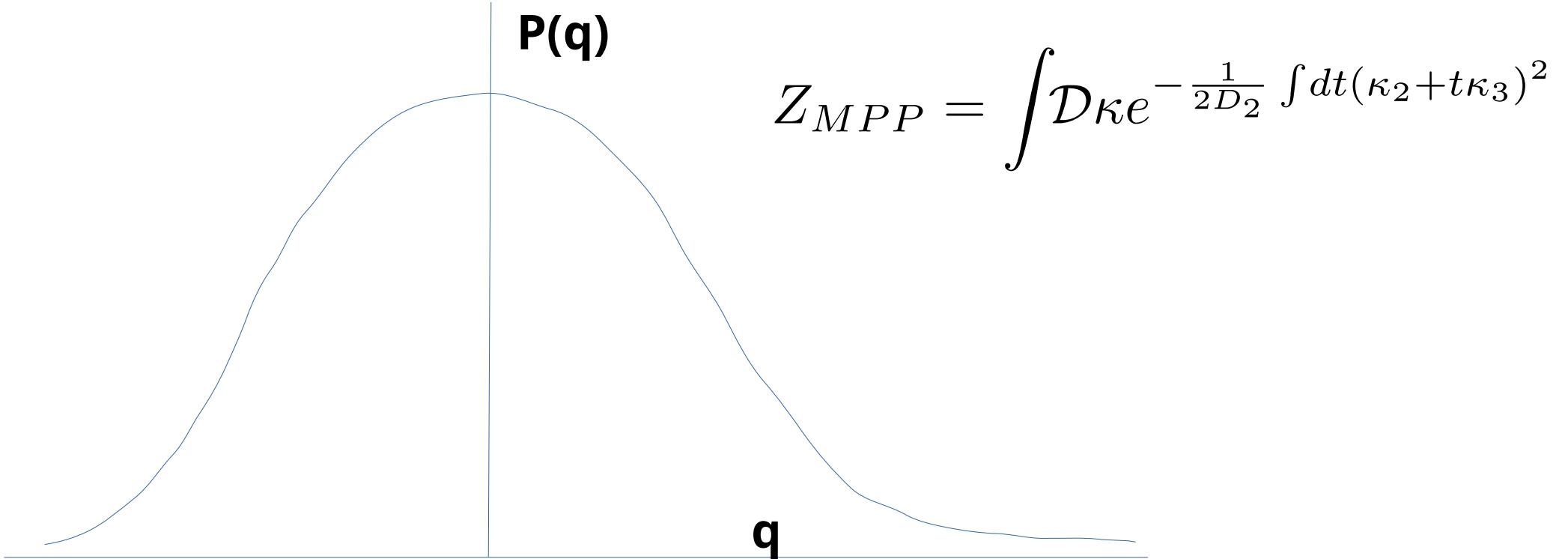
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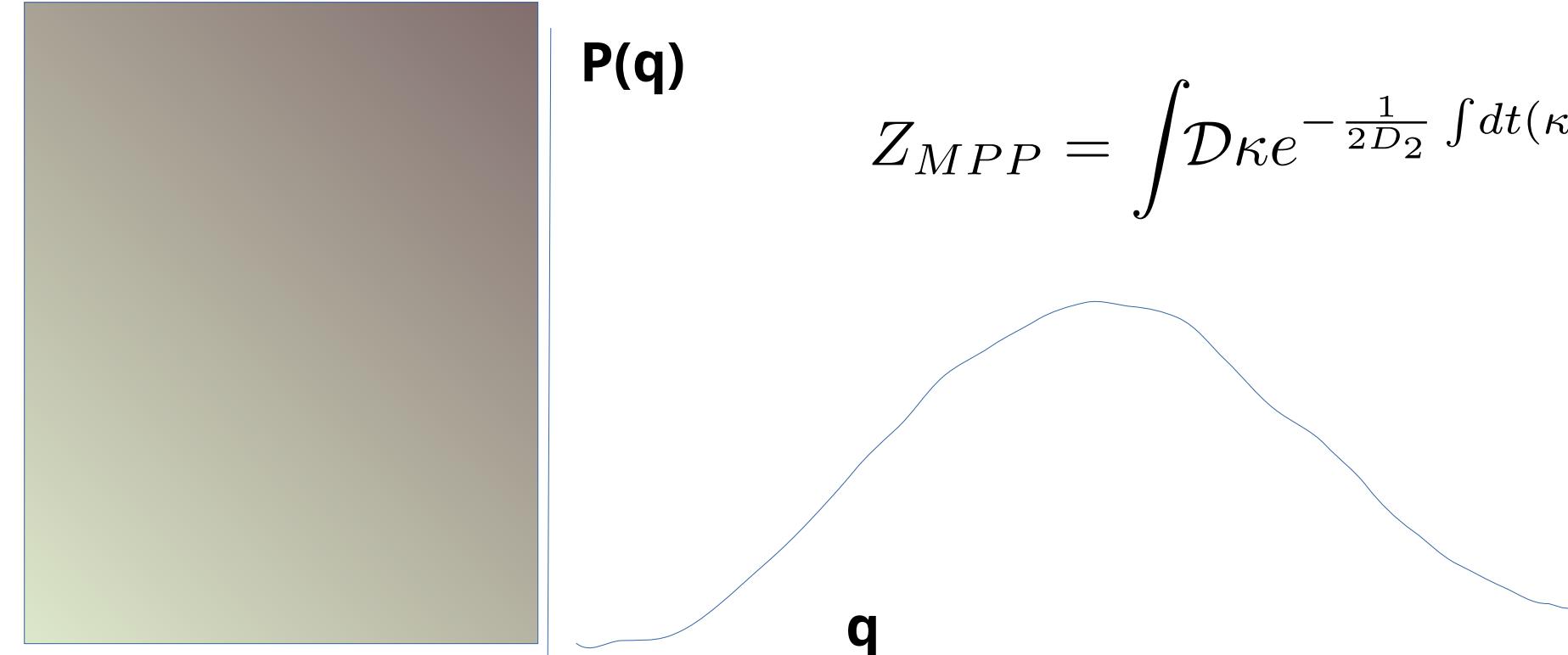
$$Z_{MPP} = \int \mathcal{D}\kappa e^{-\frac{1}{2D_2} \int dt (\kappa_2 + t\kappa_3)^2}$$

$$q_{MPP} = \kappa_0 + \kappa_1 t + \frac{1}{2}\kappa_2 t^2 + \frac{1}{6}\kappa_3 t^3$$

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# Anomalous contribution (Newtonian gravity)

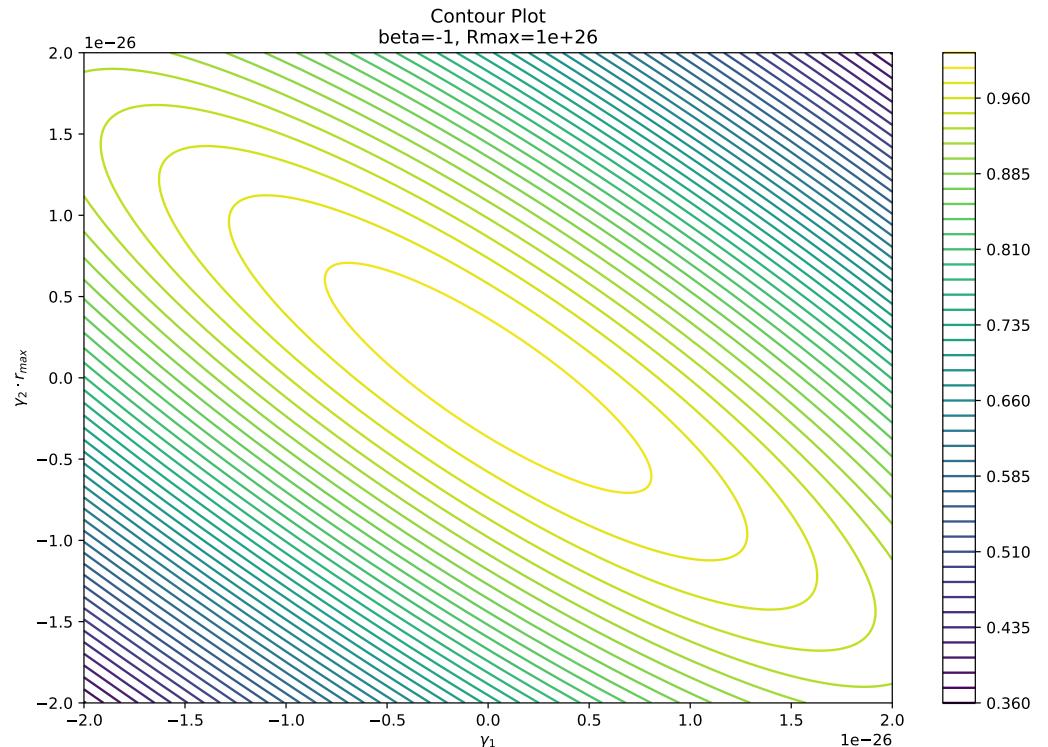
**The most probably path (MPP)**

$$\mathcal{I} = -\frac{1}{2D_2} \int d^3x (\nabla^2 \Phi - 4\pi Gm)^2$$

$$\nabla^4 \Phi = 0$$

$$\Phi_{\text{MPP}} = -\frac{\kappa_m}{r} + \kappa_1 8\pi r + \kappa_2 r^2$$

# Anomalous contribution (Schwarschild-deSitter)



$$\mathcal{I}_\gamma = -\frac{6\pi VT}{D_2} \left( \frac{5 - 18\beta}{r_{\max}^2} \gamma_1^2 + 6(1 - 4\beta) \gamma_2^2 + \frac{9(1 - 4\beta)}{r_{\max}} \gamma_1 \gamma_2 \right)$$

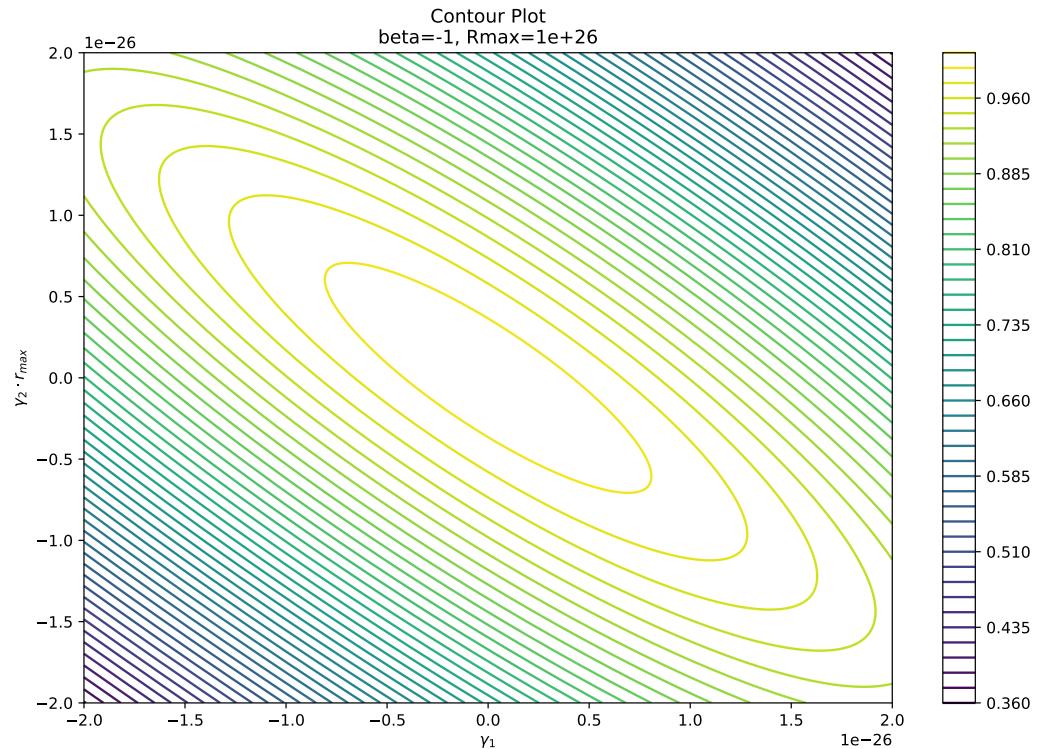
JO, Russo, 2402.19495

$$ds^2 = - \left( 1 - \frac{2MG_N}{r} - \gamma_1 r - \gamma_2 r^2 \right) dt^2 + \left( 1 - \frac{2MG_N}{r} - \gamma_1 r - \gamma_2 r^2 \right)^{-1} dr^2 + r^2 d\Omega^2$$

Riegert (1984)

Mannheim, Kazanas (1989)

# Anomalous contribution (Schwarschild-deSitter)



$$a_0 \approx \frac{1}{2\pi} \sqrt{\frac{\Lambda}{3}}$$

Milgrom (1983)

JO, Russo, 2402.19495

$$ds^2 = - \left(1 - \frac{2MG_N}{r} - \gamma_1 r - \gamma_2 r^2\right) dt^2 + \left(1 - \frac{2MG_N}{r} - \gamma_1 r - \gamma_2 r^2\right)^{-1} dr^2 + r^2 d\Omega^2$$

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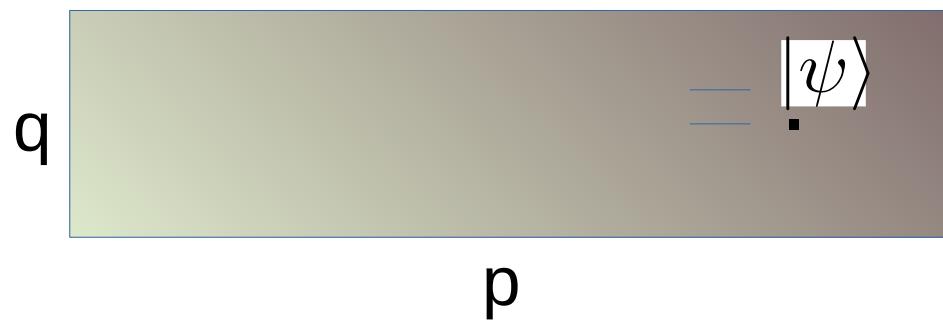
# Frameworks

## Quantum Mechanics

$$\frac{\partial \hat{\sigma}}{\partial t} = -i[\hat{H}, \hat{\sigma}]$$

## Classical Mechanics

$$\frac{\partial \rho(q,p)}{\partial t} = \{H(q,p), \rho(q,p)\}$$



# Frameworks

## Quantum Mechanics

$$\frac{\partial \hat{\sigma}}{\partial t} = -i[\hat{H}, \hat{\sigma}]$$

A diagram illustrating the state vector  $|\psi\rangle$ . It consists of a horizontal bar divided into two colored segments: a light green segment on the left labeled 'q' and a brown segment on the right labeled 'p'. A blue double-headed arrow symbol is positioned between the two segments, indicating a relationship or mapping between the position and momentum domains.

## Classical Mechanics

$$\frac{\partial \rho(q,p)}{\partial t} = \{H(q,p), \rho(q,p)\}$$

$$(q,p) = \rho(q,p) \begin{pmatrix} p(0|q,p) & \alpha(q,p) \\ \alpha^*(q,p) & p(1|q,p) \end{pmatrix}$$

# Classical, quantum, & CQ States

Q

HILBERT SPACE

 $\hat{\sigma}$ 

$$\text{tr } \hat{\sigma} = 1$$

POSITIVE MATRIX

C

PHASE SPACE

 $\rho(q, p)$ 

$$\int dq dp \rho(q, p) = 1$$

POSITIVE DISTRIBUTION

CQ

$$\hat{\rho}(z; t) = \rho(z; t) \hat{\sigma}(z; t)$$

$$z := (q, p)$$

$$\int dz \text{tr} \hat{\rho}(z) = 1$$

POSITIVE MATRIX AT EACH Z

# Classical, quantum, & CQ States

Q

HILBERT SPACE

 $\hat{\sigma}$ 

$$\text{tr } \hat{\sigma} = 1 \quad = \begin{pmatrix} p(0) & \alpha \\ \alpha^* & p(1) \end{pmatrix}$$

POSITIVE MATRIX

C

PHASE SPACE

 $\rho(q, p)$ 

$$\int dq dp \rho(q, p) = 1$$

POSITIVE DISTRIBUTION

CQ

$$\hat{\rho}(z; t) = \rho(z; t) \sigma(z; t)$$

$$(z) = \rho(z) \begin{pmatrix} p(0|z) & \alpha(z) \\ \alpha^*(z) & p(1|z) \end{pmatrix}$$

 $\hat{\rho}(z)$ 

$$\int dz \text{tr} \hat{\rho}(z) = 1$$

POSITIVE MATRIX AT EACH Z

# Classical, quantum, & CQ States

Q

HILBERT SPACE

 $\hat{\sigma}$ 

$$\text{tr } \hat{\sigma} = 1 \quad \hat{\sigma} = \begin{pmatrix} p(0) & \alpha \\ \alpha^* & p(1) \end{pmatrix}$$

POSITIVE MATRIX

C

PHASE SPACE

 $\rho(q, p)$ 

$$\int dq dp \rho(q, p) = 1$$

POSITIVE DISTRIBUTION

CQ

$$\hat{\sigma}_{cq} = \int dz \rho(z; t) |z\rangle\langle z| \otimes \sigma(z; t)$$

 $\hat{\rho}(z)$ 

$$\hat{\rho}(z) = \rho(z) \begin{pmatrix} p(0|z) & \alpha(z) \\ \alpha^*(z) & p(1|z) \end{pmatrix}$$

$$\int dz \text{tr} \hat{\rho}(z) = 1$$

POSITIVE MATRIX AT EACH Z

# Dynamics must be linear and preserve state-space

Must be positive

$$1 \otimes \mathcal{L}$$

Norm preserving

positive matrix at each  $z$

$$\int dz \text{tr} \hat{\rho}(z) = 1$$

$$|\psi\rangle = \frac{1}{\sqrt{2}} (|0\rangle|\psi_L\rangle - |1\rangle|\psi_R\rangle)$$

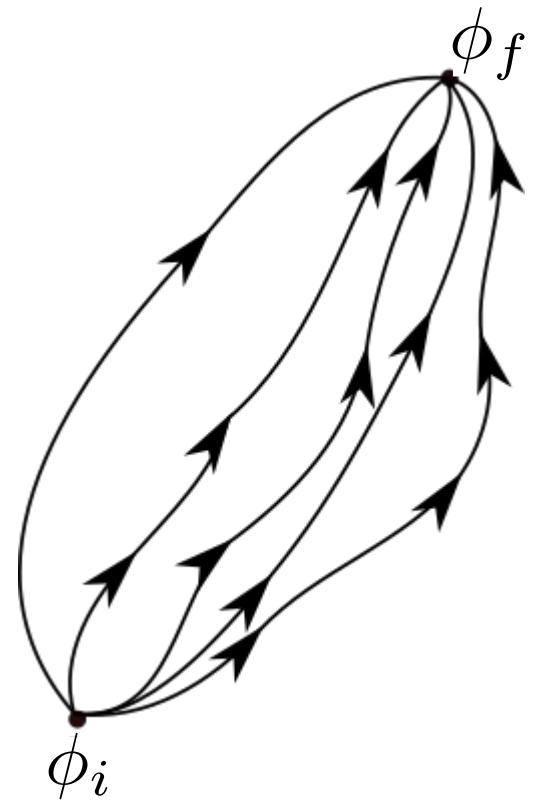
$$\sigma^{(m)} = \frac{1}{2}|\psi_L\rangle\langle\psi_L| + \frac{1}{2}|\psi_R\rangle\langle\psi_R|$$

$$\frac{1}{2}\mathcal{L}(|\psi_L\rangle\langle\psi_L|) + \frac{1}{2}\mathcal{L}(|\psi_R\rangle\langle\psi_R|) = \mathcal{L}\left(\frac{1}{2}|\psi_L\rangle\langle\psi_L| + \frac{1}{2}|\psi_R\rangle\langle\psi_R|\right)$$

# Path Integrals

## Quantum Mechanics

$$\langle \phi_f, t_f | \phi_i, t_i \rangle = \int_{\phi(t_i) = \phi_i}^{\phi(t_f) = \phi_f} \mathcal{D}\phi \ e^{iS[\phi]}$$



# Path Integrals

## Quantum Mechanics

$$\langle \phi^-, t_f | \phi_i, t_i \rangle \langle \phi_i, t_i | \phi^+, t_f \rangle = \int_{\phi^+ = \phi^- = \phi_i}^{\phi_f^+, \phi_f^-} \mathcal{D}\phi^+ \mathcal{D}\phi^- e^{iS[\phi^+] - iS[\phi^-]}$$

# Path Integrals

## Quantum Mechanics

$$\hat{\sigma}(\phi_f^+, \phi_f^-, t_f) = \int_{\phi^+ = \phi^- = \phi_i}^{\phi_f^+, \phi_f^-} \mathcal{D}\phi^+ \mathcal{D}\phi^- e^{iS[\phi^+] - iS[\phi^-]}$$

# Path Integrals

## Quantum Mechanics (open systems)

$$\hat{\sigma}(\phi_f^+, \phi_f^-, t_f) = \int \mathcal{D}\phi^+ \mathcal{D}\phi^- e^{iS[\phi^+] - iS[\phi^-] + iS_{FV}[\phi^+, \phi^-]}$$

$$iS_{FV} = D_0 \int_{t_i}^{t_f} d^4x \left( \phi^+ \phi^- - \frac{1}{2} (\phi^- \phi^- + \phi^+ \phi^+) \right)$$

Feynman-Vernon

# Path Integrals

## Quantum Mechanics (open systems)

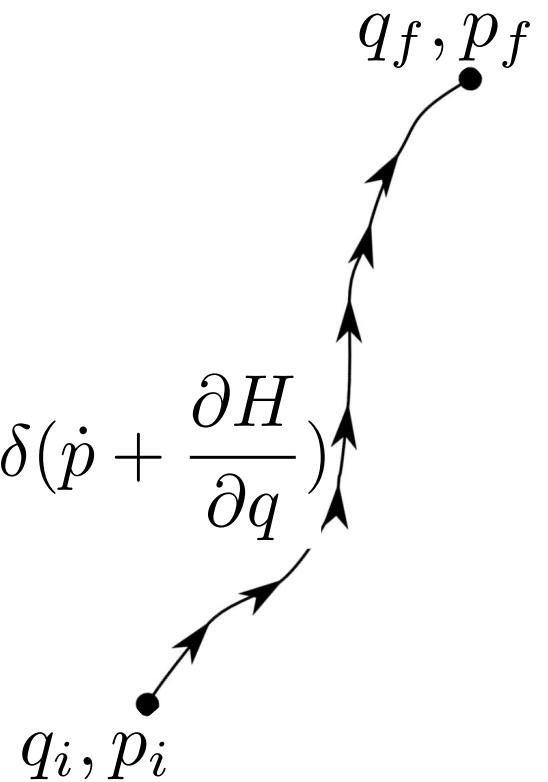
$$\hat{\sigma}(\phi_f^+, \phi_f^-, t_f) = \int \mathcal{D}\phi^+ \mathcal{D}\phi^- e^{iS[\phi^+] - iS[\phi^-] + iS_{FV}[\phi^+, \phi^-]}$$

$$iS_{FV} = -\frac{1}{2} D_0 \int_{t_i}^{t_f} d^4x (\phi^+ - \phi^-)^2$$

# Path Integrals

## Classical Mechanics

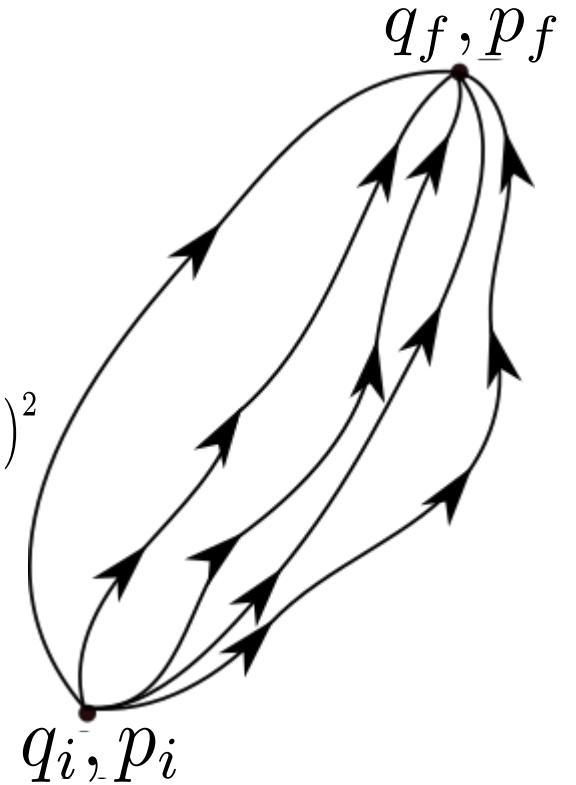
$$\rho(q_f, p_f, t_f | q_i, p_i, t_i) = \int_{q_i, p_i}^{q_f, p_f} \mathcal{D}q \mathcal{D}p \Pi_t \delta(\dot{q} - \frac{\partial H}{\partial p}) \delta(\dot{p} + \frac{\partial H}{\partial q})$$



# Path Integrals

## Classical Mechanics (stochastic)

$$\rho(q_f, p_f, t_f | q_i, p_i, t_i) = \mathcal{N} \int_{q_i, p_i}^{q_f, p_f} \mathcal{D}q \mathcal{D}p \delta(\dot{q} - \frac{\partial H}{\partial p}) e^{-\frac{1}{2D_2} \int dt \left( \dot{p} + \frac{\partial H}{\partial q} \right)^2}$$

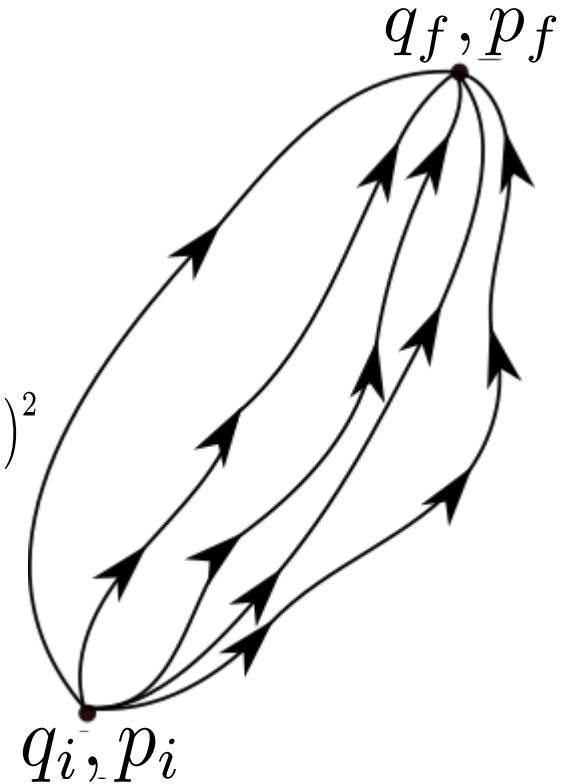


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# Path Integrals

## Classical Mechanics (stochastic)

$$\rho(q_f, p_f, t_f | q_i, p_i, t_i) = \mathcal{N} \int_{q_i, p_i}^{q_f, p_f} \mathcal{D}q \mathcal{D}p \delta(\dot{q} - \frac{\partial H}{\partial p}) e^{-\frac{1}{2D_2} \int dt (\dot{p} + \frac{\partial H}{\partial q})^2}$$



Onsager-Machlup

# Q, C & CQ Path integral

**Q**

$$iS_Q[\phi^+, \Phi^-] := iS[\phi^+] - iS[\phi^-] + iS_{FV}[\phi^+, \phi^-]$$

$$iS_{FV} = -\frac{1}{2}D_0 \int_{t_i}^{t_f} d^4x (\phi^+ - \phi^-)^2$$

**C**

$$iS_C[q, p] = -\frac{1}{2D_2} \int dt (\dot{p} + \frac{\partial H}{\partial q})^2$$

Onsager, Machlup (1953);  
Freidlin, Wentzell (1998)

**CQ**

$$iS[\phi^+, \phi^-, q, p] = S_Q[\phi^+, \phi^-] - \frac{1}{2D_2} \int dt (\dot{p} + \frac{1}{2} \frac{\partial \hat{H}^+}{\partial q} + \frac{1}{2} \frac{\partial \hat{H}^-}{\partial q})^2$$

$$iS_{FV} = -\frac{1}{2} \int dt D_0 \left( \frac{\partial \hat{H}^+}{\partial q} - \frac{\partial \hat{H}^-}{\partial q} \right)^2$$

$$4D_2 \succeq D_0^{-1}$$

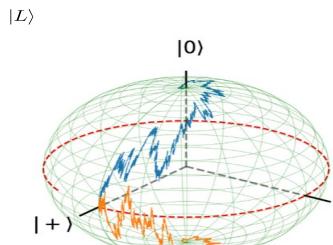
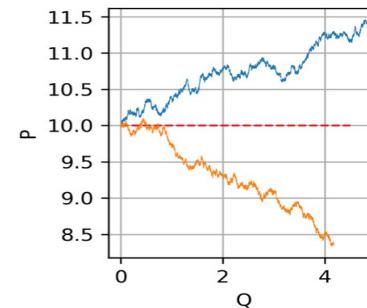
# CQ Dynamics

## Path Integral

$$\rho(q, p, \phi^\pm, t_f) = \int \mathcal{D}q \mathcal{D}p \mathcal{D}\phi^\pm e^{iS_C[q,p] + iS[\phi^+] - iS[\phi^-] + iS_{FV}[\phi^\pm] + iS_{CQ}[q,p,\phi^\pm]} \delta(\dot{q} - \frac{p}{m}) \rho(q, p, \phi^\pm, t_i)$$

JO, Zach Weller-Davies

## Trajectories



JO, I. Layton, Z. Weller-Davies

## Master Eqn

$$\begin{aligned} \frac{\partial \hat{\rho}}{\partial t} &\approx \{H^{(grav)}, \hat{\rho}\} - i[\hat{H}^{(m)}, \hat{\rho}] + \frac{1}{2}\{\hat{H}^{(m)}, \hat{\rho}\} - \frac{1}{2}\{\hat{\rho}, \hat{H}^{(m)}\} \\ &+ \int dxdx' \frac{\delta^2}{\delta\pi_\Phi(x)\delta\pi_\Phi(x')} (D_2(x, x')\hat{\rho}) + \frac{1}{2} \int dxdx' D_0(x, x') ([\hat{m}(x), [\hat{\rho}, \hat{m}(x')]]) \end{aligned}$$

CPTP MAP

JO, Sparaciari, Soda, Weller-Davies

# Q, C & CQ Dynamics (jumps)

Q

$$\frac{\partial \hat{\sigma}}{\partial t} = -i[\hat{H}, \hat{\sigma}] + \sum_{\alpha\beta} W^{\alpha\beta} \hat{L}_\alpha \hat{\sigma} \hat{L}_\beta^\dagger - \frac{1}{2} W^{\alpha\beta} \{ \hat{L}_\beta^\dagger \hat{L}_\alpha, \hat{\sigma} \}_+$$

$$W^{\alpha\beta} \succeq 0$$

C

$$\frac{\rho(z)}{\partial t} = \{ H(z), \rho(z) \} + \int dz' (W(z|z')\rho(z') - W(z)\rho(z))$$

$$W(z|z') \geq 0 \quad \forall z, z'$$

CQ

$$\frac{\partial \hat{\rho}(z; t)}{\partial t} = -i[\hat{H}(z), \hat{\rho}(z; t)] + \sum_{\alpha\beta} \int dz' W^{\alpha\beta}(z|z') \hat{L}_\alpha \hat{\rho}(z'; t) \hat{L}_\beta^\dagger - \frac{1}{2} W^{\alpha\beta}(z) \{ \hat{L}_\beta^\dagger \hat{L}_\alpha, \hat{\rho}(z; t) \}$$

$$W^{\alpha\beta} \succeq 0 \quad \forall z, z'$$

Blanchard, Jadzyk (1994);  
Poulin (2017);  
JO (2018)

# Q, C & CQ Dynamics (continuous)

**Q**

$$\frac{\partial \hat{\sigma}}{\partial t} = -i[\hat{H}, \hat{\sigma}] + \sum_{\alpha\beta} W^{\alpha\beta} \hat{L}_\alpha \hat{\sigma} \hat{L}_\beta^\dagger - \frac{1}{2} W^{\alpha\beta} \{ \hat{L}_\beta^\dagger \hat{L}_\alpha, \hat{\sigma} \}_+$$

$$W^{\alpha\beta} \succeq 0$$

**C**

$$\frac{\partial \rho(z)}{\partial t} = - \sum_i \frac{\partial}{\partial z_i} (D_{1,i} \rho(z)) + \sum_{ij} \frac{\partial^2}{\partial z_i \partial z_j} (D_{2,ij} \rho(z))$$

$$D_2(z) \succeq 0$$

Fokker-Planck Equation

**CQ**

$$\begin{aligned} \frac{\partial \hat{\rho}(z)}{\partial t} = & -i[H(z), \hat{\rho}(z)] + \mathcal{L}_{FP} \hat{\rho}(z) + \frac{\partial}{\partial z_i} (D_{1,i}^{0\alpha} \hat{\rho}(z) L_\alpha^\dagger) + \frac{\partial}{\partial z_i} (D_{1,i}^{\alpha 0} L_\alpha \hat{\rho}(z)) \\ & + D_0^{\alpha\beta}(z) L_\alpha \hat{\rho}(z) L_\beta^\dagger - \frac{1}{2} D_0^{\alpha\beta} \{ L_\beta^\dagger L_\alpha, \hat{\rho}(z) \}_+ \end{aligned}$$

$$D_2(z) - D_1^\dagger(z) D_0^{-1}(z) D_1(z) \succeq 0$$

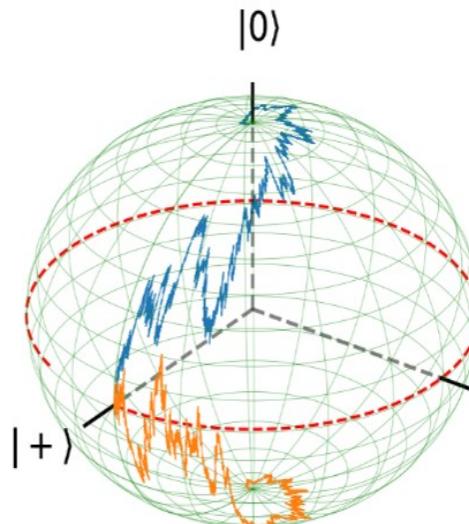
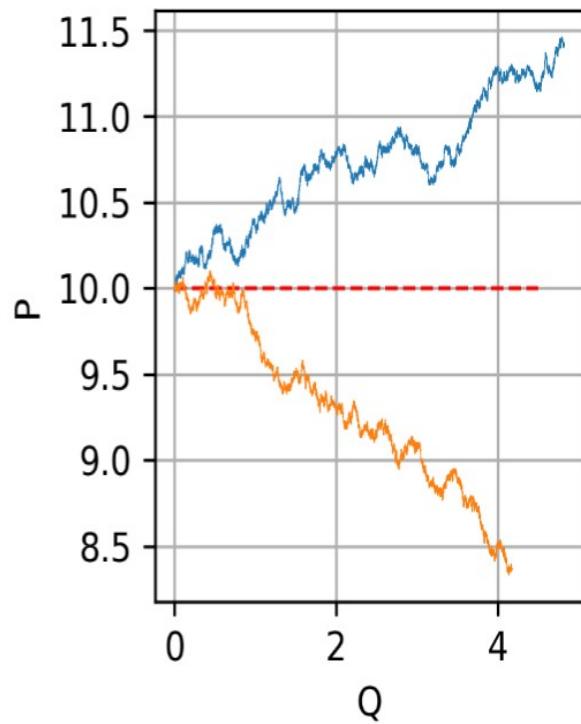
JO (2018);

JO, Soda, Sparaciari, Weller-Davies (2022)

# Example of continuous master-equation

**Stern-Gerlach**

$$\hat{H} = \frac{p^2}{2m} + q\hat{\sigma}$$



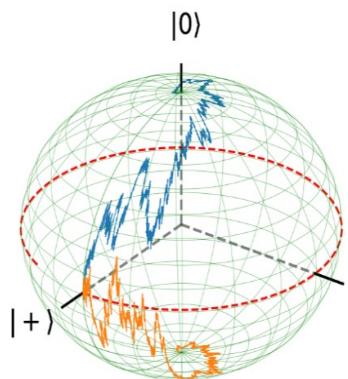
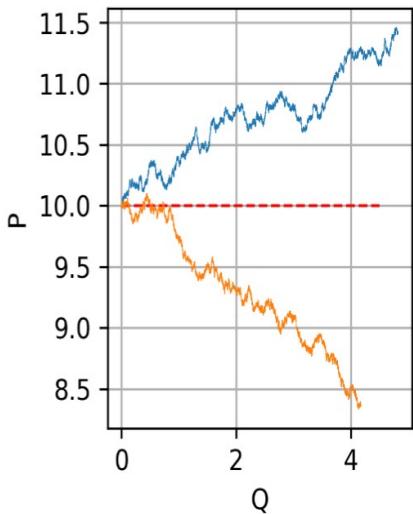
$$D_2 D_0 \succeq D_1^2$$

# Example of continuous master-equation

**Stern-Gerlach**

$$\hat{H} = \frac{p^2}{2m} + D_1 q \hat{\sigma}$$

$$\frac{\partial \hat{\rho}}{\partial t} = -i[\hat{H}, \hat{\rho}] + \frac{1}{2}\{\hat{H}, \hat{\rho}\} - \frac{1}{2}\{\hat{\rho}, \hat{H}\} + \frac{D_2}{2}\{q, \{q, \hat{\rho}\}\} + \frac{D_0}{2}[\hat{\sigma}, [\hat{\rho}, \hat{\sigma}]]$$



$$D_2 D_0 \succeq D_1^2$$

Diosi (1995)

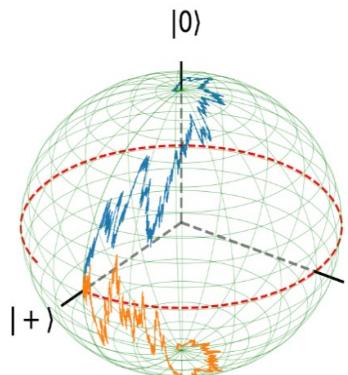
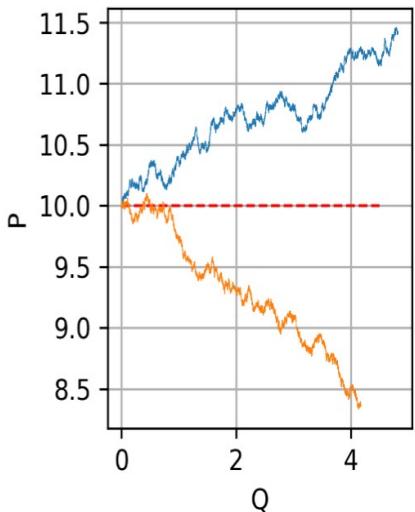
Isaac Layton, JO, Zach Weller-Davies (2022)

# Example of continuous master-equation

**Stern-Gerlach**

$$\hat{H} = \frac{p^2}{2m} + D_1 q \hat{\sigma}$$

$$\frac{\partial \hat{\rho}}{\partial t} = -i[\hat{H}, \hat{\rho}] + \frac{D_1}{2} \left( \hat{\sigma} \frac{\partial \hat{\rho}}{\partial p} + \frac{\partial \hat{\rho}}{\partial p} \hat{\sigma} \right) + \frac{D_2}{2} \frac{\partial^2 \hat{\rho}}{\partial p^2} + \frac{D_0}{2} [\hat{\sigma}, [\hat{\rho}, \hat{\sigma}]]$$



$$D_2 D_0 = D_1^2$$

Diosi (1995)  
Isaac Layton, JO, Zach Weller-Davies

# A post-quantum theory of classical gravity?

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Renormalisable  
without Ghosts!

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Anomalous  
contribution to the  
metric (dark matter,  
dark energy?)

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What is the most  
general form of  
CQ dynamics?

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Decoherence vs  
Diffusion: testing  
quantum gravity

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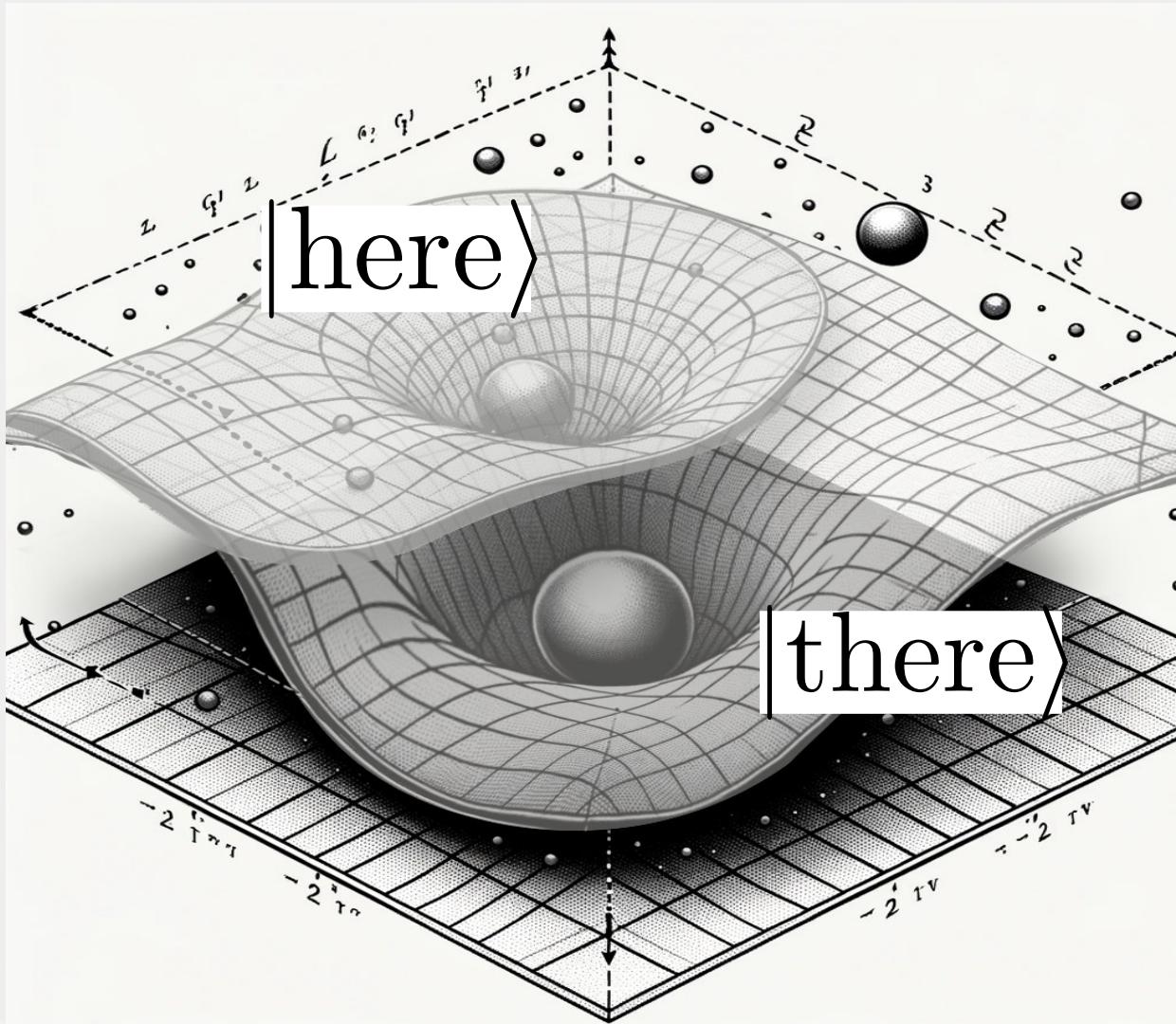
# Decoherence vs diffusion trade-off

$|{\text{here}}\rangle$



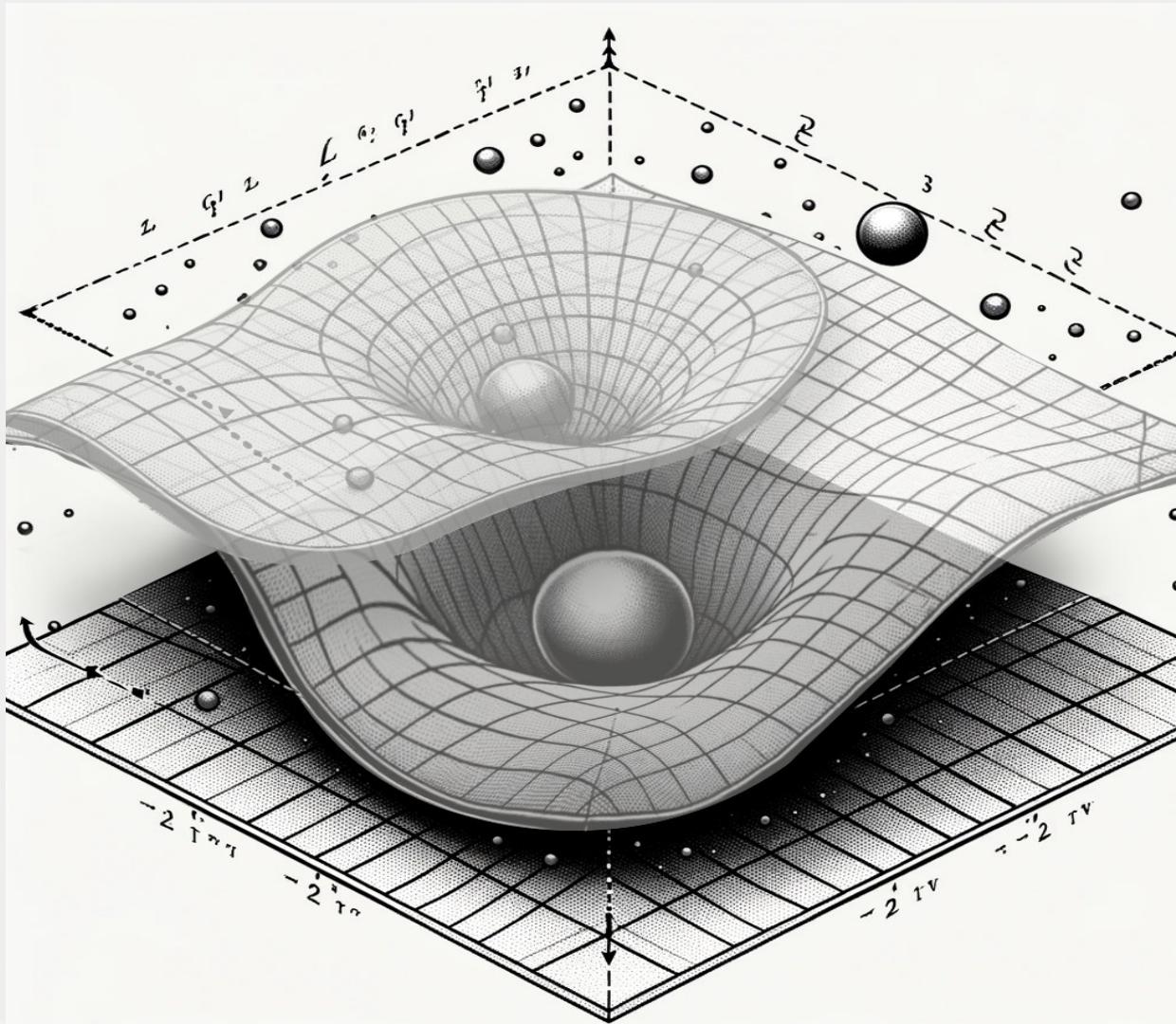
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# Decoherence vs diffusion trade-off



# Decoherence vs diffusion trade-off

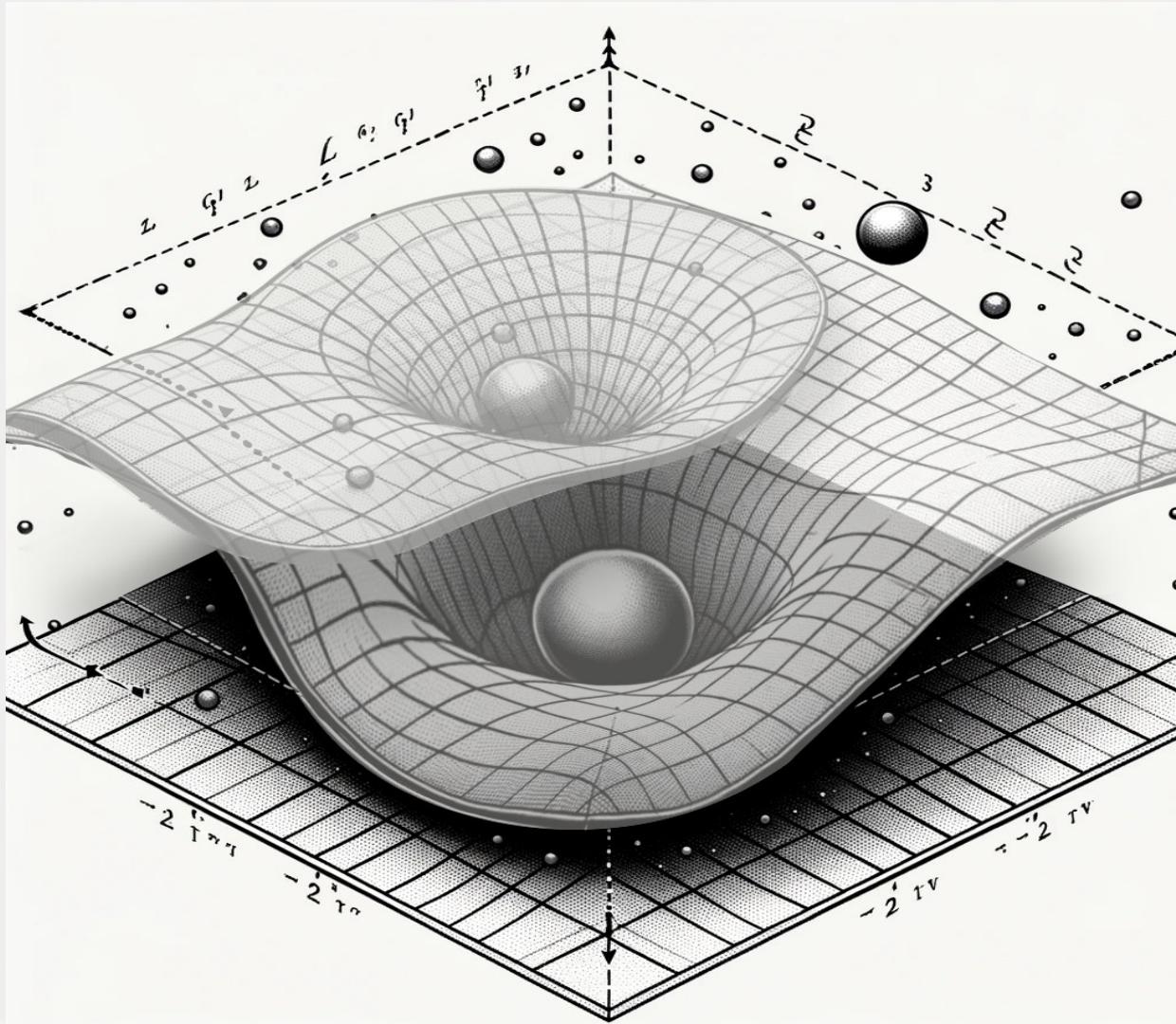
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# Decoherence vs diffusion trade-off

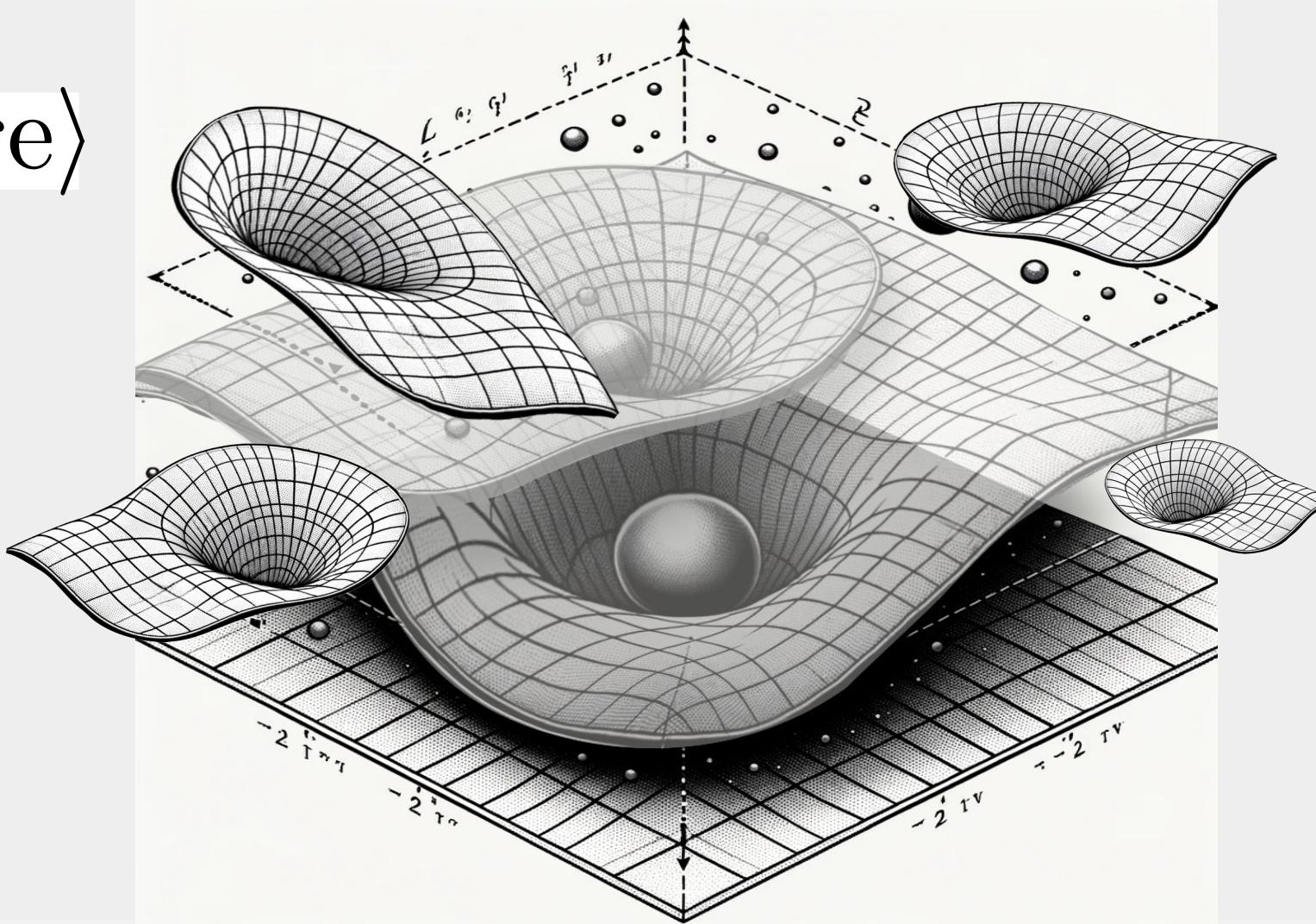
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$|there\rangle$

## Decoherence vs diffusion trade-off

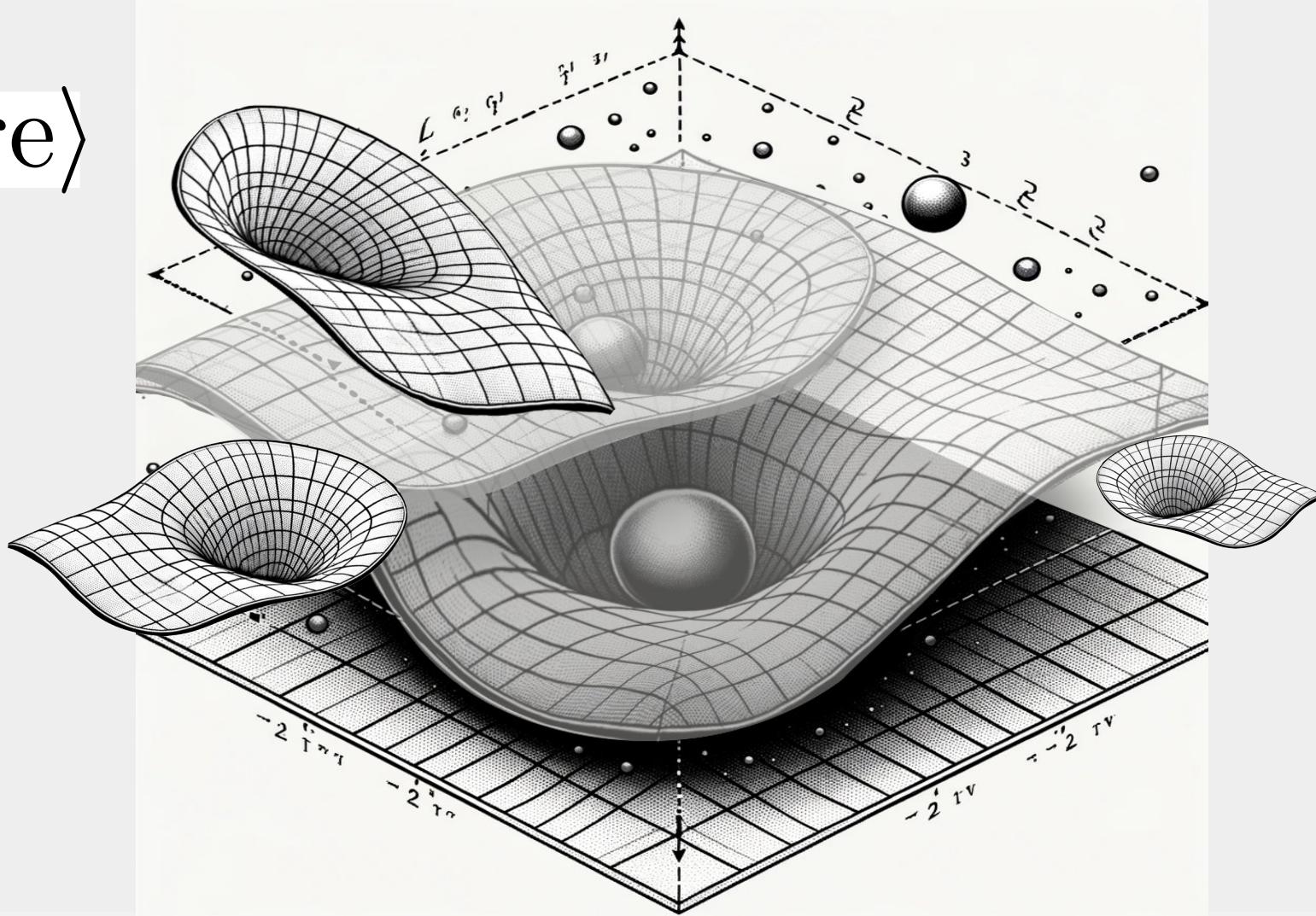
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$|there\rangle$

# Decoherence vs diffusion trade-off

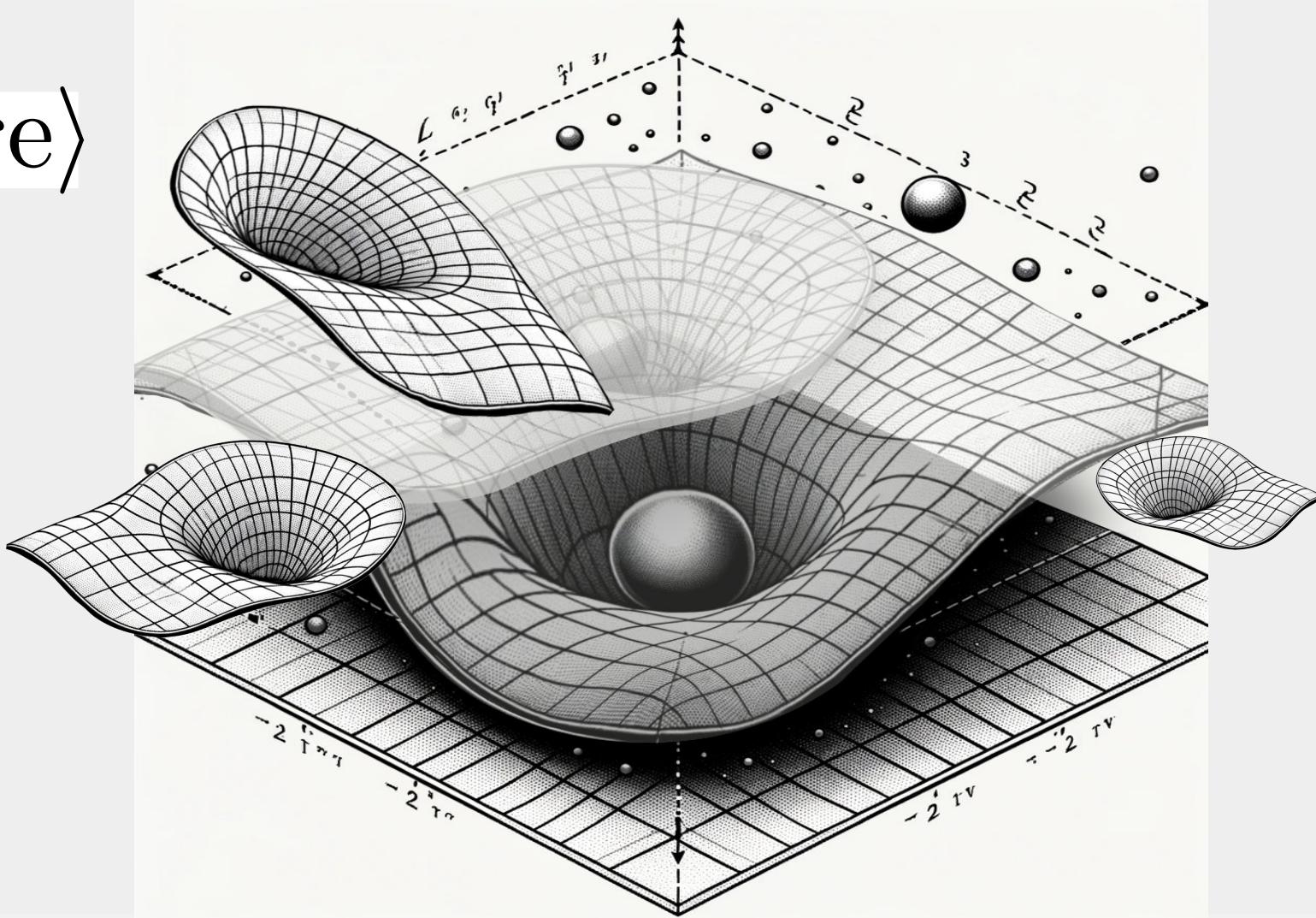
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$|there\rangle$

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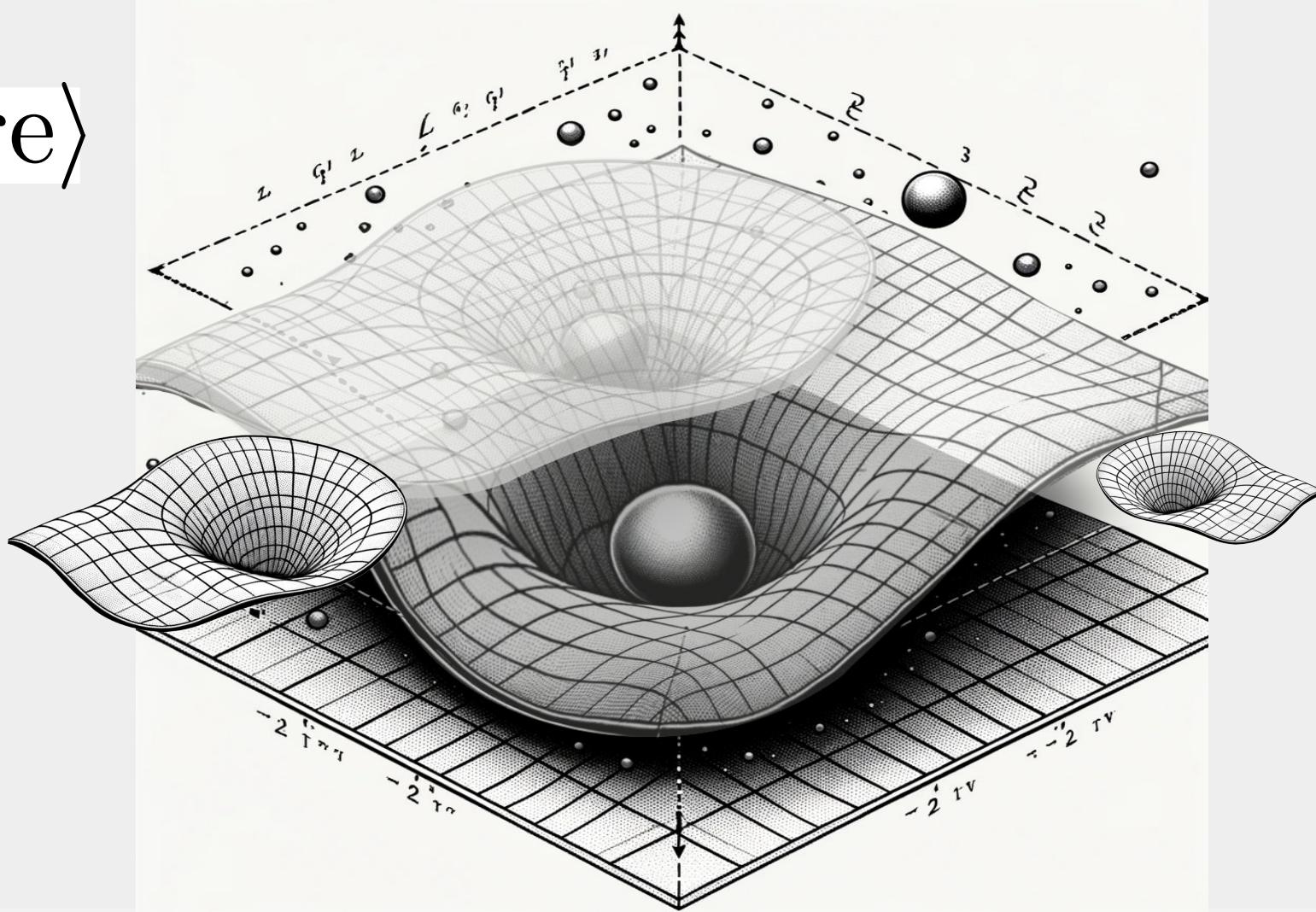
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$|there\rangle$

# Decoherence vs diffusion trade-off

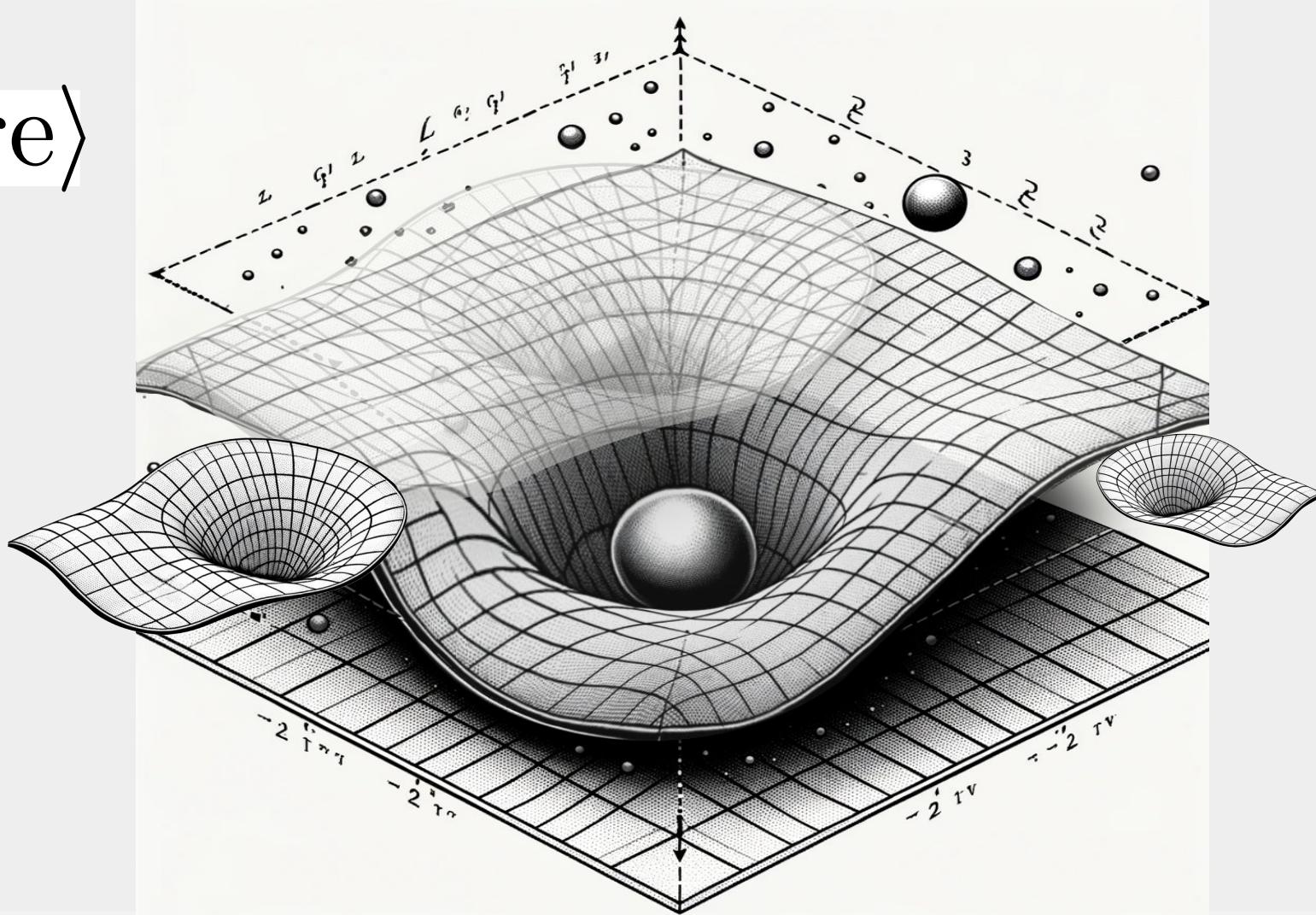
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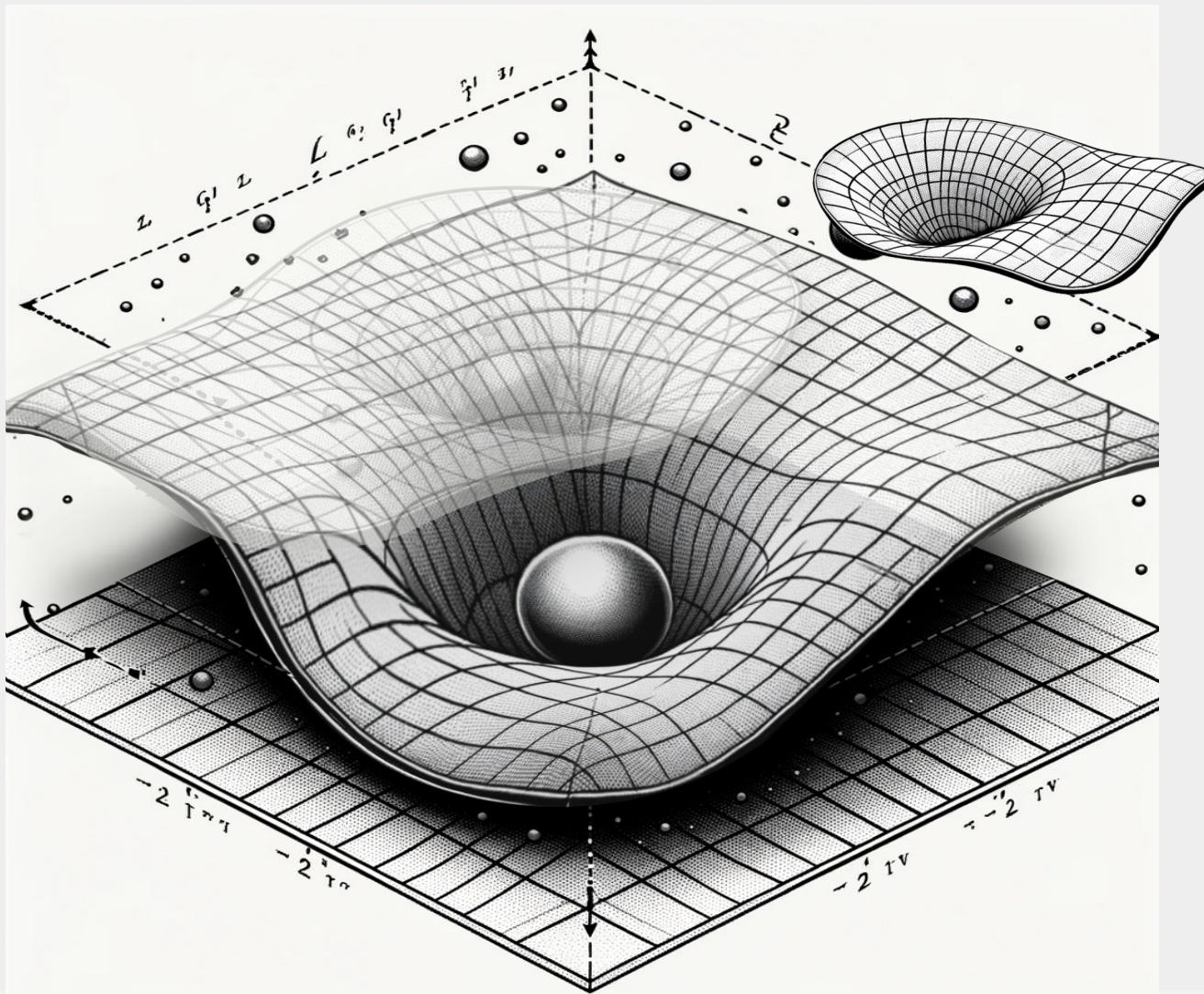
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$|here\rangle$



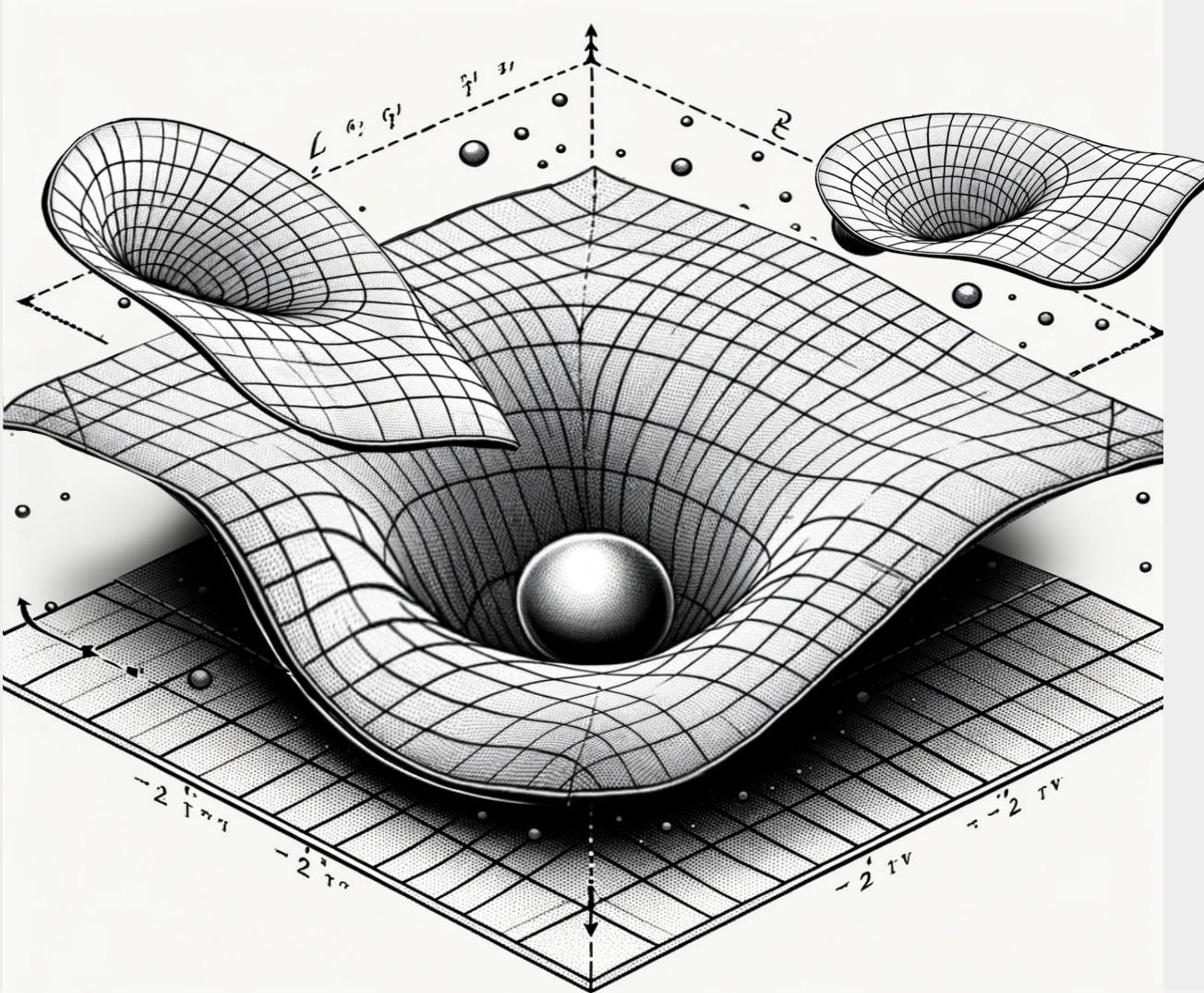
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# Decoherence vs diffusion trade-off



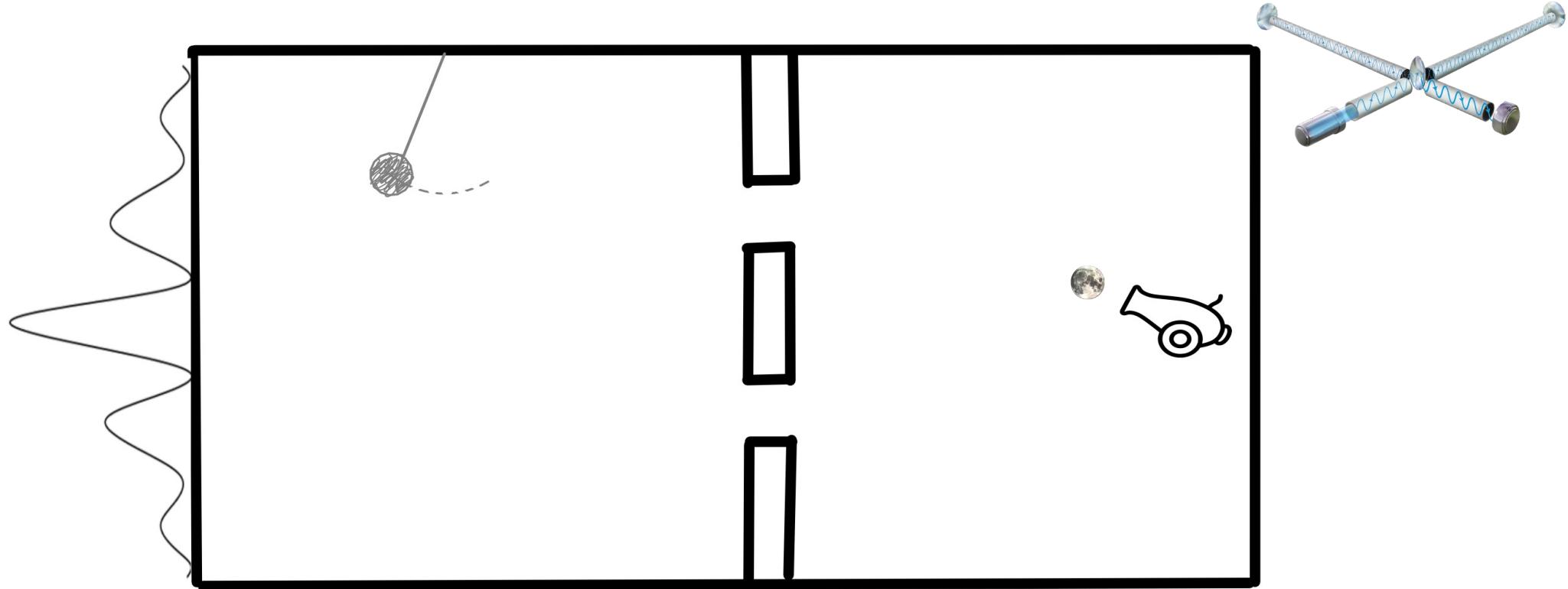
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## Decoherence vs diffusion trade-off



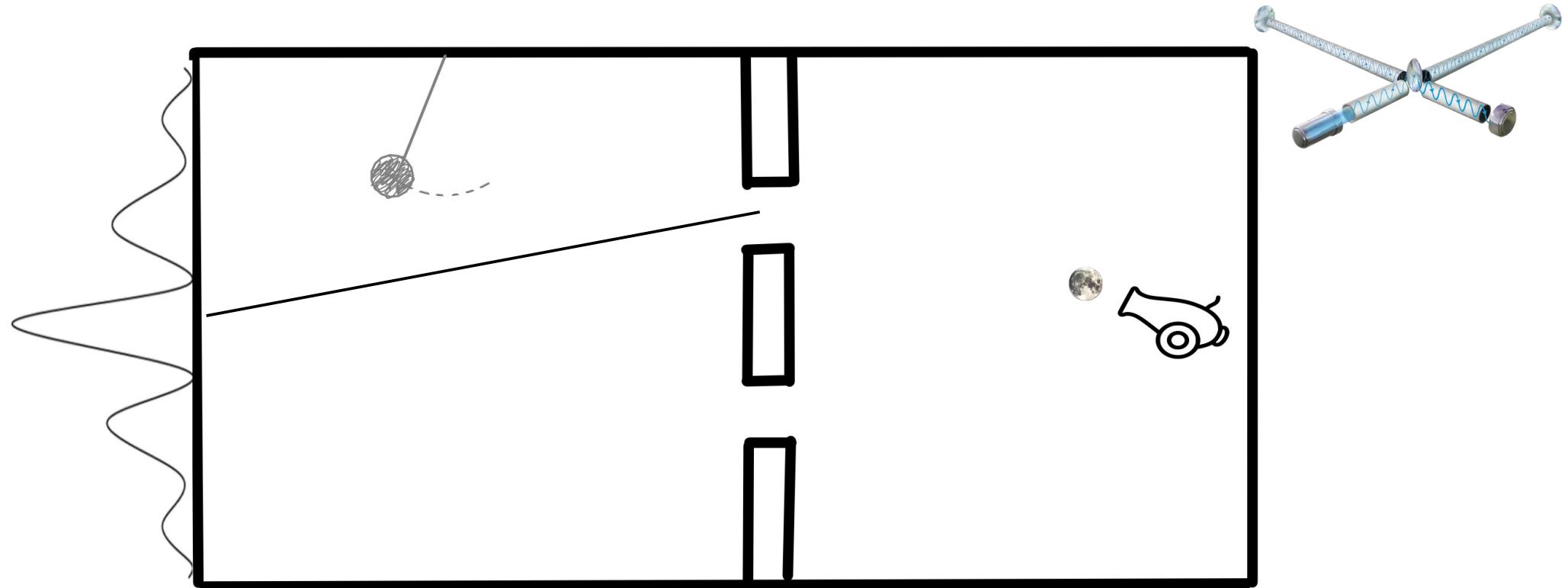
|there>

# Decoherence vs diffusion



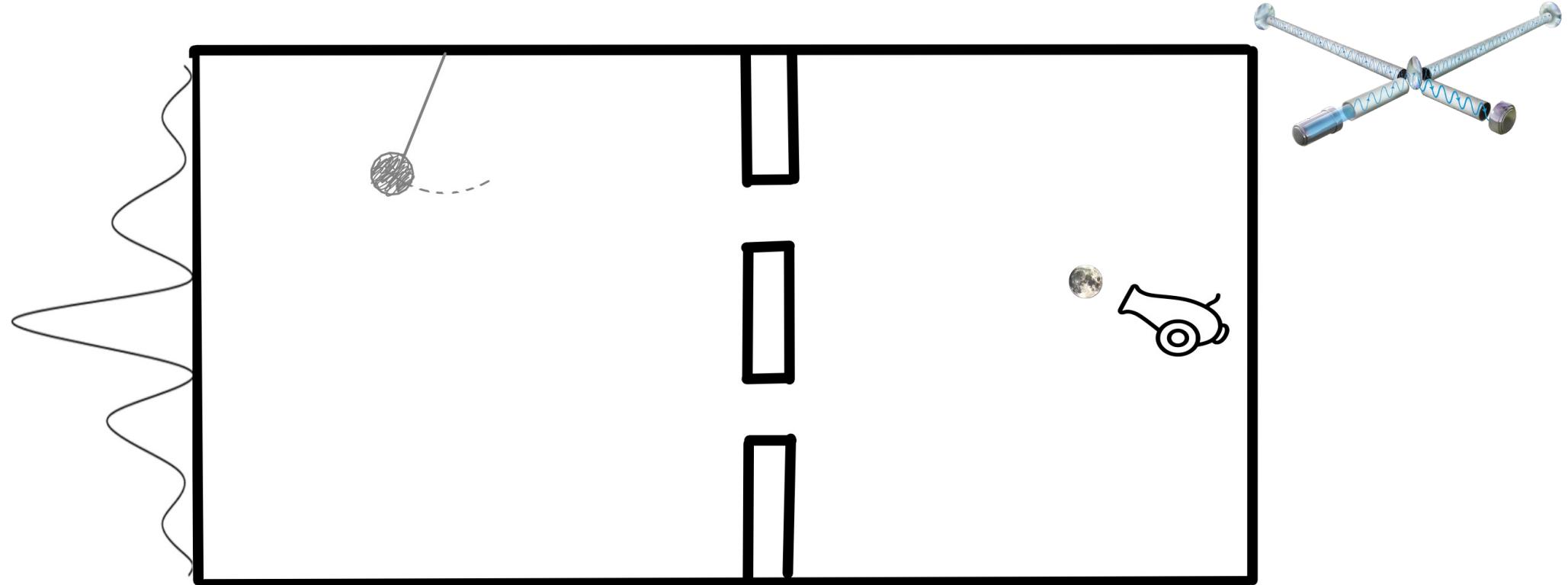
Feynman, Chapel Hill Conference (1957); Aharonov (~1986);  
Eppley & Hannah (1977); Marletto & Vedral (2017)

# Decoherence vs diffusion



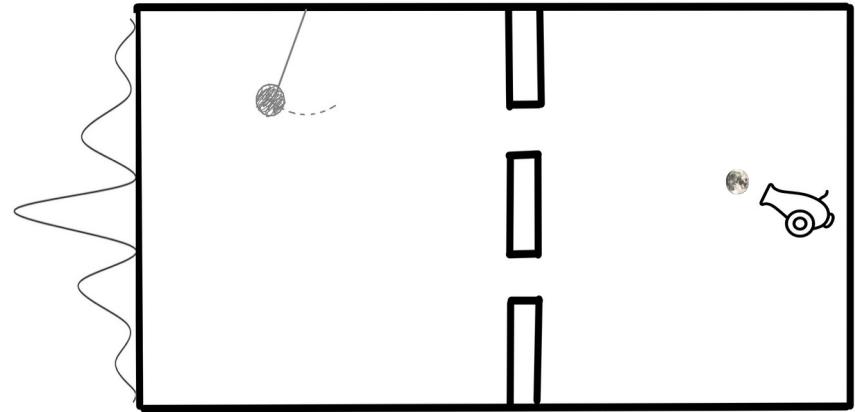
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# Decoherence vs diffusion



$$|\psi\rangle = \frac{1}{\sqrt{2}} (|E_L\rangle |L\rangle + |E_R\rangle |R\rangle)$$

# Decoherence vs diffusion

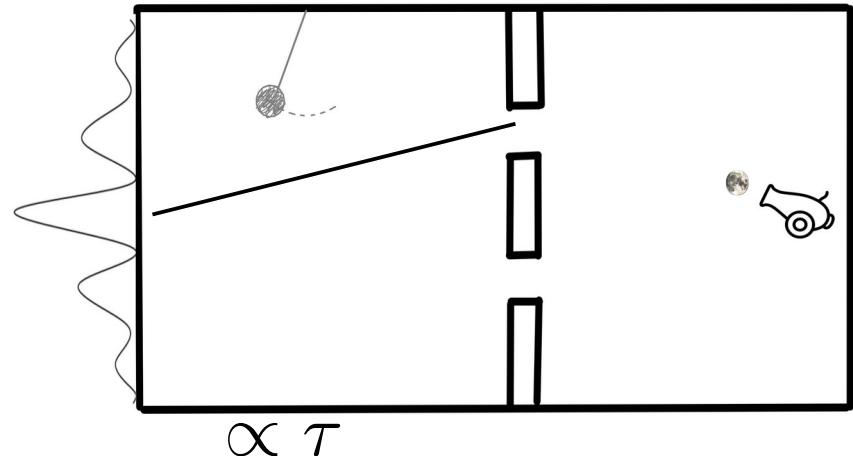


$$\hat{\sigma}(t) = \begin{pmatrix} \frac{1}{2} & \alpha^*(t) \\ \alpha(t) & \frac{1}{2} \end{pmatrix}$$

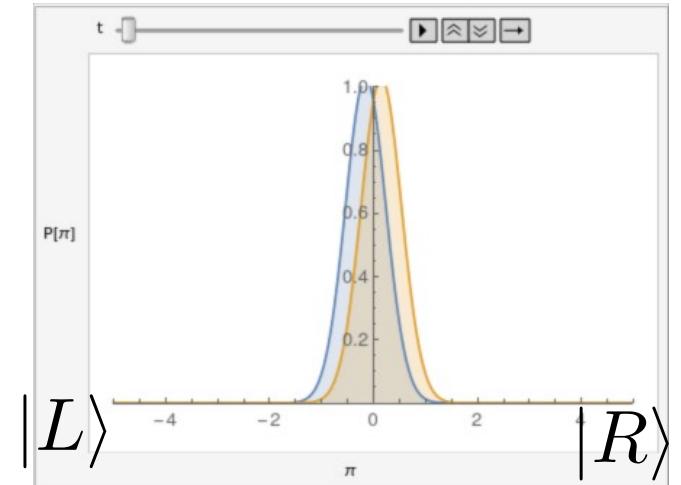
$$\alpha(t) = \langle E_L(t) | E_R(t) \rangle$$

$$|\psi\rangle = \frac{1}{\sqrt{2}} (|E_L\rangle |L\rangle + |E_R\rangle |R\rangle)$$

# Decoherence vs diffusion



$$\tau \leq \frac{D_2}{\langle F \rangle^2}$$

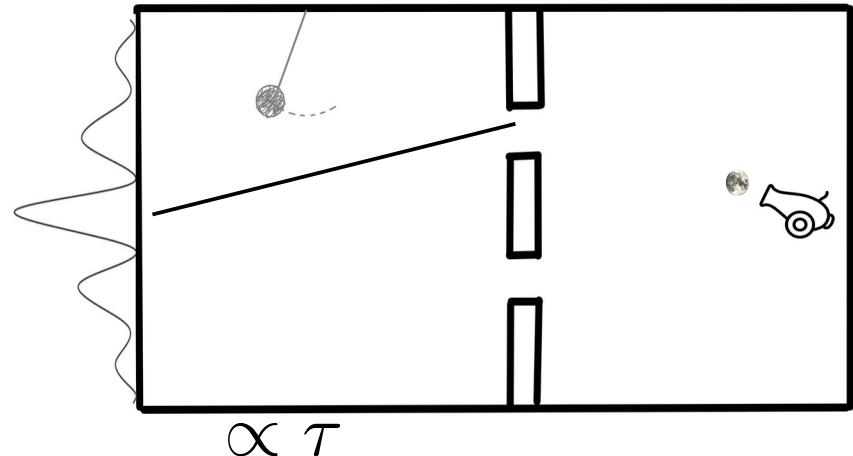


$$D_2(z) - D_1^\dagger(z)D_0^{-1}(z)D_1(z) \succeq 0$$

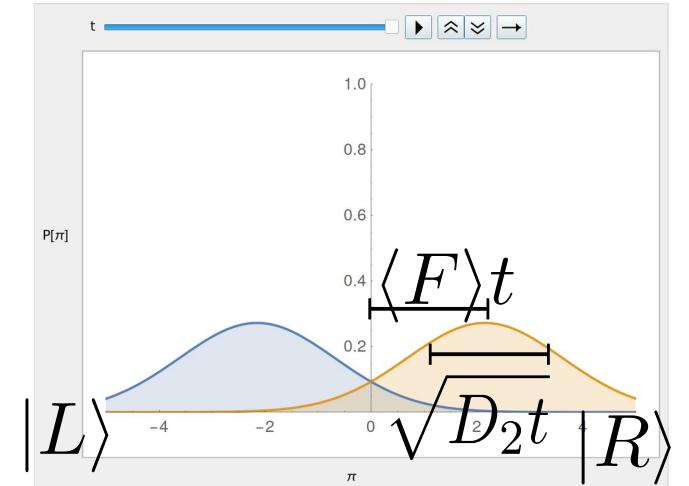
Holds for all classical-quantum dynamics

JO, Soda, Sparaciari, Weller-Davies (2022)

# Decoherence vs diffusion



$$\tau \leq \frac{D_2}{\langle F \rangle^2}$$



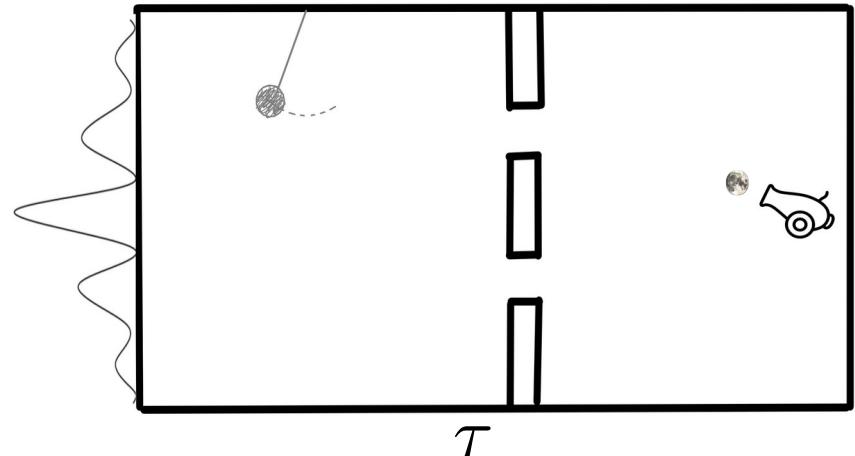
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Holds for all classical-quantum dynamics

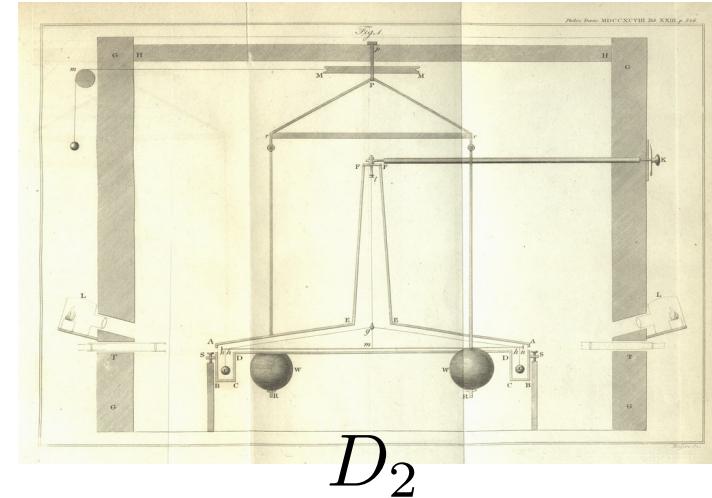
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# Decoherence vs diffusion

**Double Slit Experiment**



**Cavendish Experiment**



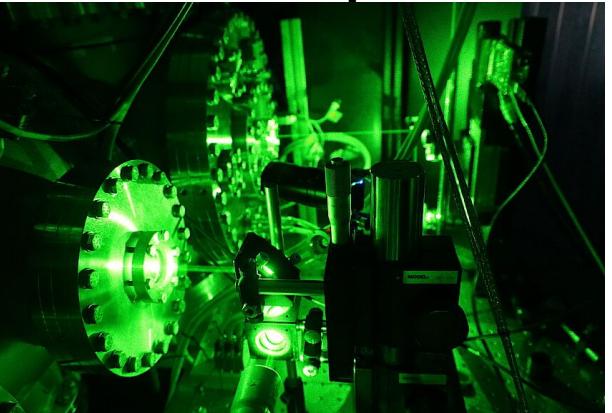
$$\langle F(x) \rangle = \langle \hat{m}(x) \rangle$$

$$D_2(z) - D_1^\dagger(z)D_0^{-1}(z)D_1(z) \succeq 0$$

$$\tau \leq \frac{D_2}{\langle F \rangle^2}$$

# Decoherence vs diffusion

**Double Slit Experiment**



$\tau$

**Cavendish Experiment**



$D_2$

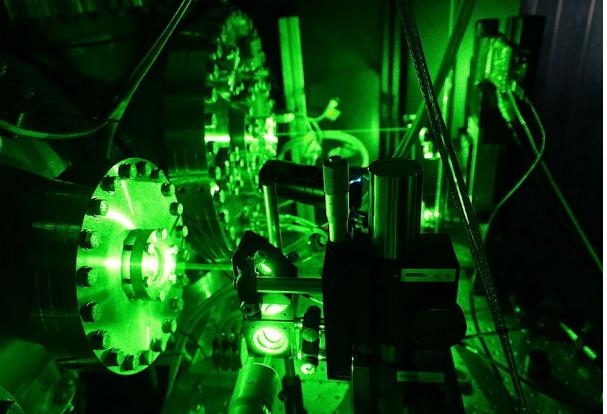
$$\langle F(x) \rangle = \langle \hat{m}(x) \rangle$$

$$D_2(z) - D_1^\dagger(z)D_0^{-1}(z)D_1(z) \succeq 0$$

$$\tau \leq \frac{D_2}{\langle F \rangle^2}$$

# Figures of merit

## Double Slit Experiment


 $\tau$ 

$$\tau \leq \frac{D_2}{\langle F \rangle^2}$$

## Gravity measurement


 $D_2$ 

$$D_2(z) - D_1^\dagger(z)D_0^{-1}(z)D_1(z) \succeq 0$$

Master Equation	Diffusion Kernel	Experimental squeeze
Continuous (ultra-local)	$D_2(\Phi; x, y) = D_2(\Phi)\delta(x, y)$ $D_2(\Phi) = \sum_n c^n \Phi^n$	$10^{-41} \geq D_2 \geq 10^{-9} \text{ kg}^2 \text{sm}^{-3}$ (Eqn (44))
Continuous (Eqn (D11) or (D13))	$D_2(\Phi; x, y) = -l_p^2 D_2(\Phi) \nabla^2 \delta(x, y)$ $D_2(\Phi) = \sum_n c^n \Phi^n$	$10^{-9} \geq l_p^2 D_2 \geq 10^{-35} \text{ kg}^2 \text{sm}^{-1}$ (Eqn (47))
Discrete (ultra-local)	$D_2(\Phi; x, y) = \frac{l_p^2}{m_P} D_2(\Phi) \delta(x, y)$ $D_2(\Phi) = \sum_n c^n \Phi^n$	$10^{-1} \geq \frac{l_p^2 D_2}{m_P} \geq 10^{-25} \text{ kgs}$ (Eqn (46))

# Challenges

## Tensions?

---

CPTP, normalisable

---

Local

---

Renormalisable in  
matter sector?

---

---

Covariant

---

Anomalous heating

---

# Is space-time classical?

I have no idea.  
But,

---

Effective vs  
fundamental vs Foil?

---

It could be

1:5000 ODDS

---

Information is lost but  
quantum state remains  
pure

DIFFUSION

---

Dark matter?  
Dark energy?

---

Measurement postulate not  
needed.

COHERENCE LIMIT

---

Decoherence vs  
Diffusion trade-off

EXPERIMENT

---

Renormalisable w/o  
Ghosts

# Is space-time classical?

I have no idea.  
But,.. THANK YOU  
for your attention

---

Effective vs  
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