A minimal SM/LCDM cosmology

Neil Turok

Higgs Centre, University of Edinburgh

with Latham Boyle

Maegan Anderson, Sam Bateman and Franz Herzog
(extension to black holes with Kostas Tzanavaris)

LCDM provides a remarkably simple description of the large-scale universe: just 5 fundamental physics parameters

the matter/energy content

1. ρ_Λ cosmological constant
2. ρ_{DM}/ρ_B DM/baryon density
3. n_B/n_γ baryons per photon

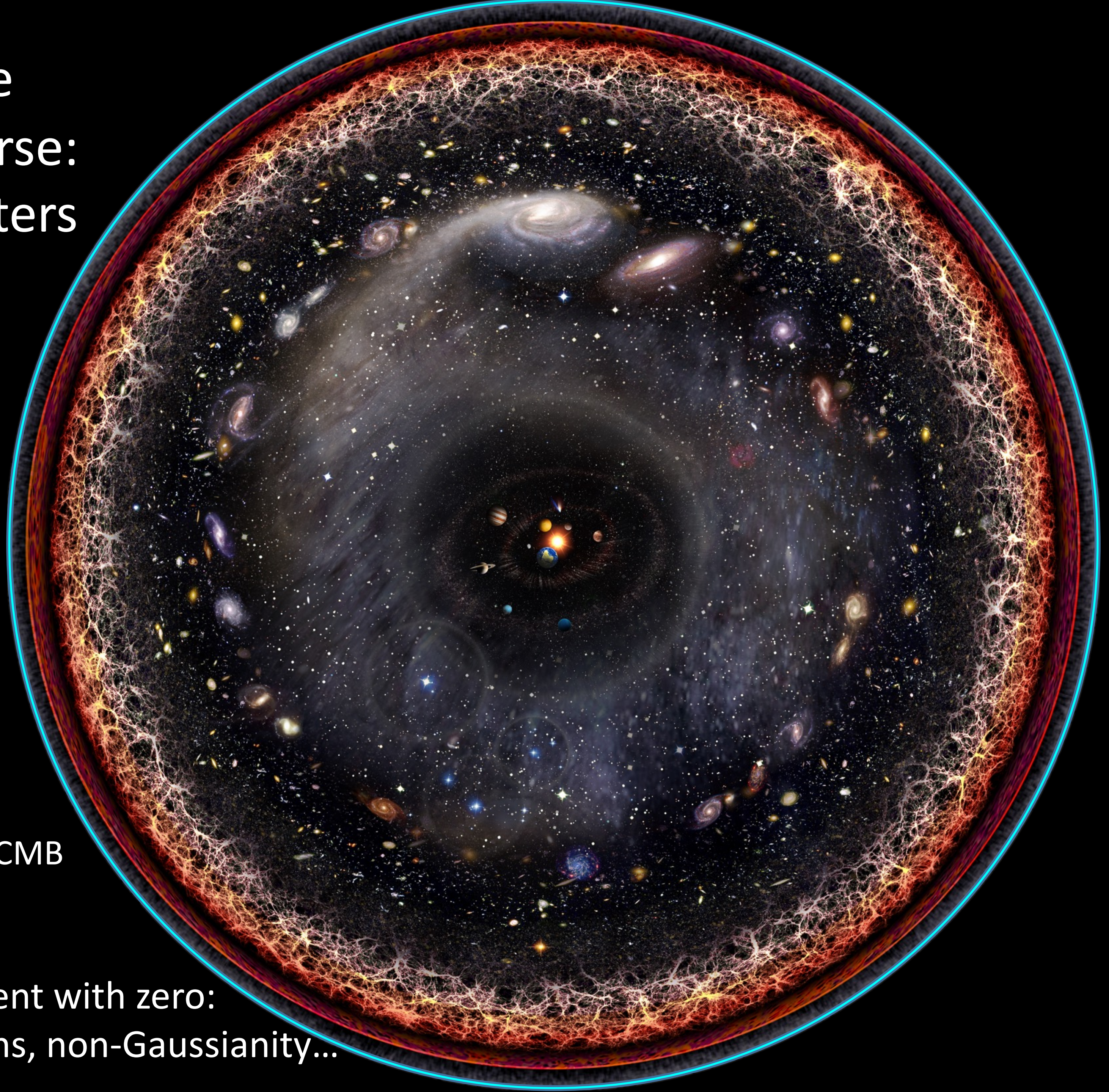
the large-scale geometry

Large scale Newtonian potential

$$\langle \Phi^2 \rangle = \int \frac{dk}{k} A_\Phi \left(\frac{k}{k_*}\right)^{n_s - 1} \quad (k_* \equiv 0.05\text{Mpc}^{-1})$$

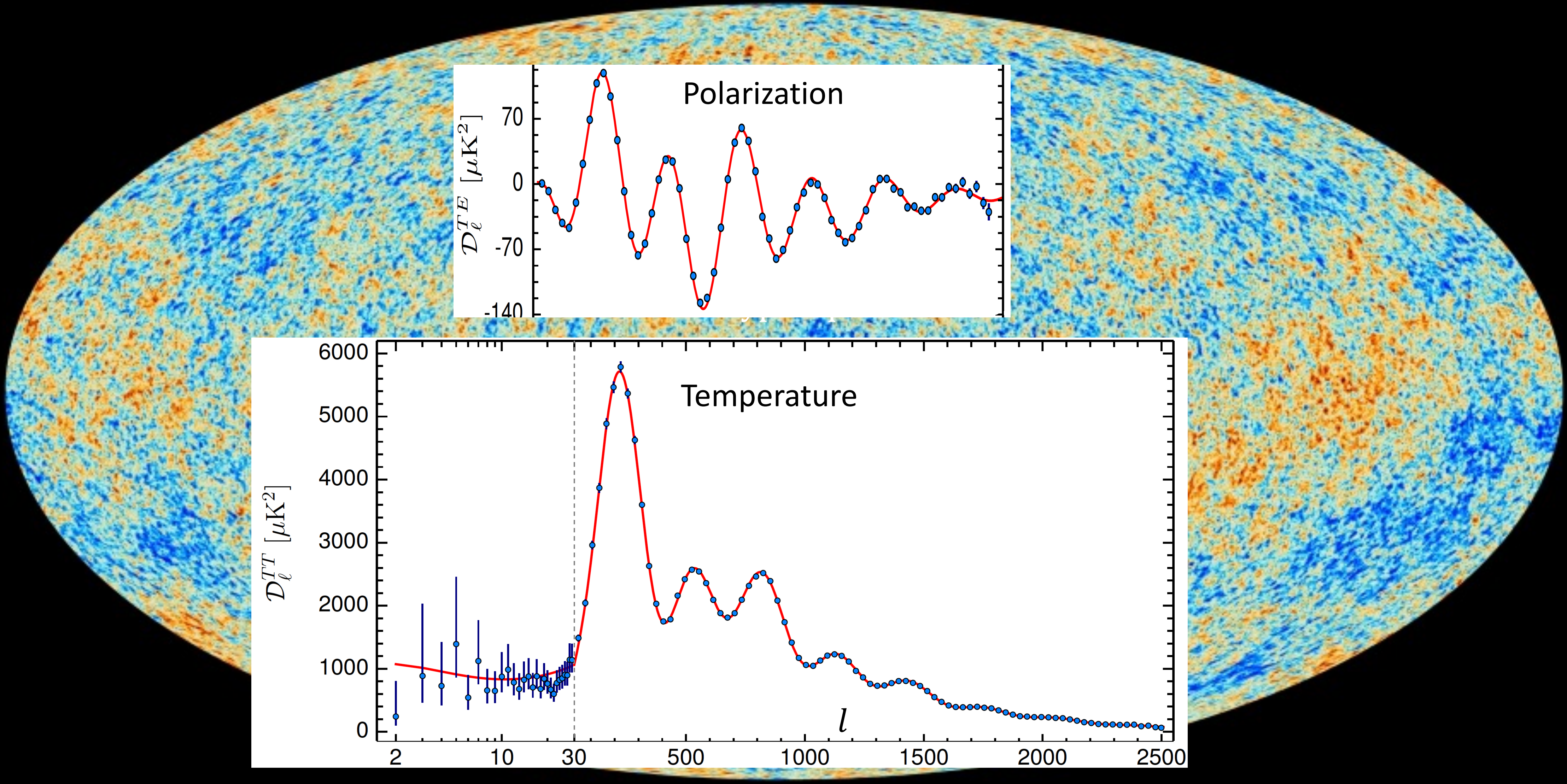
4. $A_\Phi = (7.6 \pm 0.1)\times 10^{-10}$
5. $n_s - 1 = -0.04 \pm .006$

$\Big\}$ Planck CMB



many quantities are observed to be so far consistent with zero: space curvature, tensor/isocurvature perturbations, non-Gaussianity…

Large scale perturbations



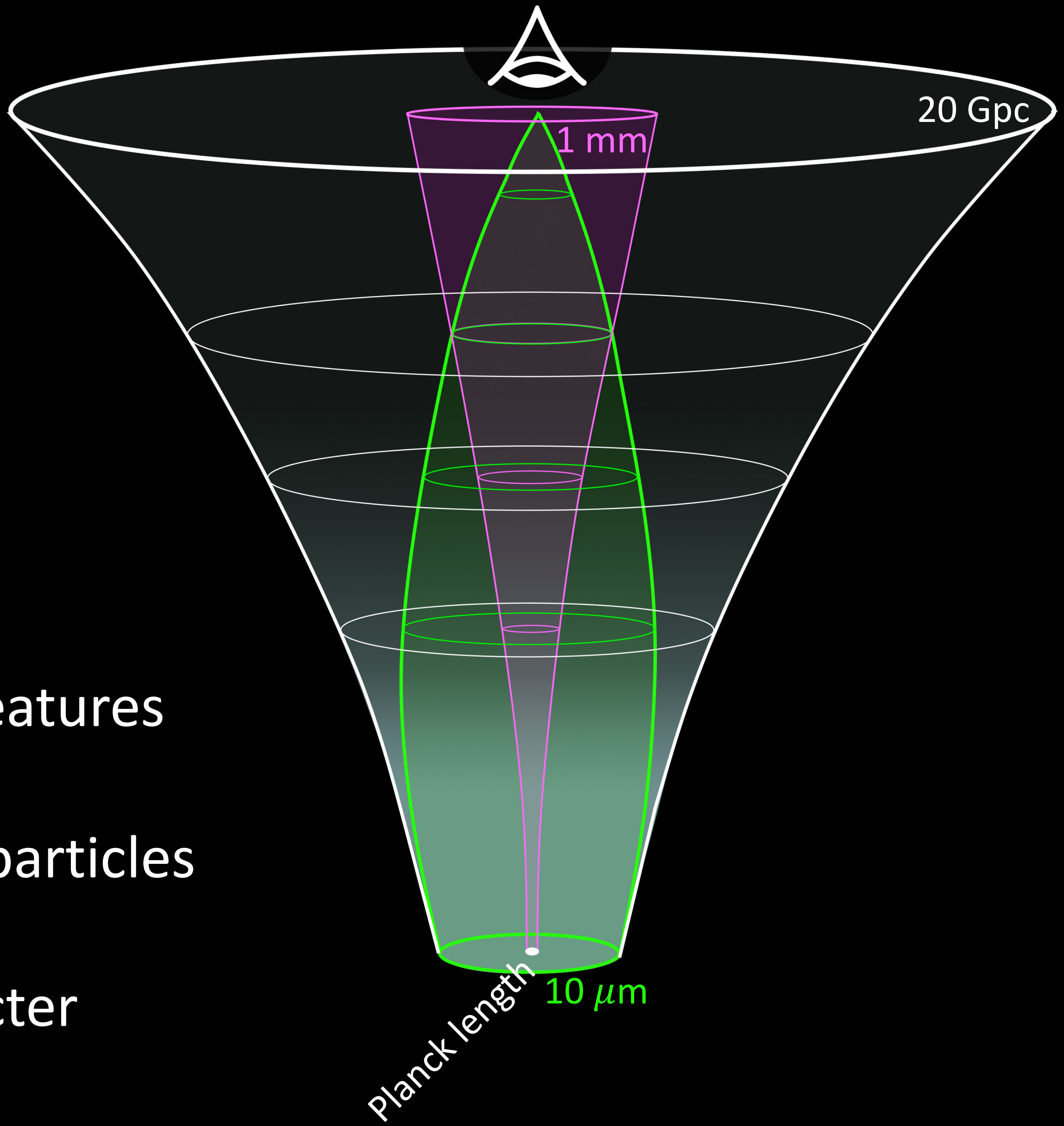
ESA Planck satellite

The simplicity of the observations suggests we look for simpler and more elegant theoretical explanations

This talk: a unified framework based on extrapolating the radiation epoch (and the SM) all the way back to the big bang singularity

Guiding principles:
CPT symmetry, analyticity, asymptotic conformal (Weyl) symmetry

new explanations for SM/LCDM's key features
3 generations including RH $\nu' s$
no need for inflation or any additional particles
with minimal assumptions we explain
the amplitude, tilt and statistical character of the primordial fluctuations



Friedmann-Lemaître-Robertson-Walker cosmology

scale factor

$$ds^2 = a(t)^2\left(-dt^2 + \gamma_{ij}dx^i dx^j\right)$$

conformal time

comoving symmetric space (assume compact) $R^{(3)} = 6\kappa$

In Planck units

$$3\dot{a}^2 = r + \mu a - 3\kappa a^2 + \lambda a^4$$

radiation   matter   space curvature   Lambda

general solution (Jacobi elliptic function) has beautiful analytic properties (single-valued in complex t-plane, only singularities are simple poles, arranged in a periodic lattice)

For a perfect radiation fluid, $T^{\mu}{}_{\mu} = 0 \ \ (P = \frac{1}{3}\rho)$, *i.e.*, local conformal symmetry,

$\exists\ \infty^3$ solutions to Einstein-fluid equations which are analytic at $t = 0$.

$$ds^2 = t^2(-dt^2 + h_{ij}(t, \boldsymbol{x}) dx^i\, dx^j);\ h_{ij}(t, \boldsymbol{x}) = h^0_{ij}(\boldsymbol{x}) + t^2\, h^2_{ij}(\boldsymbol{x}) + \ldots,$$

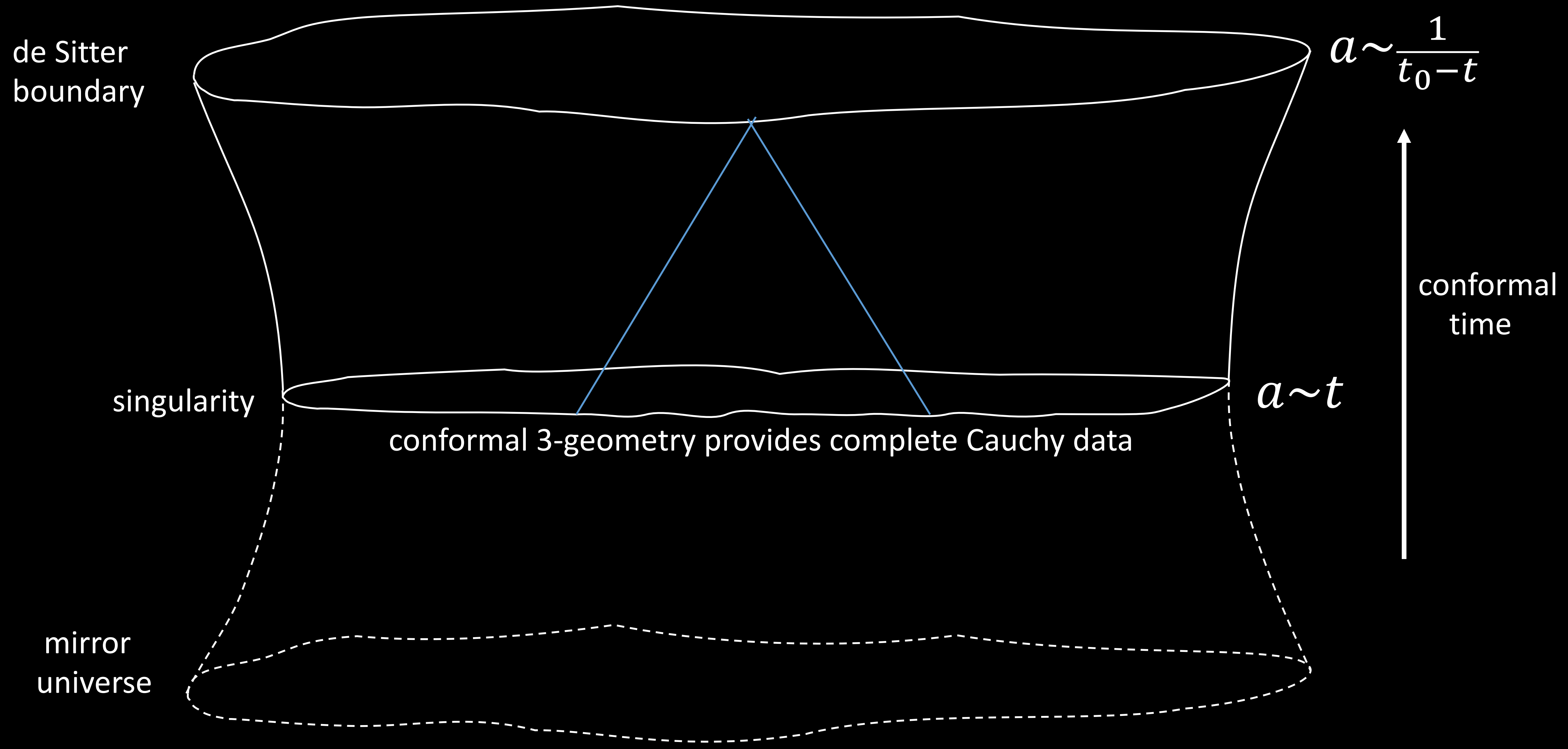
regular 4-metric — regular 3-metric — determined by Einstein eqns

They all have a global isometry $t \leftrightarrow -t$. They are saddles of the real-time path integral for gravity with CPT-symmetric boundary conditions.

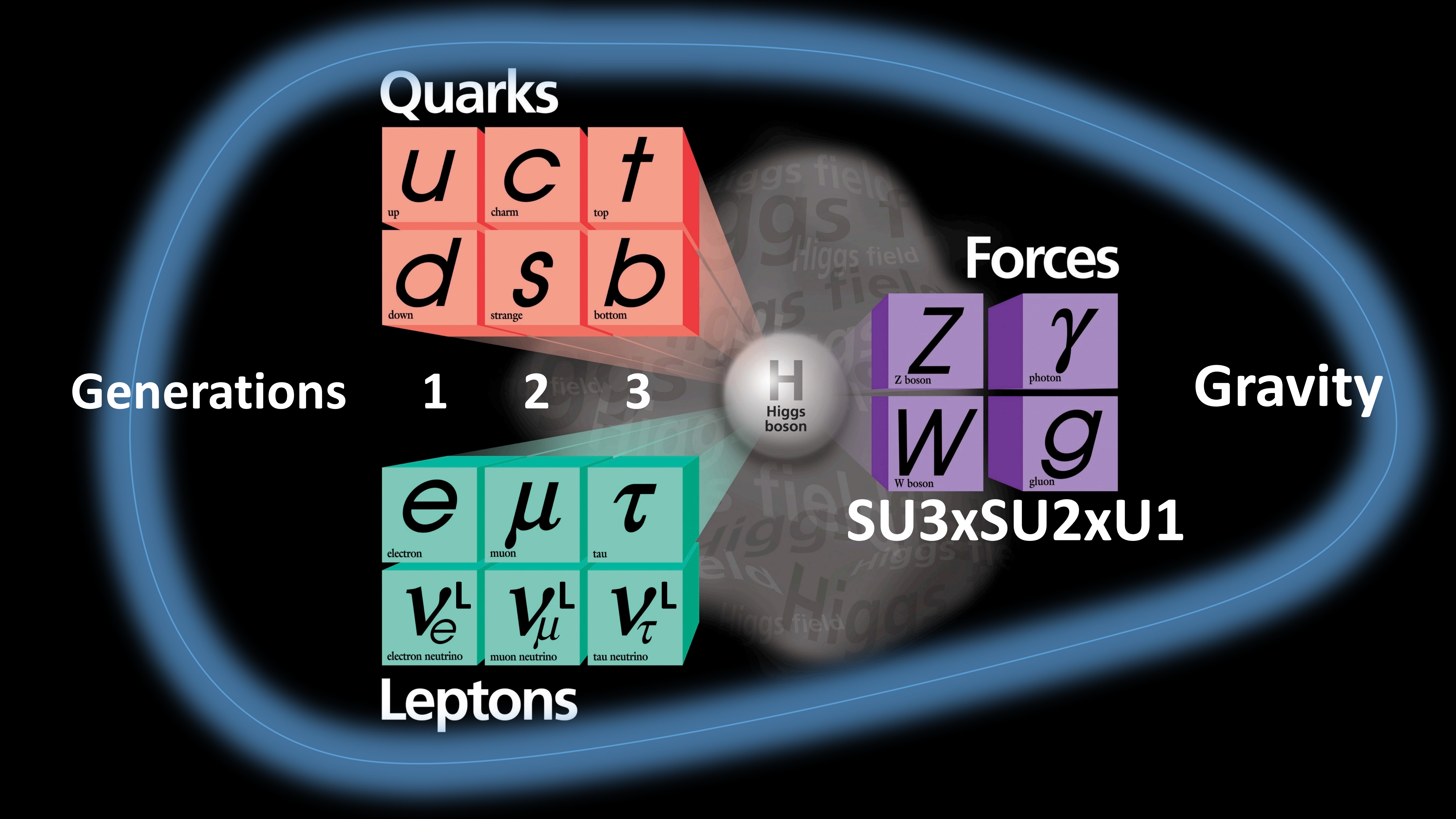
The singularity is purely *conformal* and invisible to conformally invariant matter

BKL or Mixmaster excluded because they are singular hence not genuine saddles

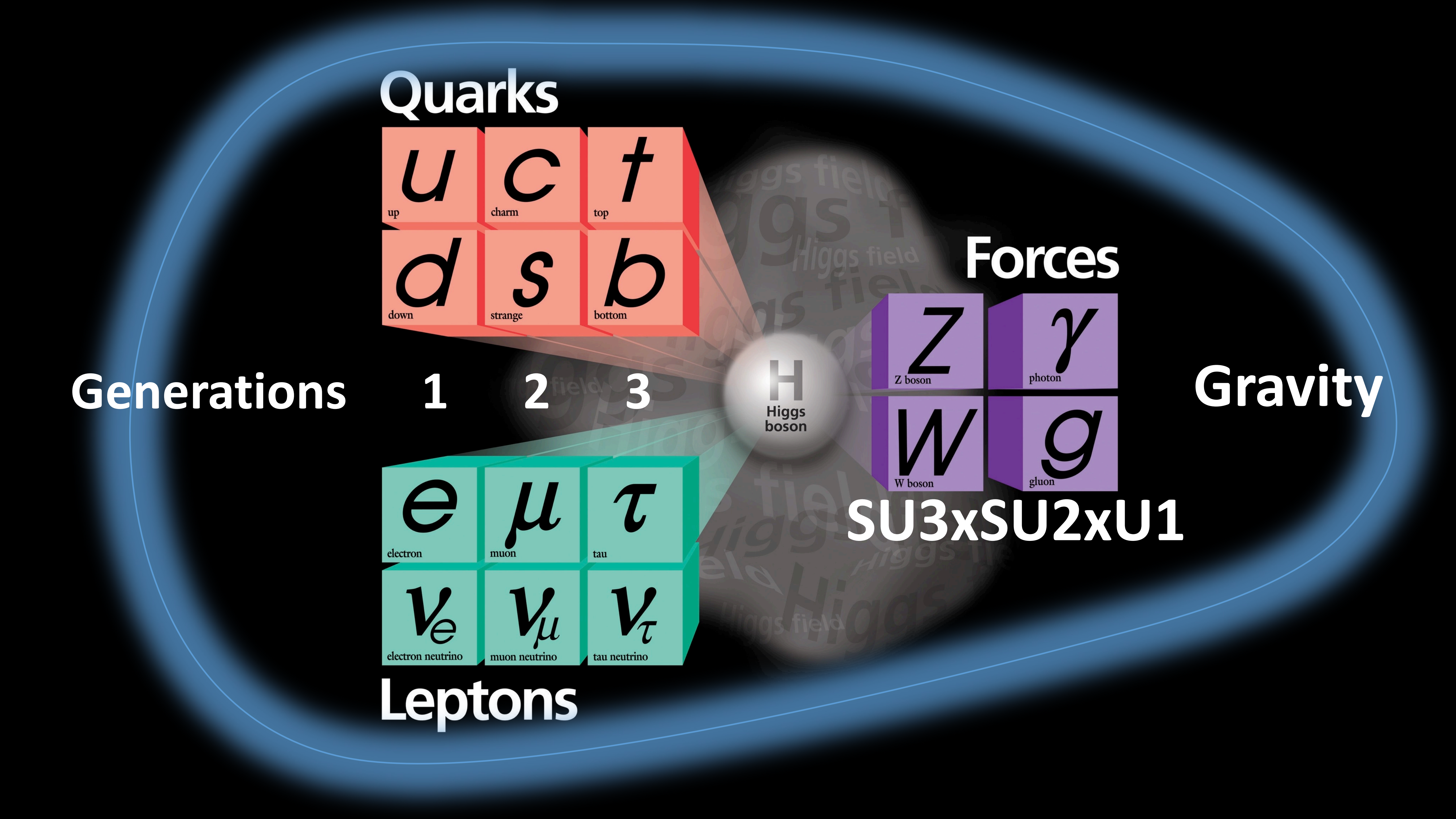
Conformal (Penrose) diagram



The full nonlinear Einstein's equations for inhomogeneous cosmology can be solved in a covariant gradient expansion by matching the two asymptotic series ("double Fefferman-Graham") (NT, in prep. 2024) a minimal explanation of the dark matter



Quarks
u up
c charm
t top
d down
s strange
b bottom
Generations 1 2 3
Higgs field
H Higgs boson
Forces
Z Z boson
γ photon
W W boson
g gluon
SU3xSU2xU1
Gravity
e electron
μ muon
τ tau
ν_e^L electron neutrino
ν_μ^L muon neutrino
ν_τ^L tau neutrino
Leptons


Quarks
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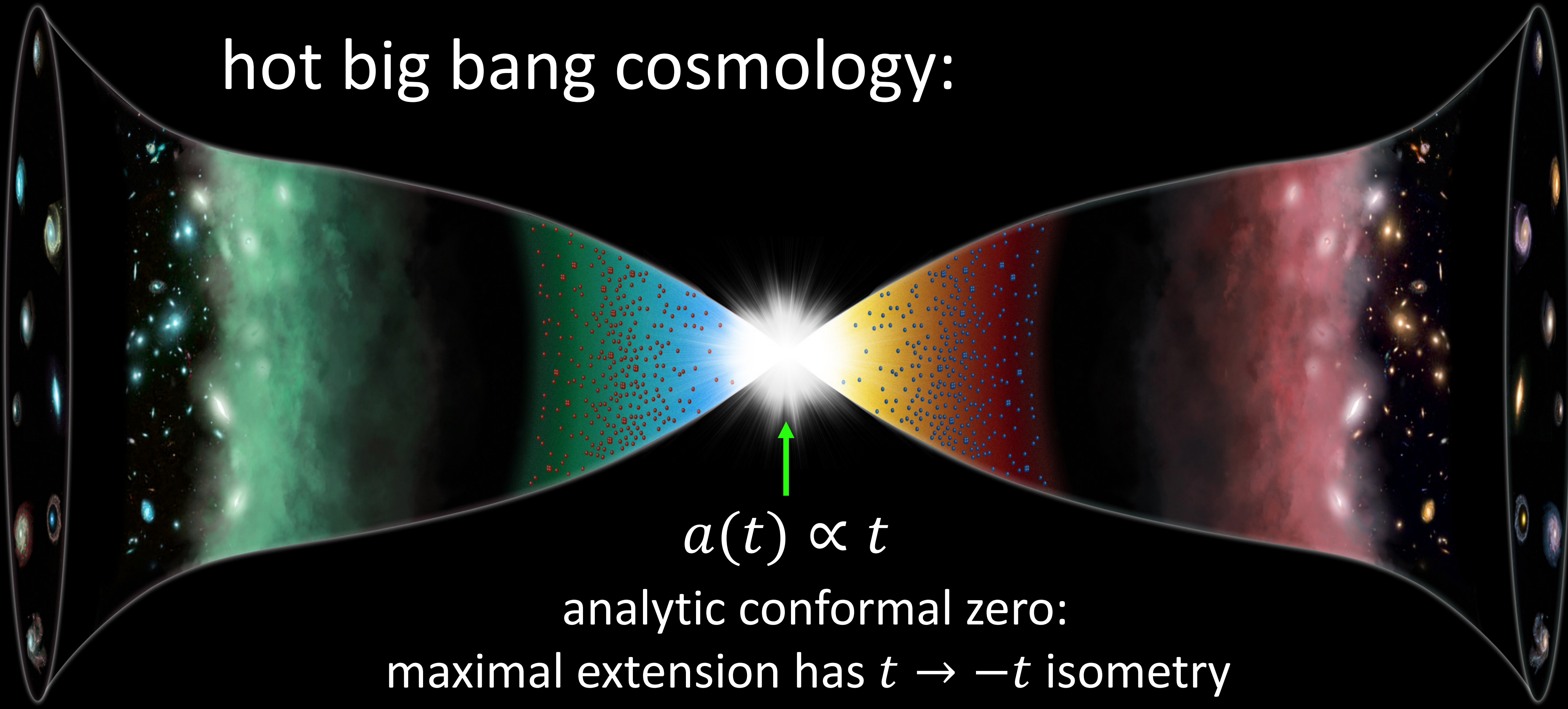
Leptons

hot big bang cosmology:



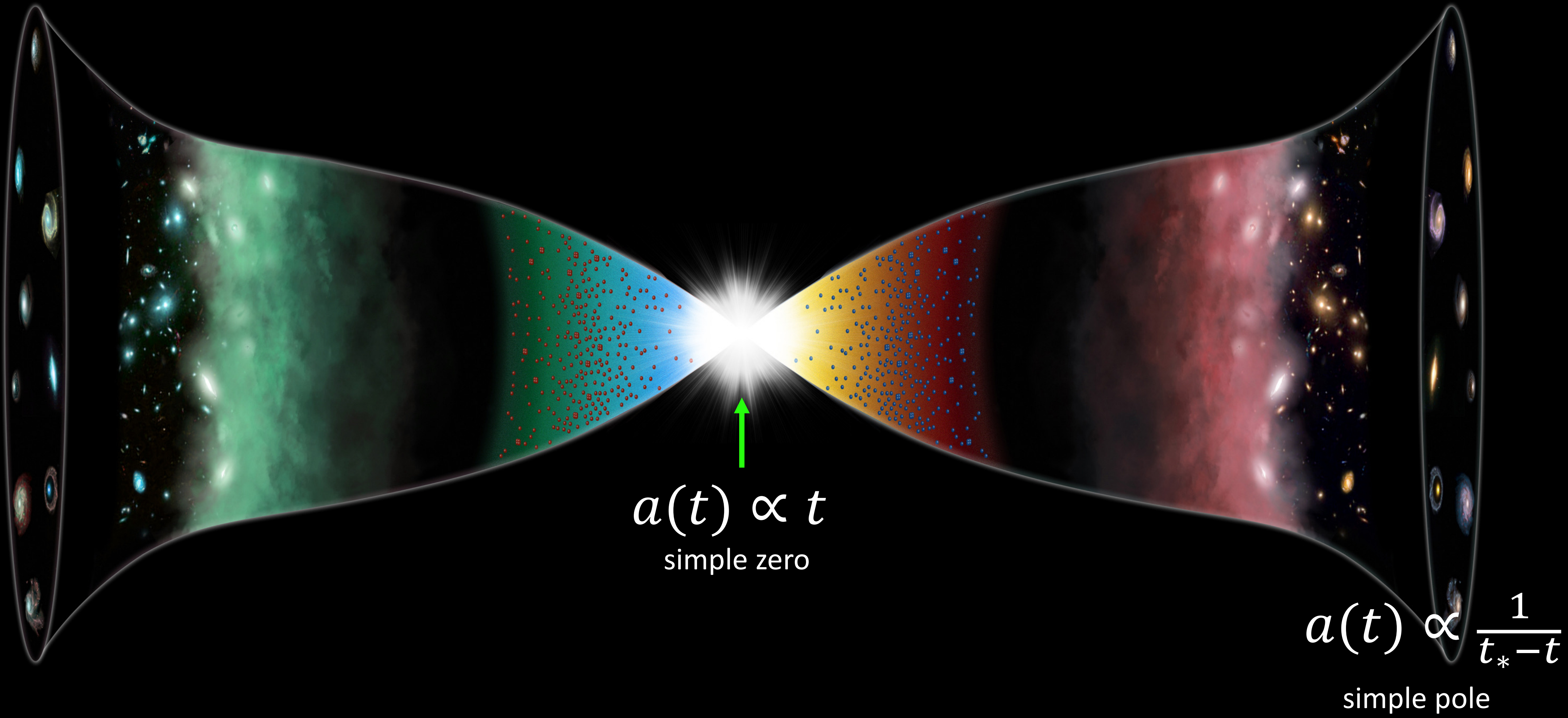
$a(t) \propto t$

analytic conformal zero:

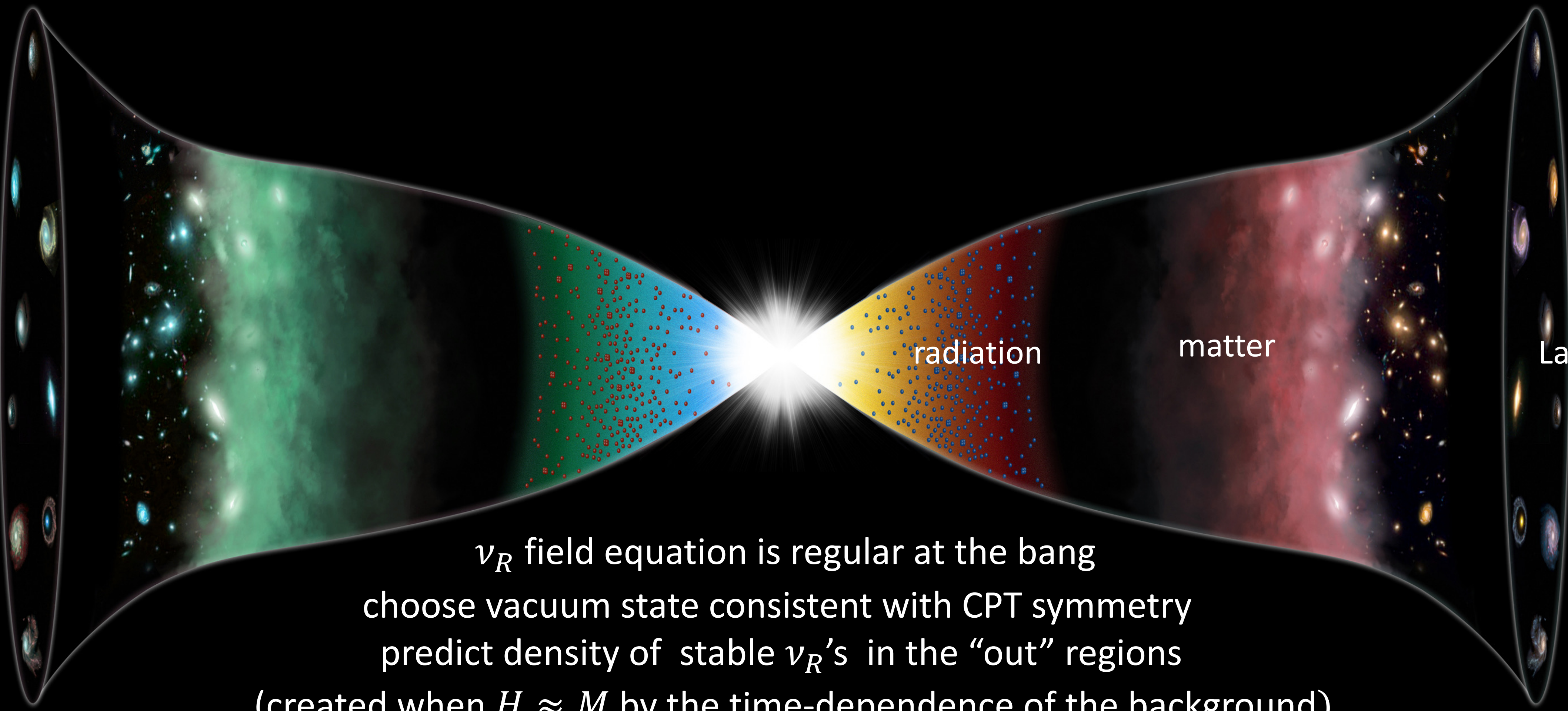
maximal extension has $t \rightarrow -t$ isometry

can impose CPT symmetry via the “method of images”

the big bang singularity is then a CPT mirror


$a(t) \propto t$
simple zero
$a(t) \propto \frac{1}{t_* - t}$
simple pole

Boyle, Finn & NT, *Phys. Rev. Lett.* 121 (2018) 251301;
Annals of Physics 438 (2022) 168767

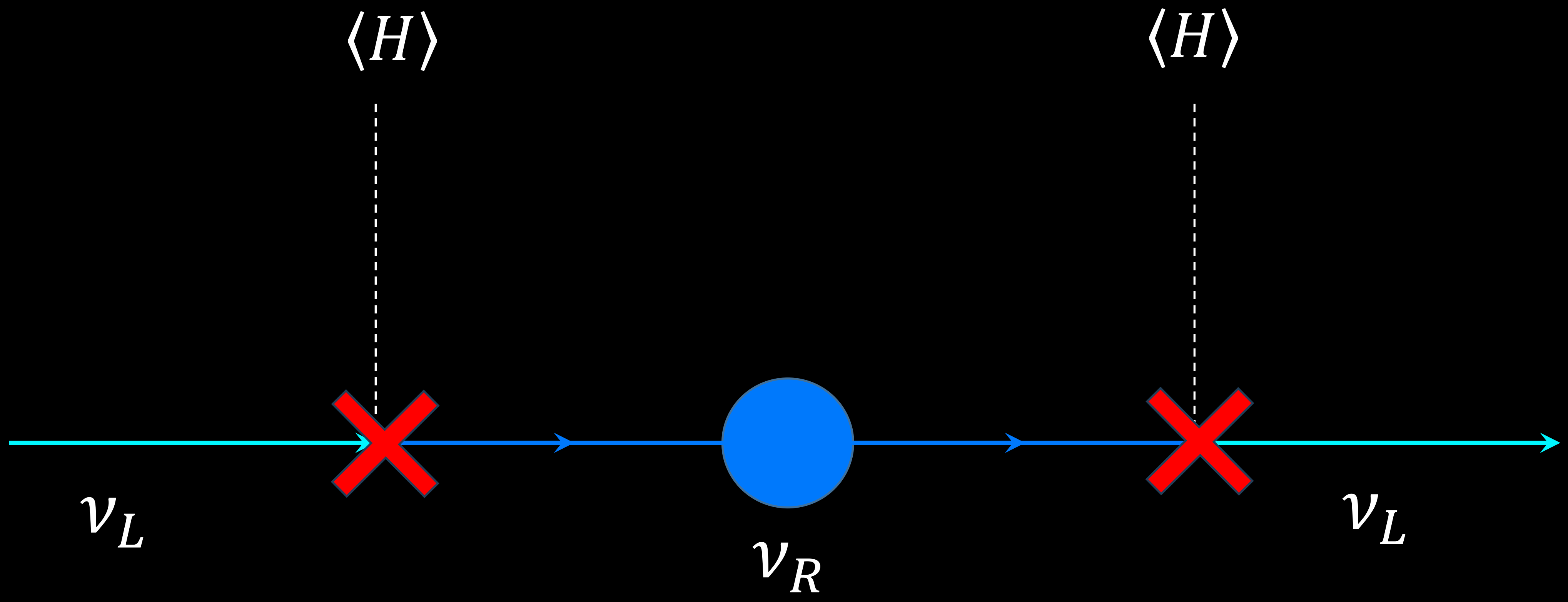


ν_R field equation is regular at the bang
choose vacuum state consistent with CPT symmetry
predict density of stable ν_R's in the "out" regions
(created when $H \approx M$ by the time-dependence of the background)

If one ν_R is stable, its density matches Ω_{DM} if its mass $M \approx 5{\times}10^8 GeV$

Also explains the baryon asymmetry as in Shaposhnikov picture. However, our ν_R DM particle is stable and much heavier. We also have a testable prediction: the lightest neutrino is massless.

Stability of one RH neutrino $\Rightarrow$ $\mathbb{Z}_2$ symm $\Rightarrow$ lightest ν massless



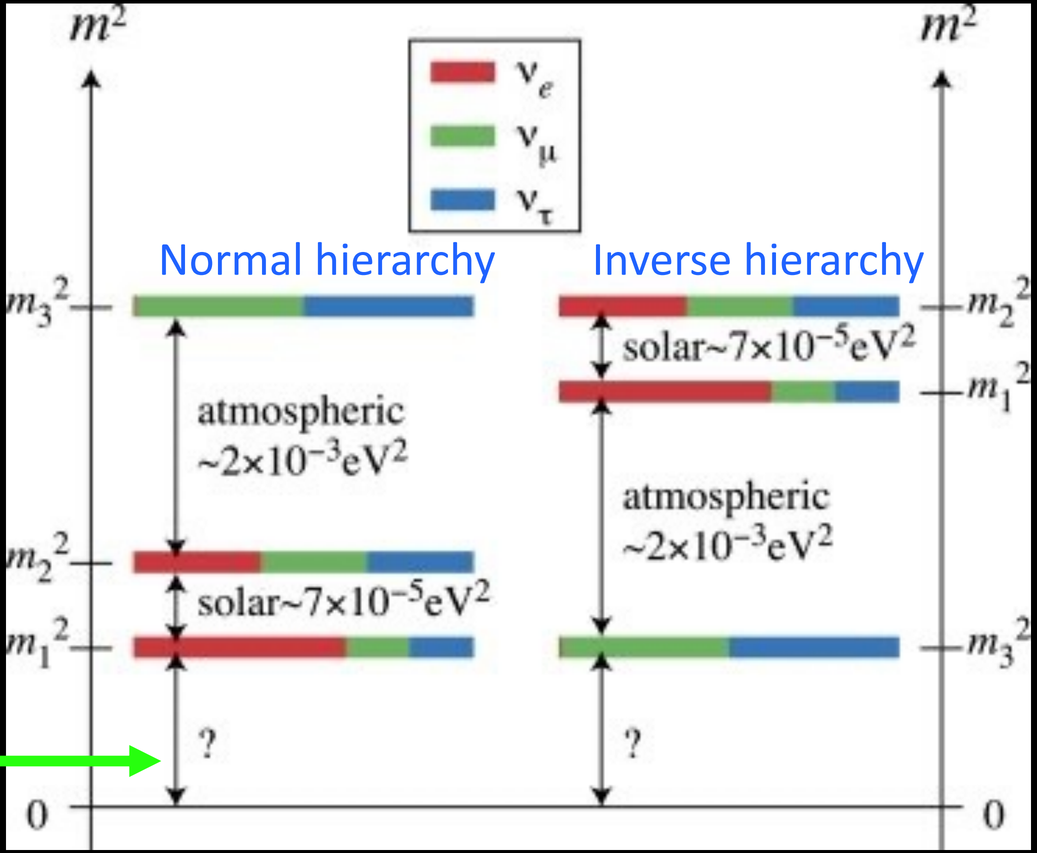
will soon be tested using EUCLID, LSST and S4

Light neutrinos: observations

current data

eBOSS 2007.08991

we predict zero



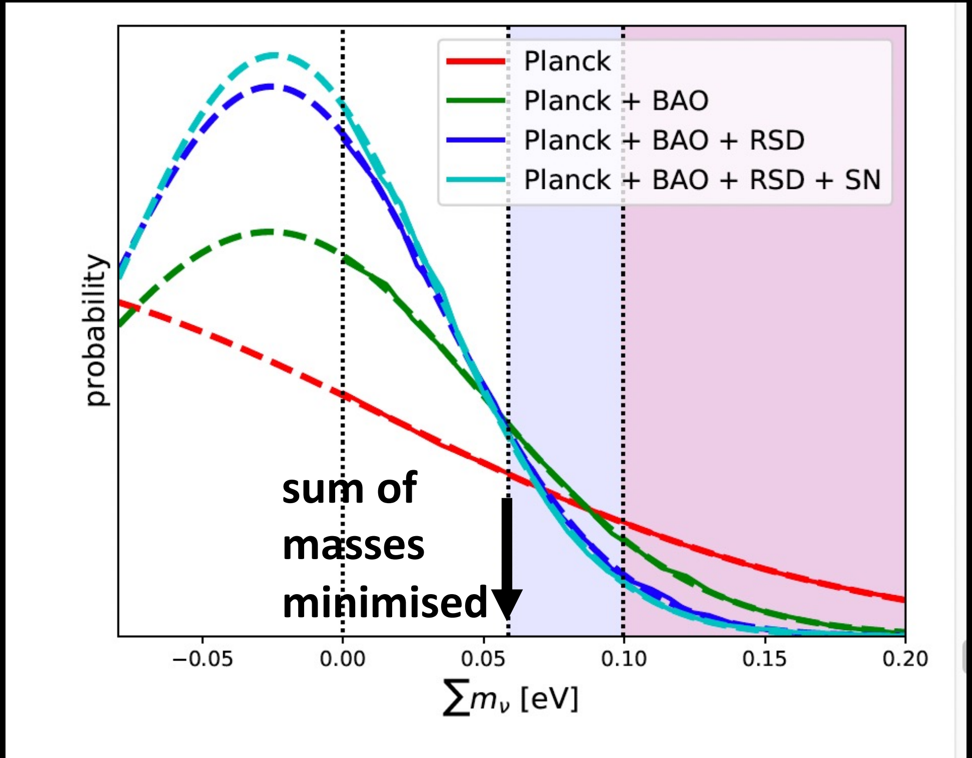


FIG. 13.— Posterior for sum of neutrino masses for selected con binations of data with a $\nu\Lambda$CDM cosmology. Dashed curves sho the implied Gaussian fits. Shaded regions correspond to lower lin its on normal and inverted hiearchies. Likelihood curves are no malized to have the same area under the curve for $\sum m_\nu > 0$.

Normal hierarchy: $M_\nu \equiv \sum m_\nu \approx 0.06\ eV$

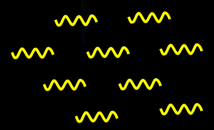
Inverted hierarchy: $M_\nu \approx 0.1\ eV$

a minimal explanation of the large-scale geometry

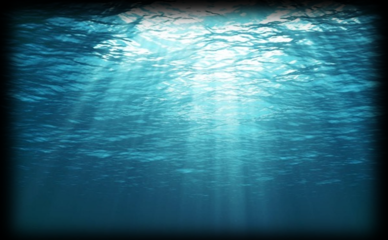
Penrose

Path integrals and gravity

gauge fields

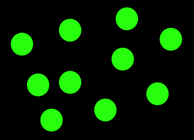


Higgs



$$\int e^{\frac{i}{\hbar}\int\left(\frac{R}{16\pi G}-\frac{1}{4}F^2+\bar{\psi} i \not{D}\psi-\lambda H\bar{\psi}\psi+|DH|^2-V(H)\right)}$$

gravity



particles

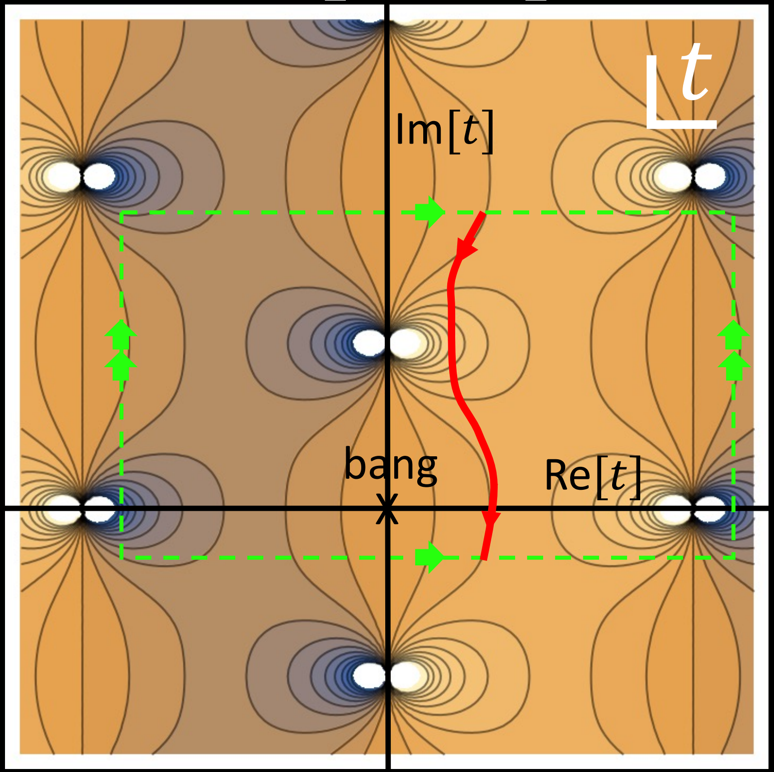
gravitational entropy

With pbcs in imaginary time, $Z = e^{S_{tot}} = e^{S_{ord} + S_g}$

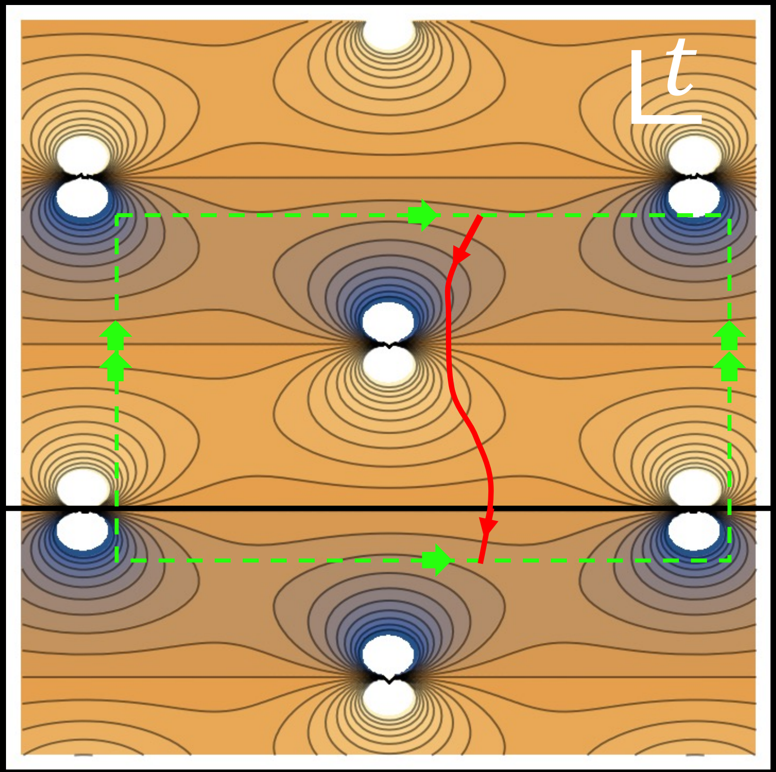
partition function

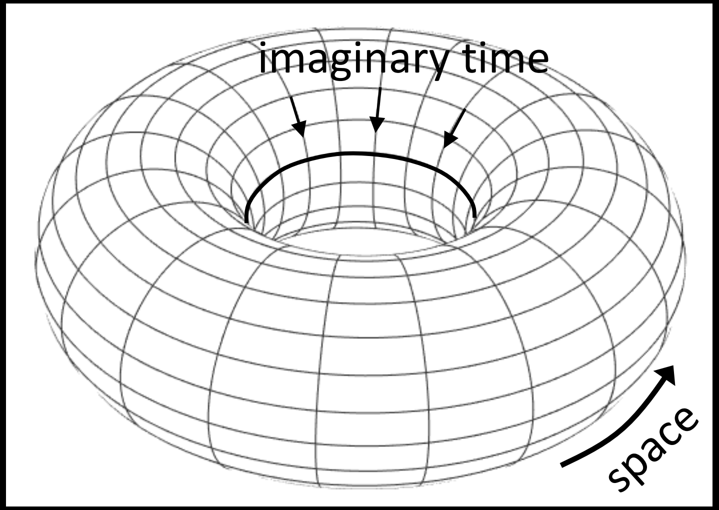
$a(t)$ is single-valued and doubly periodic in the complex t-plane. The imaginary time period and the associated action (which is topological by Cauchy's theorem) determine T_H and the gravitational entropy S_g

$\mathrm{Re}[a(t)]$



$\mathrm{Im}[a(t)]$





Euclidean instanton for a universe w/radiation, matter, curvature, Lambda

S_g can be calculated analytically for a general cosmology with radiation, matter, space curvature and a cosmological constant (*i.e.*, all globally conserved quantities).

Inhomogeneities and anisotropies treated in cosmological perturbation theory.

S_g gives the total number of states associated with a cosmology. It favours

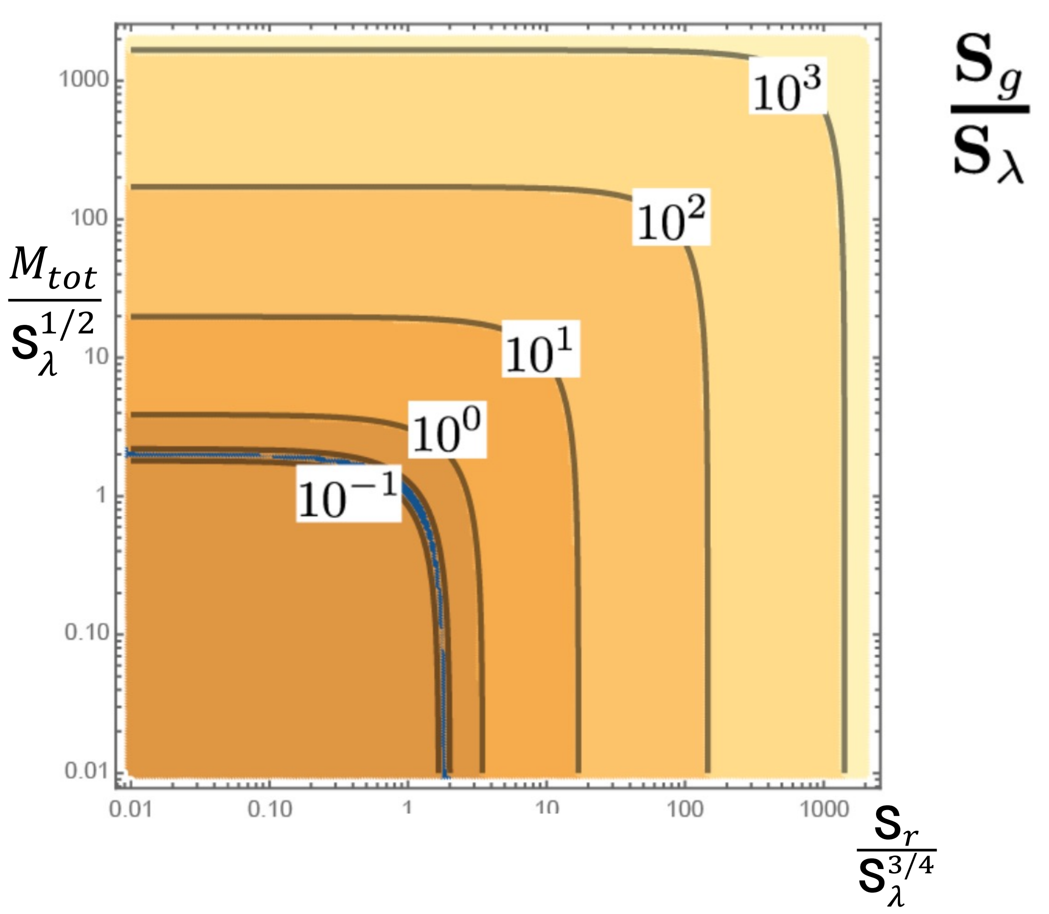
1. homogeneous, isotropic, spatially flat universes
2. a small, positive cosmological constant

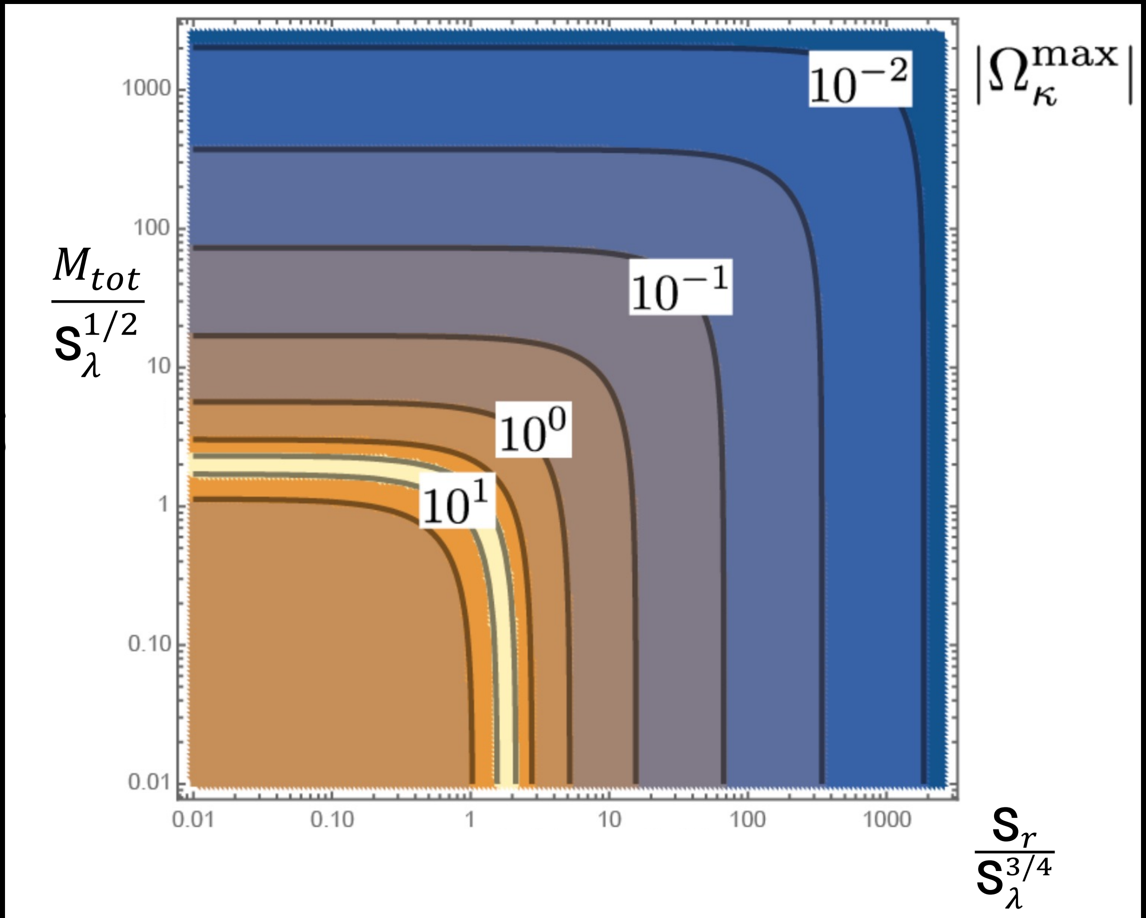
(echoing earlier arguments of Baum, Hawking, Coleman…)

This is a thermodynamic explanation of the large-scale geometry of the cosmos. No smoothing or flattening mechanism is required.

Note: S_g is the *global* entropy and counts the total number of accessible microstates. It is independent of the real time by Cauchy's theorem.

Including matter



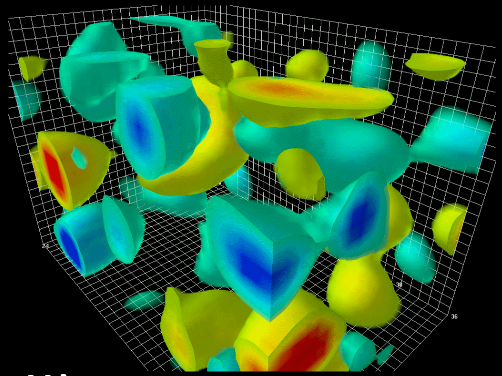


Suggests an explanation of the matter/radiation and Lambda/matter coincidence from equipartition (must also include gravitational entropy due to black holes)

We still need to understand how to impose all of the relevant constraints.

a minimal explanation of the perturbations

Quantum fields and gravity



vacuum energy and pressure are divergent in physical regularizations (*e.g.*, point-splitting, Maxwell):

B.S.DeWitt, Phys. Rep. 19 (1975) 295

$$\Longrightarrow \langle T^{\mu\nu}\rangle \sim \frac{3}{\pi^2 \Delta t^4}\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{1}{3} & 0 & 0 \\ 0 & 0 & \frac{1}{3} & 0 \\ 0 & 0 & 0 & \frac{1}{3} \end{pmatrix},$$ where Δt^2 = invart time-like separation

Can be renormalized away but leaves us without a physical understanding

In interacting theory, e.g. QED, quantum divergences spoil the local scale (Weyl) invariance of Maxwell and Dirac fields in curved backgrounds: two independent conformal anomalies.

$$\langle T^{\mu}{}_{\mu}\rangle = a\,E + c\,C^2;\quad E = R_{\alpha\beta\gamma\delta}R^{\alpha\beta\gamma\delta} - 4R_{\alpha\beta}R^{\alpha\beta} + R^2;\, C^2 = C_{\alpha\beta\gamma\delta}C^{\alpha\beta\gamma\delta}$$

Dimension zero scalars:

Described by a four-derivative, Weyl-invariant (*i.e.,* locally scale-invariant) action

$$S_4 = -\frac{1}{2}\int d^4x\sqrt{-g}\left((\Box\varphi)^2 + ..\right)$$

Canonical quantization leads to negative norm states. However, as we shall see, these can be removed using an infinite-dimensional symmetry (superselection).

The only remaining physical state is the vacuum, which has scale-invariant fluctuations and vacuum stress energy

$$\langle\varphi(0,\boldsymbol{x})\varphi(0,\boldsymbol{y})\rangle = \int \frac{d^3\boldsymbol{k}}{(2\,\pi)^3}\frac{e^{i\boldsymbol{k}.(\boldsymbol{x}-\boldsymbol{y})}}{4\,k^3}$$

cf. approximately scale-invariant Newtonian potential Φ observed in cosmology

Dim 0 scalars can cancel the above 3 anomalies in coupling the SM to gravity

L. Boyle+NT, arXiv:2110.06258

Dim 1 scalars | fermions | gauge bosons | Dim 0 scalars

1. Vacuum energy $\propto n_{s,1} - 2n_F + 2n_A + 2n_{s,0}$
2. Conformal anomaly (Euler) $\propto n_{s,1} + \frac{11}{2} n_F + 62\, n_A - 28\, n_{s,0}$
3. Conformal anomaly (Weyl^2) $\propto n_{s,1} + 3\, n_F + 12\, n_A - 8\, n_{s,0}$

1) All three vanish iff $n_{s,1} = 0 \Longrightarrow$ *no* fundamental dim 1 scalars allowed (*i.e.*, the Higgs must be composite)

2) Any two equations then give $n_F = 4n_A$ and $n_{s,0} = 3n_A$

3) For gauge group $SU3\times SU2\times U1$, cancellation of all three anomalies **predicts** n_F =48, *i.e.*, **3 fermion generations, each with a RH ν**

primordial perturbations from dimension 0 fields

Boyle+NT

arXiv: 2302.00344

Running couplings violate scale symmetry: at high temperature,

$$T_\beta^{SM} \equiv \langle T^{SM\mu}{}_\mu \rangle_\beta = 3P - \rho \approx \sum c_i \alpha_i^2 T^4 \equiv c_\beta^{SM} T^4; \quad \text{in SM, } c_\beta^{SM} \equiv \frac{125}{108}\alpha_Y^2 - \frac{95}{72}\alpha_2^2 - \frac{49}{6}\alpha_3^2$$

This anomalous trace can be cancelled by introducing a linear coupling in the effective action,

$$\Gamma^\varphi = \sum_{j=1}^{n_{s,0}} \frac{1}{2} \int -a\varphi_j \Delta_4 \varphi_j + \left[a\left(E - \tfrac{2}{3}\Box R\right) + cC^2 - n_{s,0}^{-1} T_\beta^{SM}\right]\varphi_j$$

Note: the linear term generalizes the one used in string theory to preserve conformal symmetry

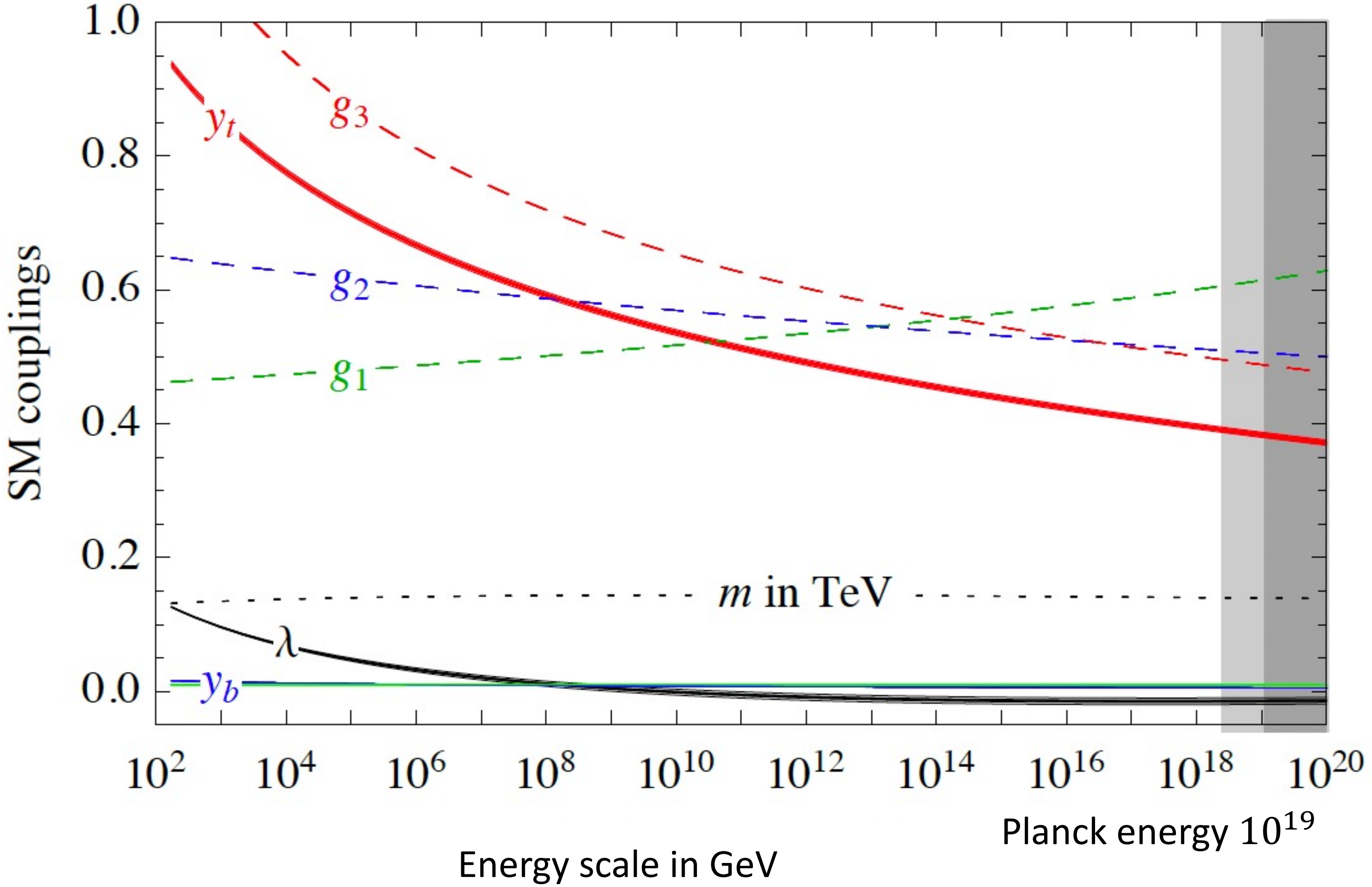
The final term corrects the Friedmann-fluid equations, converting quantum correlations in the dim-0 fields into large scale fluctuations of the conformal factor:

$$\dot{a}^2 = \frac{8\pi G}{3}\rho_r a^4 (1 + c_\varphi \bar{\varphi}(x)) \text{ with } \bar{\varphi}(x) = n_{s,0}^{-1} \sum \varphi_j(x),\ c_\varphi = c_\beta^{SM} / \left(\tfrac{\pi^2}{30}\mathcal{N}_{eff}\right), \mathcal{N}_{eff} \approx 106\tfrac{1}{4}$$

This corresponds to a “comoving curvature perturbation” $\mathcal{R}(x) = \frac{1}{4} c_\varphi \bar{\varphi}(x)$

(adiabatic, Gaussian, scalar: no primordial long-wavelength tensors)

Buttazzo et al
1307.3536
[hep-ph]



Spectral tilt

Dominated by QCD coupling α_3: asymptotic freedom $\Longrightarrow$ red tilt

We argue that $\mathcal{P}_{\mathcal{R}}(k)$ runs with k as $\alpha_3^2(k)$, as $k \to 0$

This leads to the prediction $n_s - 1 \equiv \frac{d \ln \mathcal{P}_{\mathcal{R}}(k)}{\mathrm{d} \ln k} = 2\frac{\beta_{\alpha_3}}{\alpha_3} = -\frac{7}{\pi}\alpha_3(M_P)$

Since this is a critical exponent we can extrapolate from the Planck length to today's Hubble radius traced (comoving) right back to the Planck time.

(30 orders of magnitude in length scale!)

Remarkably, the amplitude and tilt then agree with Planck's observations

Prediction for primordial perturbations

$\mathcal{P}_{\mathcal{R}}(k) = \frac{3^2 5^2}{7(2\pi)^4}\left(\frac{c_\beta^{SM}}{\mathcal{N}_{eff}}\right)^2\left(\frac{k}{k_P}\right)^{-\frac{7\alpha_3}{\pi}}$; k_P= comoving Planck wavenumber

with $c_\beta^{SM} \equiv \frac{125}{108}\alpha_Y^2 - \frac{95}{72}\alpha_2^2 - \frac{49}{6}\alpha_3^2$ and $\mathcal{N}_{eff} = 106\frac{1}{4}$

Now use $(k_P/k_*)^{1-n_s} = 14.8 \pm 5.1,$ $k_* \equiv$0.05 Mpc^{-1}

Thus, we predict $\mathcal{P}_{\mathcal{R}} = A\left(\frac{k}{k_*}\right)^{n_s-1}$, $A = (13 \pm 5)\times 10^{-10}$; $n_s = 0.958$

cf. Planck satellite: $A = (21 \pm 0.3)\times 10^{-10}$; $n_s = 0.959 \pm 0.006$

Interacting dimension zero scalars

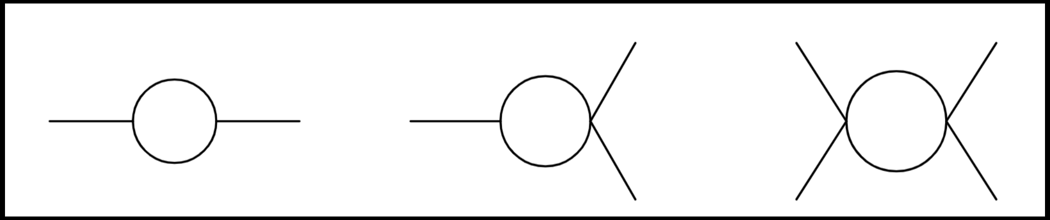
The most general renormalizable action consistent with classical scale invariance and shift symmetry $\varphi \to \varphi + c$

$$S = \int [-\tfrac{1}{2}(\Box\varphi)^2 + \lambda_3 \Box\varphi(\partial\varphi)^2 + \lambda_4((\partial\varphi)^2)^2]$$

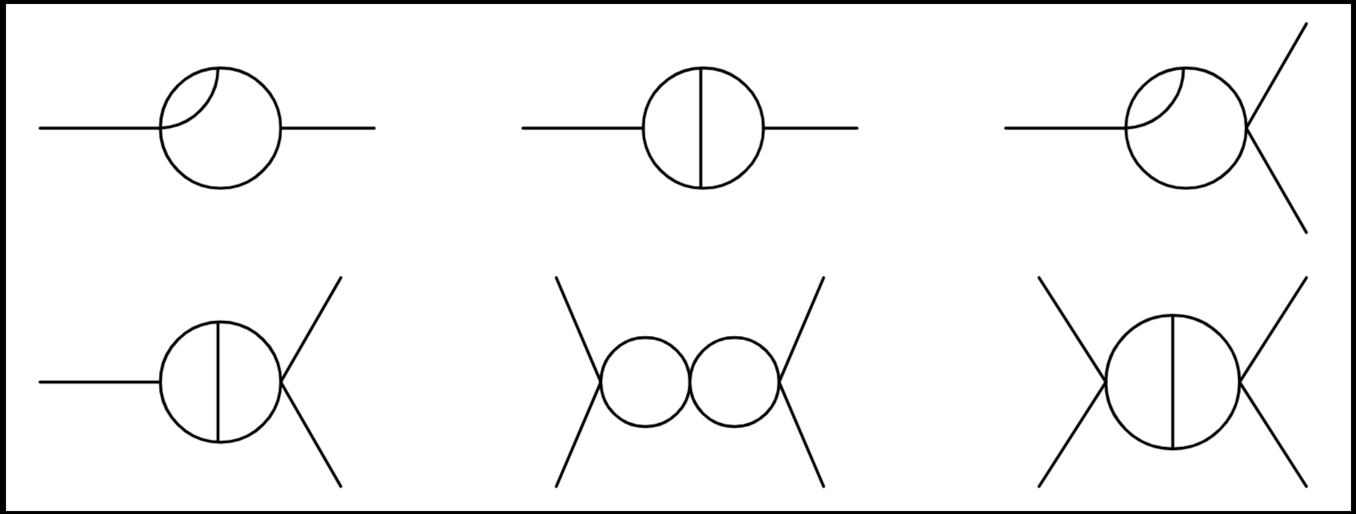
B. Holdom 2303.06723

$$S = \int[-\tfrac{1}{2}(\Box\varphi)^2+\lambda_3\Box\varphi(\partial\varphi)^2+\lambda_4((\partial\varphi)^2)^2]$$

w/ Maegan Anderson
and Franz Herzog



1 loop divergences



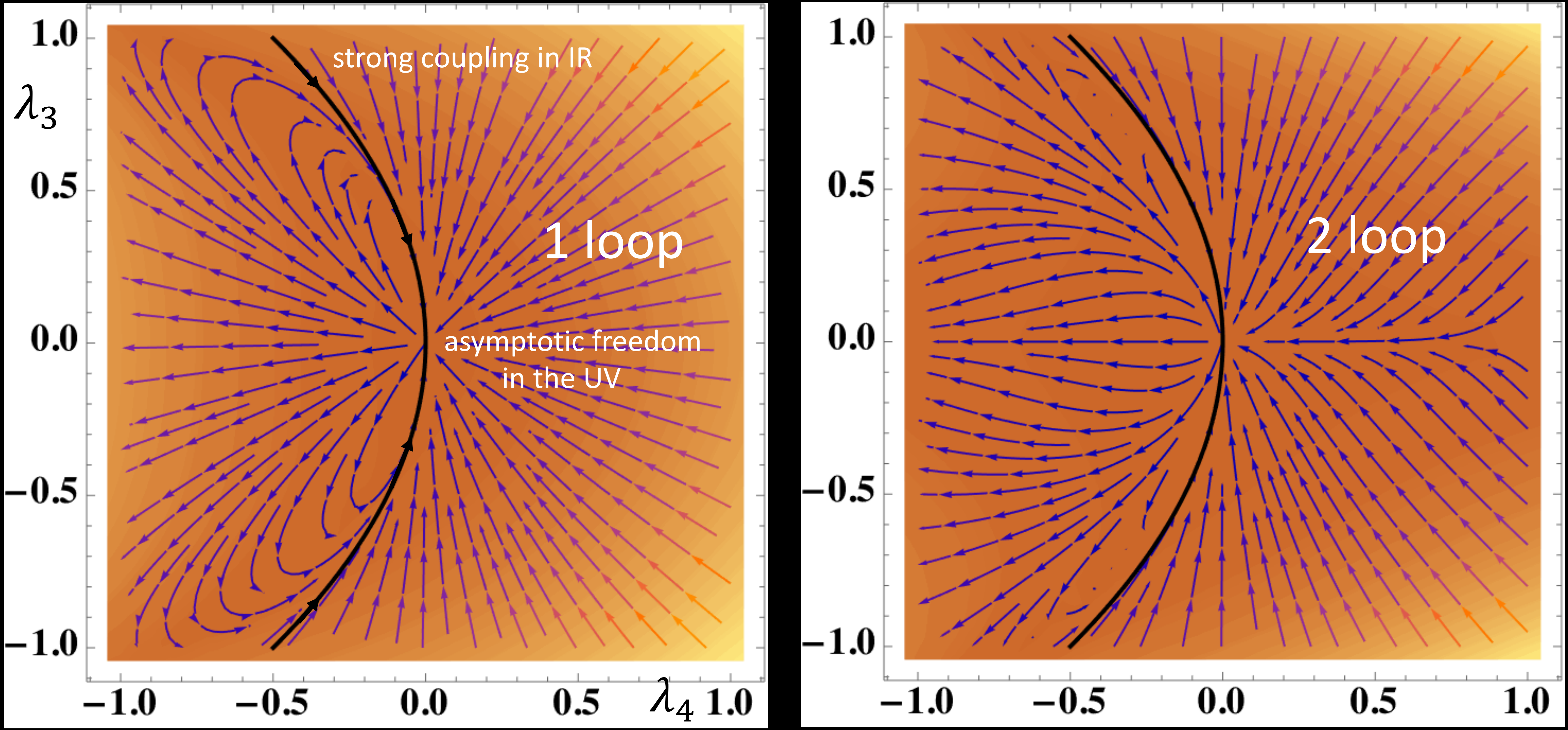
2 loop divergences

$$\beta(\lambda_3) = -\frac{5}{4\pi^2}(\lambda_3\lambda_4 + \frac{3}{4}\lambda_3^3) - \frac{5}{16\pi^4}(\frac{154}{9}\lambda_3^3\lambda_4 + \frac{44}{9}\lambda_3\lambda_4^2 + \frac{53}{6}\lambda_3^5)$$

$$\beta(\lambda_4) = -\frac{5}{4\pi^2}(\lambda_4^2 + \lambda_3^2\lambda_4) - \frac{5}{16\pi^4}(\frac{52}{3}\lambda_3^2\lambda_4^2 + \frac{106}{9}\lambda_3^4\lambda_4 - \frac{4}{9}\lambda_4^3)$$

The curve $\lambda_4 = -\frac{1}{2}\lambda_3^2$, *i.e.*, $S = \int -\frac{1}{2}\left[\Box\varphi - \lambda_3(\partial\varphi)^2\right]^2$

is special: it is preserved under RG flow ($Z_4 = Z_3^2$)



- On the special curve, action is a perfect square $S = -\frac{1}{2}\int((\Box\varphi)^2 + (\partial\varphi)^2)^2$

- Alternative formulation $S = \int(\partial U \partial V + \frac{1}{2}(UV)^2)$; $U \equiv e^{\varphi}$
- Equations of motion $\Box U = -(UV)U$, $\Box V = -(UV)V$
- Infinite number of conservation laws: if $\Box\alpha = -(UV)\alpha$, *i.e*, α sats KG eq with spacetime-dependent "mass squared", then both of the currents

$J_\mu^U = U\partial_\mu\alpha - \alpha\, \partial_\mu U$ and $J_\mu^V = V\partial_\mu\alpha - \alpha\, \partial_\mu V$ are conserved

Setting all the corresponding conserved charges to zero eliminates the negative norm states, leaving the vacuum as the unique physical state.

(with S. Bateman)

Summary

- a minimal, testable explanation for the cosmic dark matter
- a thermodynamic explanation for the large scale geometry
- a minimal mechanism to cancel the SM's vacuum energy and trace anomalies
  (as well as nasty SM corrections to the graviton propagator)
- requires 3 generations of fermions, each with a RH neutrino
  (and a composite Higgs which may help to explain the gauge hierarchy)
- the same mechanism quantitatively seems to explain the cosmological density perturbations and predicts zero tensors – a complete treatment is under way

Thank you for listening!

Boyle, Finn & NT, *Phys. Rev. Lett.* 121 (2018) 251301; *Annals of Physics* 438 (2022) 168767
Boyle & NT arXiv: 2110.06258, *Phys. Lett.* B849 (2024) 138442
arXiv: 2201.07279, *Phys. Lett.* B849 (2024) 138443
arXiv: 2302.00344 and references therein