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Reheating at strong coupling

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Bibliography

Ongoing work with:

C. Ecker (Frankfurt), T. Ishii (Tokyo), C. Rosen (Crete), W. Van der Schee (CERN and Utrecht)

C. Ecker (Frankfurt) and W. Van der Schee (CERN and Utrecht)

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Introduction

• Cosmological Inflation is considered as an important part of early universe history.

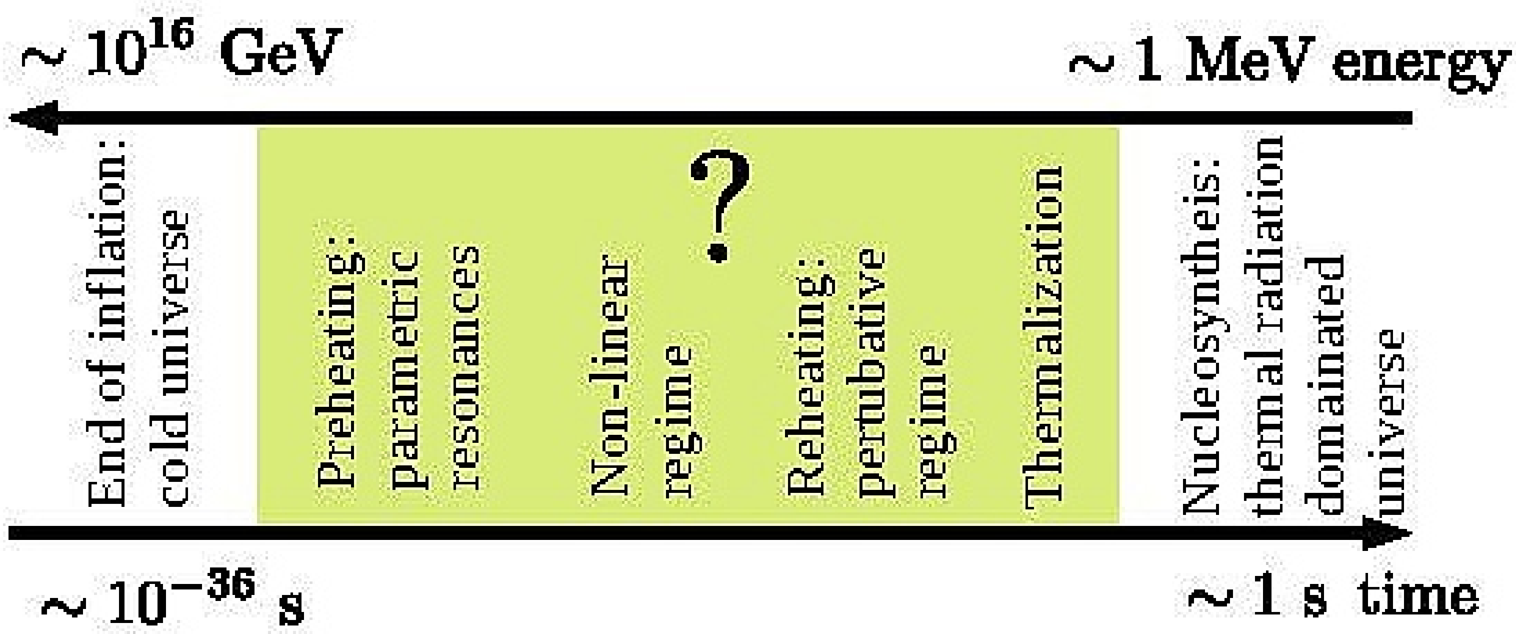
• Inflation has to end after at least 55 e–foldings.

• By that time, in almost all scenarios, the universe is cold and empty.

• The potential energy has to come back to the dynamics and eventually thermalize, above than $T \sim 10 - 40$ MeV, before nucleosynthesis.

• This is the "Big Bang".

• The standard framework used so far (with some variations) involves three stages all happening at weak coupling:
(a)Preheating (particle production via Floquet resonances)
(b)(Classical) non-linear evolution
(c) (Very slow) thermalisation.



Dolgov+Kirilova, Traschen+Brandenberger, Kofman+Linde+Starobinsky

Preheating and Thermalisation at strong coupling ?

• The question we shall address is how this can happen in strongly coupled QFTs.

• Strongly coupled QFTs are difficult to handle and solve.

• In the past 20+ years a new class of QFTs was studied: holographic QFTs.

• Such theories are dual to weakly-curved (semiclassical) generalized gravity theories.

• They typically have large N_c and strong coupling.

• Several types of calculation are possible for such theories using gravitational tools:

♠ Ground-state and RG-flow calculations

♠ Dynamical data like Minkowski signature correlation functions and associated hydrodynamic data like viscosity coefficients and even non-hydrodynamic data like thermal poles and residues.

♠ Far from equilibrium dynamics (quenches)

♠ Thermalisation and/or hydrodynamisation as well as the determination of hydrodynamic attractors.

♠ Calculations of entanglement via the Ryu-Takayanagi formula and its connection to geometric bridges (a.k.a. wormholes).

♠ The characterization of chaos and the calculation of chaos-related observables.

Thermalisation

- The process of thermalisation in QFT is poorly understood even today.

- It has been brought forward recently with the heavy-ion collisions at RHIC and CERN.

- The data indicate unusually rapid thermalisation of the initial energy density and the formation of a quark gluon plasma.

- The thermalisation time is an order of magnitude smaller than what was expected at RHIC and is even smaller at LHC.

- The theory (QCD) is in a strongly coupled regime for most of the energy range of the experiments.

- Holography has been instrumental to understand this rapid thermalisation process.

Hydrodynamization

• It was always assumed that in order for hydrodynamics to be applicable, we must have local thermalisation first.

• We now understand that in holographic theories this is not necessary!

• It was shown in many examples that far out-of equilibrium evolution first enters a "hydrodynamic attractor" and is described eventually by second order (relativistic) hydro.

Heller+Janik+Witaszczyk, Heller+Spalinski, Romatschke+Romatschke

• It is only much later it evolves to near-thermal equilibrium. $(p_x \simeq p_y \simeq pz)$.

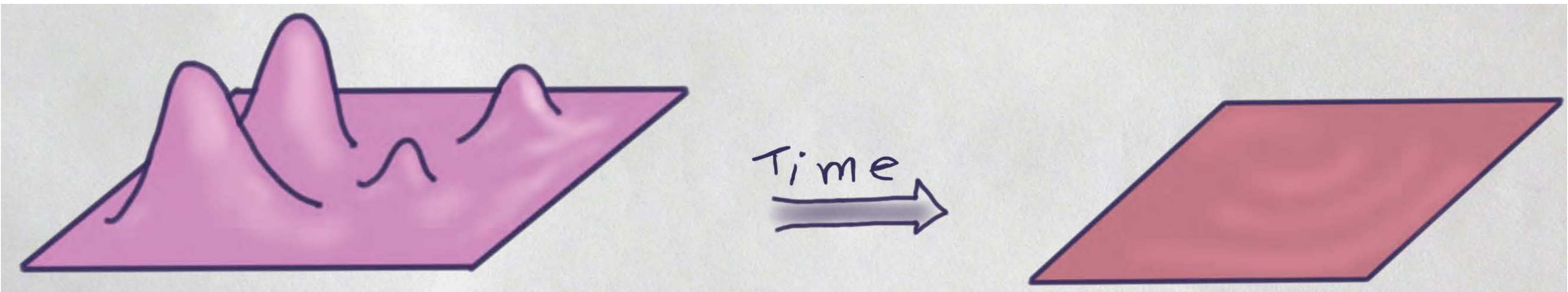
• In heavy-ion collisions it is estimated that hydrodynamization happens at least ten times faster than thermalisation.

The setup to study thermalisation

• We consider a QFT in its vacuum state and then perturb it by a time dependent coupling constant.

• This is known as a "quantum quench". It is a simplification.

$$L_{QFT} + f_0(t) \int d^4x \; O(x) \qquad \to \qquad \nabla^t \langle T_{tt} \rangle = \dot{f}_0 \; \langle O \rangle$$



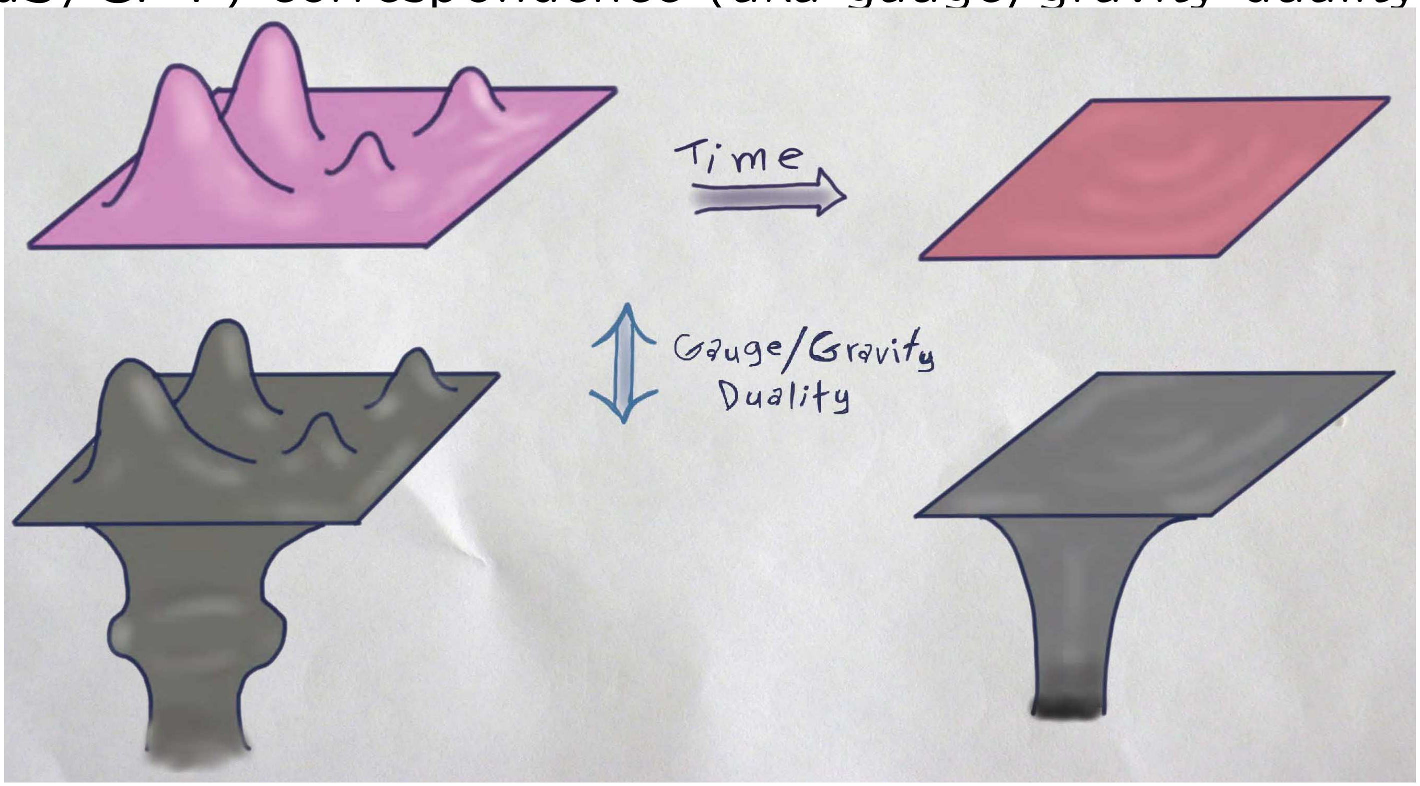
• The approach to equilibration is controlled by the expectation values $\langle T_{tt} \rangle(t)$, $\langle O \rangle(t)$.

• We expect that, if the system thermalizes, then

$$\langle O \rangle(t \to \infty) \quad \to \quad Tr[\rho_{\text{thermal}} \; O]$$

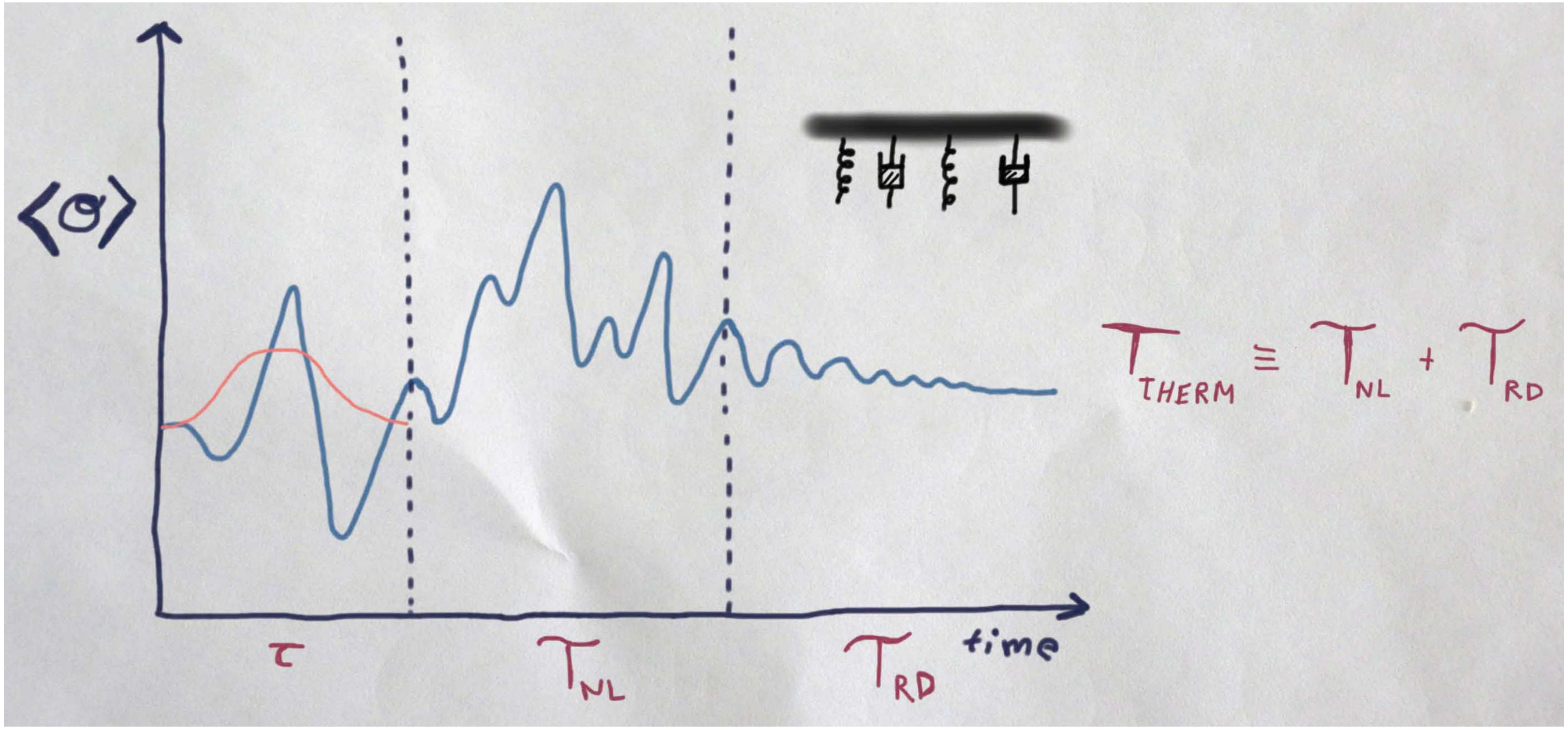
Thermalisation at strong coupling

• To calculate the observables at strong coupling we will assume the holographic (AdS/CFT) correspondence (aka gauge/gravity duality).



• Thermalisation corresponds to black hole formation in the bulk spacetime.

Gravitational expectations



- There are three possible characteristic times involved:

$\tau \to$ duration of quench,

$T_{NL} \to$ non-linear gravitational evolution,

$T_{RD} \to$ ring-down of final black hole.

Thermalisation calculations

• There have been studies of this setup in holographic CFTs (AdS space). There is no consensus yet but in most cases there is thermalisation.

Chessler+Yaffe, Heller+Janik+Witaszczyk
, Bizon+Rostorowski, Buchel+Liebling+Lehner

• There are similarities between a conformal (scale-invariant) gauge theory and a confining gauge theory (like QCD) that has a non-trivial scale, Λ_{QCD} but there are also important differences.

• The confining theories have a discrete and gapped spectrum instead of a continuous/massless spectrum.

• Confinement is tracked by the Wilson loop that has area behavior in the confining phase.

Quench dynamics

• We consider a quench profile in Improved Holographic QCD (a holographic model for YM):

$$f_0(v) = \tilde{f}_0 - \delta f_0 \, e^{-\frac{v^2}{2\tau^2}}$$

• For numerical simplicity we start with the theory in a thermal state that corresponds to low temperature = the small black hole branch.

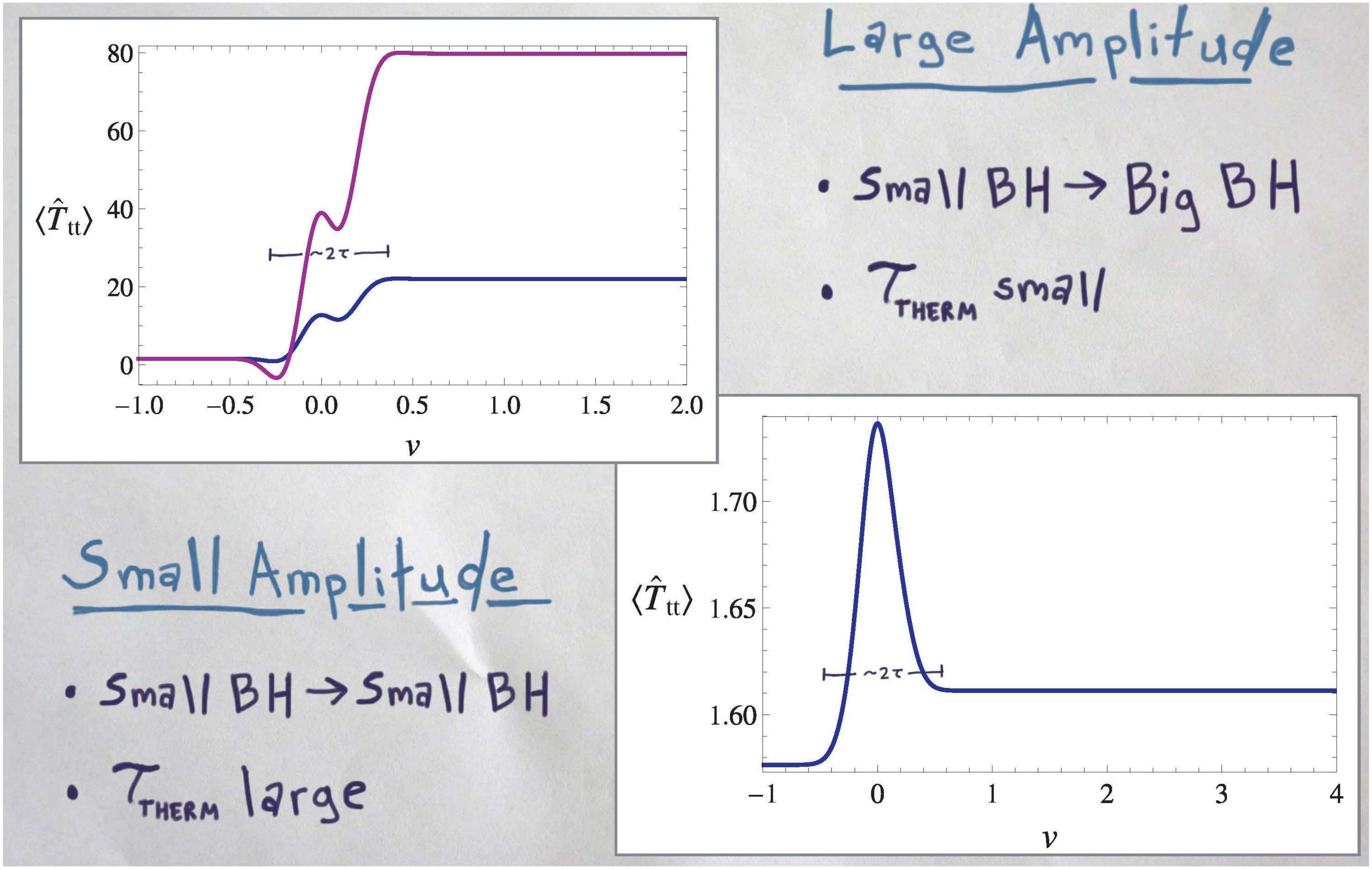
• The "smallest" the initial black hole, the closest we are to the ground state of the theory.

• The characteristic time associated with the intermediate non-linear regime is negligible compared to τ and T_{RD}. Why?

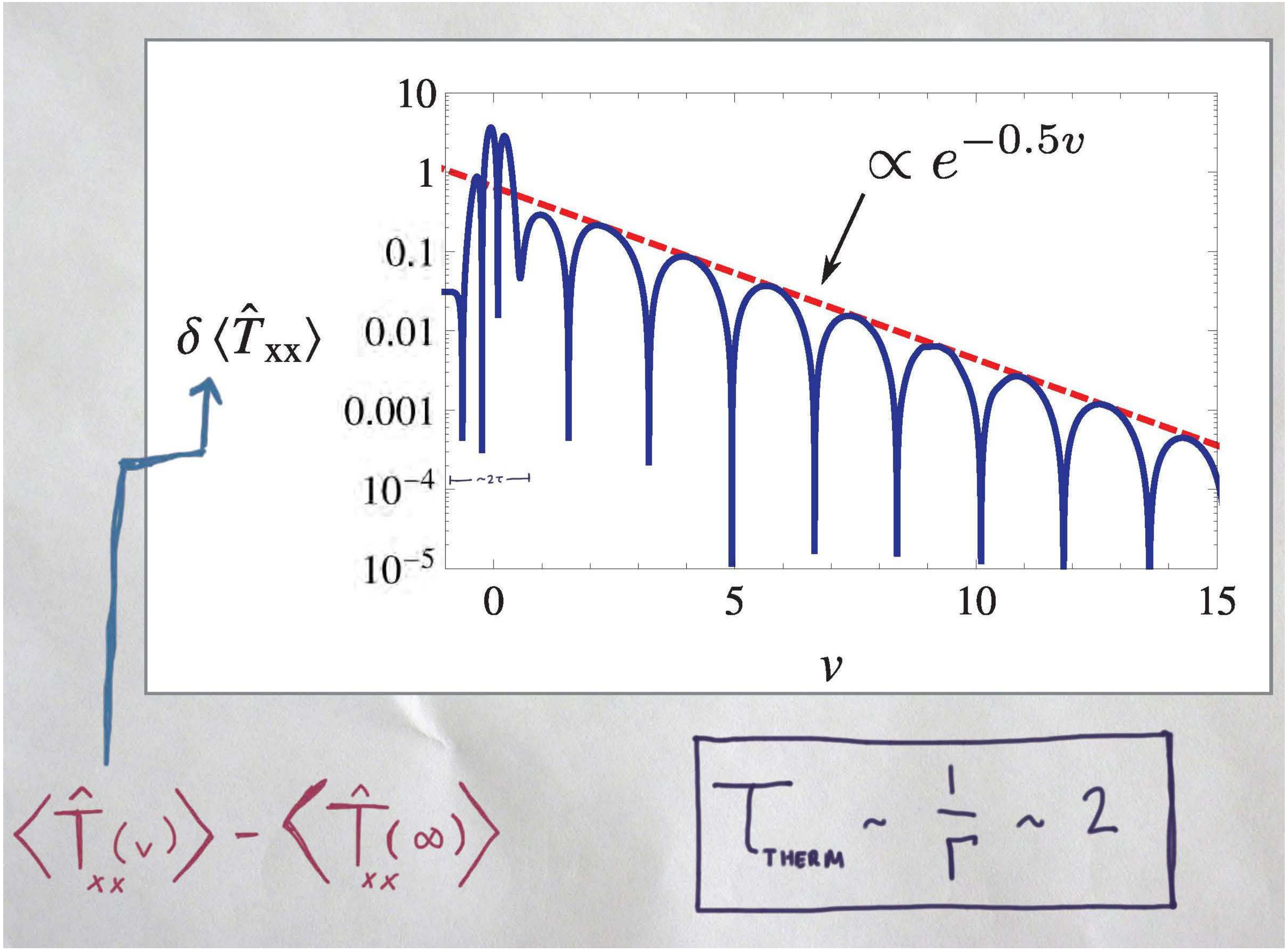
• Therefore

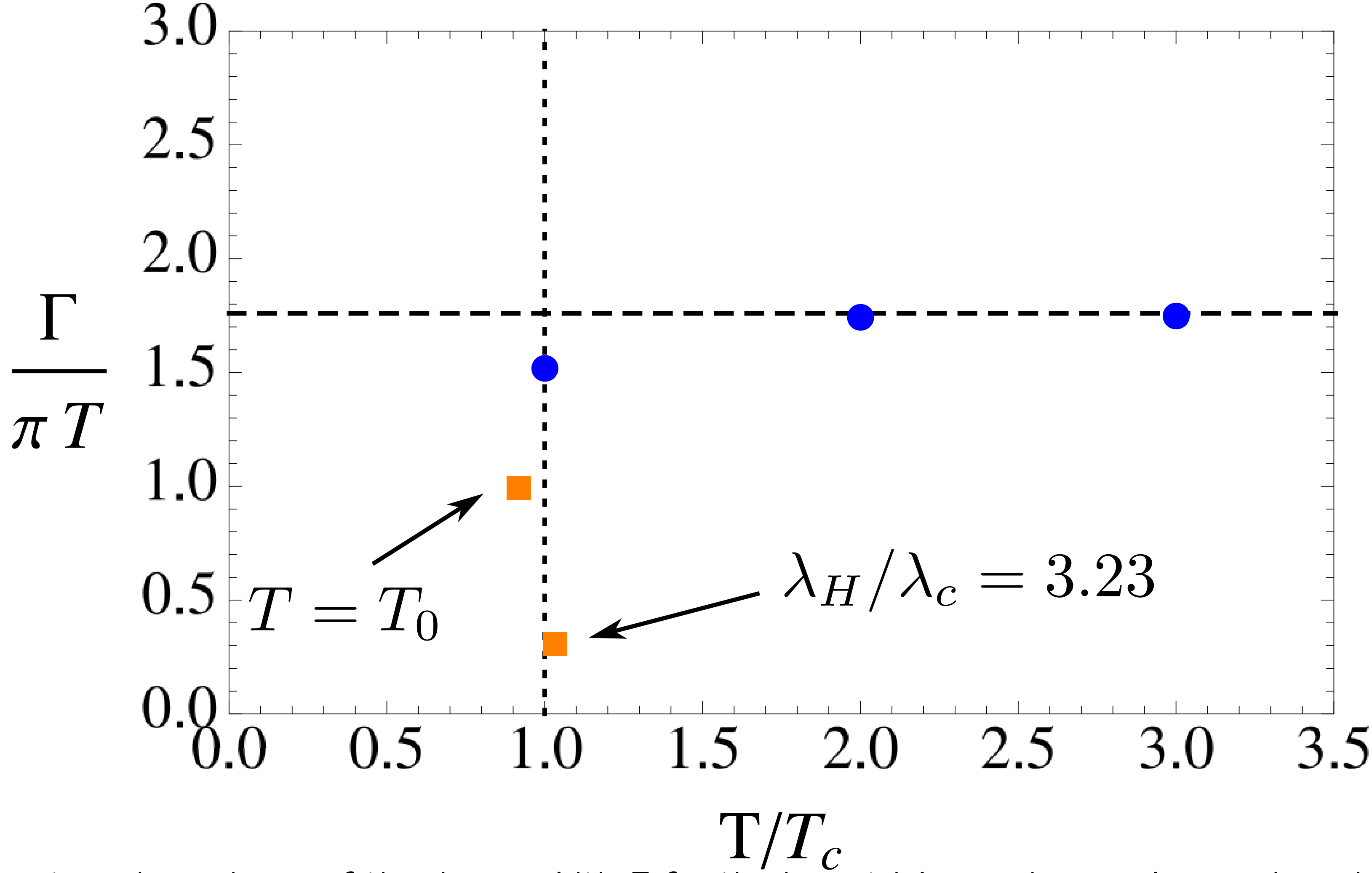
$$T_{\text{thermalisation}} \quad \simeq \quad \frac{1}{\Gamma_{RD}}$$

• For adiabatic perturbations, $\tau \gg \Lambda^{-1}$ the system does NOT oscillate but goes continuously to the final-state black hole.


Large Amplitude
• Small BH → Big BH
• τ_THERM small
Small Amplitude
• Small BH → Small BH
• τ_THERM large
⟨T̂_tt⟩
v
~2τ
80
60
40
20
0
−1.0
−0.5
0.0
0.5
1.0
1.5
2.0
1.70
1.65
1.60
−1
0
1
2
3
4

The ring-down phase





The temperature dependence of the decay width Γ for the lowest lying scalar quasi-normal mode in several states of our theory. The blue circles are large black branes whose temperature is an integer multiple of T_c. The orange squares correspond to the minimum temperature black brane (top) and the smallest black hole we perturb in our study (bottom). The ratio $\Gamma/\pi T$ approaches 1.75953 (the dashed line) at high temperatures, which coincides with the expected value for perturbations of AdS_5 Schwarzschild by a dimension 3 scalar operator

The cosmological setup

- To build a cosmological model we need (generically) three ingredients:

♠ 4d-gravity

♠ An inflaton theory

♠ A strongly coupled (holographic) "matter theory" (QFT)

♠ A coupling between the inflaton and the QFT

- There are variations on this but we stick to this setup here.

The total action is

$$S = S_{grav} + S_{infl} + S_{holo} + S_{int}$$

with

$$S_{grav} = -\int d^4x\sqrt{g}\left[M_P^2 R + \alpha R^2 + \beta\left(R_{\mu\nu}R^{\mu\nu} - \frac{R^2}{4}\right) + ...\right]$$

$$S_{infl} = -\int d^4x\sqrt{g}\left[\frac{1}{2}(\partial\phi)^2 + V(\phi)\right]$$

$$S_{int} = \int d^4x U(\phi) O(x)$$

- We must renormalize, and therefore the couplings are the renormalized couplings.

- Renormalisation affects up to the R^2 terms in Gravity.

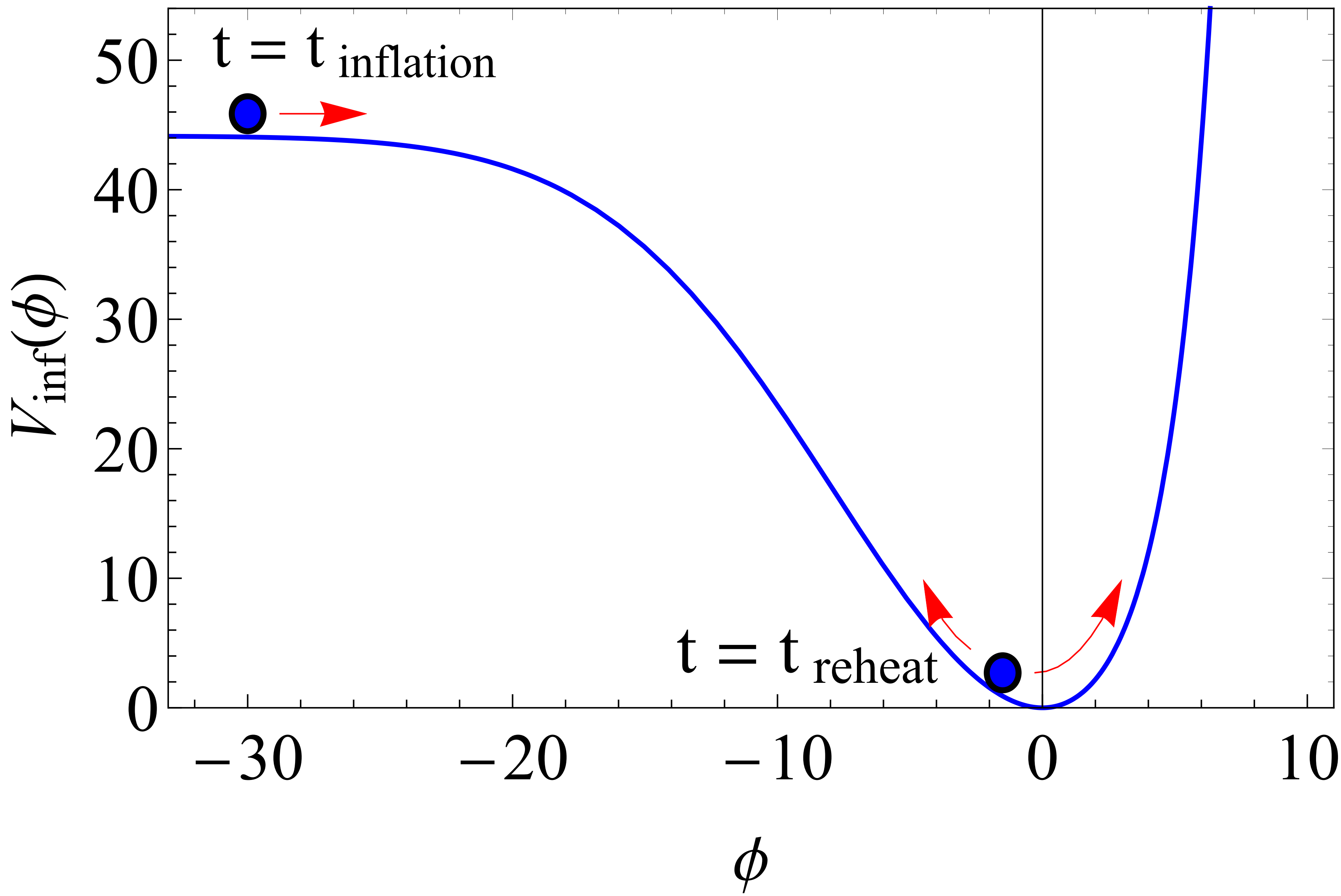
- It also renormalizes the inflaton theory.

- We choose a typical (renormalized) inflaton potential

- We choose a typical non-conformal holographic theory. Conformal invariance is broken also by S_{int}.

- The "CFT" case is similar, as the inflaton coupling breaks typically conformal symmetry.

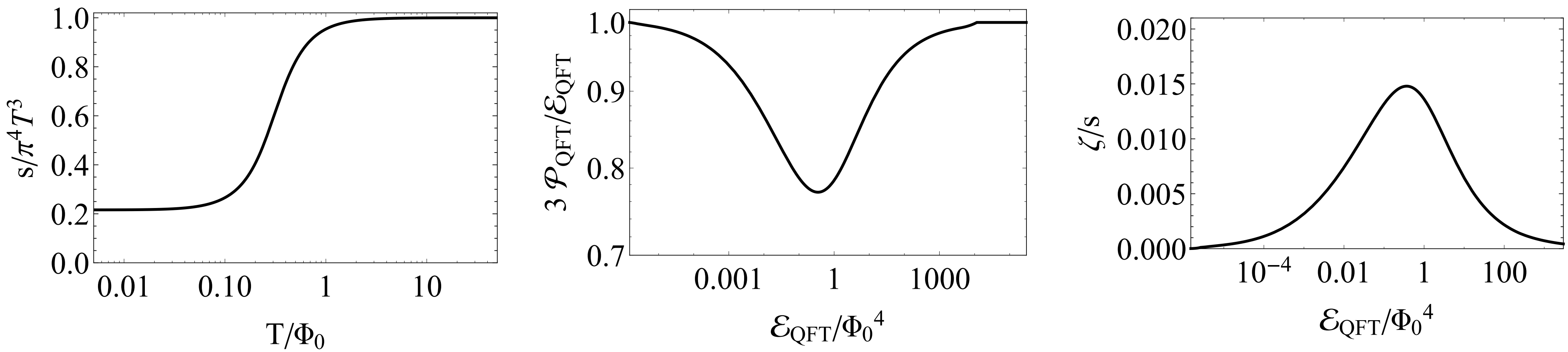
The inflaton Potential



The holographic matter theory

• We choose a bottom-up non-conformal theory with a single mass scale, driven by a relevant operator of dimension $\Delta = 3$.

• There is no phase transition in flat space (thermal ensemble) but there is a fast crossover (like QCD)



The Cosmological Evolution Equations

- Homogeneous and isotropic ansatz for the 4d metric

$$ds^2 = -dt^2 + a(t)^2 d\vec{x}^2$$

- 4d Einstein equations

$$H^2 \equiv \left(\frac{\dot{a}}{a}\right)^2 = \frac{\mathcal{E}}{3} \quad , \quad \frac{\ddot{a}}{a} = -\frac{1}{2}\left(\mathcal{P} + H^2\right)$$

with

$$\mathcal{E} = \mathcal{E}_{QFT} + \mathcal{E}_{infl} + U(\phi)\mathcal{O}_{QFT} \quad , \quad \mathcal{E}_{infl} = V_{infl}(\phi) + \frac{\dot{\phi}^2}{2}$$

$$\mathcal{P} = \mathcal{P}_{QFT} + \mathcal{P}_{infl} - U(\phi)\mathcal{O}_{QFT} \quad , \quad \mathcal{P}_{infl} = -V_{infl}(\phi) + \frac{\dot{\phi}^2}{2}$$

$$\ddot{\phi} + 3H\dot{\phi} + V'_{infl}(\phi) = U'(\phi)\mathcal{O}_{QFT} \quad , \quad \mathcal{O}_{QFT} \equiv \langle O \rangle(t)$$

• The unknown parts from the QFT, $\mathcal{E}_{QFT}, \mathcal{P}_{QFT}, \mathcal{O}_{QFT}$ are constrained by energy conservation and the conformal anomaly

$$\dot{\mathcal{E}}_{QFT} - 3H(\mathcal{E}_{QFT} + \mathcal{P}_{QFT}) = U'(\phi)\dot{\phi}\mathcal{O}_{QFT}$$

$$\mathcal{E}_{QFT} - 3\mathcal{P}_{QFT} = (4 - \Delta_O)U(\phi)\mathcal{O}_{QFT} + \mathcal{A}(H,\phi)$$

• Therefore, if we know $\mathcal{A}(H,\phi)$ and $\mathcal{O}_{QFT}(H,\phi)$ the system of cosmological equations is closed and can be solved relatively easy.

• To do this we must solve the dynamics of the QFT in the presence of two (arbitrary, time-dependent) sources: $a(t), U(\phi)$

• In the case, where $\Delta_O < 4$, $\mathcal{A}$ can be calculated and the only non-trivial function to be determined is $\mathcal{O}_{QFT}$

The holographic picture

- The observable gravity and the inflaton are four dimensional .

- The (holographic) matter QFT is replaced by its 5d "gravitational" dual theory.

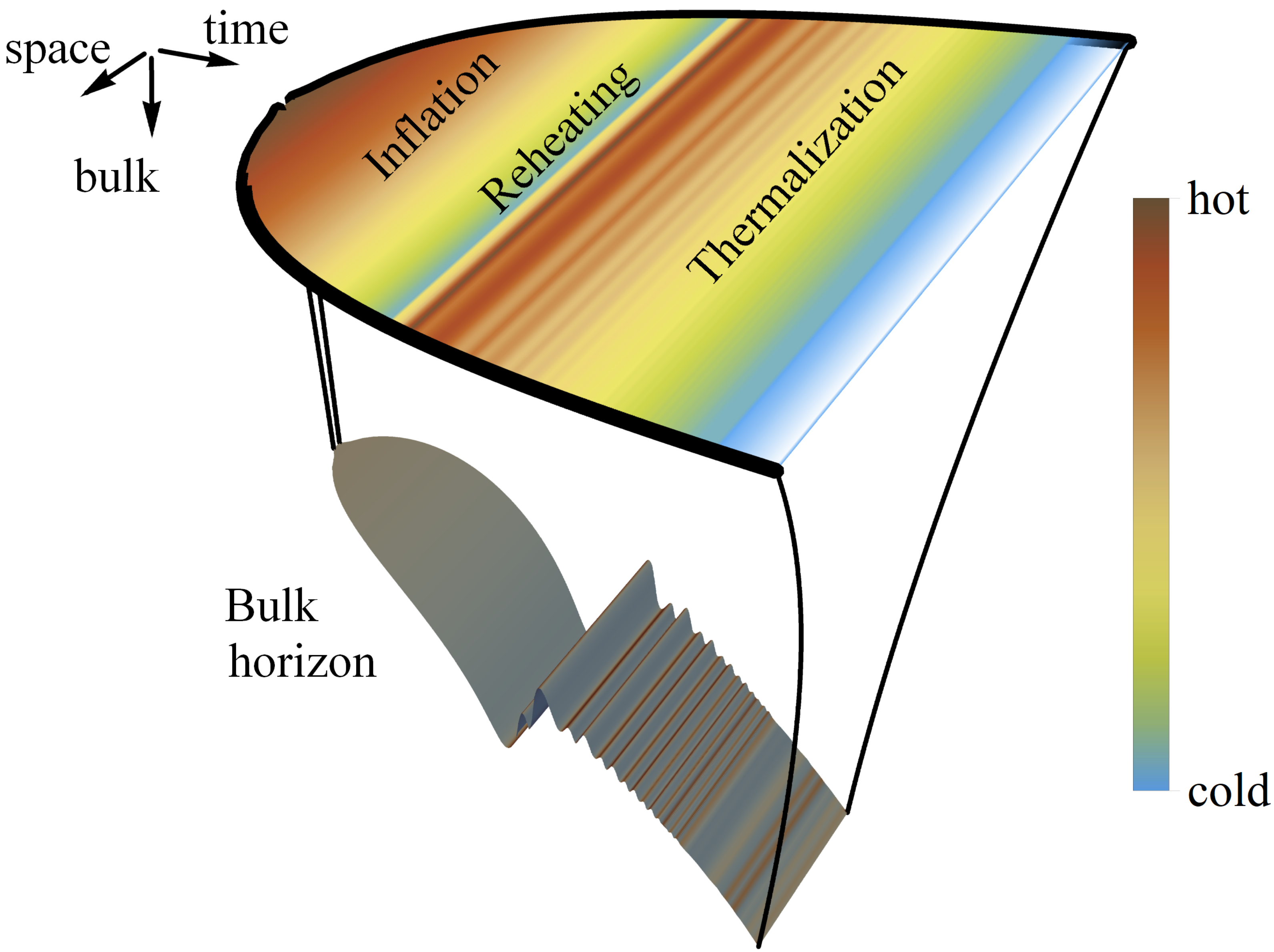
- According to holography, the $g^{(4)}_{\mu\nu}$ is the leading boundary condition for $g^{(5)}_{MN}$ at the asymptotically AdS boundary.

- One needs then to solve both, bulk 5d Einstein-scalar equations coupled via boundary conditions to the 4d Einstein-inflaton equations.

- In our maximally symmetric ansatz, these are PDEs in (t,r).

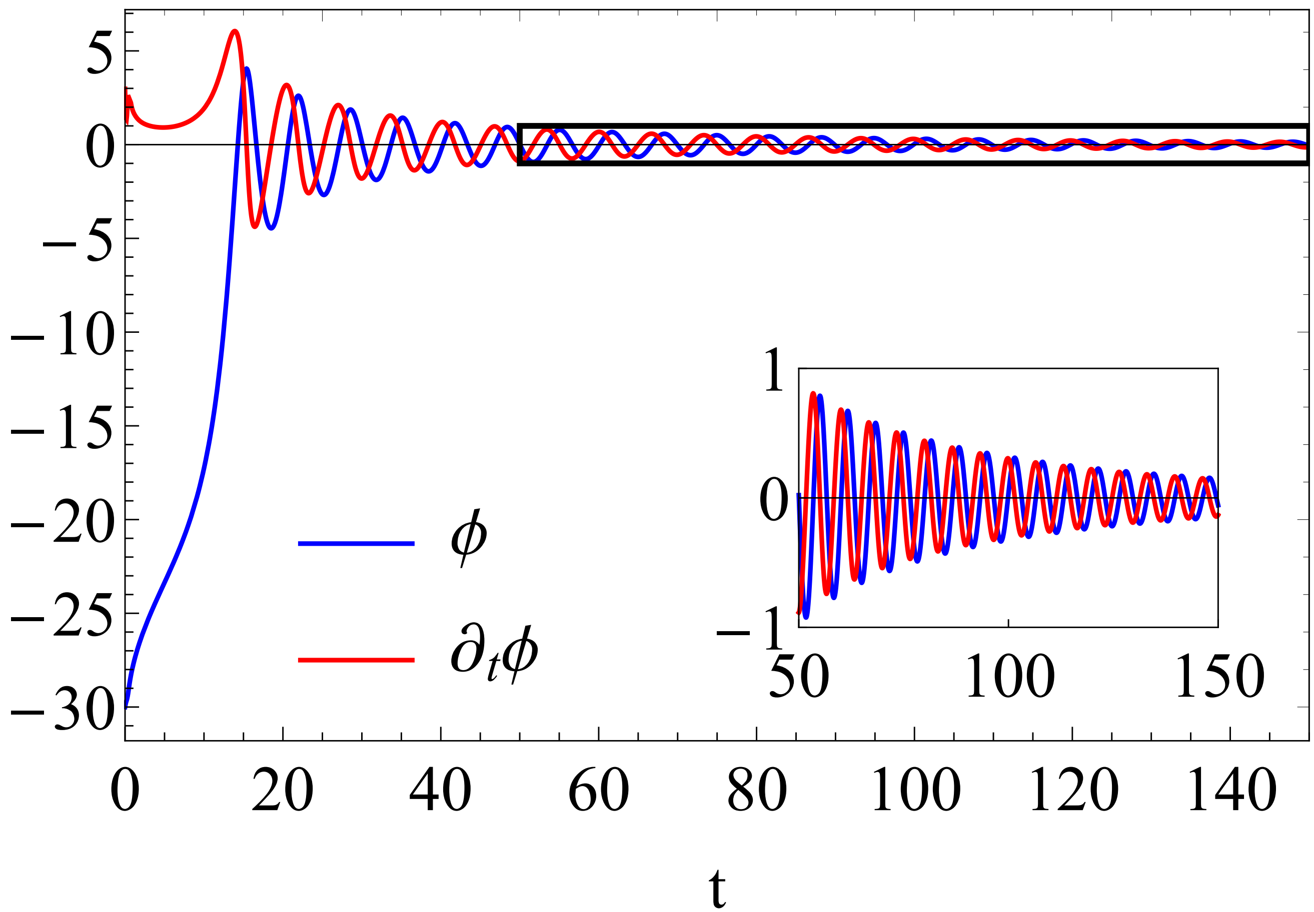
- The algorithm we use for this is the Chessler-Yaffe algorithm.

Cartoons


space
time
bulk
Inflation
Reheating
Thermalization
hot
cold
Bulk
horizon

The Inflaton

- Early phase $(t \le 3)$ dominated by QFT energy. After the universe embarks in an exponential expansion.
- At $t \simeq 14$ the inflaton reaches the bottom of the potential and starts oscillating.

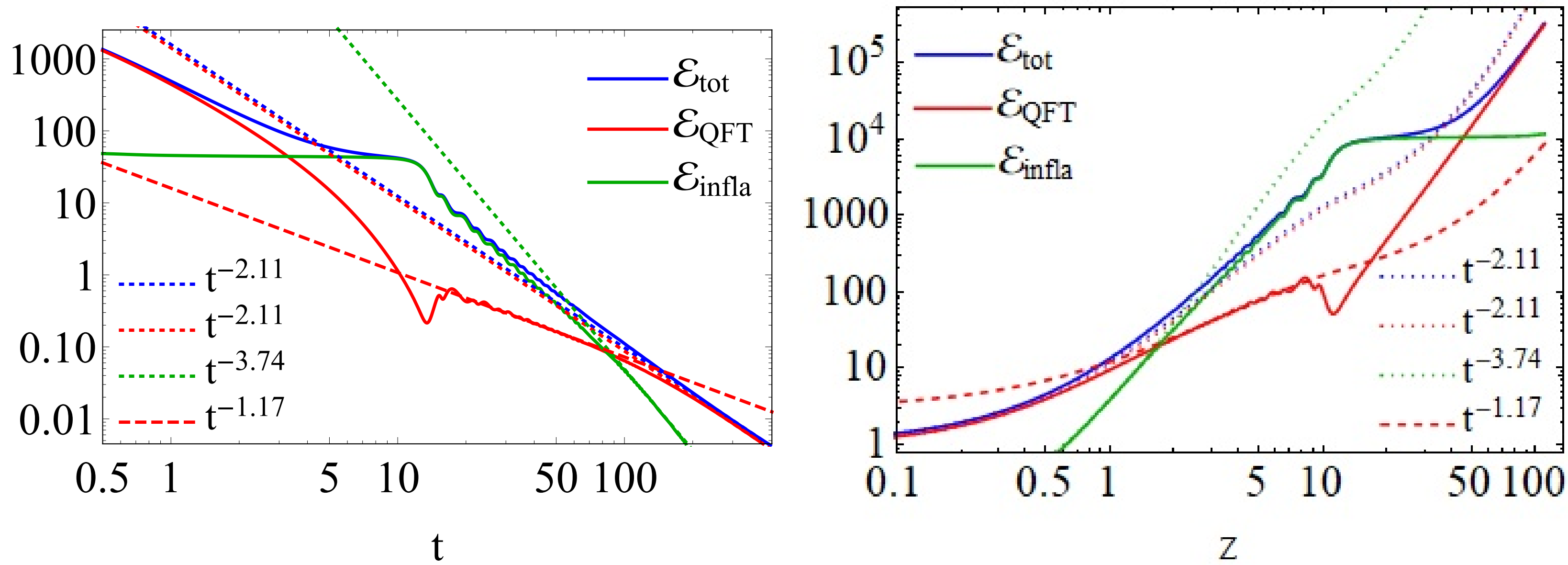


The Energy Density

• QFT energy is dominant until $t \simeq 3$, then the inflaton dominates, until it reaches the bottom of the potential ($t \simeq 14$).

• Inflaton oscillations reheat the QFT from $\mathcal{E}_{QFT} \simeq 0.21$ at $t \simeq 13.5$ to a subsequent maximum of $\mathcal{E}_{QFT} \simeq 0.64$ at $t \simeq 17.3$.

• Reheating continues: relatively slow scaling $\mathcal{E}_{QFT} \sim t^{-1.17}$ of the QFT

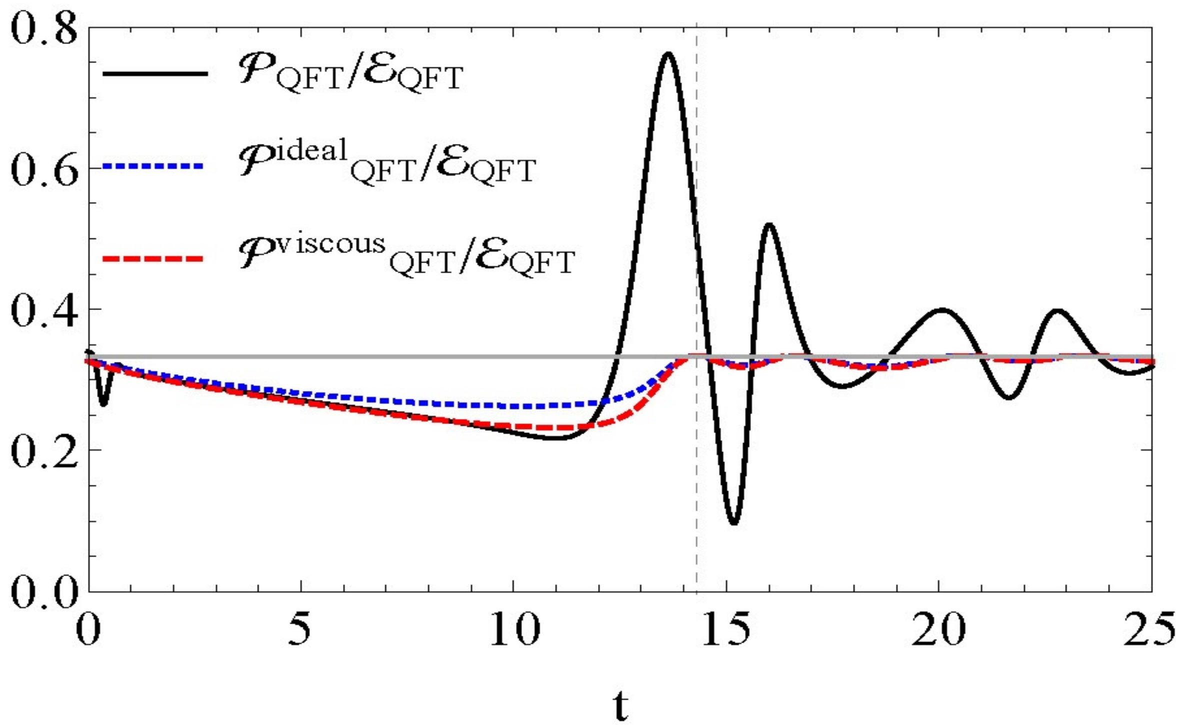


The QFT Pressure

• After an initial short far-out-of-equilibrium stage, the system is well described by hydrodynamics until the inflaton drives back the QFT out-of-equilibrium.

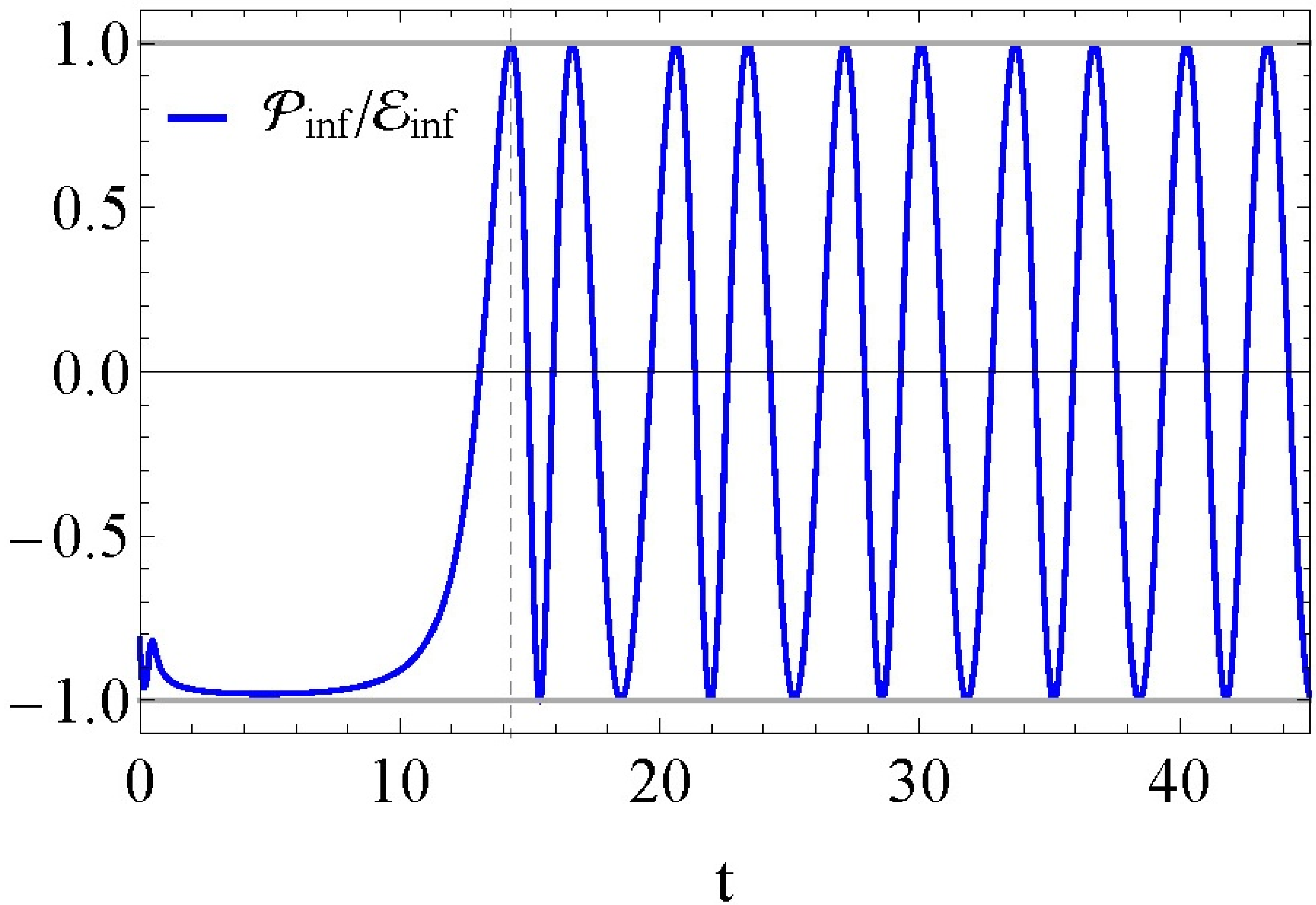
$$\mathcal{P}_{QFT}^{viscous}(t) = \mathcal{P}_{QFT}^{ideal}(t) - 3H\zeta\mathcal{E}_{QFT}(t) + \mathcal{O}(H^2)$$

• The QFT evolves from the UV to the IR fixed point where $\mathcal{P}_{QFT} = \frac{1}{3}\mathcal{E}_{QFT}$.



The Inflaton Pressure

- In the initial (inflation) stage $\mathcal{E}_{infl} \simeq -\mathcal{P}_{infl}$



Temperature

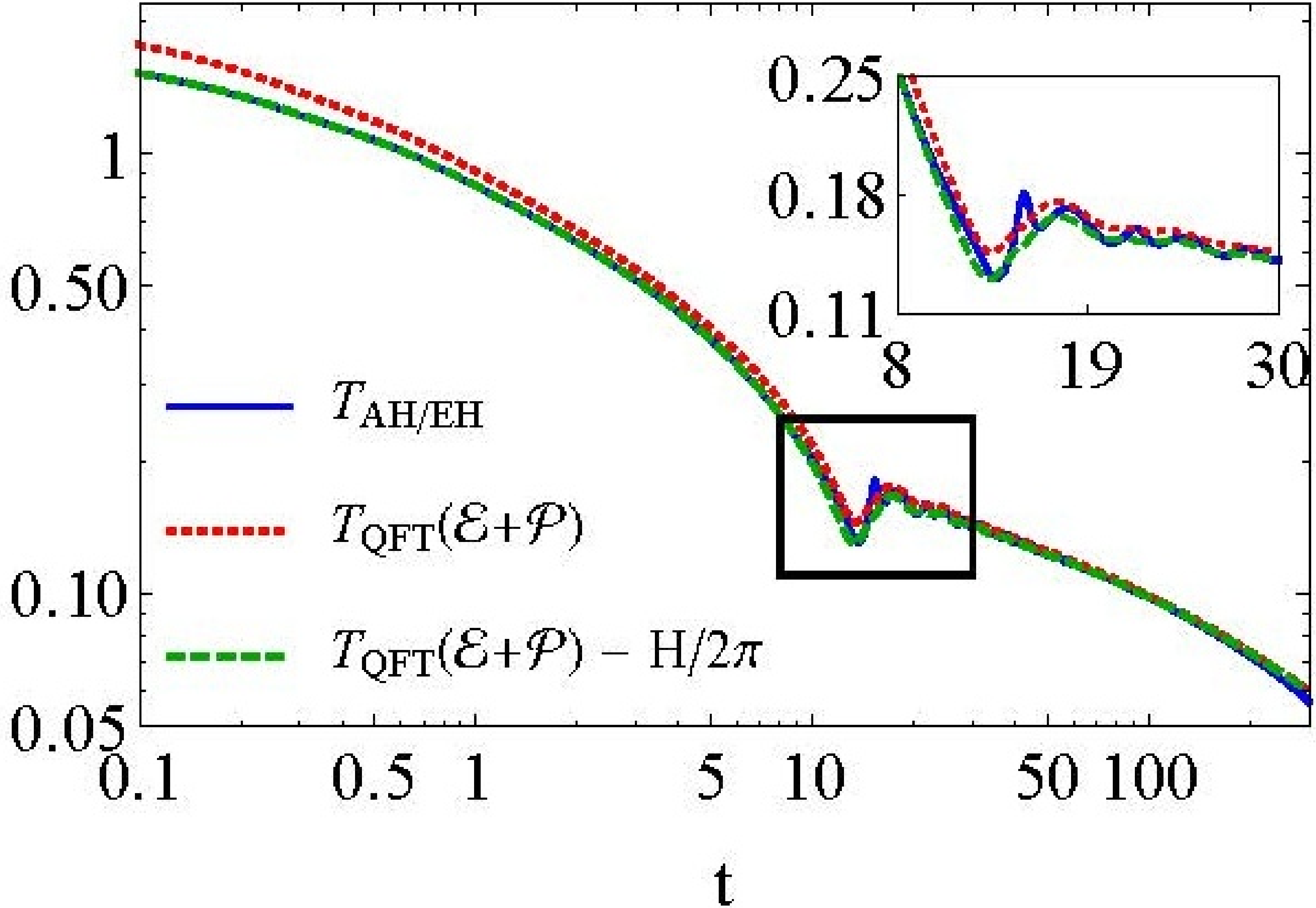
• During the evolution, the bulk solution is a time-dependent solution with an apparent horizon that is different from the (final) event horizon.

• The QFT temperature can be computed from the surface gravity of the bulk apparent horizon.

$$T_{AH} = \frac{\kappa}{2\pi}$$

• Except for a short non-equilibrium period (that can be identified as "reheating") the apparent horizon temperature and the event horizon temperature are numerically indistiguishable.

• The hydrodynamic approximation combined with the equilibrium EoS works well after subtracting the cosmological temperature, $T_{dS} = \frac{H}{2\pi}$.



0.25
0.18
0.11
8
19
30
1
0.50
0.10
0.05
$T_{\mathrm{AH/EH}}$
$T_{\mathrm{QFT}}(\mathcal{E}+\mathcal{P})$
$T_{\mathrm{QFT}}(\mathcal{E}+\mathcal{P}) - \mathrm{H}/2\pi$
0.1
0.5
1
5
10
50
100
t

Conclusions

- We have studied cosmological reheating via a strongly coupled (holographic) theory.

- Such theories thermalize/hydrodynamize very fast.

- Moreover, they provide many more options of both couplings and dynamics compared to weakly-coupled theories.

- We have provided a "proof of principle" with a system that had manageable numerics (warm inflation).

- We have found a fast transfer of energy as well as an almost immediate thermalisation.

- We have also found that, apart from two relatively short periods, the evolution is well described by viscous (homogeneous) hydrodynamics.

- This opens the way for a systematic study of models and protocols for reheating at strong coupling.

Open Ends

- The most important open problem with reheating, in general, is whether it can leave measurable relics.

- In the context of the present framework, there are many variations that will provide different scenarios for reheating.

♠ One can use variations on initial conditions.

♠ Variations of the type of strongly-coupled theory (existence of thermal phase transitions, confinement, massive IR etc) as well as of the inflaton portal (different scaling dimensions).

♠ Such variations may provides either fast quenches to the QFT (that can be computed analytically in UV Perturbation theory), or adiabatic quenches that can also be computed analytically.

♠ Tools can be developed to use viscous hydro for the evolution and transition protocols for the short non-hydro periods.

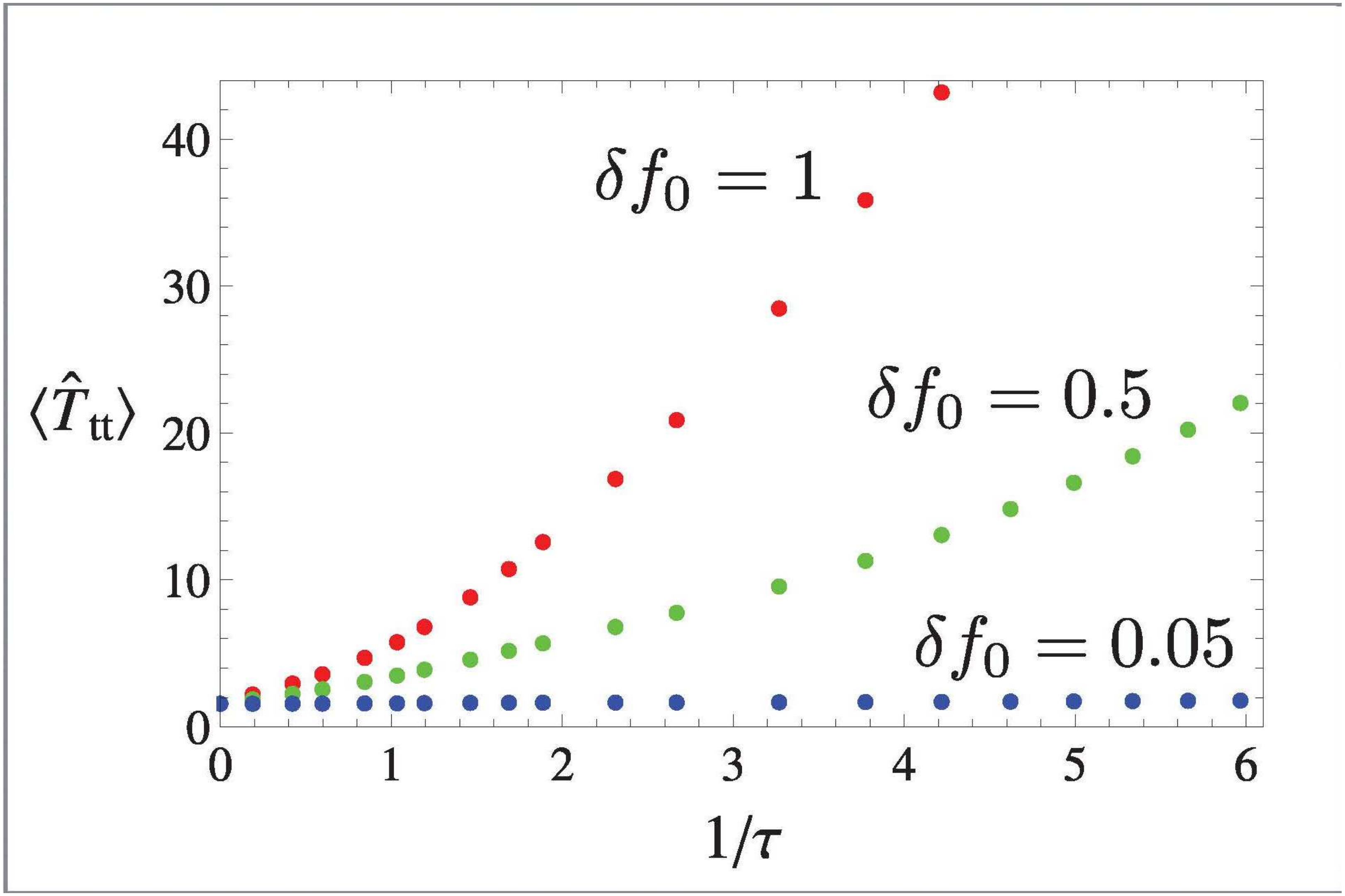
♠ In this connection, the holographic universal hydro attractors may be of use.

♠ On can contemplate using as inflaton a scalar of the QFT, that is associated with a phase transition.

- The role of such a QFT can be played by QCD (this is marginally acceptable) or a higher energy theory like "technicolor".

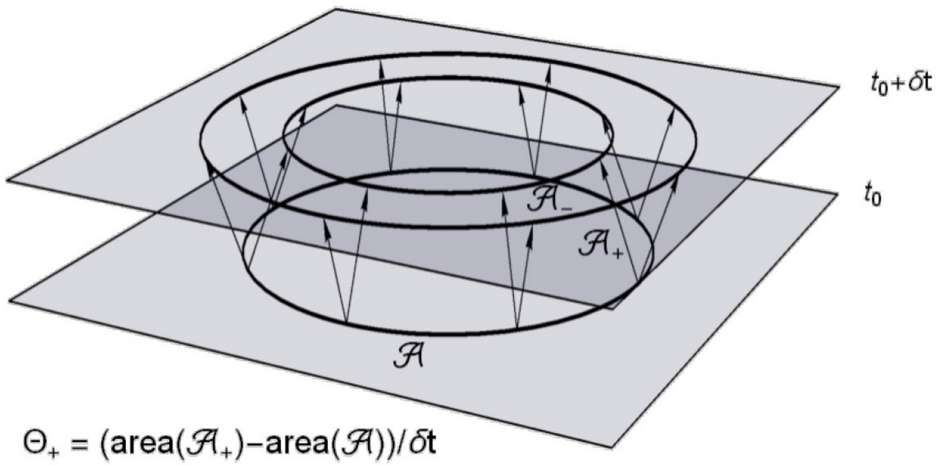
THANK YOU!

Quench numerical data



EVENT AND APPARENT HORIZONS

Trapped surface: expanding light surfaces contract



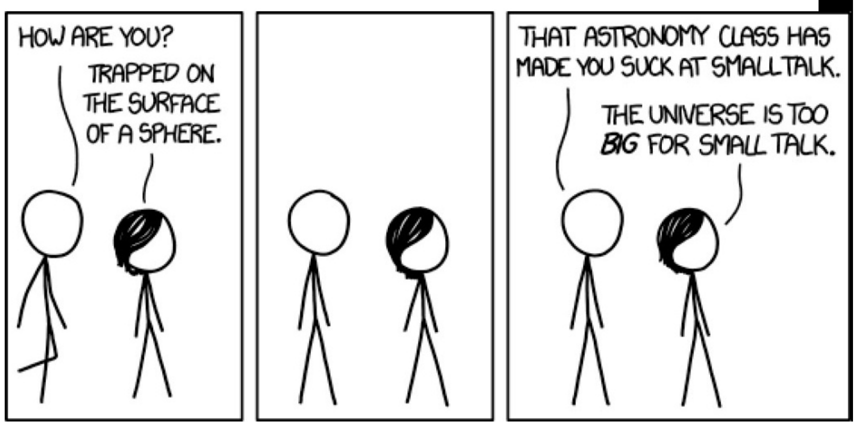
Apparent horizon: outermost trapped surface

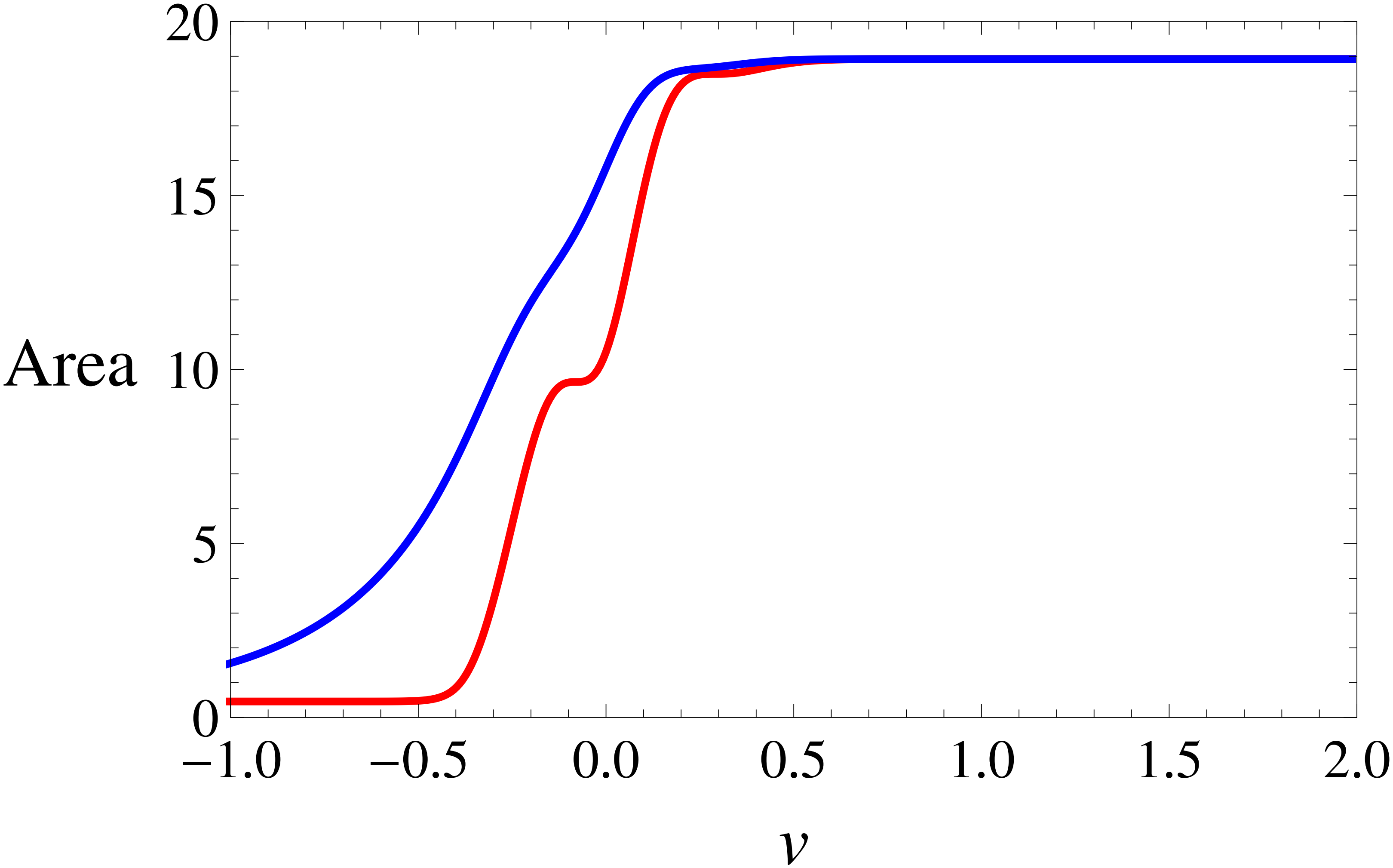
- Subtlety: depends on time slicing

Event horizon: outermost surface to causally reach infinity

- Subtlety: depends on entire future

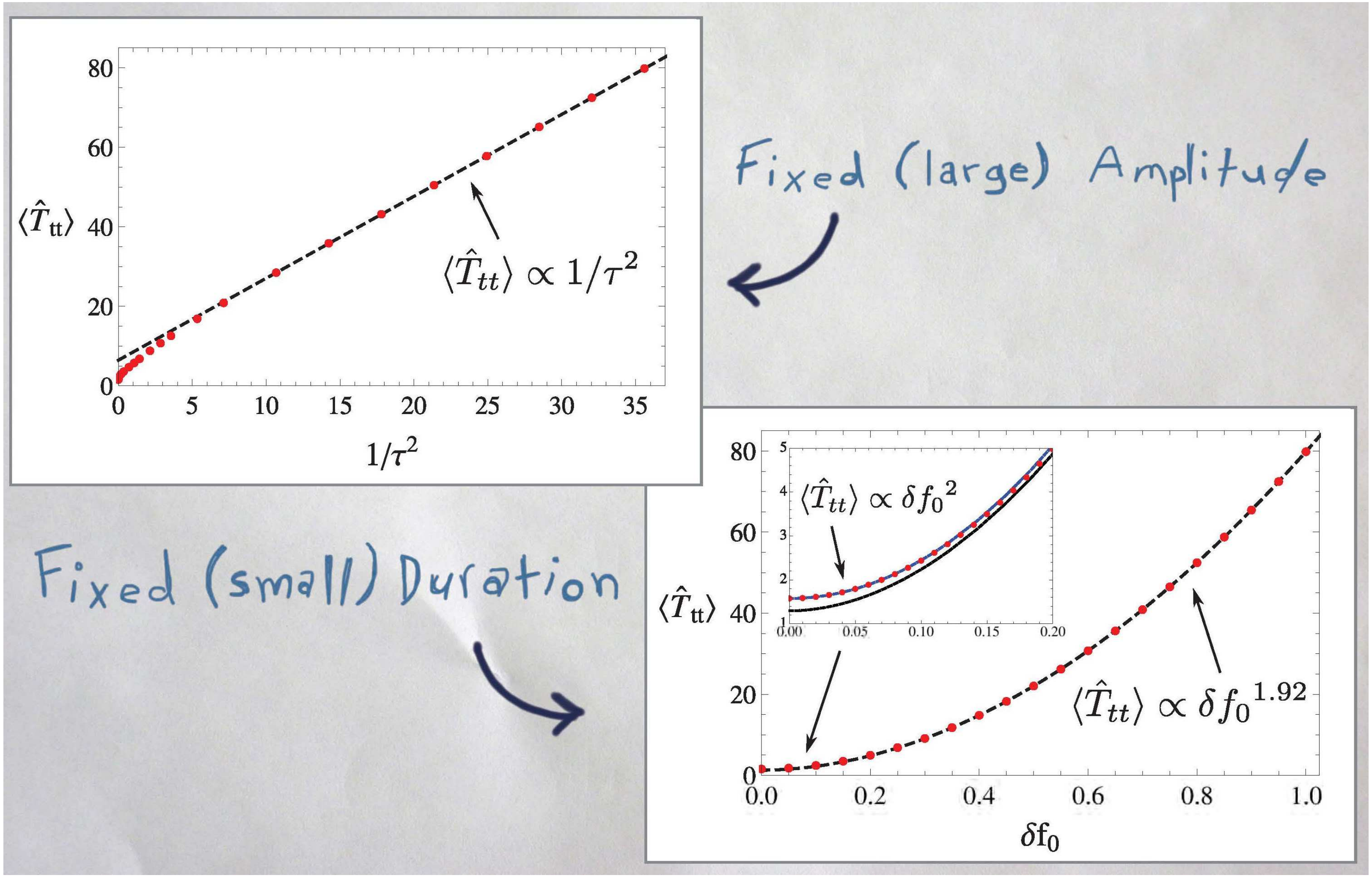
Event and apparent coincide when stationary





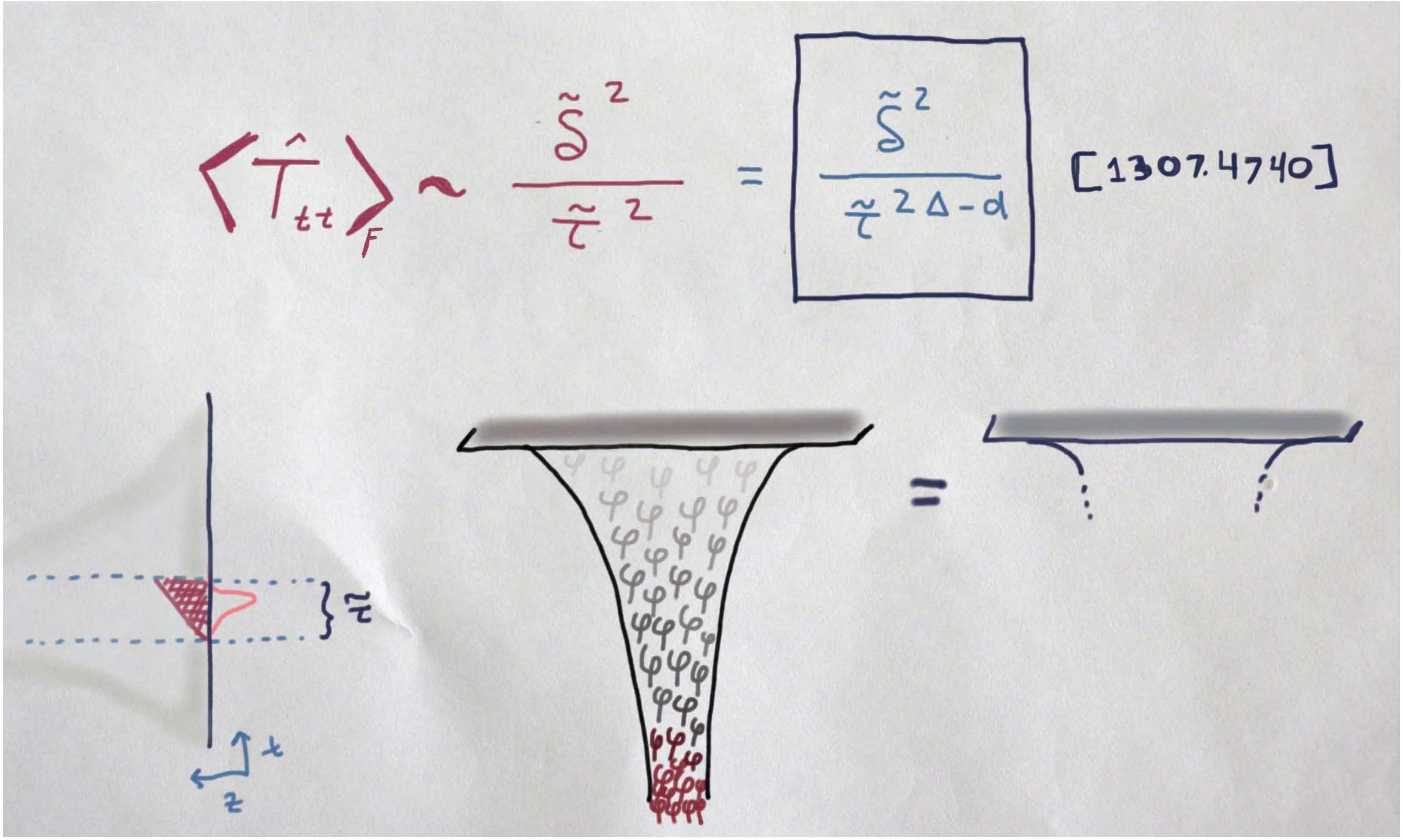
Example of the time evolution of the apparent (red) and event (blue) horizons. The event horizon coincides with the apparent horizon when the bulk solution is static, at $v \to \pm\infty$.

Scaling



80
60
$\langle\hat{T}_{tt}\rangle$
40
20
0
0 5 10 15 20 25 30 35
$1/\tau^2$
$\langle\hat{T}_{tt}\rangle \propto 1/\tau^2$
Fixed (large) Amplitude
Fixed (small) Duration
80
60
$\langle\hat{T}_{tt}\rangle$
40
20
0
0.0 0.2 0.4 0.6 0.8 1.0
δf_0
$\langle\hat{T}_{tt}\rangle \propto \delta f_0^{1.92}$
$\langle\hat{T}_{tt}\rangle \propto \delta f_0{}^2$
0.00 0.05 0.10 0.15 0.20

Fast Quenches



Buchel+Lehner+Myers+Niekerk, Das+Galante+Myers

The holographic theory:details

• We choose a bottom-up non-conformal theory with a single mass scale, driven by arelevant operator of dimension $\Delta = 3$.

• There is no phase transition in flat space (thermal ensemble) but there is a fast crossover (like QCD)

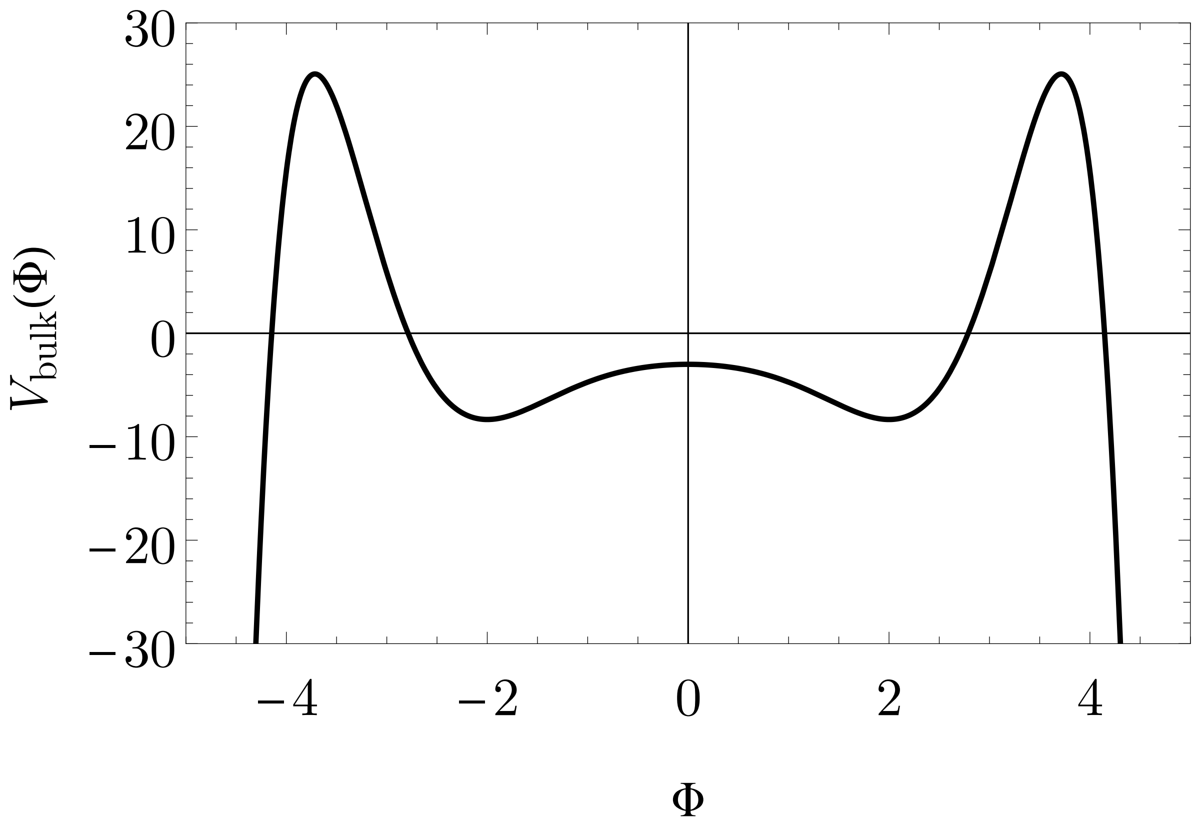
The bulk 5d action is Einstein-Dilaton gravity

$$S_{bulk} = \frac{2}{\kappa_5} \int d^5 \sqrt{g} \left[\frac{R}{4} - \frac{1}{2}(\partial\Phi)^2 - V(\Phi) \right]$$

and the bulk potential is

$$V(\Phi) = \frac{1}{\ell^2} \left[-3 - \frac{3}{2}\Phi^2 - \frac{\Phi^4}{3} + \frac{11}{96}\Phi^6 - \frac{\Phi^8}{192} \right]$$

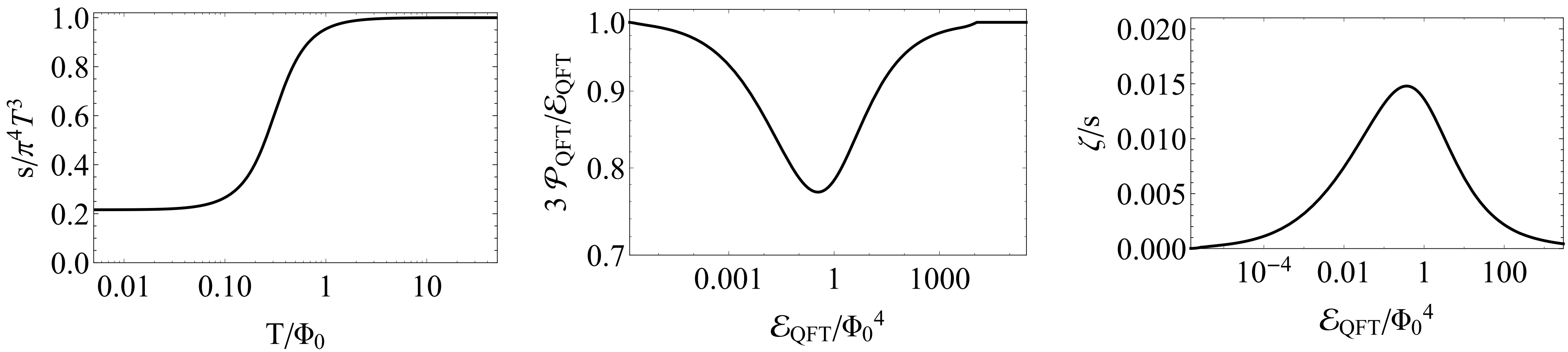
which looks like:



• The holographic theory corresponds to the flow between the maximum and one of the minima.

• The theory is massless in the IR due to the non-triviality of the IR CFT

• The thermodynamics in flat space is shown below



Holographic vev's

$$ds^2 = L^2 \frac{d\rho^2}{4\rho^2} + \bar{g}_{ij}(\rho, x) dx^i dx^j ,$$

$$\begin{aligned} \bar{g}_{ij}(\rho,x) &= \frac{1}{\rho}\Big[\gamma_{ij}(x) + \rho\,\gamma_{(2)ij}(x) + \rho^2\gamma_{(4)ij}(x) \\ &+ \rho^2 \log\rho\, h_{(4)ij}(x) + O(\rho^3)\Big], \\ \Phi(\rho,x) &= \rho^{1/2}\Big[\Phi_{(0)}(x) + \rho\,\Phi_{(2)}(x) + \rho\,\log\rho\,\psi_{(2)}(x) + O(\rho^2)\Big]. \end{aligned}$$

$$\begin{aligned} \langle T^{\rm QFT}_{ij}\rangle &= \frac{2}{\kappa_5}\Bigg\{\gamma_{(4)ij} + \frac{1}{8}\left[{\rm Tr}\gamma^2_{(2)} - ({\rm Tr}\gamma_{(2)})^2\right]\gamma_{ij} \\ &- \frac{1}{2}\gamma^2_{(2)} + \frac{1}{4}\gamma_{(2)ij}{\rm Tr}\gamma_{(2)} + \frac{1}{2}\partial_i\Phi_{(0)}\partial_j\Phi_{(0)} \\ &+ \left(\Phi_{(0)}\Phi_{(2)} - \frac{1}{2}\Phi_{(0)}\psi_{(2)} - \frac{1}{4}\partial_k\Phi_{(0)}\partial^k\Phi_{(0)}\right)\gamma_{ij} \\ &+ \alpha\left(\mathcal{T}^{\gamma}_{ij} + \mathcal{T}^{\phi}_{ij}\right) + \left(\frac{1}{18} + \beta\right)\Phi^4_{(0)}\gamma_{ij}\Bigg\}. \end{aligned}$$

$$\langle \mathcal{O}\rangle = \frac{2}{\kappa_5}\left[(1-4\alpha)\psi_{(2)} - 2\Phi_{(2)} - 4\beta\Phi^3_{(0)}\right].$$

Bulk Equations of Motion

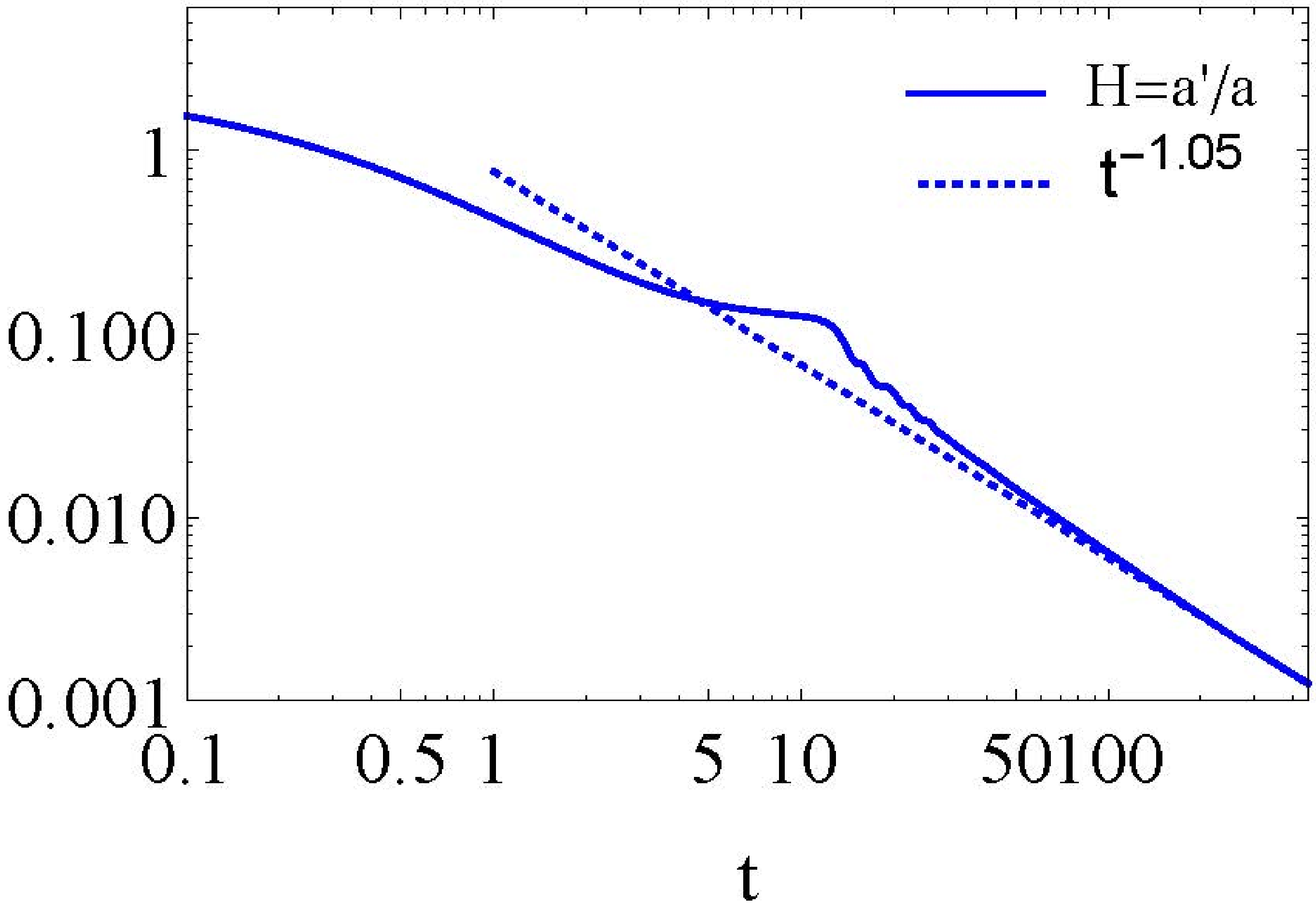
$$\begin{aligned} ds^2_{\rm bulk} &= g_{\mu\nu}dx^\mu dx^\nu = -A(r,t)dt^2 + 2drdt + S(r,t)^2 d\vec{x}^2\,, \\ \Phi &= \Phi(r,t)\,, \end{aligned}$$

$$\begin{aligned} S'' &= -\frac{2}{3}S\,(\Phi')^2\,, \\ \dot{S}' &= -\frac{2\dot{S}S'}{S} - \frac{2SV}{3}\,, \\ \dot{\Phi}' &= \frac{V'}{2} - \frac{3\dot{S}\Phi'}{2S} - \frac{3S'\dot{\Phi}}{2S}\,, \\ A'' &= \frac{12\dot{S}S'}{S^2} + \frac{4V}{3} - 4\dot{\Phi}\Phi'\,, \\ \ddot{S} &= \frac{\dot{S}A'}{2} - \frac{2S\dot{\Phi}^2}{3}\,, \end{aligned}$$

$$f' \equiv \partial_r f\,, \quad \dot{f} \equiv \partial_t f + \frac{1}{2}A\partial_r f\,.$$

Hubble Rate

Hubble Rate



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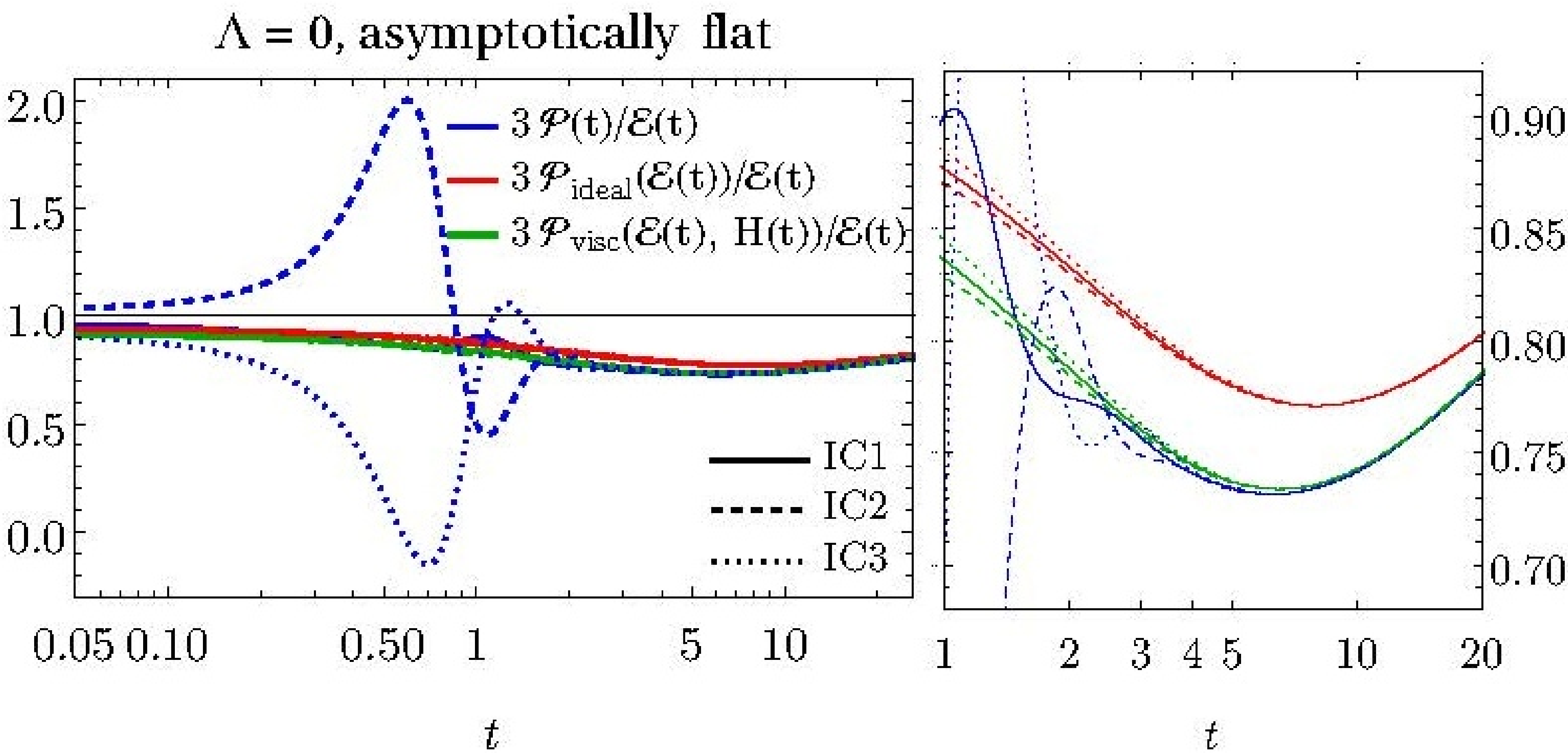
Hydrodynamization with Frozen Inflaton



Ecker+Casalderrey-Solana+Mateos+Van der Schee

Comparison between full evolution, ideal hydro and second-order viscous hydro for three different initial conditions. Viscous corrections can be important.

The conformal anomaly

• The conformal anomaly in a QFT$_4$ gives the trace of the energy-momentum tesnor

$$T^{\mu}{}_{\mu} = \mathcal{A}$$

• It depends on all external sources: background metric, coupling functions etc.

• The metric dependence is universal

$$\mathcal{A}_g = a(Gauss - Bonnet) + c(Weyl^2) + b\Box R$$

• The coupling dependent part depends crucially on the QFT

$$\mathcal{A}_{extra} = \beta(\Phi)\langle O\rangle + \mathcal{A}_{\Phi}$$

• $\mathcal{A}_{\Phi}$ exists when $\mathcal{O}$ is "anomalous". In our case ($\Delta = 3$)

$$\mathcal{A}_g = \frac{1}{16}(R_{\mu\nu}R^{\mu\nu} - \frac{1}{3}R^2) \quad , \quad \beta(\Phi) = -\Phi \quad , \quad \mathcal{A}_{\Phi} = -\frac{1}{2}\left(\partial_\mu\Phi\partial^\mu\Phi + \frac{1}{6}R\Phi^2\right)$$

The model data

- We use the following values and initial conditions

$$\kappa_5 = \frac{1}{9} \quad , \quad \kappa_4 = \frac{2\pi}{5625} \quad , \quad U(\phi) = \frac{1}{30}\phi$$

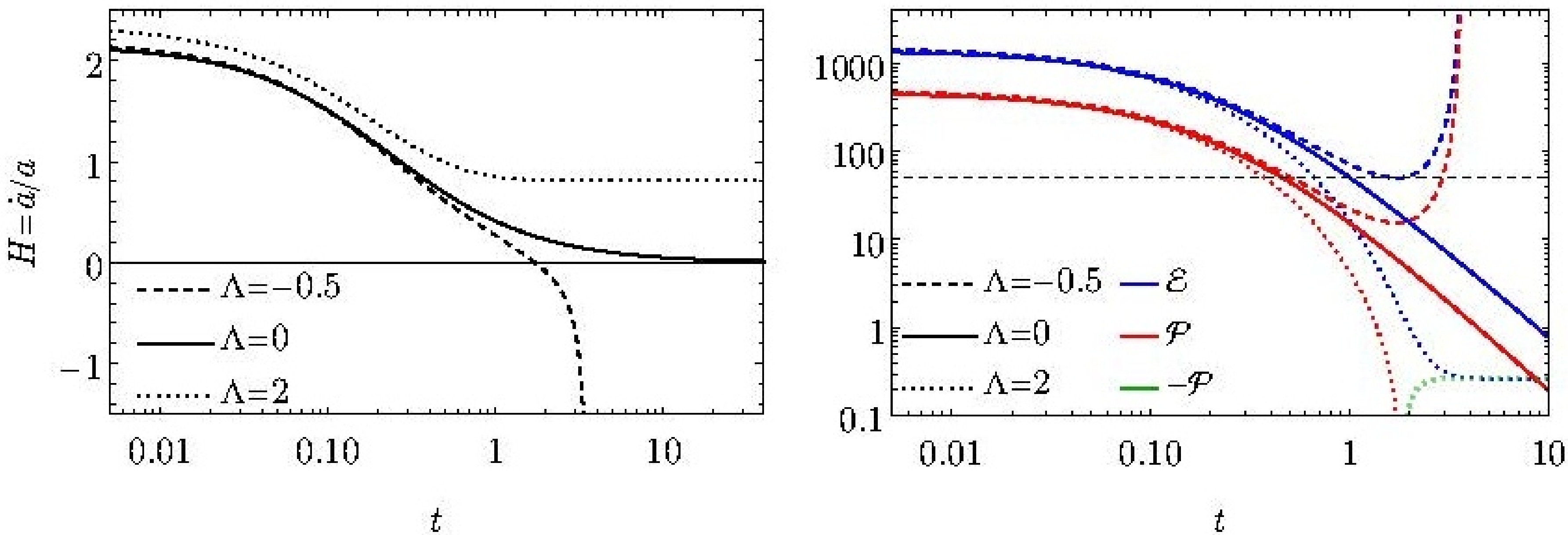
$$\mathcal{E}_{QFT}^{ini} = 13275 \quad , \quad \phi_{ini} = -30 \quad , \quad \dot{\phi}_{ini} = \frac{3}{10}$$

$$\Phi_{ini}(r) = r\left[\Phi_0(t) + r^2\Phi_2 + 2r^2 \log r \Psi_2 + \cdots + \frac{1}{r^3}\left(-6 + \frac{120}{r} - \frac{300}{r^3}\right)\right]$$

$$\Psi_2 = \frac{1}{4}\left(\Box\Phi_0 - \frac{1}{6}R\Phi_0\right)$$

Static Inflaton

- Let's first see what happens when we freeze the inflaton: $\phi(t) = \text{const.}$.
- Pick some value for cosmological constant Λ and initialize QFT.
- Depending on the value of Λ, the universe ends up in a Big Crunch ($\Lambda < 0$), in flat space ($\Lambda = 0$) or in de Sitter ($\Lambda > 0$).
- de Sitter solution has some Casimir energy $\mathcal{E}_{\text{dS}} = -\mathcal{P}_{\text{dS}}$.



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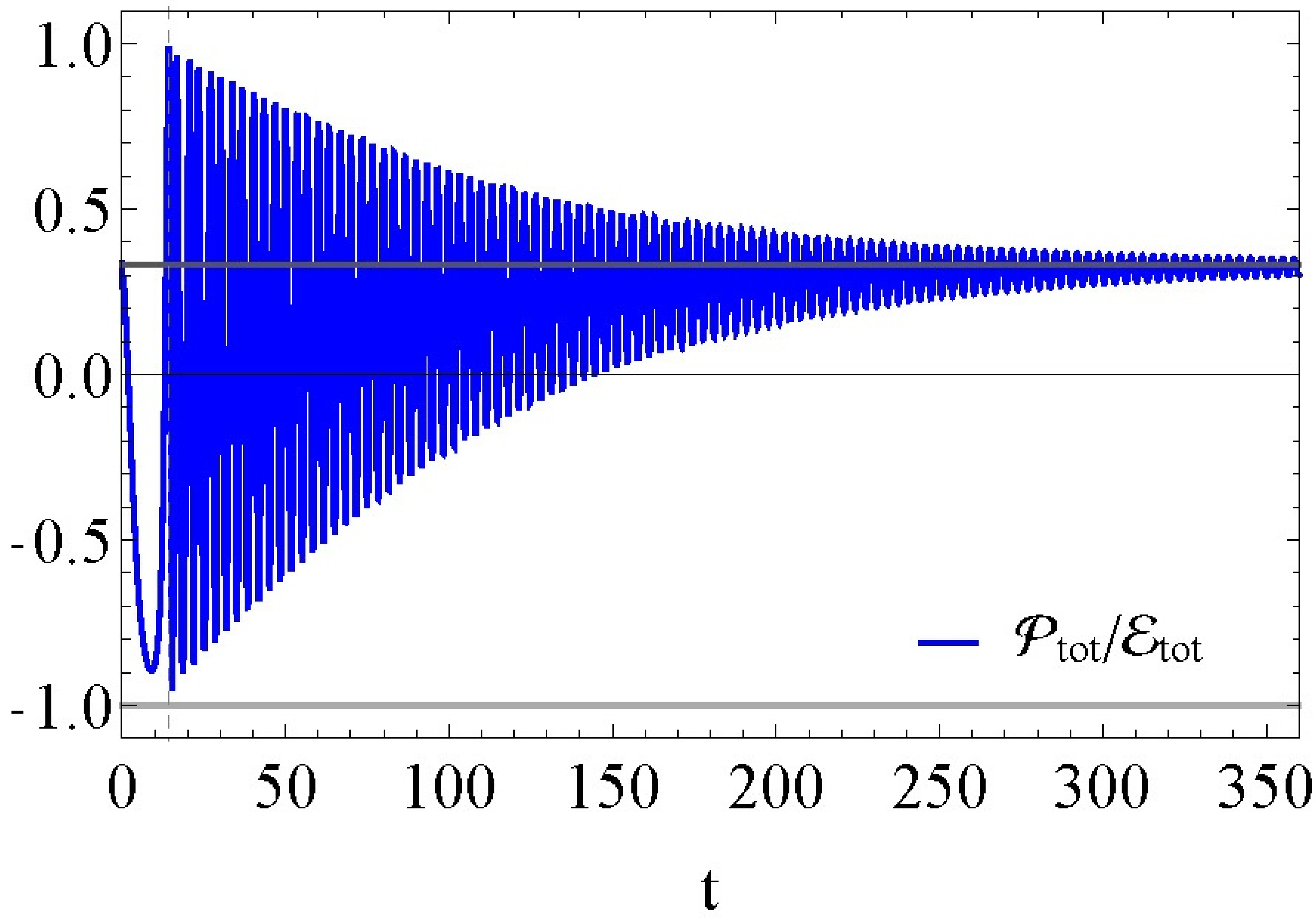
The total pressure

• The total pressure is initially dominated by the QFT, then by the inflaton, and finally by the reheated QFT.



Detailed plan of the presentation

- The Cosmological Evolution Equations 25 minutes
- The holographic picture 26 minutes
- Cartoons 27 minutes
- The Inflaton 28 minutes
- The Energy Density 29 minutes
- The QFT pressure 30 minutes
- The inflaton pressure 31 minutes
- Temperature 33 minutes
- Conclusions 34 minutes
- Open ends 36 minutes