ACV, BCJ, EFT, EOB, MPM, NR, PM, PN, QFT, SF, TF, WQFT and All That

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Gravitational Self-Force and Amplitudes Workshop,

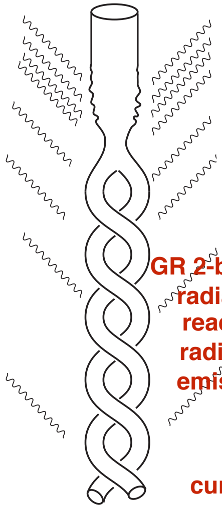
Higgs Centre, James Clerk Maxwell Building,

19-22 March 2024, Edinburgh, Scotland

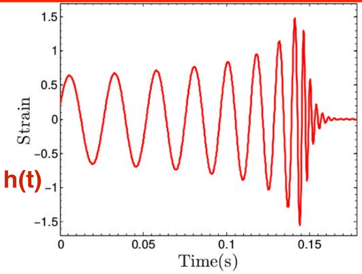
$$R_{\mu\nu} - \frac{1}{2} R\, g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu} \qquad R_{\mu\nu} = 0$$

$$ds^2 = g_{\mu\nu}(x^\lambda)\, dx^\mu dx^\nu$$

$$-g^{\mu\nu} g_{\alpha\beta,\,\mu\nu} + g^{\mu\nu} g^{\rho\sigma}\left(g_{\alpha\mu,\,\rho}\, g_{\beta\nu,\,\sigma} - g_{\alpha\mu,\,\rho}\, g_{\beta\sigma,\,\nu} + g_{\alpha\mu,\,\rho}\, g_{\nu\sigma,\,\beta} + g_{\beta\mu,\,\rho}\, g_{\nu\sigma,\,\alpha} - \frac{1}{2} g_{\mu\rho,\,\alpha}\, g_{\nu\sigma,\,\beta}\right) = 0$$



GR 2-body pb, radiation-reaction, radiation emission. → **waveform h(t)**



needed with ever-decreasing unfaithfulness: currently 10^-2 (0.1 rad); future 10^-4 (0.01 rad) or more

2

Tools used for the GR 2-body pb

Post-Newtonian (PN) approximation (**expansion in 1/c; ie v^2/c^2 and GM/(c^2r)**)

Post-Minkowskian (PM) approximation (**expansion in G; ie in GM/(c^2b)**) and its recent **Worldline EFT avatars**

Multipolar post-Minkowskian (MPM) approximation theory to the GW emission of binary systems

Matched Asymptotic Expansions useful both for the motion of strongly self-gravitating bodies, and for the nearzone-wavezone matching

Gravitational Self-Force (SF): expansion in m1/m2, with « first law of BH mechanics » (LeTiec-Blanchet-Whiting'12,…)

Effective One-Body (EOB) Approach

Numerical Relativity (NR)

Effective Field Theory (EFT) à la Goldberger-Rothstein

Recent:

Quantum scattering amplitude aided by Double-Copy, Generalized Unitarity,« Feynman-integral Calculus » (IBP, DE, regions, reverse unitarity,…), Kosower-Maybee-O'Connell, eikonal, exponentiated representation
+ Worldline QFT or Classical Worldline

Tutti Frutti method

3

LIGO's bank of search templates
O1: 200 000 EOB + 50 000 PN
O2: 325 000 EOB + 75 000 PN
(+ phenom + surrogate templates)

LISA's templates via EOB[SF] ?

PN (or NRGR) $v \ll c$, $R \gg GM/c^2$ → **EOB**

NR $v \sim c$, $R \sim GM/c^2$ → **EOB**

PM $R \gg GM/c^2$ → **EOB**

SF → **EOB**

MPM → **EOB**

BH perturbation $m_1 \ll m_2$ → **EOB**

QFT, EFT → **EOB**

WQFT → **EOB**

TF → **EOB**

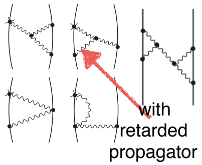
Quantum (or Classical) Scattering + Bremsstrahlung waveform

Tutti-Frutti strategy combining PN,PM,MPM,SF,EFT within EOB (Bini-TD-Geralico'19)

State of the art for PN dynamics

- 1PN (including v^2/c^2) [Lorentz-Droste '17], Einstein-Infeld-Hoffmann '38
- 2PN (inc. v^4/c^4) Ohta-Okamura-Kimura-Hiida '74, Damour-Deruelle '81 Damour '82, Schäfer '85, Kopeikin '85
- 2.5 PN (inc. v^5/c^5) Damour-Deruelle '81, Damour '82, Schäfer '85, Kopeikin '85 **LO-radiation-reaction**
- 3 PN (inc. v^6/c^6) Jaranowski-Schäfer '98, Blanchet-Faye '00, Damour-Jaranowski-Schäfer '01, Itoh-Futamase '03, Blanchet-Damour-Esposito-Farèse' 04, Foffa-Sturani '11
- 3.5 PN (inc. v^7/c^7) Iyer-Will '93, Jaranowski-Schäfer '97, Pati-Will '02, Königsdörffer-Faye-Schäfer '03, Nissanke-Blanchet '05, Itoh '09
- **4PN** (inc. v^8/c^8) Jaranowski-Schäfer '13, Foffa-Sturani '13,'16 Bini-Damour '13, Damour-Jaranowski-Schäfer '14, Marchand+'18, Foffa+'19

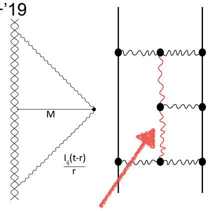
First complete 2PN and 2.5PN dynamics obtained by using 2PM (G^2) EOM of Bel et al.'81



with retarded propagator

New feature at G^4/c^8 (4PN and 4PM) : **non-locality in time** (linked to IR divergences of formal PN-expansion) (Blanchet,TD '88)

- **5PN** (inc. v^{10}/c^{10} and **G^6**) Bini-Damour-Geralico'19: complete **modulo two numerical** parameters; Bluemlein et al'21: potential-graviton contrib. and partial determination of radiation-graviton contrib.
- **6PN** (inc. v^{12}/c^{12} and **G^7**) Bini-Damour-Geralico'20: complete **modulo four** additional parameters



soft (radiation) gravitons

Inclusion of **spin-dependent effects**: Barker-O' Connell'75, Faye-Blanchet-Buonanno'06, Damour-Jaranowski-Schaefer'08, Porto-Rothstein '06, Levi '10, Steinhoff-Hergt-Schaefer '10, Steinhoff'11, Levi-Steinhoff'15-18, Bini-TD, Vines , Guevara-Ochirov-Vines,….

Perturbative computation of GW flux from binary system

- lowest order : Einstein 1918 Peters-Mathews 63
- 1 + (v^2/c^2) : Wagoner-Will 76
- … + (v^3/c^3) : Blanchet-Damour 92, Wiseman 93
- … + (v^4/c^4) : Blanchet-Damour-Iyer Will-Wiseman 95
- … + (v^5/c^5) : Blanchet 96
- … + (v^6/c^6) : Blanchet-Damour-Esposito-Farèse-Iyer 2004
- … + (v^7/c^7) : Blanchet
- … + (v^8/c^8) + (v^9/c^9) : Blanchet et al 2023

$$\nu = \frac{m_1 m_2}{(m_1+m_2)^2}$$

$$x = \left(\frac{v}{c}\right)^2 = \left(\frac{G(m_1+m_2)\Omega}{c^3}\right)^{\frac{2}{3}} = \left(\frac{\pi G(m_1+m_2)f}{c^3}\right)^{\frac{2}{3}}$$

LO quadrupole radiation

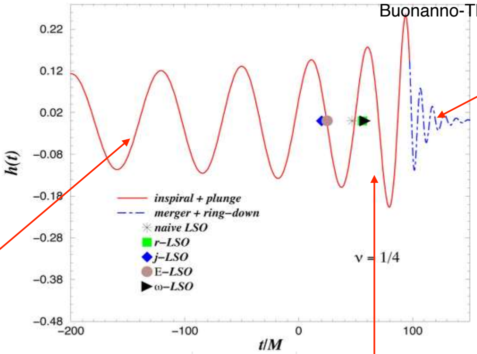
4PN

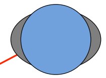
4.5PN

$$\begin{aligned}\mathcal{F} = \frac{32c^5}{5G}\nu^2 x^5 \Bigg\{ & 1 + \left(-\frac{1247}{336} - \frac{35}{12}\nu\right)x + 4\pi x^{3/2} + \left(-\frac{44711}{9072} + \frac{9271}{504}\nu + \frac{65}{18}\nu^2\right)x^2 + \left(-\frac{8191}{672} - \frac{583}{24}\nu\right)\pi x^{5/2} \\ & + \left[\frac{6643739519}{69854400} + \frac{16}{3}\pi^2 - \frac{1712}{105}\gamma_E - \frac{856}{105}\ln(16\,x) + \left(-\frac{134543}{7776} + \frac{41}{48}\pi^2\right)\nu - \frac{94403}{3024}\nu^2 - \frac{775}{324}\nu^3\right]x^3 \\ & + \left(-\frac{16285}{504} + \frac{214745}{1728}\nu + \frac{193385}{3024}\nu^2\right)\pi x^{7/2} \\ & + \left[-\frac{323105549467}{3178375200} + \frac{232597}{4410}\gamma_E - \frac{1369}{126}\pi^2 + \frac{39931}{294}\ln 2 - \frac{47385}{1568}\ln 3 + \frac{232597}{8820}\ln x \right. \\ & \quad + \left(-\frac{1452202403629}{1466942400} + \frac{41478}{245}\gamma_E - \frac{267127}{4608}\pi^2 + \frac{479062}{2205}\ln 2 + \frac{47385}{392}\ln 3 + \frac{20739}{245}\ln x\right)\nu \\ & \quad \left. + \left(\frac{1607125}{6804} - \frac{3157}{384}\pi^2\right)\nu^2 + \frac{6875}{504}\nu^3 + \frac{5}{6}\nu^4\right]x^4 \\ & + \left[\frac{265978667519}{745113600} - \frac{6848}{105}\gamma_E - \frac{3424}{105}\ln(16\,x) + \left(\frac{2062241}{22176} + \frac{41}{12}\pi^2\right)\nu \right. \\ & \quad \left. - \frac{133112905}{290304}\nu^2 - \frac{3719141}{38016}\nu^3\right]\pi x^{9/2} + \mathcal{O}(x^5)\Bigg\}. \end{aligned} \tag{4}$$

The Effective One-Body (EOB) approach to the GW signal emitted by the Merger of two Black Holes

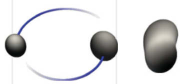
Buonanno-TD'2000





Ringdown (BBH): « vibration modes » of final BH (QNM); perturbation of BHs à la Regge-Wheeler-Zerilli-Teukolsky +Vishveshwara

Inspiral: perturbative computation of higher-order contributions to E=H and F (expansion in v^2/c^2 + tidal polarizability of NS)



Late inspiral, « plunge » and merger: first estimated by the Effective One-Body method (AB-TD 2000) later confirmed and improved by using numerical simulations (Pretorius…2005)

From EOB vs NR to EOB-NR waveforms

Buonanno-Cook-Pretorius 2007

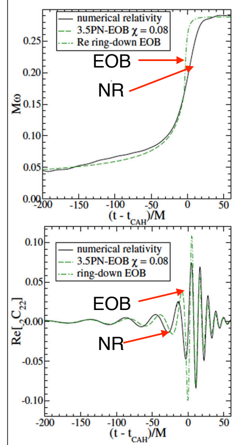
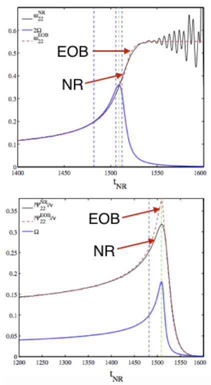
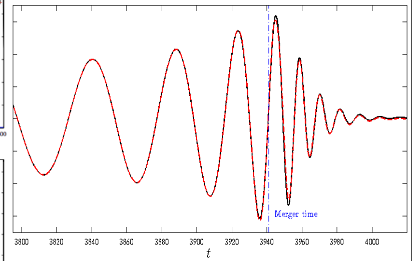


FIG. 21 (color online). We compare the NR and EOB frequency and Re[$_{-2}C_{22}$] waveforms throughout the entire inspiral–merger–ring-down evolution. The data refers to the $d = 16$ run.

TD-Nagar-Dorband-Pollney-Rezzolla 2008

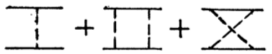


EOB-NR vs NR

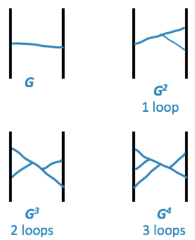


Effective One-Body (EOB) approach: H + Rad-Reac Force

Historically rooted in QM: Brezin-Itzykson-ZinnJustin'70
eikonal scattering amplitude+ Wheeler's:`Think quantum mechanically'



Real 2-body system
(in the c.o.m. frame)



An effective particle of mass mu in some effective metric

$$\mu = \frac{m_1 m_2}{m_1 + m_2}$$

$$g^{\mu\nu}_{\rm eff}(X)$$

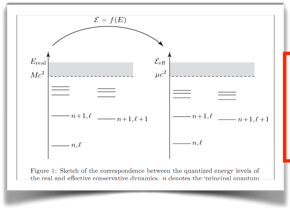
1:1 map

mass-shell constraint

$$0 = g^{\mu\nu}_{\rm eff}(X) P_\mu P_\nu + \mu^2 + Q(X, P)$$

Level correspondence
in the semi-classical limit:
Bohr-Sommerfeld ->
identification of
quantized action variables

$$J = \ell\hbar = \frac{1}{2\pi}\oint p_\varphi d\varphi$$
$$N = n\hbar = I_r + J$$
$$I_r = \frac{1}{2\pi}\oint p_r dr$$



Crucial energy map

$$\mathcal{E}_{\rm eff} = \frac{(\mathcal{E}_{\rm real})^2 - m_1^2 - m_2^2}{2(m_1 + m_2)}.$$

as functions of I_r and I_phi=J

EOB

$$\frac{dr}{dt} = \left(\frac{A}{B}\right)^{1/2} \frac{\partial \hat{H}_{\rm EOB}}{\partial p_{r_*}},$$

$$\frac{dp_{r_*}}{dt} = -\left(\frac{A}{B}\right)^{1/2} \frac{\partial \hat{H}_{\rm EOB}}{\partial r},$$

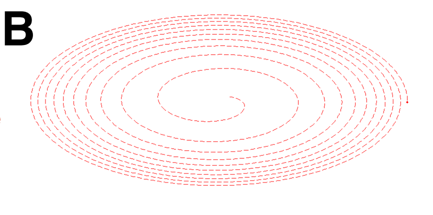
$$\Omega \equiv \frac{d\varphi}{dt} = \frac{\partial \hat{H}_{\rm EOB}}{\partial p_\varphi},$$

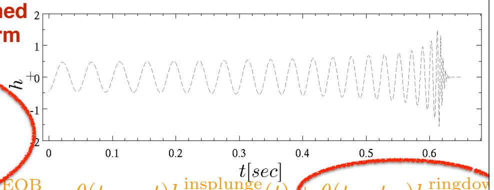
$$\frac{dp_\varphi}{dt} = \hat{\mathcal{F}}_\varphi .$$

Hamiltonian: conservative dynamics

Rad Reac Force

Resummed waveform





$$h_{\ell m} \equiv h^{(N,\epsilon)}_{\ell m} \hat{h}^{(\epsilon)}_{\ell m} \hat{h}^{\rm NQC}_{\ell m}$$

$$\hat{h}^{(\epsilon)}_{\ell m} = \hat{S}^{(\epsilon)}_{\rm eff} T_{\ell m} e^{i\delta_{\ell m}} \rho^\ell_{\ell m}$$

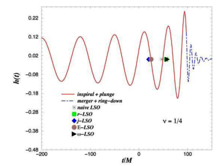
$$\mathcal{F}_\varphi \equiv -\frac{1}{8\pi\Omega} \sum_{\ell=2}^{\ell_{\max}} \sum_{m=1}^{\ell} (m\Omega)^2 |R h^{(\epsilon)}_{\ell m}|^2$$

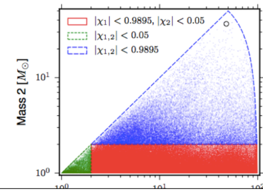
$$h^{\rm EOB}_{\ell m} = \theta(t_m - t) h^{\rm insplunge}_{\ell m}(t) + \theta(t - t_m) h^{\rm ringdown}_{\ell m}$$

$$h^{\rm ringdown}_{\ell m}(t) = \sum_N C^+_N e^{-\sigma^+_N (t - t_m)}$$

$$T_{\ell m} = \frac{\Gamma(\ell + 1 - 2i\hat{\hat{k}})}{\Gamma(\ell + 1)} e^{\pi \hat{\hat{k}}} e^{2i\hat{\hat{k}} \log(2kr_0)},$$

Complete waveforms for BBH coalescenceS:





Explicit 4PN EOB dynamics (Damour-Jaranowski-Schaefer '14)

A **simple**, but crucial transformation between the real energy and the effective one:

$$\mathcal{E}_{\rm eff} = \frac{(\mathcal{E}_{\rm real})^2 - m_1^2 - m_2^2}{2(m_1 + m_2)}.$$

A **simple post-geodesic** effective mass-shell:

$$g^{\mu\nu}_{\rm eff} P'_\mu P'_\nu + \mu^2 c^2 + Q(P'_\mu) = 0\,,$$

$$M = m_1 + m_2, \qquad \mu = \frac{m_1 m_2}{m_1 + m_2}, \qquad \nu = \frac{m_1 m_2}{(m_1 + m_2)^2}$$

$$ds^2_{\rm eff} = -A(R;\nu) dt^2 + B(R;\nu) dR^2 + R^2 (d\theta^2 + \sin^2\theta d\phi^2)$$

Padé resummed

$$a_4 = \frac{94}{3} - \frac{41\pi^2}{32}\ ;\ a_5^c = \frac{2275\pi^2}{512} + \ldots\ ;\ a'_5 = \ldots$$

$$u \equiv \frac{GM}{R c^2}$$

$$A^{\rm PN}(u;\nu) = 1 - 2u + 2\nu u^3 + \nu a_4 u^4 + (\nu a_5^c + \nu^2 a'_5 + \frac{64}{5}\nu \ln u) u^5$$

$$(AB)^{-1} = \bar{D}(u) = 1 + 6\nu u^2 + (52\nu - 6\nu^2) u^3 + \left(\left(-\frac{533}{45} - \frac{23761\pi^2}{1536} + \frac{1184}{15}\gamma_{\rm E} - \frac{6496}{15}\ln 2 + \frac{2916}{5}\ln 3\right)\nu + \left(\frac{123\pi^2}{16} - 260\right)\nu^2 + \frac{592}{15}\nu \ln u\right) u^4,$$

$$\hat{Q}(\mathbf{r}', \mathbf{p}') = \left(2(4 - 3\nu)\nu u^2 + \left(\left(-\frac{5308}{15} + \frac{496256}{45}\ln 2 - \frac{33048}{5}\ln 3\right)\nu - 83\nu^2 + 10\nu^3\right) u^3\right)(\mathbf{n}' \cdot \mathbf{p}')^4 + \left(\left(-\frac{827}{3} - \frac{2358912}{25}\ln 2 + \frac{1399437}{50}\ln 3 + \frac{390625}{18}\ln 5\right)\nu - \frac{27}{5}\nu^2 + 6\nu^3\right) u^2 (\mathbf{n}' \cdot \mathbf{p}')^6 + \mathcal{O}[\nu u (\mathbf{n}' \cdot \mathbf{p}')^8].$$

NR-completed resummed 5PN EOB radial A potential

« We think, however, that a suitable "numerically fitted" and, if possible, "analytically extended" EOB Hamiltonian should be able to fit the needs of upcoming GW detectors. » (TD 2001)

here Damour-Nagar-Bernuzzi '13, Nagar-et al '16; alternative: Taracchini et al '14, Bohe et al '17

4PN analytically complete + 5 PN logarithmic term in the A(u, nu) function,
With u = GM/R and nu = m1 m2 / (m1 + m2)^2
[Damour 09, Blanchet et al 10, Barack-Damour-Sago 10, Le Tiec et al 11, Barausse et al 11, Akcay et al 12, Bini-Damour 13, Damour-Jaranowski-Schäfer 14, Nagar-Damour-Reisswig-Pollney 15]

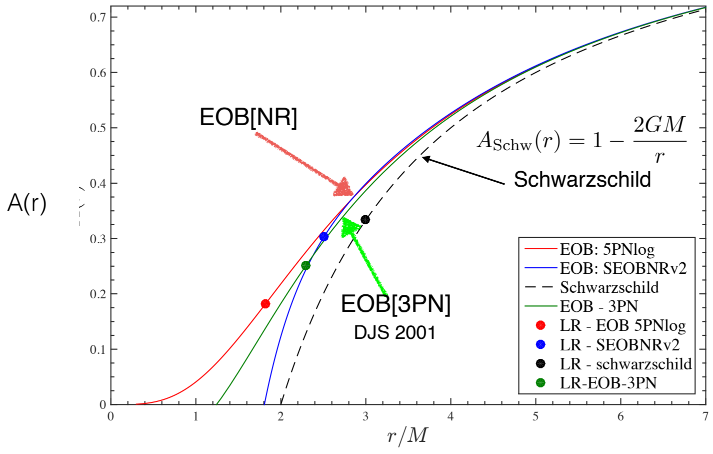
$$A(u;\nu, a_6^c) = P^1_5\left[1 - 2u + 2\nu u^3 + \nu\left(\frac{94}{3} - \frac{41}{32}\pi^2\right) u^4 + \nu\left[-\frac{4237}{60} + \frac{2275}{512}\pi^2 + \left(-\frac{221}{6} + \frac{41}{32}\pi^2\right)\nu + \frac{64}{5}\ln(16 e^{2\gamma} u)\right] u^5 + \nu\left[a_6^c(\nu) - \left(\frac{7004}{105} + \frac{144}{5}\nu\right)\ln u\right] u^6\right]$$

$$u = \frac{GM}{c^2 R} \qquad \nu = \frac{m_1 m_2}{(m_1 + m_2)^2}$$

$$a_6^{c\,\rm NR\text{-}tuned}(\nu) = 81.38 - 1330.6\,\nu + 3097.3\,\nu^2$$

MAIN RADIAL EOB POTENTIAL $A(r;\nu)$

m1=m2 case [nu=m1 m2/(m1+m2)^2=1/4]



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Spinning EOB effective Hamiltonian

Damour'01,Damour-Jaranowski-Schaefer'08,Barausse-Buonanno'11,Taracchini etal'12,Damour-Nagar'14,………

$$H_{\rm eff} = H_{\rm orb} + H_{\rm so} \quad \rightarrow H_{\rm EOB} = Mc^2\sqrt{1+2\nu\left(\frac{H_{\rm eff}}{\mu c^2}-1\right)}$$

$$\hat{H}^{\rm eff}_{\rm orb} = \sqrt{A\left(1 + B_p \boldsymbol{p}^2 + B_{np}(\boldsymbol{n}\cdot\boldsymbol{p})^2 - \frac{1}{1+\frac{(\boldsymbol{n}\cdot\boldsymbol{\chi}_0)^2}{r^2}}\frac{(r^2+2r+(\boldsymbol{n}\cdot\boldsymbol{\chi}_0)^2)}{\mathcal{R}^4+\Delta(\boldsymbol{n}\cdot\boldsymbol{\chi}_0)^2}((\boldsymbol{n}\times\boldsymbol{p})\cdot\boldsymbol{\chi}_0)^2 + Q_4\right)}.$$

$$H_{\rm so} = G_S \boldsymbol{L}\cdot\boldsymbol{S} + G_{S^*}\boldsymbol{L}\cdot\boldsymbol{S}^*,$$

$$\mathbf{S} = \mathbf{S_1} + \mathbf{S_2}\,;\; \mathbf{S}_* = \frac{m_2}{m_1}\mathbf{S_1} + \frac{m_1}{m_2}\mathbf{S_2}\,,$$

Gyrogravitomagnetic ratios (when neglecting spin^2 effects)

$$r^3 G_S^{\rm PN} = 2 - \frac{5}{8}\nu u - \frac{27}{8}\nu p_r^2 + \nu\left(-\frac{51}{4}u^2 - \frac{21}{2}up_r^2 + \frac{5}{8}p_r^4\right) + \nu^2\left(-\frac{1}{8}u^2 + \frac{23}{8}up_r^2 + \frac{35}{8}p_r^4\right)$$

$$r^3 G_{S_*}^{\rm PN} = \frac{3}{2} - \frac{9}{8}u - \frac{15}{8}p_r^2 + \nu\left(-\frac{3}{4}u - \frac{9}{4}p_r^2\right) - \frac{27}{16}u^2 + \frac{69}{16}up_r^2 + \frac{35}{16}p_r^4 + \nu\left(-\frac{39}{4}u^2 - \frac{9}{4}up_r^2 + \frac{5}{2}p_r^4\right) + \nu^2\left(-\frac{3}{16}u^2 + \frac{57}{16}up_r^2 + \frac{45}{16}p_r^4\right)$$

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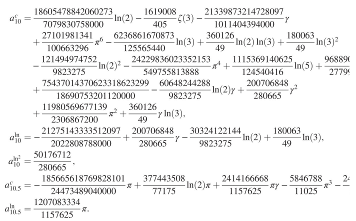
Informing the EOB Hamiltonian with high-PN-order 1SF data (using LeTiec+…)

Bini-Damour 15 $\quad A(u;\nu) = 1 - 2u + \nu a(u) + O(\nu^2)$ $\quad$ Kavanagh et al 15

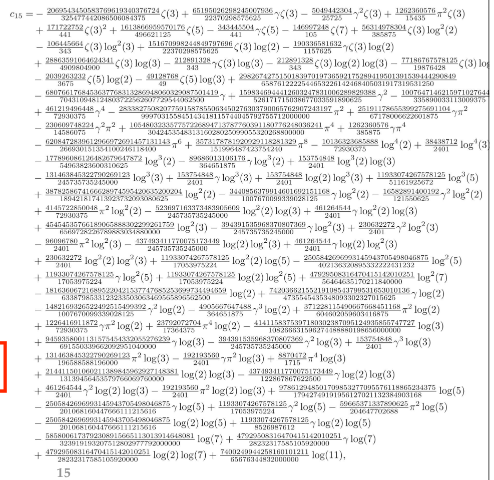


Numerical GSF computation of the O(nu) A(u;nu)-potential: with singularity at u=1/3 (Akcay et al 2012)

$$a(u) \simeq 0.25(1-3u)^{-1/2}$$

GSF can also bring scattering information

(Damour '10, Barack et al'19,Damour'19)



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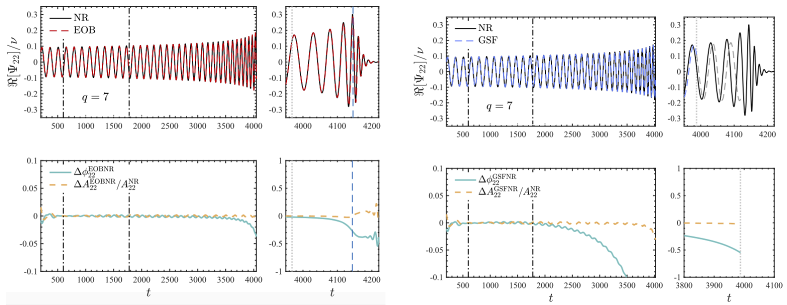
2022

Comparing second-order gravitational self-force, numerical relativity, and effective one body waveforms from inspiralling, quasicircular, and nonspinning black hole binaries

Angelica Albertini,[1,2] Alessandro Nagar,[3,4] Adam Pound,[5] Niels Warburton,[6] Barry Wardell,[6] Leanne Durkan,[6] and Jeremy Miller[7]

Comparing second-order gravitational self-force and effective one body waveforms from inspiralling, quasicircular and nonspinning black hole binaries. II. The large-mass-ratio case

Angelica Albertini,[1,2] Alessandro Nagar,[3,4] Adam Pound,[5] Niels Warburton,[6] Barry Wardell,[6] Leanne Durkan,[6] and Jeremy Miller[7]



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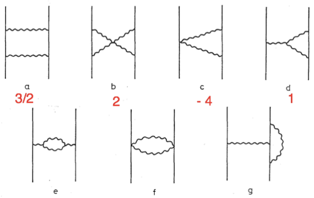
Quantum Scattering Amplitudes and 2-body Dynamics



- Quantum Scattering Amplitudes —> Potential

one-graviton exchange :
Corinaldesi '56 '71,
Barker-Gupta-Haracz 66,
Barker-O'Connell 70, Hiida-Okamura72

Nonlinear: Iwasaki 71 [1PN],
Okamura-Ohta-Kimura-Hiida 73[2 PN]

Using modern amplitude techniques: Bjerrum-Bohr+..2003-

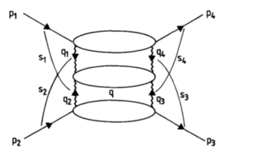


Fig. 3. The "H" diagram that provides the leading correction to the eikonal.

Amati-Ciafaloni-Veneziano 1987-2008

Ultra-High-Energy (s >> M_Planck^2)
Four-graviton Scattering at 2 loops

Eikonal phase \delta in D=4

with one- and two-loop corrections using the Regge-Gribov approach

confirmed by DiVecchia+'19

$$\delta = \frac{Gs}{\hbar}\left(\log\left(\frac{L_{IR}}{b}\right) + \frac{6\ell_s^2}{\pi b^2} + \frac{2G^2 s}{b^2}\left(1 + \frac{2i}{\pi}\log(\cdots)\right)\right)$$

Personally becoming aware of the ACV results in Parma 2008, plus discussions at IHES with Donoghue and Vanhove —> GSF and EOB (TD 2010): scattering and zero-binding zoom-whirl orbit (Barack et al'19)

Having so computed $\mathcal{E}$ and J one might then, for instance, compare the EOB prediction for the scattering angle $\theta(\mathcal{E}, J)$ (which follows from the EOB Hamiltonian) with GSF computations of θ for a sample of values of $\mathcal{E}$ and J. We see that, in principle, we have access here to one function of *two* real variables, which is ample information for determining the functions entering the EOB formalism.

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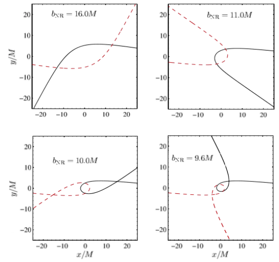
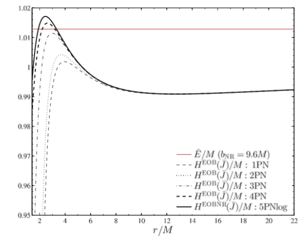
PHYSICAL REVIEW D 89, 081503(R) (2014)

Strong-field scattering of two black holes: Numerics versus analytics

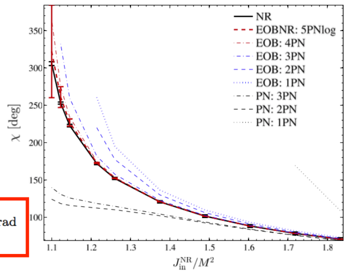
Thibault Damour,[1] Federico Guercilena,[2,3] Ian Hinder,[2] Seth Hopper,[2] Alessandro Nagar,[1] and Luciano Rezzolla[3,2]





Satisfactory comparison with 5PNlog EOB dynamics + radiation-reaction contribution using Bini-TD'12's linear response formula

$$\chi^{\rm rad}(E,J) = -\frac{1}{2}\frac{\partial \chi^{\rm cons}}{\partial E}E^{\rm rad} - \frac{1}{2}\frac{\partial \chi^{\rm cons}}{\partial J}J^{\rm rad}$$



Reviving the PM Two-Body Dynamics

(pioneered by Bertotti'56, Havas-Goldberg'62, Rosenblum'78, Westpfahl'79, Portilla'80, Bel et al.81)

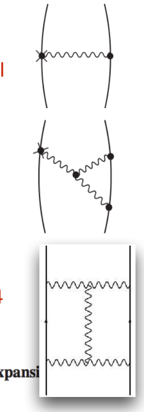
using Classical and/or Quantum Two-Body Scattering

TD 2016, 2017:

Gravitational scattering, post-Minkowskian approximation, and effective-one-body theory

High-energy gravitational scattering and the general relativistic two-body problem

A technique for translating the classical scattering function of two gravitationally interacting bodies into a corresponding (effective one-body) Hamiltonian description has been recently introduced [Phys. Rev. D 94, 104015 (2016)]. Using this technique, we derive, for the first time, to second-order in Newton's constant (i.e. one classical loop) the Hamiltonian of two point masses having an arbitrary (possibly relativistic) relative velocity. The resulting (second post-Minkowskian) Hamiltonian is found to have a tame high-energy structure which we relate both to gravitational self-force studies of large mass-ratio binary systems, and to the ultra high-energy quantum scattering results of Amati, Ciafaloni and Veneziano. We derive several consequences of our second post-Minkowskian Hamiltonian: (i) the need to use special phase-space gauges to get a tame high-energy limit; and (ii) predictions about a (rest-mass independent) linear Regge trajectory behavior of high-angular-momenta, high-energy circular orbits. Ways of testing these predictions by dedicated numerical simulations are indicated. We finally indicate a way to connect our classical results to the quantum gravitational scattering amplitude of two particles, and we urge amplitude experts to use their novel techniques to compute the two-loop scattering amplitude of scalar masses, from which one could deduce the third post-Minkowskian effective one-body Hamiltonian.



tree-level G^1

one-loop G^2

two-loop G^3+G^4

Cheung-Rothstein-Solon 2018

From Scattering Amplitudes to Classical Potentials in the Post-Minkowskian Expansi

We combine tools from effective field theory and generalized unitarity to construct a map between on-shell scattering amplitudes and the classical potential for interacting spinless particles. For general relativity, we obtain analytic expressions for the classical potential of a binary black hole system at second order in the gravitational constant and all orders in velocity. Our results exactly match all known results up to fourth post-Newtonian order, and offer a simple check of future higher order calculations. By design, these methods should extend to higher orders in perturbation theory.

one-loop G^2

Simple Map: Scattering angle <-> EOB dynamics

TD'16-18
Bini-TD-Geralico'20

$$\frac{1}{2}\chi = \Phi(E_{\rm real}, J; m_1, m_2, G) \quad \longrightarrow \quad 0 = g^{\mu\nu}_{\rm eff}P_\mu P_\nu + \mu^2 + Q$$

$$\mathcal{E}_{\rm eff} = \frac{(\mathcal{E}_{\rm real})^2 - m_1^2 - m_2^2}{2(m_1+m_2)} \qquad \frac{\mathcal{E}_{\rm eff}}{\mu} = \gamma = -\frac{p_1\cdot p_2}{m_1 m_2} \qquad \frac{1}{j} = \frac{G m_1 m_2}{J}$$

$$\chi(\gamma, j) = 2\frac{\chi_1(\gamma)}{j} + 2\frac{\chi_2(\gamma)}{j^2} + 2\frac{\chi_3(\gamma)}{j^3} + 2\frac{\chi_4(\gamma)}{j^4} + O\left[\frac{1}{j^5}\right]$$

$g^{\mu\nu}_{\rm eff}$ = Schwarzschild metric M=m1+m2

$$Q = \left(\frac{GM}{R}\right)^2 q_2(E) + \left(\frac{GM}{R}\right)^3 q_3(E) + O(G^4)$$

Map obtained from classical scattering in EOB dynamics

$$q_2 = -\frac{4}{\pi}(\chi_2 - \chi_2^{\rm Schw}) \qquad q_3 = \frac{4}{\pi}\frac{2\gamma^2-1}{\gamma^2-1}(\chi_2 - \chi_2^{\rm Schw}) - \frac{\chi_3 - \chi_3^{\rm Schw}}{\gamma^2 - 1}$$

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Quantum scattering amplitude in EOB potential

(TD 1710.10599: EOB energy-dependent potential Q(R,E) or W(R,E))

EOB

$$Q^E(u,\mathcal{E}_{\rm eff}) = u^2 q_2(\mathcal{E}_{\rm eff}) + u^3 q_3(\mathcal{E}_{\rm eff}) + u^4 q_4^E(\mathcal{E}_{\rm eff}) + O(G^5)$$

$$w(r,p_\infty) = \frac{w_1(\gamma)}{r} + \frac{w_2(\gamma)}{r^2} + \frac{w_3(\gamma)}{r^3} + \frac{w_4(\gamma)}{r^4} + \cdots$$

nonrelativistic potential scattering !

$$-\hbar^2 \Delta_{\mathbf{x}} \psi(\mathbf{x}) = \left[p_\infty^2 + \frac{w_1}{r} + \frac{w_2}{r^2} + \frac{w_3}{r^3} + O\left(\frac{1}{r^4}\right)\right]\psi(\mathbf{x})$$

$$\mathcal{M}^{QFT}_{\rm classical} = \frac{8\pi G s}{\hbar} f^{EOB} = \mathcal{M}^{EFT}$$

EFT Cheung-Rothstein-Solon 1808.02489, Bern et al'19
different EFT potential V(R,P^2) and with systematic methods for taking the classical limit at the integrand level,
and extracting the « classical part » of the scattering amplitude

$$H(\mathbf{P},\mathbf{X}) = \sqrt{m_1^2 + \mathbf{P}^2} + \sqrt{m_2^2 + \mathbf{P}^2} + V(R,\mathbf{P}^2)$$

$$V(R,\mathbf{P}^2) = G\frac{c_1(\mathbf{P}^2)}{R} + G^2\frac{c_2(\mathbf{P}^2)}{R^2} + G^3\frac{c_3(\mathbf{P}^2)}{R^3} + \cdots$$

Scattering Amplitudes and the Conservative Hamiltonian for Binary Systems at Third Post-Minkowskian Order

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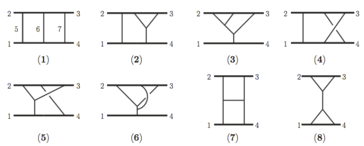
two-loop level G^3

We present the amplitude for classical scattering of gravitationally interacting massive scalars at third post-Minkowskian order. Our approach harnesses powerful tools from the modern amplitudes program such as generalized unitarity and the double-copy construction, which relates gravity integrands to simpler gauge-theory expressions. Adapting methods for integration and matching from effective field theory, we extract the conservative Hamiltonian for compact spinless binaries at third post-Minkowskian order. The resulting Hamiltonian is in complete agreement with corresponding terms in state-of-the-art expressions at fourth post-Newtonian order as well as the probe limit at all orders in velocity. We also derive the scattering angle at third post-Minkowskian order and find agreement with known results.

the eight 2-loop diagrams contributing to the O(G^3/r^3) classical potential



two-loop level

$$\mathcal{M}_3 = \frac{\pi G^3 \nu^2 m^4 \log \boldsymbol{q}^2}{6\gamma^2 \xi}\Bigg[3 - 6\nu + 206\nu\sigma - 54\sigma^2 + 108\nu\sigma^2 + 4\nu\sigma^3 - \frac{48\nu(3 + 12\sigma^2 - 4\sigma^4)\operatorname{arcsinh}\sqrt{\frac{\sigma-1}{2}}}{\sqrt{\sigma^2-1}} - \frac{18\nu\gamma(1-2\sigma^2)(1-5\sigma^2)}{(1+\gamma)(1+\sigma)}\Bigg] + \frac{8\pi^3 G^3 \nu^4 m^6}{\gamma^4 \xi}[3\gamma(1-2\sigma^2)(1-5\sigma^2)F_1 - 32 m^2\nu^2(1-2\sigma^2)^3 F_2], \quad (8)$$

arcsinh

3PM computation (Bern-Cheung-Roiban-Shen-Solon-Zeng'19)

using a combination of techniques: generalized unitarity; BCJ double-copy; 2-loop amplitude of quasi-classical diagrams; **EFT transcription** (Cheung-Rothstein-Solon'18);
resummation of PN-expanded integrals for potential-gravitons

$$\chi_3^{\rm cons} = \chi_3^{\rm Schw} - \frac{2\nu\sqrt{\gamma^2-1}}{h^2(\gamma,\nu)}\bar{C}^{\rm cons}(\gamma)$$

G^3 contrib. to H_EOB

$$q_3^{\rm cons} = \frac{3}{2}\frac{(2\gamma^2-1)(5\gamma^2-1)}{\gamma^2-1}\left(\frac{1}{h(\gamma,\nu)} - 1\right) + \frac{2\nu}{h^2(\gamma,\nu)}\bar{C}^{\rm cons}(\gamma)$$

$$\overline{C}^{\rm cons}(\gamma) = \frac{2}{3}\gamma(14\gamma^2+25) + 2(4\gamma^4 - 12\gamma^2 - 3)\frac{\mathcal{A}(v)}{\sqrt{\gamma^2-1}}$$

$$h(\gamma,\nu) \equiv \frac{\sqrt{s}}{M} = \sqrt{1+2\nu(\gamma-1)}$$

$$\mathcal{A}(v) \equiv \operatorname{arctanh}(v) = \frac{1}{2}\ln\frac{1+v}{1-v} = 2\operatorname{arcsinh}\sqrt{\frac{\gamma-1}{2}}$$

puzzling HE limits when compared to ACV and Akcay et al'12

$$\frac{1}{2}\chi^{\rm cons} = 2\frac{\gamma}{j} + (12 - 8\ln(2\gamma))\frac{\gamma^3}{j^3} + O(G^4)$$

$$q_3^{\rm cons} \approx +8\ln(2\gamma)\gamma^2 \quad \text{instead of} \quad q_3^{\rm ACV} \approx +1\gamma^2$$

confirmations: 5PN (Bini-TD-Geralico'19); 6PN (Blümlein-Maier-Marquard-Schäfer'20, Bini-TD-Geralico'20); 3PM (Cheung-Solon'20, Kälin-Porto'20)

Mass polynomiality structure in scattering (TD'20)

$$\frac{dx_a^\mu}{ds_a} = g^{\mu\nu}(x_a)u_{a\nu}, \qquad R^{\mu\nu} - \frac{1}{2}Rg^{\mu\nu} = 8\pi G T^{\mu\nu},$$

$$\frac{du_{a\mu}}{ds_a} = -\frac{1}{2}\partial_\mu g^{\alpha\beta}(x_a)u_{a\alpha}u_{a\beta}, \qquad T^{\mu\nu}(x) = \sum_{a=1,2} m_a \int ds_a u_a^\mu u_a^\nu \frac{\delta^4(x - x_a(s_a))}{\sqrt{g}}$$

$$\Delta p_{a\mu} = -\frac{m_a}{2}\int_{-\infty}^{+\infty} ds_a \partial_\mu g^{\alpha\beta}(x_a)u_{a\alpha}u_{a\beta}$$

$$\Delta p_{1\mu} = -2Gm_1m_2\frac{2(u_{10}\cdot u_{20})^2 - 1}{\sqrt{(u_{10}\cdot u_{20})^2 - 1}}\frac{b_\mu}{b^2} + \frac{Gm_1m_2}{b}\Delta_\mu.$$

polynomial in Gm1/b and Gm2/b

conservative scattering case

$$\frac{1}{2}\chi(E_{\rm real}, J) = \frac{\chi_1(\gamma,\nu)}{j} + \frac{\chi_2(\gamma,\nu)}{j^2} + \frac{\chi_3(\gamma,\nu)}{j^3} + \frac{\chi_4(\gamma,\nu)}{j^4} + \cdots,$$

at G^n

$$h^{n-1}(\gamma,\nu)\chi_n(\gamma,\nu) = P^\chi_{d(n)}(\nu).$$

polynomial in nu of degree [(n-1)/2]

0SF scattering gives access to full G^2 dynamics !
1SF scattering gives access to full G^3 and G^4 conservative dynamics !
2SF scattering gives access to full G^5 and G^6 conservative dynamics

Tutti-Frutti method

(Bini-TD-Geralico '19,'20'21)
combines
PN
MPM
EOB
Delaunay
Self-Force
Scattering
properties

SIXTH POST-NEWTONIAN LOCAL-IN-TIME DYNAMIC

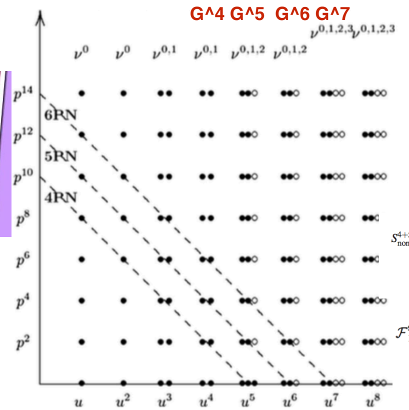


FIG. 1. Schematic representation of the irreducible information contained, at each post-Minkowskian level (keyed by a power of $u = GM/r$), in the local dynamics. Each vertical column of dots describes the post-Newtonian expansion (keyed by powers of p^2) of an energy-dependent function parametrizing the scattering angle. The various columns at a given post-Minkowskian level correspond to increasing powers of the symmetric mass-ratio ν. See text for details.

6PN conservative dynamics complete at 3PM and 4PM

$$S^{4+5\mathrm{PN}}_{\mathrm{nonloc}}[x_1(s_1), x_2(s_2)] = \frac{G^2\mathcal{M}}{c^3}\int dt \mathrm{PF}_{2r^h_{12}(t)/c} \times \int \frac{dt'}{|t-t'|}\mathcal{F}^{\mathrm{split}}_{1\mathrm{PN}}(t,t').$$

$$\mathcal{F}^{\mathrm{split}}_{1\mathrm{PN}}(t,t') = \frac{G}{c^5}\left(\frac{1}{5}I^{(3)}_{ab}(t)I^{(3)}_{ab}(t') + \frac{1}{189c^2} + \frac{16}{45c^2}J^{(3)}_{ab}(t)J^{(3)}_{ab}(t')\right).$$

Conservative and Radiative Aspects of the Dynamics

selected by Bern's integration method of regions

$$\dot{\mathbf{p}}_a \sim G\left(1 + \frac{1}{c^2} + \frac{1}{c^4} + \frac{1}{c^6} + \frac{1}{c^8} + \frac{1}{c^{10}}\right) +$$
$$+G^2\left(\frac{1}{c^2} + \frac{1}{c^4} + \frac{1}{c^5} + \frac{1}{c^6} + \frac{1}{c^7} + \frac{1}{c^8} + \frac{1}{c^9} + \frac{1}{c^{10}}\right) +$$
$$+G^3\left(\frac{1}{c^4} + \frac{1}{c^5} + \frac{1}{c^6} + \frac{1}{c^7} + \frac{1}{c^8} + \frac{1}{c^9} + \frac{1}{c^{10}}\right) +$$
$$+G^4\left(\frac{1}{c^6} + \frac{1}{c^7} + \frac{1}{c^8} + \frac{1}{c^9} + \frac{1}{c^{10}}\right) +$$
$$+G^5\left(\frac{1}{c^8} + \frac{1}{c^9} + \frac{1}{c^{10}}\right) +$$
$$+G^6\frac{1}{c^{10}}$$

1PM, 2PM, 1PN, 2PN, **2.5PN LO rad reac**

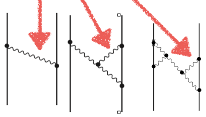
4PN (G^4,G^5) separation well-defined

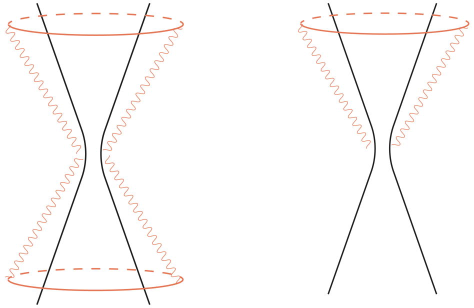
5PN same order as F_radreac^2!

Black: time-even and conservative
Red: time-odd and dissipative
Blue: nonlocal-in-time but decomposable in conservative and dissipative
Purple: ambiguous in various ways ??

Conservative vs Radiation-reacted Classical Gravitational Scattering

Fokker-Wheeler-Feynman conservative action using G_sym=1/2(G_ret+Gadv) =Re[G_F]=PV(1/p^2)





Radiation-reaction effects enter scattering at G^3/c^5 (Bini-TD'12)

$$\frac{1}{2}\chi^{\mathrm{rad}} = +\frac{8G^3}{5c^5}\frac{m_1^3 m_2^3}{J^3}\nu v^2 + \cdots$$

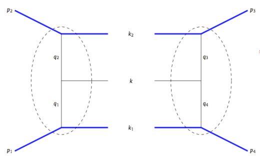
Radiation-reaction effects in scattering play a crucial role at **high-energy**
(DiVecchia-Heissenberg-Russo-Veneziano'20, TD'21, Hermann-Parra-Martinez-Ruf-Zeng'21,….)
they resolve the O(G^3) puzzle of the discrepancy between the HE limit of Amati-Ciafaloni-Veneziano'90(+ Ciafaloni-Colferai'14), and the G^3 result of Bern et al'19,20

Universality of ultra-relativistic gravitational scattering from analyticity/crossing (DiVecchia-Heissenberg-Russo-Veneziano 2008.12743)

ultra-relativistic eikonal phase: $\delta(s,b) = \delta_0(s,b) + \delta_2(s,b)$

$$\mathrm{Re}(2\delta_2) = \frac{\pi}{2\log s}\mathrm{Im}(2\delta_2) - \frac{\delta_0}{s}(\nabla 2\delta_0)^2 + \mathcal{O}\left(\frac{1}{\log s}\right)$$

IR finite — IR divergent



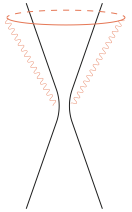
$$\mathrm{Im}\,\widetilde{A_2^{(3p)}}(s,b) \simeq \frac{1}{2s}\frac{(8G_N s)^3 \log s\,\Gamma^3(1-\epsilon)}{16(\pi b^2)^{1-3\epsilon}}\left[-\frac{1}{4\epsilon} + \frac{1}{2} + \mathcal{O}(\epsilon)\right]$$

$$\mathrm{Re}(2\delta_2) \simeq \frac{4G_N^3 s^2}{\hbar b^2}$$

universality of HE result = ACV, thanks to radiative effects

+ explicit eikonal of m1,m2 in N=8 SUGRA —> chi_sugra

Radiation-Reaction Contribution to the (transverse) Classical Scattering Angle at G^3 (TD 2010.01641)



$$\chi^{\rm tot} = \chi^{\rm cons} + \chi^{\rm rad}$$

where, to first order in Rad-Reac, one has (Bini-TD'12)

linear response formula

$$\chi^{\rm rad}(E,J) = -\frac{1}{2}\frac{\partial\chi^{\rm cons}}{\partial E}E^{\rm rad} - \frac{1}{2}\frac{\partial\chi^{\rm cons}}{\partial J}J^{\rm rad}$$

O(G^2) [TD-Deruelle'81]

chi^cons=O(G^1)   O(G^3)   O(G^4)   O(G^3)

$$h_{ij}^{\rm TT} = \frac{f_{ij}(t-r,\theta,\phi)}{r} + O\left(\frac{1}{r^2}\right)$$

$$J_k^{\rm rad} = \frac{\epsilon_{kij}}{16\pi G}\int du\, d\Omega \left[f_{ia}\partial_u f_{ja} - \frac{1}{2}x^i\partial_j f_{ab}\partial_u f_{ab}\right]$$

DeWitt'71, Thorne'80
Kovacs-Thorne'77, Bel et al'81, Westpfahl'85

$$\mathcal{I}(v) = -\frac{16}{3} + \frac{2}{v^2} + \frac{2(3v^2-1)}{v^3}\mathcal{A}(v) \qquad \mathcal{A}(v) \equiv \mathrm{arctanh}(v) = \frac{1}{2}\ln\frac{1+v}{1-v}$$

$$\frac{1}{2}\chi^{\rm rad}(\gamma,j,\nu) = +\frac{\nu}{h^2(\gamma,\nu)j^3}(2\gamma^2-1)^2\mathcal{I}(v) + O(G^4)$$

$$\frac{1}{2}(\chi^{\rm cons}+\chi^{\rm rad}) = 2\frac{\gamma}{j} + \frac{16}{3}\frac{\gamma^3}{j^3} = \chi^{\rm ACV}$$

Radiation-reaction and angular momentum loss at the second post-Minkowskian order (Bini-TD'22)

The linear-response formula assumes a balance between E and J GW-losses at infinity and the mechanical E and J of the 2-body system. It also relies on using the « standard » Peters-DeWitt-Thorne formula for the GW J-flux: $J_k^{\rm rad} = \frac{\epsilon_{kij}}{16\pi G}\int du\, d\Omega \left[f_{ia}\partial_u f_{ja} - \frac{1}{2}x^i\partial_j f_{ab}\partial_u f_{ab}\right]$ which is valid only in the (instantaneous) c.m. frame of the system, and which crucially depends on the zero-frequency part of the waveform

$$f_{ij}(u,\theta,\phi) = Gf_{ij}^{(1)}(\theta,\phi) + G^2 f_{ij}^{(2)}(u,\theta,\phi) + O(G^3)$$

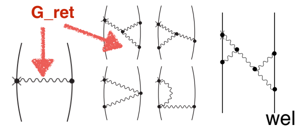
This raises several subtle issues: Ashtekar et al'20, Veneziano-Vilkovisky'22, Riva-Vernizzi, Riva-Vernizzi-Wong'23

Similarly to the resolution of the binary-pulsar « quadrupole controversy », it is useful to clarify the physics by a direct EOM-based approach, fully within a retarded PM approach (Bini-TD'22)

$$m_a\frac{d^2 z_a^\mu(\tau_a)}{d\tau_a^2} = F^\mu_{aR}[z_a(\tau_a), u_a(\tau_a); z_{bR}(\tau_a), u_{bR}(\tau_a)]$$

Considering an auxiliary Fokker-Wheeler-Feynman-type PM dynamics allows one to define Noether« conserved » quantities

G_ret



$$P_\mu^{\rm sys}(\tau_1,\tau_2) = P_\mu^{\rm kin}(\tau_1,\tau_2) + P_\mu^{\rm int}(\tau_1,\tau_2),$$
$$J_{\mu\nu}^{\rm sys}(\tau_1,\tau_2) = J_{\mu\nu}^{\rm kin}(\tau_1,\tau_2) + J_{\mu\nu}^{\rm int}(\tau_1,\tau_2).$$

$$I = -\sum_a m_a\int d\tau_a(-\dot z_{a\mu}\dot z_a^\mu)^{1/2} + \sum_{a<b}\iint d\tau_a d\tau_b\Lambda_{ab} + \sum_{a<b<c}\iiint d\tau_a d\tau_b d\tau_c\Lambda_{abc} + O(G^3),$$

Poincaré-covariant interaction terms Noether-derived from the Fokker action

well-defined O(G^2) PM rad-reac force

$$dP^\mu_{\rm sys} = \sum_a \mathcal{F}^\mu_{a\,\rm rr}(\tau_a)\frac{d\tau_a}{d\sigma}d\sigma + O(G^3),$$

$$dJ^{\mu\nu}_{\rm sys} = \sum_a (z_a^\mu\mathcal{F}^\nu_{a\,\rm rr}(\tau_a) - z_a^\nu\mathcal{F}^\mu_{a\,\rm rr}(\tau_a))\frac{d\tau_a}{d\sigma}d\sigma + O(G^3).$$

Poincaré-covariant final results: $[P^{\rm sys}]^{+\infty}_{-\infty} = O(G^3).$

$$[J^{\mu\nu}_{\rm sys}]^{+\infty}_{-\infty} = \frac{G^2 m_1 m_2}{b_{12}^2}c_I(v)\mathcal{I}(v)[(p_1-p_2)\wedge b_{12}]^{\mu\nu} + O(G^3).$$

O(G^3) momentum transfer (impulse) (Hermann-Parra-Martinez-Ruf-Zeng'21)

$$\Delta p^{\mu,(2)}_{1,\perp,\rm cons} = \frac{G^3M^4\nu}{|b|^3}\frac{2}{\sqrt{\sigma^2-1}}\frac{b^\mu}{|b|}\left[h^2(\sigma,\nu)\left(16\sigma^2 - \frac{1}{(\sigma^2-1)^2}\right)\right. \qquad (7$$
$$\left. -\frac{4}{3}\nu\sigma(14\sigma^2+25) - 8\nu(4\sigma^4-12\sigma^2-3)\frac{\mathrm{arcsinh}\sqrt{\frac{\sigma-1}{2}}}{\sqrt{\sigma^2-1}}\right]$$

$$\Delta p^{\mu,(2)}_{1,u,\rm cons} = \frac{G^3M^5\nu^2}{|b|^3}\frac{3\pi(2\sigma^2-1)(5\sigma^2-1)}{2(\sigma^2-1)}\left[\frac{1}{m_1}\check u_1^\mu - \frac{1}{m_2}\check u_2^\mu\right]$$

$$\Delta p^{\mu,(2)}_{1,\rm rad} = \frac{G^3M^4\nu^2}{|b|^3}\left\{\frac{4}{\sqrt{\sigma^2-1}}\frac{b^\mu}{|b|}\left[f_1^{\rm LS}(\sigma) + f_3^{\rm LS}(\sigma)\frac{\sigma\,\mathrm{arcsinh}\sqrt{\frac{\sigma-1}{2}}}{\sqrt{\sigma^2-1}}\right]\right.$$
$$\left. + \pi\check u_2^\mu\left[f_1(\sigma) + f_2(\sigma)\log\left(\frac{\sigma+1}{2}\right) + f_3(\sigma)\frac{\sigma\,\mathrm{arcsinh}\sqrt{\frac{\sigma-1}{2}}}{\sqrt{\sigma^2-1}}\right]\right\}.$$

$$f_1^{\rm LS}(\sigma) = -\frac{(2\sigma^2-1)^2(5\sigma^2-8)}{3(\sigma^2-1)^{3/2}},$$
$$f_3^{\rm LS}(\sigma) = \frac{2(2\sigma^2-1)^2(2\sigma^2-3)}{(\sigma^2-1)^{3/2}},$$
$$f_1(\sigma) = \frac{210\sigma^6 - 552\sigma^5 + 339\sigma^4 - 912\sigma^3 + 3148\sigma^2 - 3336\sigma + 1151}{48(\sigma^2-1)^{3/2}},$$
$$f_2(\sigma) = -\frac{35\sigma^4 + 60\sigma^3 - 150\sigma^2 + 76\sigma - 5}{8\sqrt{\sigma^2-1}},$$
$$f_3(\sigma) = \frac{(2\sigma^2-3)(35\sigma^4 - 30\sigma^2 + 11)}{8(\sigma^2-1)^{3/2}}.$$

O(G^3) **radiated 4-momentum**

High-energy puzzle

$$\Delta R^\mu = \frac{G^3 m_1^2 m_2^2}{|b|^3}\frac{u_1^\mu + u_2^\mu}{\sigma+1}\mathcal{E}(\sigma) + \mathcal{O}(G^4)$$

$$\frac{\mathcal{E}(\sigma)}{\pi} = f_1(\sigma) + f_2(\sigma)\log\left(\frac{\sigma+1}{2}\right) + f_3(\sigma)\frac{\sigma\,\mathrm{arcsinh}\sqrt{\frac{\sigma-1}{2}}}{\sqrt{\sigma^2-1}}$$

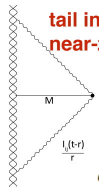
$$\mathcal{E}(\gamma) \propto \gamma^3$$

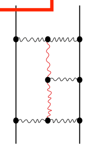
see also: DiVecchia et al, Riva-Vernizzi'21, Bjerrum-Bohr-Plante-Vanhove-Damgaard, useful results concerning the **waveform** (using QFT integration methods)…

Scattering Amplitudes, the Tail Effect, and Conservative Binary Dynamics at $\mathcal{O}(G^4)$

Zvi Bern,[1] Julio Parra-Martinez,[2] Radu Roiban,[3] Michael S. Ruf,[1] Chia-Hsien Shen,[4] Mikhail P. Solon,[1] and Mao Zeng[5]

tail in near-zone





Conservative Dynamics of Binary Systems at Fourth Post-Minkowskian Order in the Large-eccentricity Expansion

Christoph Dlapa,[1] Gregor Kälin,[1] Zhengwen Liu,[1] and Rafael A. Porto[1]

$$G^4M^7\nu^2|\boldsymbol q|\pi^2\left[\mathcal{M}_4^{\rm p} + \nu\left(4\mathcal{M}_4^{\rm t}\log\left(\frac{p_\infty}{2}\right) + \mathcal{M}_4^{\pi^2} + \mathcal{M}_4^{\rm rem}\right)\right] + \int_\ell\frac{\tilde I_{r,1}^4}{Z_1Z_2Z_3} + \int_\ell\frac{\tilde I_{r,1}^2\tilde I_{r,2}}{Z_1Z_2} + \int_\ell\frac{\tilde I_{r,1}\tilde I_{r,3}}{Z_1} + \int_\ell\frac{\tilde I_{r,2}^2}{Z_1},$$

$$\mathcal{M}_4^{\rm p} = -\frac{35(1-18\sigma^2+33\sigma^4)}{8(\sigma^2-1)},$$
$$\mathcal{M}_4^{\rm t} = r_1 + r_2\log\left(\frac{\sigma+1}{2}\right) + r_3\frac{\mathrm{arccosh}(\sigma)}{\sqrt{\sigma^2-1}}, \qquad (3)$$
$$\mathcal{M}_4^{\pi^2} = r_4\pi^2 + r_5\,\mathrm{K}\left(\frac{\sigma-1}{\sigma+1}\right)\mathrm{E}\left(\frac{\sigma-1}{\sigma+1}\right) + r_6\,\mathrm{K}^2\left(\frac{\sigma-1}{\sigma+1}\right) + r_7\,\mathrm{E}^2\left(\frac{\sigma-1}{\sigma+1}\right),$$
$$\mathcal{M}_4^{\rm rem} = r_8 + r_9\log\left(\frac{\sigma+1}{2}\right) + r_{10}\frac{\mathrm{arccosh}(\sigma)}{\sqrt{\sigma^2-1}} + r_{11}\log(\sigma) + r_{12}\log^2\left(\frac{\sigma+1}{2}\right) + r_{13}\frac{\mathrm{arccosh}(\sigma)}{\sqrt{\sigma^2-1}}\log\left(\frac{\sigma+1}{2}\right) + r_{14}\frac{\mathrm{arccosh}^2(\sigma)}{\sigma^2-1}$$
$$+ r_{15}\mathrm{Li}_2\left(\frac{1-\sigma}{2}\right) + r_{16}\mathrm{Li}_2\left(\frac{1-\sigma}{1+\sigma}\right) + r_{17}\frac{1}{\sqrt{\sigma^2-1}}\left[\mathrm{Li}_2\left(-\sqrt{\frac{\sigma-1}{\sigma+1}}\right) - \mathrm{Li}_2\left(\sqrt{\frac{\sigma-1}{\sigma+1}}\right)\right].$$

$$\mathcal{M}_4^{\rm radgrav,f} = \frac{12044}{75}p_\infty^2 + \frac{212077}{3675}p_\infty^4 + \frac{115917979}{793800}p_\infty^6 - \frac{9823091209}{76839840}p_\infty^8 + \frac{115240251793703}{1038874636800}p_\infty^{10} - \frac{411188753665637}{4155498547200}p_\infty^{12} + \cdots, \qquad (6)$$

whose first three terms match the sixth PN order result in Eq. (6.20) of Ref. [42].

matches the 6PN result of the Tutti Frutti approach (Bini-D-Geralico'21)

Radiative contributions to gravitational scattering

Donato Bini[1,2] Thibault Damour[3] and Andrea Geralico[1]
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The linear-order effects of radiation-reaction on the classical scattering of two point masses, in general relativity, are derived by a variation-of-constants method. Explicit expressions for the radiation-reaction contributions to the changes of 4-momentum during scattering are given to linear order in the radiative losses of energy, linear-momentum, and angular momentum. The polynomial dependence on the masses of the 4-momentum changes is shown to lead to nontrivial identities relating the various radiative losses. At order G^3 our results lead to a streamlined classical derivation of results recently derived within a quantum approach. At order G^4 we compute the needed radiative losses to next-to-next-to-leading-order in the post-Newtonian expansion, thereby reaching the absolute fourth and a half post-Newtonian level of accuracy in the 4-momentum changes. We also provide explicit expressions, at the absolute sixth post-Newtonian accuracy, for the radiation-graviton contribution to *conservative* $O(G^4)$ scattering. At orders G^5 and G^6 we derive explicit theoretical expressions for the last two hitherto undetermined parameters describing the fifth-post-Newtonian dynamics. Our results at the fifth-post-Newtonian level confirm results of [Nucl. Phys. **B965**, 115352 (2021)] but exhibit some disagreements with results of [Phys. Rev. D **101**, 064033 (2020)].

X. NONLINEAR RADIATION-REACTION CONTRIBUTIONS TO SCATTERING

$$\frac{d^2\mathbf{x}_1}{dt^2} = -Gm_2\frac{\mathbf{x}_1-\mathbf{x}_2}{r_{12}^3} - \epsilon_{rr}\frac{v_{12}^2}{r_{12}^3}[\mathbf{v}_{12} - 3(\mathbf{v}_{12}\cdot\mathbf{n}_{12})\mathbf{n}_{12}]$$

rad-reac force=O(G^2)

$$\epsilon_{rr} \equiv +\frac{4}{5}\frac{G^2 m_1 m_2}{c^5}.$$

rad-reac^2 =O(G^4) contribution to scattering

$$\lim_{t\to+\infty}\epsilon_{rr}^2\mathbf{v}_1^{(2)}(t) = -\frac{3\pi}{8}\epsilon_{rr}^2\frac{v_0^3}{b^4}\hat{\mathbf{b}},$$

Radiation Reaction and Gravitational Waves at Fourth Post-Minkowskian Order

Christoph Dlapa[1] Gregor Kälin[1] Zhengwen Liu[2,1] Jakob Neef[3,4] and Rafael A. Porto[1] (PRL 10 March 2023)

confirmed by recent QFT computation
Damgaard-Hansen-Plant'e-Vanhove'23

G^4 term

$$\Delta^{(n)}p_1^\mu = c_{1b}^{(n)}\frac{\hat{b}^\mu}{b^n} + \frac{1}{b^n}\sum_a c_{1\check{u}_a}^{(n)}\check{u}_a^\mu,$$

$$\begin{aligned}\frac{c_{1b}^{(4)\mathrm{tot}}}{\pi} = &-\frac{3h_1 m_1 m_2(m_1^3+m_2^3)}{64(\gamma^2-1)^{5/2}} + m_1^2m_2^2(m_1+m_2)\Bigg[\frac{21h_2\mathrm{E}^2(\frac{\gamma-1}{\gamma+1})}{32(\gamma-1)\sqrt{\gamma^2-1}} + \frac{3h_3\mathrm{K}^2(\frac{\gamma-1}{\gamma+1})}{16(\gamma^2-1)^{3/2}} - \frac{3h_4\mathrm{E}(\frac{\gamma-1}{\gamma+1})\mathrm{K}(\frac{\gamma-1}{\gamma+1})}{16(\gamma^2-1)^{3/2}} + \frac{\pi^2h_5}{8\sqrt{\gamma^2-1}}\\
&+\frac{h_6\log(\frac{\gamma-1}{2})}{16(\gamma^2-1)^{3/2}} + \frac{3h_7\mathrm{Li}_2\left(\sqrt{\frac{\gamma-1}{\gamma+1}}\right)}{(\gamma-1)(\gamma+1)^2} - \frac{3h_7\mathrm{Li}_2(\frac{\gamma-1}{\gamma+1})}{4(\gamma-1)(\gamma+1)^2}\Bigg] + m_1^3m_2^2\Bigg[\frac{h_8}{48(\gamma^2-1)^3} + \frac{\sqrt{\gamma^2-1}\,h_9}{768(\gamma-1)^3\gamma^9(\gamma+1)^4} + \frac{h_{10}\log(\frac{\gamma+1}{2})}{8(\gamma^2-1)^2}\\
&-\frac{h_{11}\log(\frac{\gamma+1}{2})}{32(\gamma^2-1)^{5/2}} + \frac{h_{12}\log(\gamma)}{16(\gamma^2-1)^{5/2}} - \frac{h_{13}\mathrm{arccosh}(\gamma)}{8(\gamma-1)(\gamma+1)^4} + \frac{h_{14}\mathrm{arccosh}(\gamma)}{16(\gamma^2-1)^{7/2}} - \frac{3h_{15}\log(\frac{\gamma+1}{2})\log(\frac{\gamma-1}{\gamma+1})}{8\sqrt{\gamma^2-1}} + \frac{3h_{16}\mathrm{arccosh}(\gamma)\log(\frac{\gamma-1}{\gamma+1})}{16(\gamma^2-1)^2}\\
&-\frac{3h_{17}\mathrm{Li}_2(\frac{\gamma-1}{\gamma+1})}{64\sqrt{\gamma^2-1}} - \frac{3}{32}\sqrt{\gamma^2-1}\,h_{18}\mathrm{Li}_2\left(\frac{1-\gamma}{\gamma+1}\right)\Bigg] + m_1^2m_2^3\Bigg[\frac{3h_{15}\log(\frac{2}{\gamma-1})\log(\frac{\gamma+1}{2})}{8\sqrt{\gamma^2-1}} + \frac{3h_{16}\log(\frac{\gamma-1}{2})\mathrm{arccosh}(\gamma)}{16(\gamma^2-1)^2} + \frac{h_{19}}{48(\gamma^2-1)^3}\\
&+\frac{h_{20}}{192\gamma^7(\gamma^2-1)^{5/2}} + \frac{h_{21}\log(\frac{\gamma+1}{2})}{8(\gamma^2-1)^2} + \frac{h_{22}\log(\frac{\gamma+1}{2})}{16(\gamma^2-1)^{3/2}} + \frac{h_{23}\log(\gamma)}{2(\gamma^2-1)^{3/2}} - \frac{h_{24}\mathrm{arccosh}(\gamma)}{16(\gamma^2-1)^3} + \frac{h_{25}\mathrm{arccosh}(\gamma)}{16(\gamma^2-1)^{7/2}} - \frac{3h_{26}\mathrm{arccosh}^2(\gamma)}{32(\gamma^2-1)^{7/2}}\\
&+\frac{3h_{27}\log^2(\frac{\gamma+1}{2})}{2\sqrt{\gamma^2-1}} + \frac{3h_{28}\log(\frac{\gamma+1}{2})\mathrm{arccosh}(\gamma)}{16(\gamma^2-1)^2} + \frac{h_{29}\mathrm{Li}_2(\frac{1-\gamma}{\gamma+1})}{4\sqrt{\gamma^2-1}} + \frac{3h_{30}\mathrm{Li}_2(\frac{\gamma-1}{\gamma+1})}{8\sqrt{\gamma^2-1}}\Bigg],\end{aligned}$$

Its PN expansion agrees with Bini-TD-Geralico'23 notably for the **nu^2 =O(RR^2) contribution.**
Moreover, Bini-TD-Geralico'23 went beyond the linear-response formula by using balance+mass-polynomiality

$$\Delta p_{a\mu} = \Delta p_{a\mu}^{\mathrm{cons}} + \Delta p_{a\mu}^{\mathrm{rr\,lin}} + \Delta p_{a\mu}^{\mathrm{rr\,nonlin}}$$

$$\Delta p_{1\,\mu G^4}^{\mathrm{rr}} = \Delta p_{1\,\mu G^4}^{\mathrm{rr\,lin-odd}} + \frac{G^4}{b^4}m_1^3m_2^2\,p_x^{G^4}(\gamma)\hat{b}_{12}^\mu,$$

$$\Delta p_{1\,\mu G^4}^{\mathrm{rr}} = \Delta p_{1\,\mu G^4}^{\mathrm{rr\,lin-odd}} + \frac{m_1}{m_2-m_1}P_{x\,G^4}^{\mathrm{rad}}\hat{b}_{12}^\mu.$$

relation between the rad-reac^2 term and P^rad_x

Strong-field scattering of two black holes: Numerical relativity meets post-Minkowskian gravity

Thibault Damour[1] and Piero Rettegno[2,3] (PRD March 2023)

$$\chi_{n\mathrm{PM}}(\gamma,j) \equiv \sum_{i=1}^{n}2\frac{\chi_i(\gamma)}{j^i}.$$

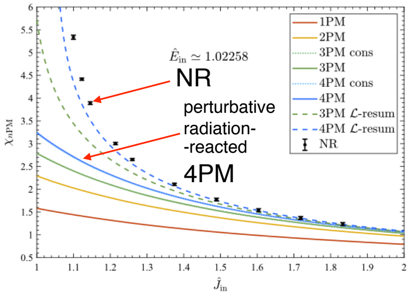
$$\mu^2 + g_{\mathrm{eff}}^{\mu\nu}P_\mu P_\nu + Q(X^\mu,P_\mu) = 0,$$

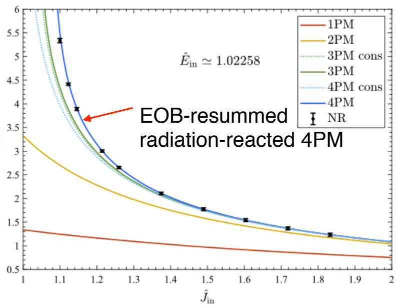
$$p_{\bar{r}}^2 + \frac{j^2}{\bar{r}^2} = p_\infty^2 + w(\bar{r},\gamma).$$

$$w(\bar{r},\gamma) = \frac{w_1(\gamma)}{\bar{r}} + \frac{w_2(\gamma)}{\bar{r}^2} + \frac{w_3(\gamma)}{\bar{r}^3} + \frac{w_4(\gamma)}{\bar{r}^4} + O\left[\frac{1}{\bar{r}^5}\right]$$

$$\chi_{n\mathrm{PM}}^{w\,\mathrm{eob}}(\gamma,j) \equiv 2j\int_0^{\bar{u}_{\max}(\gamma,j)}\frac{d\bar{u}}{\sqrt{p_\infty^2 + w_{n\mathrm{PM}}(\bar{u},\gamma) - j^2\bar{u}^2}} - \pi.$$

Newtonianlike EOB radial potential

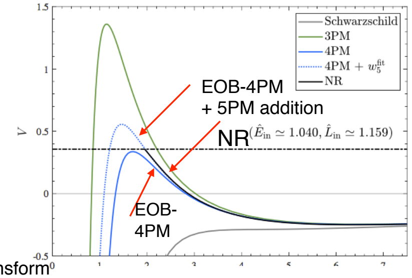




Strong-field scattering of two spinning black holes: Numerical Relativity versus post-Minkowskian gravity

(Rettegno et al.'23)

Higher-energy non-spinning: Comparison between the effective potential V=L^2/r^2-w(r) extracted from NR simulations and its EOB-PM equivalent

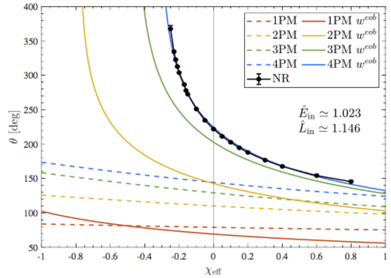


In the spin-aligned case, one can transform the PM-expanded scattering angle

$$\theta_{n\mathrm{PM}}(\gamma,\ell,S_i) \equiv \sum_{k=1}^{n}2\frac{\theta_k(\gamma,\ell,S_i)}{\ell^k}$$

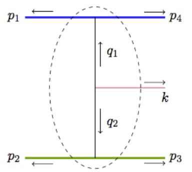
into an equivalent spin-dependent EOB potential

$$w_{n\mathrm{PM}}(\bar{r},\gamma,\ell,S_i) = w^{\mathrm{orb}}(\bar{r},\gamma) + \frac{\ell\,w_{n\mathrm{PM}}^{\mathrm{S}}(\bar{r},\gamma)}{\bar{r}^2} + \frac{w_{n\mathrm{PM}}^{\mathrm{S}^2}(\bar{r},\gamma)}{\bar{r}^2} + \frac{\ell\,w_{n\mathrm{PM}}^{\mathrm{S}^3}(\bar{r},\gamma)}{\bar{r}^4} + \frac{w_{n\mathrm{PM}}^{\mathrm{S}^4}(\bar{r},\gamma)}{\bar{r}^4}.$$



PM waveform computation $W(k^\mu) = \epsilon^\mu \epsilon^\nu h_{\mu\nu}(\omega,\theta,\phi)$

G^1=1PM (linearized, Einstein 1918) stationary $\propto \delta(\omega)$



LO (tree level) waveform

G^2=2PM: classical time-domain W(t,n): Kovacs-Thorne 1977

quantum-based: yields W(k,p1,p2,p3,p4)=W(k,p1,p2,q1)

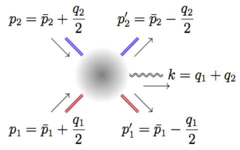
Johansson-Ochirov'15, GoldbergerRidgway'17
Luna-Nicholson-OConnellWhite'18
Mougiakakos-Riva-Vernizzi'21, Bautista-Siemonsen'22

Recent NLO (one-loop) waveform

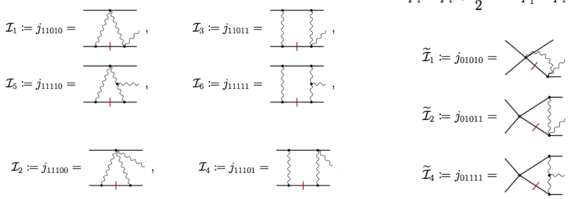
G^3=3PM

Andreas Brandhuber[a], Graham R. Brown[a], Gang Chen[b], Stefano De Angelis[c], Joshua Gowdy[a] and Gab
Aidan Herderschee,[a] Radu Roiban[b,c] and Fei Teng[b,c]

Alessandro Georgoudis[a] Carlo Heissenberg[b,a] Ingrid Vazquez-Holm[b,a]



5-point HEFT amplitude at one-loop O(G^3) waveform



$$\mathcal{M} = \kappa\, m_1 \mathcal{M}_1^{\rm lin} + \kappa^3 m_1 m_2 \mathcal{M}_1^{\rm tree} + \kappa^5 m_1^2 m_2 \mathcal{M}_1^{\rm one\,loop} + (1\to 2)$$
$$= -i\frac{\kappa}{2}\epsilon^*_\mu \epsilon^*_\nu \tilde{T}^{\mu\nu}(k) = -i\frac{\kappa}{2}\epsilon^*_\mu\epsilon^*_\nu \int \mu_{12}(q_1,q_2)\tilde{T}^{\mu\nu}(k,q_1,q_2)$$

$$\mu_{1,2}(k) \equiv e^{i(q_1\cdot b_1 + q_2\cdot b_2)}\hat{\delta}^{(4)}(k - q_1 - q_2)\hat{\delta}(q_1\cdot u_1)\hat{\delta}(q_2\cdot u_2)$$

$\mathcal{M}_1^{\rm lin}$
$$\tilde{T}^{\mu\nu}_{\rm Fig.\,1a}(k) = \sum_a m_a u_a^\mu u_a^\nu e^{ik\cdot b_a}\hat{\delta}(\omega_a)$$

$\mathcal{M}_1^{\rm tree}$ to be integrated

$$\tilde{T}^{\mu\nu}_{\rm Fig.\,1b}(k) = \frac{m_1 m_2}{4m_{\rm Pl}^2}\int_{q_1,q_2}\mu_{1,2}(k)\frac{1}{q_2^2}\left[\frac{2\gamma^2-1}{\omega_1+i\epsilon}q_2^{(\mu}u_1^{\nu)} - 4\gamma u_2^{(\mu}u_1^{\nu)} - \left(\frac{2\gamma^2-1}{2}\frac{k\cdot q_2}{(\omega_1+i\epsilon)^2} - \frac{2\gamma\omega_2}{\omega_1+i\epsilon} - 1\right)u_1^\mu u_1^\nu\right],$$

$$\tilde{T}^{\mu\nu}_{\rm Fig.\,1c}(k) = \frac{m_1 m_2}{4m_{\rm Pl}^2}\int_{q_1,q_2}\mu_{1,2}(k)\frac{1}{q_1^2 q_2^2}\left[\frac{2\gamma^2-1}{2}q_2^\mu q_2^\nu + (2\omega_2^2 - q_1^2)u_1^\mu u_1^\nu + 4\gamma\omega_2 q_2^{(\mu}u_1^{\nu)} - \eta^{\mu\nu}\left(\gamma\omega_1\omega_2 + \frac{2\gamma^2-1}{4}q_2^2\right) + 2(\gamma q_1^2 - \omega_1\omega_2)u_1^{(\mu}u_2^{\nu)}\right],$$

$\mathcal{M}_1^{\rm one\,loop}$

$$\mathcal{M}^{(1)}_{m_1^3 m_2^2} = \frac{\mathfrak{D}_{\rm IR}}{\epsilon} + \mathfrak{R} + i\pi\,\mathfrak{i}_1 + \frac{i\pi}{\sqrt{\bar{y}^2-1}}\mathfrak{i}_2 + c_{1,0}\mathcal{I}_1 + c_{2,0}\mathcal{I}_2$$
$$+ \mathfrak{l}_{\bar{w}_1}\log\frac{\bar{w}_1^2}{\mu_{\rm IR}^2} + \mathfrak{l}_{\bar{w}_2}\log\frac{\bar{w}_2^2}{\mu_{\rm IR}^2} + \mathfrak{l}_q\log\frac{q_1^2}{q_2^2} + \mathfrak{l}_{\bar{y}}\frac{\log\left(\sqrt{\bar{y}^2-1}+\bar{y}\right)}{\sqrt{\bar{y}^2-1}} + \mathcal{O}(\epsilon^0)$$

complicated rational functions of 8 variables with spurious poles

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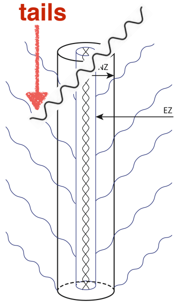
Comparing one-loop amplitude to MPM waveform

(Bini-TD-Geralico'23)

tails



$$g = \eta + Gh_1 + G^2h_2 + G^3h_3 + ...,$$
$$\Box h_1 = 0,$$
$$\Box h_2 = \partial\partial h_1 h_1,$$
$$\Box h_3 = \partial\partial h_1 h_1 h_1 + \partial\partial h_1 h_2,$$
$$h_1 = \sum_\ell \partial_{i_1 i_2 \dots i_\ell}\left(\frac{M_{i_1 i_2\dots i_\ell}(t-r/c)}{r}\right) + \partial\partial\dots\partial\left(\frac{\epsilon_{j_1 j_2 k}S_{k j_3\dots j_\ell}(t-r/c)}{r}\right),$$
$$h_2 = FP_B\Box^{-1}_{\rm ret}\left(\left(\frac{r}{r_0}\right)^B\partial\partial h_1 h_1\right) + ...,$$
$$h_3 = FP_B\Box^{-1}_{\rm ret}....$$

algorithmic STF tensors encoding multipole moments (related to the source moments I_L,J_L)

radiative multipole moments (observable at infinity) U_L, V_L

$$rh_{ij}^{\rm TT} = \frac{4G}{c^2}P(n)_{ijab}\sum_{l=2}^{\infty}\frac{1}{c^l}\frac{1}{l!}\left(U_{abL-2}n_{L-2} - \frac{2l}{c(l+1)}n_{cL-2}\epsilon_{cd(a}V_{b)dL-2}\right)$$

$$\mathcal{M}^{\rm MPM}(k,b,u_1,u_2,m_1,m_2) = -i\frac{\kappa}{2}\epsilon^\mu\epsilon^\nu h^{\rm MPM}_{\mu\nu}(\omega,\theta,\phi) = -i\frac{\kappa}{2}\int dt e^{i\omega t}\epsilon^\mu\epsilon^\nu h^{\rm MPM}_{\mu\nu}(t,\theta,\phi)$$

$$\mathcal{M}^{\rm HEFT}(k,b,u_1,u_2,m_1,m_2) =$$
$$e^{i\frac{b_1+b_2}{2}\cdot k}\int\frac{d^Dq}{(2\pi)^{D-2}}\delta\left(2p_1\cdot\left(q+\frac{k}{2}\right)\right)\delta\left(2p_2\cdot\left(-q+\frac{k}{2}\right)\right)e^{iq\cdot(b_1-b_2)}\mathcal{M}^{(1)}_{5,\rm HEFT}\left(q+\frac{k}{2},-q+\frac{k}{2};h\right)$$

Comparison one-loop amplitude vs MPM waveform

$$W(t,\theta,\phi) \sim \frac{1}{c^4}\left(G\,(\text{stationary}) + G^2(1 + \frac{1}{c^1} + \frac{1}{c^2} + \frac{1}{c^3} + \cdots) + G^3(1 + \frac{1}{c^1} + \frac{1}{c^2} + \frac{1}{c^3} + \cdots) + O(G^4)\right)$$

tree-level — one-loop

Aim: accuracy up to radiation-reaction effects: O(1/c^5) beyond LO quadrupole

$$U_{ij}(\omega) \sim \left(G(1 + \frac{1}{c^2} + \frac{1}{c^4}) + G^2(1 + \frac{1}{c^2} + \frac{1}{c^3} + \frac{1}{c^4} + \frac{1}{c^5}) + O(G^4)\right) + O(\frac{1}{c^6})$$

Newtonian G^2 — LO tail — rad-reac plus similar effects

Main results of the initial EFT-MPM comparison (Bini-TD-Geralico, 2023):
mismatch at the Newtonian level, except if one refers the one-loop amp. to classical averaged momenta, rather than incoming momenta; then the terms linked to time-even PN corrections to multipoles agree but there are many mismatches at the G^2/c^5 level

Updated comparisons (Georgoudis-Heissenberg-Russo'23,'24, Bini et al. 24) lead to perfect agreement after taking into account three subtle effects:
(1) the bilinear-in-amplitude KMOC term generates the needed rotation
(2) IR divergences generate an additional (D-4)/(D-4) contribution
(3) zero-frequency gravitons contribute additional terms at h~G and h~G^3

Current Puzzles

high-energy limits?

G^3 energy loss too large

G^3 angular momentum loss too large (Manohar-Ridgway-Shen'22)

Rad-reacted G^4 scattering diverges (Porto..,Damgaard..)

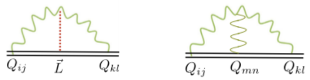
cf ACV motivation: BH formation in HE scattering

Subtleties in defining/computing angular momentum flux

(Ashtekar et al., Veneziano-Vilkovisky, Riva-Vernizzi,…)

low-energy discrepancy at 5PN between

Foffa-Sturani'19,21,22 Bluemlein et al'21 and Bini-TD-Geralico



$$S_{QQL} = C_{QQL} G^2 \int dt I_{is}^{(4)} I_{js}^{(3)} \varepsilon_{ijk} L_k$$
$$S_{QQQ_1} = C_{QQQ_1} G^2 \int dt I_{is}^{(4)} I_{js}^{(4)} I_{ij} ,$$
$$S_{QQQ_2} = C_{QQQ_2} G^2 \int dt I_{is}^{(3)} I_{js}^{(3)} I_{ij}^{(2)} .$$

TF-constraint on 5PN O(nu^2) EFT radiative terms

$$0 = \frac{2973}{350} - \frac{69}{2} C_{QQL} + \frac{253}{18} C_{QQQ_1} + \frac{85}{9} C_{QQQ_2}$$

not solved by recent in-in results (Foffa-Sturani'22)

Conclusions

The synergy between ACV, BCJ, EFT, EOB, MPM, NR, PM, PN, QFT, SF, TF, WQFT has led to many very interesting new vistas on the gravitational 2-body interaction.

Many impressive new results have been derived and more are in store, though one is close to reaching the limits of the new techniques

There remains puzzles to clarify

I think that the flexible analytical nature of the EOB formalism can be useful to absorb the recently acquired new information (as well as new results to come) and to incorporate them in LIGO-grade waveform templates