# Analytic self-force: Bound and Unbound

-PN, PM, SF and just that

Gravitational self-force and scattering amplitudes workshop, Higgs centre, Edinburgh

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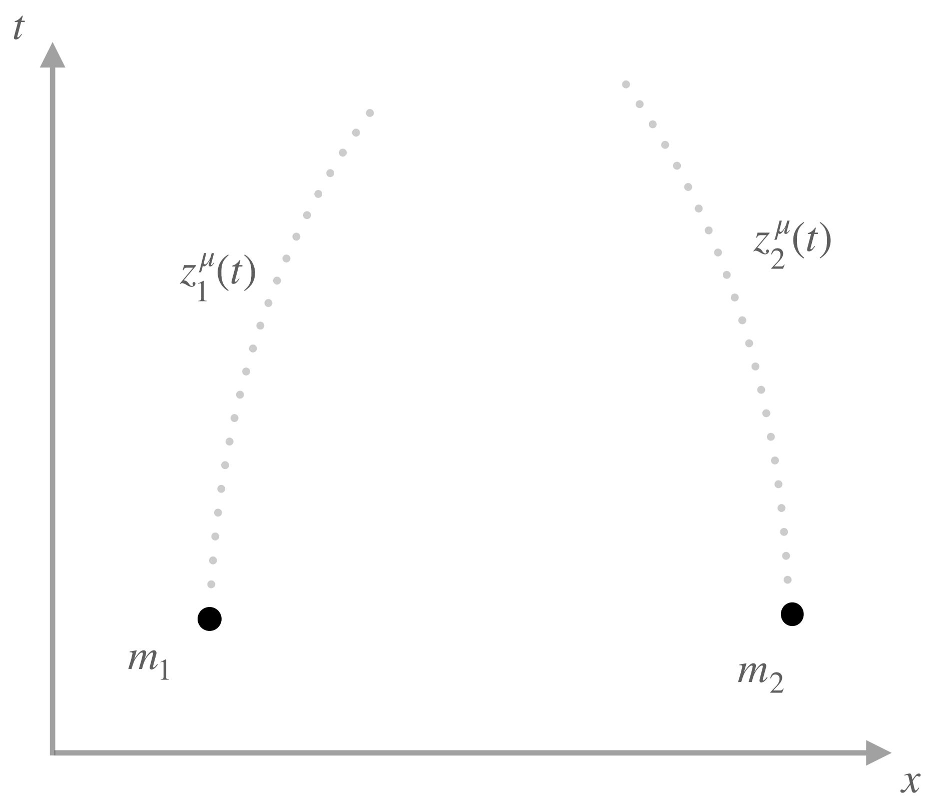
# PART 1: Self-Force, GWs & computational strategy

# Binary motion PN/PM

‘Point particles’ endowed with **multipole moments**, fields constrained by EFEs




(1) $S = S_M[m_i] + S_{GR}[g]$

(2) $G_{\mu\nu}[g] = 8\pi T_{\mu\nu}$

$$T_{\mu\nu} = \sum_{i=1,2} m_i \int d\tau \delta^4(x, z_i) u^i_\mu u^i_\nu$$

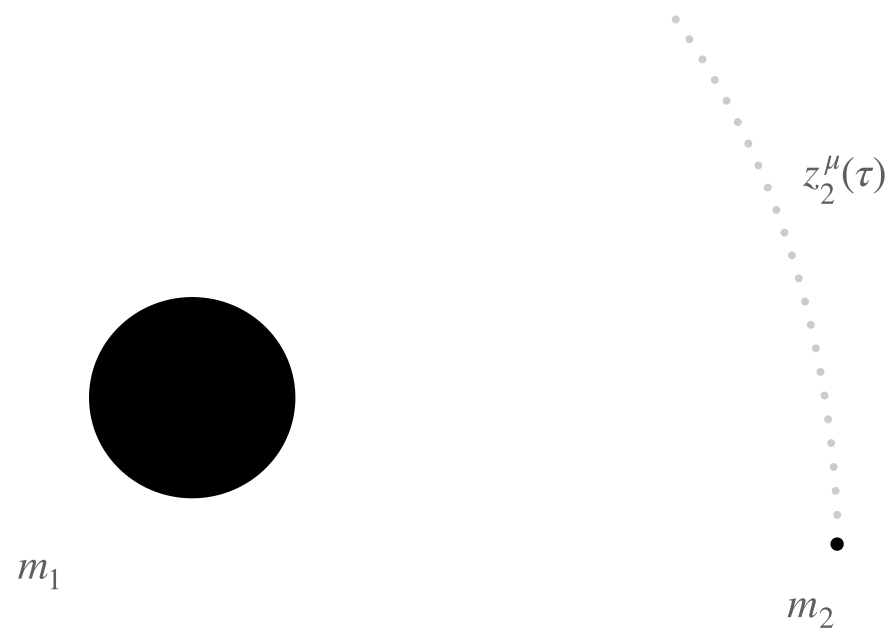
$$g_{\mu\nu} = \eta_{\mu\nu} + G h^{(1)}_{\mu\nu} + G^2 h^{(2)}_{\mu\nu} + \ldots$$

$$\Box^0 \, [h^i]_{\mu\nu} = S[h^{i-1}, \ldots] \quad \to G_{\text{flat}}(x, x')$$

- Both approaches need delicate treatments of point particle limit
- Expansion typically not valid throughout space — require ‘far field’ expansions, tail terms..

# Binary motion using self-force

'Point particle' endowed with multipole moments + Exact Kerr BH




$$(1) \quad S = S_M[m_i] + S_{GR}[g]$$

See Thursdays talks.

$$(2) \quad G_{\mu\nu}[g] = 8\pi T_{\mu\nu}$$

$$T_{\mu\nu} = m_2 \int d\tau \delta^4(x, z_2) u_\mu u_\nu$$

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + \epsilon h^{(1)}_{\mu\nu} + \epsilon^2 h^{(2)}_{\mu\nu} + \ldots$$

$$\epsilon = \frac{m_2}{m_1}$$

Lorenz gauge field equations:

$$\bar{\nabla}^a \bar{\nabla}_a h^{(1)}_{\mu\nu} + 2\bar{R}_\mu{}^a{}_\nu{}^b h^{(1)}_{ab} = 16\pi T_{\mu\nu}$$

$$\bar{\nabla}^a \bar{\nabla}_a h^{(2)}_{\mu\nu} + 2\bar{R}_\mu{}^a{}_\nu{}^b h^{(2)}_{ab} = S[h^{(1)}_{\mu\nu}, h^{(1)}_{\mu\nu}]$$

Equations of motion:

$$u^a \bar{\nabla}_a u^\mu = -\frac{1}{2}(\bar{g}^{\mu\nu} - h^{\mu\nu}_R)(2h^R_{\nu\rho;\sigma} - h^R_{\rho\sigma;\nu})u^\rho u^\sigma$$

$$h^R_{\mu\nu} = \epsilon h^{R,(1)}_{\mu\nu} + \epsilon^2 h^{R,(2)}_{\mu\nu}$$

# GW Phase evolution: what is important

For a small mass ratio binary self force gives the following approximation [Hinderer, Flanagan 2008]:

$$\phi = \frac{\phi_0}{\epsilon} + \frac{\phi_{1/2}}{\epsilon^{1/2}} + \phi_1 + O(\epsilon)$$

Cannot have an $O(1)$ error in the phase — the approximate signal will not match that of an observed signal

$\phi_0$ : dissipative 1SF (e.g. fluxes) — 0PA (post-Adiabatic expansion)

$\phi_{1/2}$ : orbital resonances (lets ignore this for our conversation)

$\phi_1$ : conservative 1SF, dissipative 2SF, linear-in-spin dissipative SF — 1PA

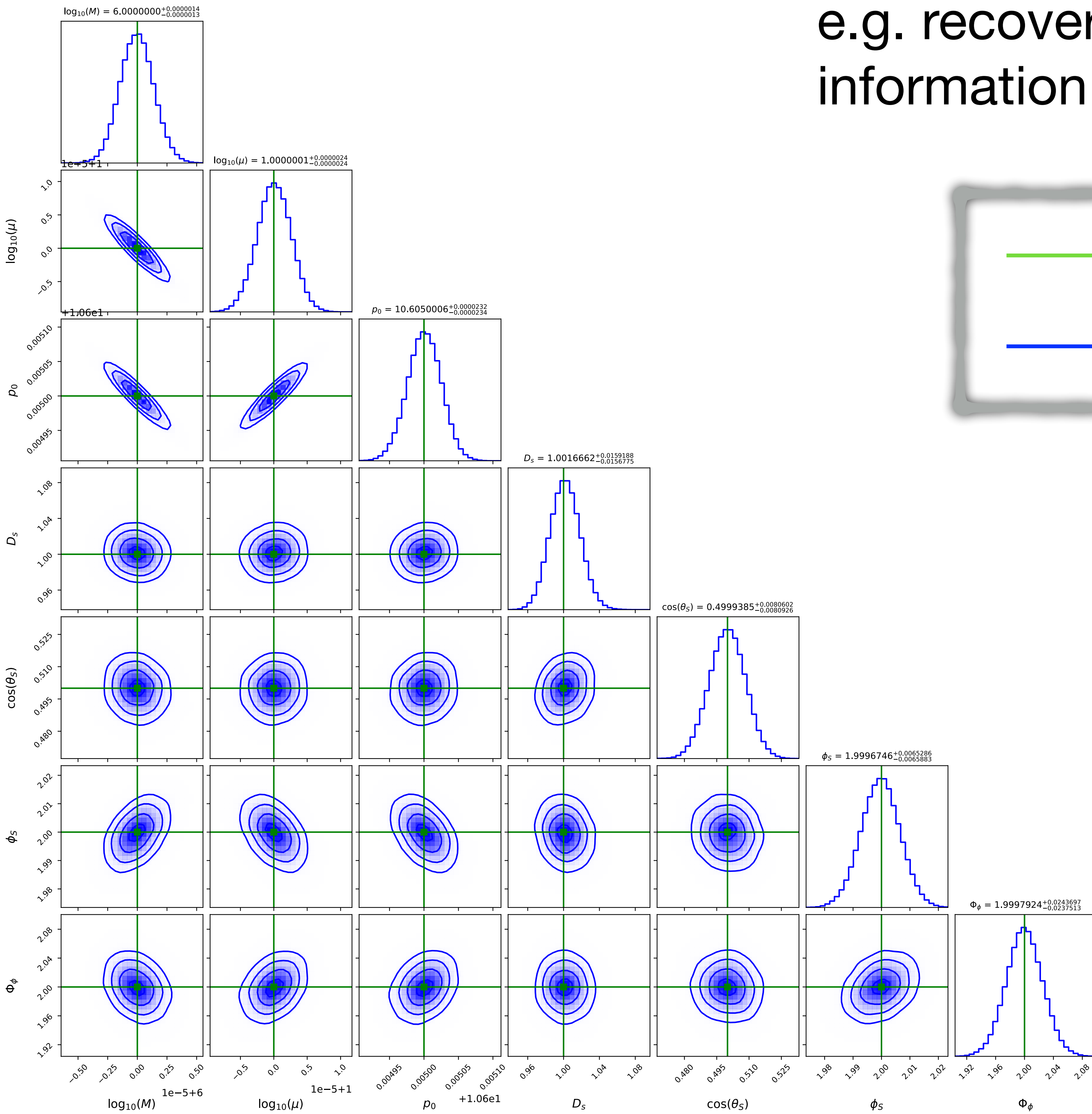
Key observation: For 1PA terms we need $O(\epsilon)$ fewer digits of accuracy

# GW Phase evolution: accuracy at 1PA

Burke et al: arXiv:2310.08927

Idea: Inject 1PA waveform into data stream, and attempt to recover parameters of the system with various approximate waveform models, check **biases** on recovered parameters

# GW Phase evolution: accuracy at 1PA




e.g. recovering with full 1PA information - **no approximation**

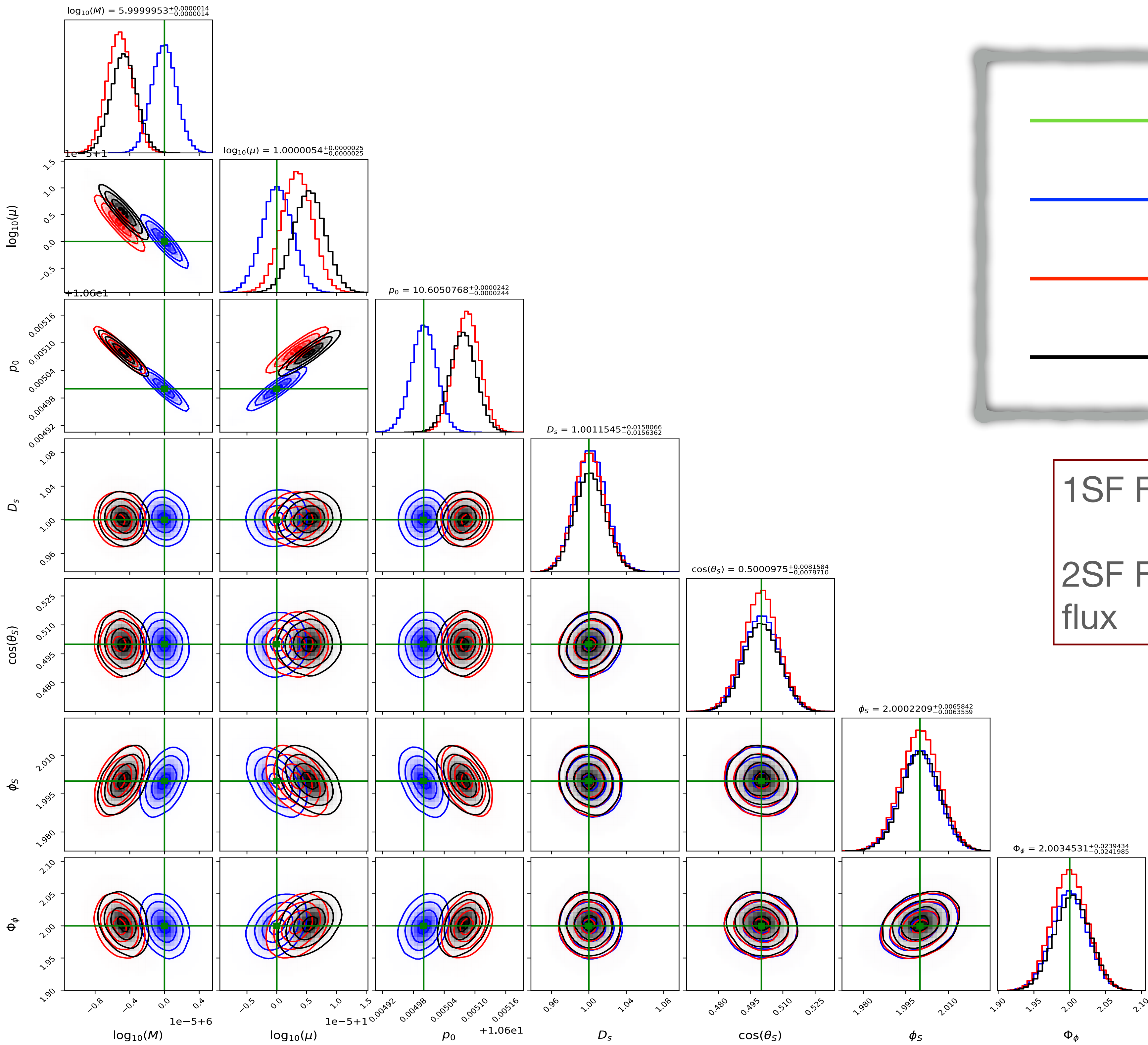
True values

Posterior

no biases

*with thanks to O Burke for plots. More detail see Capra 27 talk by O Burke*

# 1.1 GW Phase evolution: what is important




True

Full 1PA

Only 0PA

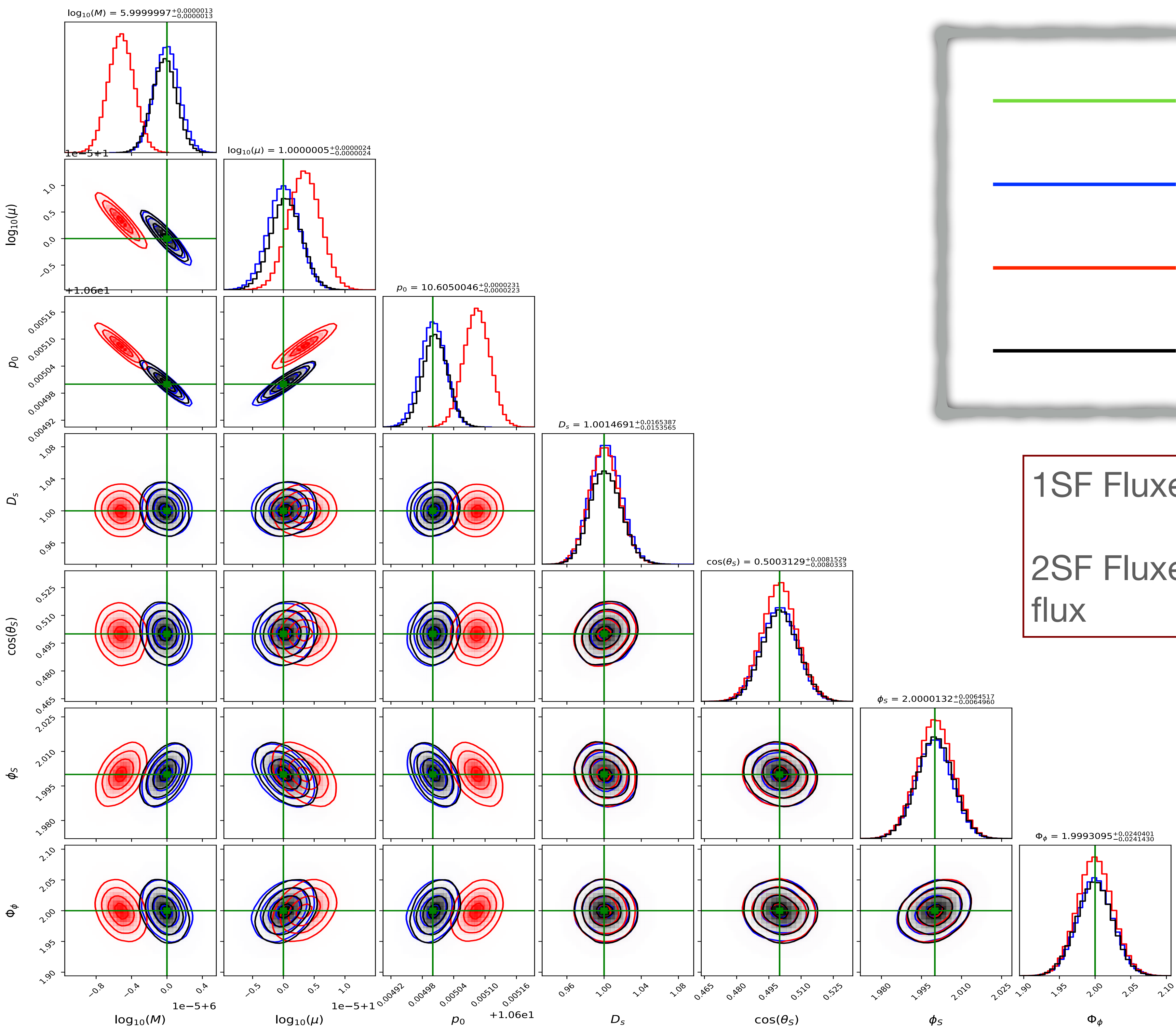
1PA3PN

1SF Fluxes — high accuracy numerics

+

2SF Fluxes — second order terms in 3PN flux

# GW Phase evolution: accuracy at 1PA




True values
Full 1PA
Only 0PA
Resummation

1SF Fluxes — high accuracy numerics
+
2SF Fluxes — second order terms in 3PN flux

# GW Phase evolution: accuracy at 1PA

Takeaways:

- ~~2SF program is obsolete, 3PN is enough!~~

- weak-field approximations have significant potential for realistically reducing the load on numerics **at 1PA**
- We will only know when we have all the information

# The road to observables

Step 1: Geodesic equations

$$\frac{dr}{d\tau} = R[r, E, L]$$

$$\frac{d\varphi}{d\tau} = \Phi[r, E, L]$$

$$\frac{dt}{d\tau} = T[r, E, L]$$

# The road to observables

Step 2: Field Equations

$$\bar{\nabla}^a \bar{\nabla}_a h^{(1)}_{\mu\nu} + 2\bar{R}_\mu{}^a{}_\nu{}^b h^{(1)}_{ab} = 16\pi T_{\mu\nu}$$

- not separable in Kerr
- no analytic information known about a Green function
- solving for something gauge dependent (always cause for concern..)

# The road to observables

Step 2: Field Equations —> Teukolsky Equation

$$O_s \psi = \mathcal{S}\left[T_{\mu\nu}\left(z, z_p(\tau)\right)\right]$$

$${}_s\psi = \sum_{lm} \int d\omega e^{-i\omega t} {}_sR_{lm\omega}(r)\ {}_sS_{lm}(\theta, \phi; a\omega)$$

$$\mathcal{S}[T] = \sum_{lm} \int d\omega e^{-i\omega t} {}_sT_{lm\omega}(r)\ {}_sS_{lm}(\theta, \phi; a\omega)$$

$${}_sR_{lm\omega}(r) = \int_{r_+}^{\infty} dr\ G_{lm\omega}(r, r')\,{}_sT_{lm\omega}(r)$$

$$= \int_{r_+}^{\infty} dr\ G_{lm\omega}(r, r') \int dt' e^{i\omega t'} \mathcal{S}[T(t')]_{lm}$$

# The road to observables

Step 3: back again to the metric

$$h_{\mu\nu} = \nabla_a \zeta^4 \nabla_b C^a{}_{(\mu}{}^b{}_{\nu)}[{}_s\psi] + \nabla_{(\mu}\xi_{\nu)} + \mathcal{N}T_{\mu\nu} + g_{\mu\nu}[\delta M, \delta a]$$

[Wald, Cohen & Kegeles, Chrzanowski, Steward 70s]
[Price, Whiting; Acksteiner, Andersson, Backdahl ++;] Green, Hollands, Zimmerman +; Dolan, CK, Wardell, Dolan, CK, Wardell, Durkan]

In principle this can give the Green function also, see e.g.

- Casals, Holland, Pound , Toomani 2024
- Dolan, Durkan, Wardell, CK 2023 (less explicitly a GF, but Lorenz gauge)

→ explicit need for $s = 0, \pm 1, \pm 2$ fields to completely fix the metric without gauge divergencies/discontinuities

# PART II: Bound Orbits, post-Newtonian expansions + analytic perturbation theory

$$r_p(\tau) \gg M$$

$$ {}_sR_{lm\omega}(r) = \int_{r_+}^{\infty} dr\ G_{lm\omega}(r, r')\, {}_sT_{lm\omega}(r)$$

$$T_{lm\omega} = A^0(r_p, \omega)\delta(r - r_P) + A^1(r_p, \omega)\delta'(r - r_p) + A^2(r_p, \omega)\delta''(r - r_p)$$

All we need is the **radial GF** for asymptotically large radius.

# MST solutions give us the *exact* retarded Green function:

[Leaver 85/86, Mano Suzuki and Takasugi+ ~96]

$$R^{\rm in}_{lm\omega}(r) = C^{\rm in}_{lm\omega} \sum_{n=-\infty}^{\infty} a_n^{\nu}(\omega)\ {}_2F_1(a, b, c, 1 - r/2M)$$

$$R^{\rm up}_{lm\omega}(r) = C^{\rm up}_{lm\omega} \sum_{n=-\infty}^{\infty} a_n^{\nu}(\omega)\ U(d, e, r\omega)$$

$$G^{\rm ret}_{lm\omega}(r, r') = \frac{R^{\rm in}_{lm\omega}(r')R^{\rm up}_{lm\omega}(r)}{W[R^{\rm in}_{lm\omega}(r), R^{\rm in}_{lm\omega}(r)]}\theta(r - r') + \frac{R^{\rm in}_{lm\omega}(r)R^{\rm up}_{lm\omega}(r')}{W[R^{\rm in}_{lm\omega}(r), R^{\rm in}_{lm\omega}(r)]}\theta(r' - r)$$

Formally valid to all orders in weak-field expansions

$$a_n^{\nu} \sim (GM\omega)^{|n|}$$

Sum naturally truncates in PN/PM expansion.

$$R^{\rm in}_{lm\omega}(r) = C^{\rm in}_{lm\omega} \sum_{n=-\infty}^{\infty} a_n^\nu \; {}_2F_1(a, b, c, 1 - r/2M)$$

$$R^{\rm up}_{lm\omega}(r) = C^{\rm up}_{lm\omega} \sum_{n=-\infty}^{\infty} a_n^\nu \; U(d, e, r\omega)$$

$$G^{\rm ret}_{lm\omega}(r, r') = \frac{R^{\rm in}_{lm\omega}(r') R^{\rm up}_{lm\omega}(r)}{W[R^{\rm in}_{lm\omega}(r), R^{\rm in}_{lm\omega}(r)]}\theta(r - r') + \frac{R^{\rm in}_{lm\omega}(r) R^{\rm up}_{lm\omega}(r')}{W[R^{\rm in}_{lm\omega}(r), R^{\rm in}_{lm\omega}(r)]}\theta(r' - r)$$

Put in standard PN scalings

$$r \sim \eta^{-2}$$

$$\omega \sim \eta^{3}$$

$\longrightarrow$

- PN expansions purely from near zone
- No near zone/far zone matching
- .. no far zone integrations (the hard part!)
- solutions are pure polynomials + log(r)

MST solutions now used frequently in Amplitudes/EFT calculations. Recent examples:

- Ivanov et al (arXiv:2401.08752)— “Gravitational Raman Scattering ..”
- Saketh, Zhou, Ivanov (arXiv:2307.10391)
- Y F Bautista et al (arXiv:2312.05965) — “BHPT meets CFT”
- Bautista, Guevara, CK,  Vines (arXiv:2212.07965, arXiv:2107.10179)

# What we know analytically:

- Dissipative 1SF
  - ~10PN circular equatorial fluxes for Kerr (i.e. 'aligned spin') [Fujita++]
  - low eccentricity limit but high order $\sim e^{10}$ or higher (equatorial) e.g. [Evans, Munna++]
  - small particle spin -7PN, aligned, Schwarzschild
  - Generic Kerr, closed form in inclinations 5PN [Fujita, Sago et al]

- Conservative 1SF
  - ~10PN circular Redshift for Kerr (i.e. 'aligned spin') [CK, Wardell, Kavanagh]
  - low eccentricity limit + low spin limit [Bini, Geralico +] (all orders in spin possible!)
  - Linear in spin + quadratic in spin redshift [Bini ++]

⟶
- Successful program generating high order PN terms for EOB
- Damour et al 'tutti frutti' approach

# What we don't know analytically:

- **High eccentricies i.e.** $e \sim 1$

  Is it possible to gain some control of the poor numerical convergence beyond $e \sim 0.3$ ?

- **Precession effects to all orders**

  Kerr fluxes known to all-orders-in-inclination + Kerr spin parameter.
  Conservative sector?

- **2SF**

  Scalar model demonstration [Pound + CK - in prep]
  GR calculation at v. early stages
  —independent calculations are going to be essential.

# PART III: Unbound Orbits + PM expansion

Aim: can we do a PM expansion of the mp along a scattering trajectory:

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + \epsilon h^{(1)}_{\mu\nu} + \epsilon^2 h^{(2)}_{\mu\nu} + \ldots$$

$$h^{(1)}_{\mu\nu} = \sum G^i M^i h^{(1,i)}_{\mu\nu}$$

i.e. can we construct Teukolsky solutions along a weak-field scattering orbit?

Barack+Long, Barack, Whitall, Bern+ have been paving the way for this type of question

—See C Whittall, O Long's talks on Wednesday.

# Scalar self-force

$$\bar{\nabla}^a \bar{\nabla}_a h^{(1)}_{\mu\nu} + 2\bar{R}_\mu{}^a{}_\nu{}^b h^{(1)}_{ab} = 16\pi T_{\mu\nu}$$

$$u^a \bar{\nabla}_a u^\mu = -\frac{1}{2}(\bar{g}^{\mu\nu} - h^{\mu\nu}_R)(2h^R_{\nu\rho;\sigma} - h^R_{\rho\sigma;\nu})u^\rho u^\sigma$$

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$$\bar{\nabla}^a \bar{\nabla}_a \Phi = 4\pi T$$

$$u^a \bar{\nabla}_a u^\mu = Q \bar{\nabla}^\mu \Phi^{\mathrm{R}}$$

$$T = -Q\int_{-\infty}^{\infty} \frac{\delta^4(z - z_p(\tau))}{\sqrt{-g}} d\tau$$

# Scalar self-force

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$$T = -Q \int_{-\infty}^{\infty} \frac{\delta^4(z - z_p(\tau))}{\sqrt{-g}} d\tau$$

- Same analytic GF structure
- Same geodesic eq structure
- No metric gauge ambiguities
- No metric reconstruction issues
- There are results in the literature.

$$\delta\chi_1^{\text{cons}} = 0, \tag{5.19}$$

$$\delta\chi_2^{\text{cons}} = -\frac{\pi}{4} G \frac{m_2^2}{b^2}, \tag{5.20}$$

$$\delta\chi_3^{\text{cons}} = -\frac{4}{3} G^2 \frac{\sigma(1+2\sigma^2)}{(\sigma^2-1)} \frac{m_2^3}{b^3}, \tag{5.21}$$

$$\begin{aligned}\delta\chi_4^{\text{cons}} = \pi G^3 \frac{3m_2^4}{8(\sigma^2-1)b^4} \Bigg\{ &- \left[ 4\mathcal{M}_4^{\text{t}} \log\left(\tfrac{\sqrt{\sigma^2-1}}{2}\right) + \mathcal{M}_4^{\pi^2} + \mathcal{M}_4^{\text{rem}} \right] \\ &+ c_1(\sigma^2-5) + \left( c_2(\bar{\mu}) - \frac{31}{3} + 2\log\left[2b^2 e^{2\gamma_{\text{E}}} \bar{\mu}^2\right] \right) (\sigma^2-1) \Bigg\}. \end{aligned} \tag{5.22}$$

# Solving the field equations

$$\Phi_{lm}(r,t) = \int dr' \int d\omega \int dt' \; e^{-i\omega(t-t')} G_{lm\omega}(r,r') T_{lm}(r',t')$$

# Solving the field equations

$$\Phi_{lm}(r,t) = \int dr' \int d\omega \int dt' \; e^{-i\omega(t-t')} G_{lm\omega}(r,r') T_{lm}(r',t')$$

Plan: PM expand our MST Green function

PM expansion:

- Introduce PM scaling: scale all dimensional variables with impact parameter (Large $b$ equivalent to $M \to 0$)

$$r = b\bar{r}$$

$$\omega = \frac{1}{b}\bar{\omega} \qquad\qquad b \gg M$$

$$t = b\bar{t}$$

$$G = c = 1$$

# Solving the field equations

End up with nasty functions from MST, e.g. $\partial_a^k \, U(a, b, c, \bar{r}\bar{\omega})$

- Also include scaling with velocity (at ∞) $v$ —> PN expansion

$$r = b\bar{r}$$

$$\omega = \frac{v}{b}\bar{\omega} \qquad b \gg M$$

$$t = \frac{b}{v}\bar{t} \qquad v \ll 1$$

i.e. at each PM order, expand also in low velocity.

- PM-PN expanded Green function $(l = m = 1)$:

$$
\frac{\frac{r}{3\,\mathrm{rb}^2} + \left(\frac{r}{6} - \frac{r^3}{30\,\mathrm{rb}^2}\right)\omega^2\, v^2 + O[v]^3}{b}
+ \frac{\left(\frac{2\,r}{3\,\mathrm{rb}^3} - \frac{1}{3\,\mathrm{rb}^2}\right) + \left(-\frac{1}{6} - \frac{r^3}{15\,\mathrm{rb}^3} - \frac{7\,r^2}{30\,\mathrm{rb}^2} + \frac{2\,r}{3\,\mathrm{rb}}\right)\omega^2\, v^2 + O[v]^3}{b^2} +
$$

$$
\frac{\left(\frac{6\,r}{5\,\mathrm{rb}^4} - \frac{2}{3\,\mathrm{rb}^3}\right) + \omega^2\left(-\frac{3\,r^3}{25\,\mathrm{rb}^4} - \frac{7\,r^2}{15\,\mathrm{rb}^3} - \frac{2}{3\,\mathrm{rb}} + \frac{r\left(\frac{1244}{675} - \frac{38\,\mathrm{Log}[r]}{45} + \frac{38\,\mathrm{Log}[\mathrm{rb}]}{45}\right)}{\mathrm{rb}^2}\right) v^2 + O[v]^3}{b^3} +
$$

$$
\frac{\left(\frac{32\,r}{15\,\mathrm{rb}^5} - \frac{6}{5\,\mathrm{rb}^4}\right) + \omega^2\left(-\frac{16\,r^3}{75\,\mathrm{rb}^5} - \frac{21\,r^2}{25\,\mathrm{rb}^4} + \frac{\frac{631}{675} + \frac{38\,\mathrm{Log}[r]}{45} - \frac{38\,\mathrm{Log}[\mathrm{rb}]}{45}}{\mathrm{rb}^2} + \frac{r\left(\frac{1108}{675} - \frac{76\,\mathrm{Log}[r]}{45} + \frac{76\,\mathrm{Log}[\mathrm{rb}]}{45}\right)}{\mathrm{rb}^3}\right) v^2 + O[v]^3}{b^4} +
$$

$$
\frac{\left(\frac{80\,r}{21\,\mathrm{rb}^6} - \frac{32}{15\,\mathrm{rb}^5}\right) + \frac{4\,i\,\omega\, v}{9\,r^2\,\mathrm{rb}^2} + \omega^2\left(-\frac{8\,r^3}{21\,\mathrm{rb}^6} - \frac{112\,r^2}{75\,\mathrm{rb}^5} + \frac{4}{3\,r\,\mathrm{rb}^2} + \frac{\frac{2642}{675} + \frac{76\,\mathrm{Log}[r]}{45} - \frac{76\,\mathrm{Log}[\mathrm{rb}]}{45}}{\mathrm{rb}^3} + \frac{r\left(\frac{614}{875} - \frac{76\,\mathrm{Log}[r]}{25} + \frac{76\,\mathrm{Log}[\mathrm{rb}]}{25}\right)}{\mathrm{rb}^4}\right) v^2 + O[v]^3}{b^5} + O\left[\frac{1}{b}\right]^6
$$

This can be taken to essentially arbitrary PM order.

# GF structure and the field modes

$$G_{lm\omega} = \sum_{k=0} G^{k,0}_{lm\omega}(\bar{r}, \bar{r}', b, v)\bar{\omega}^k + \sum_{k=2} G^{k,1}_{lm\omega}(\bar{r}, \bar{r}', b, v)\bar{\omega}^k \log(\bar{\omega}) + \sum_{k=4} G^{k,2}_{lm\omega}(\bar{r}, \bar{r}', b, v)\bar{\omega}^k \log^2(\bar{\omega}) + \ldots$$

4PM

5PM

Working at 4PM we just need the following distributional Fourier transforms:

$$\int \omega^k e^{i\omega t} d\omega = \frac{2\pi}{i^k}\delta^k(t)$$

$$\int \omega^k e^{i\omega t} \log(\omega) d\omega = \frac{1}{2}\left(\frac{1}{t}\right)_1 - \frac{1}{2}\left(\frac{1}{|t|}\right)_1 - \gamma_{\mathrm{E}}\delta^k(t)$$

# GF structure and the field modes

$$G_{lm\omega} = \sum_{k=0} G^{k,0}_{lm\omega}(\bar{r}, \bar{r}', b, v)\bar{\omega}^i + \sum_{k=2} G^{k,1}_{lm\omega}(\bar{r}, \bar{r}', b, v)\bar{\omega}^k \log(\bar{\omega}) + \sum_{k=4} G^{k,2}_{lm\omega}(\bar{r}, \bar{r}', b, v)\bar{\omega}^k \log^2(\bar{\omega}) + \dots$$

4PM

5PM

Working at 4PM the time-domain field is then:

$$\Phi_{lm}(t, r) = \sum_k \frac{i^k}{k!}\frac{d^k}{d\bar{t}^k}\int d\bar{r}' G^{k,0}_{lm\omega}(\bar{r}, \bar{r}')T_{lm}(\bar{r}', \bar{t}') - i^k \int d\bar{r}' G^{k,1}_{lm\omega}(\bar{r}, \bar{r}')\left[\left(\gamma_E - \log\left(\frac{v}{b}\right)\right)\frac{d^k}{d\bar{t}^k}T_{lm}(\bar{r}', \bar{t}') + F^{(k)}_{lm}(\bar{r}', \bar{t}')\right]$$

Evaluate functions on geodesic worldline with no explicit integrations—‘**easy**’

$$F^{(k)}_{lm} = \int_0^\infty dy \log(y) T^{(k+1)}_{lm}\left(\bar{t} - y, r'\right)$$

*can be reduced to a set of master integrals*

# Results: *preliminary*!!

w/ Adam Pound, Davide Usseglio, Donato Bini, Andrea Geralico

Putting everything together, we can use Barack and Long formulation to calculate e.g. the conservative scattering angle:

$$\delta\chi = \delta\chi^{2\mathrm{PM}} + \delta\chi^{3\mathrm{PM}} + \delta\chi^{4\mathrm{PM}} + \ldots$$

$$\delta\chi^{\mathrm{Cons}}_{2\mathrm{PM}} = -\frac{\pi q^2 M^2}{4b^2},$$

$$\delta\chi^{\mathrm{Cons}}_{3\mathrm{PM}} = -\frac{q^2 M^3}{b^3 v^2}\left(4 + \frac{2v^2}{3} + \frac{5v^4}{6} + O\left(v^6\right)\right),$$

$$\delta\chi^{\mathrm{Cons}}_{4\mathrm{PM}} = \frac{\pi q^2 M^4}{b^4 v^4}\left[\frac{9}{4} + v^2\left(\frac{91}{24} + \frac{21\pi^2}{128} - \log(2) + \log(v)\right) + v^4\left(\frac{493}{480} + \frac{4335\pi^2}{8192} - 2\log(2) - \frac{3\log(b/M)}{2} + 2\log(v)\right) + O\left(v^6\right)\right]$$

— free from any undetermined constants.

# Results: *preliminary*!!

Compare our result with Barack, Bern et al, at 4PM we have:

$$\delta\chi^{\text{Cons}}_{4\text{PM}} = \frac{\pi q^2 M^4}{b^4 v^4}\left[\frac{9}{4} + v^2\left(\frac{91}{24} + \frac{21\pi^2}{128} - \log(2) + \log(v)\right) + v^4\left(\frac{493}{480} + \frac{4335\pi^2}{8192} - 2\log(2) - \frac{3\log(b/M)}{2} + 2\log(v)\right) + O\left(v^6\right)\right]$$

Expanding in low $v$, their result gives:

$$v^0: \quad \frac{9}{4} \quad \checkmark$$

$$v^2: \quad \left(-\frac{3c_1}{2} - \frac{85}{24}\right) - \log(v) - \frac{21\pi^2}{128} + \log(2) \quad \Longrightarrow \quad c_1 = \frac{1}{6}$$

$$v^4: \quad -\frac{313}{480} + \frac{15c_1}{8} + \frac{3c_2(\mu)}{8} + \frac{3\log(b\mu)}{2} - 2\log(v) - \frac{4335\pi^2}{8192} + \frac{3\gamma_{\text{E}}}{2} + \frac{11\log(2)}{4}$$

$$\Longrightarrow \quad c_2(\mu) = -\frac{11}{6} - 4\gamma_E - 2\log(2\mu^2 M^2)$$

# Results: *preliminary*!!

$$c_1 = \frac{1}{6} \qquad c_2(\mu) = -\frac{11}{6} - 4\gamma_E - 2\log(2\mu^2 M^2)$$

$$S^{\text{tidal}} = G^3 \int d^D x \sqrt{-\text{g}} \Big[ (4\pi c_1) \left[ m_2^2 (\partial_\mu \phi_2 \partial^\mu \psi)^2 - m_2^4 \phi_2^2 (\partial_\mu \psi)(\partial^\mu \psi) \right] + (4\pi c_2^{\text{bare}})\, m_2^2 \,(\partial_\mu \phi_2 \partial^\mu \psi)^2 \Big] + \mathcal{O}(G^4), \qquad (3.11)$$

$c_1$ — scalar equivalent of static love number ~scalarizability?

$c_2$ — related to UV divergence

**Gravitational Raman Scattering in Effective Field Theory: a Scalar Tidal Matching at $\mathcal{O}(G^3)$**

Mikhail M. Ivanov,[1,*] Yue-Zhou Li,[2,†] Julio Parra-Martinez,[3,‡] and Zihan Zhou[2,§]

$$S_{\text{fs}}^{\text{ct}} = \sum_\ell \frac{1}{2\ell!} \int d\tau \Big[ C_\ell (\boldsymbol{\partial}_L \phi)^2 + C_{\ell,\omega^2} (\boldsymbol{\partial}_L \dot{\phi})^2 + \cdots \Big] \quad (6)$$

$$= \frac{1}{2} \int d\tau \Big[ C_1 (\boldsymbol{\partial}\phi)^2 + C_{0,\omega^2} \dot{\phi}^2 + C_{1,\omega^2} (\boldsymbol{\partial}\dot{\phi})^2 + \cdots \Big],$$

can we easily relate the coefficients?

# Takeaways

- 1 post-Adiabatic effects may be modelled well by analytic approximations for sizeable portions of the parameter space

- Still more places to meet, e.g. Precession effects? can the road to non-linear SF (analytic or not) be made more efficient using other methods?

- SF can now give PM expansions. GR version should come soon. What can we do with these?
  - low-$\nu$ to all-orders
  - GR wilson coefficients
  - B-2-B mappings in SF
  - how bad will our master integrals get..?
  - higher power logs seem doable, are there surprises waiting?
  - WQFT have 5PM 1SF!! This will be a big target of comparison. (arXiv:2403.07781)

*Thanks for listening*

# extra slide: Master Integrals

$$F^{(k)}_{lm} = \int_0^\infty dy \log(y) T^{(k+1)}_{lm}\left(\bar{t} - y, r'\right)$$

$$I_1 = \int du \frac{\text{ArcSinh}(u)}{(t+u)^k \sqrt{1+u^2}} \qquad \to \left\{\text{ArcSinh}[t],\ \text{ArcTan}[t],\ \text{PolyLog}\left[2, \frac{t+\sqrt{1+t^2}}{t-\sqrt{1+t^2}}\right]\right\}$$

$$I_2 = \int du \frac{\text{ArcSinh}^2(u)}{(t+u)^k}$$

$$I_2 = \int du \log(1+u^2) u^k$$