# Gravitational Self Force from Scattering Amplitudes in Curved Space

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Département de Physique Théorique, Université de Genève

Based on 2308.15304 with Mikhail Solon

Gravitational Self-Force and Scattering Amplitudes - March 21, 2024

# Gravitational Waves and Amplitudes

Artistic Impression
Credit: MARK GARLICK / SCIENCE PHOTO / Getty images

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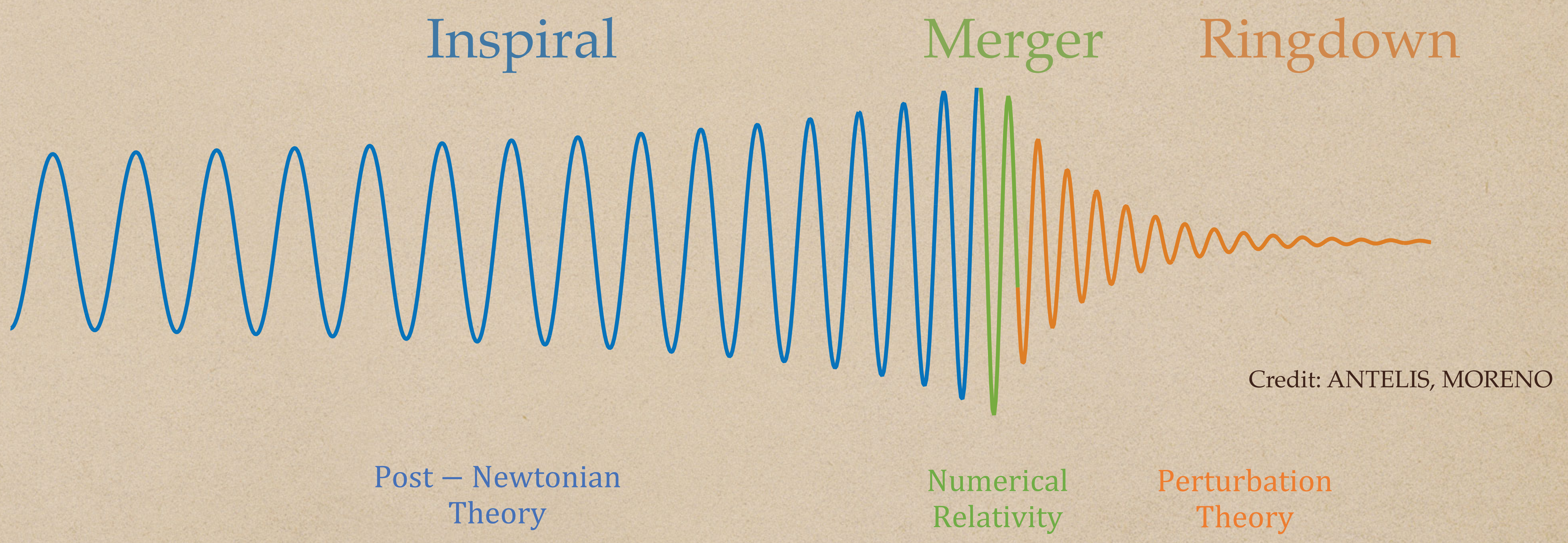

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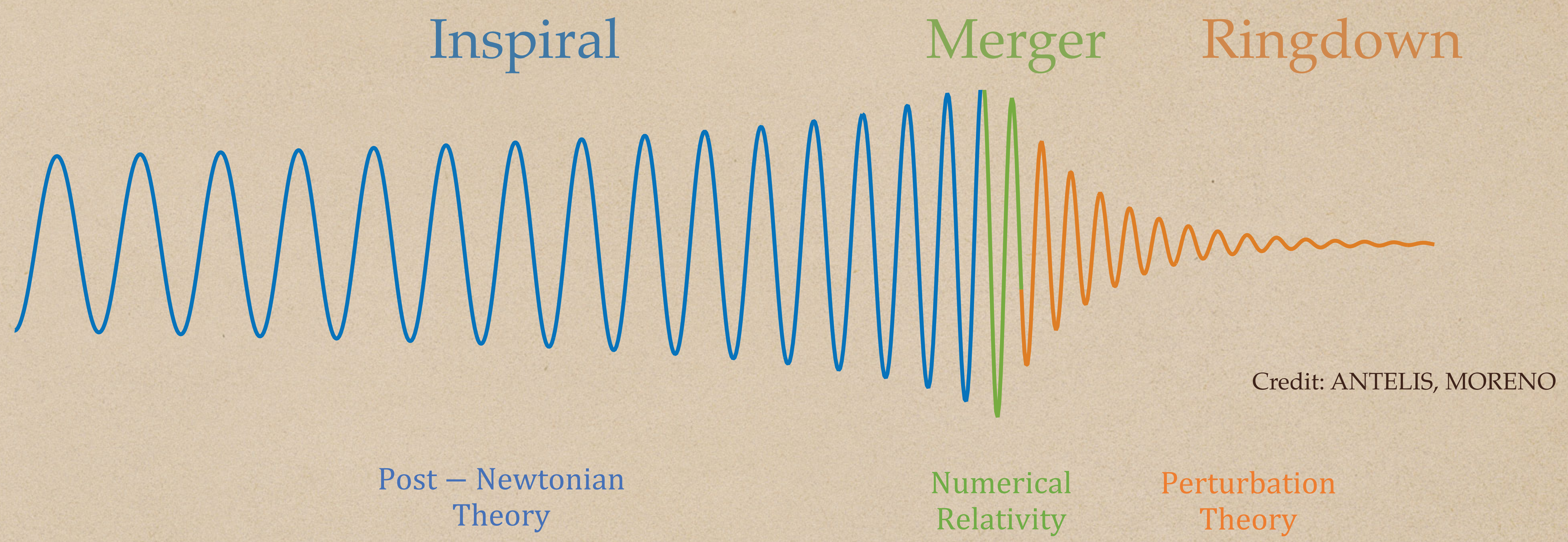
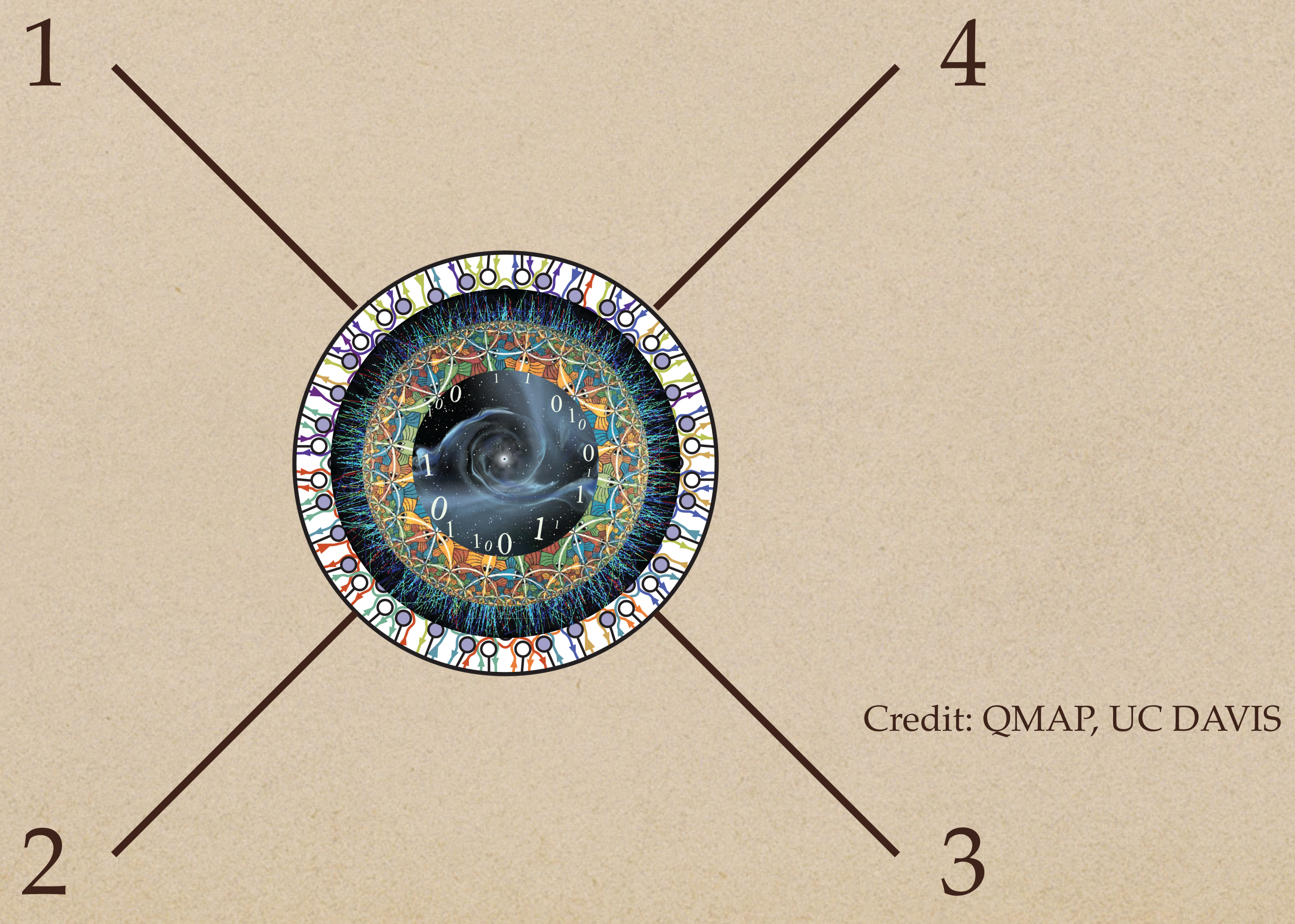



$$\mathcal{A}_4 \sim {}_{\rm out}\langle 34|12\rangle_{\rm in} \sim$$



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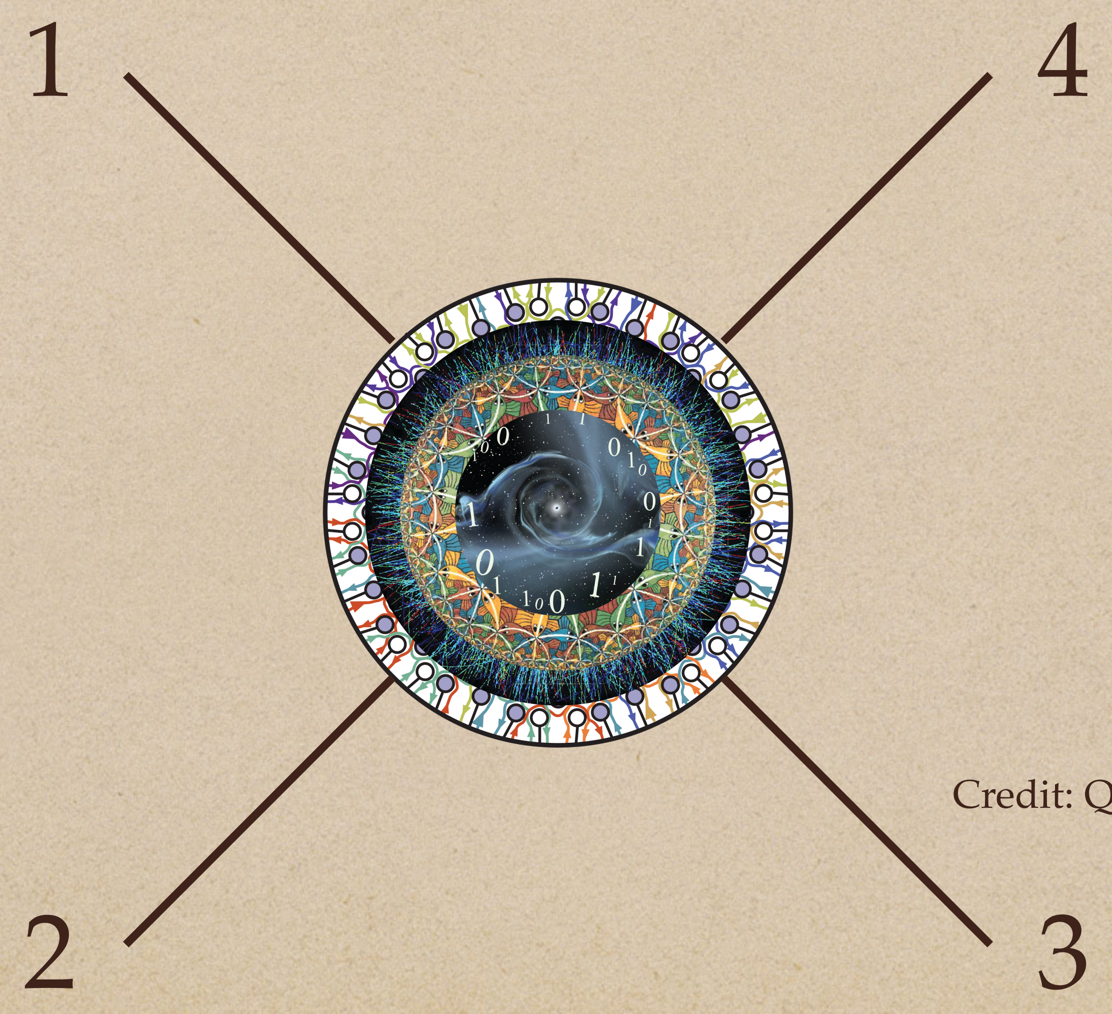


Credit: QMAP, UC DAVIS

# Gravitational Waves and Amplitudes in Curved Space

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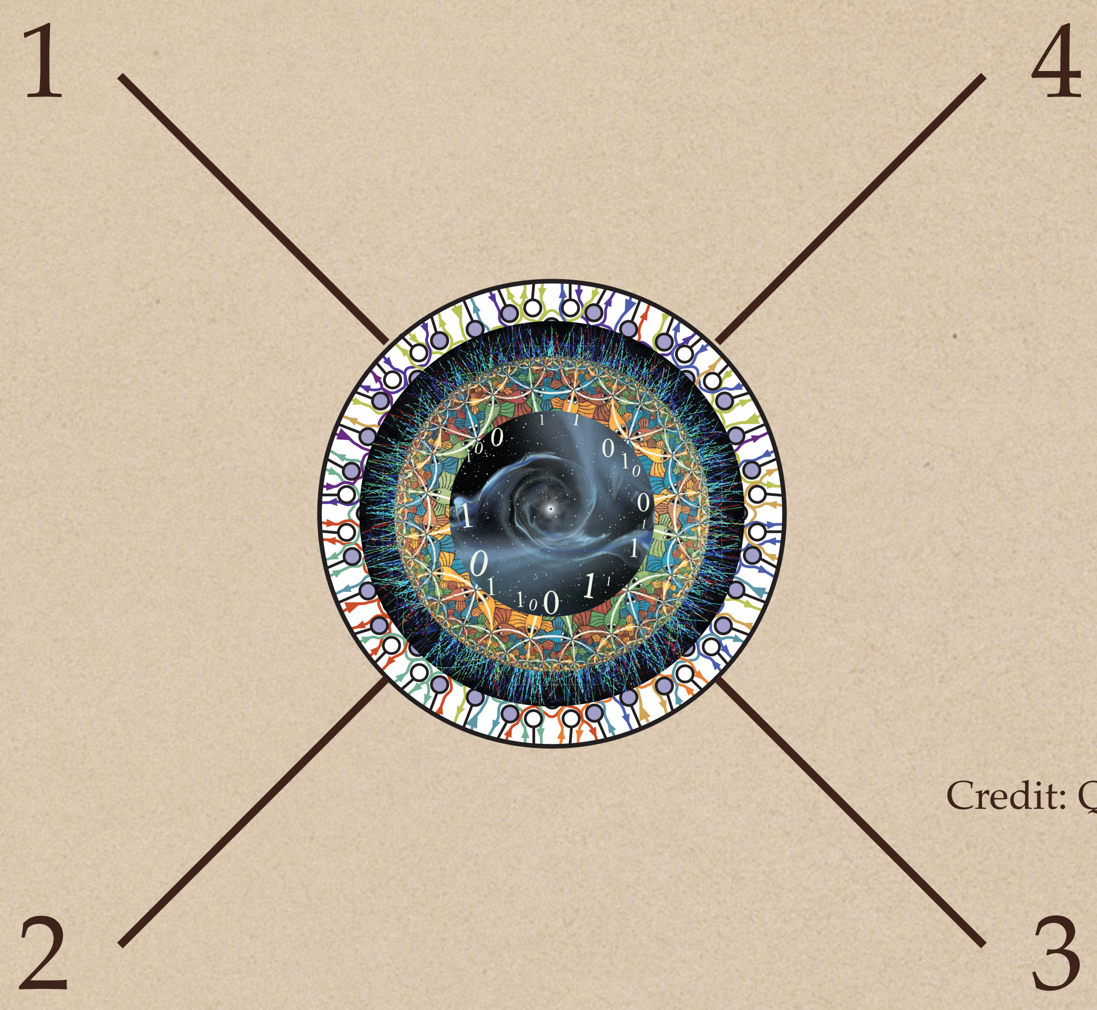


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## General Relativity

Black Holes

$$\bar{g}_{\mu\nu}dx^\mu dx^\nu = (1 - \frac{2GM}{r})dt^2 - (1 - \frac{2GM}{r})^{-1}dr^2 - r^2 d\Omega^2$$

## Quantum Field Theory

Point Particles

$$\mathscr{L} = \frac{1}{2}\left(\partial\phi\right)^2 - \frac{1}{2}m^2\phi^2$$

Incorporating the Black Hole Metric in a QFT Framework

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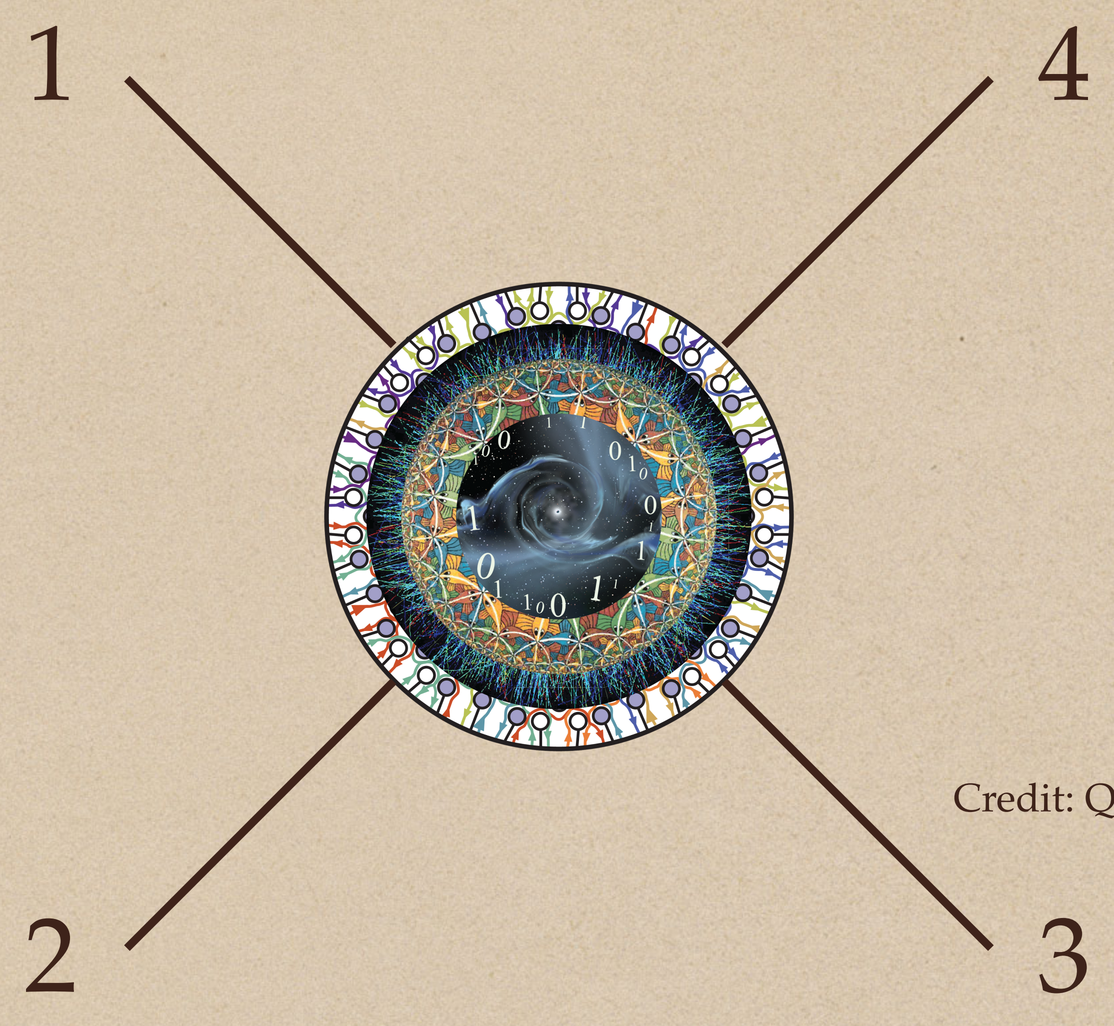


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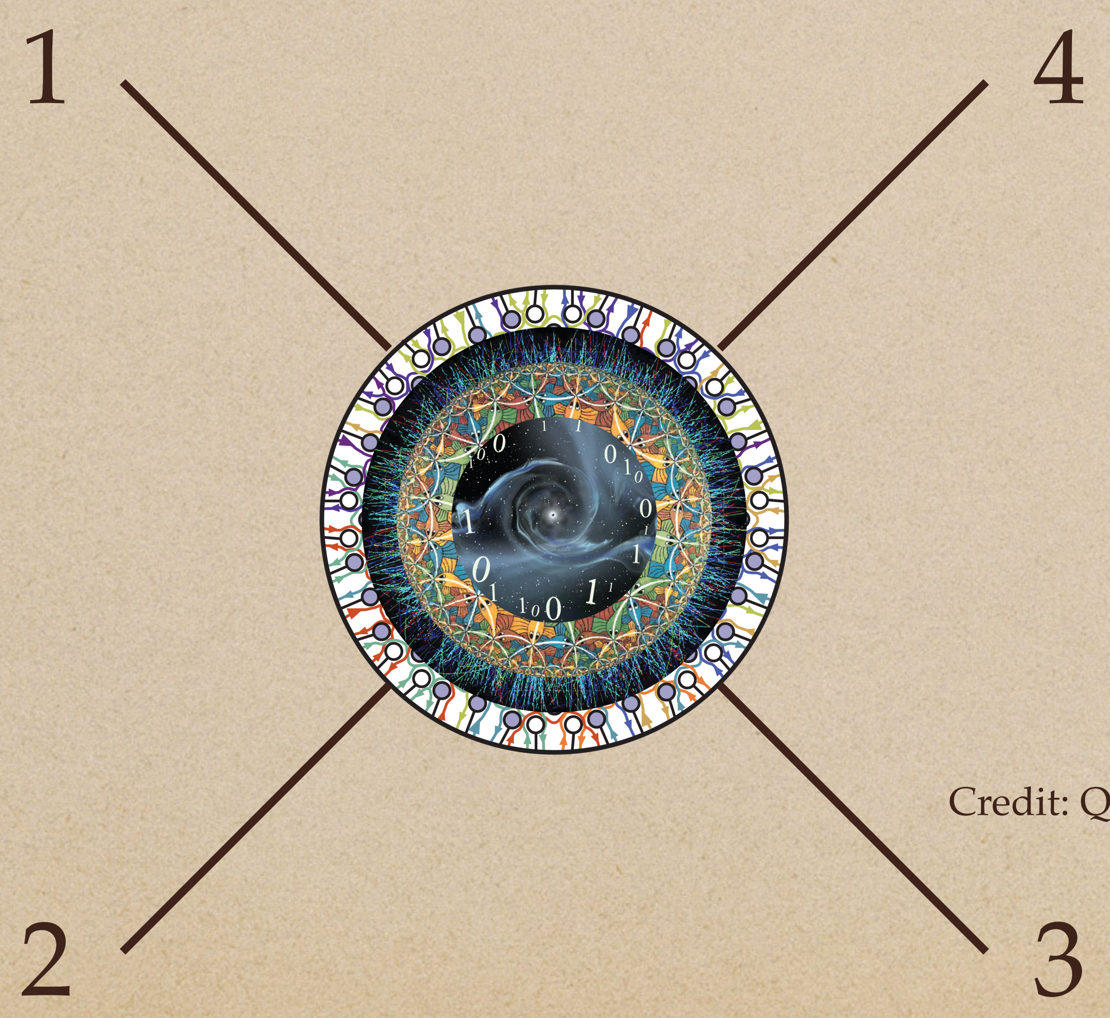


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Quantum Field Theory around a Black Hole


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# Outline

- Motivation
- From Quantum Amplitudes to Classical Hamiltonians
- Quantum Field Theory around a Black Hole
- Outlook

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See also [Cheung, Parra-Martinez, Rothstein, Shah and Wilson-Gerow]

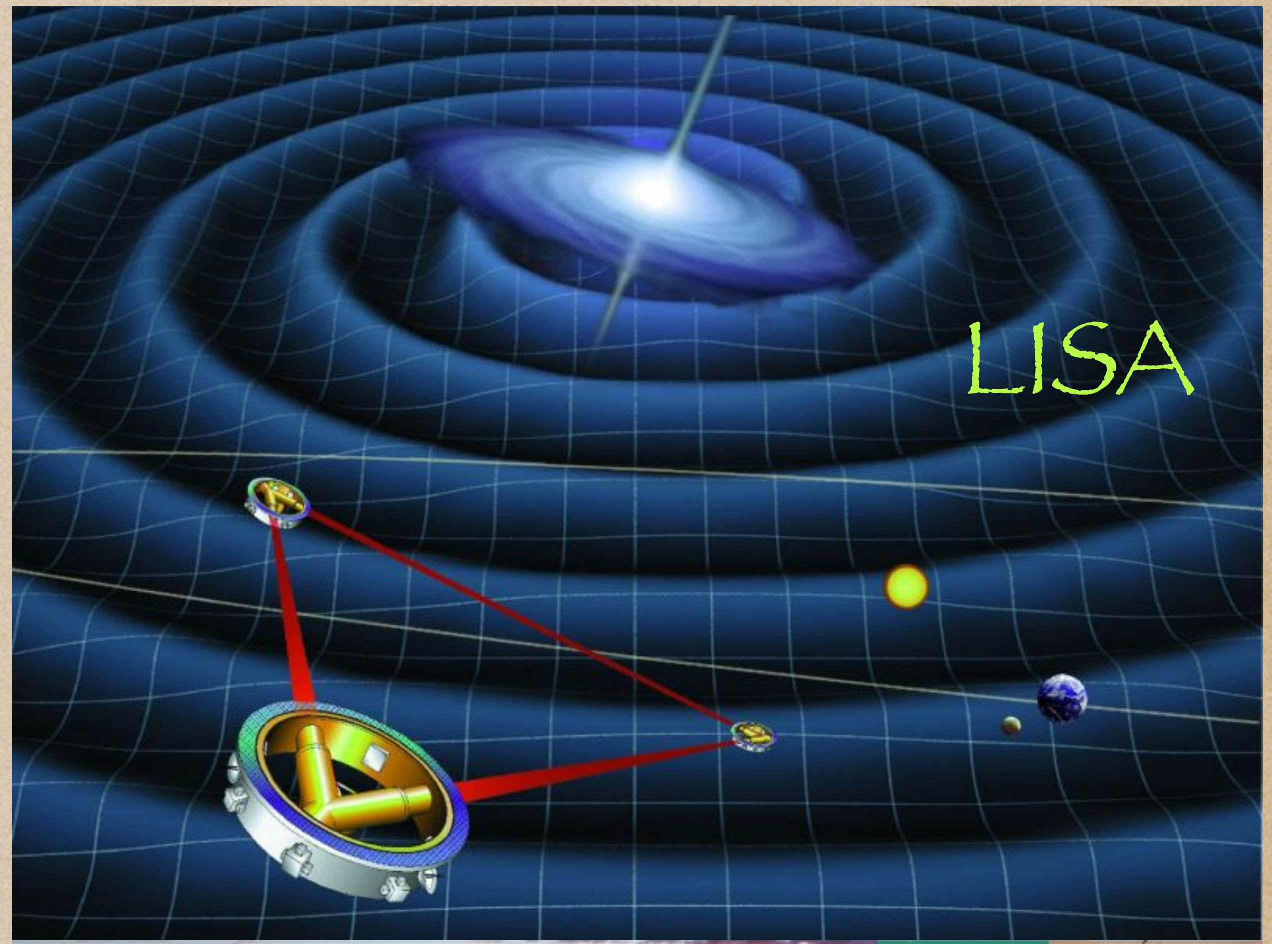
and talk by Cliff Cheung

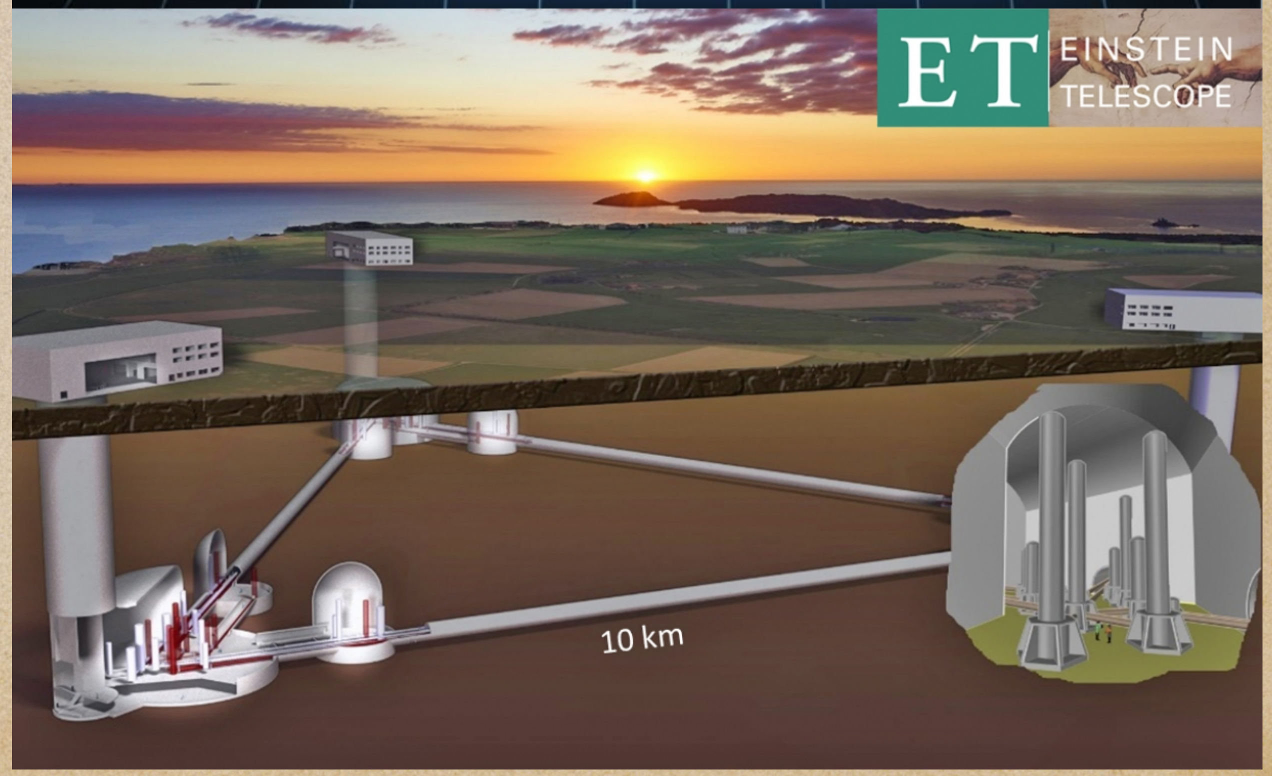
- Outlook

# Era of Gravitational Wave Astronomy

Present Experiments


Future Experiments




And More!

# Gravitational-Wave Science

- Nuclear & Atomic Physics
- Dark Matter & Axions
- Cosmology & The Primodial Universe
- Black-Hole Horizons & Singularities
- Exotic QCD Matter (Neutron-Star Mergers)
    - Neutron-Star Equation of State
    - QCD Phase Diagram

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# Gravitational Wave Analysis

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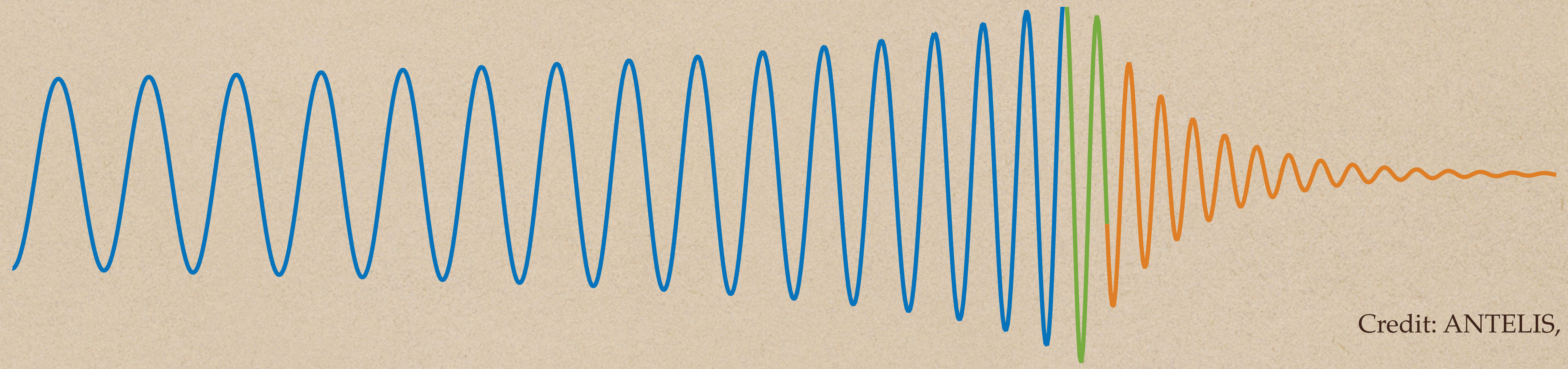

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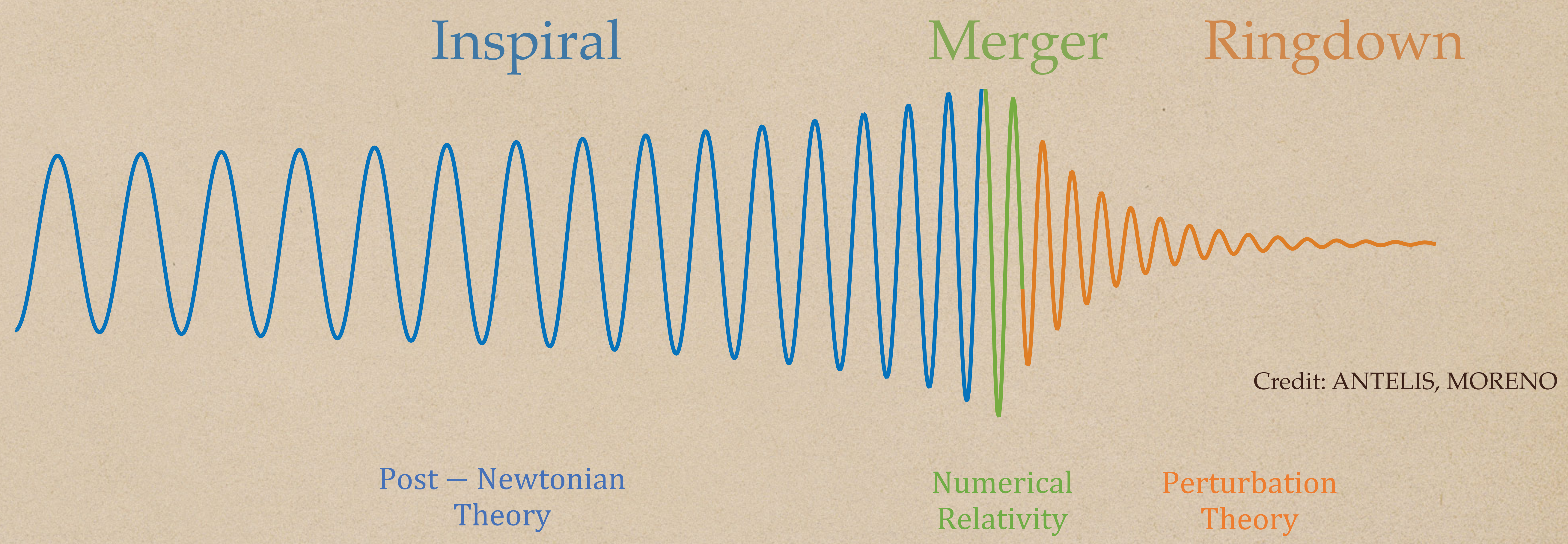


Quadruple radiation:

$$h \sim \frac{G}{r}\ddot{Q}$$

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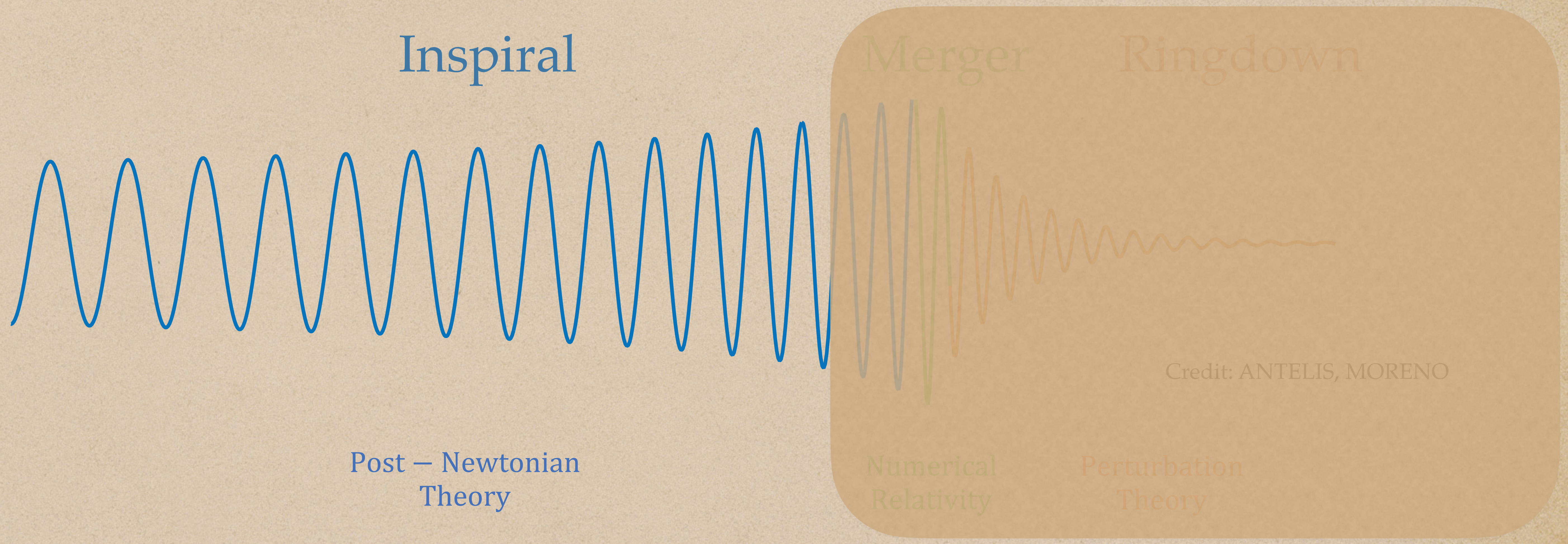


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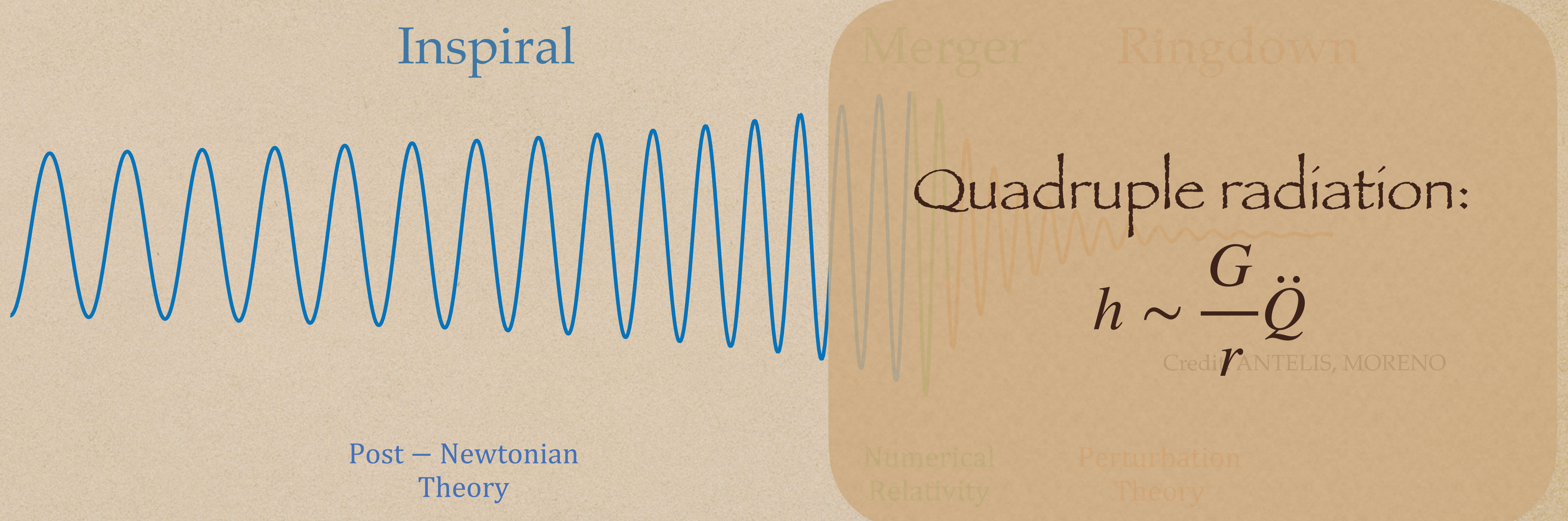


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Adiabatic Approximation →

- Conservative motion via a potential $V$
- Energy loss due to radiation

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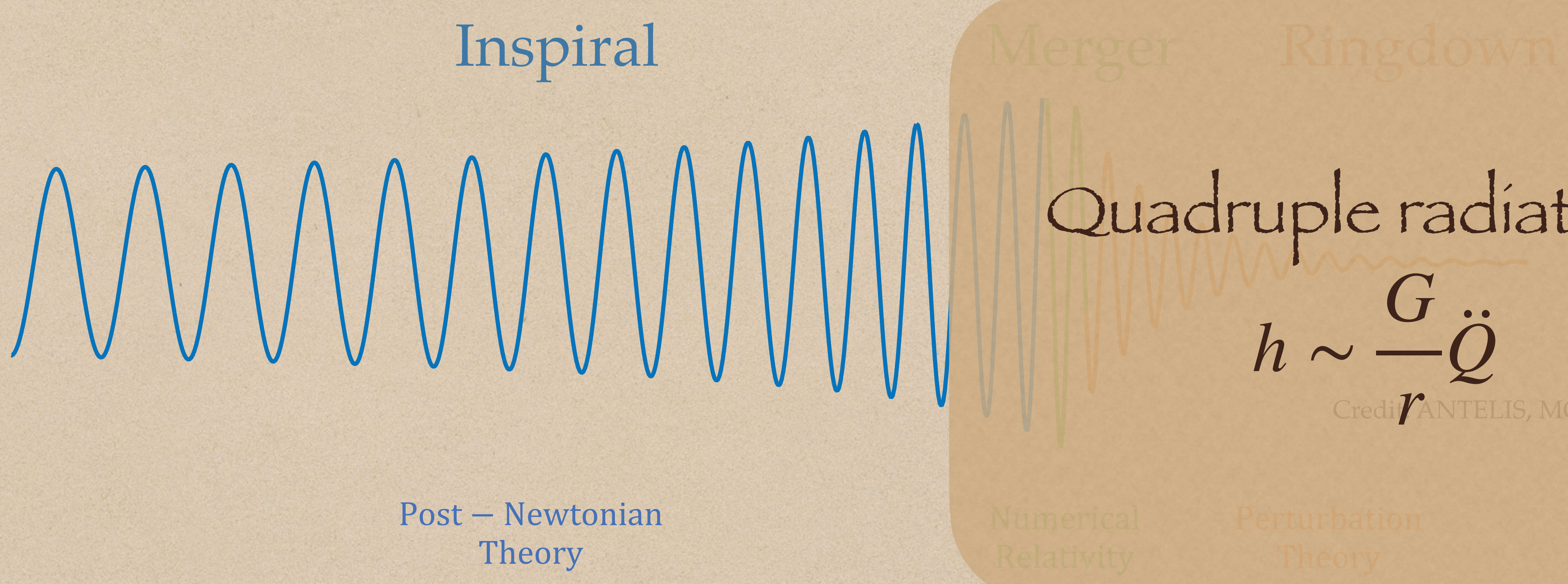


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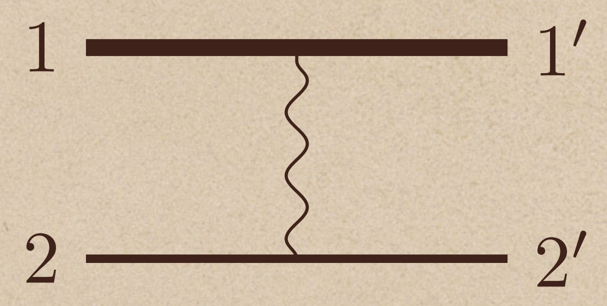
Adiabatic Approximation

Conservative motion via a potential $V$

Energy loss due to radiation

# From Quantum Amplitudes to Classical Hamiltonians

# Amplitude to Potential: Tree-Level Matching

$\mathcal{A}_4 \sim$ 

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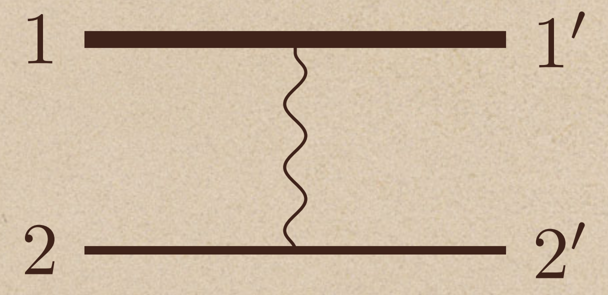
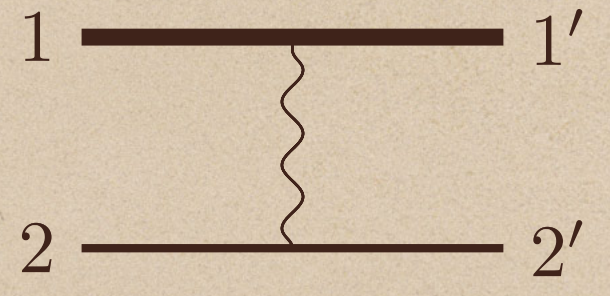



$$\sim G\frac{p_1^{\mu} p_1^{\nu} \mathcal{P}_{\mu\nu\alpha\beta} p_2^{\alpha} p_2^{\beta}}{q^2}$$

# Amplitude to Potential: Tree-Level Matching

$\mathscr{A}_4 \sim$ 


$$\sim G\frac{p_1^\mu p_1^\nu \mathscr{P}_{\mu\nu\alpha\beta} p_2^\alpha p_2^\beta}{q^2} \sim E_1 E_2 \left(\frac{G\, n(\mathbf{p})}{\mathbf{q}^2}\right)$$

(CoM)

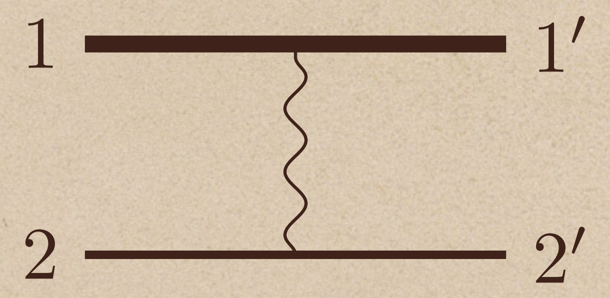
(CoM)

$p_1 = (E_1, \mathbf{p})$

$p_2 = (E_2, -\mathbf{p})$

# Amplitude to Potential: Tree-Level Matching

$\mathscr{A}_4 \sim$




$$\sim G\frac{p_1^\mu p_1^\nu \mathscr{P}_{\mu\nu\alpha\beta}p_2^\alpha p_2^\beta}{q^2} \sim E_1E_2\left(\frac{G\,n(\mathbf{p})}{\mathbf{q}^2}\right) \sim E_1E_2V(\mathbf{p},\mathbf{q})$$

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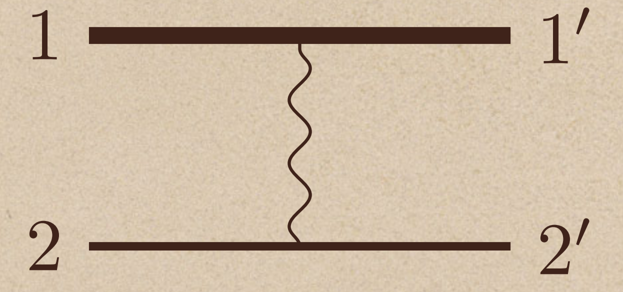


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(CoM)

1PM Potential

$$V(\mathbf{p}, \mathbf{r}) = -\frac{G\, n(\mathbf{p})}{r}, \quad n(\mathbf{p}) = m_1 m_2 + \mathscr{O}\left(\mathbf{p}^2\right)$$

(CoM)

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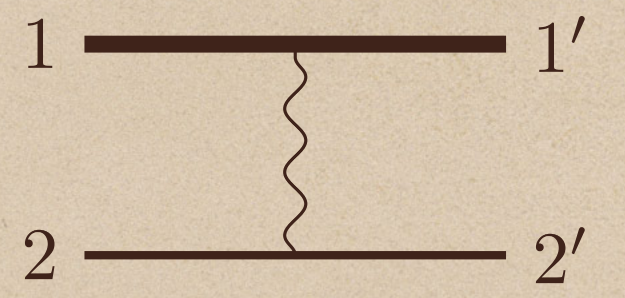
# Amplitude to Potential: Tree-Level Matching

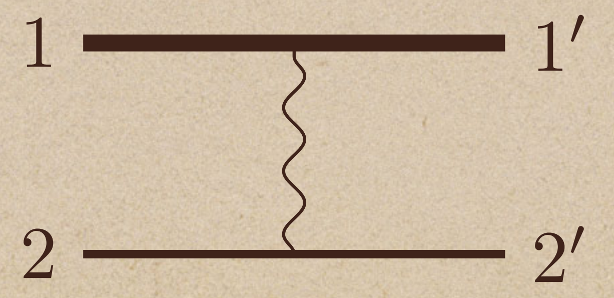
$\mathscr{A}_4 \sim$ 

$\sim G\dfrac{p_1^\mu p_1^\nu \mathscr{P}_{\mu\nu\alpha\beta}p_2^\alpha p_2^\beta}{q^2} \sim E_1E_2\left(\dfrac{G\,n(\mathbf{p})}{\mathbf{q}^2}\right) \sim E_1E_2V(\mathbf{p},\mathbf{q})$

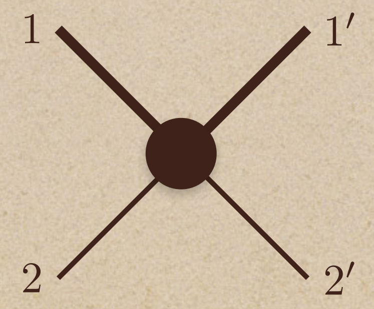
(CoM)

1PM Potential $\quad V(\mathbf{p},\mathbf{r}) = -\dfrac{G\,n(\mathbf{p})}{r}, \quad n(\mathbf{p}) = m_1m_2 + \mathcal{O}\left(\mathbf{p}^2\right)$

Full Theory

$\mathscr{A}_4^{\text{F.T.}} \sim$ 


Effective Theory

$\mathscr{A}_4^{\text{EFT}} \sim$ 

$\sim V(\mathbf{p},\mathbf{q})$

(CoM)

$p_1 = \left(E_1, \mathbf{p}\right)$

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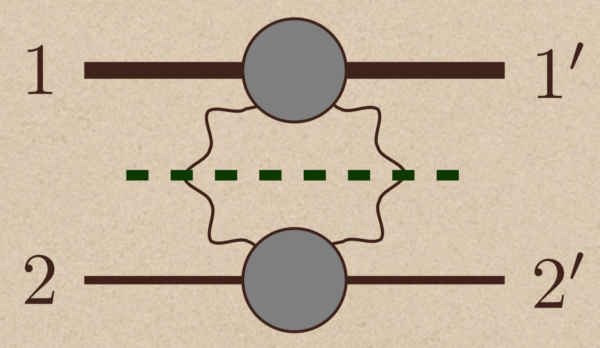
# Amplitude to Potential: Loop-Level Matching

Full Theory

$\mathscr{A}_4^{\mathrm{F.T.}} \sim$ 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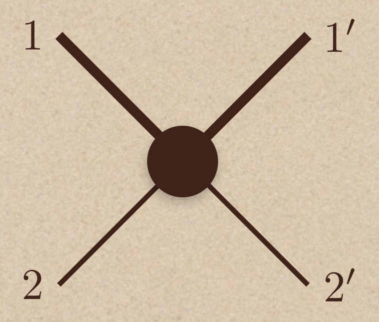
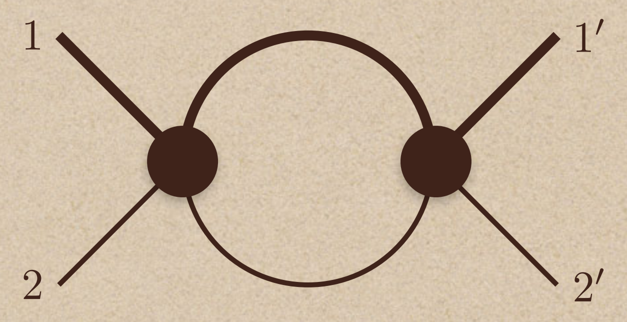
$\mathscr{A}_4^{\mathrm{EFT}} \sim$ 

$+$ 

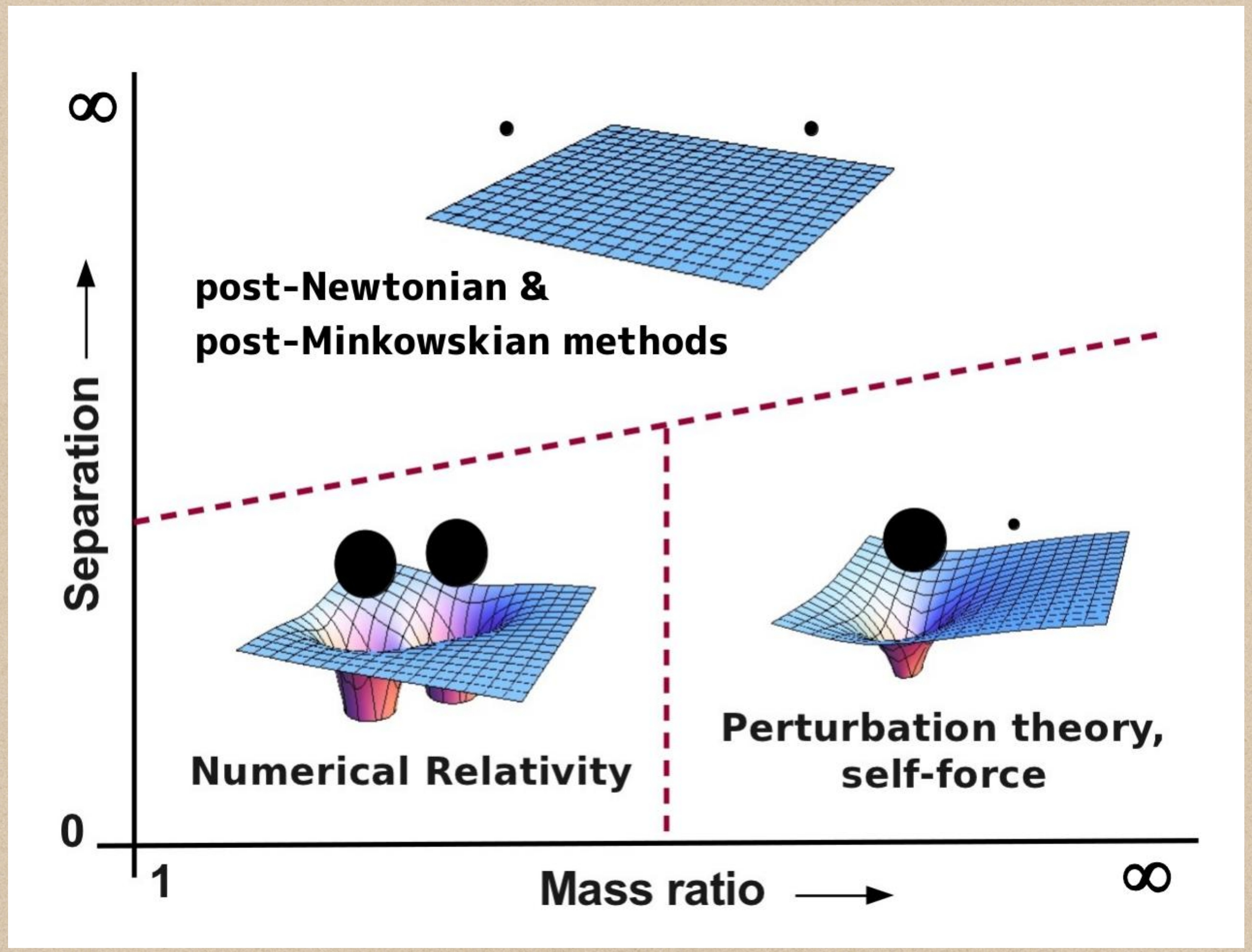
$+ \quad \dots$

2PM Potential $\quad \delta V(\mathbf{p}, \mathbf{r}) = -\dfrac{G^2\, \tilde{n}(\mathbf{p})}{r^2}$

[Cheung, Rothstein, Solon]

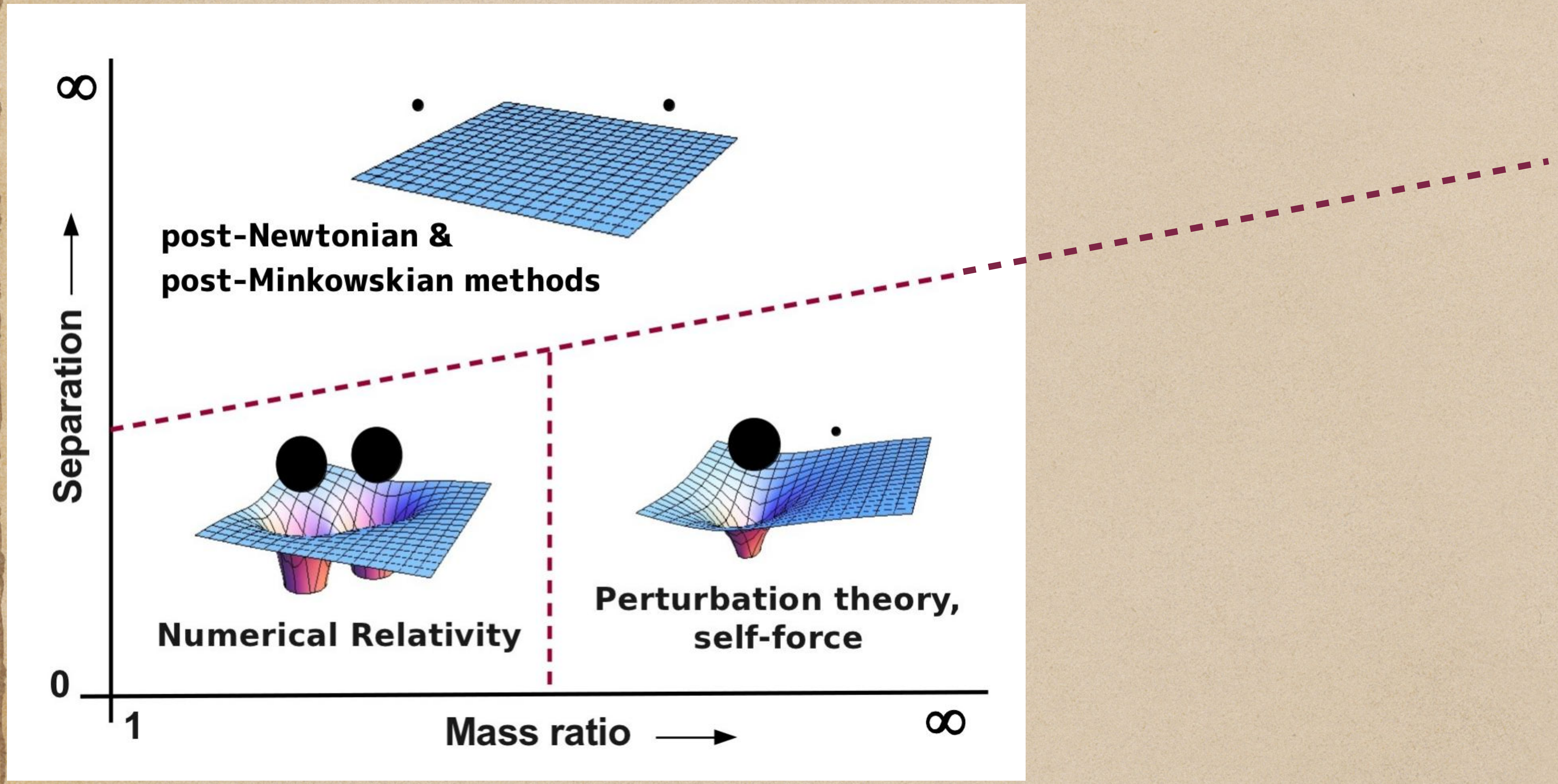
# Quantum Field Theory around a Black Hole

# Approaches to the Two-Body Problem




Credit: BARACK & POUND

# Approaches to the Two-Body Problem

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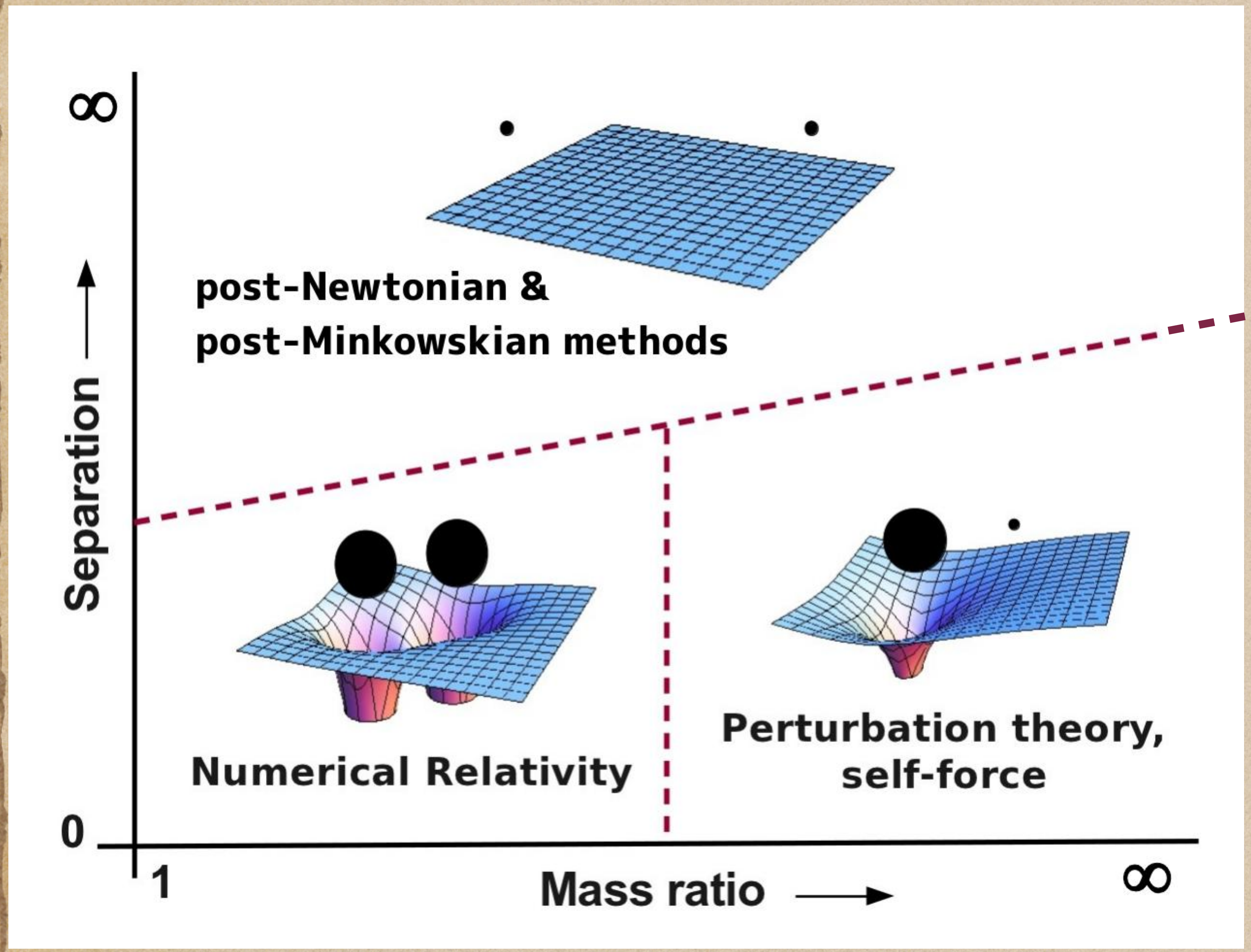
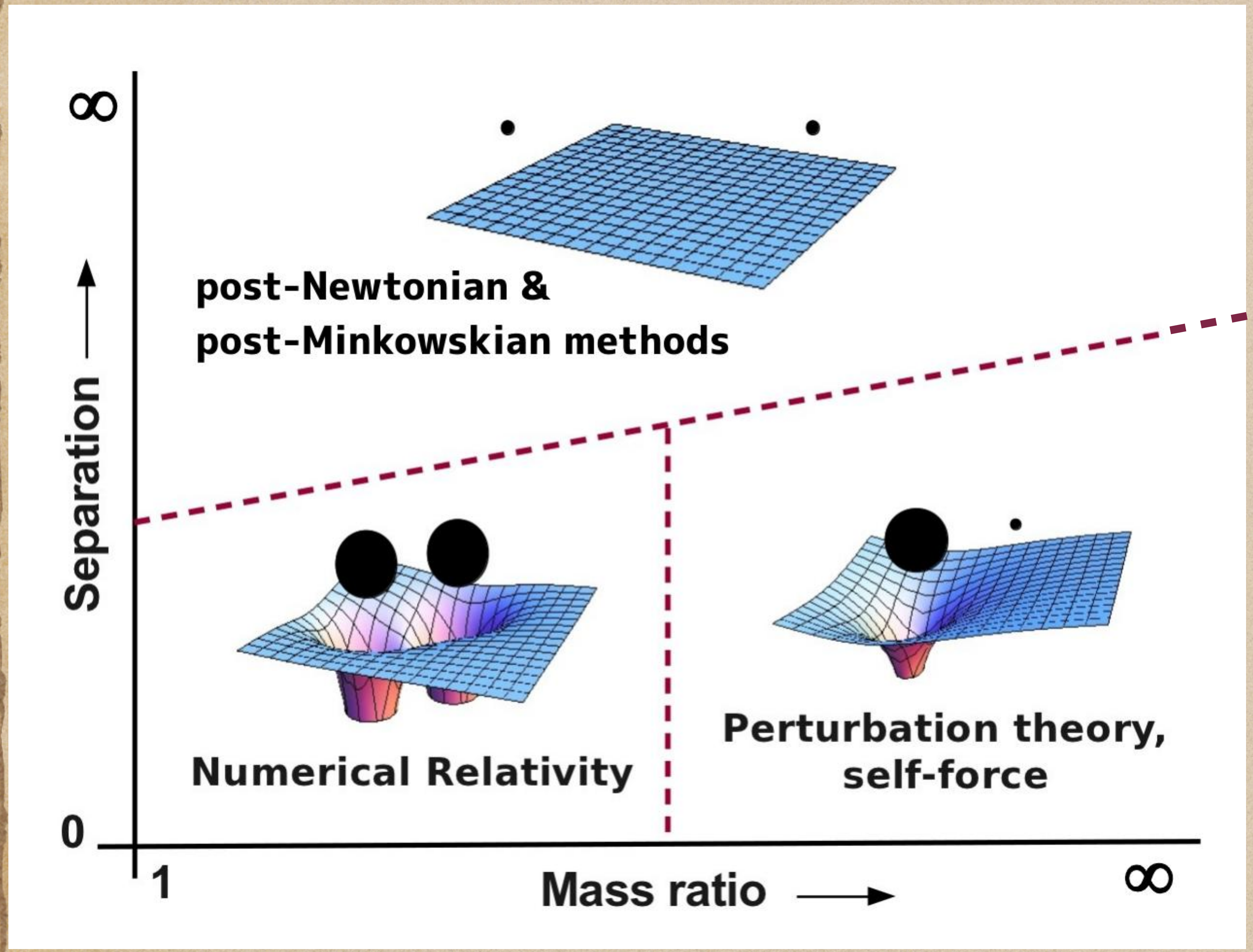



Perturbative in G

Exact in G

Credit: BARACK & POUND

# Approaches to the Two-Body Problem

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Analytical

Perturbative in G

Numerical

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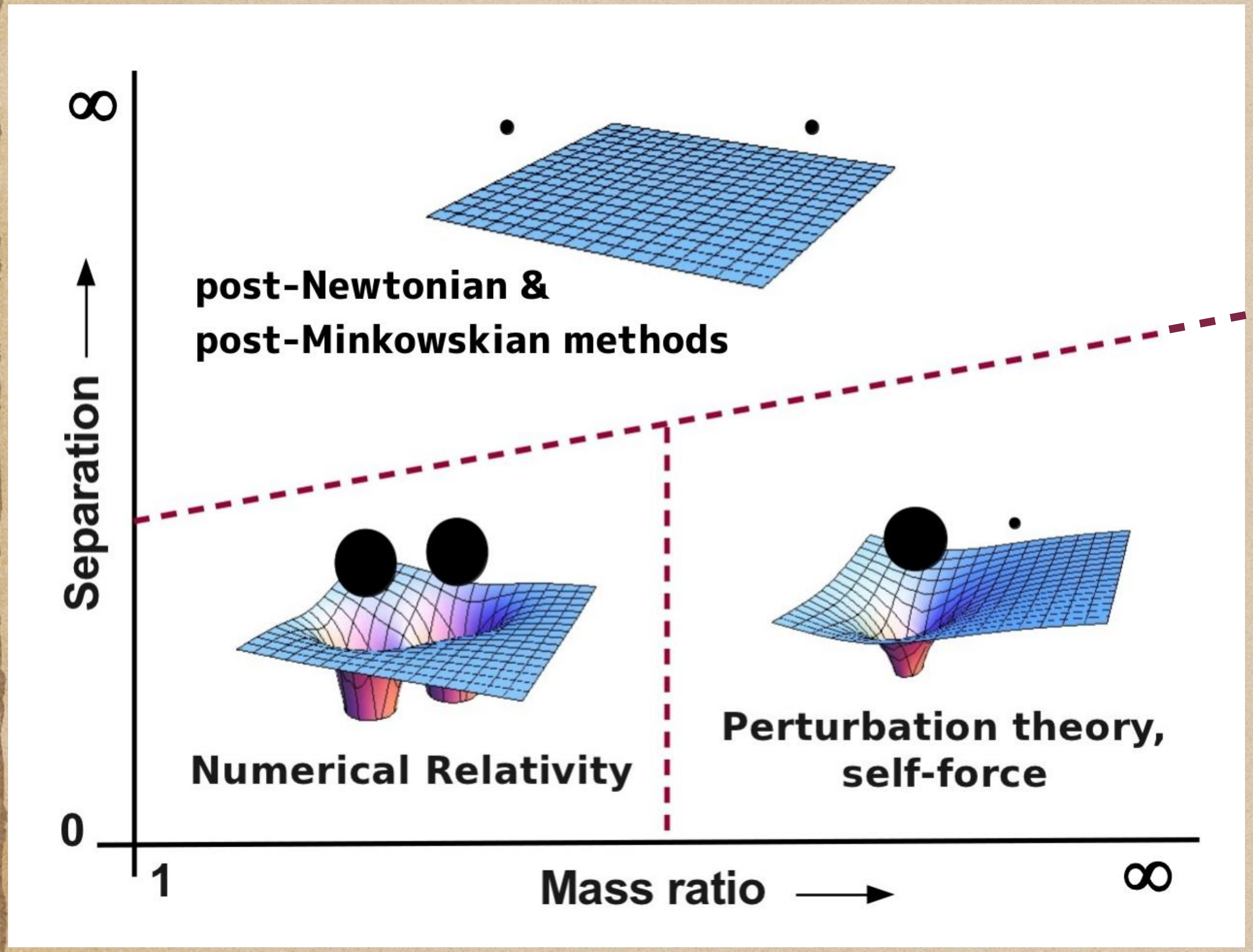
# Approaches to the Two-Body Problem

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Perturbative in G

Numerical

Exact in G

Can we inform the analytical methods with ideas from self force?

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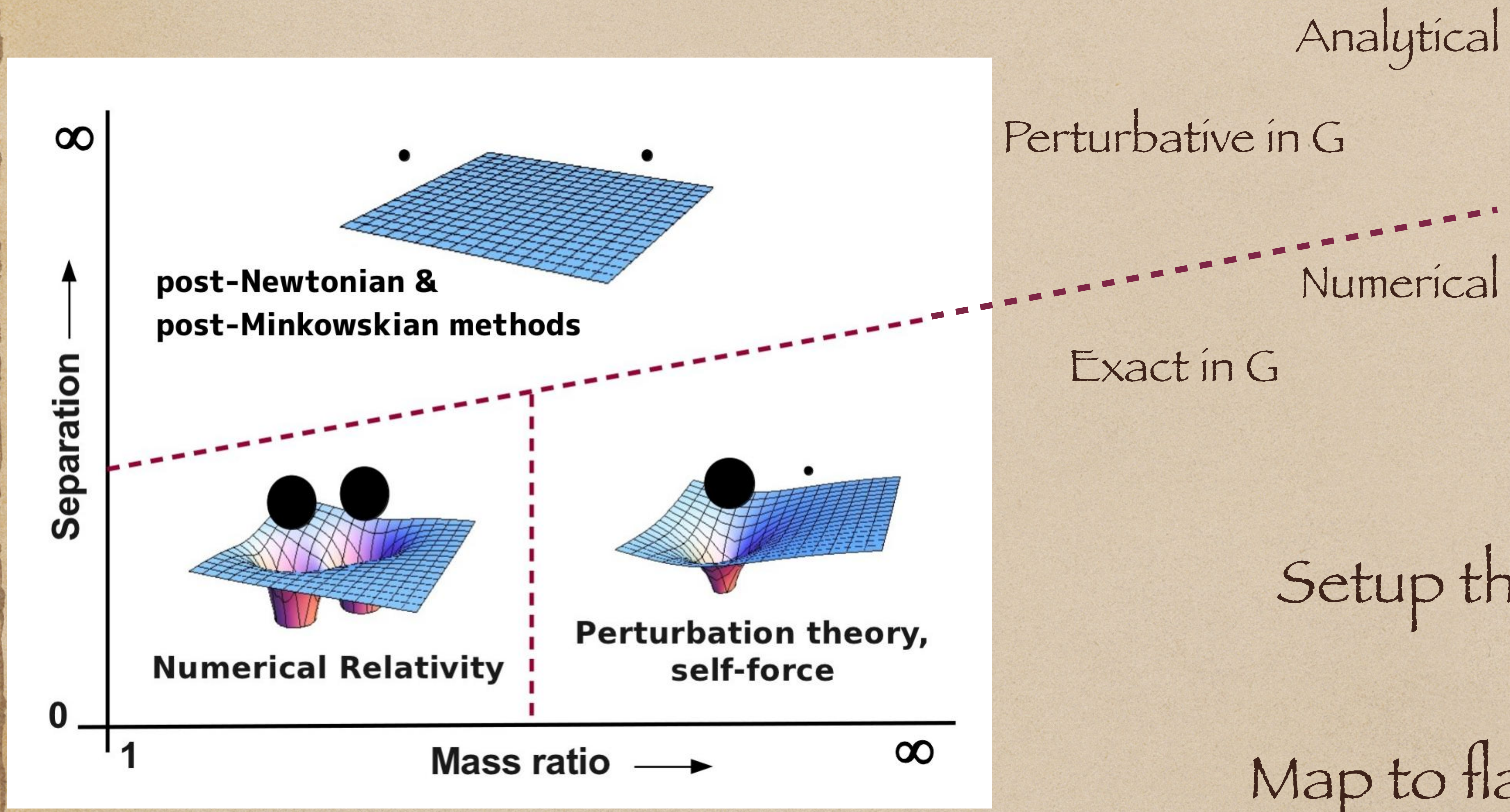


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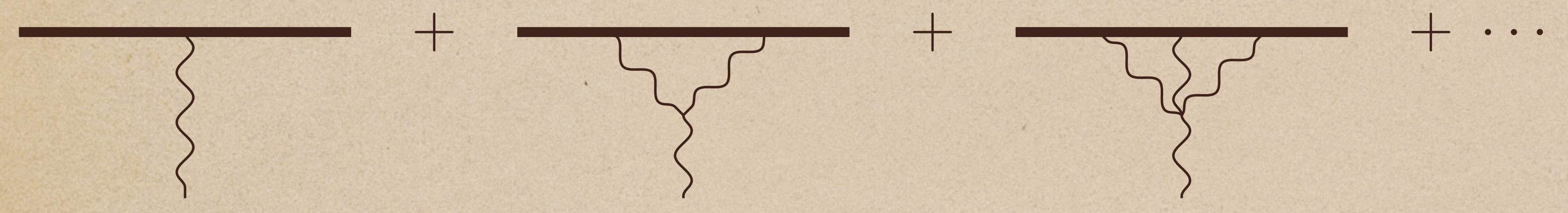
Can we inform the analytical methods with ideas from self force?

Strategy:

Setup theory in curved space → retain partially exact-in-G information

Map to flat space → use analytical amplitudes toolkit

# Harnessing the Information in the Metric



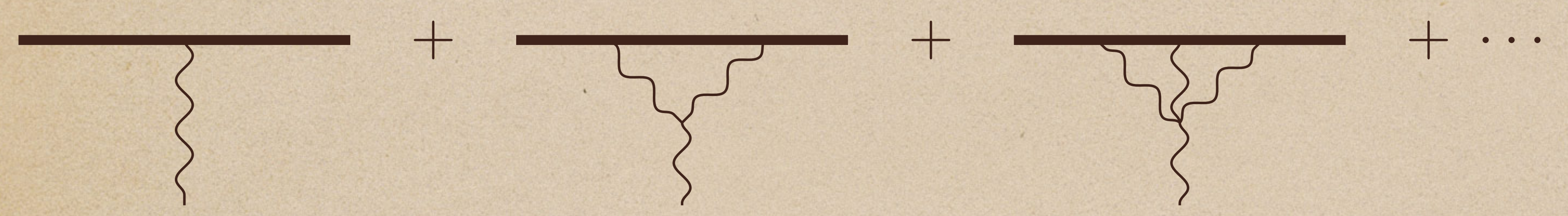
$\approx \bar{g}_{\mu\nu}$

$$\bar{g}_{\mu\nu} dx^\mu dx^\nu = (1 - \frac{2GM}{r}) dt^2 - (1 - \frac{2GM}{r})^{-1} dr^2 - r^2 d\Omega^2$$

[Duff]

Systematic framework for incorporating the all-orders-in-$G$ information in the metric

# Harnessing the Information in the Metric



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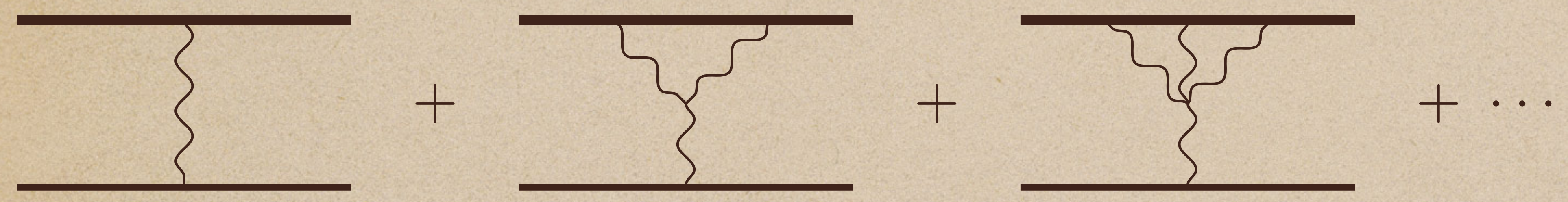

$\leftrightarrow$ Geodesic Motion

✓

Systematic framework for incorporating the all-orders-in-$G$ information in the metric

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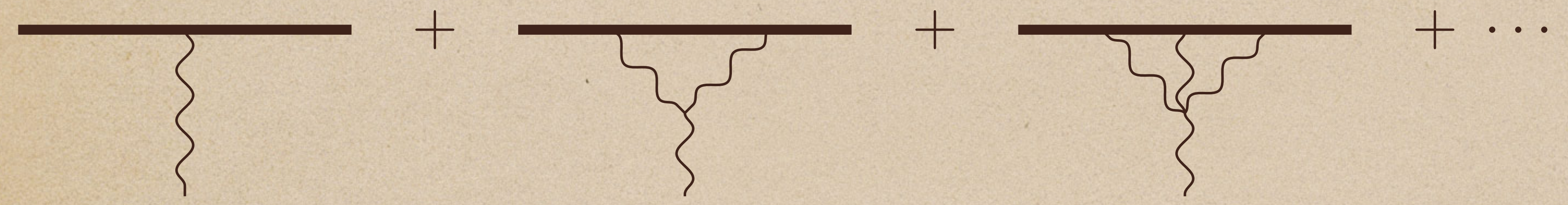

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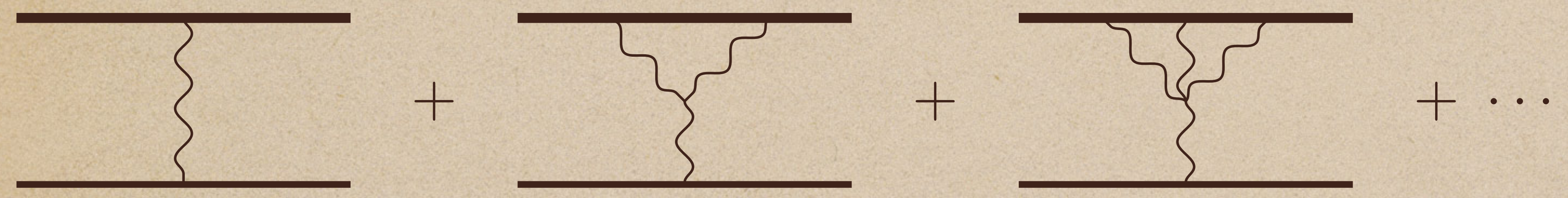

$\leftrightarrow$ Geodesic Motion

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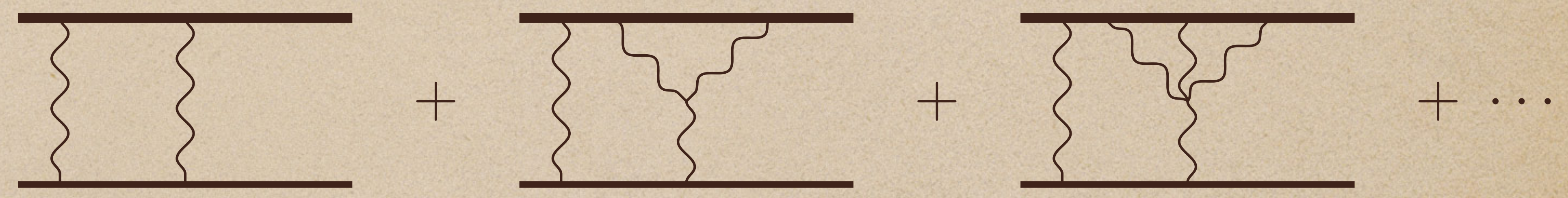

$\leftrightarrow$ Beyond Geodesic Motion

?

*Systematic framework for incorporating the all-orders-in-$G$ information in the metric*

# The Self-Force Nomenclature

Geodesic Motion $\leftrightarrow$ 0SF

$n$th Correction Beyond Geodesic Motion $\leftrightarrow$ $n$SF $\leftrightarrow$ Suppressed by $(m/M)^n$

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Geodesic Motion ↔ 0SF

$n$th Correction Beyond Geodesic Motion ↔ $n$SF ↔ Suppressed by $(m/M)^n$

$$\begin{aligned}\mathcal{A}_4 = &\, G\ M^2 m^2\ \ a_{1,0} \\ &+ G^2 M^2 m^2 \left(a_{2,0} M \ \ + a_{2,1} m\right) \\ &+ G^3 M^2 m^2 \left(a_{3,0} M^2 + a_{3,1} M m + a_{3,2} m^2\right) \\ &\vdots\end{aligned}$$

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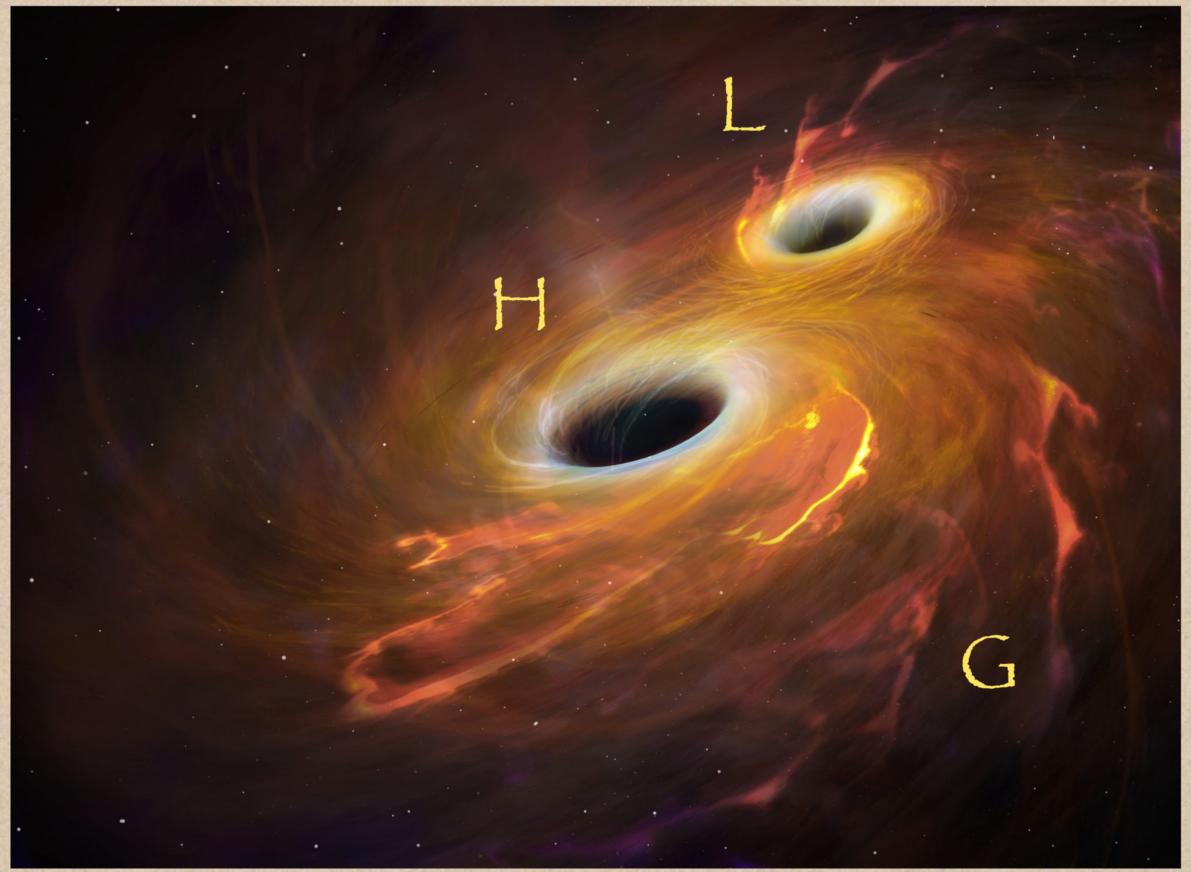
0SF 1SF 2SF

PM: Post Minkowski

SF: Self Force

# QFT in Curved Space: The Setup



Artistic Impression
Credit: MARK GARLICK / SCIENCE PHOTO / Getty images

$$S = S_G + S_L + S_H$$

The Action for our System

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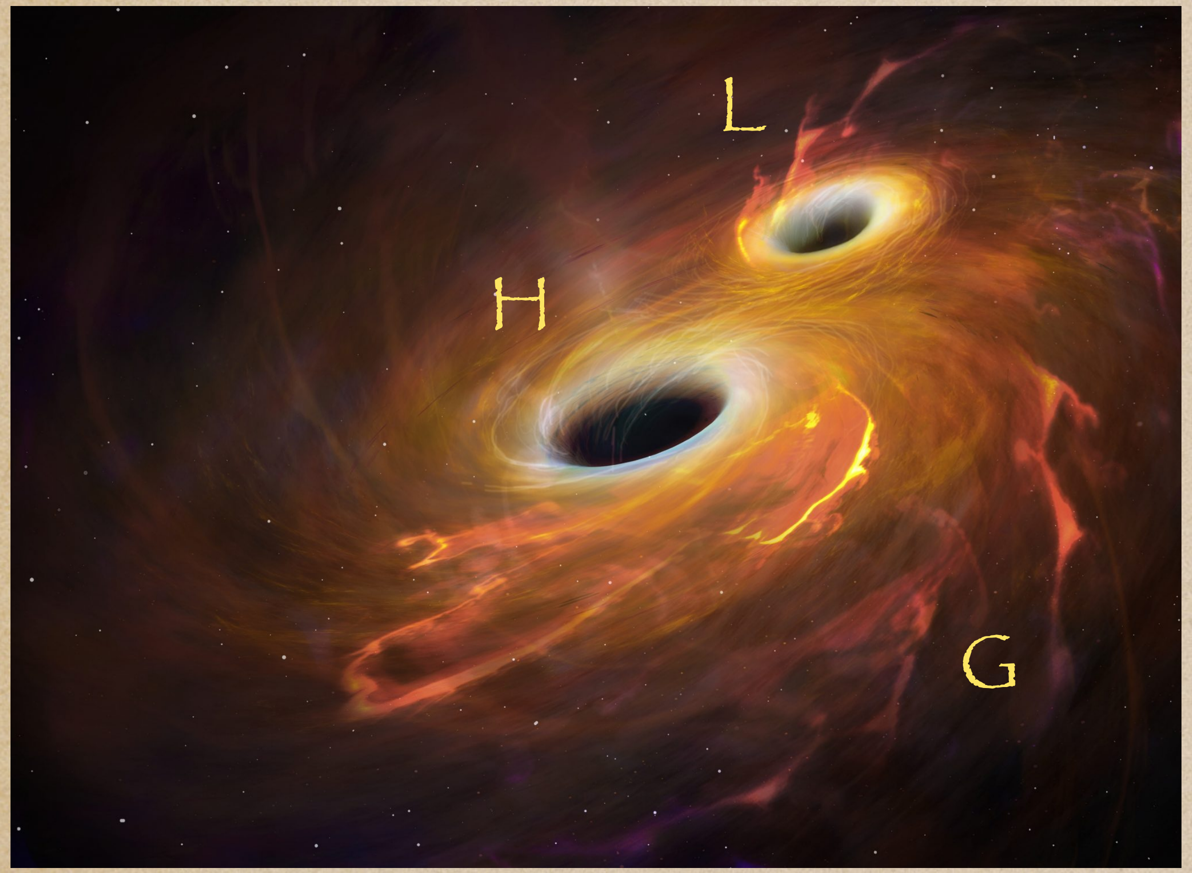

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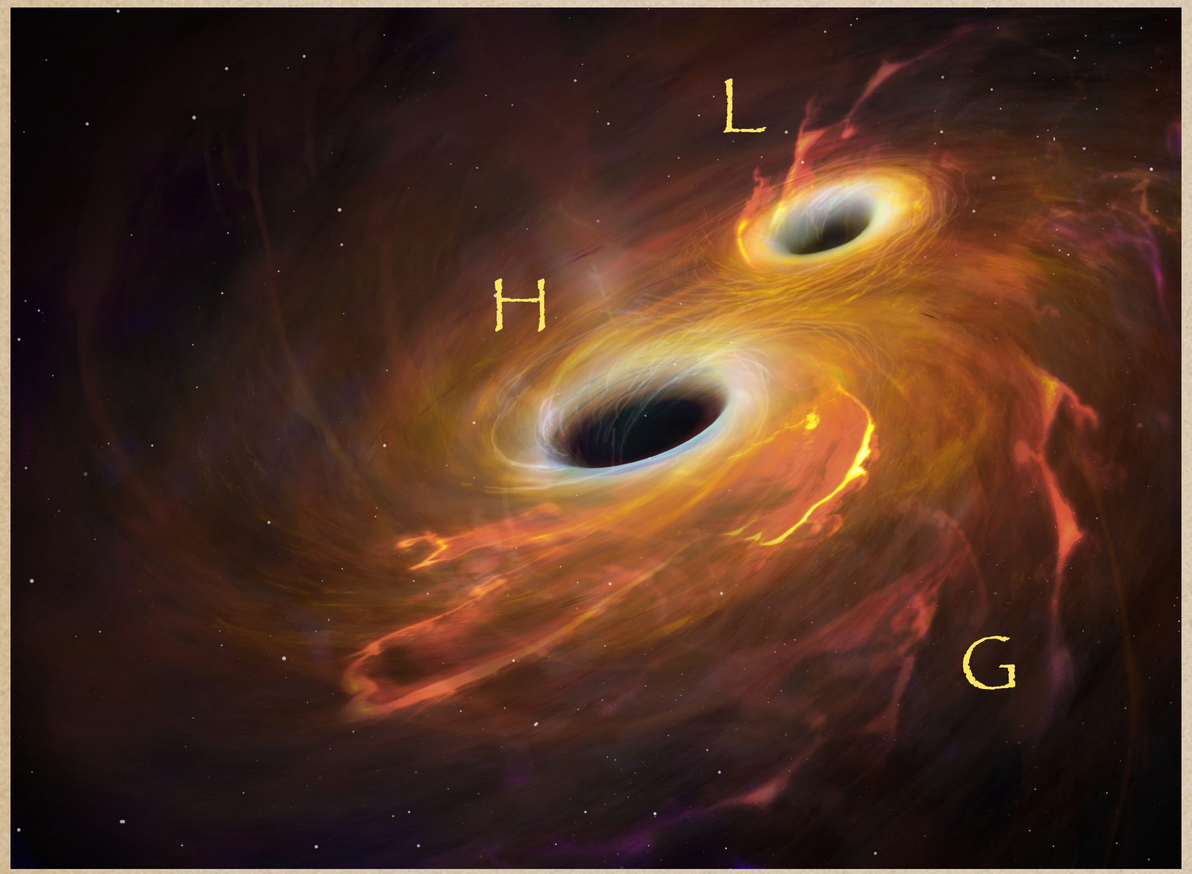


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$$S_{\mathrm{G}} = S_{\mathrm{EH}} + S_{\mathrm{GF}}$$

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + \kappa\, h_{\mu\nu}$$

$$S = S_{\mathrm{G}} + S_{\mathrm{L}} + S_{\mathrm{H}}$$

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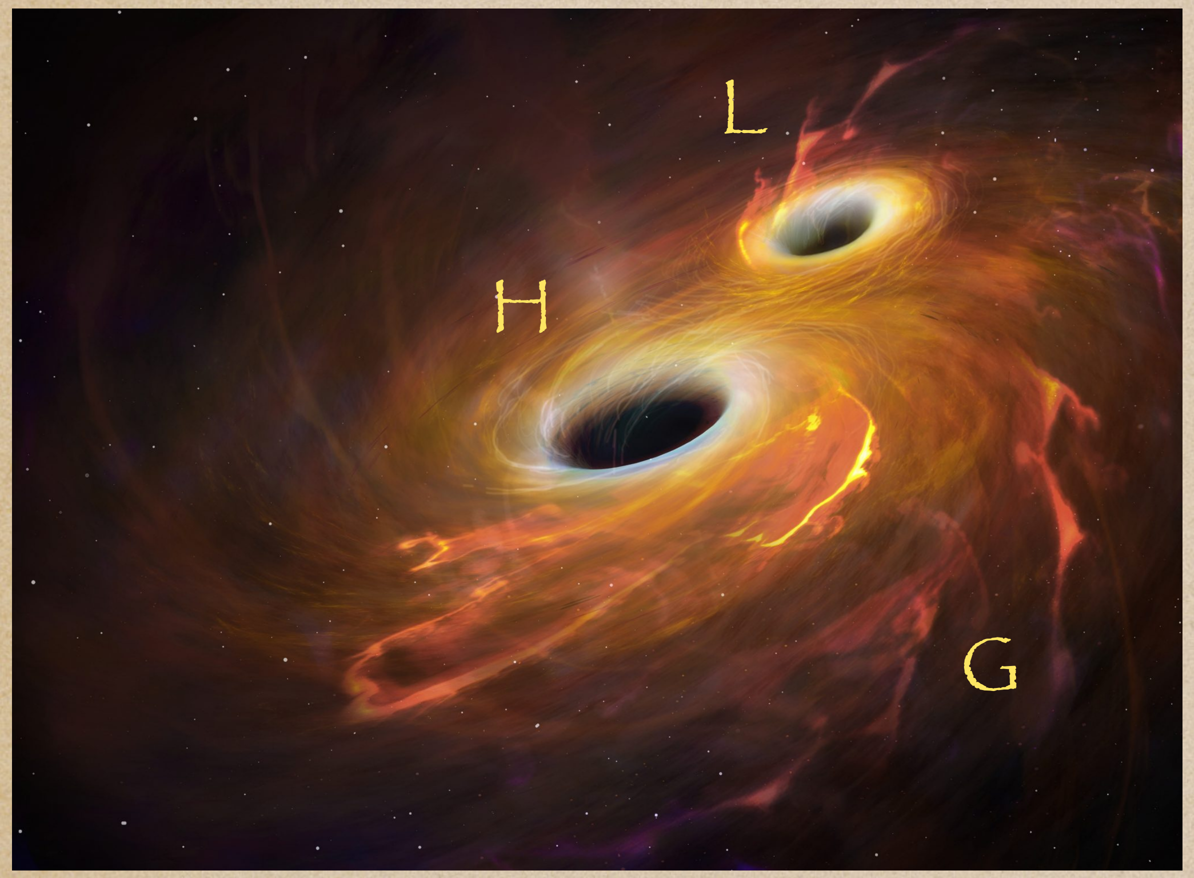


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$$S_{\mathrm{G}} = S_{\mathrm{EH}} + S_{\mathrm{GF}}$$

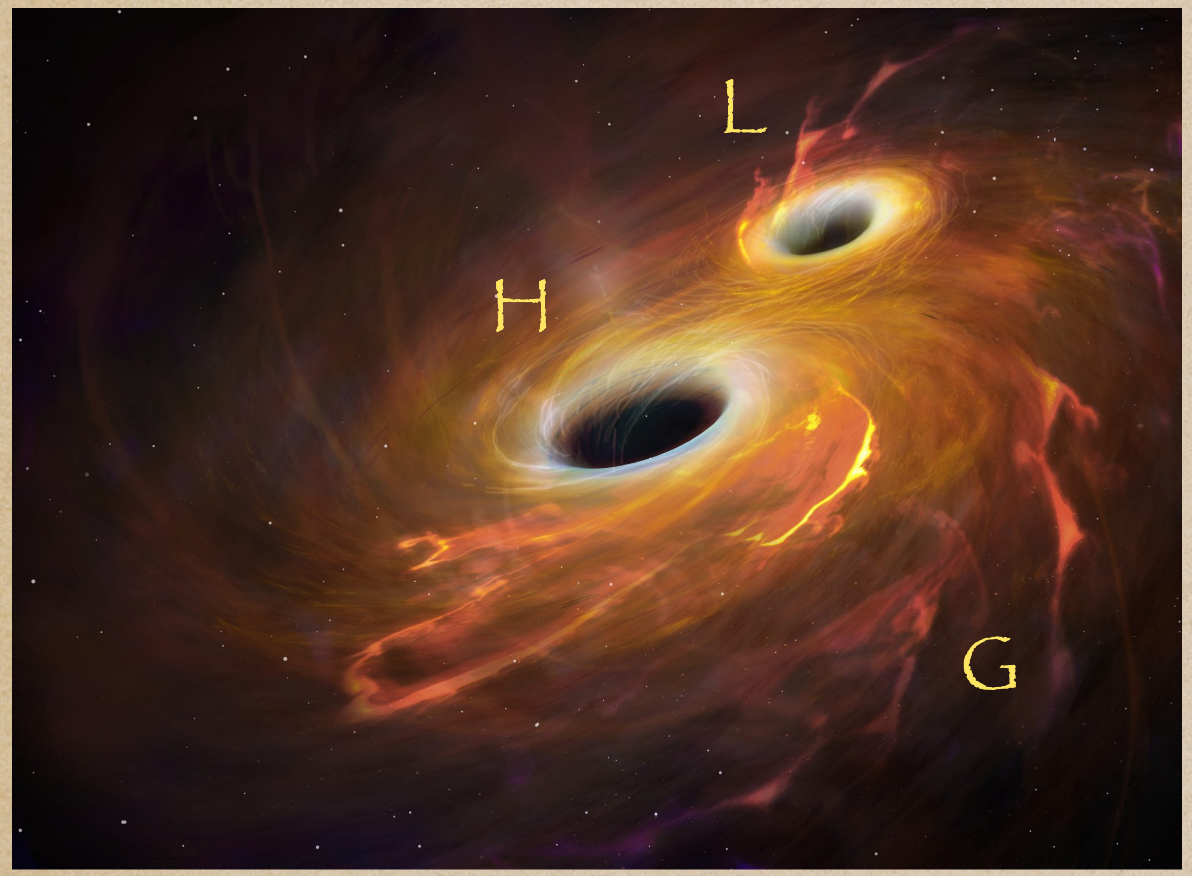
$$g_{\mu\nu} = \bar{g}_{\mu\nu} + \kappa h_{\mu\nu}$$

$$S_{\mathrm{L}} = \int d^4x\sqrt{-g}\left(\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - \frac{1}{2}m^2\phi^2\right)$$

$$S = S_{\mathrm{G}} + S_{\mathrm{L}} + S_{\mathrm{H}}$$

The Action for our System

# QFT in Curved Space: The Setup



Artistic Impression
Credit: MARK GARLICK / SCIENCE PHOTO / Getty images

$$S_{\mathrm{G}} = S_{\mathrm{EH}} + S_{\mathrm{GF}}$$

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + \kappa h_{\mu\nu}$$

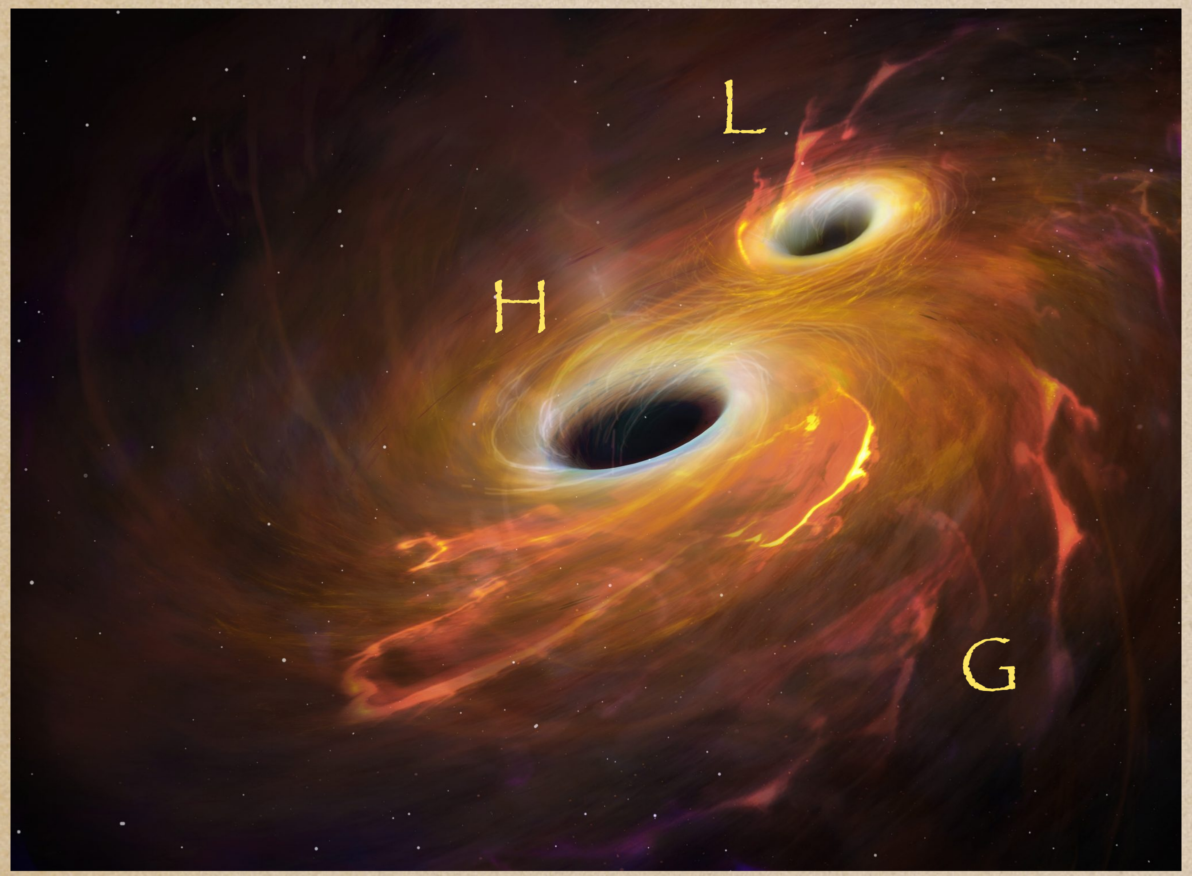
$$S_{\mathrm{L}} = \int d^4x\sqrt{-g}\left(\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - \frac{1}{2}m^2\phi^2\right)$$

$$S = S_{\mathrm{G}} + S_{\mathrm{L}} + S_{\mathrm{H}}$$

$$S_{\mathrm{H}} = -M\int d\tau$$

The Action for our System

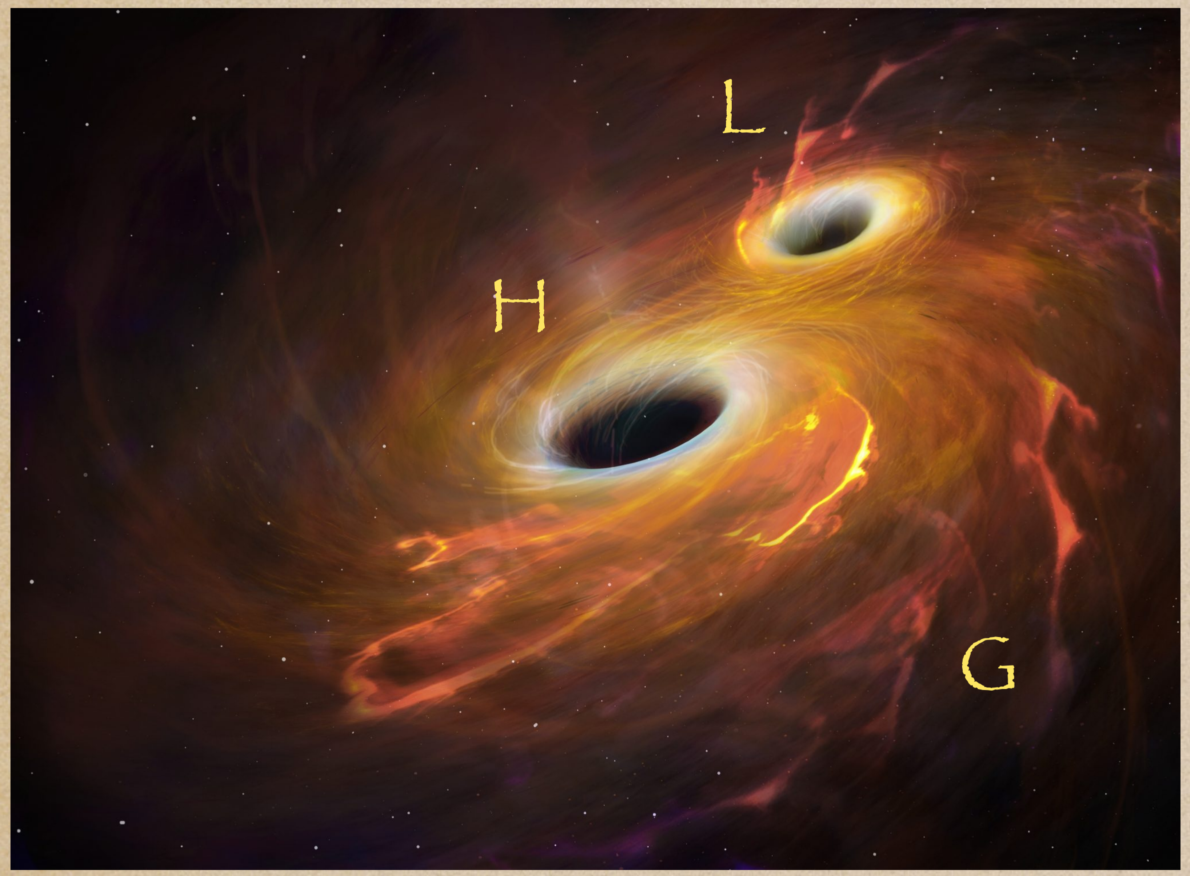
# Geodesic Limit 1: OSF at $\mathcal{O}(G)$



Artistic Impression
Credit: MARK GARLICK / SCIENCE PHOTO / Getty images

$$S = S_{\mathrm{G}} + S_{\mathrm{L}} + S_{\mathrm{H}}$$

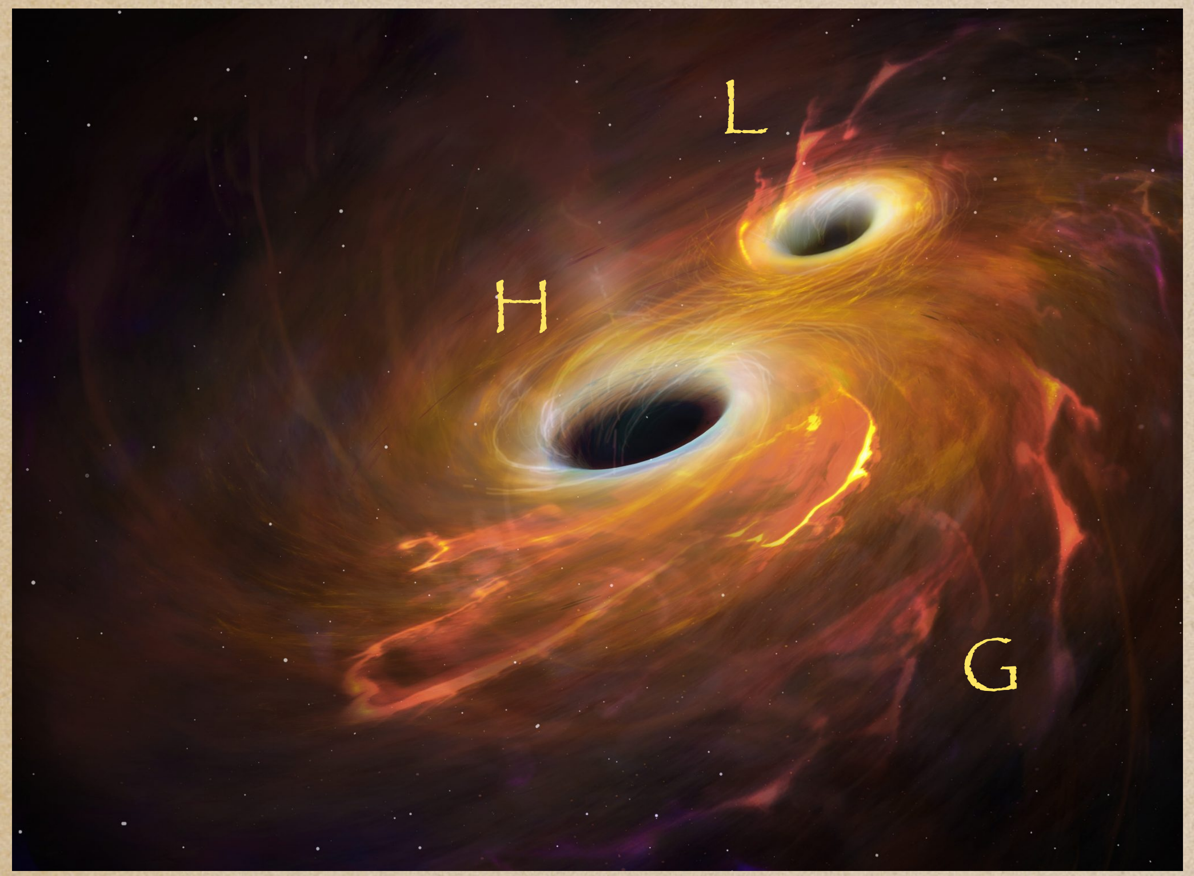
# Geodesic Limit 1: OSF at $\mathcal{O}(G)$




Artistic Impression
Credit: MARK GARLICK / SCIENCE PHOTO / Getty images

$$S = \cancel{S_{\mathrm{G}}} + S_{\mathrm{L}} + \cancel{S_{\mathrm{H}}}$$

# Geodesic Limit 1: OSF at $\mathcal{O}(G)$



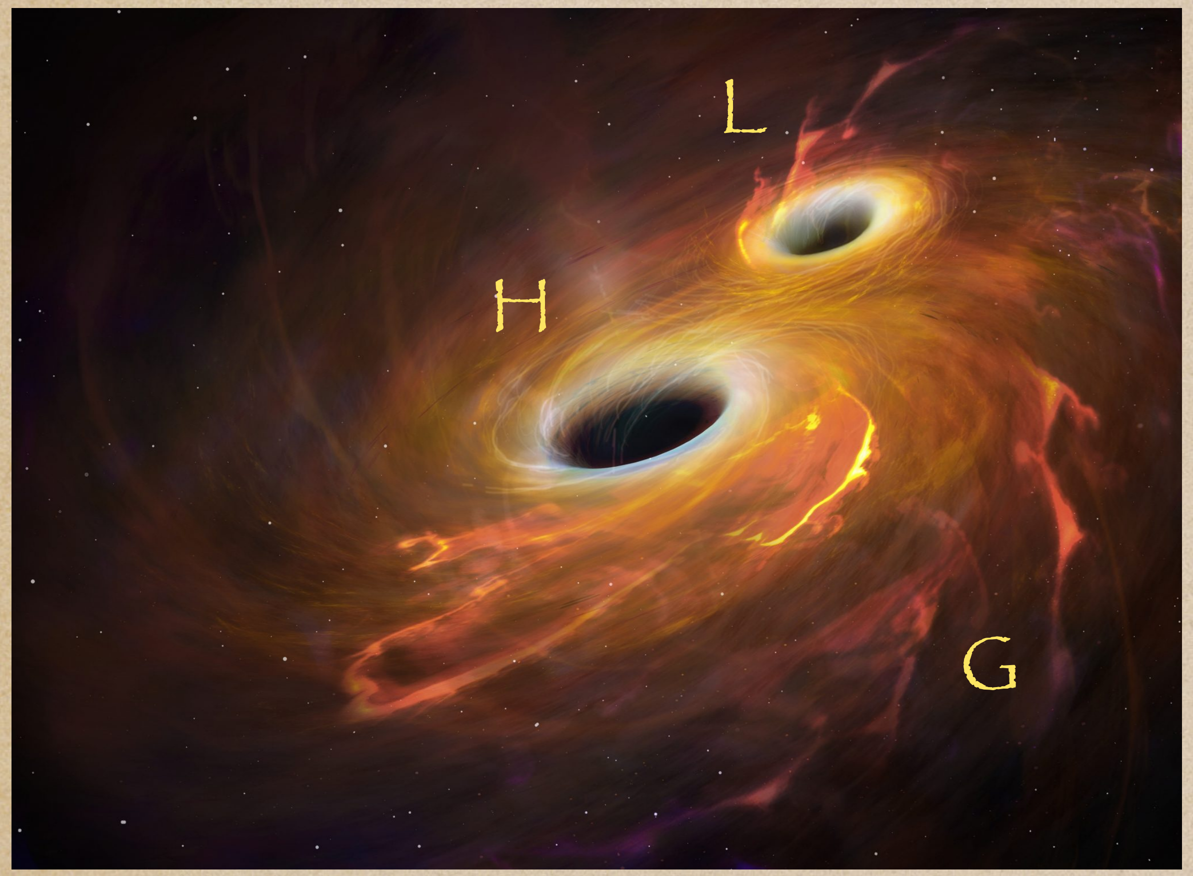

Artistic Impression
Credit: MARK GARLICK / SCIENCE PHOTO / Getty images

$$\mathscr{L}_{\mathrm{L}}^{(0)} = \sqrt{-\bar{g}}\left(\frac{1}{2}\bar{g}^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - \frac{1}{2}m^2\phi^2\right)$$

$$S = S_{\mathrm{G}} + S_{\mathrm{L}} + S_{\mathrm{H}}$$

$S_{\mathrm{L}}^{(0)}$

# Geodesic Limit 1: OSF at $\mathcal{O}(G)$




Artistic Impression
Credit: MARK GARLICK / SCIENCE PHOTO / Getty images

$$\mathscr{L}^{(0)}_{\mathrm{L}} = \sqrt{-\bar{g}}\left(\frac{1}{2}\bar{g}^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - \frac{1}{2}m^2\phi^2\right)$$

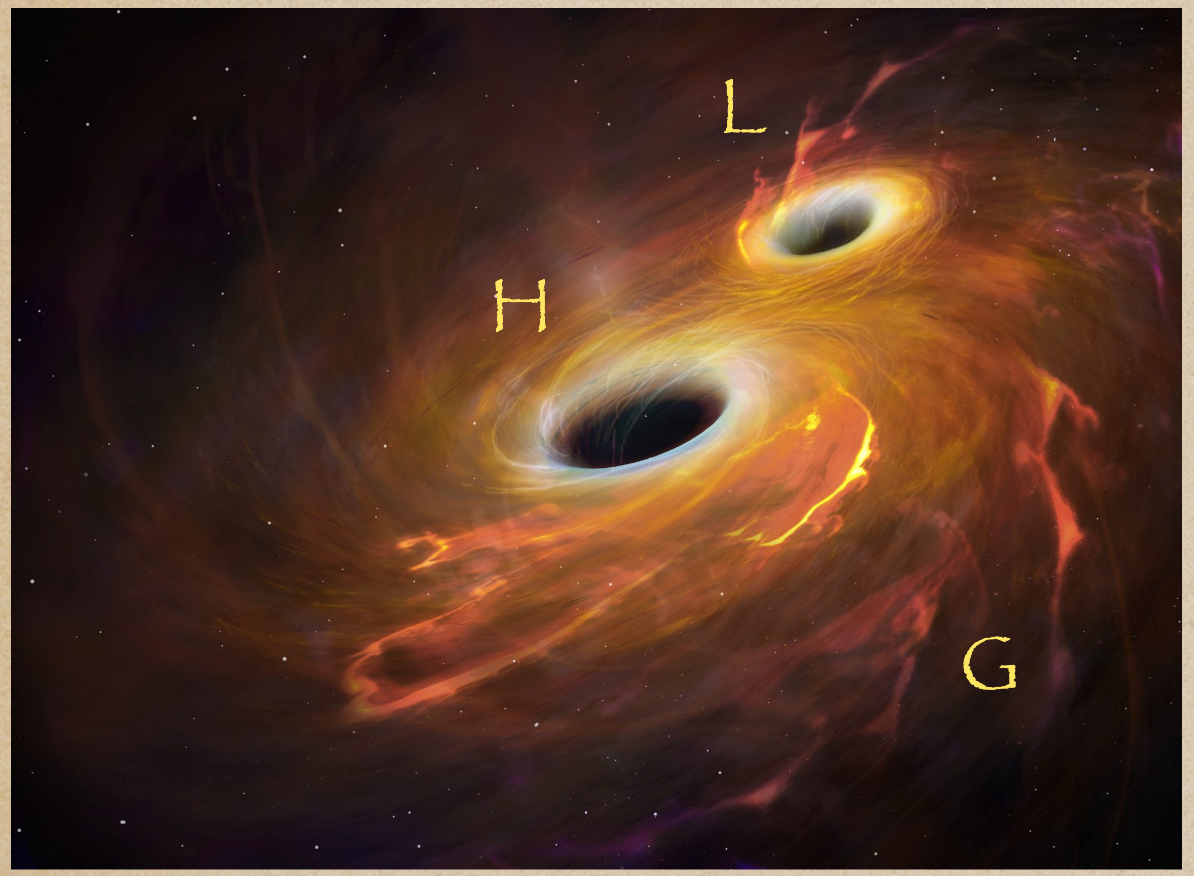
$$\mathscr{L}^{(0)}_{\mathrm{L,\,free}} = \frac{1}{2}\eta^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - \frac{1}{2}m^2\phi^2$$

$$\mathscr{L}^{(0)}_{\mathrm{L,\,int}} = \underbrace{\frac{1}{2}\left(\sqrt{-\bar{g}}\,\bar{g}^{\mu\nu} - \eta^{\mu\nu}\right)}_{C^{\mu\nu}}\partial_\mu\phi\partial_\nu\phi \underbrace{-\frac{1}{2}m^2\left(\sqrt{-\bar{g}} - 1\right)}_{C}\phi^2$$

$$S_{\mathrm{L}}^{(0)}$$

$$S = \cancel{S_{\mathrm{G}}} + \cancel{S_{\mathrm{L}}} + \cancel{S_{\mathrm{H}}}$$

# Geodesic Limit 1: OSF at $\mathcal{O}(G)$



Artistic Impression
Credit: MARK GARLICK / SCIENCE PHOTO / Getty images

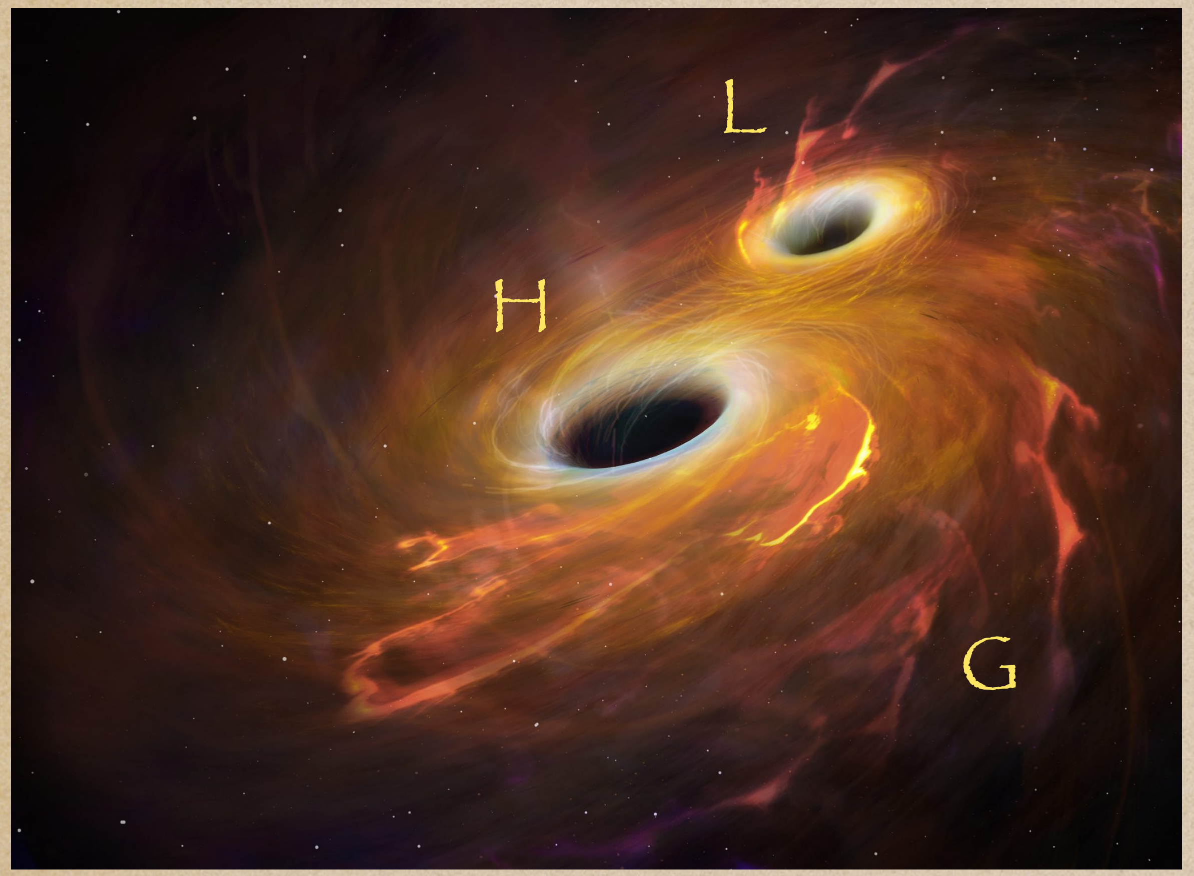
$$\mathscr{L}^{(0)}_{\mathrm{L}} = \sqrt{-\bar{g}}\left(\frac{1}{2}\bar{g}^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - \frac{1}{2}m^2\phi^2\right)$$

$$\mathscr{L}^{(0)}_{\mathrm{L,\,free}} = \frac{1}{2}\eta^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - \frac{1}{2}m^2\phi^2 \qquad \mathscr{L}^{(0)}_{\mathrm{L,\,int}} = \underbrace{\frac{1}{2}\left(\sqrt{-\bar{g}}\,\bar{g}^{\mu\nu} - \eta^{\mu\nu}\right)}_{C^{\mu\nu}}\partial_\mu\phi\partial_\nu\phi \underbrace{-\frac{1}{2}m^2\left(\sqrt{-\bar{g}} - 1\right)}_{C}\phi^2$$

$$\xrightarrow[p]{\quad\quad} = \frac{i}{p^2 - m^2}$$

$$S = \cancel{S_{\mathrm{G}}} + \cancel{S_{\mathrm{L}}} + \cancel{S_{\mathrm{H}}}, \qquad S_{\mathrm{L}} \to S^{(0)}_{\mathrm{L}}$$

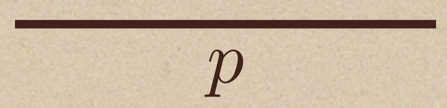
# Geodesic Limit 1: OSF at $\mathcal{O}(G)$




Artistic Impression
Credit: MARK GARLICK / SCIENCE PHOTO / Getty images

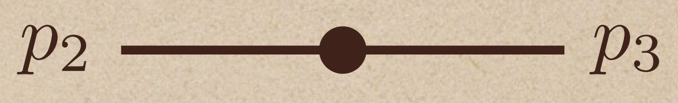
$$\mathscr{L}^{(0)}_{\mathrm{L}} = \sqrt{-\bar{g}}\left(\frac{1}{2}\bar{g}^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - \frac{1}{2}m^2\phi^2\right)$$

$$\mathscr{L}^{(0)}_{\mathrm{L,\,free}} = \frac{1}{2}\eta^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - \frac{1}{2}m^2\phi^2$$




$= \frac{i}{p^2-m^2}$

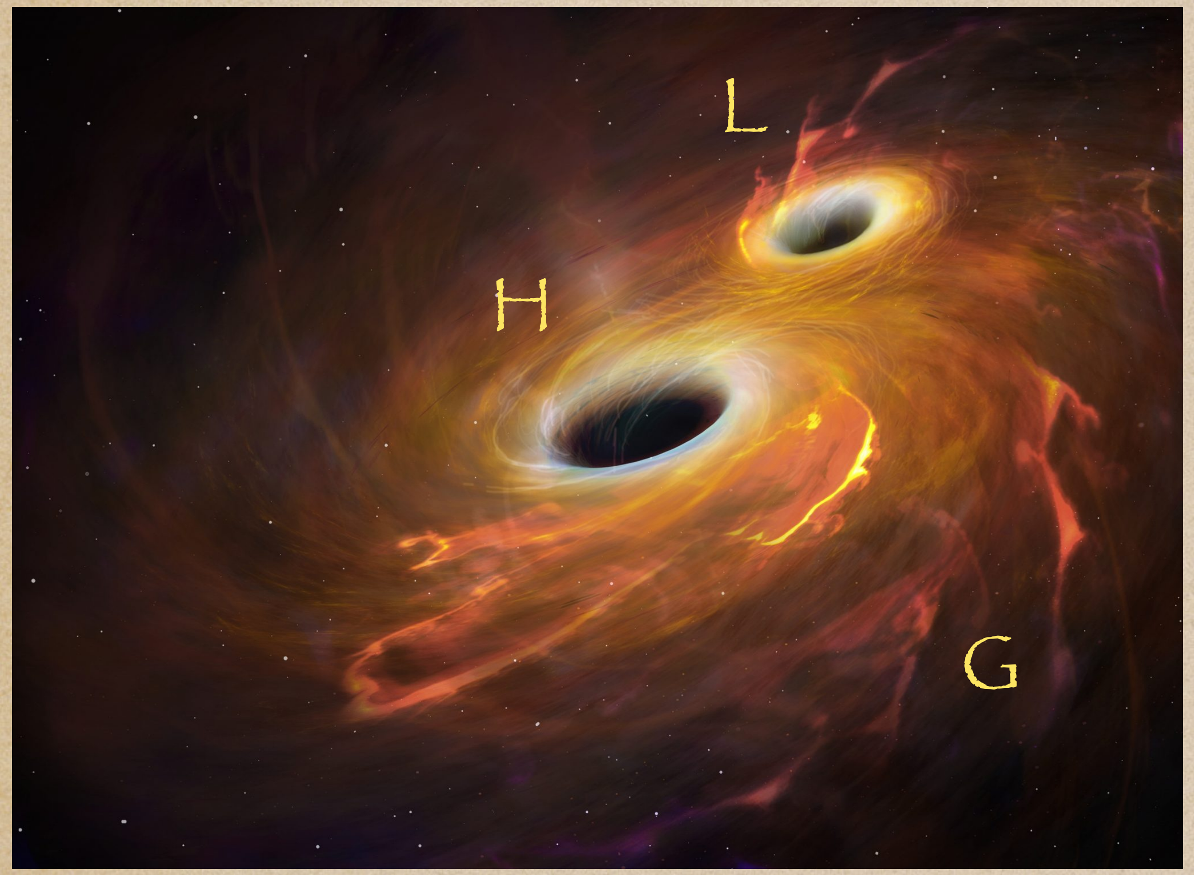
$$\mathscr{L}^{(0)}_{\mathrm{L,\,int}} = \underbrace{\frac{1}{2}\left(\sqrt{-\bar{g}}\,\bar{g}^{\mu\nu} - \eta^{\mu\nu}\right)}_{C^{\mu\nu}}\partial_\mu\phi\partial_\nu\phi \underbrace{-\frac{1}{2}m^2\left(\sqrt{-\bar{g}} - 1\right)}_{C}\phi^2$$




$S^{(0)}_{\mathrm{L}}$

$$S = S_{\mathrm{G}} + S_{\mathrm{L}} + S_{\mathrm{H}}$$

# Geodesic Limit 1: 0SF at $\mathcal{O}(G)$




Artistic Impression
Credit: MARK GARLICK / SCIENCE PHOTO / Getty images

$$\mathcal{L}^{(0)}_{\mathrm{L}} = \sqrt{-\bar{g}}\left(\frac{1}{2}\bar{g}^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - \frac{1}{2}m^2\phi^2\right)$$

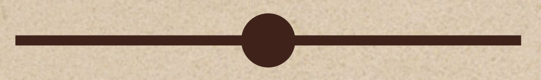
$$\mathcal{L}^{(0)}_{\mathrm{L,\,free}} = \frac{1}{2}\eta^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - \frac{1}{2}m^2\phi^2 \quad\longrightarrow\quad \overline{\quad p \quad} = \frac{i}{p^2-m^2}$$

$$\mathcal{L}^{(0)}_{\mathrm{L,\,int}} = \underbrace{\frac{1}{2}\left(\sqrt{-\bar{g}}\bar{g}^{\mu\nu} - \eta^{\mu\nu}\right)}_{C^{\mu\nu}}\partial_\mu\phi\partial_\nu\phi \underbrace{-\frac{1}{2}m^2\left(\sqrt{-\bar{g}} - 1\right)}_{C}\phi^2 \quad\longrightarrow\quad p_2 \;—\!\bullet\!—\; p_3$$

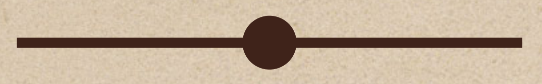
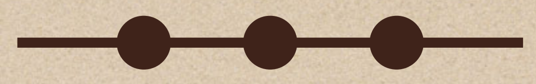
$$S = \cancel{S_{\mathrm{G}}} + \cancel{S_{\mathrm{L}}} + \cancel{S_{\mathrm{H}}}, \qquad S_{\mathrm{L}} \rightarrow S^{(0)}_{\mathrm{L}}$$

$$\boxed{\mathcal{A}_2^{\mathrm{s.in.}}(p_2, \mathbf{q}) = 2p_{2\mu}(p_{2\nu}+q_\nu)\tilde{C}^{\mu\nu}(\mathbf{q}) + 2\tilde{C}(\mathbf{q}) = \frac{8\pi G M m^2\left(1-2\gamma^2\right)}{\mathbf{q}^2} + \mathcal{O}\left(G^2\right)}$$

# Geodesic Limit 2: OSF at All Orders in $G$

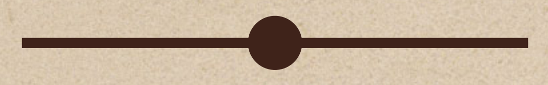
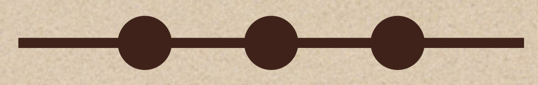
$\mathscr{A}_2^{\mathrm{OSF}} =$ $p_2$  $p_3$ $+$ $p_2$  $p_3$ $+$ $p_2$  $p_3 + \ldots$

# Geodesic Limit 2: OSF at All Orders in $G$

$\mathscr{A}_2^{\text{OSF}} =$ $p_2$  $p_3$ $+$ $p_2$  $p_3$ $+$ $p_2$  $p_3$ $+ \ldots$

$$\tilde{\mathscr{A}}_2^{\text{OSF}}(p_2, \mathbf{x}) = m^2 \left( -\gamma^2 \left( \frac{\bar{g}_{rr}}{\bar{g}_{tt}} + 1 \right) + \left(1 + \bar{g}_{rr}\right) \right)$$

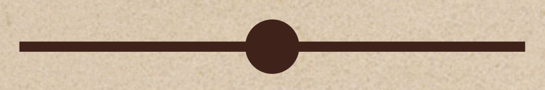
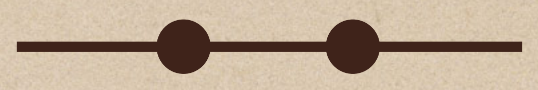
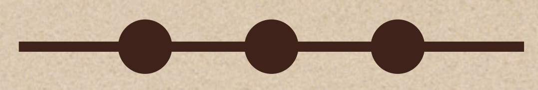
# Geodesic Limit 2: OSF at All Orders in $G$

$\mathscr{A}_2^{\mathrm{OSF}} =$ $p_2$  $p_3$ $+$ $p_2$  $p_3$ $+$ $p_2$  $p_3 + \dots$

$$\tilde{\mathscr{A}}_2^{\mathrm{OSF}}(p_2, \mathbf{x}) = m^2 \left( -\gamma^2 \left( \frac{\bar{g}_{rr}}{\bar{g}_{tt}} + 1 \right) + \left(1 + \bar{g}_{rr}\right) \right)$$

$$= \frac{\left(2\gamma^2 - 1\right) GMm^2}{r} + \frac{3\left(5\gamma^2 - 1\right) G^2M^2m^2}{4r^2} + \frac{\left(18\gamma^2 - 1\right) G^3M^3m^2}{4r^3} + \frac{\left(129\gamma^2 - 1\right) G^4M^4m^2}{32r^4} + \mathscr{O}\left(G^5\right)$$

# Geodesic Limit 2: OSF at All Orders in $G$

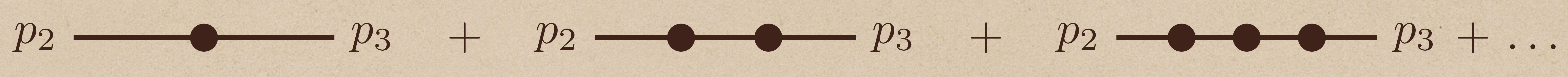
$\mathscr{A}_2^{\mathrm{OSF}} =$ $p_2$  $p_3$ $+$ $p_2$  $p_3$ $+$ $p_2$  $p_3 + \ldots$

$$\tilde{\mathscr{A}}_2^{\mathrm{OSF}}(p_2, \mathbf{x}) = m^2 \left( -\gamma^2 \left( \frac{\bar{g}_{rr}}{\bar{g}_{tt}} + 1 \right) + \left(1 + \bar{g}_{rr}\right) \right)$$

Agrees with [Kälin, Porto; Cheung, Shah, Solon]

$$= \frac{\left(2\gamma^2 - 1\right) GMm^2}{r} + \frac{3\left(5\gamma^2 - 1\right) G^2M^2m^2}{4r^2} + \frac{\left(18\gamma^2 - 1\right) G^3M^3m^2}{4r^3} + \frac{\left(129\gamma^2 - 1\right) G^4M^4m^2}{32r^4} + \mathcal{O}\left(G^5\right)$$

# Geodesic Limit 2: OSF at All Orders in $G$

$\mathscr{A}_2^{\mathrm{OSF}} =$ 

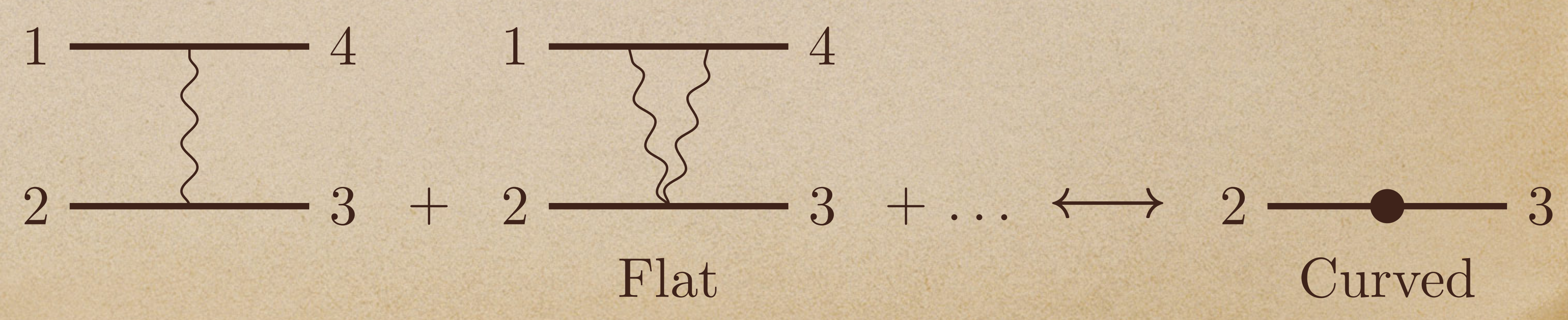


$$\tilde{\mathscr{A}}_2^{\mathrm{OSF}}(p_2, \mathbf{x}) = m^2 \left( -\gamma^2 \left( \frac{\bar{g}_{rr}}{\bar{g}_{tt}} + 1 \right) + \left(1 + \bar{g}_{rr}\right) \right)$$

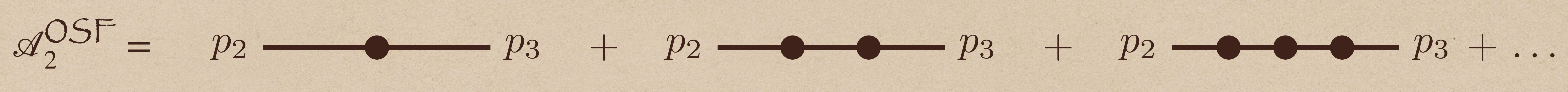
Agrees with [Kälin, Porto; Cheung, Shah, Solon]

$$= \frac{\left(2\gamma^2 - 1\right) GMm^2}{r} + \frac{3\left(5\gamma^2 - 1\right) G^2M^2m^2}{4r^2} + \frac{\left(18\gamma^2 - 1\right) G^3M^3m^2}{4r^3} + \frac{\left(129\gamma^2 - 1\right) G^4M^4m^2}{32r^4} + \mathcal{O}\left(G^5\right)$$

- Geometric resummation ↔ Field redefinition



# Geodesic Limit 2: OSF at All Orders in $G$

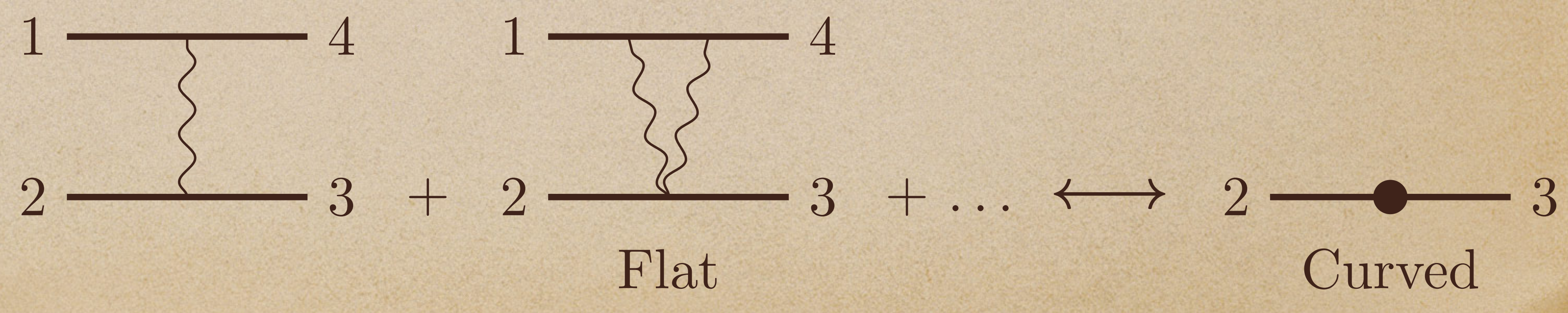



$$\tilde{\mathscr{A}}_2^{\text{OSF}}(p_2, \mathbf{x}) = m^2 \left( -\gamma^2 \left( \frac{\bar{g}_{rr}}{\bar{g}_{tt}} + 1 \right) + (1 + \bar{g}_{rr}) \right)$$

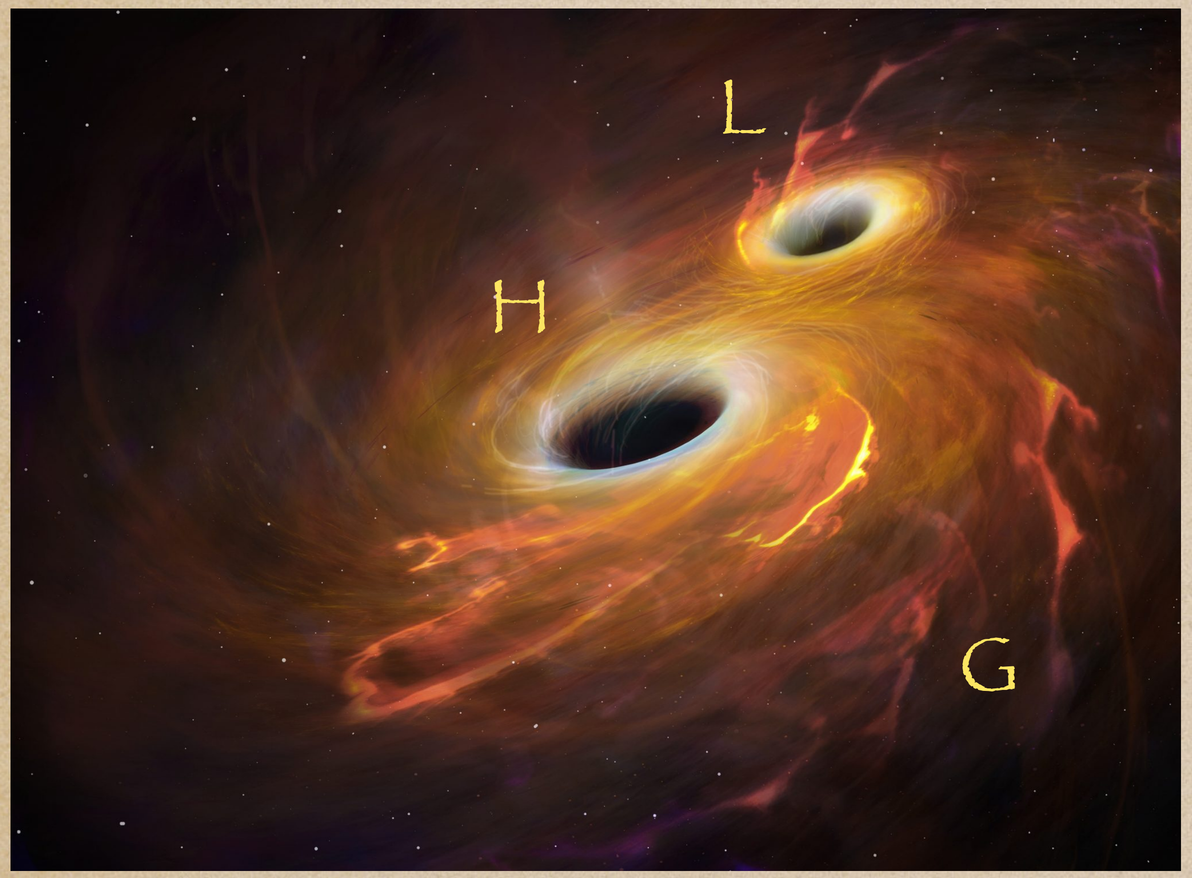
Agrees with [Kälin, Porto; Cheung, Shah, Solon]

$$= \frac{(2\gamma^2 - 1)\, GMm^2}{r} + \frac{3\,(5\gamma^2 - 1)\, G^2M^2m^2}{4r^2} + \frac{(18\gamma^2 - 1)\, G^3M^3m^2}{4r^3} + \frac{(129\gamma^2 - 1)\, G^4M^4m^2}{32r^4} + \mathcal{O}\left(G^5\right)$$

- Geometric resummation $\leftrightarrow$ Field redefinition
- Dropped iteration



# Beyond OSF 1: The Graviton

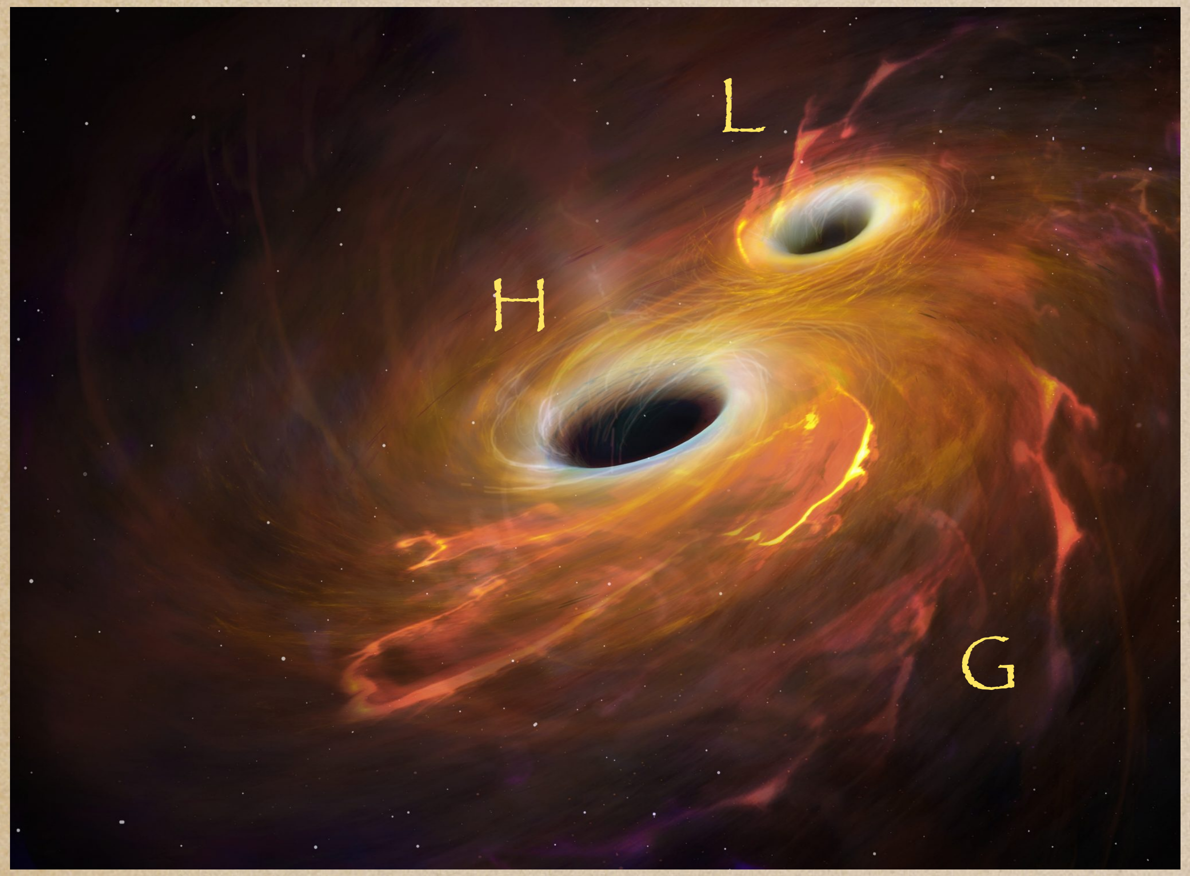



Artistic Impression
Credit: MARK GARLICK / SCIENCE PHOTO / Getty images

$$S = \underline{S_G} + S_L + S_H$$

# Beyond OSF 1: The Graviton

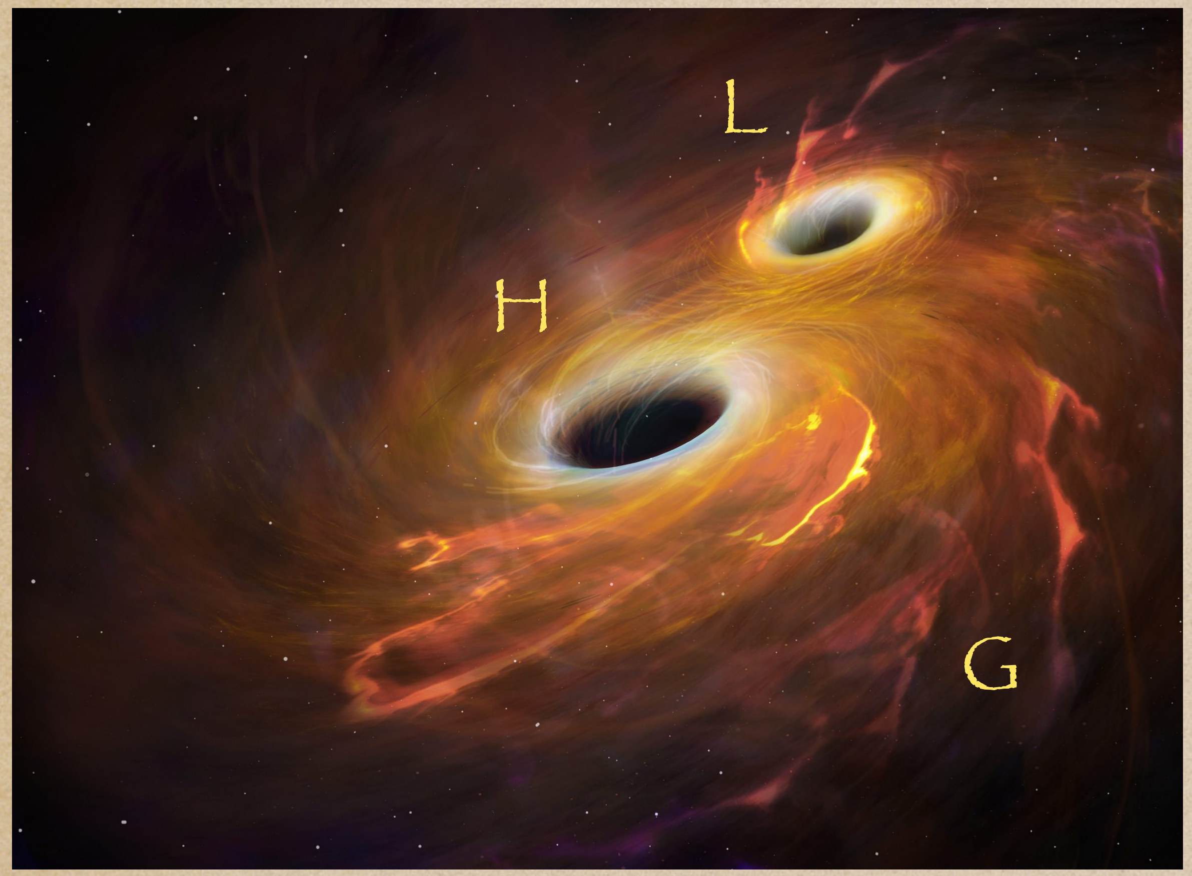
$$S_{\mathrm{G}} = S_{\mathrm{EH}} + S_{\mathrm{GF}} = -\frac{1}{16\pi G}\int d^4x\sqrt{-g}R + \int d^4x\sqrt{-\bar{g}}F^{\mu}F_{\mu}$$



Artistic Impression
Credit: MARK GARLICK / SCIENCE PHOTO / Getty images

$$S = \underline{S_{\mathrm{G}}} + S_{\mathrm{L}} + S_{\mathrm{H}}$$

# Beyond OSF 1: The Graviton



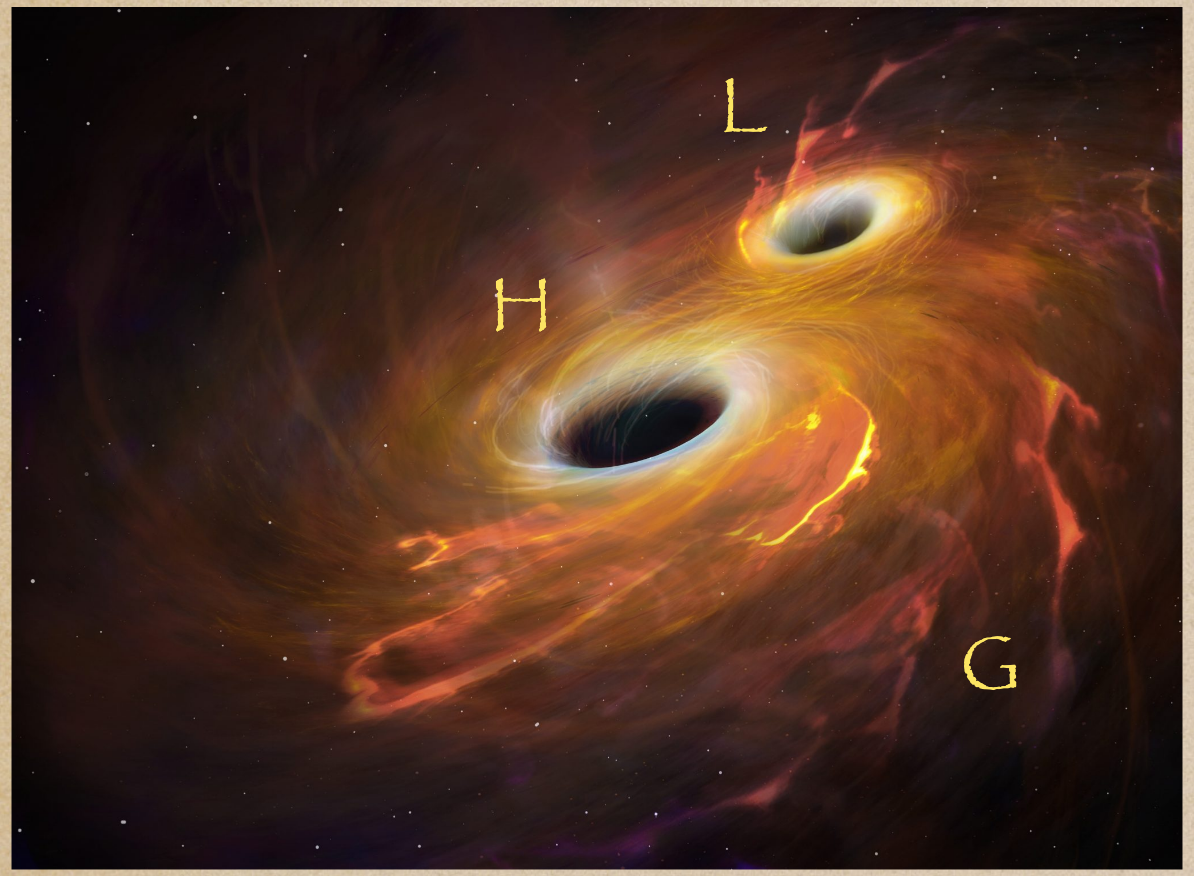

Artistic Impression
Credit: MARK GARLICK / SCIENCE PHOTO / Getty images

$$S_{\mathrm{G}} = S_{\mathrm{EH}} + S_{\mathrm{GF}} = -\frac{1}{16\pi G}\int d^4x\sqrt{-g}R + \int d^4x\sqrt{-\bar{g}}F^{\mu}F_{\mu}$$

$$F_{\mu} = \bar{\mathscr{D}}^{\nu}h_{\mu\nu} - \frac{1}{2}\bar{\mathscr{D}}_{\mu}h\,, \quad h = \bar{g}^{\alpha\beta}h_{\alpha\beta}\,, \quad g_{\mu\nu} = \bar{g}_{\mu\nu} + \kappa\, h_{\mu\nu}$$

$$S = \underline{S_{\mathrm{G}}} + S_{\mathrm{L}} + S_{\mathrm{H}}$$

# Beyond 0SF 1: The Graviton




Artistic Impression
Credit: MARK GARLICK / SCIENCE PHOTO / Getty images

$$S_{\mathrm{G}} = S_{\mathrm{EH}} + S_{\mathrm{GF}} = -\frac{1}{16\pi G}\int d^4x\sqrt{-g}R + \int d^4x\sqrt{-\bar{g}}F^{\mu}F_{\mu}$$

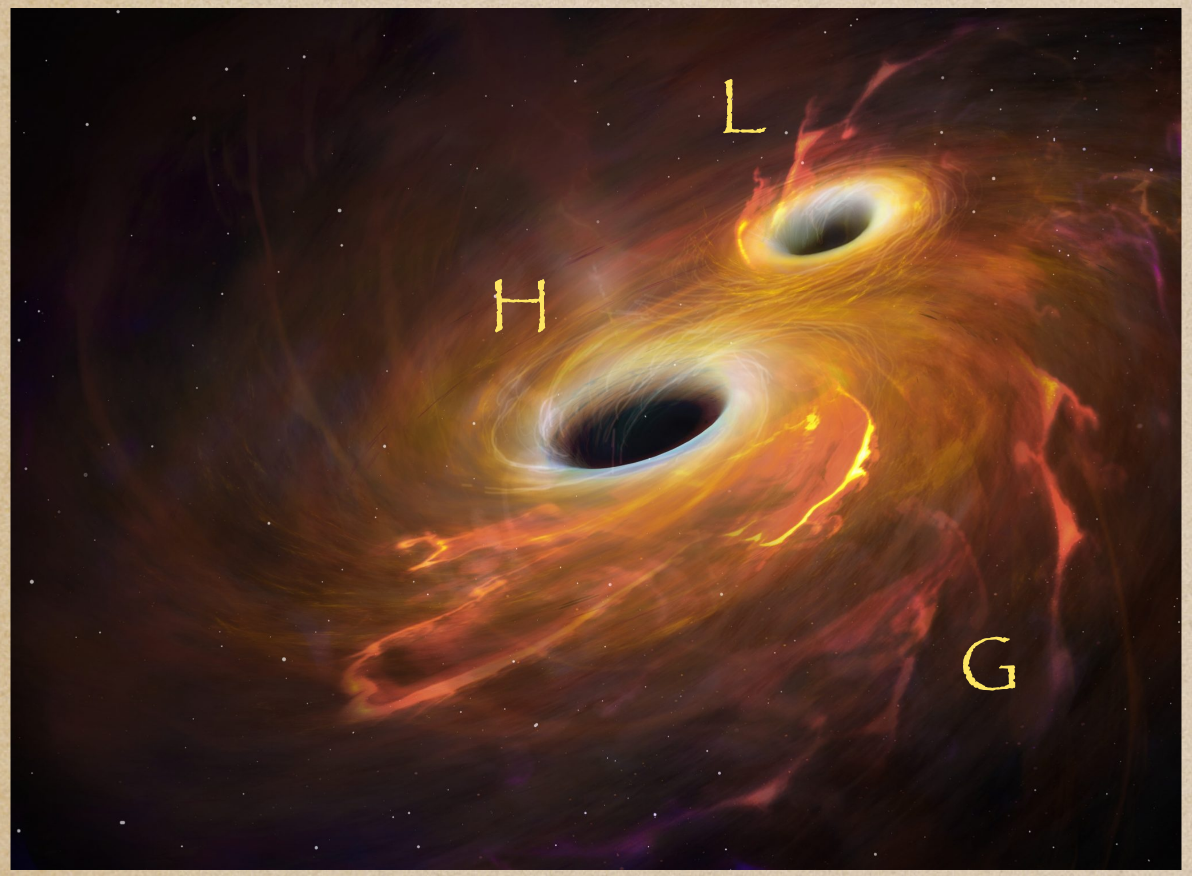
$$F_{\mu} = \bar{\mathscr{D}}^{\nu}h_{\mu\nu} - \frac{1}{2}\bar{\mathscr{D}}_{\mu}h\,, \quad h = \bar{g}^{\alpha\beta}h_{\alpha\beta}\,, \quad g_{\mu\nu} = \bar{g}_{\mu\nu} + \kappa\, h_{\mu\nu}$$

$$n\mathrm{SF} \leftrightarrow h^{n+1}$$

$$S = \underline{S_{\mathrm{G}}} + S_{\mathrm{L}} + S_{\mathrm{H}}$$

Backreaction of the Gravitational Field

# Beyond OSF 1: The Graviton




Artistic Impression
Credit: MARK GARLICK / SCIENCE PHOTO / Getty images

$$S_{\mathrm{G}} = S_{\mathrm{EH}} + S_{\mathrm{GF}} = -\frac{1}{16\pi G}\int d^4x\sqrt{-g}R + \int d^4x\sqrt{-\bar{g}}F^{\mu}F_{\mu}$$

$$F_{\mu} = \bar{\mathscr{D}}^{\nu}h_{\mu\nu} - \frac{1}{2}\bar{\mathscr{D}}_{\mu}h, \quad h = \bar{g}^{\alpha\beta}h_{\alpha\beta}, \quad g_{\mu\nu} = \bar{g}_{\mu\nu} + \kappa h_{\mu\nu}$$

$$n\mathrm{SF} \leftrightarrow h^{n+1}$$

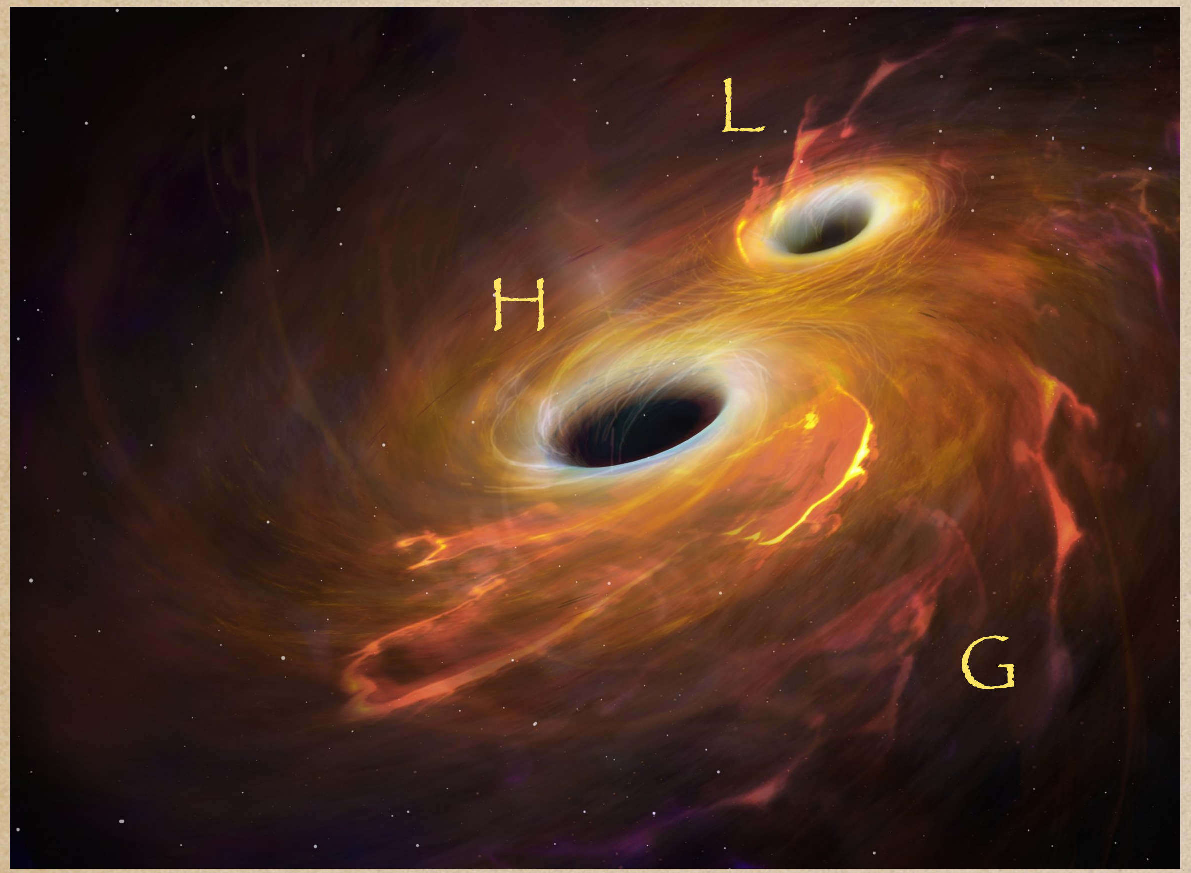
1SF:

$$\frac{\mathscr{L}_{\mathrm{G}}}{\sqrt{-\bar{g}}} = \frac{\kappa}{16\pi G}\bar{G}^{\alpha\beta}h_{\alpha\beta} - \frac{1}{4}h^{;\beta}h_{;\beta} + \frac{1}{2}h_{\alpha\beta;\gamma}h^{\alpha\beta;\gamma} + \frac{1}{4}\bar{R}\left(2h^{\alpha\beta}h_{\alpha\beta} - h^2\right) + \bar{R}_{\beta\gamma}\left(h^{\beta\gamma}h - h^{\gamma}_{\alpha}h^{\alpha\beta}\right) - \bar{R}_{\alpha\gamma\beta\lambda}h^{\alpha\beta}h^{\gamma\lambda} + \mathcal{O}\left(h^3\right)$$

$$S = \underline{S_{\mathrm{G}}} + S_{\mathrm{L}} + S_{\mathrm{H}}$$

Backreaction of the Gravitational Field

# Beyond OSF 1: The Graviton



Artistic Impression
Credit: MARK GARLICK / SCIENCE PHOTO / Getty images

$$S_{\mathrm{G}} = S_{\mathrm{EH}} + S_{\mathrm{GF}} = -\frac{1}{16\pi G}\int d^4x\sqrt{-g}R + \int d^4x\sqrt{-\bar{g}}F^{\mu}F_{\mu}$$

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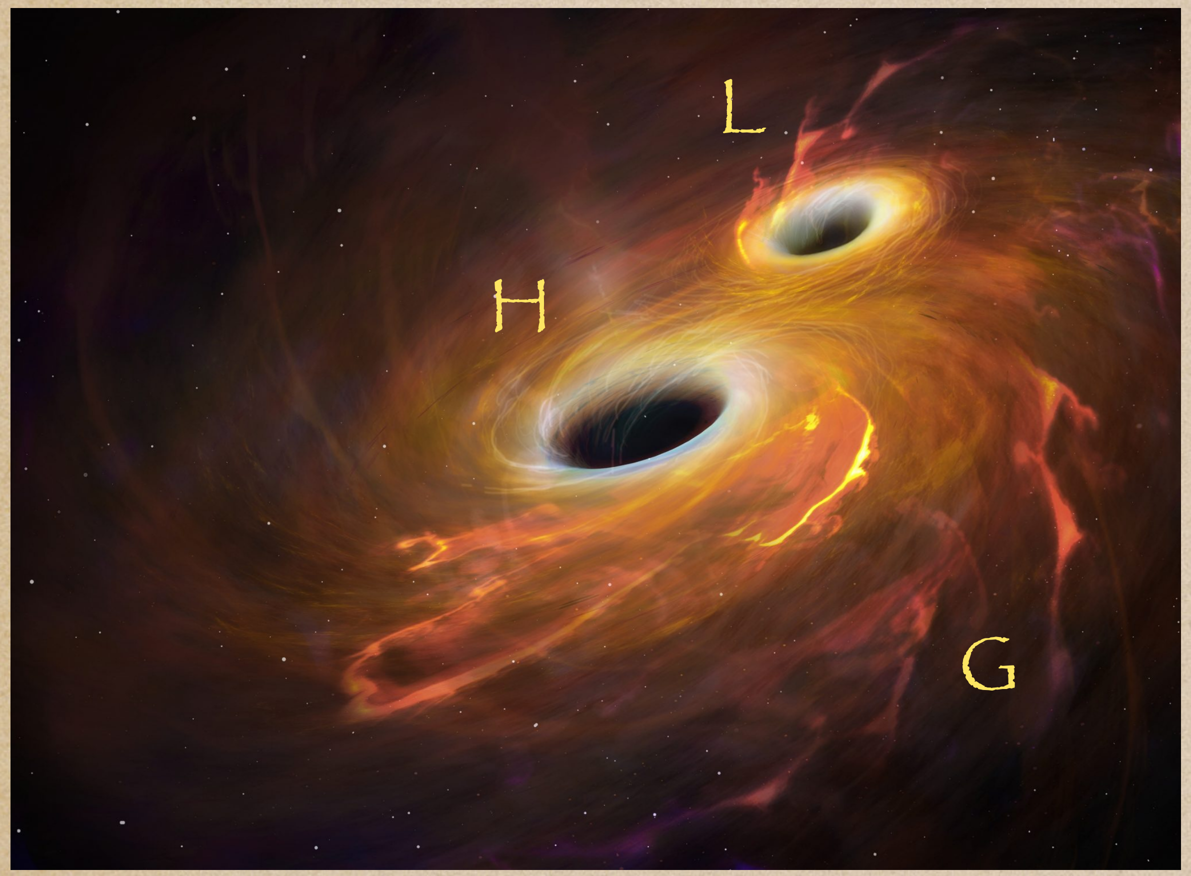
$$n\mathrm{SF} \leftrightarrow h^{n+1}$$

1SF: $$\frac{\mathscr{L}_{\mathrm{G}}}{\sqrt{-\bar{g}}} = \frac{\kappa}{16\pi G}\bar{G}^{\alpha\beta}h_{\alpha\beta} - \frac{1}{4}h^{;\beta}h_{;\beta} + \frac{1}{2}h_{\alpha\beta;\gamma}h^{\alpha\beta;\gamma} + \frac{1}{4}\bar{R}\left(2h^{\alpha\beta}h_{\alpha\beta} - h^2\right) + \bar{R}_{\beta\gamma}\left(h^{\beta\gamma}h - h^{\gamma}_{\alpha}h^{\alpha\beta}\right) - \bar{R}_{\alpha\gamma\beta\lambda}h^{\alpha\beta}h^{\gamma\lambda} + \mathcal{O}\left(h^3\right)$$

$$S = \underline{S_{\mathrm{G}}} + S_{\mathrm{L}} + S_{\mathrm{H}}$$

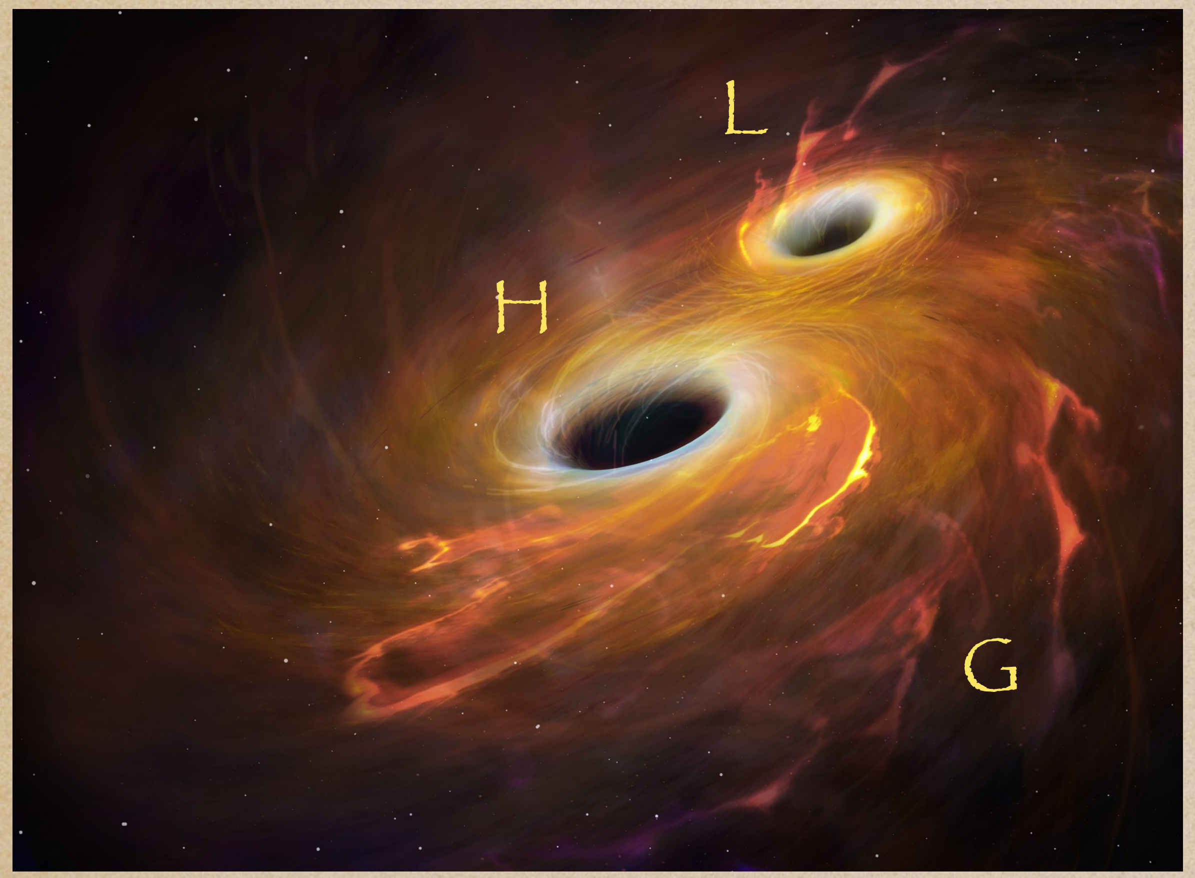
# Beyond OSF 2: The Spacetime Goldstones




Artistic Impression
Credit: MARK GARLICK / SCIENCE PHOTO / Getty images

$$S = S_G + S_L + \underline{S_H}$$

# Beyond OSF 2: The Spacetime Goldstones



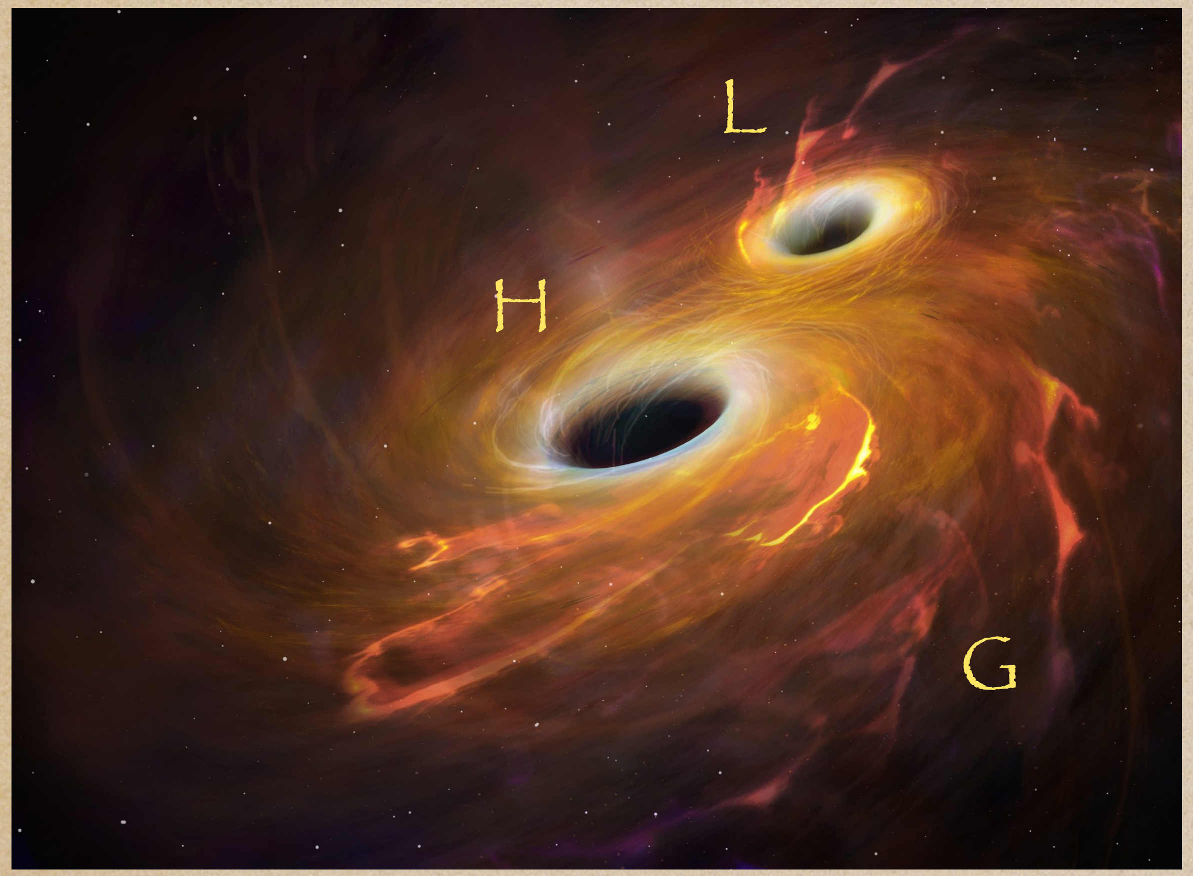

Artistic Impression
Credit: MARK GARLICK / SCIENCE PHOTO / Getty images

Black Hole at the Origin

$$S = S_G + S_L + \underline{S_H}$$

Recoil of the Heavy Source

# Beyond OSF 2: The Spacetime Goldstones



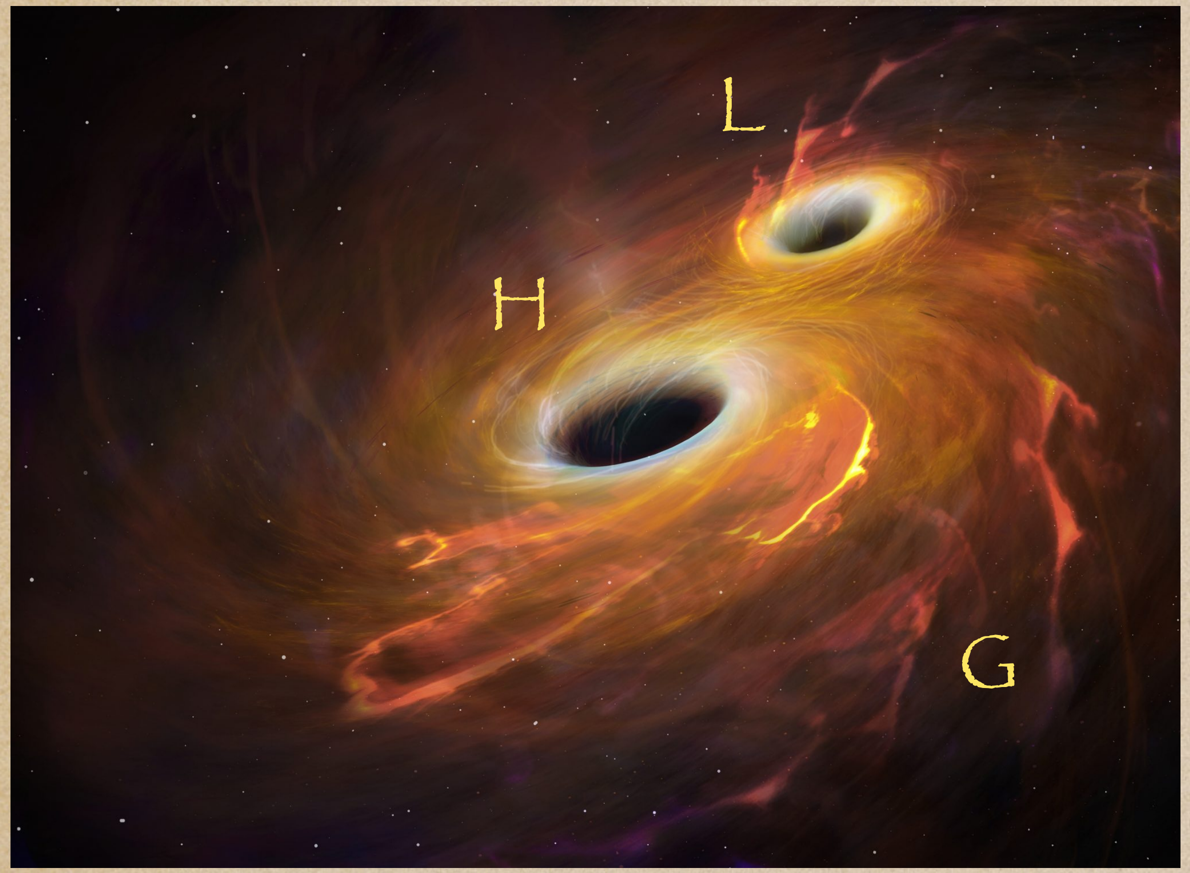

Artistic Impression
Credit: MARK GARLICK / SCIENCE PHOTO / Getty images

Black Hole at the Origin
- → Breaks Spatial Translations & Boosts
- → Preserves Time Translation & Rotations

$$S = S_G + S_L + \underline{S_H}$$

Recoil of the Heavy Source

# Beyond OSF 2: The Spacetime Goldstones



Artistic Impression
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- Breaks Spatial Translations & Boosts
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[Low and Manohar] [Watanabe and Murayama]

↓

Goldstone Bosons $\zeta^i(t)$, $i = 1,2,3$

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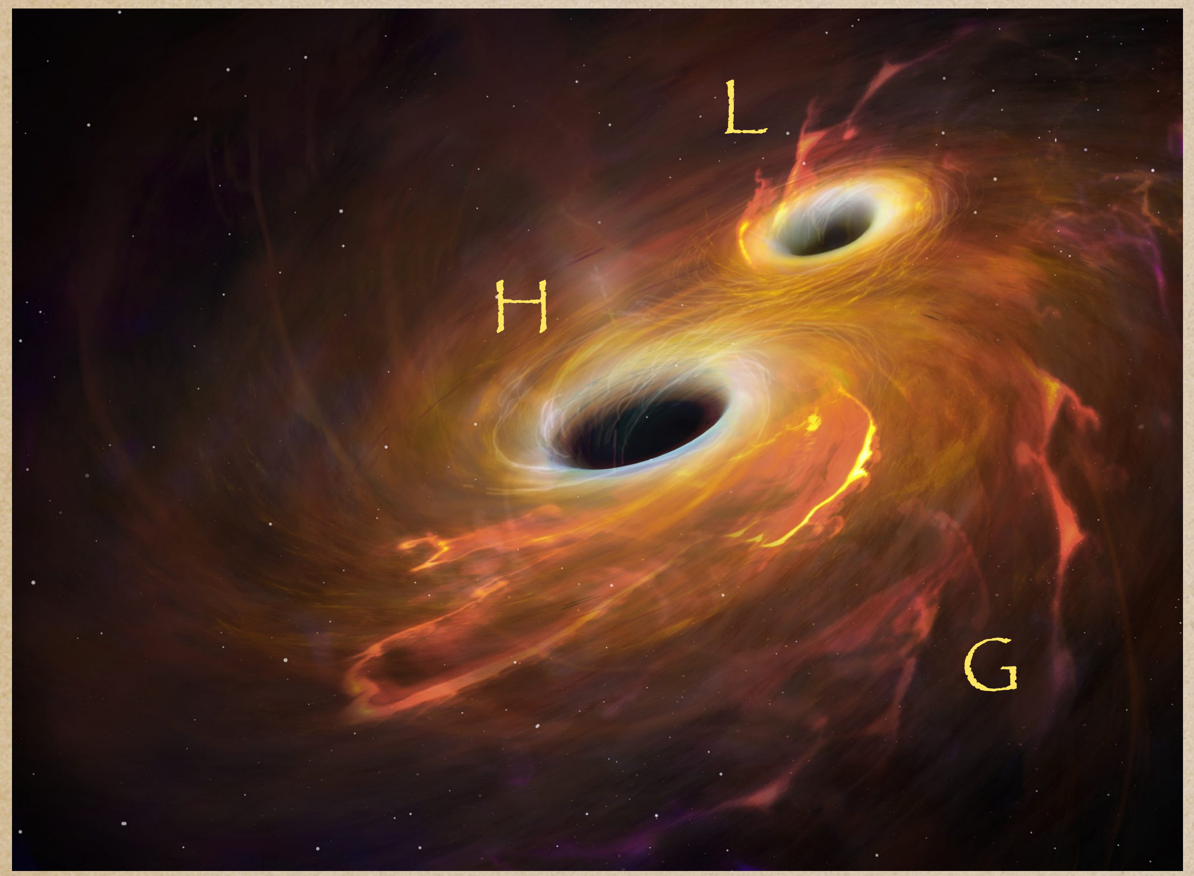


Artistic Impression

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$S = S_{\mathrm{G}} + S_{\mathrm{L}} + \underline{S_{\mathrm{H}}}$

Coset Construction: $S_{\mathrm{H}} = -M \int d\tau$

[Delacrétaz, Endlich, Monin, Penco and Riva]

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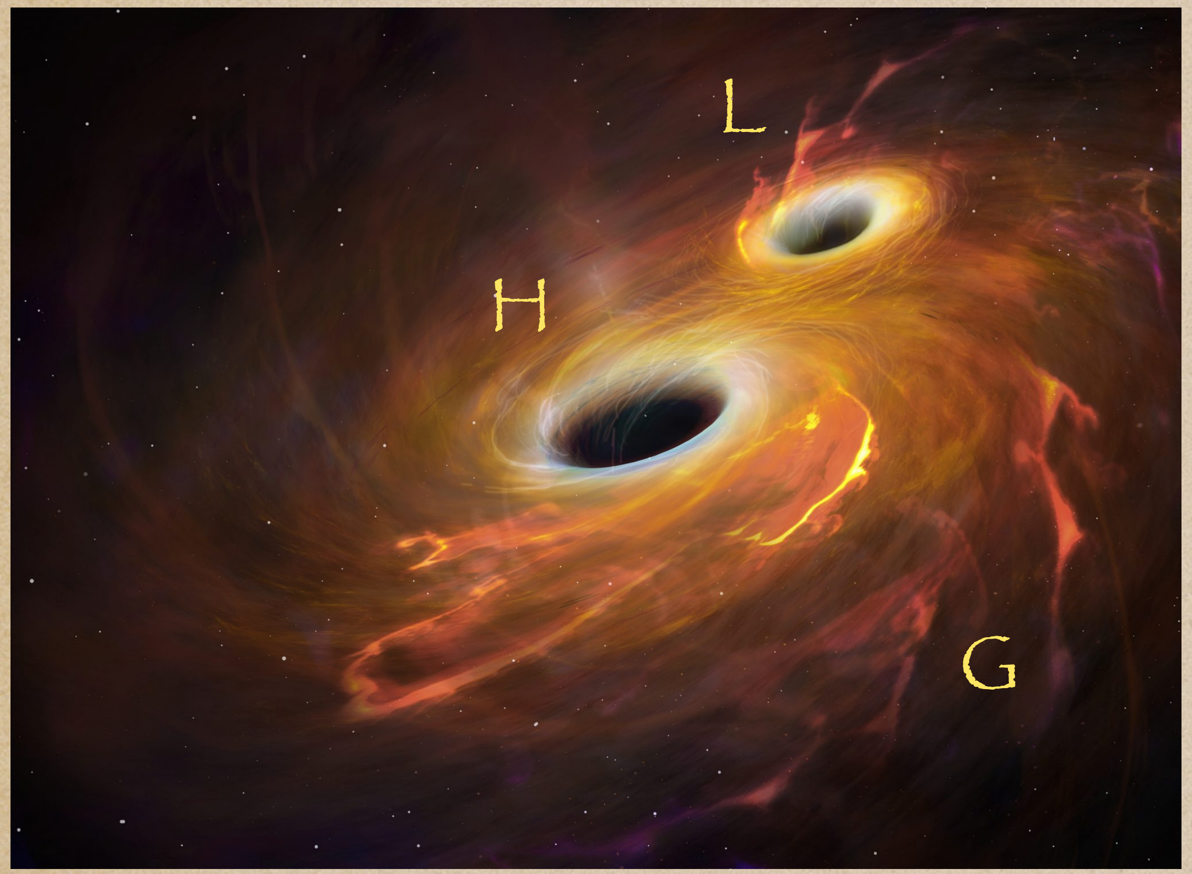


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Recoil of the Heavy Source

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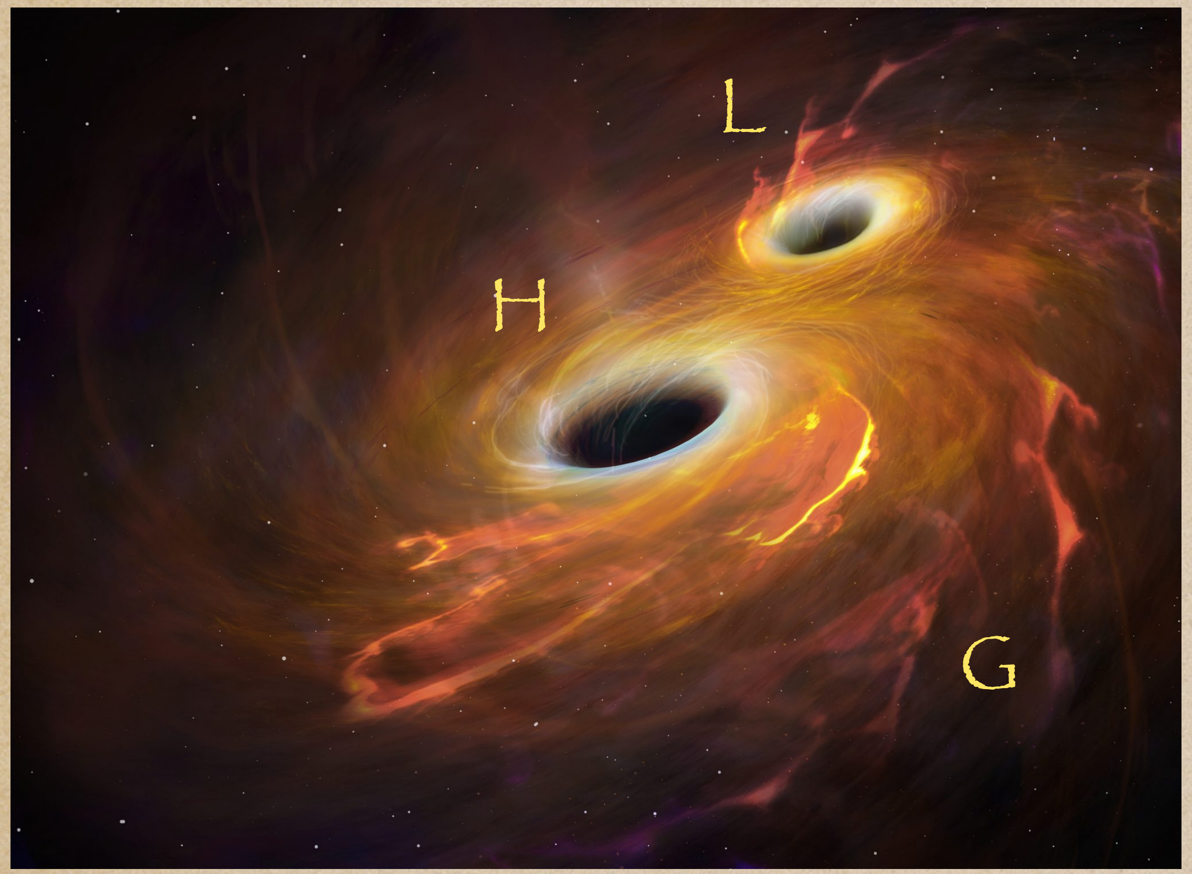


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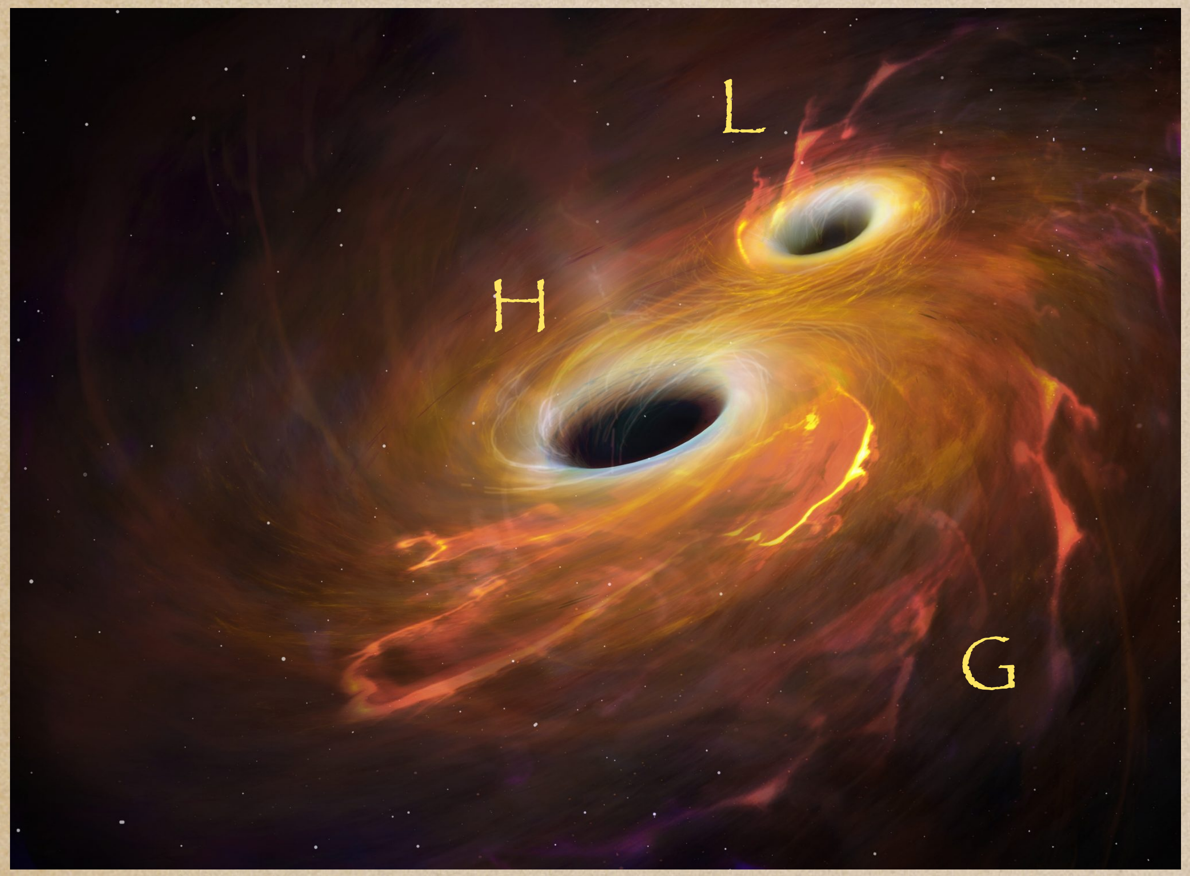


Artistic Impression
Credit: MARK GARLICK / SCIENCE PHOTO / Getty images

Follow WQFT [Mogull, Plefka and Steinhoff, +Jakobsen, +Sauer, +Xu]

$$S = S_{\text{G}} + S_{\text{L}} + \underline{S_{\text{H}}}$$

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Recoil of the Heavy Source

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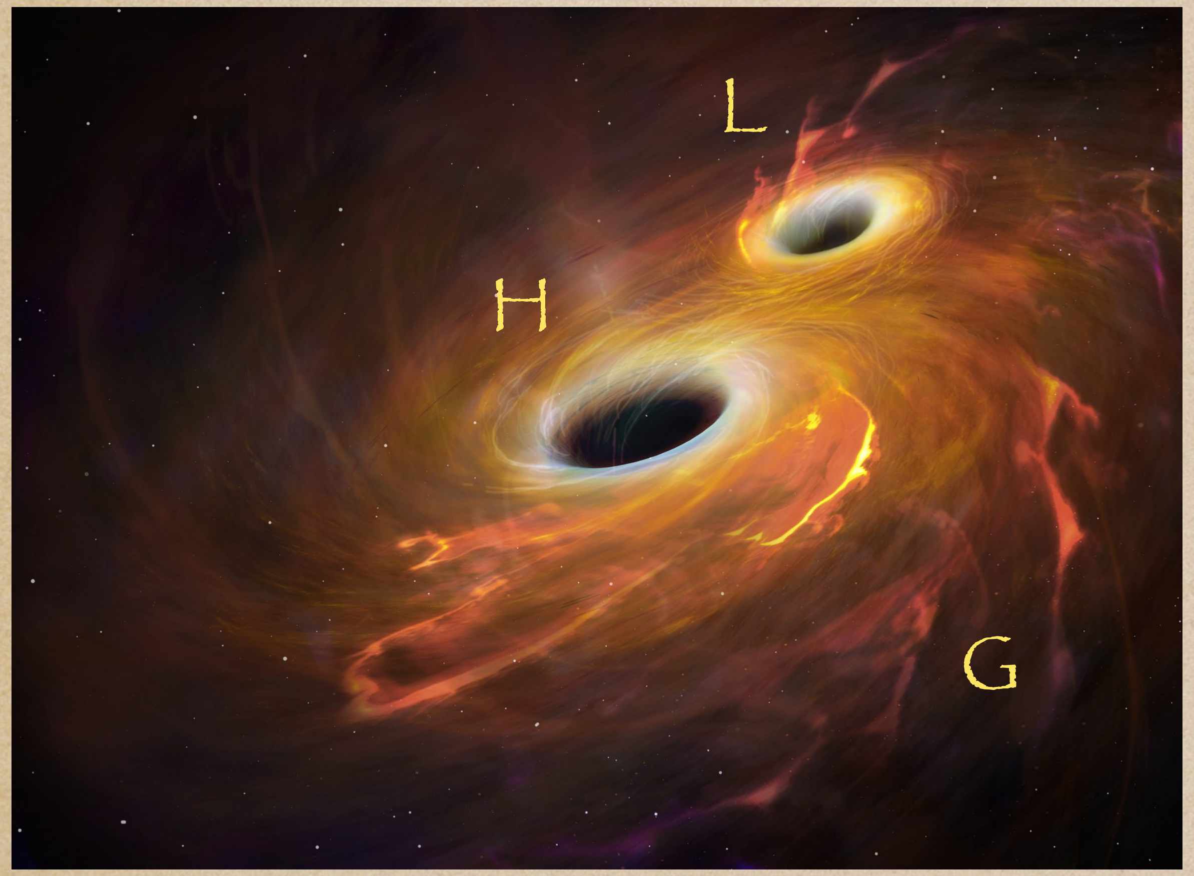


Artistic Impression
Credit: MARK GARLICK / SCIENCE PHOTO / Getty images

Follow WQFT [Mogull, Plefka and Steinhoff, +Jakobsen, +Sauer, +Xu]

$$S_{\mathrm{H}} = -\frac{M}{2}\int d\tau \left(g_{\mu\nu}\dot{x}^\mu \dot{x}^\nu + 1\right), \quad x^\mu = v^\mu \tau + z^\mu, \quad v^\mu = (1,0,0,0)$$

$$S = S_{\mathrm{G}} + S_{\mathrm{L}} + \underline{S_{\mathrm{H}}}$$

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Recoil of the Heavy Source

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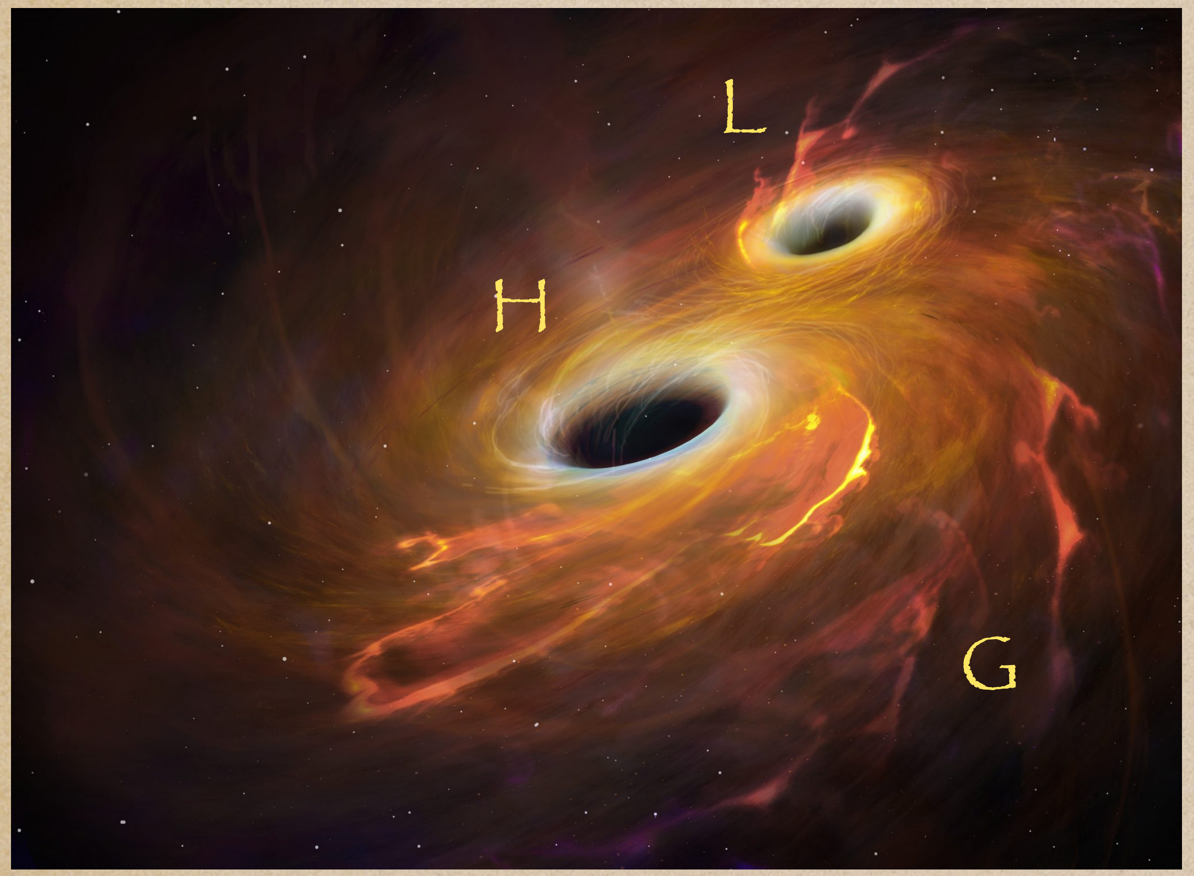

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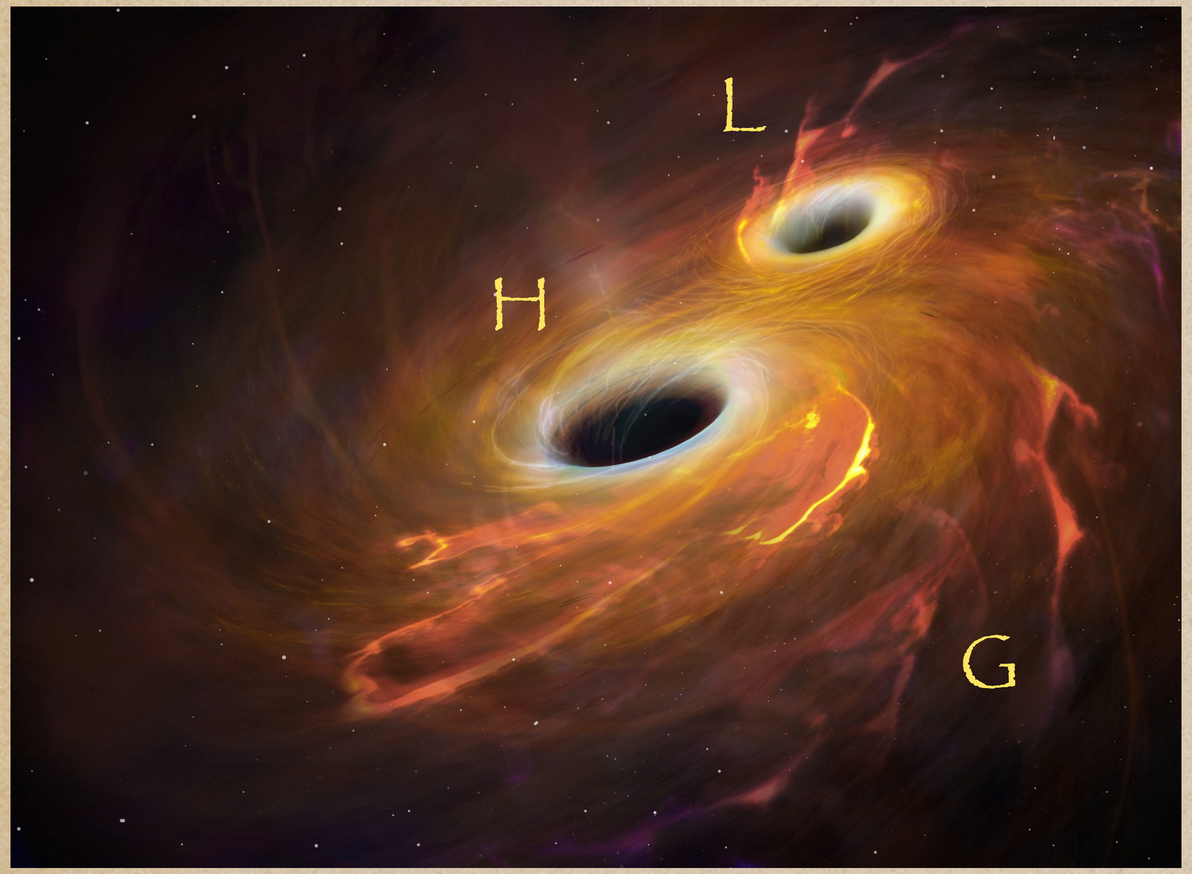
$$S_{\mathrm{H}} = -M \int d\tau$$

$$S_{\mathrm{H}} = -\frac{M}{2} \int d\tau \left( g_{\mu\nu} \dot{x}^\mu \dot{x}^\nu + 1 \right), \quad x^\mu = v^\mu \tau + z^\mu, \quad v^\mu = (1,0,0,0)$$

$$\approx -\frac{M}{2} \int d\tau \left( \bar{g}_{\mu\nu} \dot{x}^\mu \dot{x}^\nu + \kappa h_{\mu\nu} \dot{x}^\mu \dot{x}^\nu \right)$$

Recoil of the Heavy Source

# Beyond OSF 2: The Spacetime Goldstones

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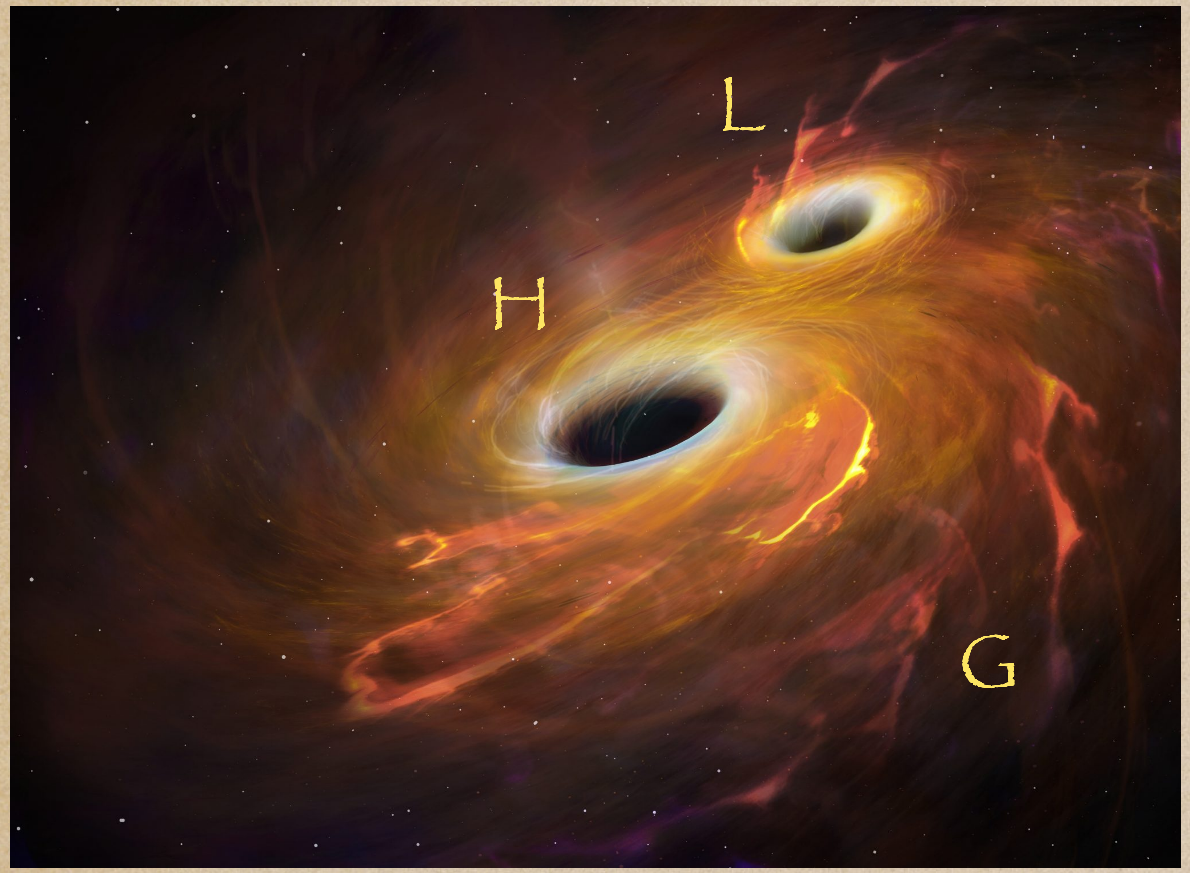
$$= -\frac{M}{2}\int d\tau\, \delta\bar{g}_{\mu\nu}\dot{x}^{\mu}\dot{x}^{\nu} - \frac{M}{2}\int d\tau \left( \eta_{\mu\nu}\dot{x}^{\mu}\dot{x}^{\nu} + \kappa h_{\mu\nu}\dot{x}^{\mu}\dot{x}^{\nu} \right)$$

Recoil of the Heavy Source

# Beyond OSF 2: The Spacetime Goldstones

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Recoil of the Heavy Source

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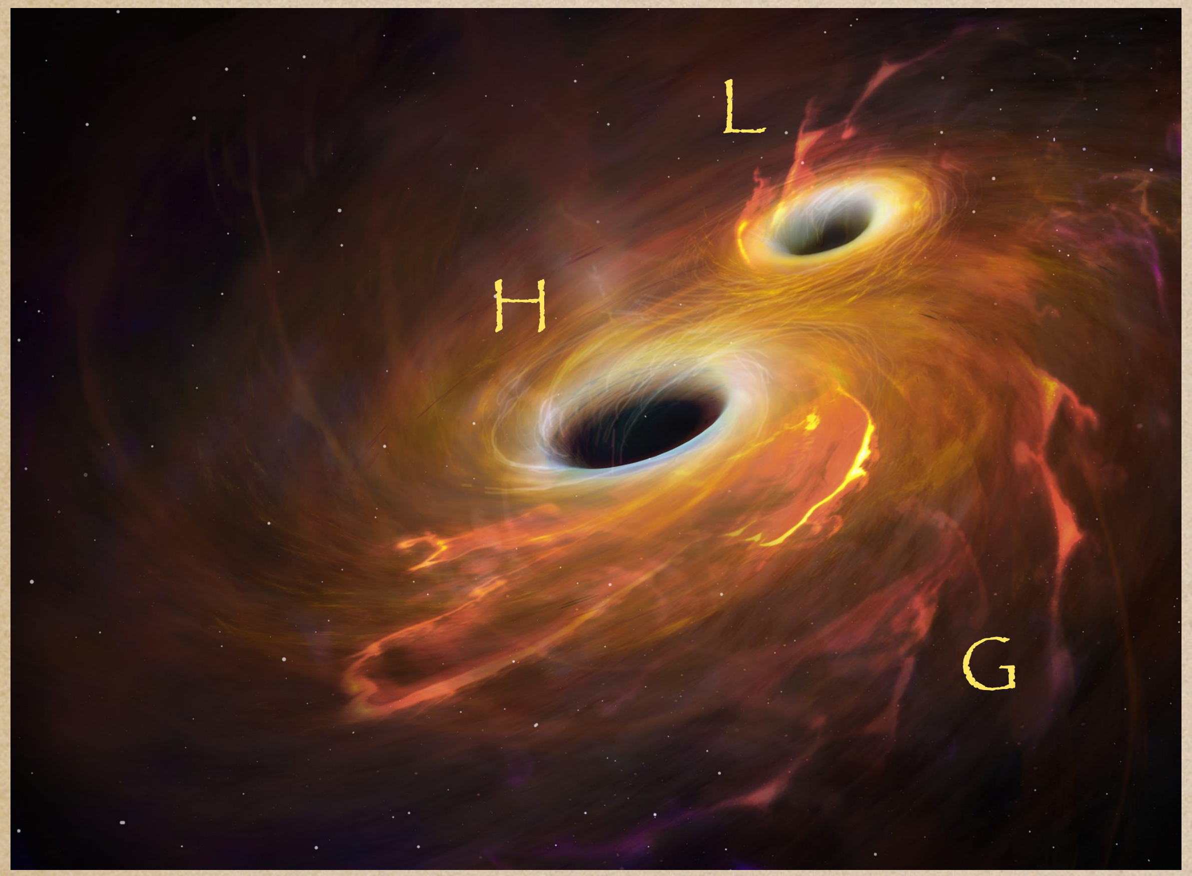


Artistic Impression
Credit: MARK GARLICK / SCIENCE PHOTO / Getty images

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Recoil of the Heavy Source

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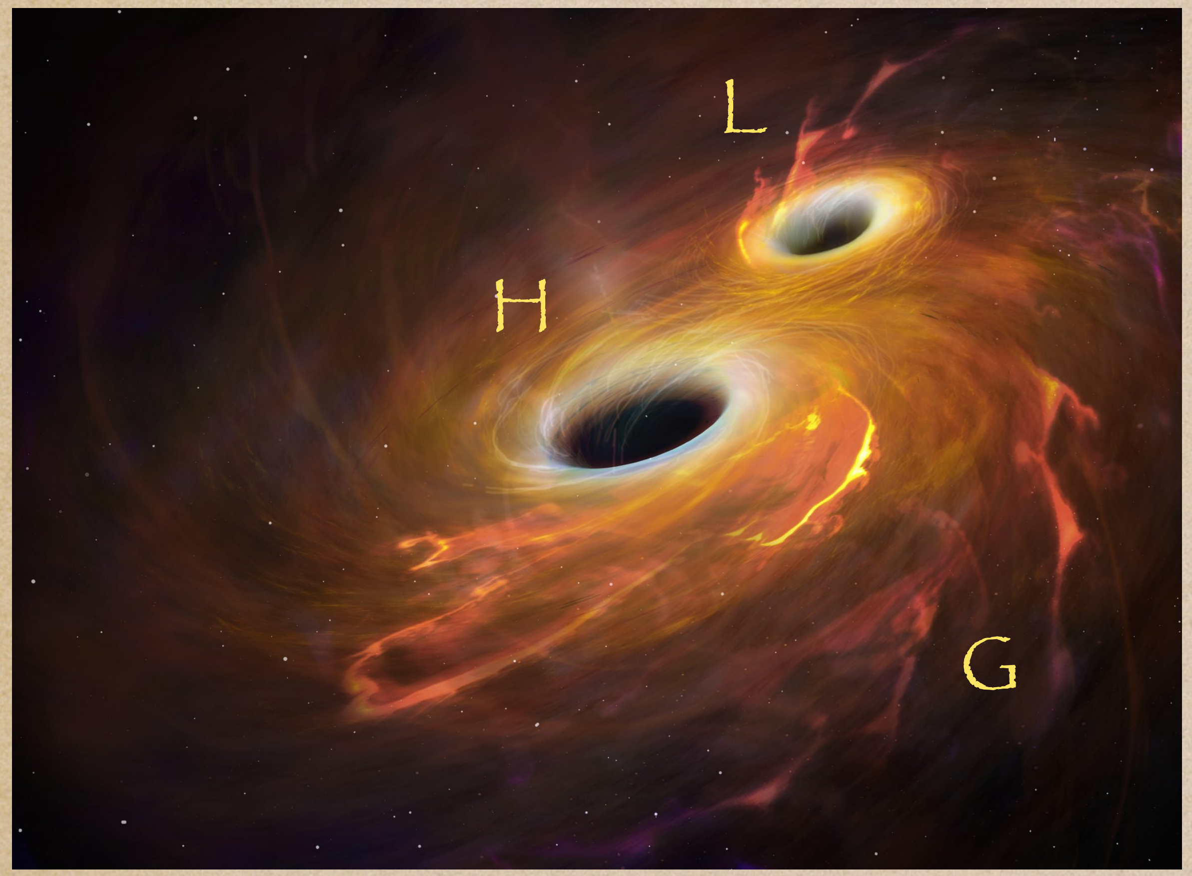


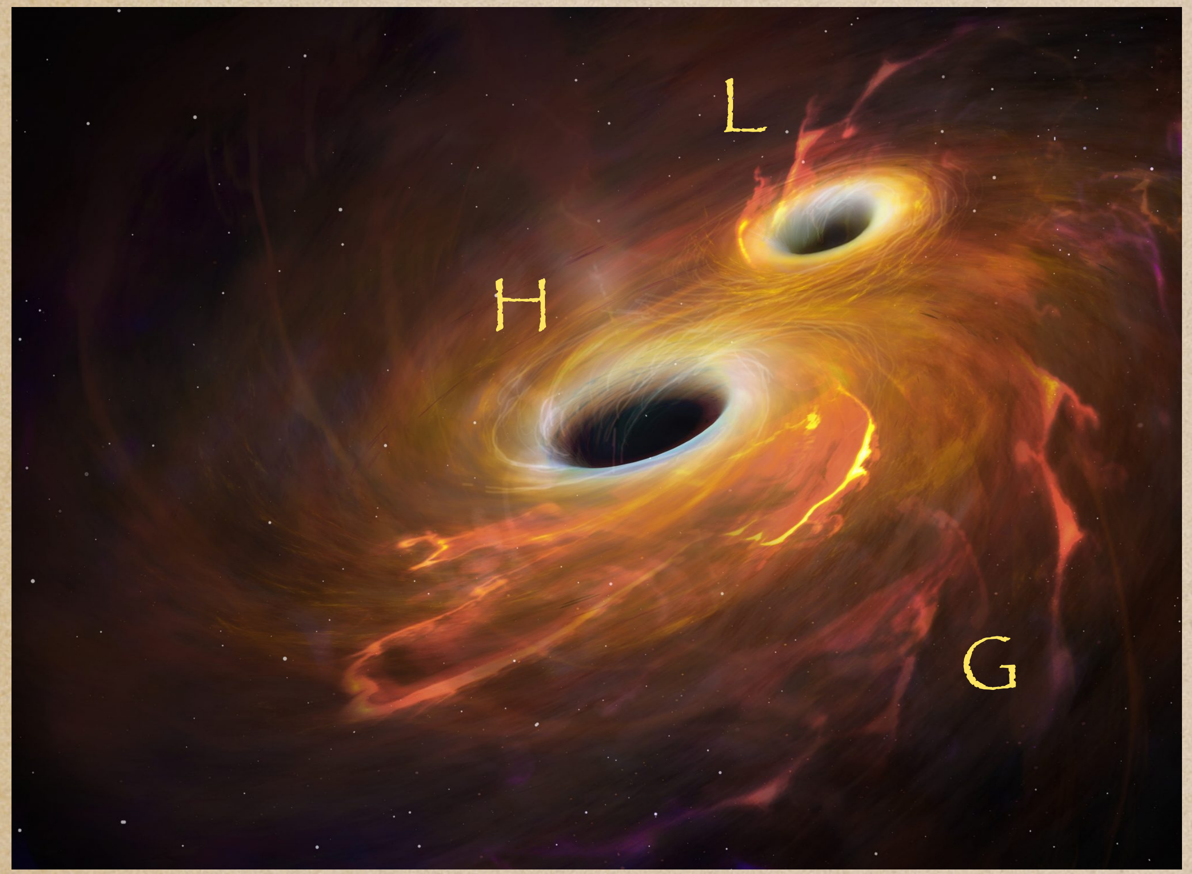

Artistic Impression
Credit: MARK GARLICK / SCIENCE PHOTO / Getty images

$$-\frac{M}{2}\int d\tau\, \delta\bar{g}_{\mu\nu}\dot{x}^{\mu}\dot{x}^{\nu}$$

Divergent

Recoil of the Heavy Source — Understanding the Divergence

# Beyond OSF 2: The Spacetime Goldstones



Artistic Impression
Credit: MARK GARLICK / SCIENCE PHOTO / Getty images

1) It works! Confirmation up to 3PM.

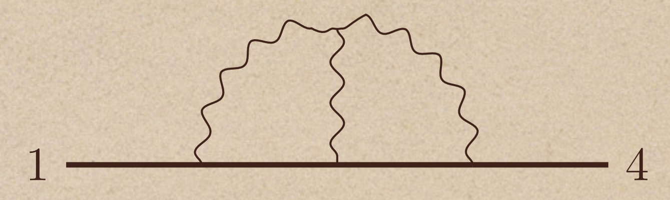
$$-\frac{M}{2}\int d\tau\, \delta\bar{g}_{\mu\nu}\dot{x}^\mu \dot{x}^\nu$$

Divergent

# Beyond OSF 2: The Spacetime Goldstones

1) It works! Confirmation up to 3PM.

2) Would-be flat-space diagrams…


… are zero in the classical limit.

$$\frac{-\frac{M}{2}\int d\tau\, \delta\bar{g}_{\mu\nu}\dot{x}^{\mu}\dot{x}^{\nu}}{\text{Divergent}}$$

# Beyond OSF 2: The Spacetime Goldstones

1) It works! Confirmation up to 3PM.

2) Would-be flat-space diagrams are zero.

3) Dimensional regularization:

$$\delta\bar{g}_{\mu\nu}(0) = \left(-\frac{2GM}{r}\Big|_{r=0}\right)\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&1\end{pmatrix}_{\mu\nu} + \mathcal{O}\left(G^2\right)$$

Isotropic gauge
Take BH at the origin

$$-\frac{M}{2}\int d\tau\,\delta\bar{g}_{\mu\nu}\dot{x}^\mu\dot{x}^\nu$$

Divergent

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Isotropic gauge

Take BH at the origin

$$\frac{1}{r} \doteq \int \frac{d^{3-2\epsilon}\mathbf{q}}{(2\pi)^{3-2\epsilon}} e^{i\mathbf{r}\cdot\mathbf{q}} \frac{4\pi}{\mathbf{q}^2}$$

$$\frac{-\frac{M}{2}\int d\tau\, \delta\bar{g}_{\mu\nu}\dot{x}^\mu \dot{x}^\nu}{\text{Divergent}}$$

# Beyond OSF 2: The Spacetime Goldstones

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$$-\frac{M}{2}\int d\tau\, \delta\bar{g}_{\mu\nu}\dot{x}^\mu\dot{x}^\nu$$

Divergent

$$\left(\frac{GM}{r}\Big|_{r=0}\right) \doteq \int \frac{d^{3-2\epsilon}\mathbf{q}}{(2\pi)^{3-2\epsilon}} \left(e^{i\mathbf{r}\cdot\mathbf{q}}\Big|_{r=0}\right)\frac{4\pi GM}{\mathbf{q}^2} = \int \frac{d^{3-2\epsilon}\mathbf{q}}{(2\pi)^{3-2\epsilon}} \frac{4\pi GM}{\mathbf{q}^2} = 0$$

# Beyond OSF 2: The Spacetime Goldstones

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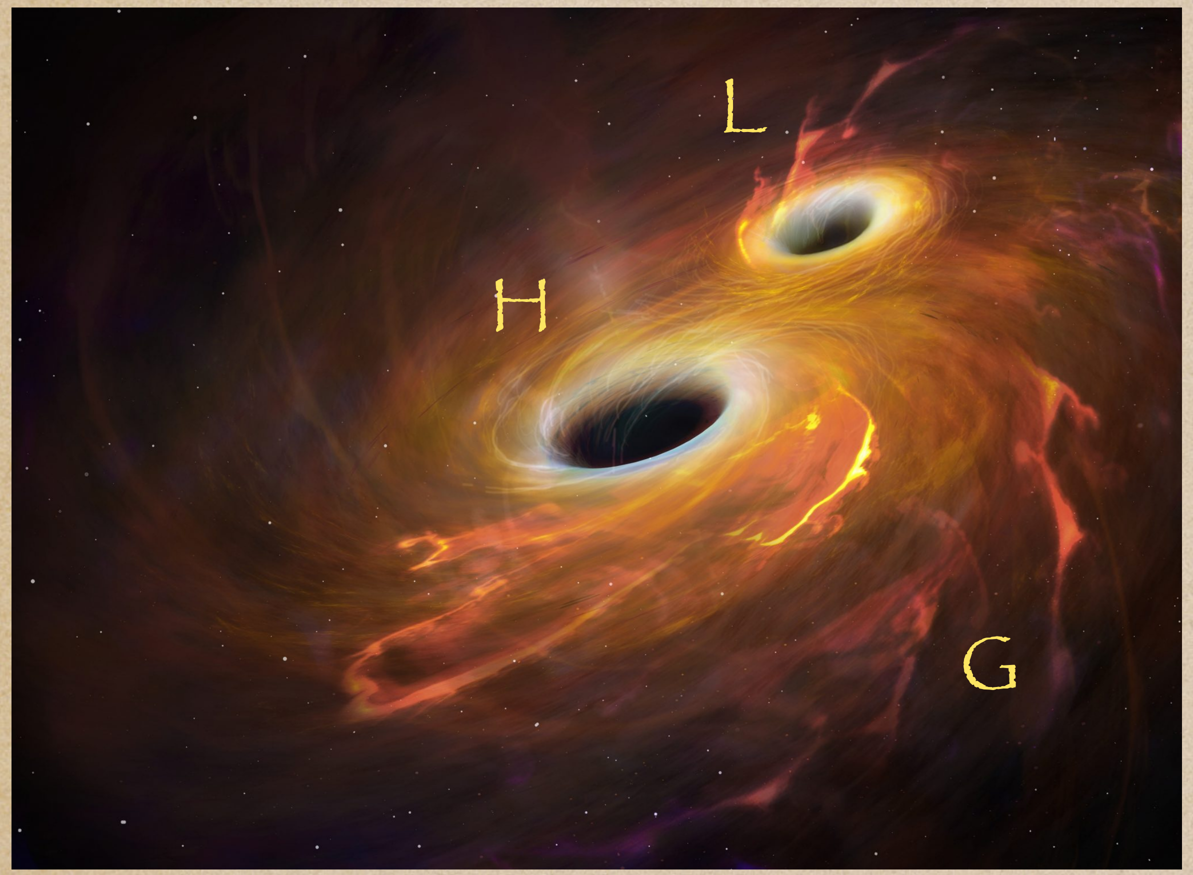
$$\left(\frac{GM}{r}\Big|_{r=0}\right) \doteq \int \frac{d^{3-2\epsilon}\mathbf{q}}{(2\pi)^{3-2\epsilon}} \left(e^{i\mathbf{r}\cdot\mathbf{q}}\big|_{r=0}\right) \frac{4\pi GM}{\mathbf{q}^2} = \int \frac{d^{3-2\epsilon}\mathbf{q}}{(2\pi)^{3-2\epsilon}} \frac{4\pi GM}{\mathbf{q}^2} = 0$$

Power-law divergencies are zero in dim. reg.

# Stability of the Background



Artistic Impression
Credit: MARK GARLICK / SCIENCE PHOTO / Getty images

$$S_G\Big|_{\text{linear-in-h}} = \int d^4x\sqrt{-\bar{g}}\left(\frac{1}{16\pi G}\bar{G}^{\mu\nu}\right)\kappa h_{\mu\nu}$$

$$S_H\Big|_{\text{linear-in-h}} = \int d\tau\left(-\frac{1}{2}Mv^{\mu}v^{\nu}\right)\kappa h_{\mu\nu}$$

*verified to leading order in G

Linear-in-h terms cancel against each other

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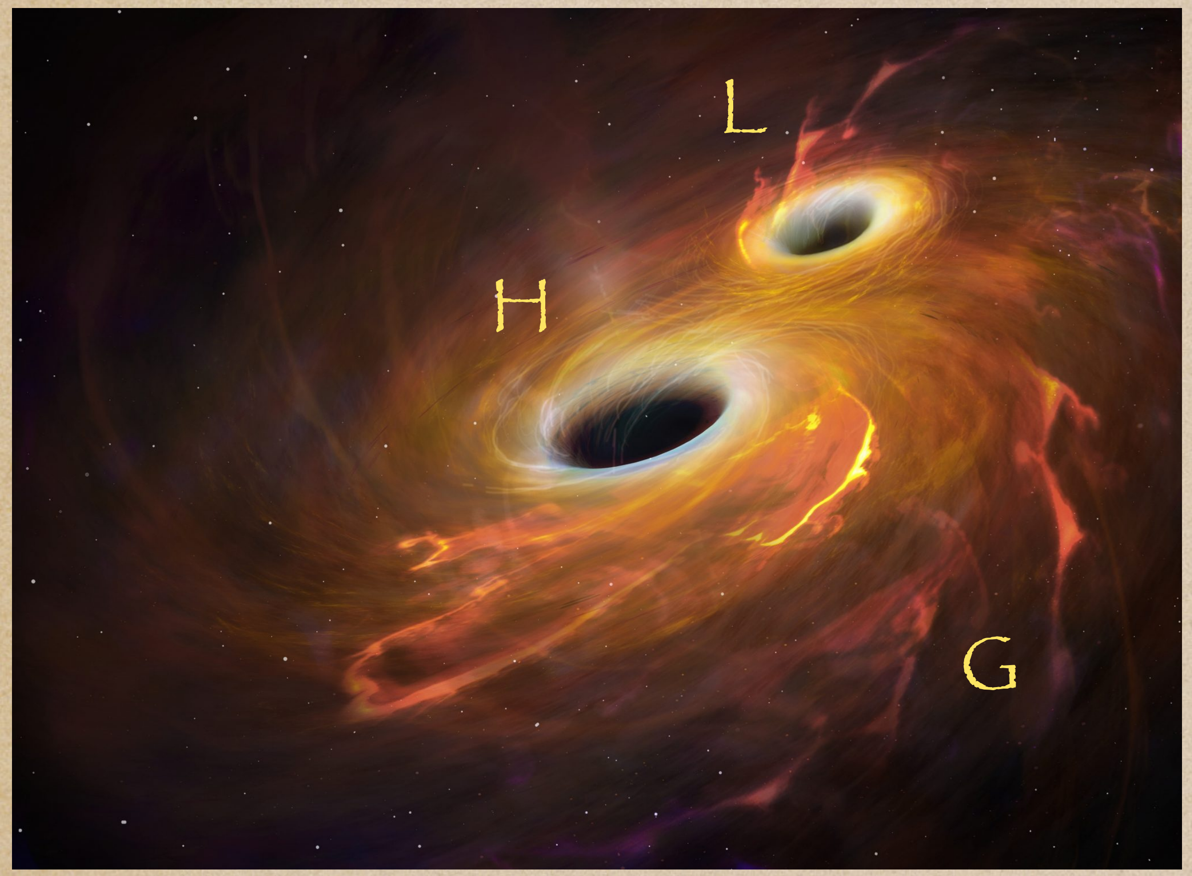

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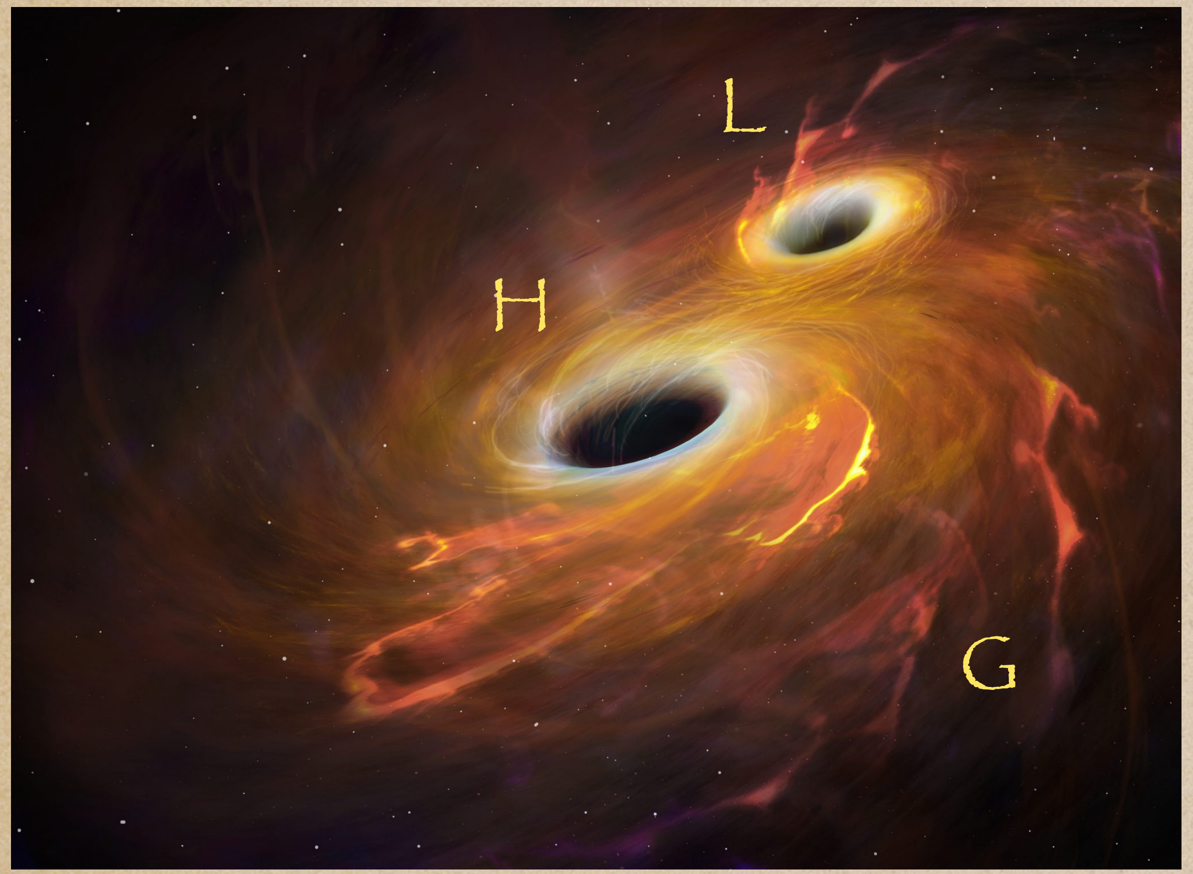

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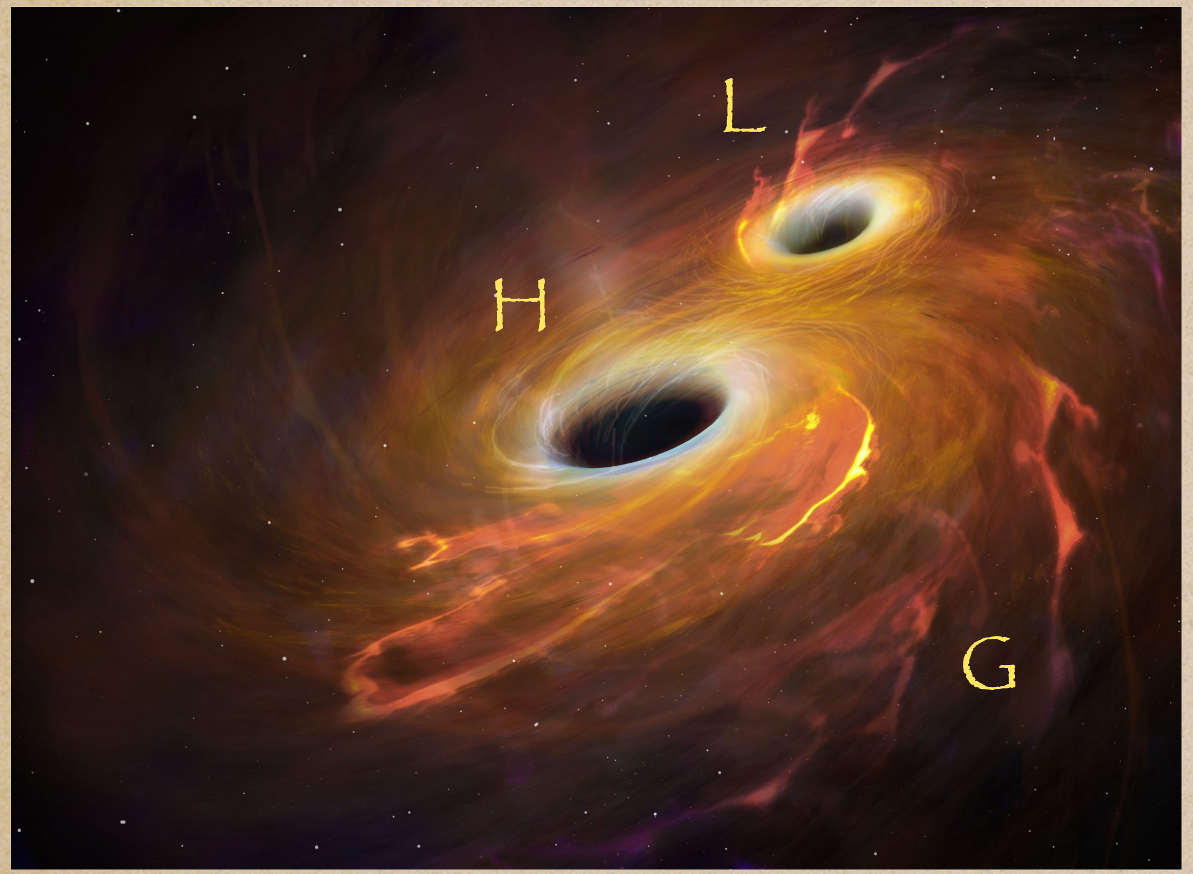
$$\sqrt{-\bar{g}}\left(\frac{1}{16\pi G}\bar{G}^{\mu\nu}\right) = -\frac{\kappa M}{8\pi}v^\mu v^\nu\left(\nabla^2\frac{1}{r}\right) + \mathcal{O}\left(\kappa^3\right)$$

Isotropic gauge
Take BH at the origin

Linear-in-h terms cancel against each other

# Stability of the Background




Artistic Impression
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$$S_{\mathrm{G}}\Big|_{\text{linear-in-h}} = \int d^4x\sqrt{-\bar{g}}\left(\frac{1}{16\pi G}\bar{G}^{\mu\nu}\right)\kappa h_{\mu\nu}$$

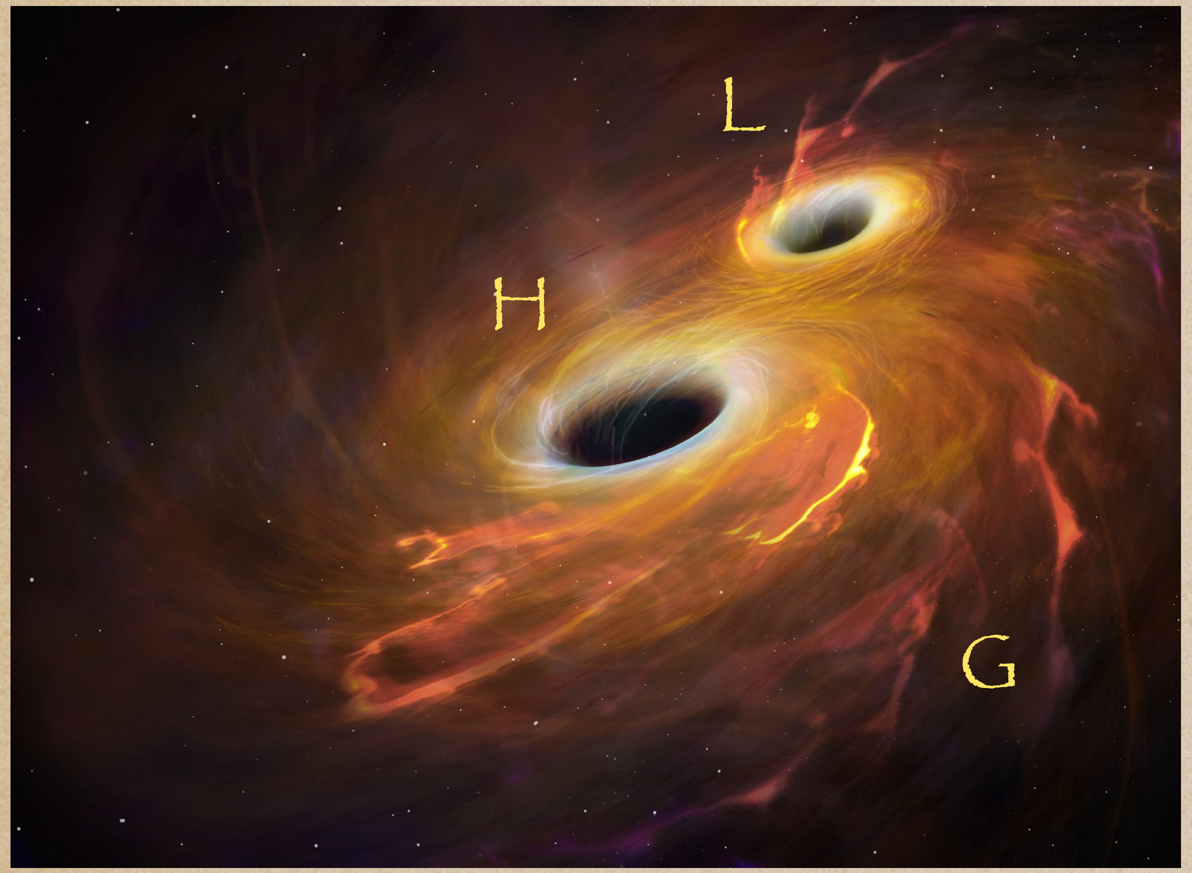
$$S_{\mathrm{H}}\Big|_{\text{linear-in-h}} = \int d\tau\left(-\frac{1}{2}Mv^{\mu}v^{\nu}\right)\kappa h_{\mu\nu}$$

$$\sqrt{-\bar{g}}\left(\frac{1}{16\pi G}\bar{G}^{\mu\nu}\right) = -\frac{\kappa M}{8\pi}v^{\mu}v^{\nu}\left(\nabla^2\frac{1}{r}\right) + \mathcal{O}\left(\kappa^3\right)$$

$$\left(\nabla^2\frac{1}{r}\right) \doteq \int\frac{d^{3-2\epsilon}\mathbf{q}}{(2\pi)^{3-2\epsilon}}\frac{4\pi}{\mathbf{q}^2}\nabla^2 e^{i\mathbf{r}\cdot\mathbf{q}} = -4\pi\int\frac{d^{3-2\epsilon}\mathbf{q}}{(2\pi)^{3-2\epsilon}}e^{i\mathbf{r}\cdot\mathbf{q}}$$

Linear-in-h terms cancel against each other

# Feynman Rules at 1SF




Artistic Impression
Credit: MARK GARLICK / SCIENCE PHOTO / Getty images

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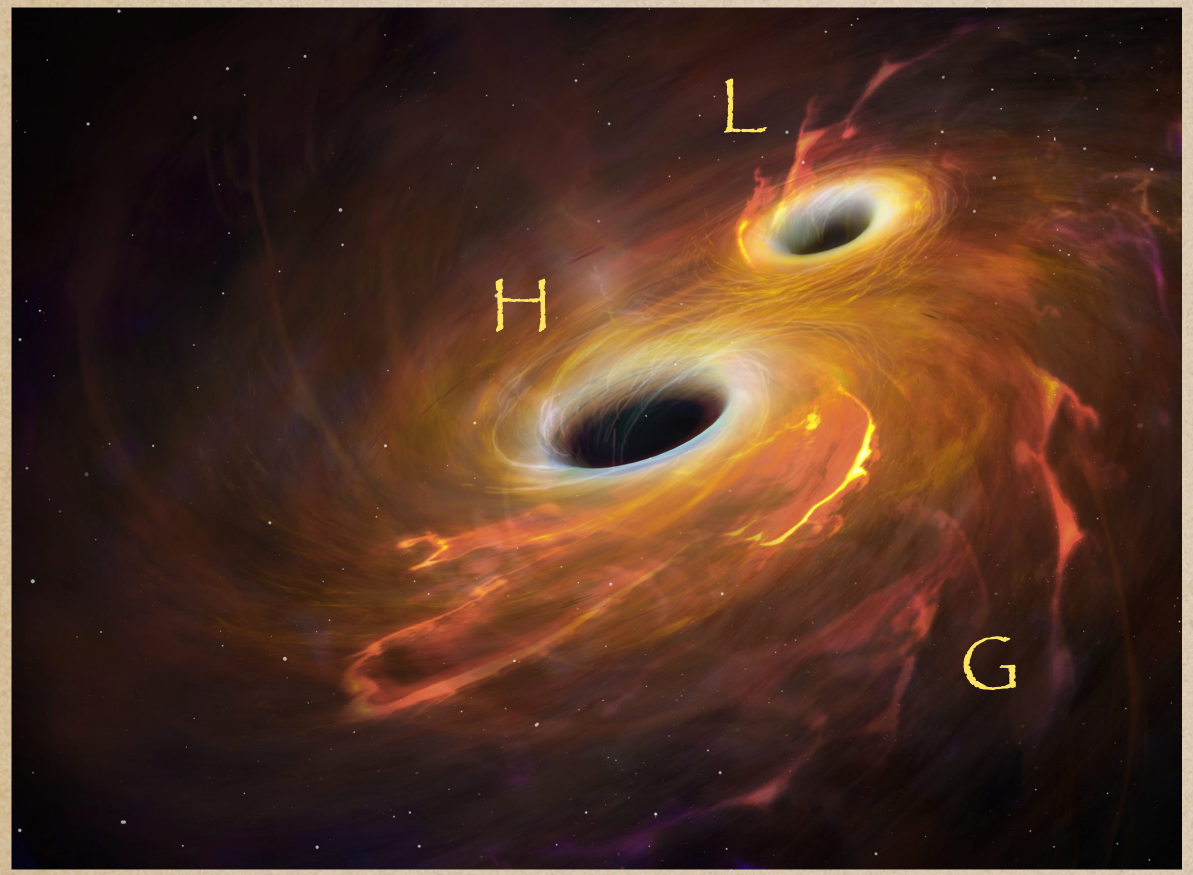
Rules Sufficient for 1SF Calculations at Any Order in $G$

# Feynman Rules at 1SF




Artistic Impression
Credit: MARK GARLICK / SCIENCE PHOTO / Getty images

$S_G$

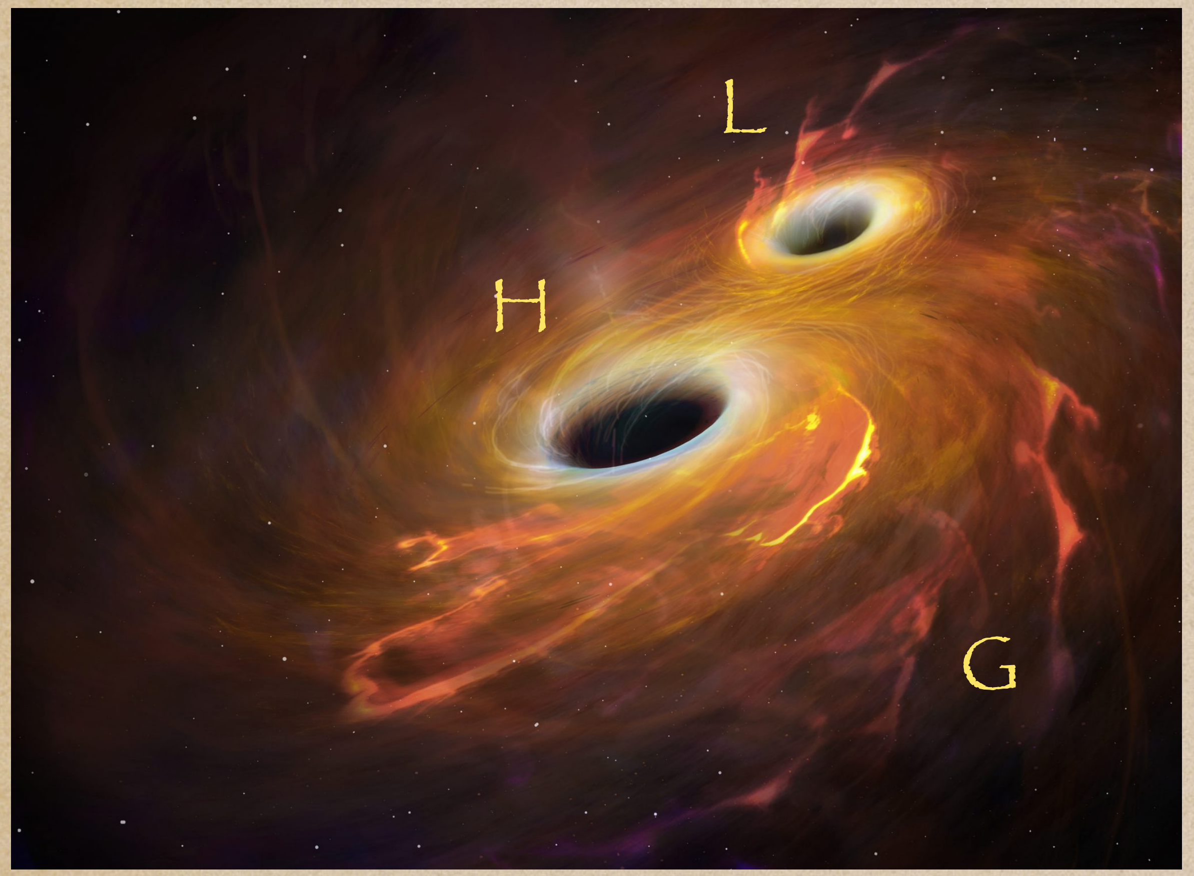
$S_{G,\,\text{free}} = S_G\big|_{\tilde{g}\to\eta}$ → $\mu\nu \;\overset{q}{\sim\sim\sim}\; \alpha\beta \;=\; \frac{i}{q^2}\mathcal{P}^{\mu\nu\alpha\beta}\,, \quad \mathcal{P}^{\mu\nu\alpha\beta} = \frac{1}{2}\left(\eta^{\mu\alpha}\eta^{\nu\beta} + \eta^{\nu\alpha}\eta^{\mu\beta} - \eta^{\mu\nu}\eta^{\alpha\beta}\right)$

$S_{G,\,\text{int}} = S_G - S_{G,\,\text{free}}$ → $(q_1, \mu\nu) \sim\sim\bullet\sim\sim (q_2, \alpha\beta)$

$S = S_G + S_L + S_H$

Rules Sufficient for 1SF Calculations at Any Order in $G$

# Feynman Rules at 1SF

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$S_G$

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$S_{G,\,\text{int}} = S_G - S_{G,\,\text{free}}$ → $(q_1, \mu\nu) \sim\!\sim\!\bullet\!\sim\!\sim (q_2, \alpha\beta)$

$S_H$

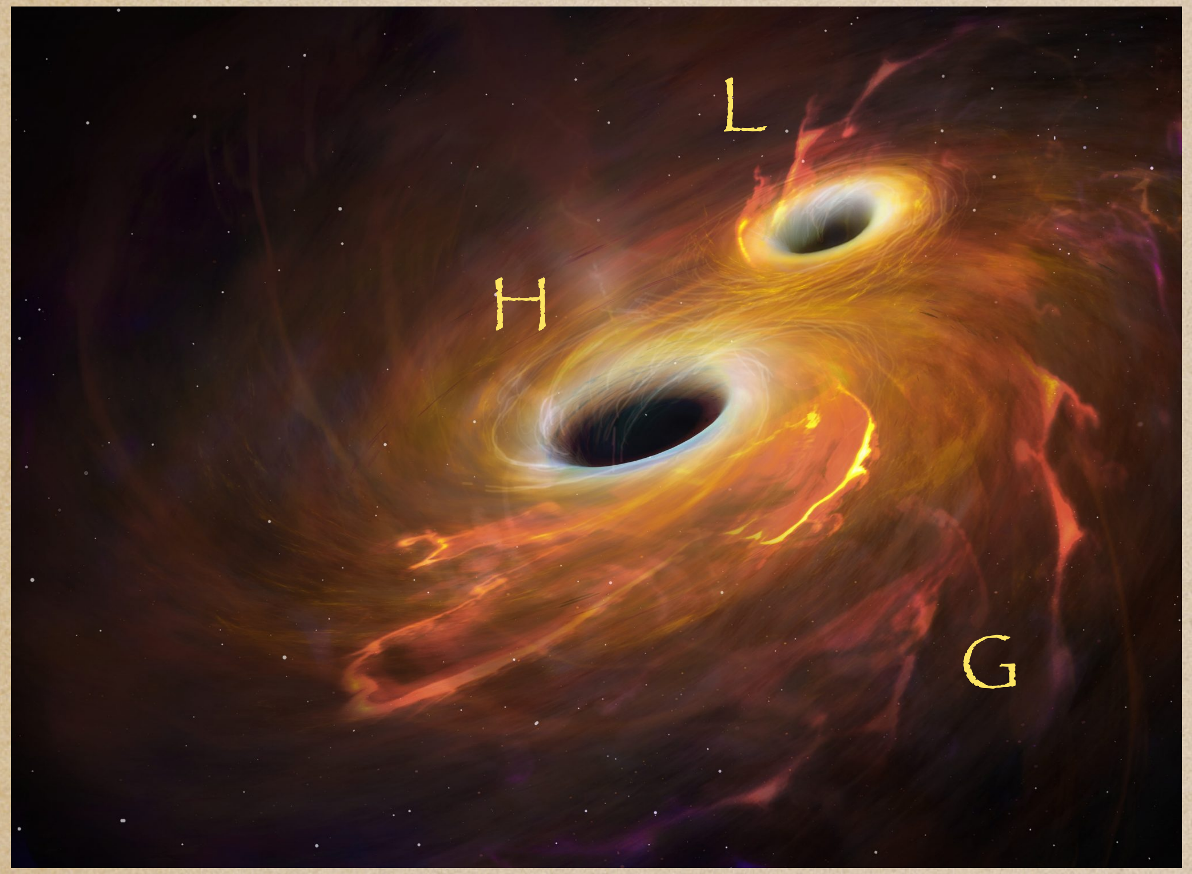
→ $\alpha$ - - - - - $\omega$ - - - - - $\beta$ $= -\frac{i}{M\omega^2}\eta^{\alpha\beta}$

→ $(\omega, \alpha)$ - - - - - $\sim\!\sim\!\sim$ $(q, \mu\nu)$

$$S = S_G + S_L + S_H$$

Rules Sufficient for 1SF Calculations at Any Order in $G$

# Feynman Rules at 1SF




Artistic Impression
Credit: MARK GARLICK / SCIENCE PHOTO / Getty images

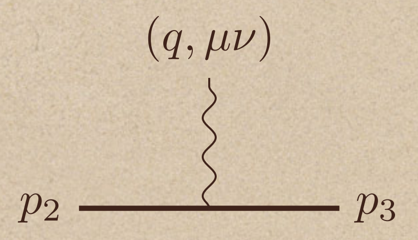
$S_G$ → ($S_{G,\text{free}} = S_G\big|_{\bar{g}\to\eta}$) → $\mu\nu \sim\sim_q\sim\sim\, \alpha\beta \;=\; \frac{i}{q^2}\mathcal{P}^{\mu\nu\alpha\beta}\,, \qquad \mathcal{P}^{\mu\nu\alpha\beta} = \frac{1}{2}\left(\eta^{\mu\alpha}\eta^{\nu\beta} + \eta^{\nu\alpha}\eta^{\mu\beta} - \eta^{\mu\nu}\eta^{\alpha\beta}\right)$

$S_G$ → ($S_{G,\text{int}} = S_G - S_{G,\text{free}}$) → $(q_1, \mu\nu) \sim\sim\bullet\sim\sim (q_2, \alpha\beta)$

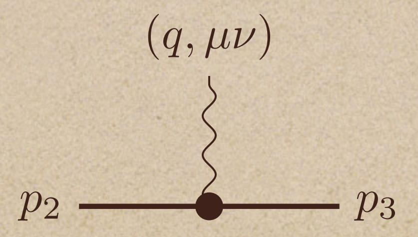
$S_H$ → $\alpha \cdots_\omega\cdots \beta \;=\; -\frac{i}{M\omega^2}\eta^{\alpha\beta}$

$S_H$ → $(\omega, \alpha) \cdots\sim\sim (q, \mu\nu)$

$S_L$ →



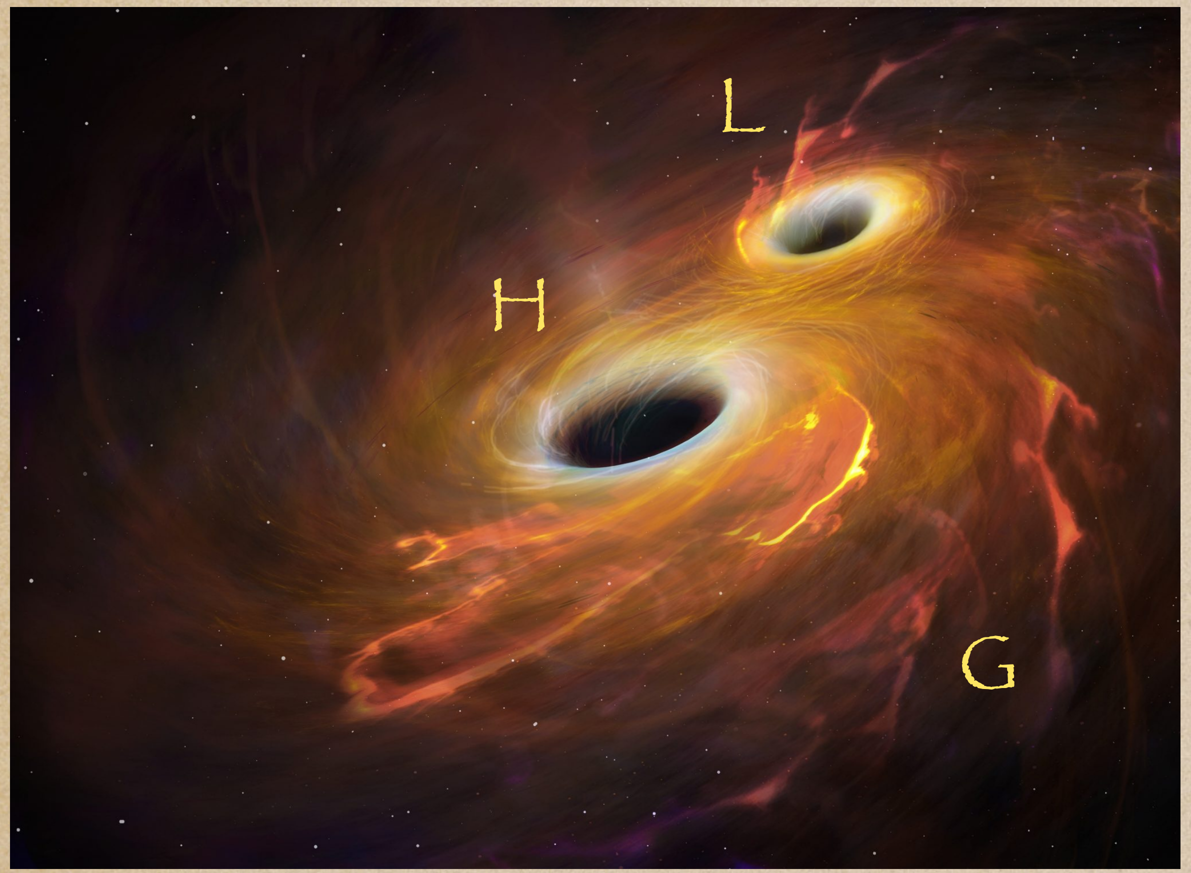

$S_L$ →



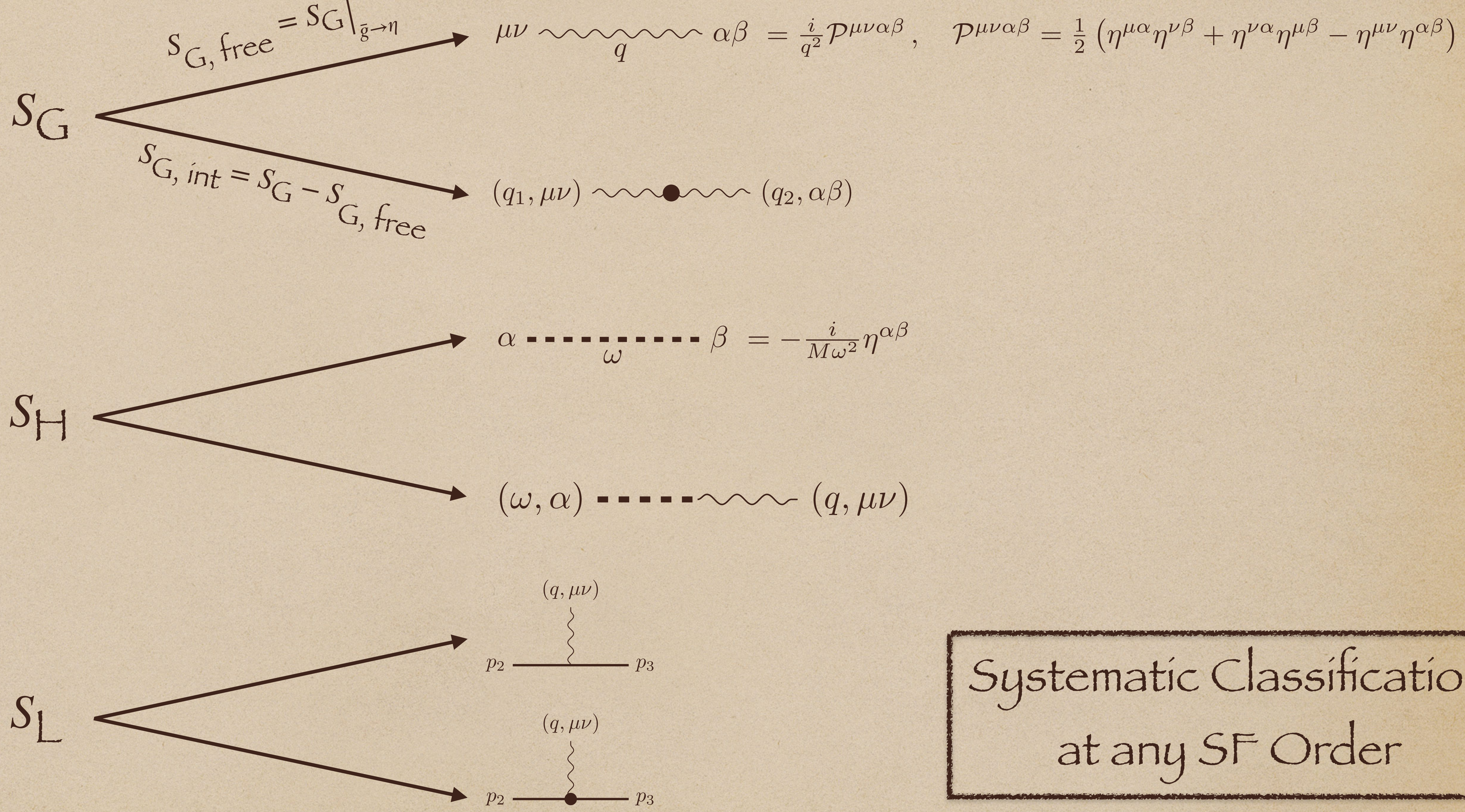

$$S = S_G + S_L + S_H$$

Rules Sufficient for 1SF Calculations at Any Order in ***G***

# Feynman Rules at 1SF




Artistic Impression
Credit: MARK GARLICK / SCIENCE PHOTO / Getty images




$$S = S_G + S_L + S_H$$

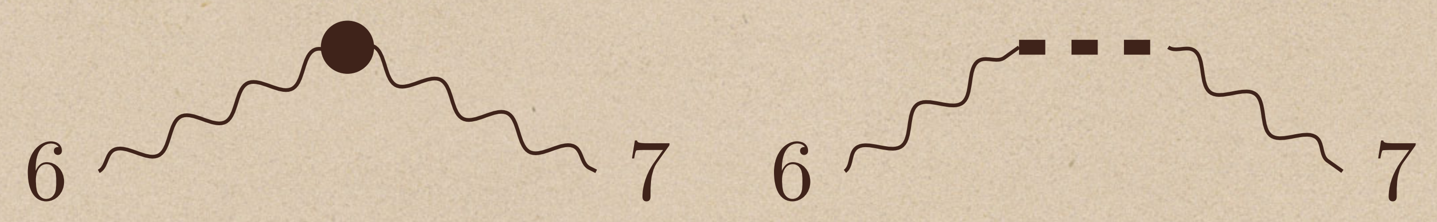
Systematic Classification at any SF Order

Rules Sufficient for 1SF Calculations at Any Order in $G$

# 1SF Amplitude at $\mathcal{O}(G^2)$

# 1SF Amplitude at $\mathcal{O}(G^2)$

Compton



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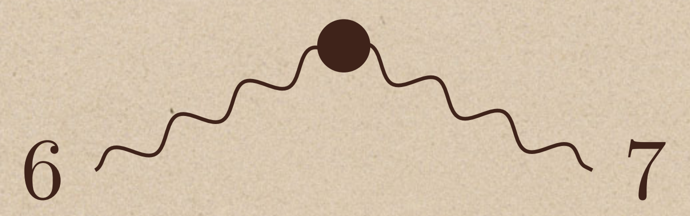



$$\mathcal{A}_{\rm C} = -\frac{8\pi GM}{\omega^2(q_6\cdot q_7)}\Big(\omega^2(\varepsilon_6\cdot\varepsilon_7) - (v\cdot\varepsilon_6)(v\cdot\varepsilon_7)(q_6\cdot q_7) + \omega\big((v\cdot\varepsilon_7)(\varepsilon_6\cdot q_7) - (v\cdot\varepsilon_6)(\varepsilon_7\cdot q_6)\big)\Big)^2$$

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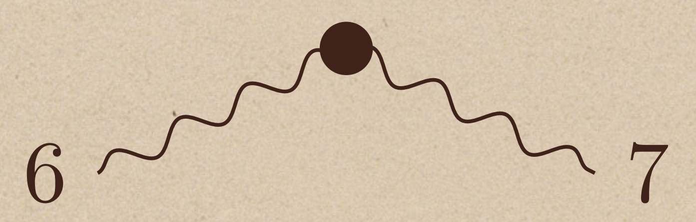
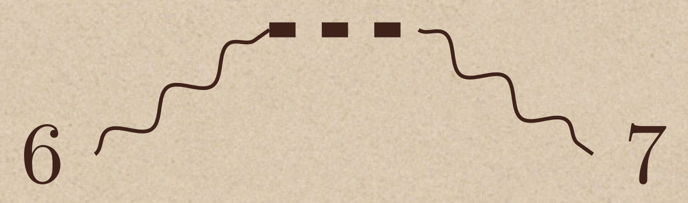
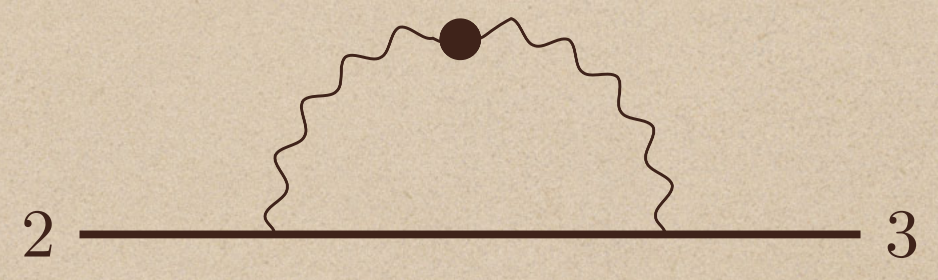


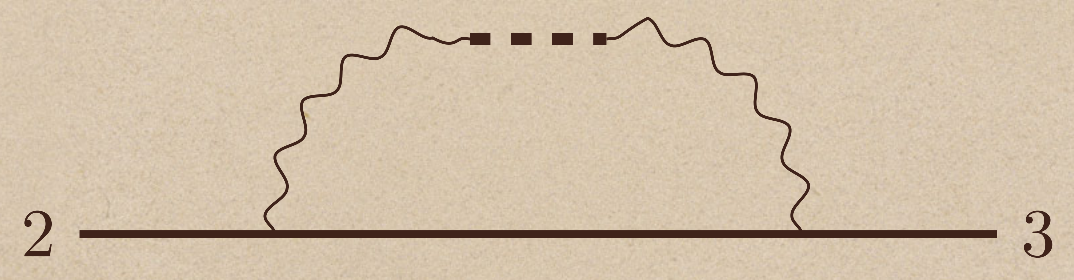




$$\mathcal{A}_C = -\frac{8\pi GM}{\omega^2(q_6\cdot q_7)}\Big(\omega^2(\varepsilon_6\cdot\varepsilon_7) - (v\cdot\varepsilon_6)(v\cdot\varepsilon_7)(q_6\cdot q_7) + \omega\big((v\cdot\varepsilon_7)(\varepsilon_6\cdot q_7) - (v\cdot\varepsilon_6)(\varepsilon_7\cdot q_6)\big)\Big)^2$$

$1 \rightarrow 1$ Amplitude

($2 \rightarrow 2$ in flat space)



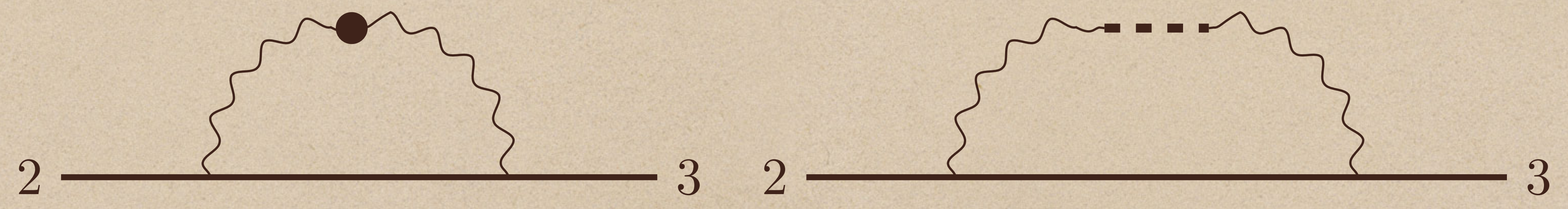
# 1SF Amplitude at $\mathcal{O}(G^2)$

Compton


$$\mathcal{A}_C = -\frac{8\pi G M}{\omega^2 (q_6 \cdot q_7)} \Big( \omega^2 (\varepsilon_6 \cdot \varepsilon_7) - (v \cdot \varepsilon_6)(v \cdot \varepsilon_7)(q_6 \cdot q_7) + \omega \big( (v \cdot \varepsilon_7)(\varepsilon_6 \cdot q_7) - (v \cdot \varepsilon_6)(\varepsilon_7 \cdot q_6) \big) \Big)^2$$

1 → 1 Amplitude

(2 → 2 in flat space)




$$\mathcal{A}_2^{1\mathrm{SF},\, \mathcal{O}(G^2)} = \frac{3\pi^2 G^2 M m^3 (5\gamma^2 - 1)}{|q|}$$

# 1SF Amplitude at $\mathcal{O}(G^3)$

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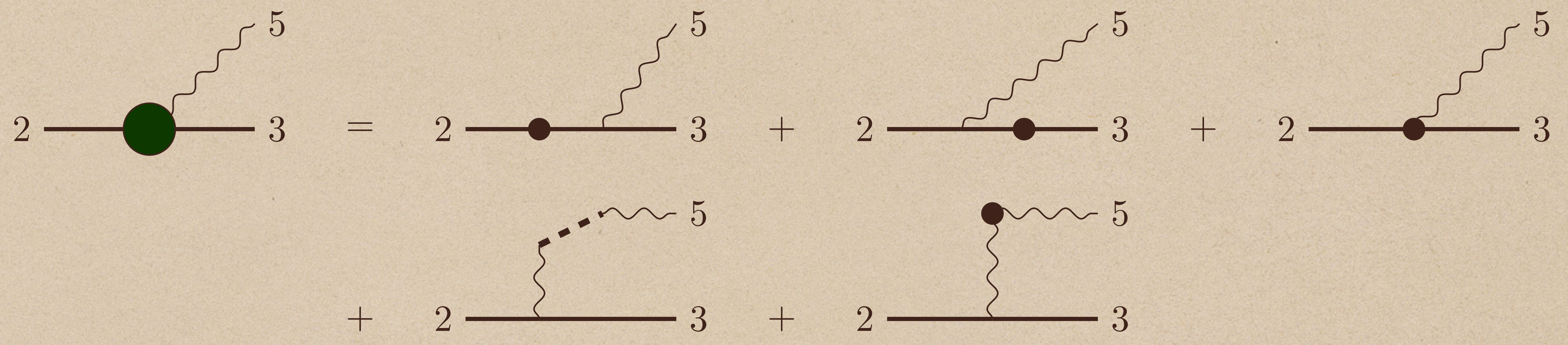
Graviton Emission Amplitude

(2 → 3 in flat space)



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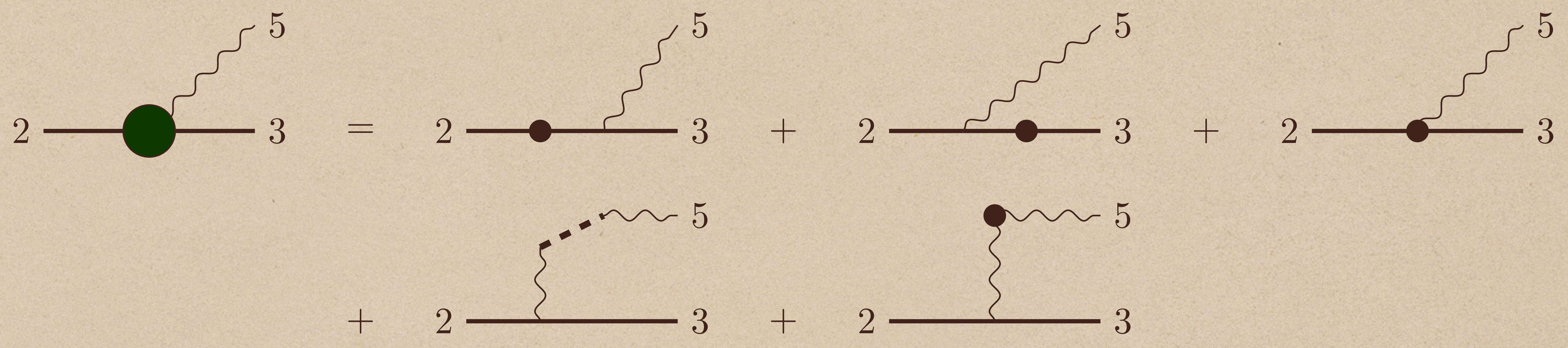
Graviton Emission Amplitude

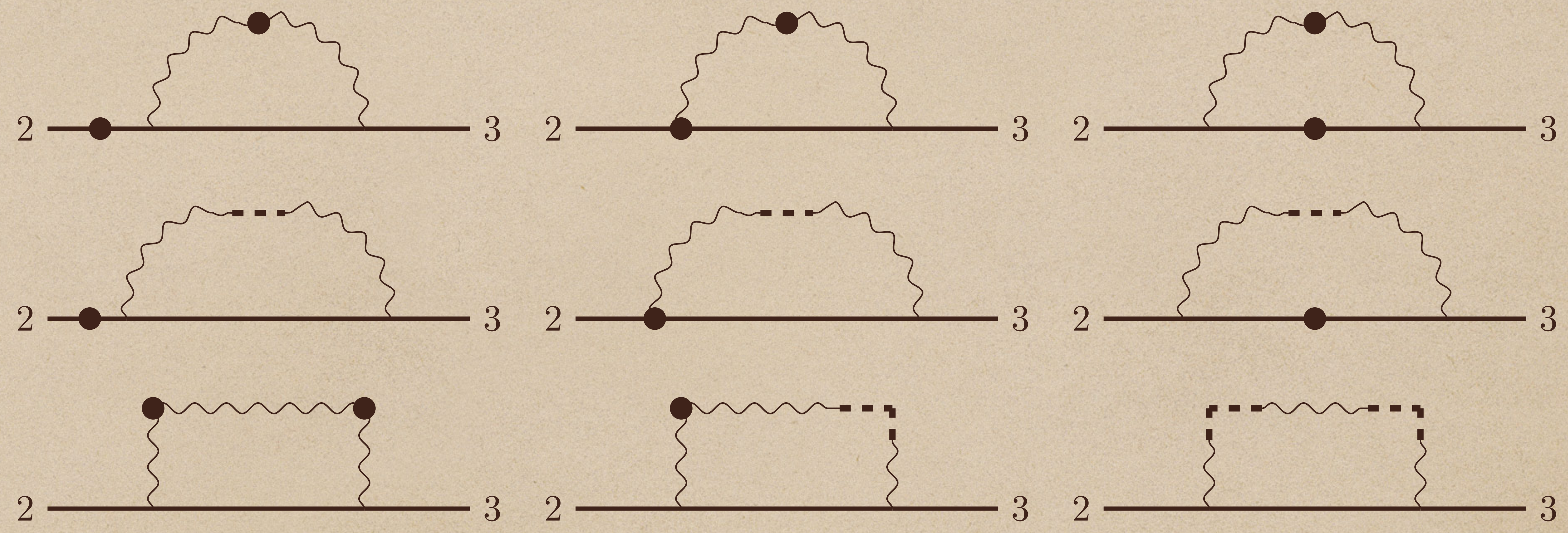
(2 → 3 in flat space)



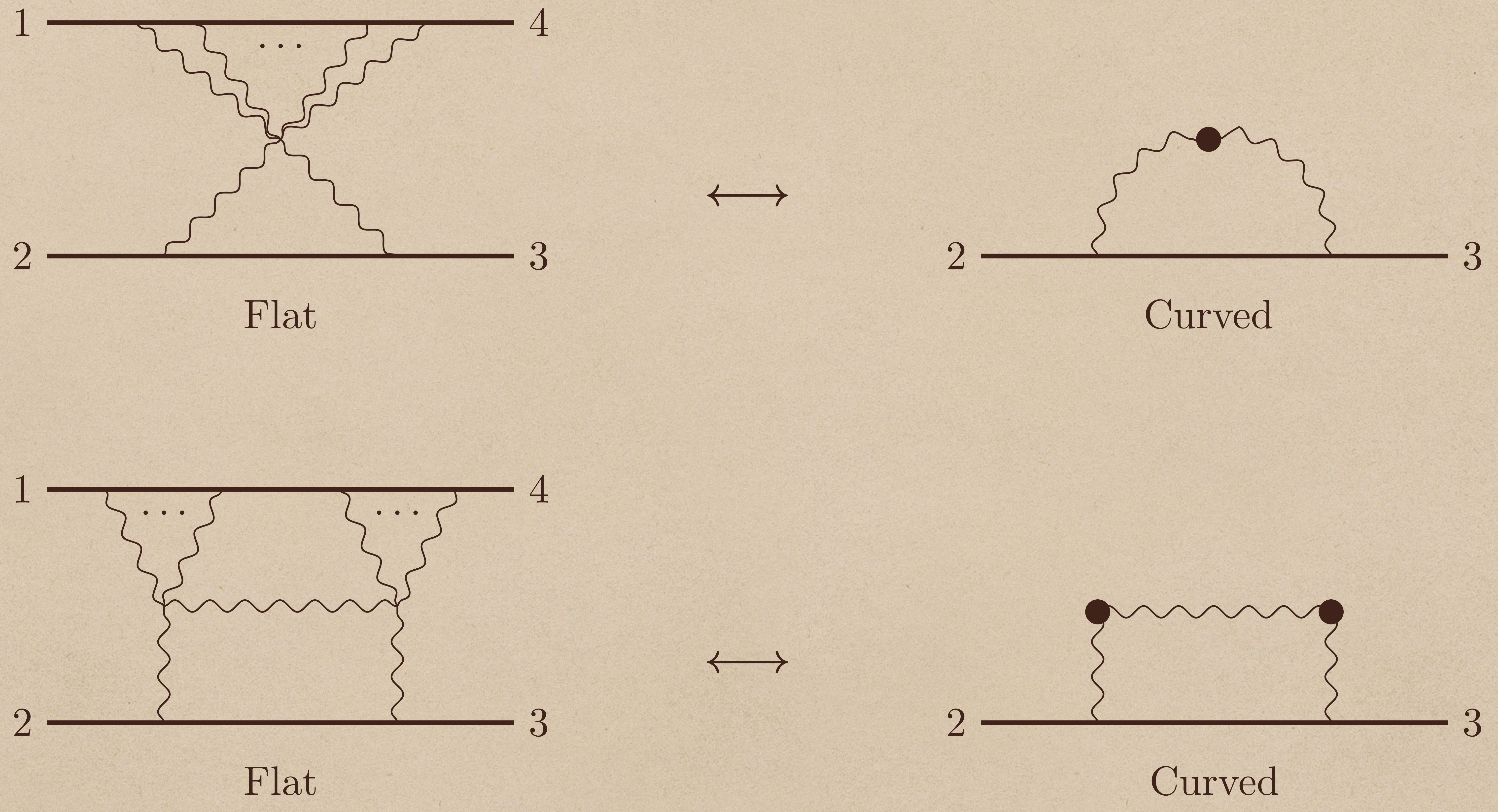

1 → 1 Amplitude

(2 → 2 in flat space)

*integrand from unitarity and Feynman rules



# Comparison to Flat Space




Benefits: Classes of Diagrams Combined & Generically Fewer Integrals

# Outlook

# Future Directions

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- Can we do 1SF at All Orders in $G$ ?

# Conclusions

- New Formalism for calculations relevant to Gravitational-Wave detection from Binary Mergers
- Combines Non-Perturbative information in the Metric with flat-space Amplitudes techniques
- Few Feynman Rules for given SF Order to All Orders in $G$
- Diagrams Combine & Fewer Integrations Generically Necessary


Thank You