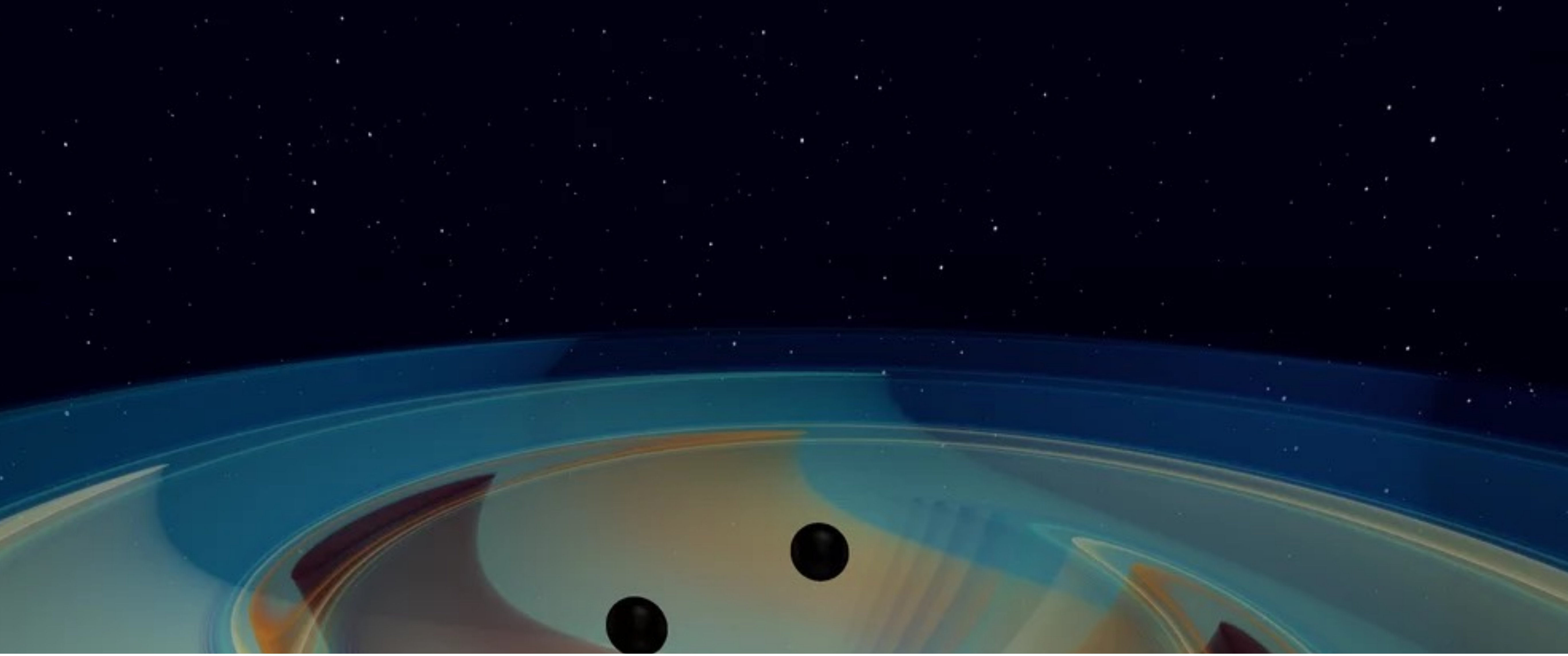
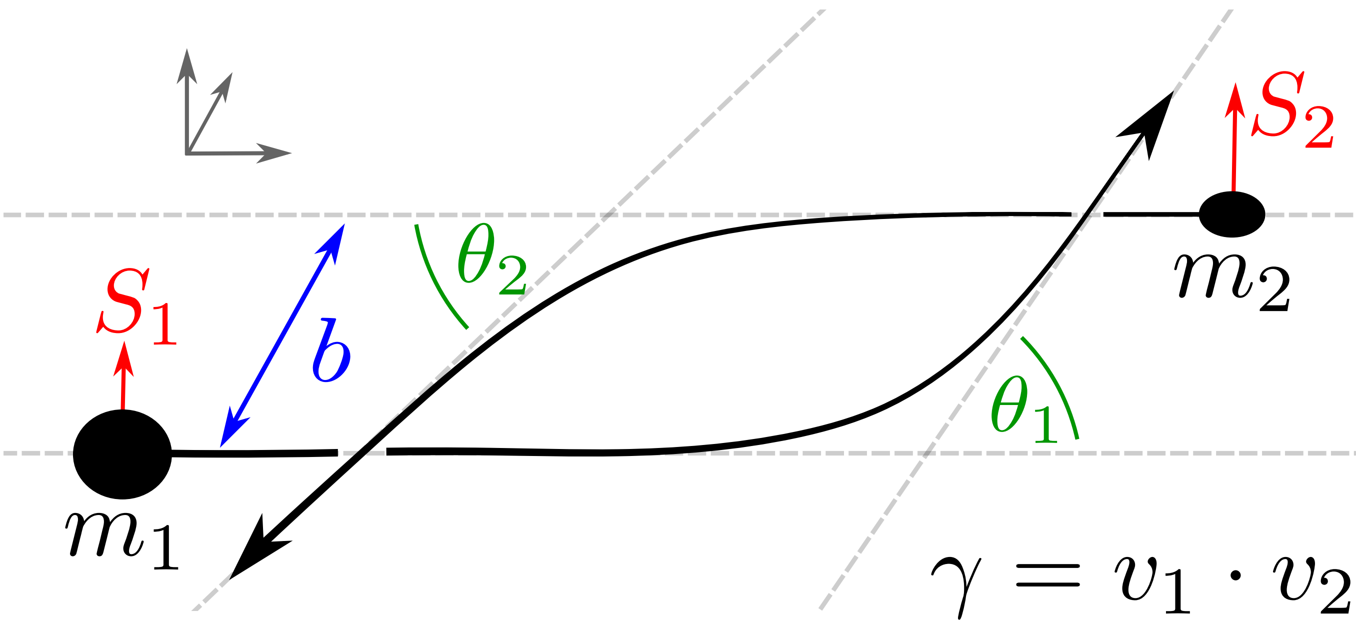


CONSERVATIVE BLACK HOLE SCATTERING AT 5PM–1SF ORDER

RTG 2575: Rethinking Quantum Field Theory

Gustav Mogull

SPINNING BLACK HOLE SCATTERING



$$\Delta p_1^\mu = p_1(\tau = +\infty) - p_1(\tau = -\infty)$$

$$\Delta S_1^\mu = S_1(\tau = +\infty) - S_1(\tau = -\infty)$$

$$\Delta p_1^\mu = G \Delta p_1^{(1)\mu} + G^2 \Delta p_1^{(2)\mu} + \cdots$$

Non-spinning 4PM scattering:

- *Dlapa, Kälin, Liu, Neef, Porto* (complete, using worldline)
- *Damgaard, Hansen, Planté, Vanhove* (complete, using KMOC + exponential S-matrix)
- *Bern, Parra-Martinez, Roiban, Ruf, Shen, Solon, Zeng* (conservative only, KMOC)

Spinning 3PM scattering:

- *Cordero, Kraus, Lin, Ruf, Zeng* (conservative only, KMOC)

Talk outline

1. **Non-spinning black hole scattering at 5PM-1SF (4 loops)**
2. **Spinning black hole scattering at 4PM (3 loops)**
3. **SEOB-PM resummation** for **large-angle scattering**

WORLDLINE QUANTUM FIELD THEORY

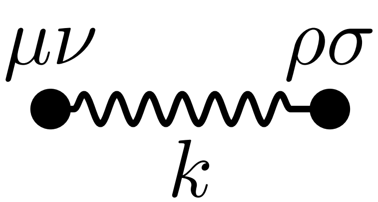
GM, Plefka, Steinhoff *JHEP* **02** (2021)

$$S=-\frac{2}{\kappa^2}\int \mathrm{d}^4x\sqrt{-g}R-\sum_{i=1}^{2}\int \mathrm{d}\tau_i\frac{m_i}{2}g_{\mu\nu}\dot{x}_i^\mu\dot{x}_i^\nu$$

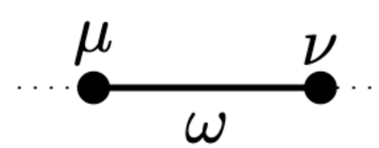
$$g_{\mu\nu}(x)=\eta_{\mu\nu}+\kappa h_{\mu\nu}(x)$$

$$x_i^\mu(\tau_i)=b_i^\mu+\tau_i v_i^\mu+z_i^\mu(\tau_i)$$

- Promote gravitons, deflections to propagating d.o.f.'s, with propagators:



$$=i\frac{P_{\mu\nu;\rho\sigma}}{k^2+i\epsilon}$$

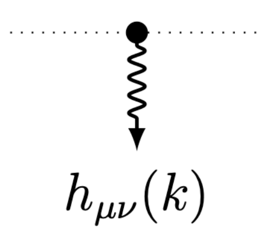


$$=-i\frac{\eta^{\mu\nu}}{m\,(\omega+i\epsilon)^2}$$

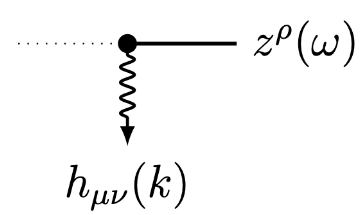
$$P_{\mu\nu;\rho\sigma}=\eta_{\mu(\rho}\eta_{\sigma)\nu}-\frac{1}{D-2}\eta_{\mu\nu}\eta_{\rho\sigma}$$

Theory trivially lifts to D=4-2ε dimensions

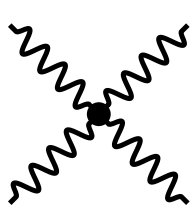
- Causality demands use of **retarded WL propagators** (from **Schwinger-Keldysh in-in formalism**)
- **Gravitons** live in the bulk, carry **momentum; deflections** live on the worldline, carry **energy**.



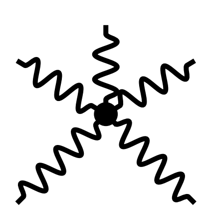
$$=-i\frac{m}{2m_{\rm Pl}}e^{ik\cdot b}\delta\!\!\!{}^-(k\cdot v)v^\mu v^\nu\,,$$



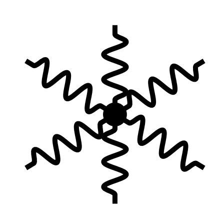
$$=\frac{m}{2m_{\rm Pl}}e^{ik\cdot b}\delta\!\!\!{}^-(k\cdot v+\omega)\left(2\omega v^{(\mu}\delta^{\nu)}_\rho+v^\mu v^\nu k_\rho\right)$$



$\sim\sqrt{G}^2k^2,$



$\sim\sqrt{G}^3k^2$



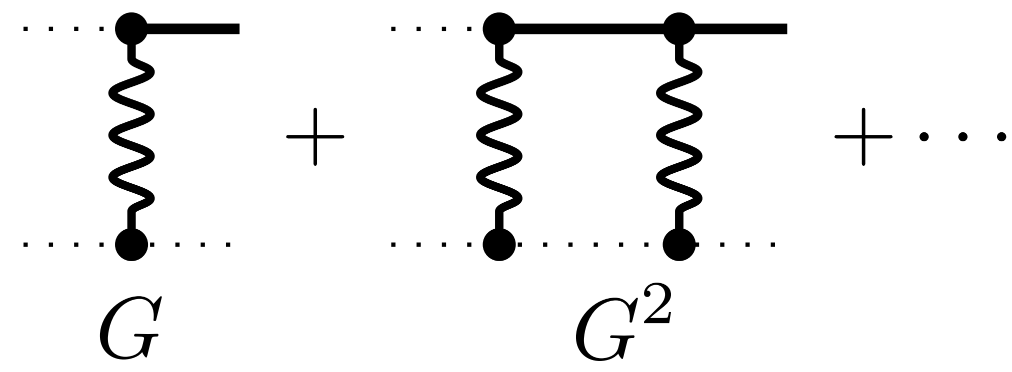
$\sim\sqrt{G}^4k^2,\ldots$

Tree-level one-point functions = Solutions to classical equations of motion

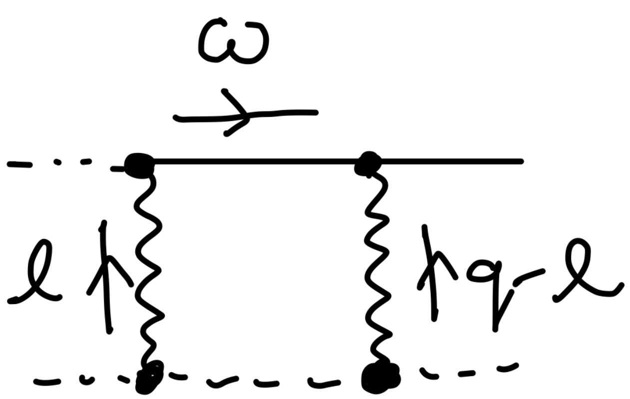
SCATTERING OBSERVABLES

- For **momentum impulse** draw tree diagrams with 1 outgoing line:

$$\Delta p_1^\mu = -m_1 \omega^2 \, \langle z_1^\mu(\omega) \rangle \big|_{\omega=0} =$$



- **All graphs are trees.** Integrate on internal energies/momenta:



$$= \int_{q,\ell,\omega} \frac{\delta(\omega - \ell\cdot v_1)\delta(\omega + (q-\ell)\cdot v_1)\delta(\ell\cdot v_2)\delta((q-\ell)\cdot v_2)}{(\omega + i0)^2 \ell^2 (\ell - q)^2} e^{iq\cdot b}$$

$$= \int_q \delta(q\cdot v_1)\delta(q\cdot v_2) e^{iq\cdot b} \int_\ell \frac{\delta(\ell\cdot v_2)}{(\ell\cdot v_1 + i0)^2 \ell^2 (\ell - q)^2}$$

- Loop integrals arise from **lack of momentum conservation:**

Loop integrals from tree-level diagrams

5PM MOMENTUM IMPULSE

0SF: 63 diagrams 1SF: 363 diagrams

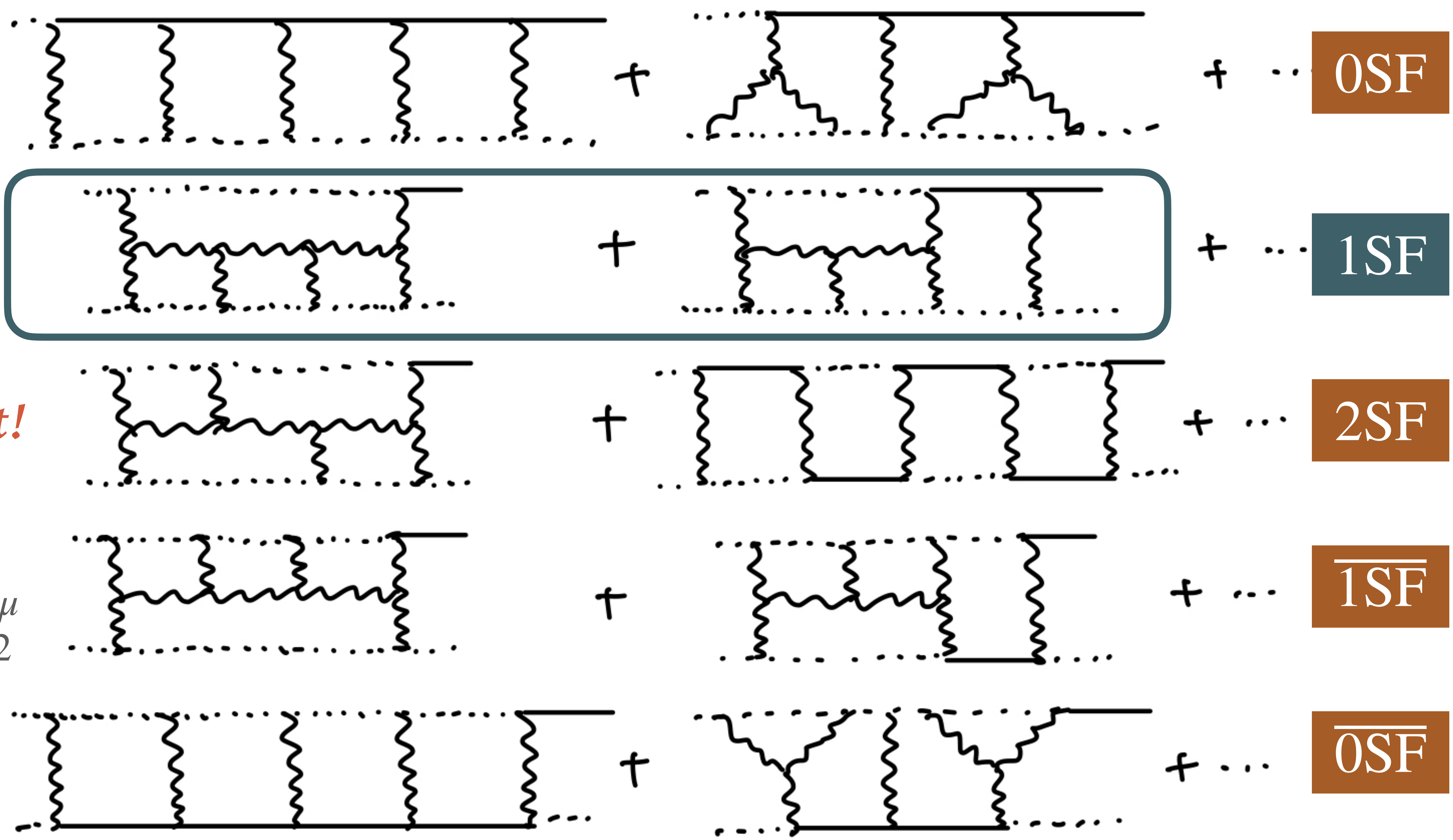
$$\Delta p_1^{(5)\mu} =$$

Our calculation

Very difficult!

Given by $\Delta p_1^\mu = -\Delta p_2^\mu$

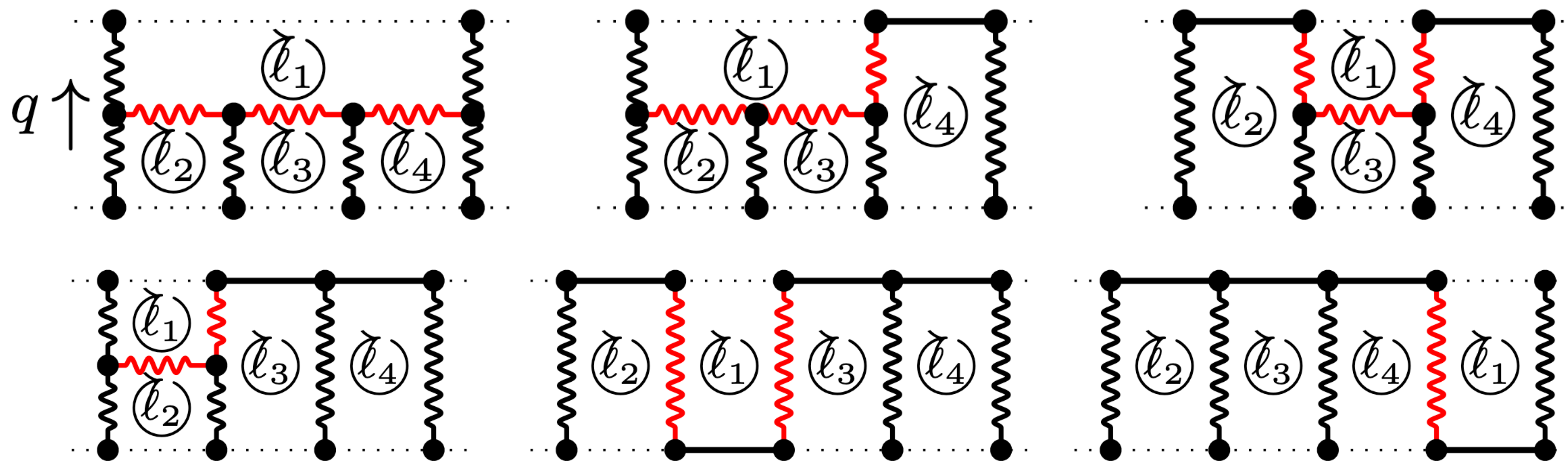
Very easy!



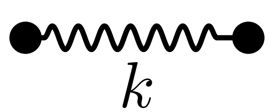
- Integrand divides naturally into **mass sectors**, following **Self-Force (SF)**
- Quickly generate from **Feynman rules** using recursive **FORM** algorithm.
- We focus on the **1SF sector, conservative dynamics**!

PLANAR 1SF INTEGRALS

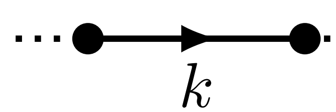
At 1SF we need only **1 planar integral basis** to handle all contributions:



Red propagators are active. Diagrammatic rules for loop integrals:



$= \dfrac{1}{k^2 + i0^+}$



$= \dfrac{1}{k \cdot v_i + i0^+}$

$= \delta(k \cdot v_i)$

Integrals depend trivially on $|q|$, non-trivially on $\gamma = v_1 \cdot v_2$

$$\mathcal{I}^{\{\sigma\}}_{\{n\}} = \int_{\ell_1 \cdots \ell_L} \frac{\delta^{(\bar{n}_1 - 1)}(\ell_1 \cdot v_1) \prod_{i=2}^{L} \delta^{(\bar{n}_i - 1)}(\ell_i \cdot v_2)}{\prod_{i=1}^{L} D_i^{n_i}(\sigma_i) \prod_{I<J} D_{IJ}^{n_{IJ}}}$$

$$D_{ij} = (\ell_i - \ell_j)^2$$

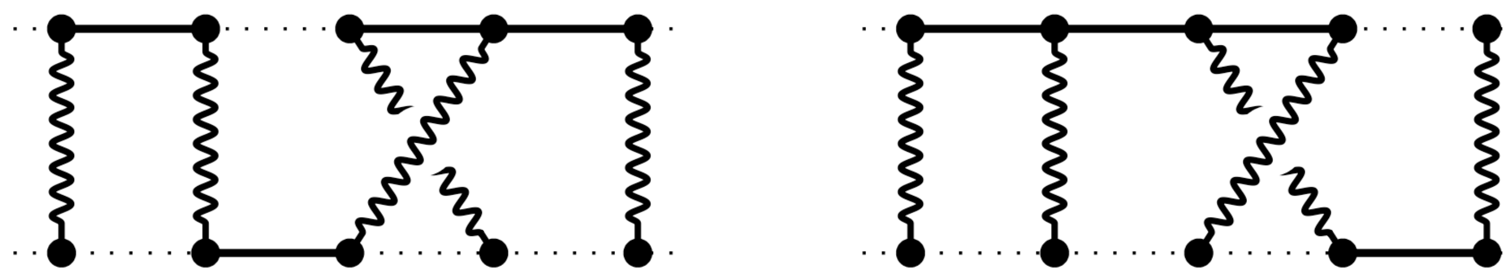
$$D_{qi} = (\ell_i + q)^2$$

$$D_{0i} = \ell_1^2$$

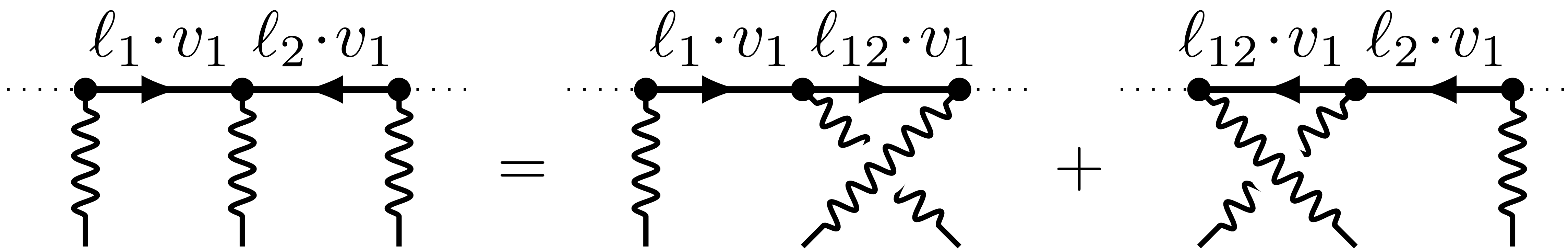
$$D_1 = \ell_1 \cdot v_2 + \sigma_1 i0^+ \,, \qquad D_{i>1} = \ell_i \cdot v_1 + \sigma_i i0^+ \,,$$

PARTIAL FRACTION IDENTITIES

What about the nonplanar diagrams, e.g.

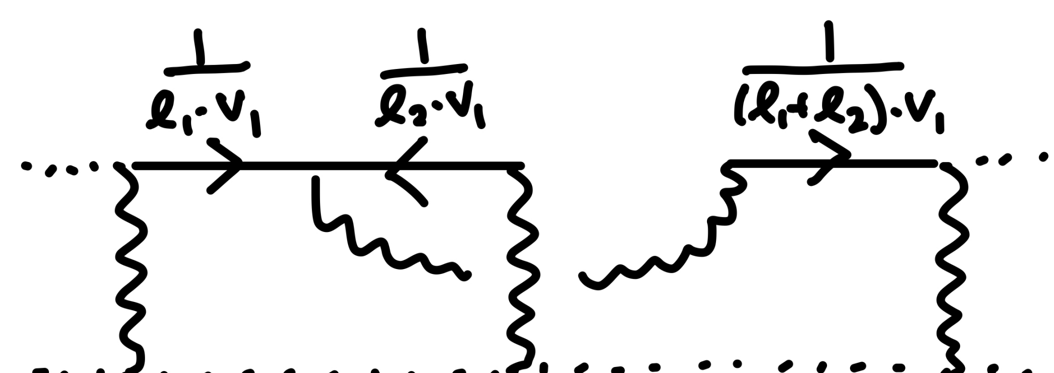


Nonplanar contributions are **systematically eliminated** using **partial fraction identities**… untangling the legs!



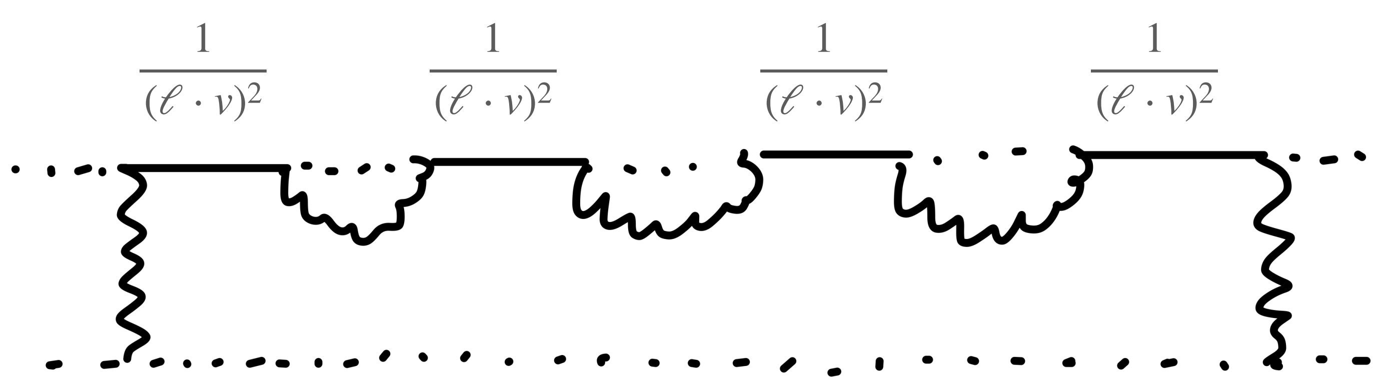
$$\frac{1}{(\ell_1 \cdot v_1 + i0^+)(\ell_2 \cdot v_1 + i0^+)} = \frac{1}{(\ell_1 \cdot v_1 + i0^+)(\ell_{12} \cdot v_1 + i0^+)} + \frac{1}{(\ell_2 \cdot v_1 + i0^+)(\ell_{12} \cdot v_1 + i0^+)}$$

Also resolve **linearly dependent worldline propagators**



INTEGRATION-BY-PARTS (IBP) IDENTITIES

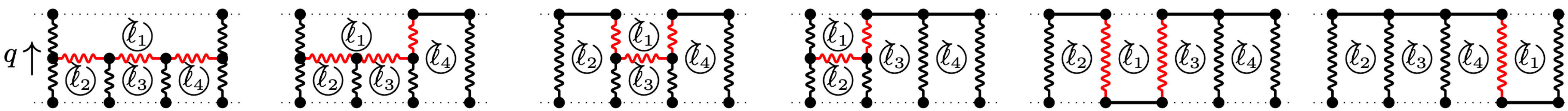
- This was **by far** the most challenging part of the calculation!
- We used **KIRA 3.0** (upcoming release) in **Finite Field (FF)** mode.
- Reconstructed coefficients of rational functions from FF samples with **FireFly**
- Key improvement: generate **fewer equations (Laporta algorithm)** by tightly controlling the **allowed powers of propagators**.
- Complete reductions took **~300k core hours** on **HPC cluster**



- Maximum of **8 allowed powers** on **worldline propagators** (7 dots)
- Maximum of **9 scalar products** overall

Johann Usovitsch

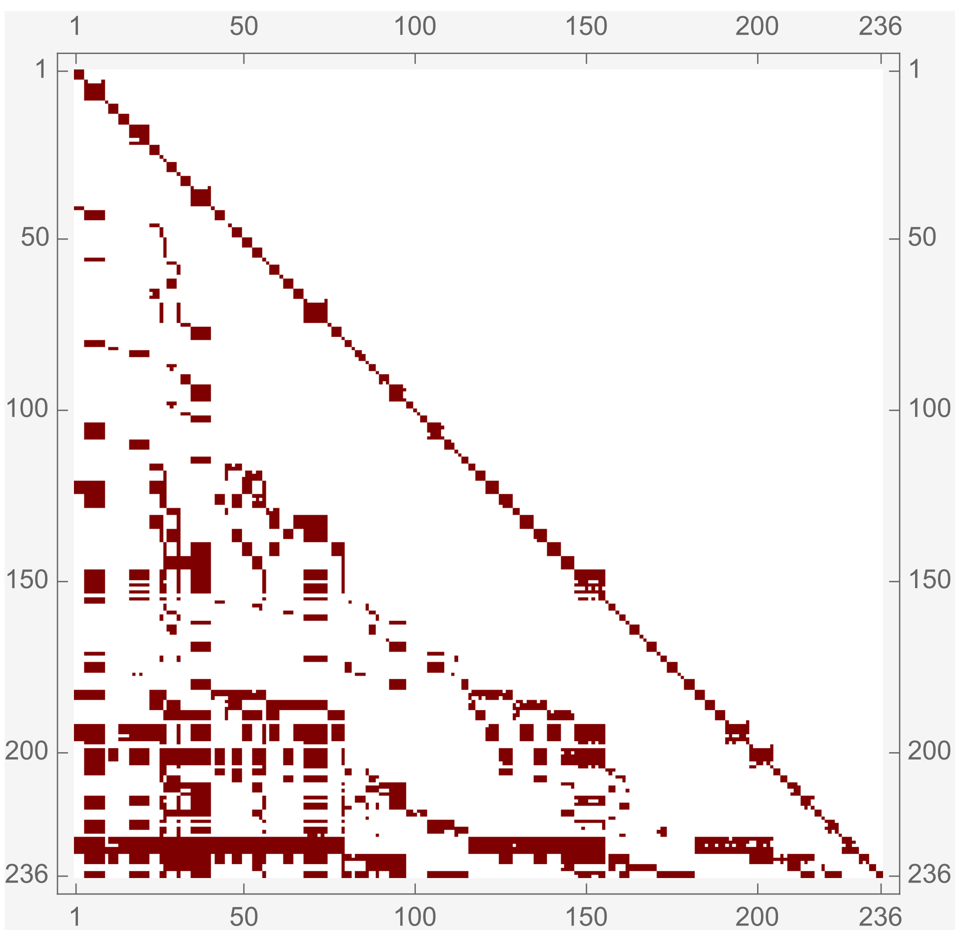
SOLVING DIFFERENTIAL EQUATIONS (1)

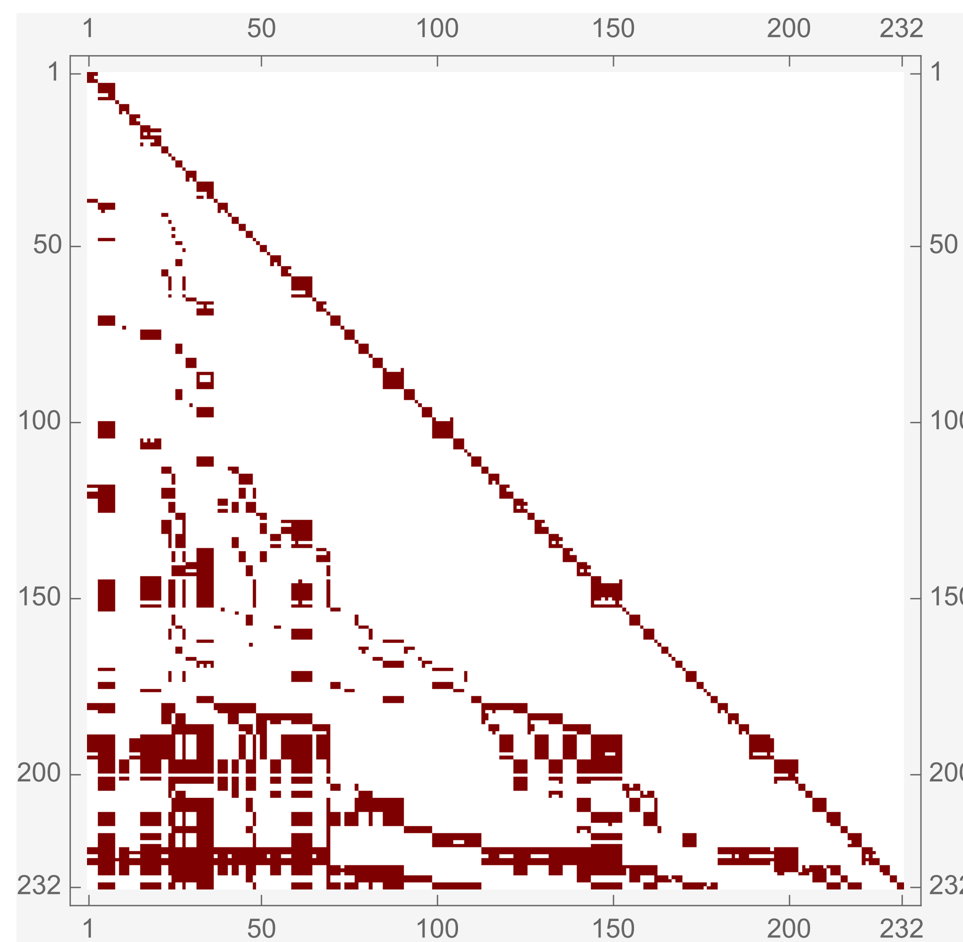


$$\frac{\mathrm{d}}{\mathrm{d}x}\underline{J} = M(x,\epsilon)\underline{J}$$

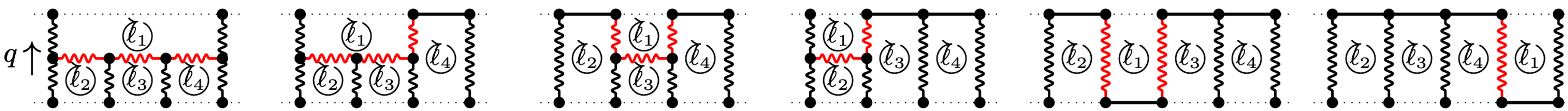
$$x = \gamma - \sqrt{\gamma^2 - 1}$$

- Total of **236 even** & **234 odd** Master Integrals (MIs)

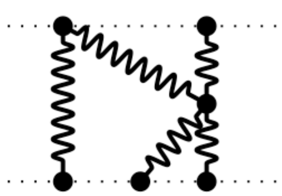
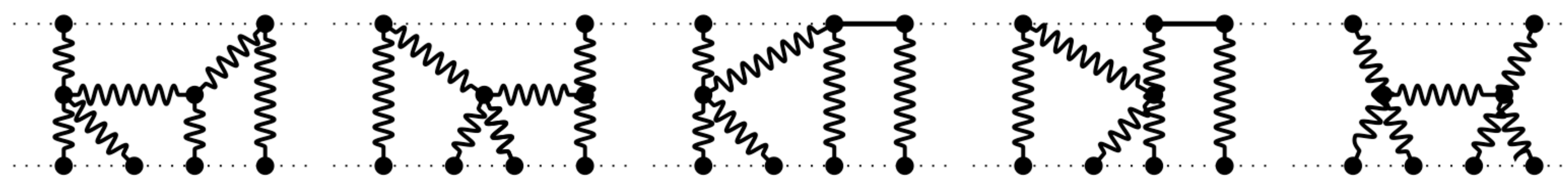




SOLVING DIFFERENTIAL EQUATIONS (2)



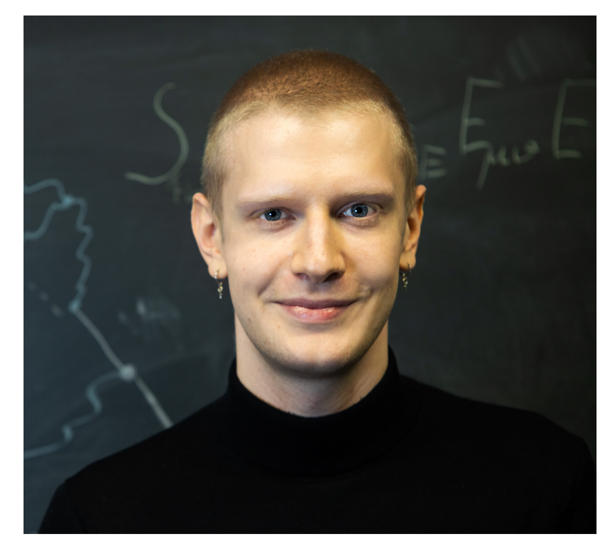
$$\frac{\mathrm{d}\vec{I}(x)}{\mathrm{d}x} = \epsilon A(x)\,\vec{I}(x) \qquad x = \gamma - \sqrt{\gamma^2 - 1}$$

4PM-1SF:  5PM-1SF: 

Klemm, Nega, Sauer, Plefka '24

We seek a **canonical form** of the DEs:

1. Organise into **block diagonals**, using **top-level sectors**
2. Most useful algorithms: **CANONICA, INITIAL, FiniteFlow** (plus **Fuchsia, Libra, Epsilon**)
3. **Complete elliptic integrals** K/E in the **canonical transformation** — solve **degree-3 Picard-Fuchs equation**

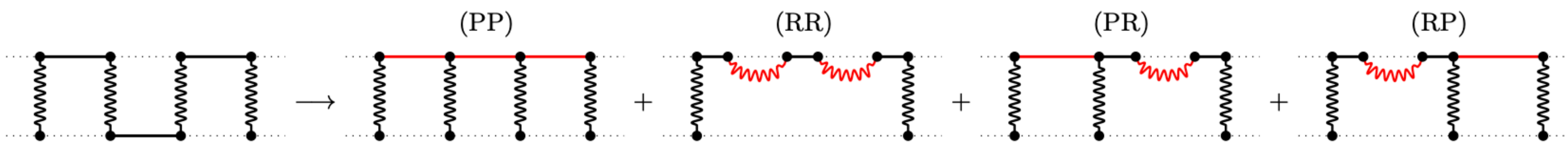


Benjamin Sauer

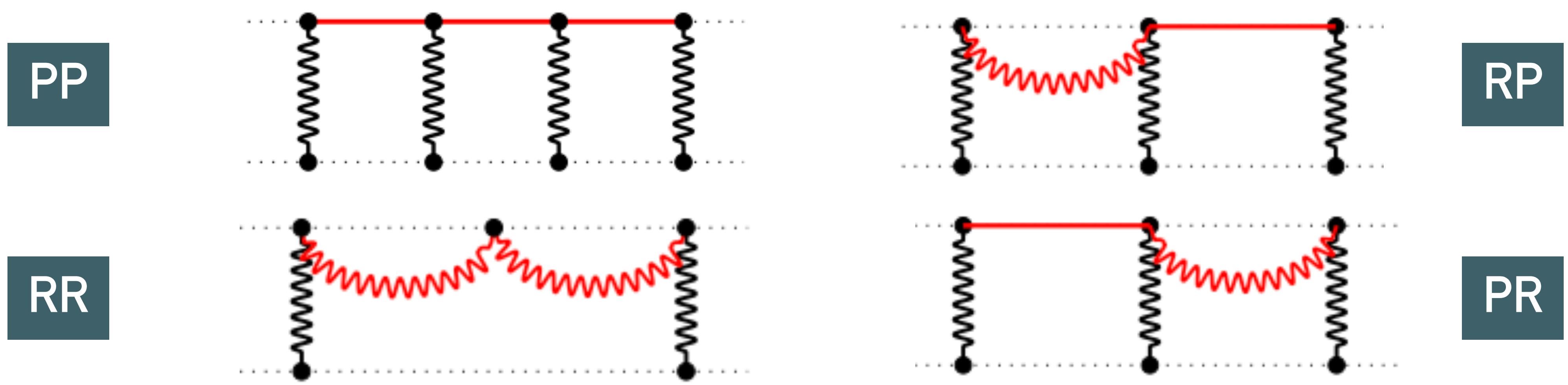
BOUNDARY FIXING

$$\ell_i^{\text{pot}} = (\ell_i^0, \boldsymbol{\ell}_i) \sim (v, 1)\,, \qquad \ell_i^{\text{rad}} = (\ell_i^0, \boldsymbol{\ell}_i) \sim (v, v)$$

In the slow-velocity limit, master integrals can be re-expressed in terms of those with a **simpler velocity dependence**:



Simpler boundary integrals are handled using IBPs, with **top-level sectors**:



In our case, at 5PM we keep **PPP** and **PRR** regions. This gives us **even-in-velocity contributions.**

$$\langle \mathcal{O} \rangle_{\text{cons}} := \text{Re} \int \mathcal{D}h_{\mu\nu} e^{iS_{\text{EH}}[h_{\mu\nu}]} \int \mathcal{D}z_i^{(1)} \mathcal{D}z_i^{(2)} e^{i(S_{\text{pp}}[h_{\mu\nu}, z_i^{(1)}] - S_{\text{pp}}^*[h_{\mu\nu}, z_i^{(2)}])} \mathcal{O}$$

ANATOMY OF RESULTS

*Driesse, Jakobsen, **GM**, Plefka, Sauer, Usovitsch '24*

- The **conservative** impulse has a **universal form:**

$$\Delta p^{\mu}_{1,\mathrm{cons}} = |p| \sin\theta_{\mathrm{cons}} \frac{b^{\mu}}{|b|} + (\cos\theta_{\mathrm{cons}} - 1) p^{\mu}$$

b-terms ***v-terms***

$$p^{\mu} = (0, \mathbf{p}) = \frac{E_2 p_1^{\mu} - E_1 p_2^{\mu}}{M}$$

- Result depends only on the **scattering angle**:

$$\theta_{\mathrm{cons}} = \frac{E}{M} \sum_{n \geq 1} \left(\frac{GM}{|b|} \right)^n \left(\theta^{(n,0)}_{\mathrm{cons}}(\gamma) + \nu \theta^{(n,1)}_{\mathrm{cons}}(\gamma) + \cdots \right) \qquad \nu = \frac{m_1 m_2}{M^2}$$

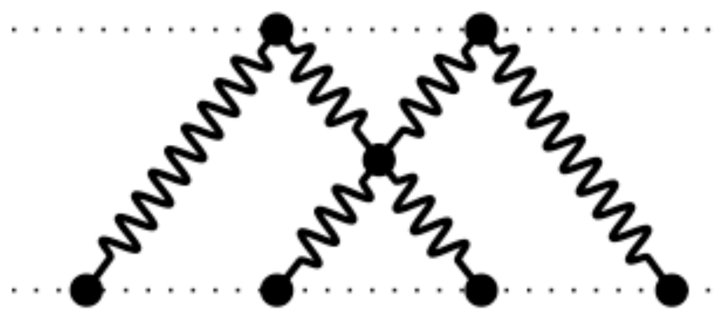
- $\theta^{(5,1)}_{\mathrm{cons}}$ consists of **Multiple PolyLogarithms (MPLs)** up to **weight-3**, **alphabet** $a_i \in \{0, \pm 1, \pm i\}$

$$\theta^{(5,1)}_{\mathrm{cons}} = \sum_{k=1}^{31} c_k(\gamma) f_k(\gamma) \qquad G(a_1, \ldots, a_n; y) = \int_0^y \frac{\mathrm{d}t}{t - a_1} G(a_2, \ldots, a_n; t)$$

No elliptic K/E functions in the conservative 5PM-1SF result!

LOOKING AHEAD: FUTURE 5PM CALCULATIONS

- Our next step will be to calculate the **complete momentum impulse**, including **dissipative (radiation) effects**.
- Will yield **full angle, radiated energy flux**.
- Means upgrading to **retarded graviton propagators,** and including the **(PPR) & (RRR) regions** (odd-in-v)

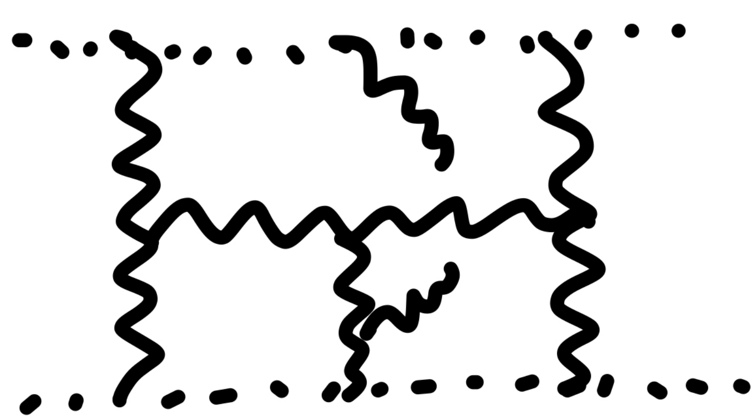


(CY3)

Frellesvig, Morales, Wilhelm '23

Klemm, Nega, Sauer, Plefka '24

- Will we see a **Calabi-Yau 3-Fold**? They appear in the odd DEs…
- Then, 2SF! M**ore difficult integrals**, including genuine **nonplanars**:



N=2 SUSY THEORY

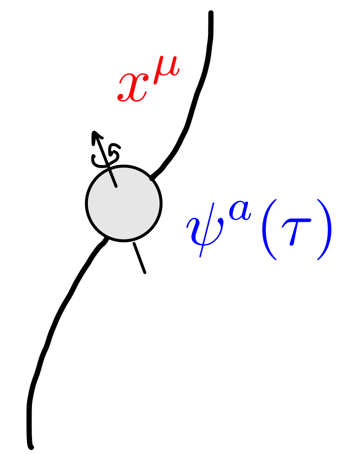
*Jakobsen, **GM**, Plefka, Steinhoff* ***Phys. Rev. Lett. 128 (2022)***

➤ To describe spinning BHs/NSs, use a **spin-1 particle**:

$$S_{\mathrm{BH/NS}} = -m\int d\tau\Big[\tfrac{1}{2}g_{\mu\nu}\dot{x}^\mu\,\dot{x}^\nu + i\bar{\psi}D_\tau\psi + \tfrac{1}{2}R_{abcd}\bar{\psi}^a\psi^b\bar{\psi}^c\psi^d + C_E R_{a\mu b\nu}\dot{x}^\mu\dot{x}^\nu\bar{\psi}^a\psi^b\,\bar{\psi}\cdot\psi\Big]$$

spin degrees of freedom *neutron star term*

➤ **Spin-1 theory** enjoys a **global SUSY:**



$$\delta x^\mu = i e^\mu_a(\bar{\epsilon}\psi^a + \epsilon\bar{\psi}^a)\,, \qquad \delta S^{\mu\nu} = 2p^{[\mu}\delta x^{\nu]}$$

$$\delta\psi^a = -\epsilon e^a_\mu\,\dot{x}^\mu - \delta x^\mu\omega_\mu{}^a{}_b\psi^b$$

➤ Symmetries imply **conserved charges**: $S^{\mu\nu} = -2i\bar{\psi}^{[\mu}\psi^{\nu]} = \epsilon^{\mu\nu\rho\sigma}p_\rho a_\sigma$

$$\dot{x}^2 = 1 + R_{abcd}\bar{\psi}^a\psi^b\bar{\psi}^c\psi^d \qquad \bar{\psi}\cdot\psi = s$$

Conserved spin length

$$p\cdot\psi = p\cdot\bar{\psi} = 0 \quad\Longrightarrow\quad p_\mu S^{\mu\nu} = 0$$

Covariant SSC

➤ Neutron star term **preserves SUSY up to O(S²)**.

SPINNING WQFT FEYNMAN RULES

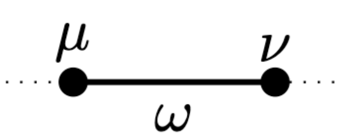
- Inclusion of spin requires **extended Feynman rules**:

$$g_{\mu\nu}(x) = \eta_{\mu\nu} + \kappa h_{\mu\nu}(x)$$

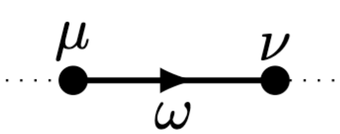
$$x_i^\mu(\tau_i) = b_i^\mu + \tau_i v_i^\mu + z_i^\mu(\tau_i)$$

$$\psi_i^\mu(\tau_i) = \Psi_i^\mu + \psi_i'^\mu(\tau_i)$$

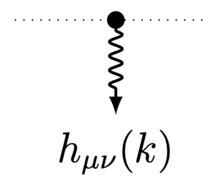
- Propagators:



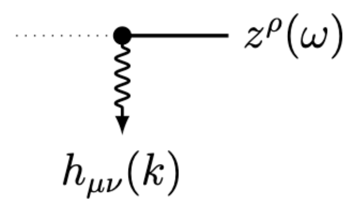
$$= -i\frac{\eta^{\mu\nu}}{m\,(\omega + i\epsilon)^2}\,,$$



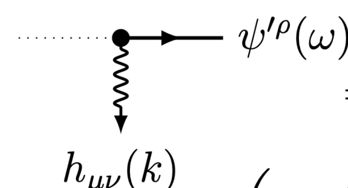
$$= -i\frac{\eta^{\mu\nu}}{m\,(\omega + i\epsilon)}\,,$$



$$= -i\frac{m\kappa}{2}e^{ik\cdot b}\bar{\delta}(k\cdot v)$$
$$\times\left(v^\mu v^\nu + i(k\cdot\mathcal{S})^{(\mu}v^{\nu)} - \frac{1}{2}(k\cdot\mathcal{S})^\mu(k\cdot\mathcal{S})^\nu + \frac{C_E}{2}v^\mu v^\nu(k\cdot\mathcal{S}\cdot\mathcal{S}\cdot k)\right),$$



$$= \frac{m\kappa}{2}e^{ik\cdot b}\bar{\delta}(k\cdot v + \omega)$$
$$\times\left(2\omega v^{(\mu}\delta_\rho^{\nu)} + v^\mu v^\nu k_\rho + i(k\cdot\mathcal{S})^{(\mu}(k_\rho v^{\nu)} + \omega\delta_\rho^{\nu)}) + \frac{1}{2}k_\rho(k\cdot\mathcal{S})^\mu(\mathcal{S}\cdot k)^\nu\right.$$
$$\left. + \frac{C_E}{2}\left(\left(2\omega v^{(\mu}\delta_\rho^{\nu)} + v^\mu v^\nu k_\rho\right)(k\cdot\mathcal{S}\cdot\mathcal{S}\cdot k) - \omega^2 k_\rho(\mathcal{S}\cdot\mathcal{S})^{\mu\nu} + 2\omega^2(k\cdot\mathcal{S}\cdot\mathcal{S})^{(\mu}\delta_\rho^{\nu)}\right)\right)$$



$$= -im\kappa e^{ik\cdot b}\bar{\delta}(k\cdot v + \omega)$$
$$\times\left(k_{[\rho}\delta_{\sigma]}^{(\mu}\left(v^{\nu)} - i(\mathcal{S}\cdot k)^{\nu)}\right) + iC_E\left(v^{(\mu}k_\lambda + \omega\delta_\lambda^{(\mu}\right)\left(v^{\nu)}k_{[\rho} + \omega\delta_{[\rho}^{\nu)}\right)\mathcal{S}^\lambda{}_{\sigma]}\right)\bar{\Psi}^\sigma\,.$$

Equivalent to solving Mattison-Papapetrou-Dixon (MPD) Equations

SPINNING 4PM RESULTS

*Jakobsen, **GM**, Plefka, Sauer, Xu* ***Phys. Rev. Lett. 131 (2023)***
*Jakobsen, **GM**, Plefka, Sauer* ***Phys. Rev. Lett. 131 (2023)***

- **4PM momentum impulse** takes the form (depending also on spin variables)

$$\Delta p_i^{(4)\mu} = \Delta p_{i,\mathrm{cons}}^{(4)\mu} + \Delta p_{i,\mathrm{rad}^1}^{(4)\mu} + \Delta p_{i,\mathrm{rad}^2}^{(4)\mu}$$

Feynman i0 prescription (PP+RR) | Odd-in-v (PR) | Even-in-v (RR)

- 1SF result involves **special functions** (even-in-v):

$$F_{1,\dots,5} = \left\{1, \frac{\log[x]}{\sqrt{\gamma^2-1}}, \log\left[\frac{\gamma_+}{2}\right], \log^2[x], \frac{\log[x]\log\left[\frac{\gamma_+}{2}\right]}{\sqrt{\gamma^2-1}}\right\},$$

$$F_{6,\dots,9} = \left\{\log[\gamma], \log^2\left[\frac{\gamma_+}{2}\right], \mathrm{Li}_2\left[\frac{\gamma_-}{\gamma_+}\right], \mathrm{Li}_2\left[-\frac{\gamma_-}{\gamma_+}\right]\right\}$$

$$F_{10,\dots,13} = \left\{\frac{\log[x]}{\sqrt{\gamma^2-1}}\log[\gamma], \frac{1}{\sqrt{\gamma^2-1}}\chi_2\left[\sqrt{\frac{\gamma_-}{\gamma_+}}\right], \mathrm{Li}_2[-x^2] - 4\mathrm{Li}_2[-x] - \log[4]\log[x] - \frac{\pi^2}{4}, \frac{\mathrm{Li}_2[-x] - \mathrm{Li}_2\left[-\frac{1}{x}\right] + \log[4]\log[x]}{\sqrt{\gamma^2-1}}\right\}$$

$$F_{14,15,16} = \left\{\mathrm{E}^2\left[\frac{\gamma_-}{\gamma_+}\right], \mathrm{K}^2\left[\frac{\gamma_-}{\gamma_+}\right], \mathrm{E}\left[\frac{\gamma_-}{\gamma_+}\right]\mathrm{K}\left[\frac{\gamma_-}{\gamma_+}\right]\right\}$$

$$F_{17,18,19} = \left\{\log\left[\frac{\gamma_-}{2}\right], \frac{\log\left[\frac{\gamma_-}{2}\right]\log[x]}{\sqrt{\gamma^2-1}}, \log\left[\frac{\gamma_-}{2}\right]\log\left[\frac{\gamma_+}{2}\right]\right\}$$

Conservative Only!

$$\gamma_\pm = \gamma \pm 1$$
$$x = \gamma - \sqrt{\gamma^2-1}$$

$$\gamma = \frac{1}{\sqrt{1-v^2}}$$

$$\chi_\nu[z] = \frac{1}{2}(\mathrm{Li}_\nu[z] - \mathrm{Li}_\nu[-z])$$

SCATTERING SPINNING BLACK HOLES: STATE-OF-THE-ART

Higher orders in spin are **suppressed by physical PM counting**:

$$a_i^\mu = G m_i \chi_i^\mu \,, \qquad |\chi_i| < 1$$

	S^0 (Spin-0)	S^1 (Spin-1/2)	S^2 (Spin-1)	S^3 (Spin-3/2)	S^4 (Spin-2)	S^5 (Spin-5/2)
1PM (tree level)	G	G^2	G^3	G^4	G^5	G^6
2PM (1 loop)	G^2	G^3	G^4	G^5	G^6	G^7
3PM (2 loops)	G^3	Jakobsen, GM '22	Jakobsen, GM '22	G^6	G^7	G^8
4PM (3 loops)	G^4	Jakobsen, GM, Plefka, Sauer, Xu '23	G^6	G^7	G^8	*Tail effect*
5PM (4 loops)	Driesse, Jakobsen, GM, Plefka, Sauer, Usovitsch '24	G^6	G^7	G^8	G^9	G^{10}

Nearing completion of full G^5 perturbative order!

EFFECTIVE-ONE-BODY RESUMMATION

Buonanno, Damour '98

Starting point is the **impetus formula** (inverse Hamiltonian):

$$\mathbf{p}^2 = p_r^2 + \frac{L^2}{r^2} = p_\infty^2 + w(E, L, r; a_i) \qquad \theta + \pi = -2\int_{r_{\min}}^{\infty} \mathrm{d}r \frac{\partial}{\partial L} p_r$$

Where $|\mathbf{p}| \to p_\infty$ as $r \to \infty$. We seek to describe the potential w.

$$w_{\mathrm{nPM}} = \sum_{m=1}^{n} \left(\frac{GM}{r}\right)^m w^{(m)}$$

wEOB model: ***Rettegno, Praten, Thomas, Schmidt, Damour '23***

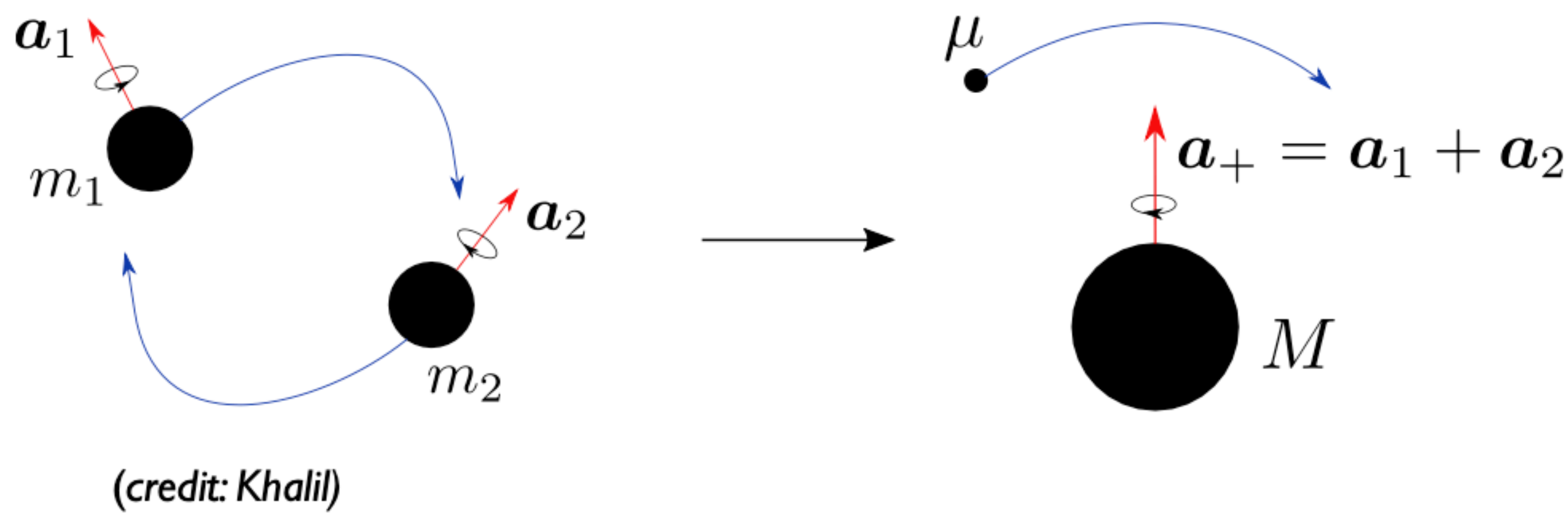
Key requirement: the scattering angle **matches the angle** for **small-angle scattering**:

$$\theta_{\mathrm{nPM}} = \theta + \mathcal{O}(G^{n+1})$$

SEOB-PM MODEL

Buonanno, Jakobsen, ***GM*** *'24*

Now, use the probe limit as a starting point



$$g^{\mu\nu}_{\mathrm{Kerr}} p_\mu p_\nu = \mu^2 \qquad \mu = \frac{m_1 m_2}{M}, \quad \nu = \frac{\mu}{M}, \quad M = m_1 + m_2$$

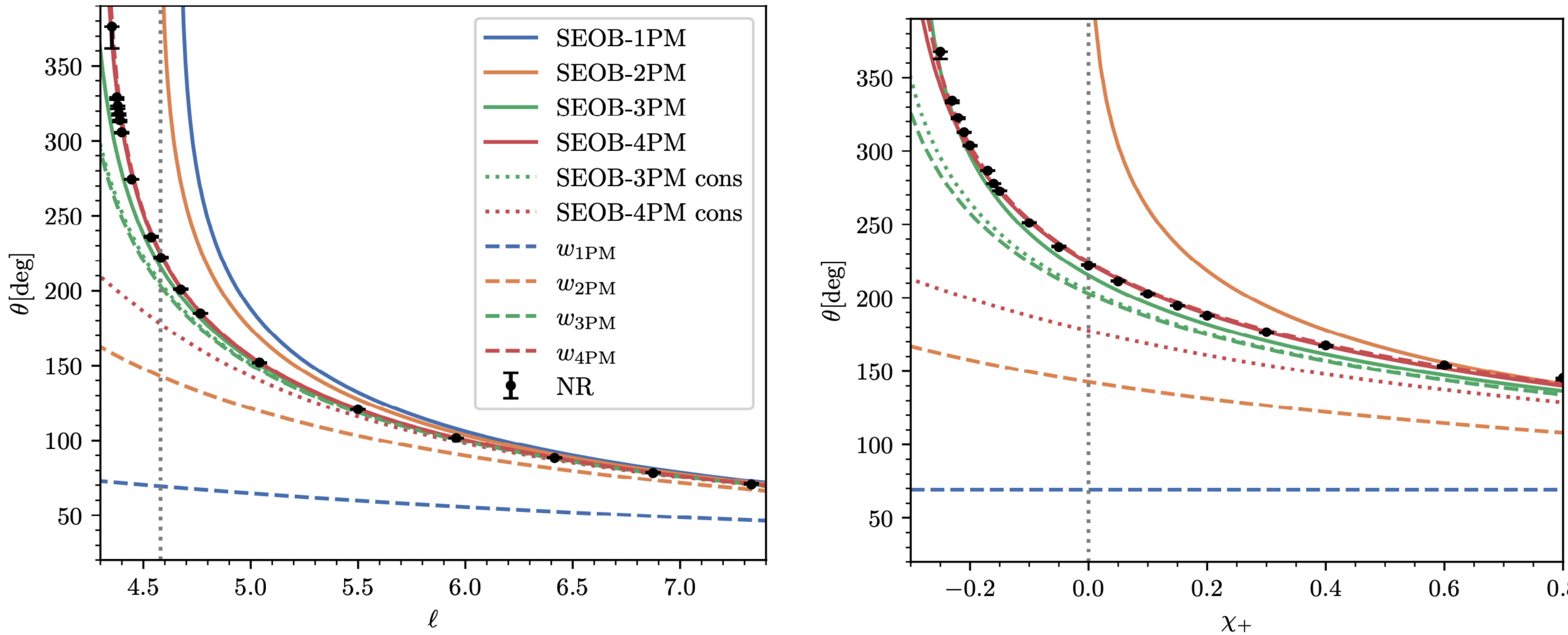
Invert to solve for full motion in the probe limit, add deformations:

$$p_r^2 = \frac{1}{A(1+B^{\mathrm{Kerr}}_{\mathrm{np}})}\left[\left(E_{\mathrm{eff}} - \frac{ML(g_{a_+}a_+ + g_{a_-}\delta a_-)}{r^3 + a_+^2(r+2M)}\right)^2 - A\left(\mu^2 + \frac{L^2}{r^2} + B^{\mathrm{Kerr}}_{\mathrm{npa}}\frac{L^2 a_+^2}{r^2}\right)\right]$$

Even-in-spin corrections to A, **odd-in-spin** gyro-gravitomagnetic factors $g_{a_\pm}$.
Inspired by deformations in **SEOBNRv5**.

NUMERICAL RELATIVITY COMPARISONS

➤ Numerically calculate all-order-in-G scattering angle, compare with NR

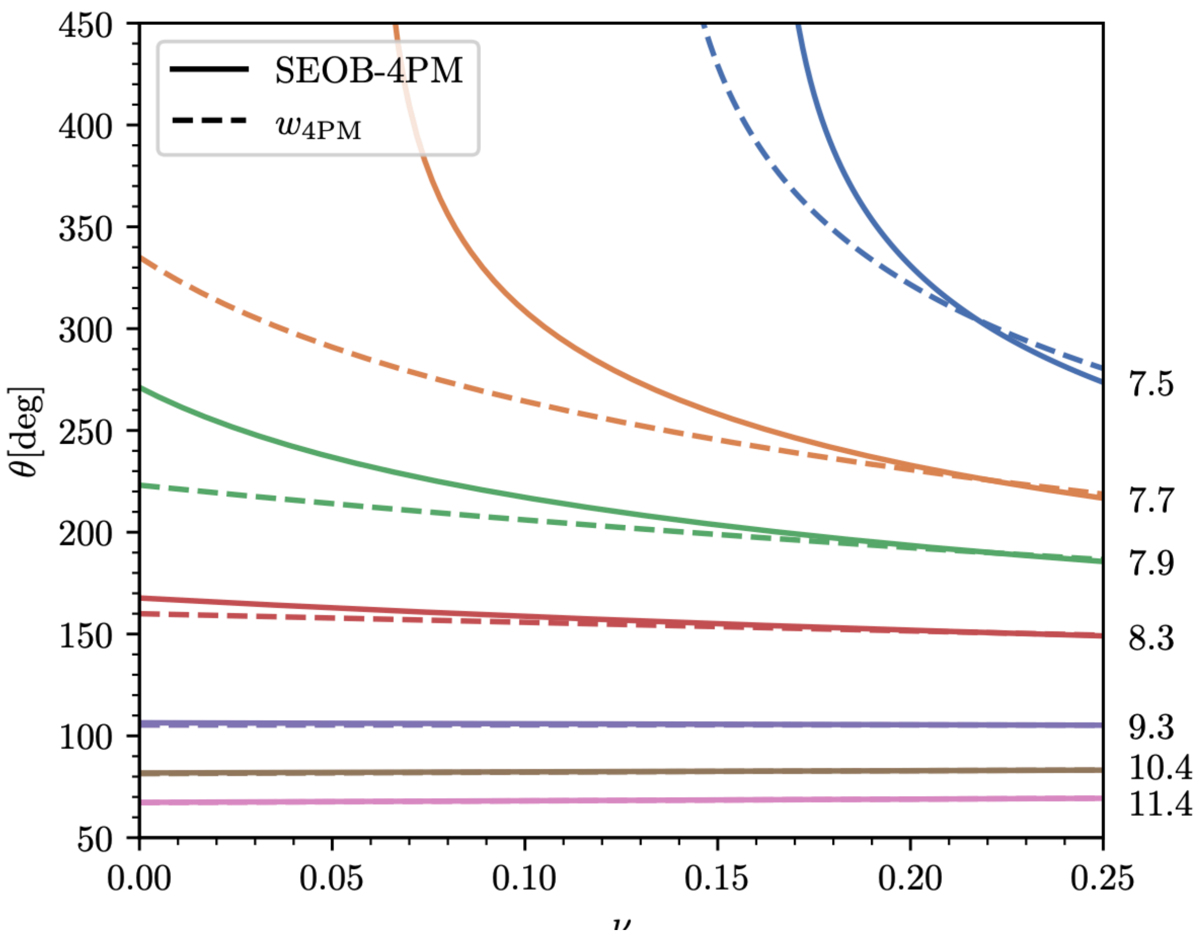


➤ For now, incorporate 2-body PM scattering data up to G^4 (complete results known)

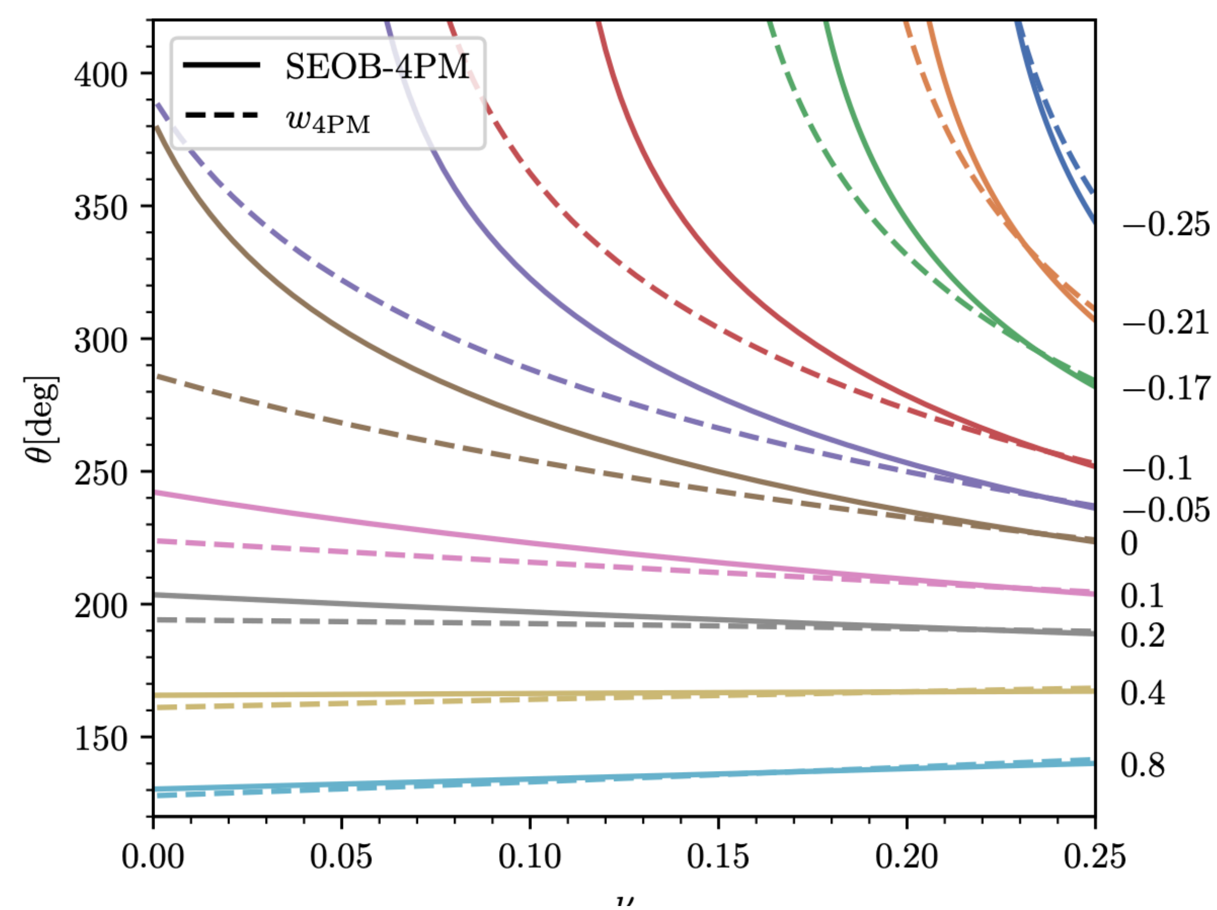
➤ NR data from ***Damour, Rettegno et. al.***, and comparison with **wEOB model**

VARYING MASS RATIO

- ➤ For different mass ratios, we see a **sharp divergence** between the two models



$\gamma = 1.228, \chi_{\pm} = 0$ varying $|b|/M$



$\gamma = 1.092, \ |b|/M = 10.7$ varying χ_+

Calls for NR data across different values of ν !

CONCLUSIONS & OUTLOOK

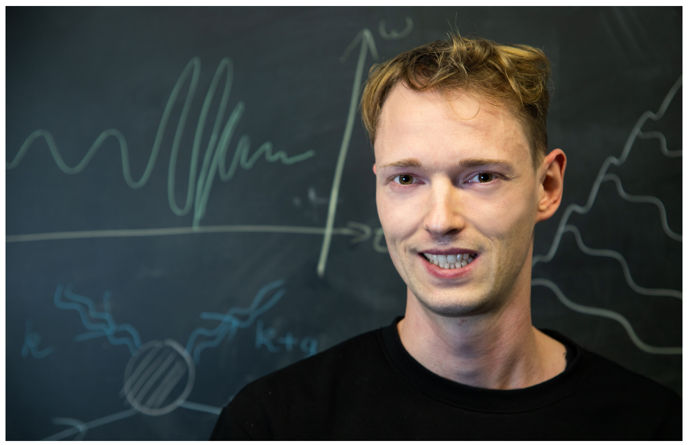
The outlook for PM-scattering methods in classical GR is **very bright**, and with WQFT technology we are **nearing completion of the full G^5 dynamics**

- **Classical observables** from **tree-level diagrams**
- **Supersymmetric** encoding of **spin degrees of freedom**
- **Flow of causality** assured by use of **retarded propagators**

> - **5PM-1SF**: very feasible, open question which special functions will appear in full result (Calabi-Yau?)
> - **5PM-2SF**: very difficult, will require another leap in IBP technology

The **SEOB-PM model** will continue to incorporate future PM-scattering results… and eventually, applications to **bound-orbit waveforms**!

THANKS TO MY COLLABORATORS!



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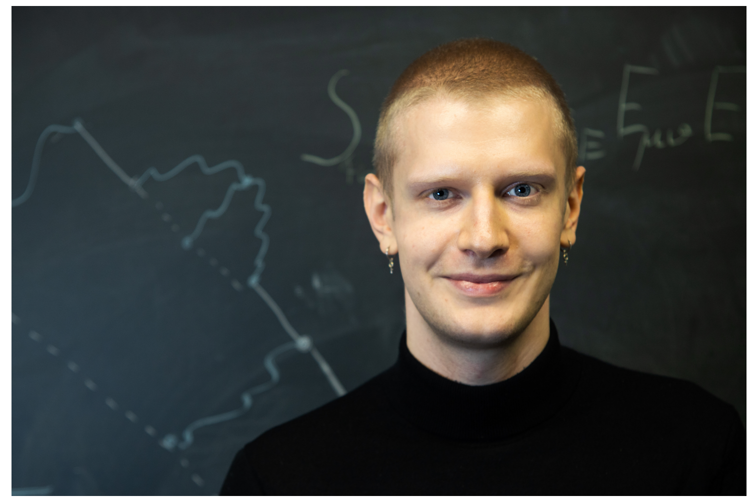

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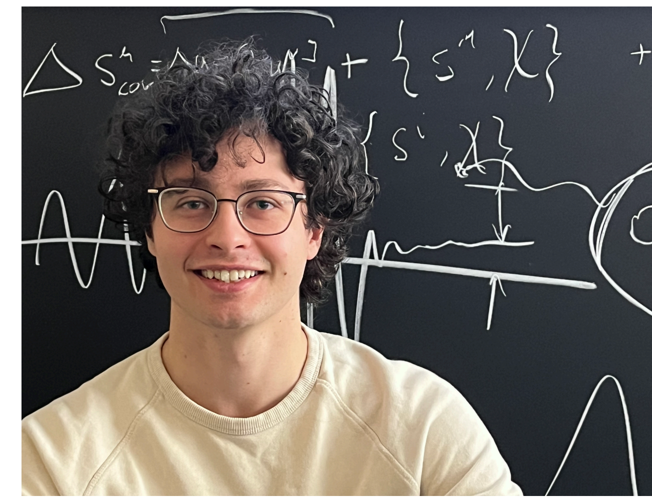

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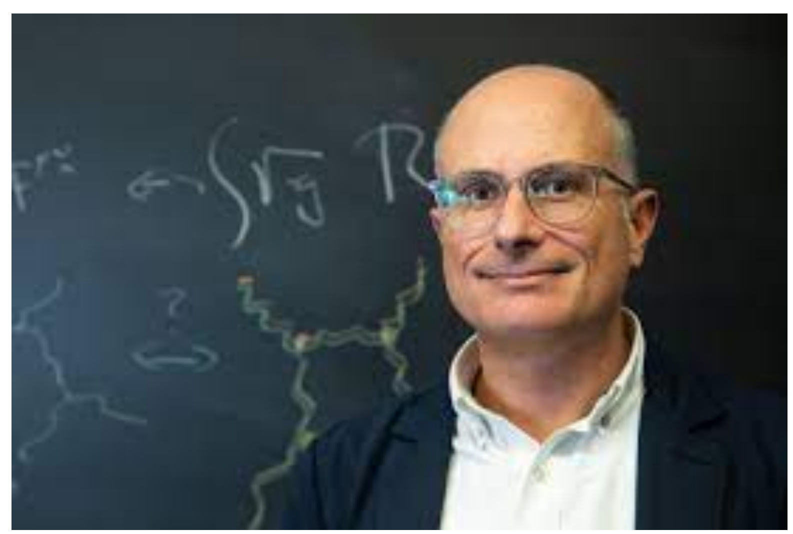

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