QCD with an IR Fixed Point and a Dilaton

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mostly based on

Del Debbio, RZ	JHEP'22 2112.1364	Dilaton new phase?
RZ	PRD, 2306.06752	broken χ-sym.@IRFP - pions
RZ	2306.12914	Dilaton improves Goldstones
Shifman RZ	PRD, 2310.16449	β'_* in N=1 confomal window
RZ	PRD 2312.13761	broken χ-sym.@IRFP - pions & dilaton

Dilaton Dynamics Workshop - Edinburgh - 26-28 June 2024

A collection of refs

more refs in my papers and surely more as an old and interesting topic

- **Pre-QCD work 69-70**

  PhD thesis: John Ellis & Rod Crewther'70 and paper resulting
  Imperial group; Isham, Salam, Strathdee also Mack

- **PDG banned σ-meson (dilaton candidate 20 years)**

  Pelaez et al'96 good values Caprini, Colangelo, Leutwyler'06 good value, small error, Roy eq. with LHC

- **Pre&post LHC model building - (not attached to gauge theories)**

  Rattazi et al 1306.4601 Grinstein et al 0708.1463 Terning et al 1406.5192} ….
  Some pragmatic, some negative conclusions

- **Conformal Window lattice efforts '07 - (walking technicolor)**

  Del Debbio, Lucini, Patella, Kuti, Holland, Sannino, Pica, Appelquist, LSD, LatKMI …….
  Some results established, finding light scalars more and more

- **σ-meson in χPT? Crewther'Tunstall, 12'-15'**
- **Dilaton (gravity explaining origin of Planck Mass) Wetterich, Zee, Shaposhnikov, Karananas..**
- **Crawling TC (connecting Pre-QCD work) Cata, Crewther'Tunstall, 18'**
- **Dilaton-EFT'15 (understand lattic results) Appelquist, Piai, Ingolby .. Golterman & Shamir ..**

Overview

- **Constraint on operator generating dilaton mass**

  $$\Delta_{\mathcal{O}} = d_{\mathcal{O}} + \gamma_{\mathcal{O}} = d - 2$$

  (model-independent - comments on QCD)

- **QCD@IRFP** $\gamma_* = 1$ **in many ways**

- **Relation to** $\mathcal{N} = 1$ **SUSY gauge theories (Seiberg dualities)**

  Inspires $\beta'_* = 0$ *(also in QCD@IRFP)*

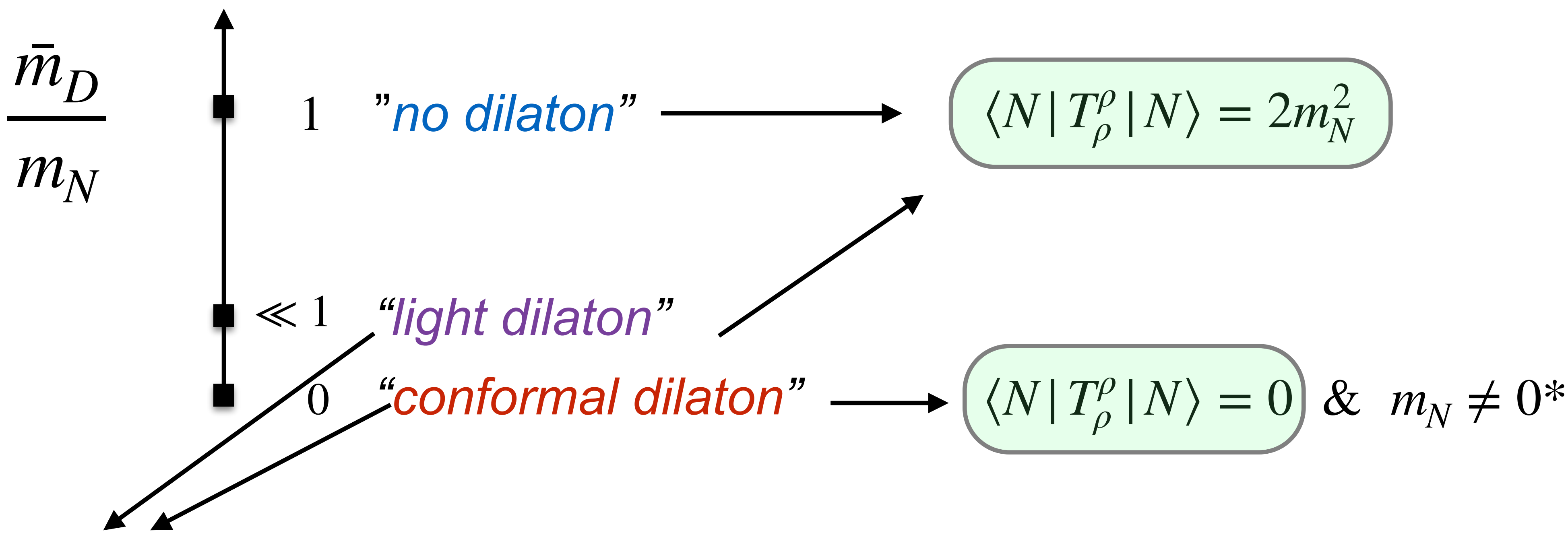
- **Outlook & Conclusions**

Dilaton: basic picture

$|D\rangle$: **lightest State with** $J^{PC} = 0^{++}$

$|N\rangle$: **non-Goldstone state** (e.g. nucleon)

- remove explicit scale breaking …



- (pseudo) Goldstone of spontaneous scale symmetry breaking

$$\langle 0 | T_{\mu\nu} | D(q) \rangle = \frac{F_D}{d-1}(m_D^2 \eta_{\mu\nu} - q_\mu q_\nu)$$

decay constant F_D
order parameter

"dilaton can hide the nucleon mass" (akin Goldberger-Treiman relation) Del Debbio, RZ JHEP'22 2112.1364 + 70's

Dilaton mass?

- Assume operator $\mathcal{O} \subset T^\rho_\rho$ responsible for dilation mass

$$2m_D^2 = \langle D|T^\rho{}_\rho|D\rangle \quad (1)$$ *

$$F_D m_D^2 = \langle 0|T^\rho{}_\rho|D(q)\rangle \quad (2)$$ ← previous slide

- Idea: using soft-dilation thm ⇒ constraint on scaling dimension $\Delta_\mathcal{O}$

RZ 2312.13761

$$\langle D(q)\beta|\mathcal{O}(0)|\alpha\rangle = -\frac{1}{F_D}\langle\beta|i[Q_D,\mathcal{O}(0)]|\alpha\rangle + \lim_{q\to 0} iq\cdot R$$

$$i[Q_D,\mathcal{O}(x)] = (\Delta_\mathcal{O} + x\cdot\partial)\mathcal{O}(x) \qquad R_\mu = -\frac{i}{F_D}\int d^dx e^{iq\cdot x}\langle\beta|TJ^D_\mu(x)\mathcal{O}(0)|\alpha\rangle$$

---

* "dilaton cannot hide its own mass", since dilation pole is needed!

Dilaton soft theorem applied

$$2m_D^2 = \langle D|\mathcal{O}(x)|D\rangle = -\frac{1}{F_D}\langle 0|i[Q_D,\mathcal{O}(x)]|D\rangle = -(\Delta_{\mathcal{O}} + x\cdot\partial)\langle 0|\mathcal{O}(x)|D\rangle$$

- There is **x-dependence** in matrix element: $\langle 0|\mathcal{O}(x)|D(p)\rangle = F_{\mathcal{O}} e^{-ipx}$,

- Interpret as distribution to be smeared out

$$\mathbb{1}_V[x\cdot\partial\langle 0|\mathcal{O}(x)|D\rangle] = -d\frac{1}{V}\int_V d^dx\langle 0|\mathcal{O}(x)|D\rangle$$

$$\mathbb{1}_V = \frac{1}{V}\int_V d^dx .$$

Physics: form **wave packet** (integration by parts ok as no bdry-contribution)

… concluding

$$2m_D^2 = \frac{1}{F_D}(d-\Delta_{\mathcal{O}})\langle 0|T^{\rho}{}_{\rho}|D(0)\rangle = (d-\Delta_{\mathcal{O}})m_D^2$$

$F_D m_D^2$ by (2)

$\Rightarrow$ Operator giving mass to dilation ought to be of scaling dimension

$$\Delta_{\mathcal{O}} = d - 2$$

1st main result

Interpretation in dilation EFT

- Dilaton EFT (dilaton sector) $\chi \equiv e^{-\hat{D}}, \ \hat{D} \equiv \frac{D}{F_D} \quad \kappa_d \equiv \frac{2}{(d-1)(d-2)}$

$$\mathcal{L}_{\rm LO} = \frac{1}{2}\hat{\chi}^{d-4}(\partial\chi)^2 + \frac{\kappa_d}{4} R\,\chi^{d-2} - V_\Delta(\hat{\chi}) + \ldots$$

$$\frac{m_D^2 F_D^2}{\Delta - d}\left(\frac{1}{\Delta}\hat{\chi}^\Delta - \frac{1}{d}\hat{\chi}^d\right)$$

- Improvement term reproduces eq (1) (RZ, 2306.12914)

$$\langle 0|T_{\mu\nu}|D(q)\rangle = \frac{F_D}{d-1}(m_D^2\eta_{\mu\nu} - q_\mu q_\nu)$$

Zumino-term

- Potential with $\Delta_{\mathcal{O}} = d-2$ reproduces eq (2) (RZ, 2312.13761)

$$\langle D|T^\rho{}_\rho|D\rangle = 2m_D^2$$

(corresponds dim. of 2 free fields … can make more formal)

Interpretation in QCD (gauge theory) in d=4

$$T^\rho_\rho\,|_{phys} = \beta/(2g)\, G^2 + N_f m_q (1+\gamma_m)\bar{q}q$$

- $m_q = 0$: only $\mathcal{O} = cG^2$ with $\Delta_{G^2} = 4 + \beta'_* = 4 \neq 2$
- $m_q \neq 0$: then $\mathcal{O} = c\bar{q}q$ with $\Delta_{\bar{q}q} = 3 - \gamma_* = 2 \quad \Leftrightarrow \quad \gamma_* \equiv \gamma_{m_q}|_{\mu=0} = 1$
- Concluding:

  - If $m_D = 0$, deforming $m_q \neq 0$ **dilation GMOR** (previous works 70' and 80' difference $\gamma_* = 1$)

$$F_D^2 m_D^2 = -4 N_f m_q \langle \bar{q}q \rangle$$

  - How G^2 can give mass to dilation seems unclear

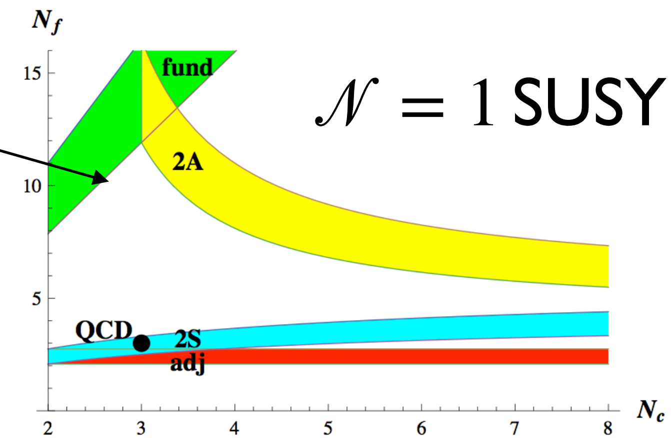
  If (1) is **correct**, as in **tension** with a) **soft theorem** b) standard **LO-EFT**

QCD@IRFP

RZ, PRD, 2306.06752

1.QCD: $\langle \bar{q} q \rangle \neq 0$ **breaks** chiral & possibly scale symmetry **spontaneously**

2.@boundary of **conformal window (CW)**
SSB scale invariance is considered to lead to dilation and IR-CFT interpretation



$\mathcal{N} = 1$ SUSY

3.This talk: **explore** IRFP **interpretation** in all of QCD phase

$\bar{m}_D \neq 0,$
deep-IR ⇒ standard **χPT**

$\bar{m}_D = 0,$
deep-IR ⇒ **dilaton-χPT**

"QCD with an IRFP - pion sector"

"QCD with an IRFP and a dilaton"

Matching scalar adjoint correlator

$$m_q = 0$$

$$S^a = \bar{q} T^a q$$

$$\langle S^a(x) S^a(0) \rangle_{\text{CD-QCD}} = \langle S^a(x) S^a(0) \rangle_{\chi\text{PT}} \,, \quad \text{for } x^2 \to \infty$$

deep-IR

$$\langle \mathcal{O}(x) \mathcal{O}^\dagger(0) \rangle_{\text{CFT}} \propto (x^2)^{-\Delta_{\mathcal{O}}}$$

$$\Delta_{\mathcal{O}} = d_{\mathcal{O}} + \gamma_{\mathcal{O}}$$

$$\langle S^a(x) S^a(0) \rangle_{\text{CD-QCD}} \propto (x^2)^{-(3-\gamma_*)}$$

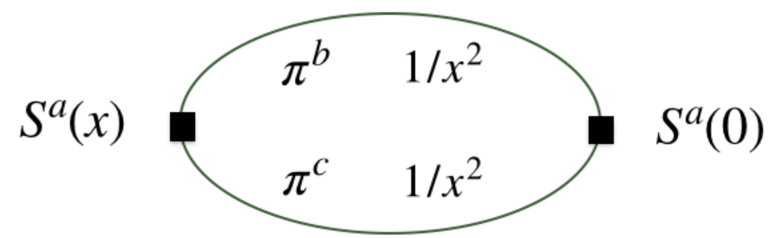
$$\Delta_{S^a} = d_{S^a} - \gamma_*$$

source theory

Gasser & Leutwyler'84

$$S^a|_{\text{LO}} = -\frac{F_\pi^2 B_0}{2} \text{Tr}[T^a U^\dagger + U T^a] \propto B_0 d^{abc} \pi^b \pi^c + \ldots$$

$$\langle S^a(x) S^a(0) \rangle_{\chi\text{PT}} \propto B_0^2 d^{abc} d^{abc} \langle \pi^a(x) \pi^a(0) \rangle^2 \propto \frac{1}{x^4}$$



CFT-scaling matches Goldstone EFT

$$\gamma_* = 1$$

2nd main result

Trace anomaly & Feynman-Hellmann thm

$m_q \neq 0$

$$2m_\pi^2 = \langle \pi^a | T^\rho{}_\rho | \pi^a \rangle$$

$$T^\rho_\rho |_{phys} = \beta/(2g)\, G^2 + N_f m_q (1+\gamma_m)\bar{q}q$$

Ellis, Chanowist, Crewther, Minkowski
Adler, Duncan, Nielsen, Collins, Jogelekar '72-75 '

$$\partial_{\ln m_q} E_\pi = N_f m_q \langle \hat{\pi} | \bar{q}q | \hat{\pi} \rangle + \mathcal{O}(m_q^2)$$

main ingredient: $\partial_{m_q} \langle \hat{\pi}(p') | \hat{\pi}(p) \rangle = 0$

rewrite using GMOR $m_\pi^2 \propto m_q$ (QCD):

$$2m_\pi^2 = \langle \pi | \beta/(2g) G^2 + N_f m_q (1+\gamma_m)\bar{q}q | \pi \rangle$$

$$2m_\pi^2|_{m_q} = 2N_f m_q \langle \pi | \bar{q}q | \pi \rangle$$

reduces to GMOR double soft-pion thm

1. Note that these two **must equate at** $\mathcal{O}(m_q)$, also in standard QCD

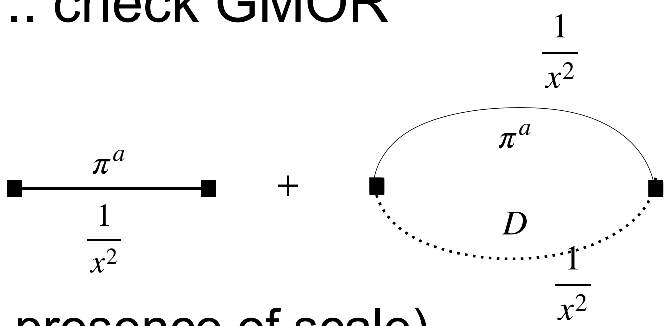
2. Note that $\beta \to \beta_* = 0\,, \gamma_m \to \gamma_* = 1$ seems a simple $\mathcal{O}(m_q)$-solution

$\Rightarrow \gamma_* = 1$ follows once more

*residue $\mathcal{O}(q^2, m_\pi^2) \Rightarrow$ pole no "dramatic" effect

Interpretation & comments

- Works with and without dilation ($m_D = 0$, $m_D \neq 0$) .. check GMOR



- 1) *Accidental?* can be **derived** in **other ways**

  i) $P^a = \bar{q}\gamma_5 T^a q$-correlator
  (breakdown of state-operator correspondence or RG in presence of scale)

  ii) hyperscaling $m_\pi^2 \propto m_q^{\frac{2}{1+\gamma_*}} \propto m_q$ (need to argue)

  iii) low energy thm for pion gravitational form factor RZ, 2306.12914v2

- 2) *Accidental?* **consistent** with **end** of **conformal window** in

  a) $\mathcal{N} = 1$ SUSY gauge theories b) other approaches & lattice

  Suggests: not accidental at boundary

*However, does it **make sense** to **extend below CW**-boundary?*
$\Rightarrow$ look at $\mathcal{N} = 1$

non-standard

$\mathcal{N} = 1$ SUSY gauge theories (Seiberg duality)

electric theory | **magnetic theory**

$SU(N)$ & $2N_f$ chiral matter fields

$SU(N_D)$ & $N_D = N_f - N$ .. matter &

$M_i^{\bar{j}}$ colour neutral **meson field** (adjoint)

Dual IR? a) **global symmetries match** IR b) some operators known to match

a) e.g. $$\langle T^\rho{}_\rho(x) T^\alpha{}_\alpha(0)\rangle_{\mathrm{el}} \overset{\mathrm{IR}}{\longleftrightarrow} \langle T^\rho{}_\rho(x) T^\alpha{}_\alpha(0)\rangle_{\mathrm{mag}}$$

b) e.g. $$\tilde{Q}^{\bar{j}} Q_i \quad \leftrightarrow \quad M_i^{\bar{j}}$$

below CW (chiral sym. broken) $$N + 1 < N_f < \frac{3}{2} N$$ **IR-free magnetic phase**

$$2 - \gamma_* = \Delta_{\tilde{Q}Q} = \Delta_M = 1 \quad \Leftrightarrow \quad \gamma_* = 1$$

- Q: *Does it make sense to extend below CW-boundary?*
  A: **At least in $\mathcal{N} = 1$ SUSY** gauge theory

- We can get **further inspiration** from $\mathcal{N} = 1$....

$$\Delta_{G^2} = 4 + \beta'_* = \Delta_{T^\rho_\rho} \quad \Rightarrow \quad \langle T^\rho{}_\rho(x) T^\rho{}_\rho(0) \rangle_{CW} \propto \frac{1}{(x^2)^{4+\beta'_*}}$$

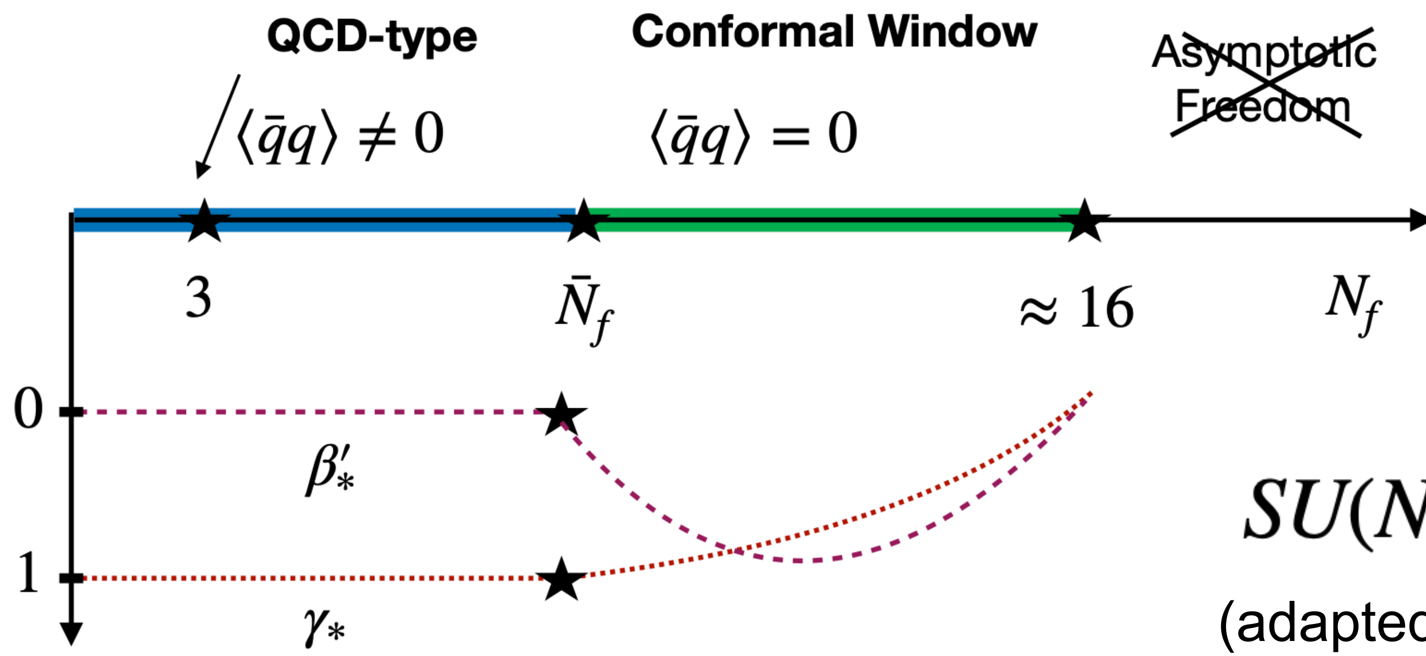
$$\langle T^\rho{}_\rho(x) T^\alpha{}_\alpha(0) \rangle_{\text{el}} \overset{\text{IR}}{\longleftrightarrow} \langle T^\rho{}_\rho(x) T^\alpha{}_\alpha(0) \rangle_{\text{mag}}$$

$$\Rightarrow \quad \boxed{\beta'_*|_{\text{el}} = \beta'_*|_{\text{mag}}}_{\text{CW}}$$

Anselmi, Grisaru, Johanson 97'
Shifman RZ '23

- ***Below CW?*** Magnetic IR-free, thus $\boxed{\beta'_*|_{\text{mag}} = 0 \;\Rightarrow\; \beta'_*|_{\text{el}} = 0}$ **by continuity**

$\mathcal{N} = 1$ CW-picture



$SU(N_c)$, $N_c = 3$
(adapted here for QCD)

RZ, 2312.13761

- Q: *Does $\beta'_* = 0$ hold in non-SUSY (QCD) case?*
  A: Yes

  - If $m_D \neq 0$ can show by matching $\langle T^\rho_\rho(x) T^\rho_\rho(0) \rangle$-correlator
  - RGE (adapted hyperscaling derivation also works)

3rd main result

$\beta'_* = 0$ important since ..

- Power-running $\delta g \propto \mu^{\beta'_*}$ $\Rightarrow$ **log-running**

$$\delta g \propto \frac{1}{|\beta''_*| \ln(\mu/\lambda_{IR})}$$

$\Rightarrow$ seems can **drop** $\mathscr{L}_{\textbf{anom}}(\beta'_*)$ from LO Lagrangian

as anomaly reproduced in extending "EMT in ✗PT" Donoghue & Leutwyler 90'

$\Rightarrow$ **log-running**, sign of **mass-gap**. QCD asymptotes into Goldstone-EFT

- Makes light (or massless) dilaton more probable since: $m_D = \mathcal{O}(\beta'_*) \to \mathcal{O}(\beta''_*)$

- Argument in favour of Seiberg dual for QCD (possibly hidden local symmetry)

Conclusions & Outlook

- Three main results allow to write **LO dilaton-chiral perturbation theory**

$$\Delta_{\bar{q}q} = 3 - \gamma_* = 2 \,,\ V_{\Delta_{\bar{q}q}} \,,\ \beta'_* = 0 \qquad U = e^{i\pi^a T^a / F_\pi} \qquad \hat{\chi} = e^{-D/F_D}$$

$$\mathcal{L}_{\rm LO}^{\rm d\chi PT} = \frac{F_\pi^2}{4}\hat{\chi}^2 {\rm Tr}[\partial^\mu U \partial_\mu U^\dagger] + \frac{1}{2}(\partial\chi)^2 + \frac{1}{12}\chi^2 R \quad + \mathcal{L}_{\rm anom}(\beta'_*)$$

$$\frac{B_0 F_\pi^2}{2}\left({\rm Tr}[\mathcal{M}U^\dagger + U\mathcal{M}^\dagger]\,\hat{\chi}^2 - {\rm Tr}[\mathcal{M} + \mathcal{M}^\dagger]\,\hat{\chi}^4\right)$$

Integer scaling dimension seem unavoidable when matched EFT

Compare

Bardeen, Leung Love'86, γ_* not fixed, not consider β'_* ($m_D = 0$ assumed), $+m_q$

Crewther Tunstall 12-15 γ_* not fixed, $V_{4+\beta'_*}$ $+m_q$

Appqelquist, Ingoldby, Piai 17' γ_* not fixed, V_Δ $+m_q$ (reminimise)

Golterman, Shamir 16' γ_* not fixed, V_4 (gluon field strength) $+m_q$

- *Q: Can the dilaton remain massless when there is a flow into IRFP?*

  A: yes it cab d=3 model Cresswell-Hogg Litim'23 and Cresswell-Hogg Litim, RZ '24

  Methods presented seem to work - consistency in the dilaton-GMOR relation

- *Q: Can $\sigma = f_0(500)$ **meson** be a **dilaton?***

  A1: likely more special than many people think (e.g. light in chiral limit)

  A2: dilaton-EFT. - **width** works qualitatively ..
  - **mass** issues with a) strange quark & b) convergence.

  A3: efforts needed: lattice, FRG, Dyson-Schwinger, Roy equations & Bootstrap?

- *Q: Can **Higgs** be a **dilaton?***

  A: probably yes, **if** $F_\pi/F_D \approx 1$ for $N_f = 2$ (weak force)

  - gauge theory G' with one doublet (narrow dilaton)
  - one does not need massless dilaton
  - coupled to SM via Yukawa-sector as EFT
  - is approximately satisfied in nucleon potential models
    puzzling as there is no symmetry reason known (yet)

Interesting open problems …
Hope to learn more during workshop - thank you!

Backup

An emerging picture

- Message seems to be: integer γ_* is special

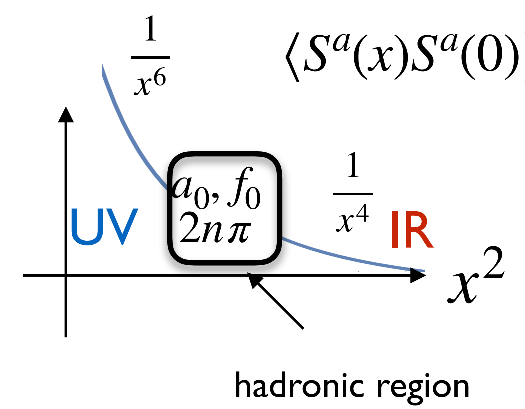
$\gamma_* = 2$ unitarity bound (Mack'77) = 1 free scalar

$\gamma_* = 1$ lower end of CW = 2 free scalars $\Delta^{UV}_{S^a} = 2$

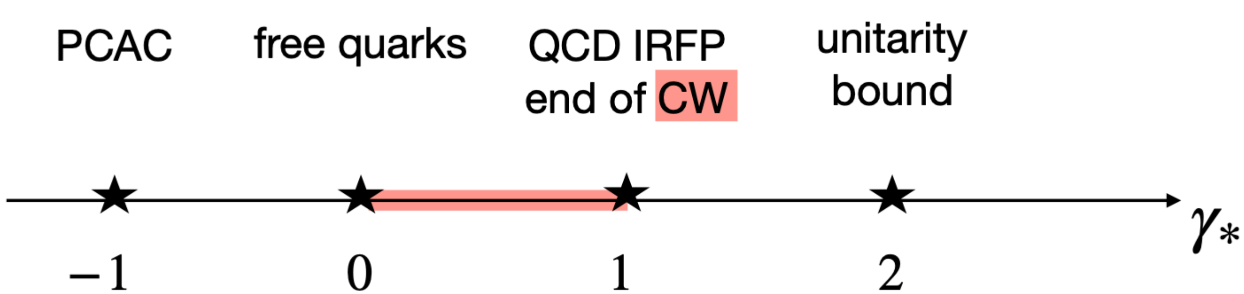
} degenerate $\mathcal{N} = 1$ SUSY

$\gamma_* = 0$ upper end of CW = 2 free quarks $\Delta^{UV}_{S^a} = 3$

$\gamma_* = -1$ PCAC bound (Wilson'69)



QCD-like theories (no scalars) $\gamma_m = -\gamma_{\bar{q}q}|_{\mu=0} = \gamma_*$



- Conformal window only uses 1/3 of allowed γ_*-range

RG derivation of $\beta'_* = 0$

RG-consideration*: $\langle \pi | G^2 | \pi \rangle \propto m_q^{\frac{2+\beta'_*}{y_m}}$

pion-GMOR $\langle \pi | G^2 | \pi \rangle = \mathcal{O}(m_q)$ $y_m = 1 + \gamma_* = 2$

$\Leftrightarrow \quad \beta_* = 0$

* $\langle \pi | G^2 | \pi \rangle \propto F_\pi^2$ since $\langle \pi | \bar{q}q | \pi \rangle \propto F_\pi^2$ by GMOR

The higgs boson as a dilaton

universal part

- If **v = 0, SM conformal** (up to log-running), Higgs like a dilaton

$$(1 + \frac{h}{v}) \to \chi = e^{-\frac{D}{F_D}} \to (1 + \frac{h}{F_D})$$

If number of **doublets = 1** $\Rightarrow v = F_\pi$

- $r = \frac{F_\pi}{F_D} = 1$ is the Standard Model limit

- One can deduce indirectly: $r_{QCD} = 1.0(2) \pm$ syst, **intriguing!**
  a) **no symmetry reason** for this to happen (however, systematics…)
  b) closeness to unity, **LO-invisible @ LHC**

Why does the dilaton couple like the Higgs?

non-universal part

1. popular just before LHC $G_{CFT} = G_{SM} \times G'$ + $\delta\mathscr{L}_{\not{CFT}} = c\mathscr{O}$

new-sector

Golfberger et al, Terning et al etc

in trouble: $\delta_{SM}(gg \to h) \propto \delta_{SM}(h \to \gamma\gamma) \propto \Delta\beta_{decoupled}$ = too large

when it is said that *"the dilaton as a Higgs has been excluded by the LHC"*.
then that's what people mean.

2. another idea (Cata, Crewther'Tunstall, 18') $G_{SM}^{\text{no Higgs}} \xleftrightarrow{\text{Yukawa}} G'$

$$\mathscr{L} \supset \frac{1}{4}v^2 tr[D^\mu U D_\mu U^\dagger] - v\bar{q}_L Y_d U \mathscr{D}_R + \dots$$

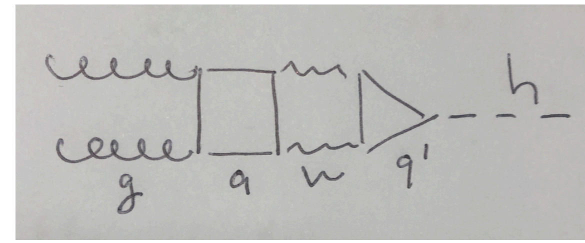
$$U = \exp(i2T^a\pi^a/F_\pi) \qquad U \to V_L U V_Y\,, \quad V_Y = e^{iyT_3}$$

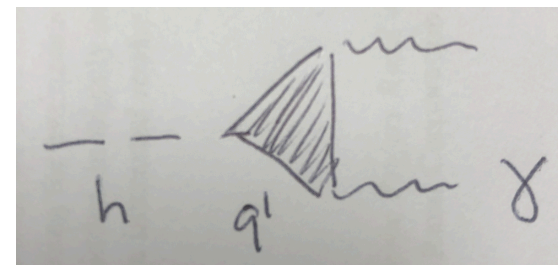
how to bring back the higgs/dilaton is not addressed in that paper,
one cannot use the compensator argument as only G' IR-CFT.

In 2312.13761 it is argued that if there is a symmetry reason for $r_2 \approx 1$, then same reason might enforce the right coupling aka

$$\mathcal{L} \supset \frac{1}{4} v^2 e^{-2D/F_D} \mathrm{Tr}[D^\mu U D_\mu U^\dagger] - v e^{-D/F_D} \bar{q}_L Y_d U \mathcal{D}_R + \dots$$

- **Constraints?**





$\delta_{SM}(gg \to h) =$ NNLO

$\delta_{SM}(h \to \gamma\gamma) =$ non-perturatbive

EWPO: e.g. S-parameter $\delta S = \mathcal{O}(2\%)$ if $r_2 = 1$

most "dangerous one" looks like $h \to \gamma\gamma$
… to be continued & discussed or other idea

- **Higgs-dilaton potential?**

radiatively induced aka composite Higgs with $\Lambda_{G'} = \mathcal{O}(1)\,\Lambda_{EW}$

What is a **dilaton**?

- Always: particle vacuum quantum numbers $J^{PC} = 0^{++}$
  Otherwise: few different meanings

1. **Goldstone boson*** of spontaneously **broken scale invariance** of strong interactions 1968-1970 then largely forgotten *(resurrected as Higgs as dilaton pre-LHC)*

this talk

2. **Scalar component of gravity (gravi-scalar)**
   Brans-Dicke, supergravity (string theory)

3. A **name** for a **light** $J^P = 0^+$ **scalar** in context of approximate scale inv.

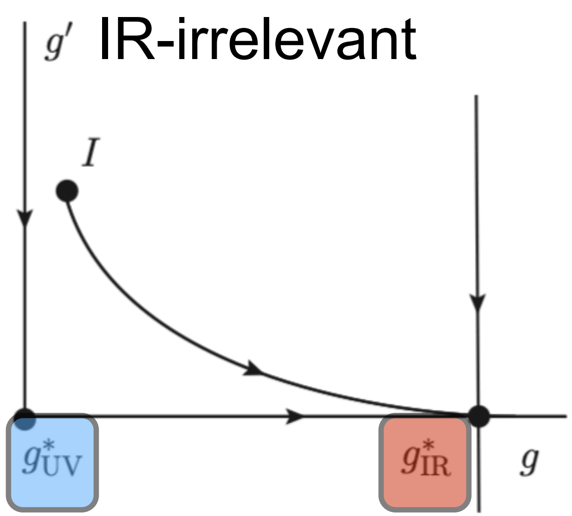
   However, it is not a Goldstone (no limit when it's massless…)

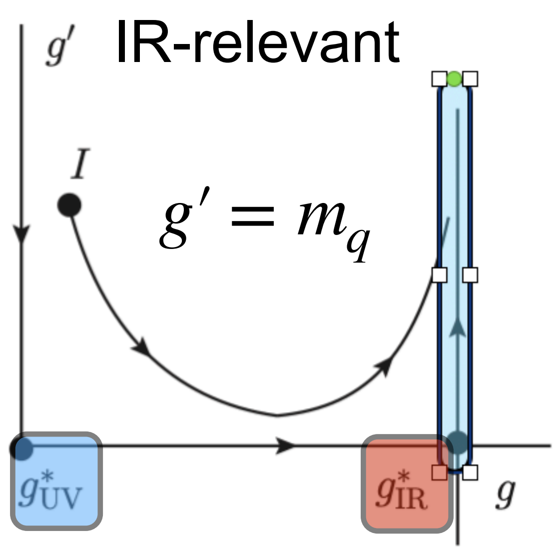
Types of Renormalisation Group (RG)-flow

- assume UV fixed point (e.g. asymptotic freedom) g^*_{UV} , IR flow?

IR fixed point g^*_{IR}

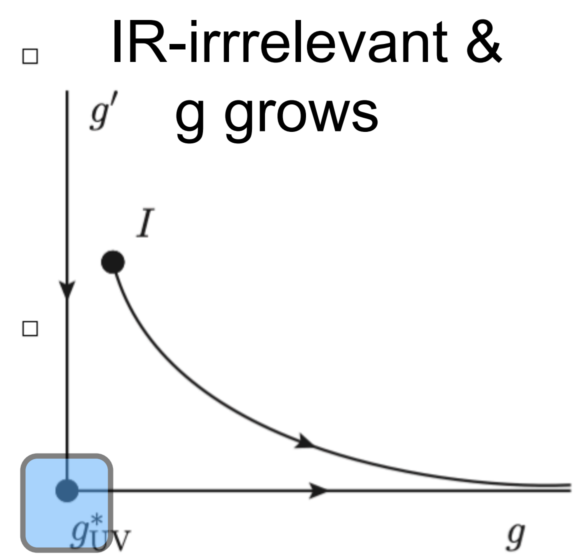
conformal window





no IR fixed point

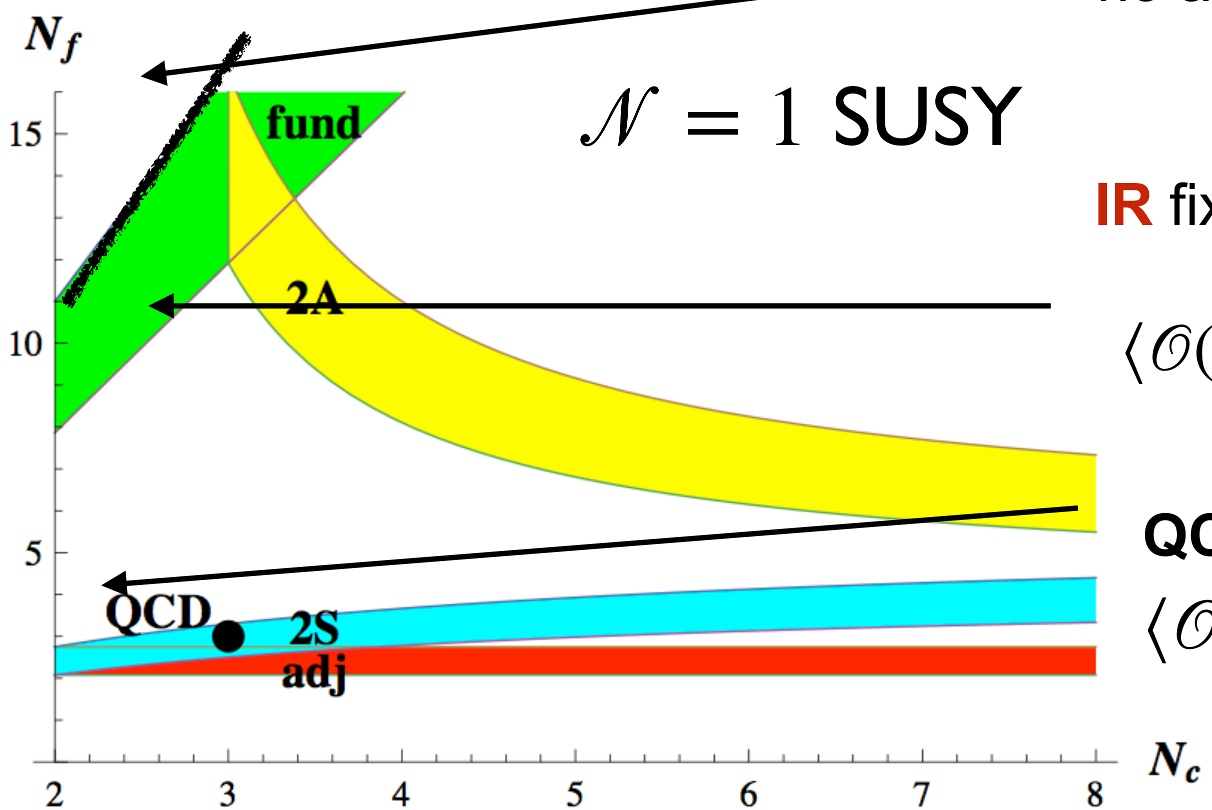
QCD-picture



Phases of gauge theories - **Conformal Window**

- gauge theory **massless quarks** in some **irrep** (e.g. fund. of say $SU(N_c)$)

- Focus on green = fund irrep



$\mathcal{N} = 1$ SUSY

no asymptotic freedom (ignore)

IR fixed point = **conformal window**

$$\langle \mathcal{O}(x)\mathcal{O}(0)\rangle_{CFT} \propto \frac{1}{(x^2)^{\Delta_{\mathcal{O}}}} \quad x^2 \to \infty$$

QCD: *chiral SSB* & *confinement*

$\langle \mathcal{O}(x)\mathcal{O}(0)\rangle_{QCD} \propto$ complicated

QCD@low energy: pion EFT = χPT

isospin

- QCD $\langle \bar{q}q \rangle \neq 0$ **breaks** chiral $SU_L(N_f) \times SU_L(N_f) \to SU_V(N_f)$ **spontaneously,** $N_f^2 - 1$ **Goldstones = pions** $[\, m_\pi^2 = \mathcal{O}(m_q)\,]$

- CCWZ construction $U = e^{i\pi^a T^a / F_\pi}$

$\mathcal{M} \equiv \text{diag}(m_{q_1}, \ldots, m_{q_{N_f}})$

$$\mathcal{L}_{LO}^{\chi PT} = \frac{F_\pi^2}{4} \text{Tr}[\partial^\mu U \partial_\mu U^\dagger] + \frac{B_0 F_\pi^2}{2} \text{Tr}[\mathcal{M} U^\dagger + U \mathcal{M}^\dagger]$$

kinetic *m_q-term (spurion technique)* *GMOR* $m_\pi^2 F_\pi^2 = -2 m_q \langle \bar{q}q \rangle$

PCAC GMOR, Goldberger-Treiman
LO: Weinberg '67
NLO: Weinberg '79
Gasser Leutweyler '84,'85
NNLO: Bijnes, Colangelo, Gasser …

- QCD $\langle \bar{q}q \rangle \neq 0$ also **breaks scale symmetry,** possibly spontaneously?
  If yes, **1** (pseudo) **Goldstones = dilaton**

$$\mathcal{L}_{LO}^{d\chi PT} = later$$

$$m_D^2 = \mathcal{O}(m_q, \beta'_*)$$

does Goldstone mass remember the flow?
(Not settled - If CFT SSB then massless)

IRFP-interpretation - assumptions

- scaling @IRFP with SSB: $\langle \bar{q} q \rangle \neq 0$

$$\langle \mathcal{O}(x)\mathcal{O}(0)\rangle \propto \frac{1}{(x^2)^{\Delta_{\mathcal{O}}}} + \text{ GB-corrections}$$

$x^2 \to \infty$

$\Delta_{\mathcal{O}} = d_{\mathcal{O}} + \gamma_{\mathcal{O}}$

- **assume** exists a scheme: $\beta_* = \beta|_{\mu=0} = 0$

$$\beta = \beta'_* \delta g + \beta''_* \frac{(\delta g)^2}{2} + \mathcal{O}((\delta g)^3) \, , \quad \delta g \equiv g - g_* \, .$$

$$T^\rho{}_\rho|_{\text{phys}} = \frac{\beta}{2g} G^2 + \sum_q m_q(1+\gamma_m)\bar{q}q$$

- idea: QCD@IRFP $\leftrightarrow$ EFT (**dilaton)-χPT** for $x^2 \to \infty$

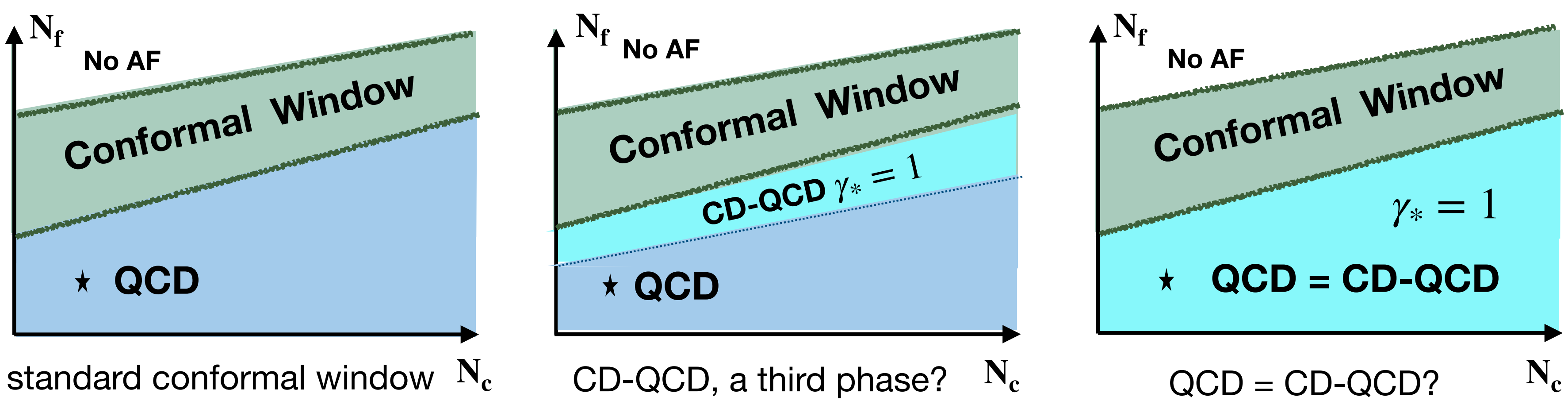
determine anomalous dimension: e.g* $\gamma_{m_q} = -\gamma_{\bar{q}q}|_{\mu=0} \equiv \gamma_*$

*** main quantity in CW-hunt. and Walking technicolor** $-1 \leq \gamma_* \leq 2$ **allowed range**

irrelevant(PCAC) unitarity

End of main part and …

- At least any of these three possibilities is logically possible.
  Option 1 is what is what is taken for granted in standard view.



- Hope, convinced you that option 2 & 3 are not as absurd as .. I thought as well.

- Important: under assumptions got back consistent results.

Before going to T^{ρ}_{ρ}-correlator ...

…. pause and introduce EFT: **dilaton-χPT**

chiral	dilatation	
$J^a_{5\mu} = \bar{q} T^a \gamma_\mu \gamma_5 q$	$J^D_\mu(x) = x^\nu T_{\mu\nu}(x)$	sym. currents
$\langle \pi^b(q) \lvert J^a_{5\mu} \rvert 0 \rangle = i F_\pi q_\mu \delta^{ab}$	$\langle D(q) \lvert J^D_\mu \rvert 0 \rangle = i F_D q_\mu$	decay constants= order parameters
$U = e^{i\pi^a T^a / F_\pi}$	$\chi \equiv F_D e^{-D/F_D}$	coset rep.
$U \to L U R^\dagger$	$\chi \to \chi e^{\alpha(x)}$	transformation
$(L, R) \in SU(N_f)_L \otimes SU(N_f)_R$	$\alpha(x) \in \mathbb{R}$	

Isham, Salam, Strathdee, Mack, Zumino ca '70

Leading order dilaton-χPT

- Building principle: enforce Weyl invariance

$$g_{\mu\nu} \to e^{-2\alpha} g_{\mu\nu} \qquad \chi \to \chi e^{\alpha} \qquad U \to U$$

$$\Delta_{\bar{q}q} = 3 - \gamma_* = 2$$

quark mass = expl. sym-breaking

$$\mathcal{L}_{\rm LO}^{\rm d\chi PT} = \frac{F_\pi^2}{4} \hat{\chi}^2 \mathrm{Tr}[\partial^\mu U \partial_\mu U^\dagger] + \frac{B_0 F_\pi^2}{2} \hat{\chi}^{\Delta_{\bar{q}q}} (\mathrm{Tr}[\mathcal{M} U^\dagger + U \mathcal{M}^\dagger] + \frac{1}{2} (\partial \chi)^2$$

standard-extend χPT + dilaton **global Weyl inv.**

$$- \frac{\Delta_{\bar{q}q}}{4} \mathrm{Tr}[\mathcal{M} + \mathcal{M}^\dagger] \hat{\chi}^4) + \frac{1}{12} \chi^2 R + \mathcal{L}_{\rm anom}(\beta'_*) + V(\chi) \,,$$

removes tadpole (**Zumino-term**)

local Weyl inv. - solves Goldstone improvement problem (another talk RZ 2306.12914)

matches trace anomaly (talk later)

potential (big unknown) last part …

Ready for T^ρ_ρ-correlator …

- Trace of EMT: $T^\rho{}_\rho|_{\rm phys} = \frac{\beta}{2g} G^2$

$$(\gamma_{G^2})_* = \beta'_* \quad \Rightarrow \Delta_{T^\rho{}_\rho} = \Delta_{G^2} = 4 + \beta'_*$$

$$\beta = \beta'_* \delta g + \beta''_* \frac{(\delta g)^2}{2} + \mathcal{O}((\delta g)^3) \;, \quad \delta g \equiv g - g_*$$

- Formally (& RG)

$$\langle T^\rho{}_\rho(x) T^\rho{}_\rho(0) \rangle \propto (\beta'_* \delta g + \beta''_* \frac{(\delta g)^2}{2})^2 \frac{1}{(x^2)^{4+\beta'_*}}$$

- EFT **difference between χPT and dilaton-χPT** (with improvement RZ 2306.12914)

$$T^\rho{}_\rho|^{\rm LO}_{\chi{\rm PT}} = -\frac{1}{2}\partial^2 \pi^a \pi^a \;, \quad T^\rho{}_\rho|^{\rm LO}_{d\chi{\rm PT}} = 0$$

$$\langle T^\rho{}_\rho(x) T^\rho{}_\rho(0) \rangle^{\rm LO}_{\chi{\rm PT}} \propto \frac{1}{x^8} \;, \quad \langle T^\rho{}_\rho(x) T^\rho{}_\rho(0) \rangle^{\rm LO}_{{\rm d}\chi{\rm PT}} \propto 0$$

- χPT implies $\beta'_* = 0$ for dχPT not obvious (need RG-tools)

2nd main result

$\beta'_* = 0$ seems important for consistency

$$\delta g \propto \frac{1}{|\beta''_*|\ln(\mu/\lambda_{IR})}$$

- Power-running $\delta g \propto \mu^{\beta'_*}$ $\Rightarrow$ **log-running**

  $\Rightarrow$ seems can **drop** $\mathscr{L}_{\textbf{anom}}(\beta'_*)$ from LO Lagrangian

  as anomaly reproduced in extending "EMT in ✗PT" Donoghue & Leutwyler 90'

  $\Rightarrow$ **log-running**, sign of **mass-gap**. QCD asymptotes into Goldstone-EFT

- Makes light (or massless) dilaton more probable since: $m_D = \mathcal{O}(\beta'_*) \to \mathcal{O}(\beta''_*)$

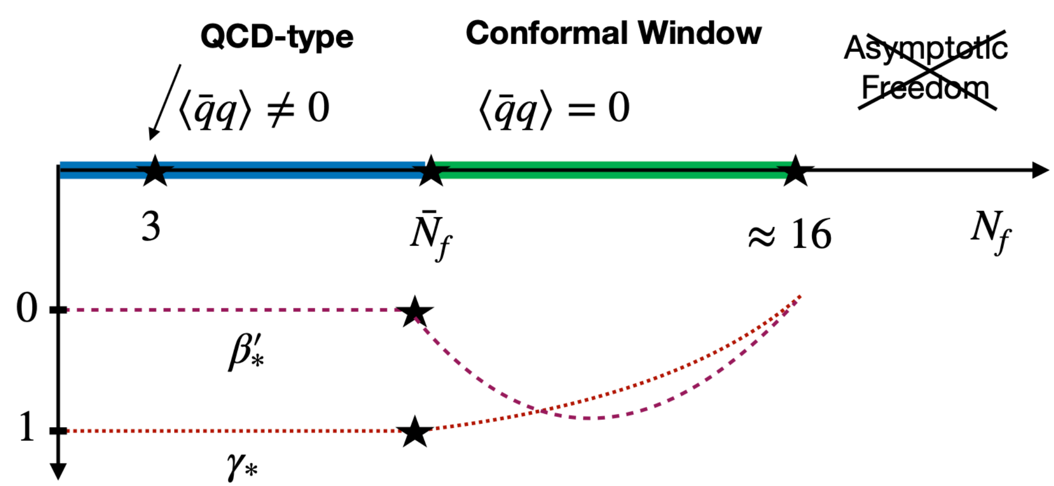
- Continuous **matching** to **N=1 SUSY** conformal window $\beta'_* \to 0$ @boundary

  Anselmi, Grisaru, Johanson 97' Shifman RZ '23

$$\beta'_*|_{\rm el} = \beta'_*|_{\rm mag} \quad \Leftrightarrow \quad \langle T^\rho{}_\rho(x) T^\alpha{}_\alpha(0)\rangle_{\rm mag} \overset{\rm IR}{\longleftrightarrow} \langle T^\rho{}_\rho(x) T^\alpha{}_\alpha(0)\rangle_{\rm el}$$

- Summary figure:

  $SU(N_c)\ , N_c = 3$



The essence of QCD and the dilaton

- A **dilaton in QCD**? Who? Consensus it would be the $\sigma \equiv f_0(500)$-meson

$$\sqrt{s_\sigma} = m_\sigma - \frac{i}{2}\Gamma_\sigma = (441^{+16}_{-8} - i272^{+9}_{-12.5})\,\text{MeV}\;,$$

Caprini, Colangelo, Leutwyler'06
Roy-equations+input

- **Question**: does m_σ become massless or nearly massless in chiral limit?
  Fact: *nobody knows*, some indication it becomes lighter.

- using **dilaton-χPT:**
  1) can reproduce width ($SU(3)_F$-analysis): $\Gamma_\sigma = 616\,^{-108}_{+146} \pm \text{syst}^*$ MeV
  2) soft-mass even too large (EFT-convergence broken)

- **Concluding**: 1) success (already 1970's) 2) inconclusive
  Hence, not bad but there could be more to it …

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* notion of σ decay constant F_σ not well-defined, took model-value needs further thought

The higgs boson as a dilaton

Attention: different ways to implement ...
some universal and some not.

- If **v = 0, SM conformal** (up to log-running), Higgs like a dilaton

$$(1+\frac{h}{v}) \to \chi = e^{-\frac{D}{F_D}} \to (1+\frac{h}{F_D})$$

universal

If number of **doublets = 1** $\Rightarrow$ $v = F_\pi$ and $r = \frac{F_\pi}{F_D}$ determines diff. to SM

- One can deduce indirectly: $r_{QCD} = 1.0(2) \pm$ syst, **intriguing!**
  a) **no symmetry reason** for this to happen (however, systematics…)
  b) closeness to unity, **LO-invisible @ LHC**

- An idea for model: **new gauge sector IRFP**,
  EWSB as in technicolor and **dilaton** as **naturally light Higgs**

$$\mathcal{L} \supset \frac{1}{4} v^2 e^{-2D/F_D} \mathrm{Tr}[D^\mu U D_\mu U^\dagger] - v e^{-D/F_D} \bar{q}_L Y_d U \mathcal{D}_R + \ldots$$

non-universal

Like SM@LO but **why** coupled in this way?
Suspect, if there is a symmetry reason for $r \approx 1$,
then same reason enforces Lagrangian as above.

to be continued …

BACKUP

Massive Hadrons in Conformal Phase

Chiral limit $m_q \to 0$ resolve the contradiction below

$$\langle \phi(p) \,|\, T^{\mu}_{\mu} \,|\, \phi(p) \rangle \begin{cases} = 2m_\phi^2 & \text{standard formula} \\ = 0 & \text{with (massless) dilaton} \end{cases}$$

"*The dilaton can hide the nucleon mass*"

Del Debbio, RZ JHEP'22 2112.1364

Gravitational Form Factors

focus scalar instead of nucleon

- parameterise using Lorentz & translation invariance ($\partial^\mu T_{\mu\nu} = 0$)

$$\langle \varphi(p') \,|\, T_{\mu\nu} \,|\, \varphi(p) \rangle = 2\mathscr{P}_\mu \mathscr{P}_\nu G_1(q^2) + (q_\mu q_\nu - q^2 \eta_{\mu\nu}) G_2(q^2)$$

$\mathscr{P} = \frac{1}{2}(p + p')\,, \quad q = p - p'$ momentum transfer

- consider soft limit $q \to 0$ then G_2 drops and using $P_\mu = \int d^3x T^0_\mu$

$$\langle \phi(p) \,|\, T^\mu_\mu \,|\, \phi(p) \rangle = 2m_\phi^2$$

$$G_1(0) = 1$$

… seems the end of the road (for massive hadrons and conformality)

- Let's have another look at*

$$\langle \varphi(p') \,|\, T_{\mu\nu} \,|\, \varphi(p) \rangle = 2P_\mu P_\nu G_1(q^2) + (q_\mu q_\nu - q^2 \eta_{\mu\nu}) G_2(q^2)$$

$\langle \phi(p) \,|\, T^\mu_\mu \,|\, \phi(p) \rangle = 2m_\phi^2$ does **not** need to **hold if**

$$G_2(q^2) = \frac{r}{q^2} + \ldots$$

Goldstone pole (the **dilaton)**

- That is already a bit of a shock - can we make this quantitative?

Yes in soft limit, as then can use $G_1(0) = 1$ and vanishing trace imposes

$$r = \frac{2m_\phi^2}{(d-1)}$$

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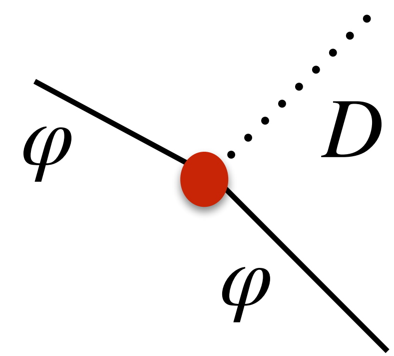
*e.g lecture notes Gell-Mann '69 (pre-QCD), no details worked out

Computation of Residue (new)

$$r = \frac{2m_\phi^2}{(d-1)}$$

- need to know $\langle D\varphi \,|\, \varphi\rangle = i(2\pi)^d \delta\left(\sum p_i\right)$ $g_{\varphi\varphi D}$
- can get it via **compensator trick** **(Weyl scaling)**



$$g_{\mu\nu} \to e^{-2\alpha} g_{\mu\nu}\,, \quad \varphi \to e^{\alpha}\varphi \quad \Rightarrow \quad D \to D - \alpha F_D$$

compensates m_φ^2 by dilaton, regain``conformal inv'': $\delta_\alpha \sqrt{-g}\,\mathscr{L}^{eff} = 0$

$$\mathscr{L}^{eff} \supset -e^{-2D/F_D}\frac{1}{2}m_\varphi^2\varphi^2 \quad \Rightarrow \quad g_{D\varphi\varphi} = \frac{2m_\varphi^2}{F_D}$$

$$r = \frac{2m_\phi^2}{(d-1)}$$

- now apply the LSZ formula (or dispersion theory)

$$\langle D\varphi|\varphi\rangle = \lim_{q^2\to 0}(-i)\frac{q^2}{Z_D}\int d^dx e^{iq\cdot x} P_2^{\mu\nu} T_{\mu\nu}^{(\varphi)}(p,p',x)$$

$$= \lim_{q^2\to 0}(-i)\frac{q^2}{Z_D} G_2(q^2)(2\pi)^d\delta\left(\sum p_i\right)$$

use EMT as dilaton interpolator

$Z_D = -F_D/(d-1)$

- from where we get exactly the right residue

$$r = \lim_{q^2\to 0} q^2 G_2(q^2) = -g_{\varphi\varphi D} Z_D = \frac{2m_\varphi^2}{d-1}$$

- Rather encouraging. The **approach** is **self-consistent**!

The dilaton improves Goldstones

based on
2306.12914 RZ

The standard improved scalar field

- Two terms curved space, no dim. couplings* $\mathcal{L} = \frac{1}{2}\left((\partial\varphi)^2 - \xi R \varphi^2\right)$

$$T^{\rho}{}_{\rho} = -d_{\varphi}(\partial\varphi)^2 + \xi(d-1)\partial^2\varphi^2 = (d-1)(\xi - \xi_d)\partial^2\varphi^2$$

↑ eom

- Conformal $T^{\rho}_{\rho} = 0$, only for $\xi = \xi_d \equiv \frac{(d-2)}{4(d-1)} \to \frac{1}{6}$ (d=4)
  - improved EMT Callan, Coleman, Jackiw'70, finite EMT (necessary as observable)
  - earlier in GR: Penrose'64 required by weak equivalence principle Chernikov&Tagirov'68
  - finite integrated Casimir-effect deWitt'75

- Heuristically, $\mathscr{L} \propto R\phi^2$, not possible to write with coset field $U = e^{i\frac{\pi^a T^a}{F_\pi}}$

Dolgov & Voloshin'82 Leutwyler-Shifman '89, Donoghue-Leutwyler' 91

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* may also work in flat space from start, but less elegant

Intermezzo on relevance for flow theorems

- Focus d=2 for simplicity, Weyl anomaly $T^{\rho}_{\rho} = cR$ reveals central charge of CFT.

  c-theorem (Zamalodchikov'86).: $\Delta c = c_{UV} - c_{IR} \geq 0$

  Cardy'88.: $\Delta c \propto \int d^2x \, x^2 \, \langle T^{\rho}_{\rho}(x) T^{\rho}_{\rho}(0) \rangle \quad \Rightarrow \quad T^{\rho}_{\rho} \to 0$ in UV and IR fast enough

  d=2 ok, Goldstone special anyway

- d=4, if **Goldstones not improvable** $T^{\rho}_{\rho} = -\frac{1}{2}\partial^2 \pi^2$, then **log-IR divergence**

  a-thm* & $\Box R$-flow analogue formula IR-divergent

$\Rightarrow$ Goldstone improvement desirable

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*for a-thm, Luty, Polchiniski, Rattazzi'12 ' provide argument formula is IR-onvergent as inclusive enough

The Goldstone improvement proposal

- dilaton-pion system improvement

$$\mathcal{L}_{\rm LO} = \mathcal{L}_{\rm kin,4} + \mathcal{L}_4^R - V_4(\chi)$$

$$\mathcal{L}_{\rm kin,d} = \frac{F_\pi^2}{4}\hat{\chi}^{d-2}{\rm Tr}[\partial^\mu U \partial_\mu U^\dagger] + \frac{1}{2}\chi^{d-4}(\partial\chi)^2$$

standard Lag.

$$\mathcal{L}_d^R = \frac{\kappa}{4} R \chi^{d-2}$$

0, no mass (later..)

improvement term, κ to be **determined**

- **locally Weyl invariant ⇒** conformal invariance.

$$\kappa = \kappa_d \equiv \frac{2}{(d-1)(d-2)} \stackrel{d\to 4}{\to} \frac{1}{3}$$

Compared to $\xi_4 = 1/6$ like a
``double improvement" (more to say)

- realises decay constant in EFT

$$\langle 0|T_{\mu\nu}|D(q)\rangle \stackrel{\rm def}{=} \frac{F_D}{d-1}(m_D^2\eta_{\mu\nu} - q_\mu q_\nu) \quad = \quad \langle 0|T_{\mu\nu}^R|D(q)\rangle = \langle 0|\frac{1}{6}(\eta_{\mu\nu}\partial^2 - \partial_\mu\partial_\nu)\chi^2|D(q)\rangle$$

3a. **Improvement $T^{\rho}_{\rho} = 0$ use of equation of motion**

- dilaton eom: $\chi \partial^2 \chi = 2\mathcal{L}^{\pi}_{\text{kin},4} - \partial_{\ln \chi} V_4$

$$T_{\mu\nu} = \frac{F_\pi^2}{2} \hat{\chi}^2 \text{Tr}[\partial_\mu U \partial_\nu U^\dagger] + \partial_\mu \chi \partial_\nu \chi - \eta_{\mu\nu}(\mathcal{L}_{\text{kin},4} - V_4) + T^R_{\mu\nu}$$

$$T^R_{\mu\nu} = \frac{\kappa}{2}(g_{\mu\nu}\partial^2 - \partial_\mu \partial_\nu)\chi^2$$

$$T^{\rho}{}_{\rho}|_{V=0} = \frac{3}{2}\kappa \partial^2 \chi^2 - 2\mathcal{L}^{\pi}_{\text{kin},4} - 2\mathcal{L}^{D}_{\text{kin},4}$$

$$\stackrel{eom}{=} \frac{3}{2}\kappa \partial^2 \chi^2 - (\partial \chi)^2 - \chi \partial^2 \chi$$

$$= (3\kappa - 1)\{\chi \partial^2 \chi + (\partial \chi)^2\} = 0$$

$$\kappa = \kappa_4 = \frac{1}{3}$$

- **works** as expected from **local Weyl invariance**, also works d-dim curved space