Workshop Dilaton Dynamics - from Theory to Applications
Edinburgh, 26 - 28 June 2024

Critical QFTs with spontaneous breaking of scale symmetry

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critical points

thermal or quantum phase transitions

continuous phase transition
Landau-Ginsburg-type universality
conformal / topological phase transitions
deconfined criticality, ``emergence”

symmetry breaking, generation of mass

conformal field theory

scale invariance vs conformal invariance
conformal bootstrap, RG, CFT data

critical points in 4d

UV critical points

fundamental definition of QFT — Wilson '71

asymptotic freedom — Gross, Wilzcek '73 , Politzer '73
asymptotic near-freedom — Bailin, Love '74, Litim, Sannino '14
asymptotic safety — Weinberg '79

IR critical points

Banks-Zaks conformal window — Caswell '74, Banks, Zaks, '82

weak-strong dualities — Seiberg '95

critical points in 3d

many applications in condensed matter physics
Dirac materials

Wilson-Fisher fixed points

mass generation and symmetry breaking

AdS/CFT conjecture

3d critical bosons and critical fermions
relate to higher-spin gauge theories on AdS4

Klebanov, Polyakov ’02
Sezgin, Sundell ’03

Giombi, Yin ‘12

CFT vs higher spin symmetry

Maldacena, Zhiboedov ’11, ‘12

Chern-Simons-matter dualities

Aharony, Giombi, Gur-Ari, Maldacena, Yacoby, ‘12
Seiberg, Senthil, Wang, Witten, ‘16

today:

3d QFTs with strongly interacting FPs
and *spontaneous* scale symmetry breaking

scalars	O(N)
fermions	Gross-Neveu
Yukawa	Gross-Neveu—Yukawa

based on 2207.10115, 2212.06815, 2311.16246, 2406.00100
and ongoing work with **Charlie Cresswell-Hogg**

recap: O(N) symmetric scalars

3d: **super**-renormalisable $(\phi^* \phi)^3_{3\mathrm{d}}$

free UV fixed point
Wilson-Fisher IR fixed point

Townsend '77
Appelquist, Heinz '82
Pisarski '82

exactly solvable at infinite N

Bardeen, Moshe, Bander '84
David, Kessler, Neuberger '84

main tool: functional RG

Wilson '71
Polchinski '84
Wetterich '92

$$\partial_t \Gamma_k = \frac{1}{2} \operatorname{tr} \left\{ \left[\Gamma_k^{(2)} + R_k \right]^{-1} \cdot \partial_t R_k \right\}$$

$t = \ln k$

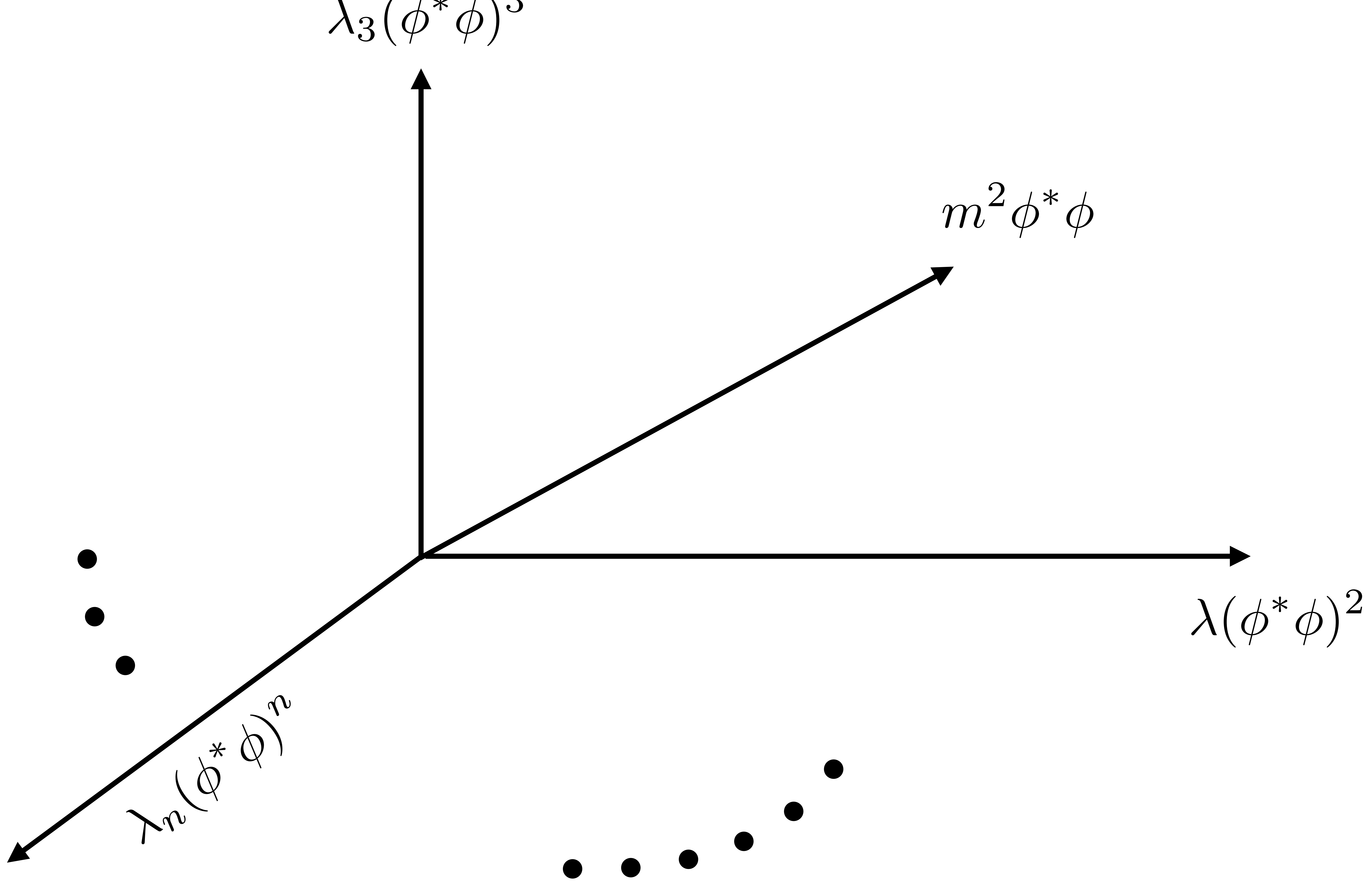
UV
$k = \Lambda$

IR
$k = 0$

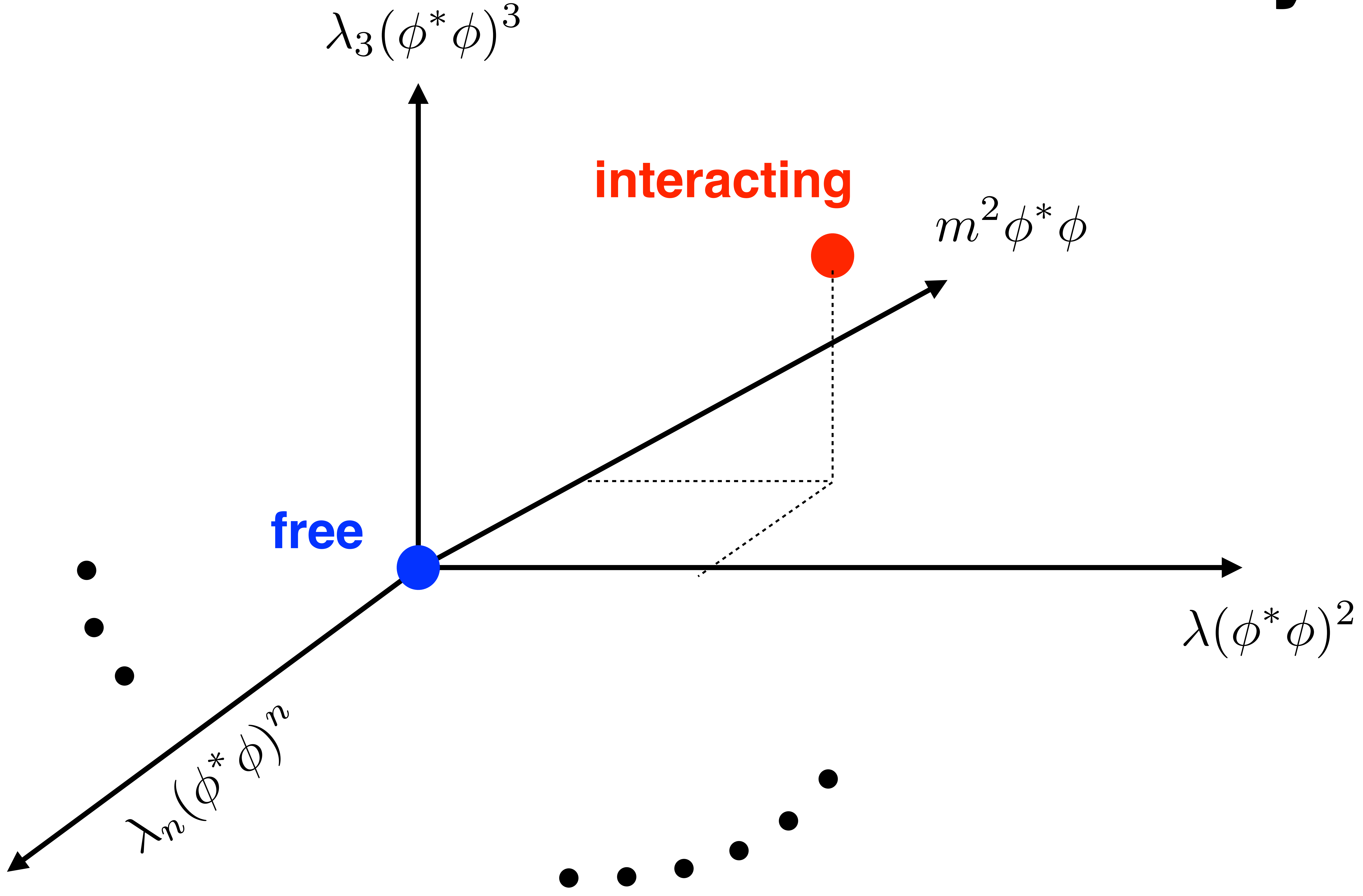
Large N: LPA flow exact, and exactly solvable

d'Attanasio, Morris '96
Cresswell-Hogg, Litim '24

"theory space"



“theory space”



“theory space”

interactions	classical
$m^2\phi^*\phi$	relevant
$\lambda(\phi^*\phi)^2$	relevant
$\lambda_3(\phi^*\phi)^3$	marginal
$\vdots$	
$\lambda_n(\phi^*\phi)^n$	irrelevant
$\vdots$	

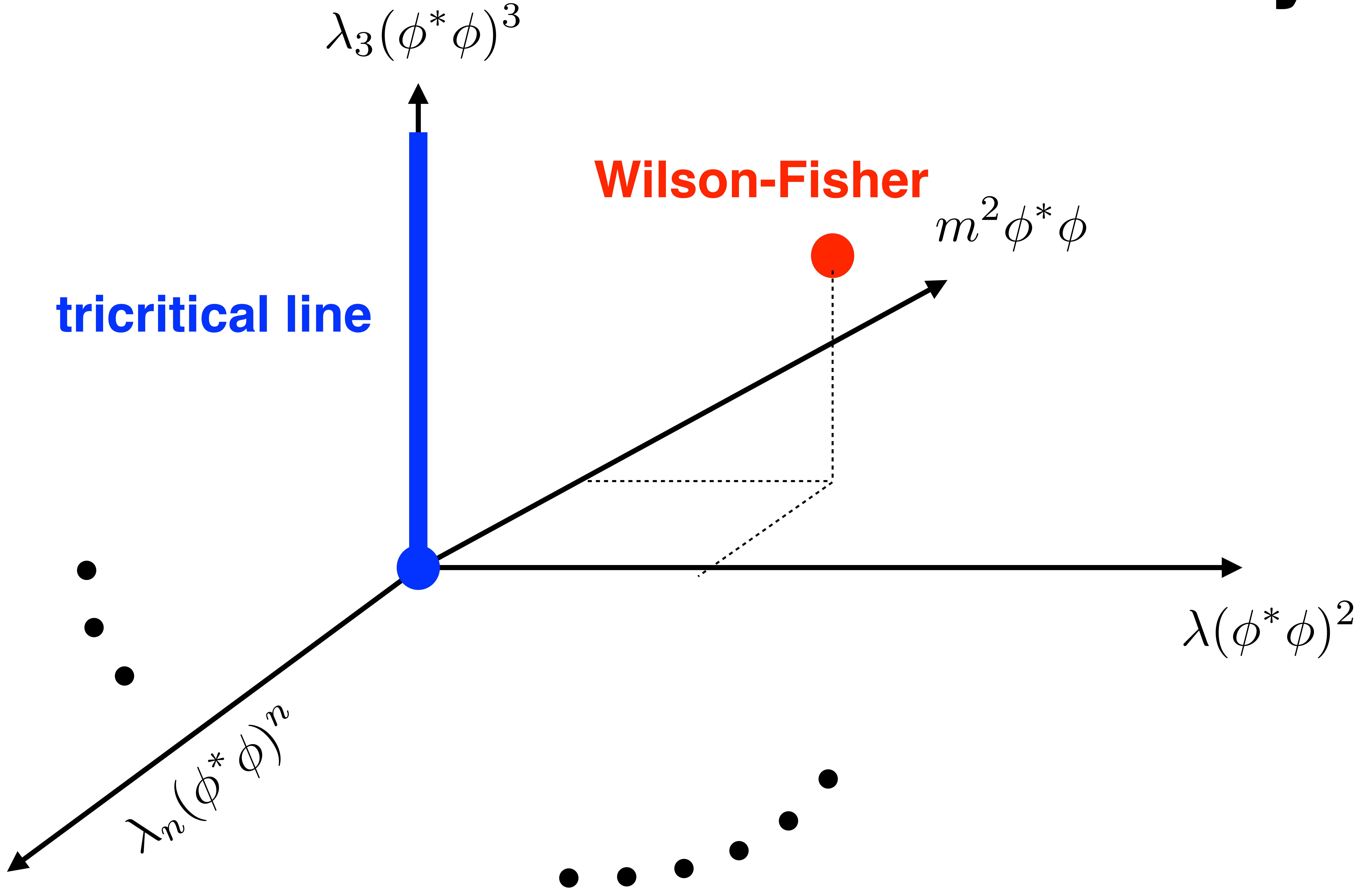
UV

“theory space”

interactions	classical	quantum
$m^2\phi^*\phi$	relevant	relevant
$\lambda(\phi^*\phi)^2$	relevant	irrelevant
$\lambda_3(\phi^*\phi)^3$	marginal	irrelevant
$\vdots$		
$\lambda_n(\phi^*\phi)^n$	irrelevant	irrelevant
$\vdots$		

UV ⟷ **IR**

"theory space"



functional RG study

Polchinski:
UV cutoff

$$P_{\rm UV} = \frac{K(q^2/k^2)}{q^2}$$

Polchinski ’84

exact map
R=R_opt

$$K(q^2/k^2) = \frac{R_k(q^2)}{q^2 + R_k(q^2)}$$

$$P_{\rm UV} + P_{\rm IR} = \frac{1}{q^2}$$

Wetterich:
IR cutoff

$$P_{\rm IR} = \frac{1}{q^2 + R_k(q^2)}$$

Wetterich ‘92

DF Litim, E Marchais, P Mati, 1702.05749/PRD
DF Litim, M Trott, 1810.01678 /PRD

functional RG study

Polchinski:
UV cutoff

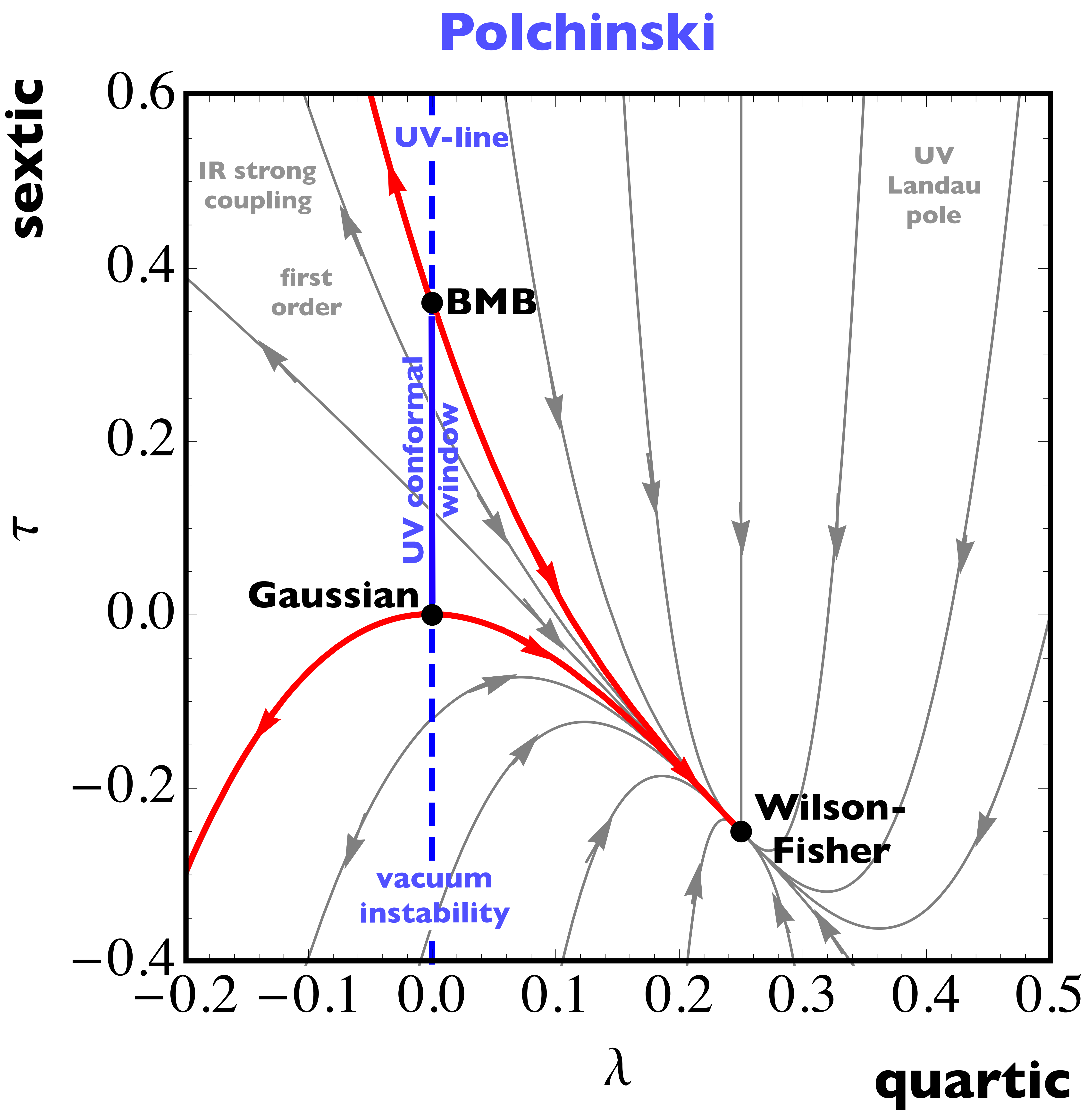
exact map
R=R_opt

$$\begin{cases} \partial_t u &= -du + (d-2)\rho u' + 2\rho(u')^2 - (N-1)u' - (u' + 2\rho u'') \\ \partial_t w &= -dw + (d-2)zw' + (N-1)\left(\frac{1}{1+w'} - 1\right) + \left(\frac{1}{1+w'+2zw''} - 1\right) \end{cases}$$

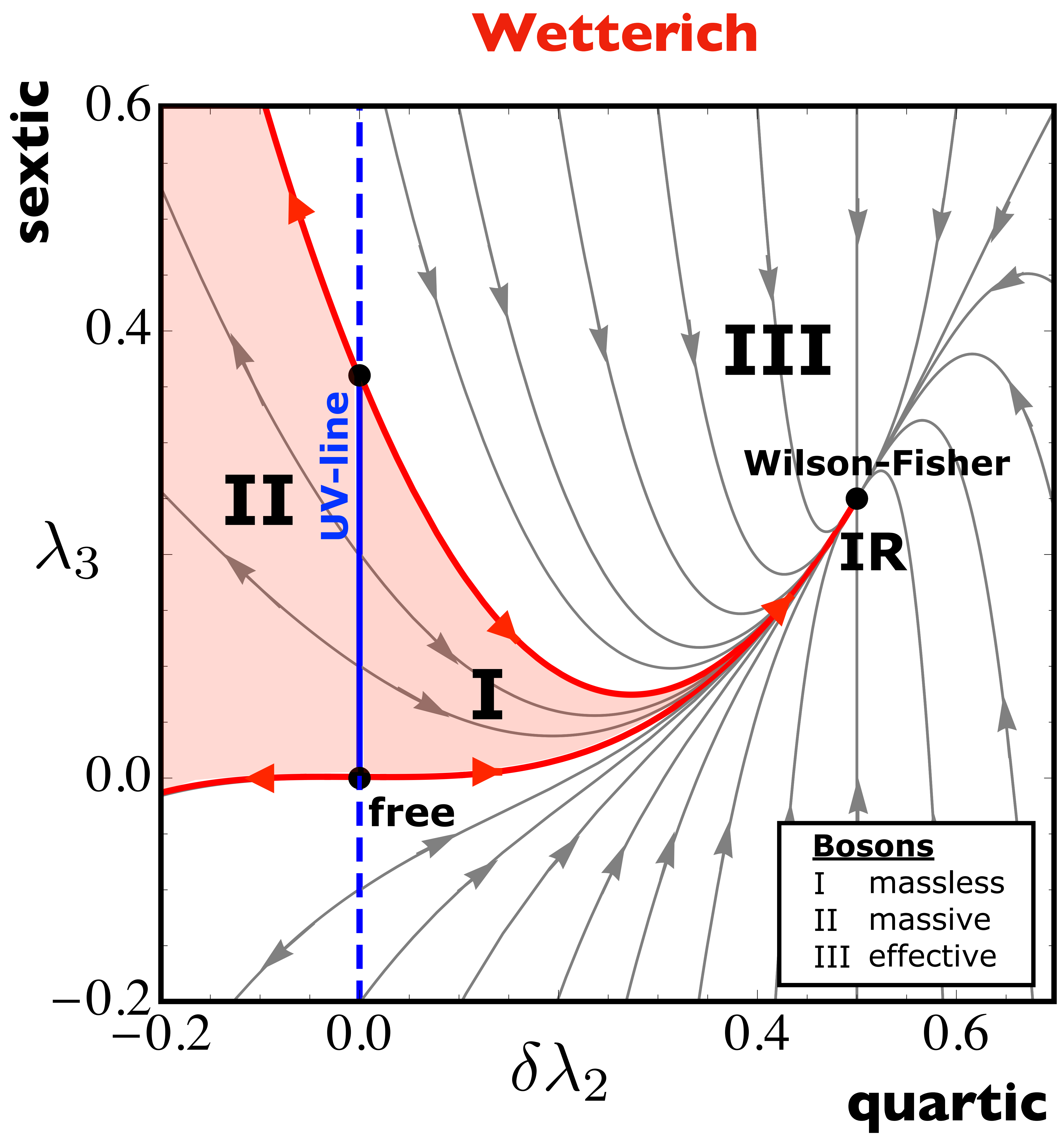
Wetterich:
IR cutoff

DF Litim, E Marchais, P Mati, 1702.05749/PRD
DF Litim, M Trott, 1810.01678 /PRD

functional RG



DF Litim, M Trott, 1810.01678 /PRD



DF Litim, E Marchais, P Mati, 1702.05749/PRD

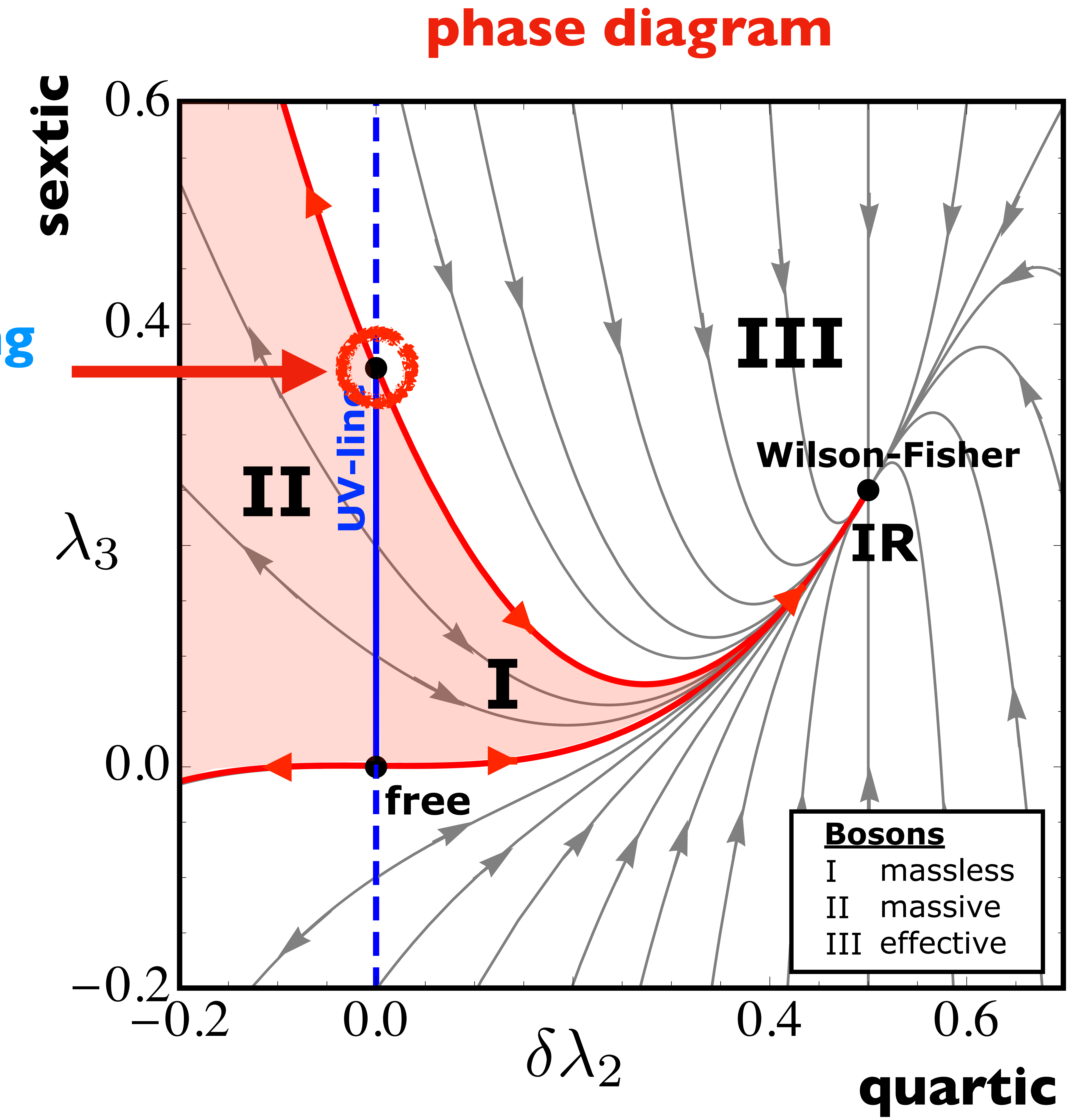
functional RG

BMB phenomenon Bardeen, Moshe, Bander '84
David, Kessler, Neuberger '84

spontaneous scale symmetry breaking
breaking of hyperscaling

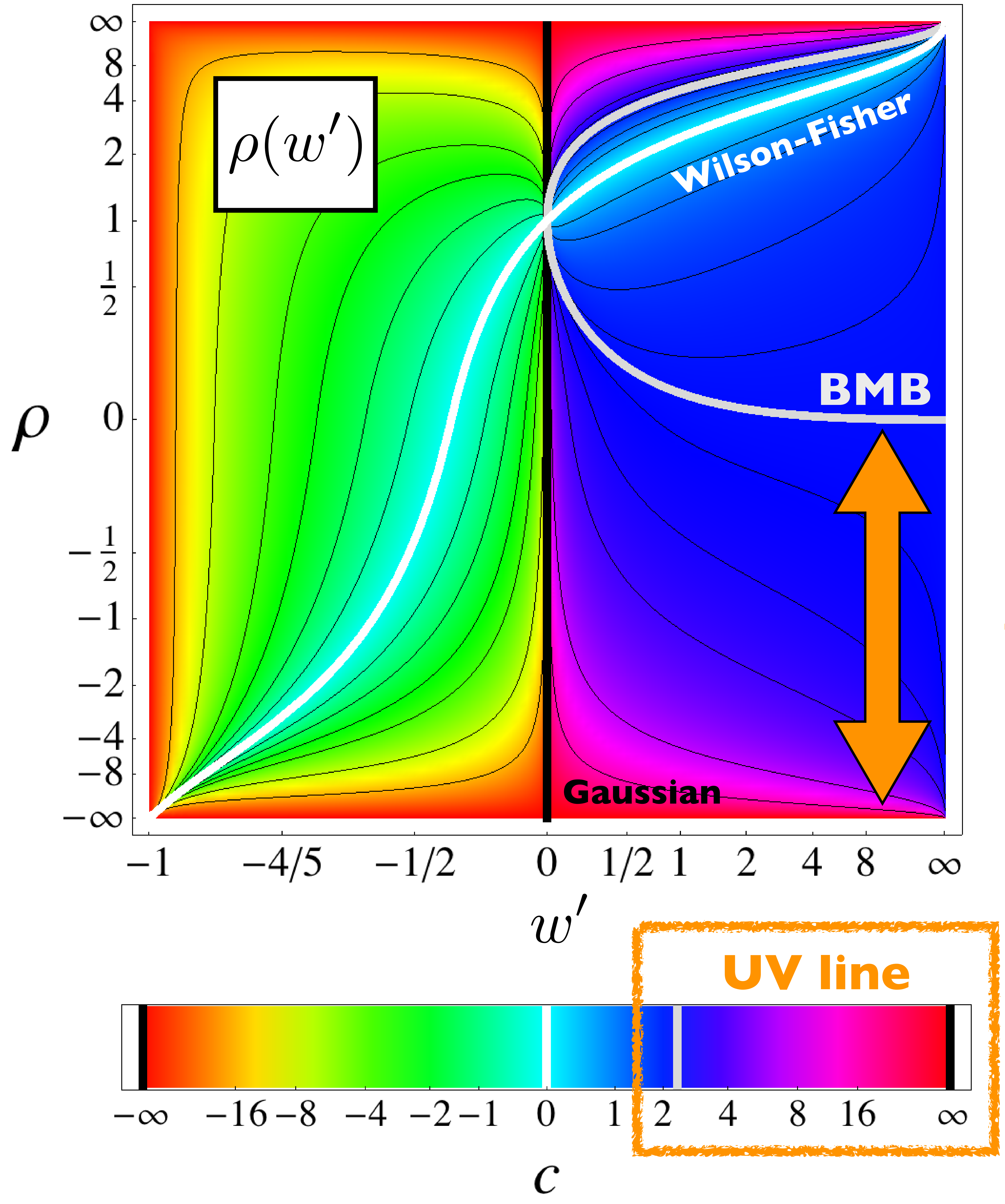
compact conformal manifold
physical mass = free parameter

non-perturbative
infinite-order in local couplings



DF Litim, E Marchais, P Mati, 1702.05749/PRD

fixed points



“global” fixed points

w’ = w’(rho) for all rho

finite UV conformal manifold

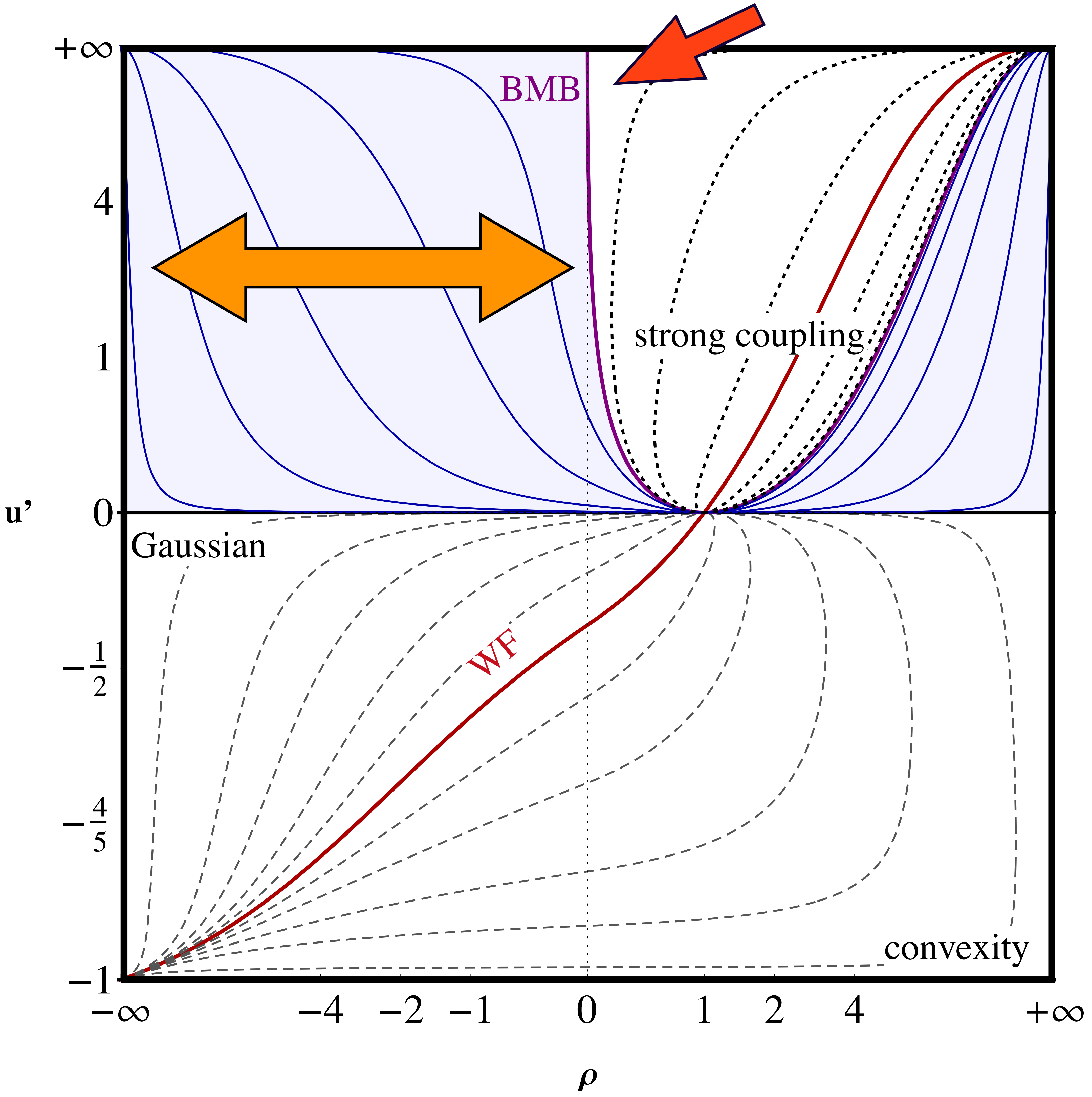
field $\rho = \phi^* \phi$

sextic $\tau \propto \dfrac{1}{c^2}$

Bardeen-Moshe-Bander phenomenon

spontaneous breaking of scale invariance

Bardeen, Moshe & Bander ('84)



fingerprint

$$\lim_{\rho\to 0} u'(\rho) \to \infty$$

DL, E Marchais, P Mati (2014)

spontaneous generation of a new mass scale

$$M^2 = \lim_{k\to 0} u'(\rho = 0)\cdot k^2 > 0$$

***M* not determined by fundamental parameters**

and now

something completely different …

… fermions

Gross-Neveu

U(N) symmetric fermions

4-fermion interactions $G(\bar{\psi}\psi)^2$

Gross, Neveu '74

chiral symmetry $\psi \to \gamma^5 \psi \quad \bar{\psi} \to -\bar{\psi}\gamma^5$

3d: perturbatively non-renormalisable… $[G] = 2 - d$

…yet non-perturbatively renormalisable
interacting UV fixed point

Gawedzki, Kupiainen '85
Rosenstein, War, Park '89
de Calan, Faria da Veiga, Magnen, de Seneor '91

functional RG

Jakovac, Patkos '13, '14
Cresswell-Hogg, Litim,'22, '23 and in prep '24

Gross-Neveu+

relax chiral symmetry

$$S = \int \mathrm{d}^d x \left\{ \bar{\psi}_a \partial\!\!\!/ \psi_a + \frac{1}{2} G (\bar{\psi}_a \psi_a)^2 + \frac{1}{3!} H (\bar{\psi}_a \psi_a)^3 \right\}.$$

mass term permitted $m\bar{\psi}\psi$

6-fermion interactions permitted

$$[H] = 3 - 2d$$

functional RG

exactly solvable at infinite Nf

Cresswell-Hogg, Litim, **Line of fixed points in Gross-Neveu theories** 2207.10115
Critical fermions with spontaneous broken sale symmetry 2212.06815
Scale-symmetry breaking and generation of mass at quantum critical points 2311.16246

“theory space”

interactions

	classical
$\lambda_1 \, \bar{\psi}\psi$	relevant
$\lambda_2 (\bar{\psi}\psi)^2$	irrelevant
$\lambda_3 (\bar{\psi}\psi)^3$	⋮
$\lambda_n (\bar{\psi}\psi)^n$	
⋮	

IR

“theory space”

interactions

	classical	quantum
$\lambda_1 \, \bar{\psi}\psi$	relevant	relevant
$\lambda_2(\bar{\psi}\psi)^2$	irrelevant	relevant
$\lambda_3(\bar{\psi}\psi)^3$	.	marginal
$\lambda_n(\bar{\psi}\psi)^n$	.	irrelevant
.	.	.
.		.
.	IR	UV

Gross-Neveu+

functional RG

$$\Gamma_k = \int d^d x \left(\sum_{i=1}^{N_\mathrm{f}} \bar\Psi_i \partial\!\!\!/ \Psi_i + V_k(\Psi, \bar\Psi) \right)$$

exact local potential

$$\partial_t v = -dv + (d-1)zv' - (4N_\mathrm{f}+1)\,\ell_d[(v')^2] + \ell_d[v' \cdot (v' + 2zv'')],$$

$v(\bar\psi\psi)$
fermion potential

classical

$z \propto \bar\psi\psi$
field variable

quantum

Gross-Neveu+

$$v(z) = \sum_n \frac{\lambda_n}{n!} z^n$$

large Nf:

mass $$\beta_1 = -\lambda_1 + \frac{2\lambda_1\lambda_2}{(1+\lambda_1^2)^2}$$

4F $$\beta_2 = (d-2)\lambda_2 + \frac{2\lambda_1\lambda_3}{(1+\lambda_1^2)^2} + \frac{(2-6\lambda_1^2)\lambda_2^2}{(1+\lambda_1^2)^3}$$

6F $$\beta_3 = (2d-3)\lambda_3 + \frac{2\lambda_1\lambda_4}{(1+\lambda_1^2)^2} + \frac{6\lambda_2\lambda_3(1-3\lambda_1^2)}{(1+\lambda_1^2)^3} + \frac{24\lambda_1\lambda_2^3(\lambda_1^2-1)}{(1+\lambda_1^2)^4}.$$

mass=0 is an exact RG fixed point

Cresswell-Hogg, Litim, **Line of fixed points in Gross-Neveu theories** 2207.10115
Critical fermions with spontaneous broken sale symmetry 2212.06815
Scale-symmetry breaking and generation of mass at quantum critical points 2311.16246

generation of mass is 1/N protected even though parity symmetry is absent

Cresswell-Hogg, Litim, **Generation of fermion mass without symmetry breaking,** 2406.00100

Gross-Neveu+

mass=0:

4F fixed point

4F $\tilde{\beta}_2 = (d - 2 + 2\lambda_2)\lambda_2,$

6F $\tilde{\beta}_3 = (2d - 3 + 6\lambda_2)\lambda_3,$

mass = 0 **renders 4F and 6F betas homogeneous**

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Gross-Neveu+

mass=0:

4F fixed point

4F $$\tilde{\beta}_2 = (d - 2 + 2\lambda_2)\lambda_2,$$

6F $$\tilde{\beta}_3 = (2d - 3 + 6\lambda_2)\lambda_3,$$

3d: 4F fixed point renders 6F coupling exactly marginal

scheme independent

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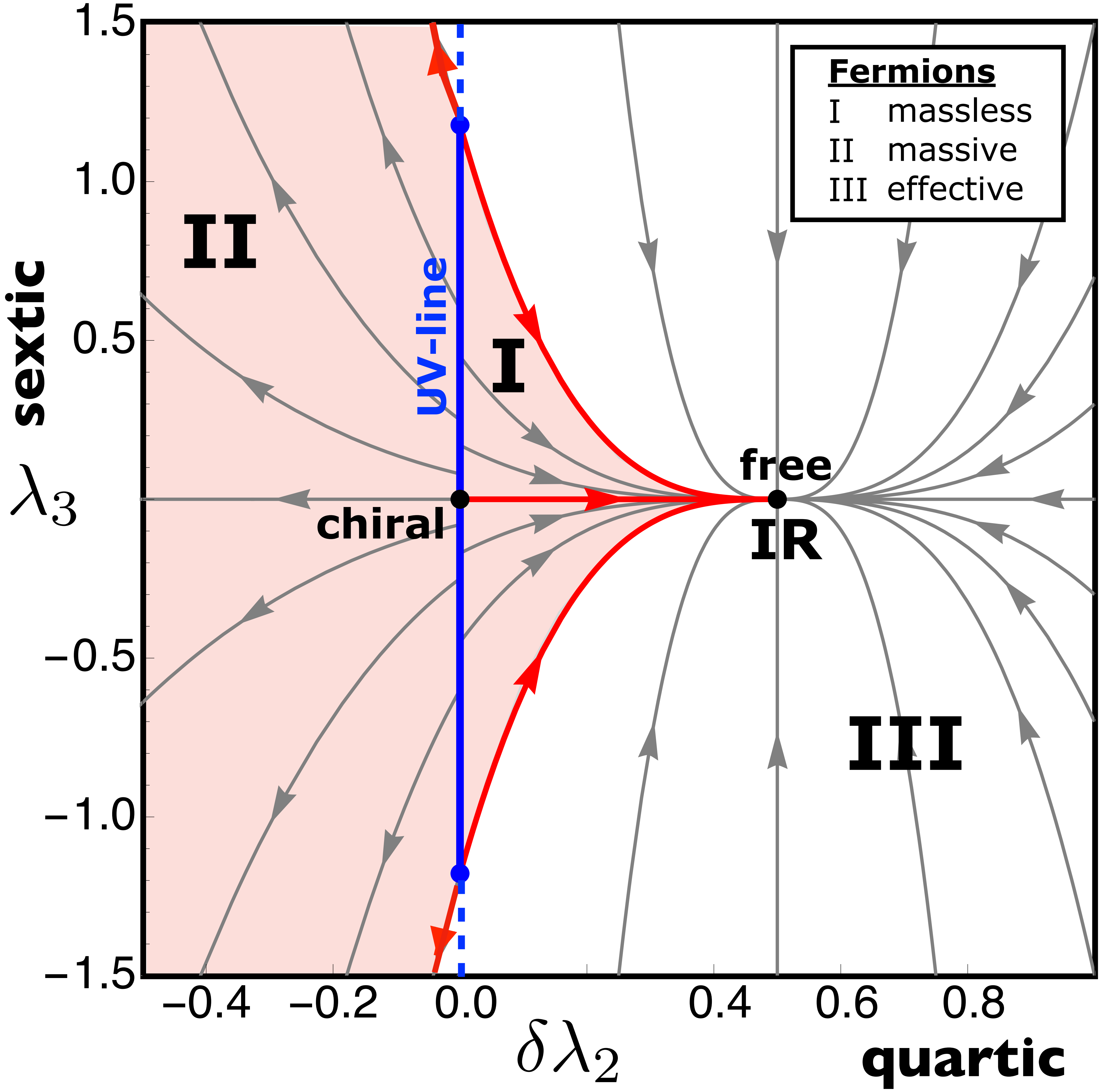
Gross-Neveu+

UV-IR connecting trajectories

exactly marginal sextic coupling



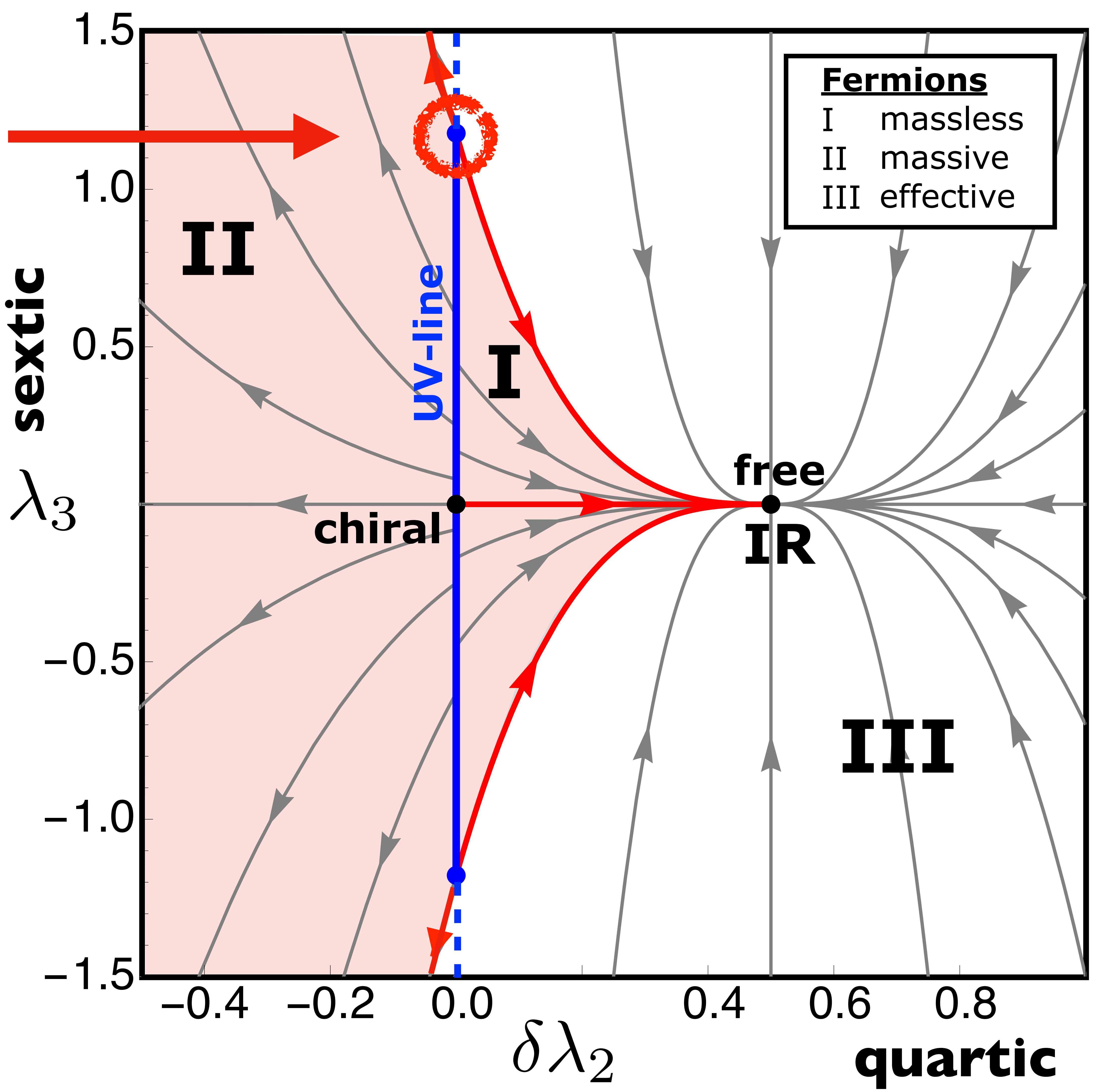
Cresswell-Hogg, Litim, **Line of fixed points in Gross-Neveu theories** 2207.10115

Gross-Neveu+

spontaneous scale symmetry breaking
breaking of hyperscaling

compact conformal manifold
physical mass = free parameter

non-perturbative
infinite-order in local couplings



Cresswell-Hogg, Litim, **Line of fixed points in Gross-Neveu theories** 2207.10115

Gross-Neveu+

global fixed points

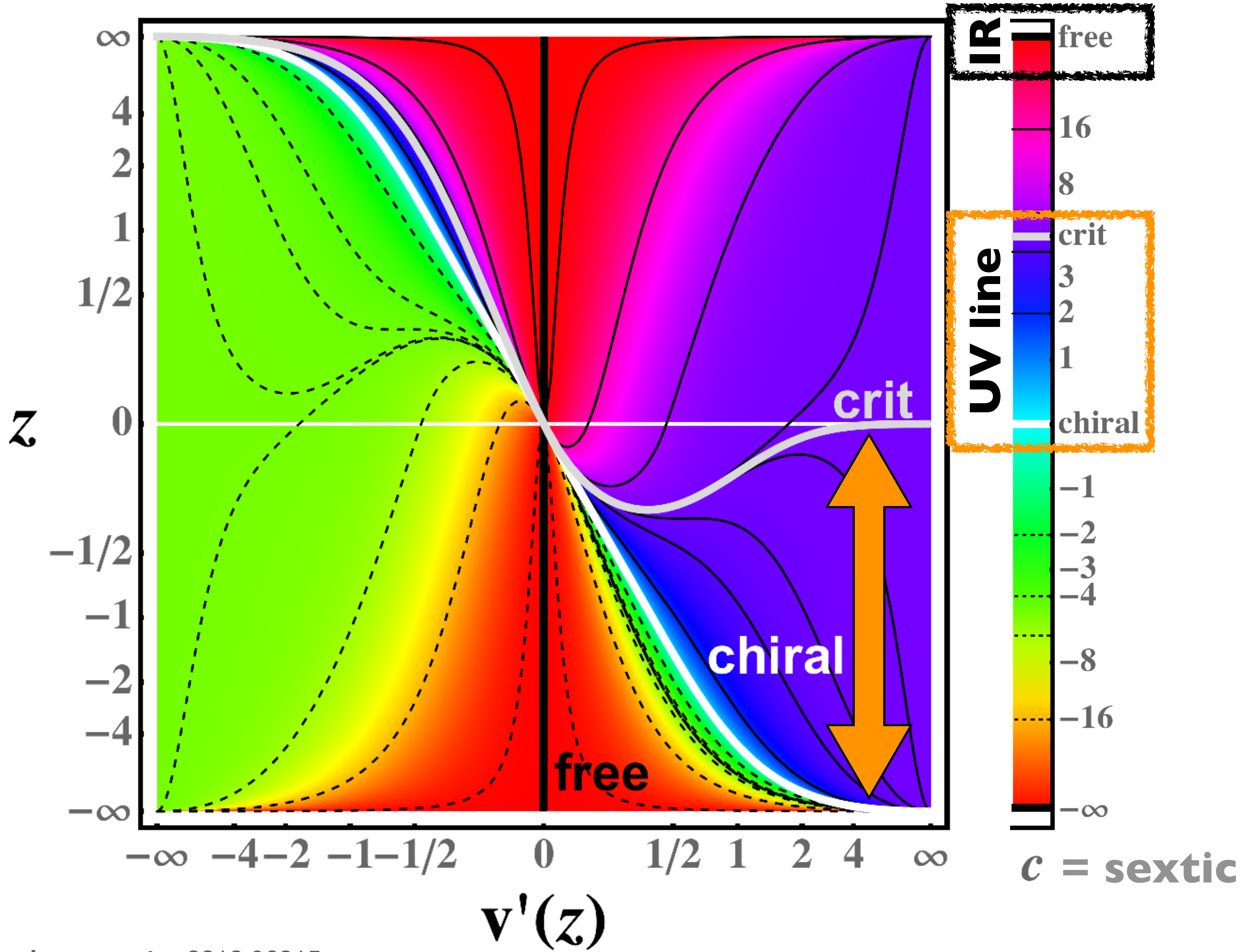
v' = v'(z) for all z

z=0: v'= 0 — **mass=0**

v''= -1/2 — **4F**

v'''= c — **6F**

finite UV conformal manifold



Cresswell-Hogg, Litim, **Critical fermions with spontaneous broken sale symmetry** 2212.06815

Gross-Neveu+

spontaneous generation of mass

$$M = \lim_{k\to 0} k \cdot v'(0)$$

gap equation

$$\left[c - \frac{3\pi}{2}\,\mathrm{sgn}(M)\right] M^2 = 0$$

two solutions

[...] = 0

M = free parameter

M = 0

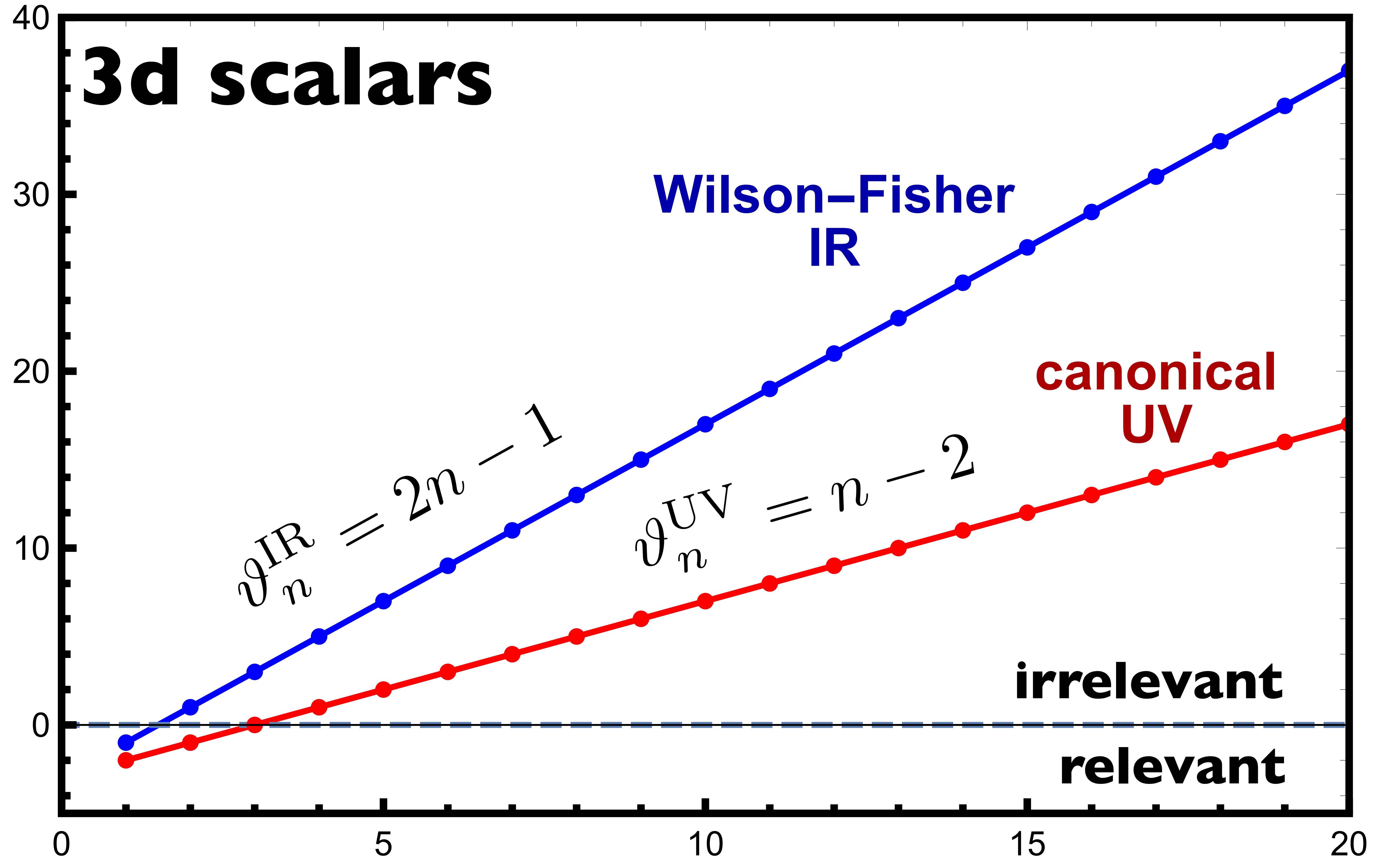
→ scale symmetry broken **spontaneously**

prerequisite: **6F interactions,** hence **no** chiral symmetry breaking

Cresswell-Hogg, Litim, **Critical fermions with spontaneous broken sale symmetry** 2212.06815

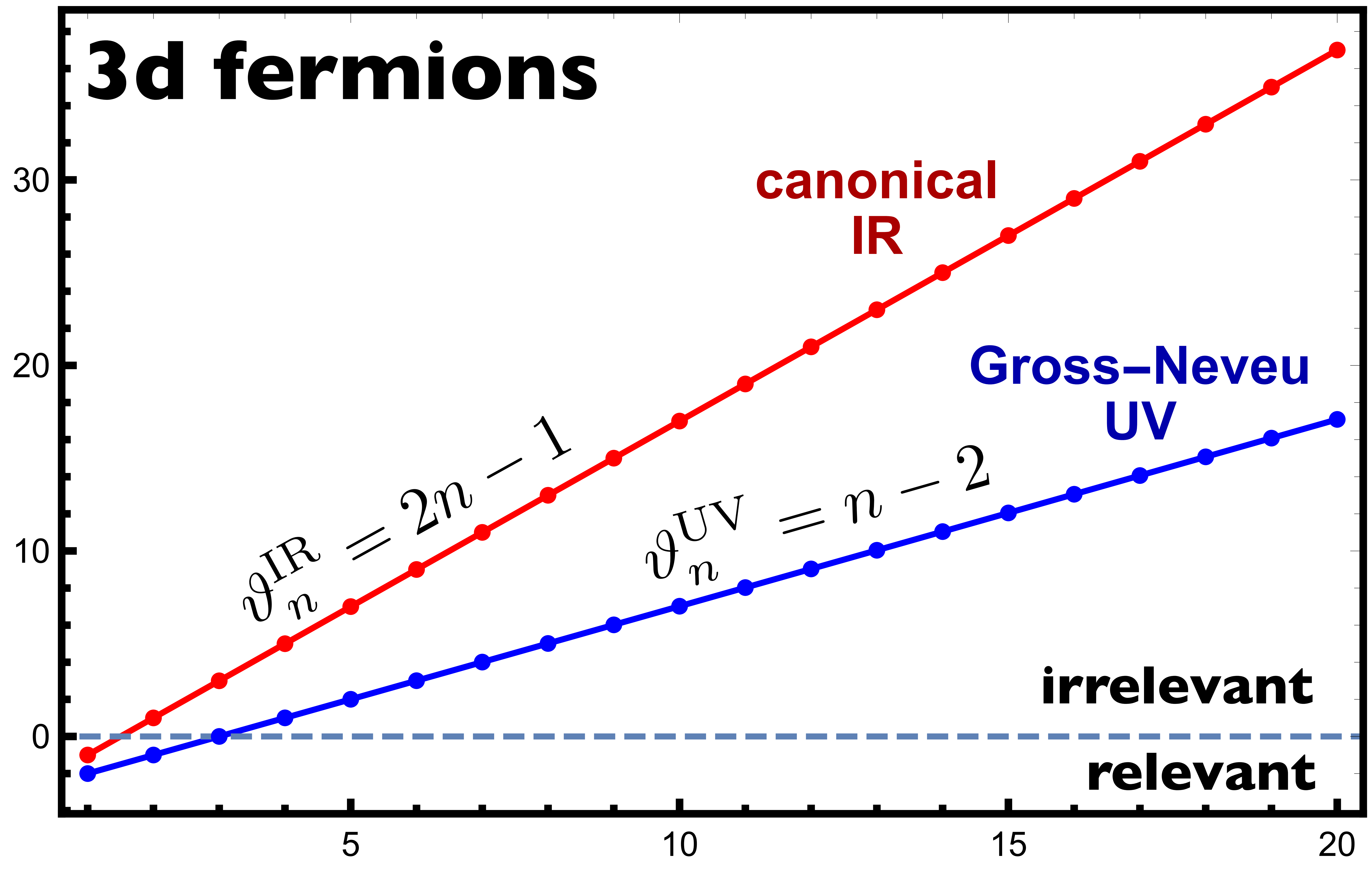
boson-fermion duality

3d O(N) or U(N) symmetric scalar QFTs at large N



boson-fermion duality

3d O(N) or U(N) symmetric fermionic QFTs at large N



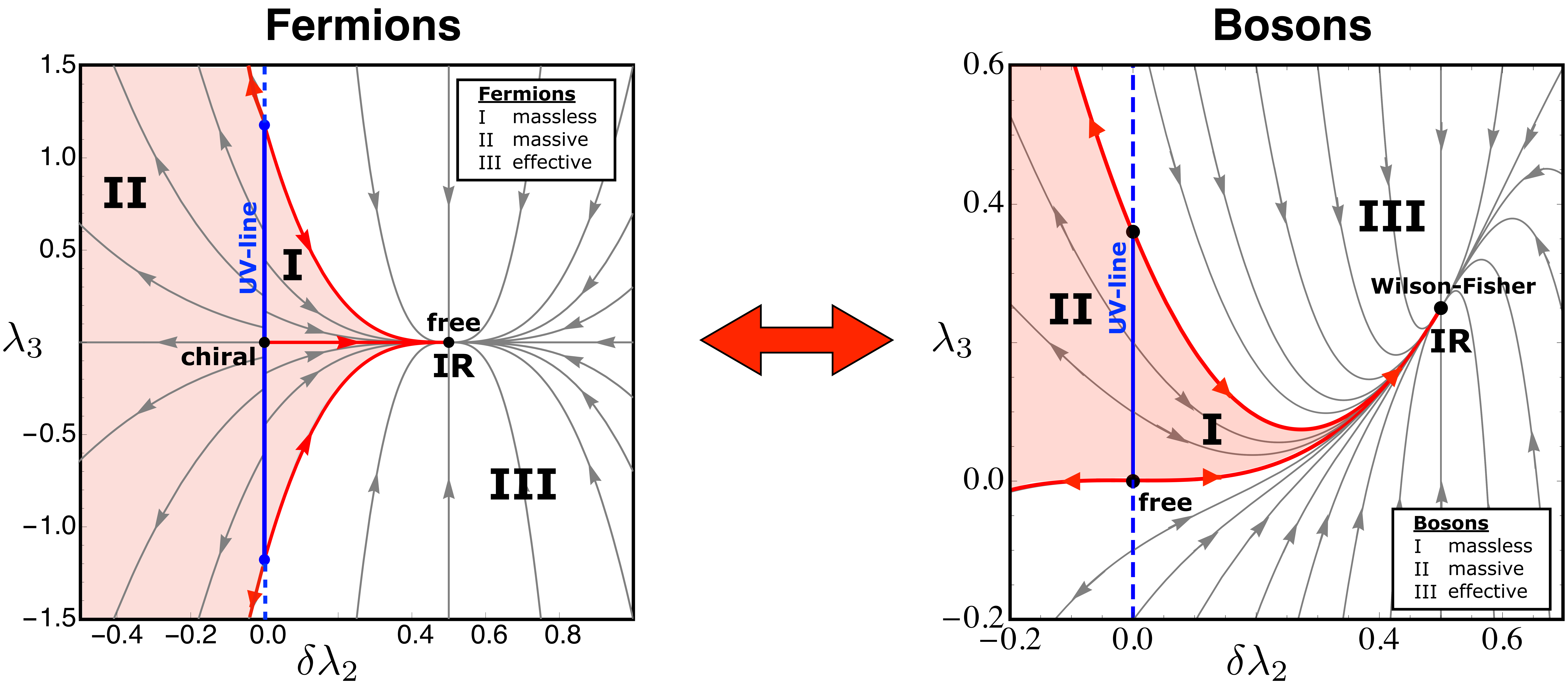
3d fermions
canonical
IR
Gross–Neveu
UV
irrelevant
relevant
3d scalars
Wilson–Fisher
IR
canonical
UV
$\vartheta_n^{\rm IR} = 2n - 1$
$\vartheta_n^{\rm UV} = n - 2$
irrelevant
relevant

boson-fermion duality

3d O(N) or U(N) symmetric QFTs at large N

UV	free scalars = interacting fermions
IR	interacting scalars = free fermions

Bona fide different QFTs, yet equivalent CFT data at critical points



AdS/CFT

higher spin gauge theories

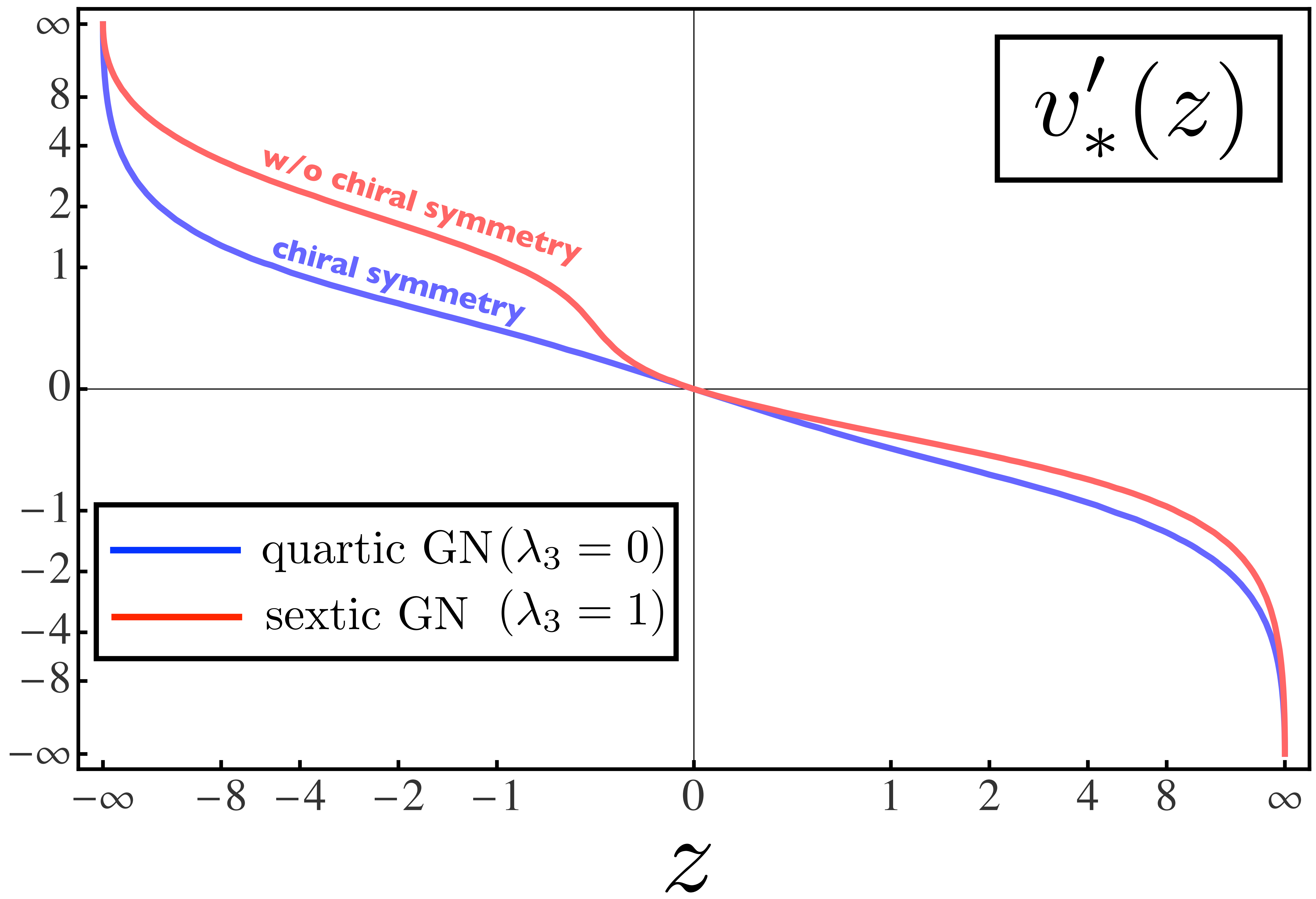
Klebanov, Polyakov, hep-th/0210114, Szegin, Sundell, hep-th/0305040
Maldacena, Zhiboedov, 1112.1016, 1204.3882
Giombi, Zin, 1208.4036

Chern-Simons-Matter

Aharony, Giombi, Gur-Ari, Maldacena, Yacoby, 1211.4843
Seiberg, Senthil, Wang, Witten, 1606.01989

outlook I

non-chiral CFT duals for higher-spin GTs on AdS4



outlook II

bosonisation duality

$$\bar{\psi}_a \partial\!\!\!/ \psi_a + V_k(\bar{\psi}_a \psi_a)$$

GN

$$\bar{\psi}_a \partial\!\!\!/ \psi_a + \tfrac{1}{2} Z_\phi (\partial\phi)^2 + H_k \phi\, \bar{\psi}_a \psi_a + U_k(\phi)$$

GNY

map

$$V_k(\bar{\psi}_a \psi_a) = H_k\, \phi\, \bar{\psi}_a \psi_a + U_k(\phi)$$

$$U_k'(\phi) = -H_k\, \phi\, \bar{\psi}_a \psi_a$$

$$\lambda_{2\mathrm{F}} = \quad h\,\sigma_0\,,$$

$$\lambda_{4\mathrm{F}} = -h^2/\lambda_2\,,$$

$$\lambda_{6\mathrm{F}} = -h^3\lambda_3/\lambda_2^3\,,$$

$$\lambda_{8\mathrm{F}} = h^4\left(\lambda_2\lambda_4 - 3\lambda_3^2\right)/\lambda_2^5\,,$$

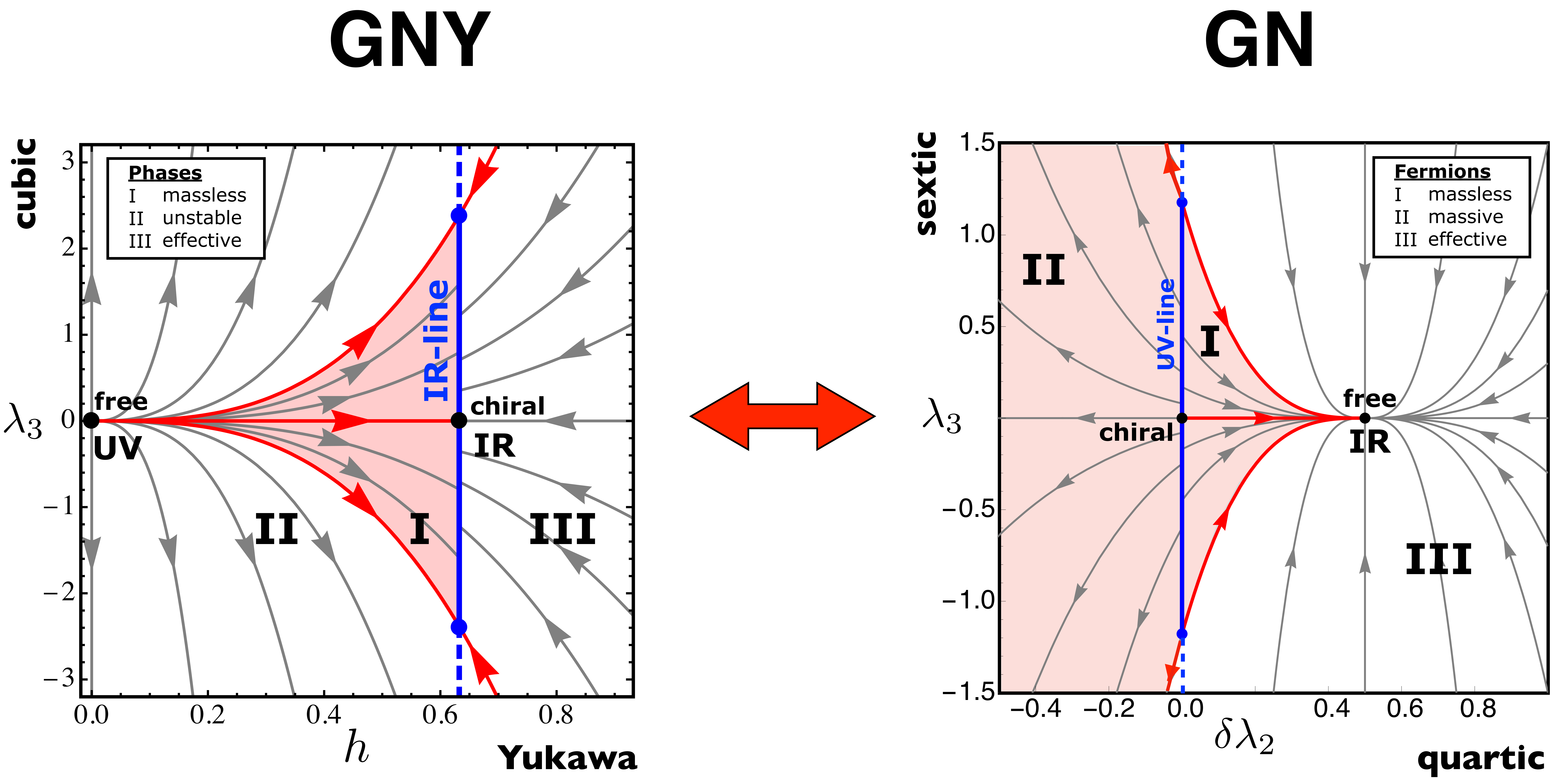
$$\vdots$$

valid along RG trajectories

Cresswell-Hogg, Litim, **Scale-symmetry breaking and generation of mass at quantum critical points** 2311.16246

functional Legendre transform

S Weinberg, **Effective field theories in the large N limit**, hep-th/9706042



Cresswell-Hogg, Litim, **Scale-symmetry breaking and generation of mass at quantum critical points** 2311.16246

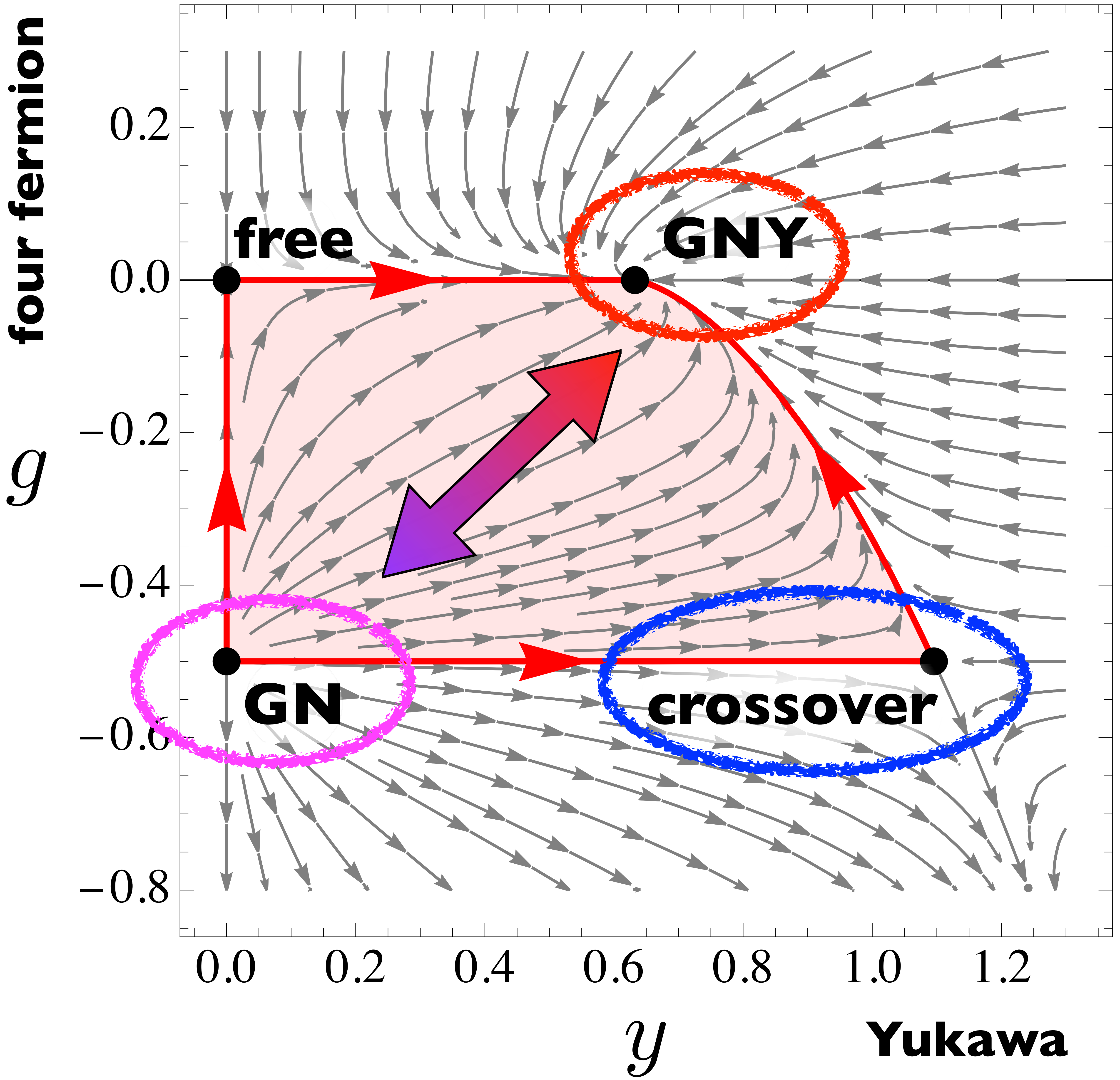
outlook III

action

$$\bar{\psi}_i \not{\partial} \psi_i + \frac{1}{2}(\partial\phi)^2 + W_k(\phi, \bar{\psi}_i\psi_i)$$

e.g.

$$W_k = \frac{1}{2}G(\bar{\psi}_i\psi_i)^2 + Y_k\,\phi\,\bar{\psi}_i\psi_i$$



Summary

critical points in 3d QFTs

fermionic theories from first principles, UV / IR fixed points,
mass generation & spontaneous scale symmetry breaking
prerequisite: interactions break parity symmetry
links with CFTs and AdS/CFT

large N dualities

maps between seemingly different 3d QFTs
new critical points, fermions vs bosons, GN vs GNY

what's next?

hunt the dilaton, extract CFT data
conformal manifolds of other 3d QFTs
exploit results in AdS4/CFT3

Thank you!