a-Anomalous Interactions of the Holographic Dilaton

[arXiv:2205.15324 [hep-ph]]

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Dilaton Dynamics...

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Dilatons Everywhere

Spontaneously broken conformal sector:

- Stringy models
- Composite Higgs models
- Hidden conformal sectors
- Continuum naturalness

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COMMON FEATURE

Dilaton-like particle

A Standard Model Analogy

Anomalous $\mathrm{U}(1)_A$ in the chiral Lagrangian:

$$\Gamma(\pi_0 \longrightarrow \gamma\gamma) = \frac{\alpha^2}{576\pi}\frac{m_\pi^3}{f_\pi^2}N_C^2$$

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- Low energy experimental probe
- High energy UV physics
- What about conformal sectors?

Outline

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1. **Effective Theory of the Dilaton**

2. The Holographic Dilaton

3. Phenomenological Signatures

Dilaton Action

Dilaton τ: Goldstone boson of conformal symmetry

- Define $\tilde{g}_{\mu\nu} = e^{-2\tau} g_{\mu\nu}$

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$$S_a = 2a \int \mathrm{d}^4 x \, (\partial\tau)^4 \quad (+\text{ higher derivative terms})$$

The a-Theorem

Monotonic function interpolating between UV and IR?

$$a(\mu) = a_{\mathsf{UV}} - \frac{f^4}{\pi} \int_{s>\mu} \mathrm{d}s \, \frac{\sigma(s)}{s^2}$$

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- Effective counting of degrees of freedom

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General Strategy

Simplest example: Randall–Sundrum geometry

- 4D strongly coupled problem to a 5D perturbative one

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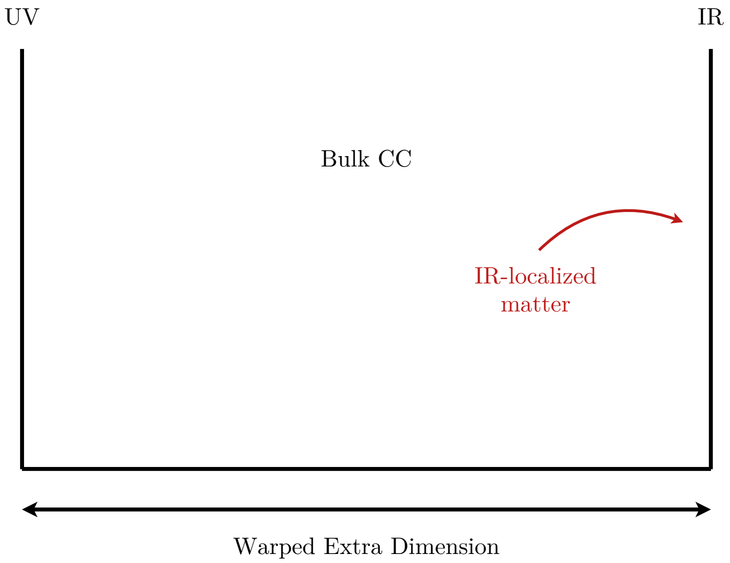
$$S = S_{\text{bulk}} + S_{\text{branes}} + S_{\text{matter}},$$
$$\mathrm{d}s^2 = e^{-2A(x,y)}(\eta_{\mu\nu} + h_{\mu\nu}(x,y))\mathrm{d}x^\mu \mathrm{d}x^\nu - B^2(x,y)\mathrm{d}y^2$$

Holographic Setup



The a-Anomaly

We get a total y-derivative:

$$S_{\text{radion}} = \int \mathrm{d}^4x \, \frac{f^2}{2} e^{-2\tau} (\partial\tau)^2 - \lambda f^4 e^{-4\tau} + \tau T^\mu_\mu$$

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We can read off the a-anomaly:

$$a_{\text{RS}} = \frac{1}{8\kappa^2 k^3} = \frac{N^2}{4(16\pi^2)}$$

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Collider Probes

Expand around a VEV, $\exp(-\tau) = 1 - \varphi/f$:

$$S_{\text{radion}} \supseteq \int \mathrm{d}^4x \, \frac{\pi^2}{3N^2 M_{\text{KK}}^4} \partial^\mu \varphi \partial^\nu \varphi \left(T_{\mu\nu} - \frac{1}{6} \eta_{\mu\nu} T \right)$$

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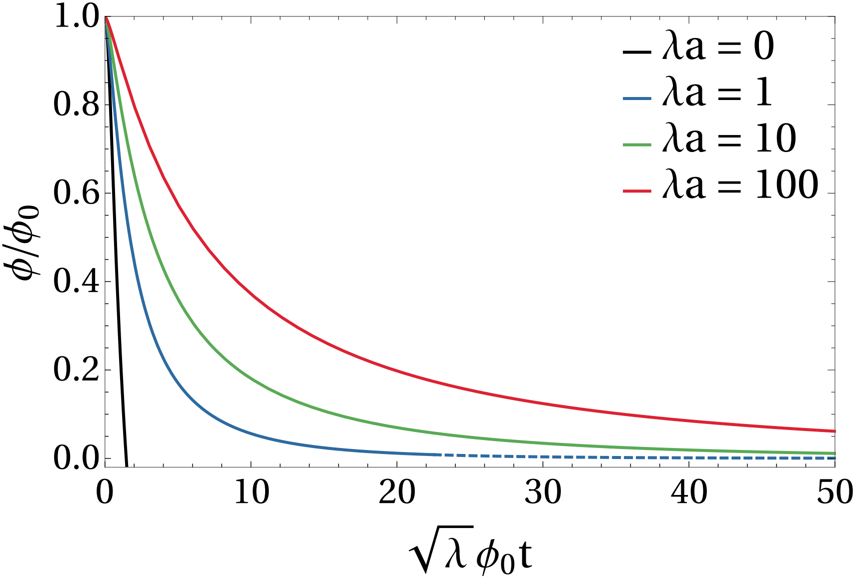
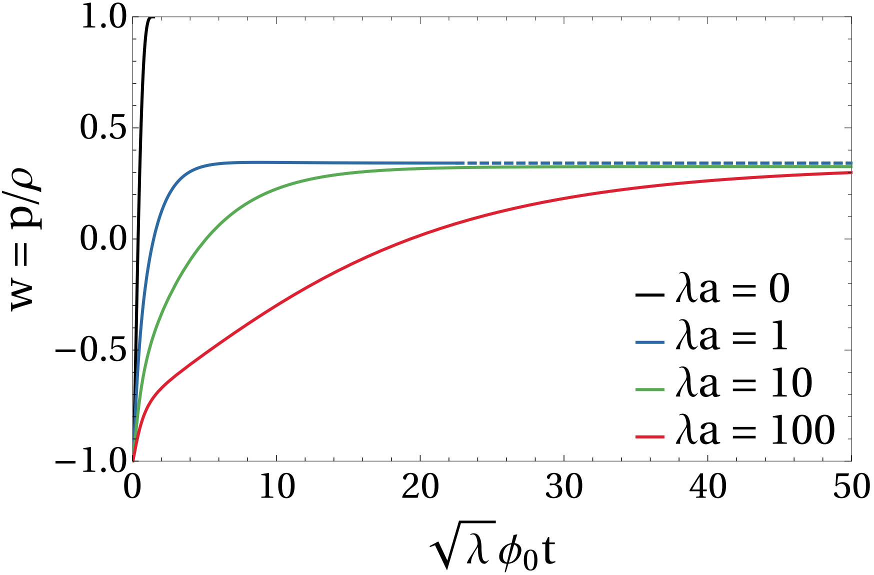
- Also for classically scale-invariant fields
- Relevant for collider studies?

Cosmological Evolution



Equation of State



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- Other geometries: soft wall?

Thank you!

Explicit Breaking and $1/N$

What about corrections to this result?

- Explicit breaking suppressed by $\exp(A_0 - A_1)$
- $1/N$ probed by Gauss–Bonnet

$$a_{\mathsf{GB}} = a_{\mathsf{RS}} \left(1 - 12\lambda_{\mathsf{GB}} \left[\frac{2\kappa^2 k^3}{24\pi^3} \right]^{2/3} \right)$$