θ angle & Scalings for QCD Charged Sectors

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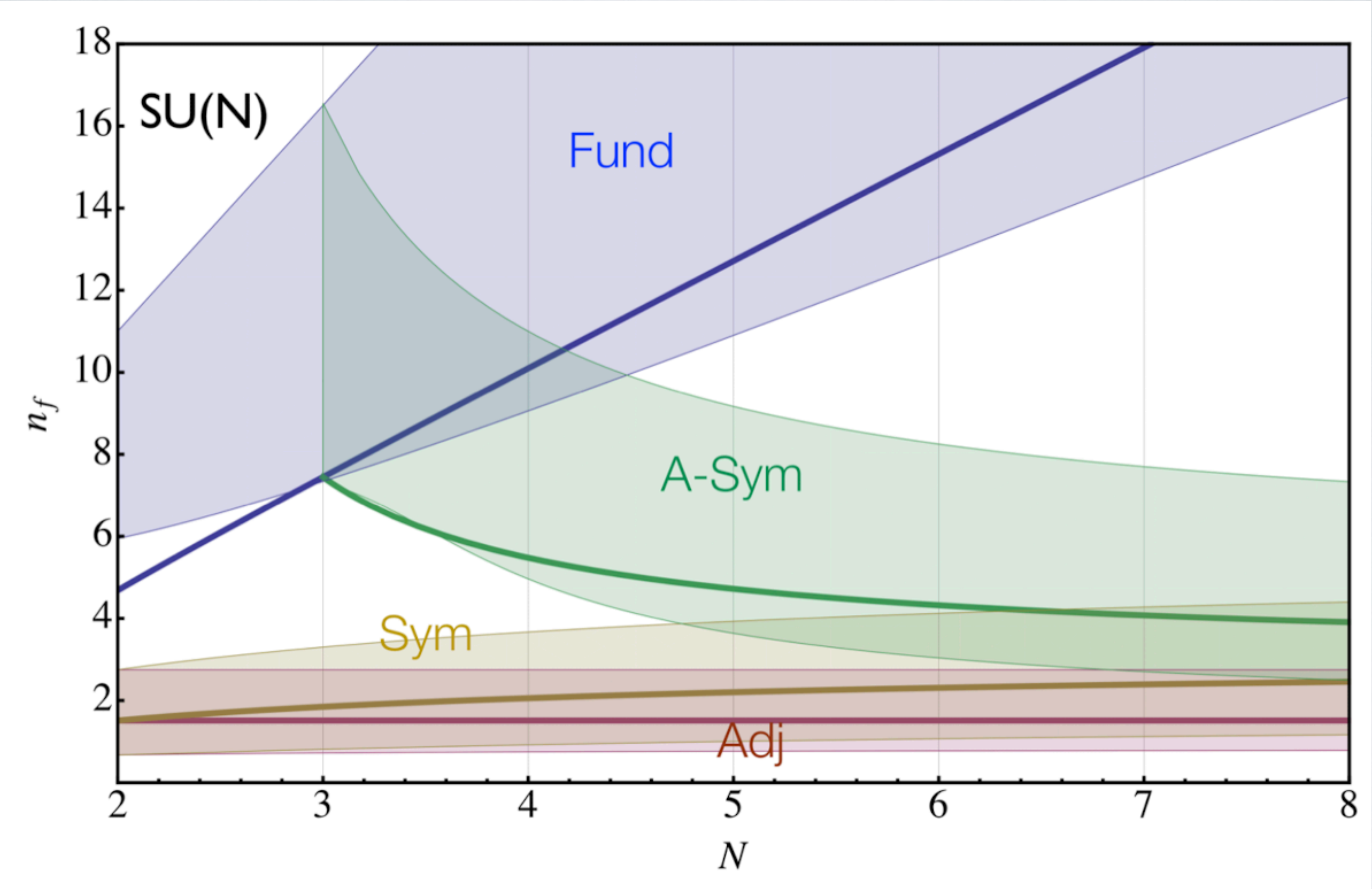
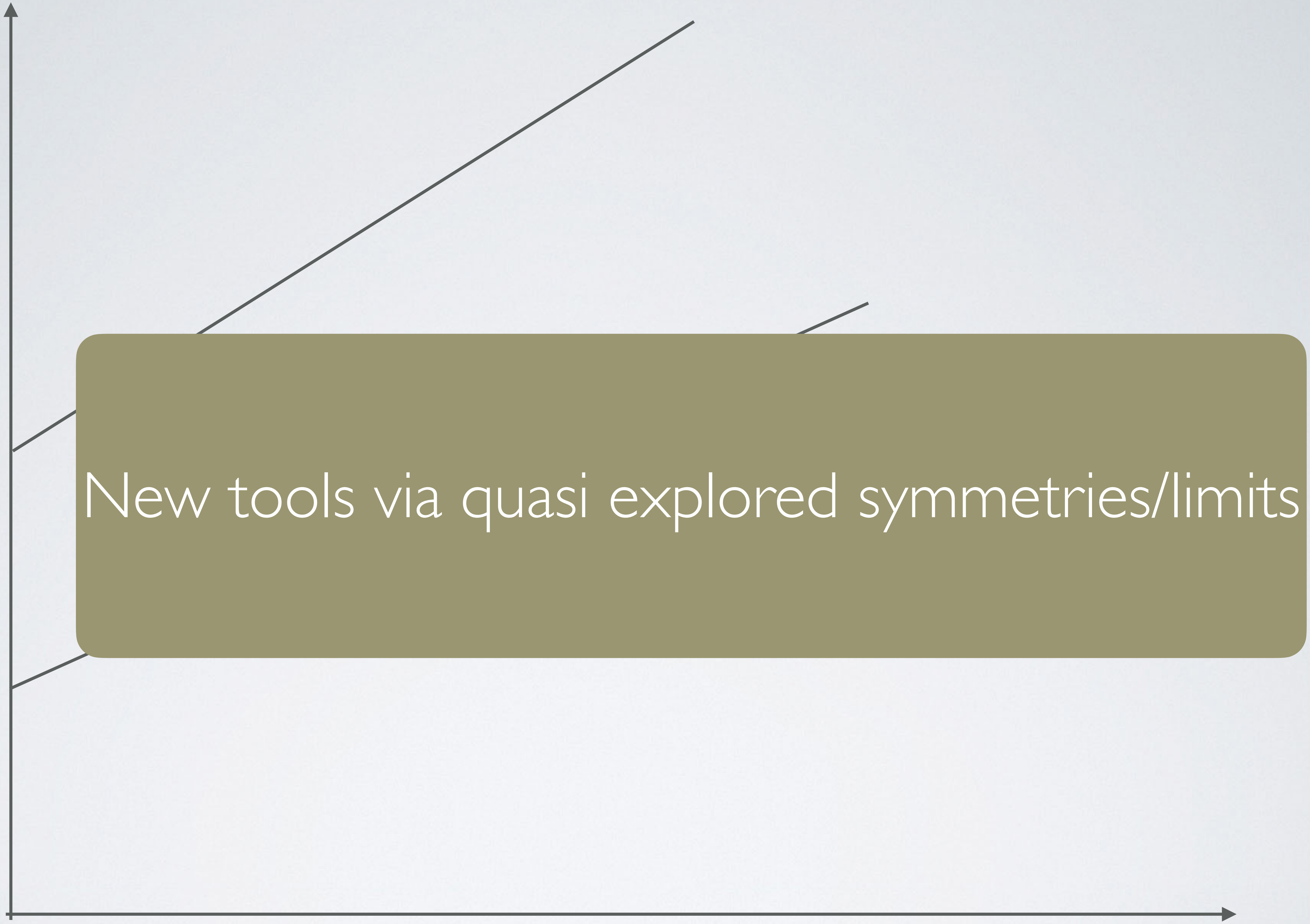


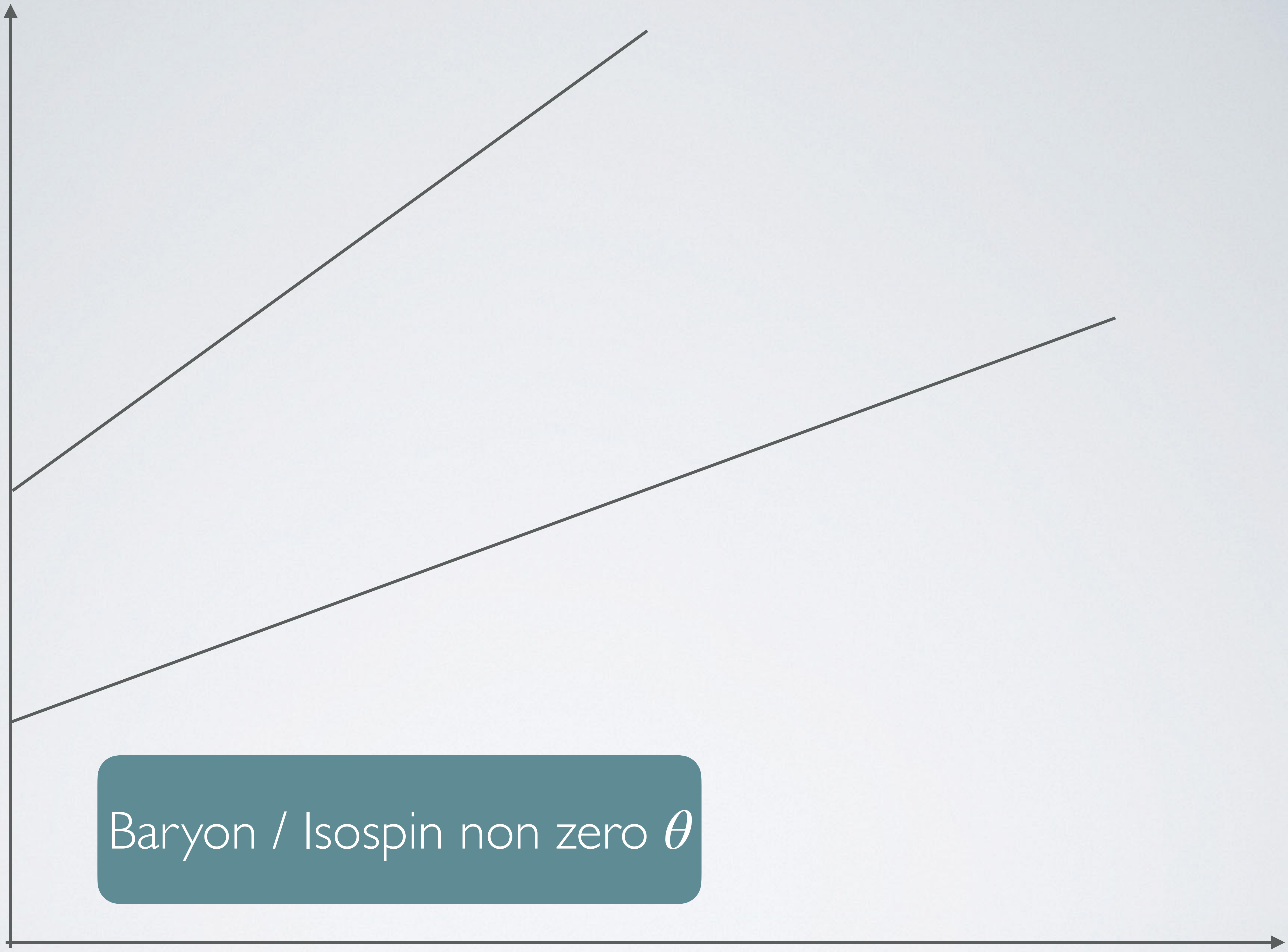
Appelquist, Karabali, Wijewardhana 86

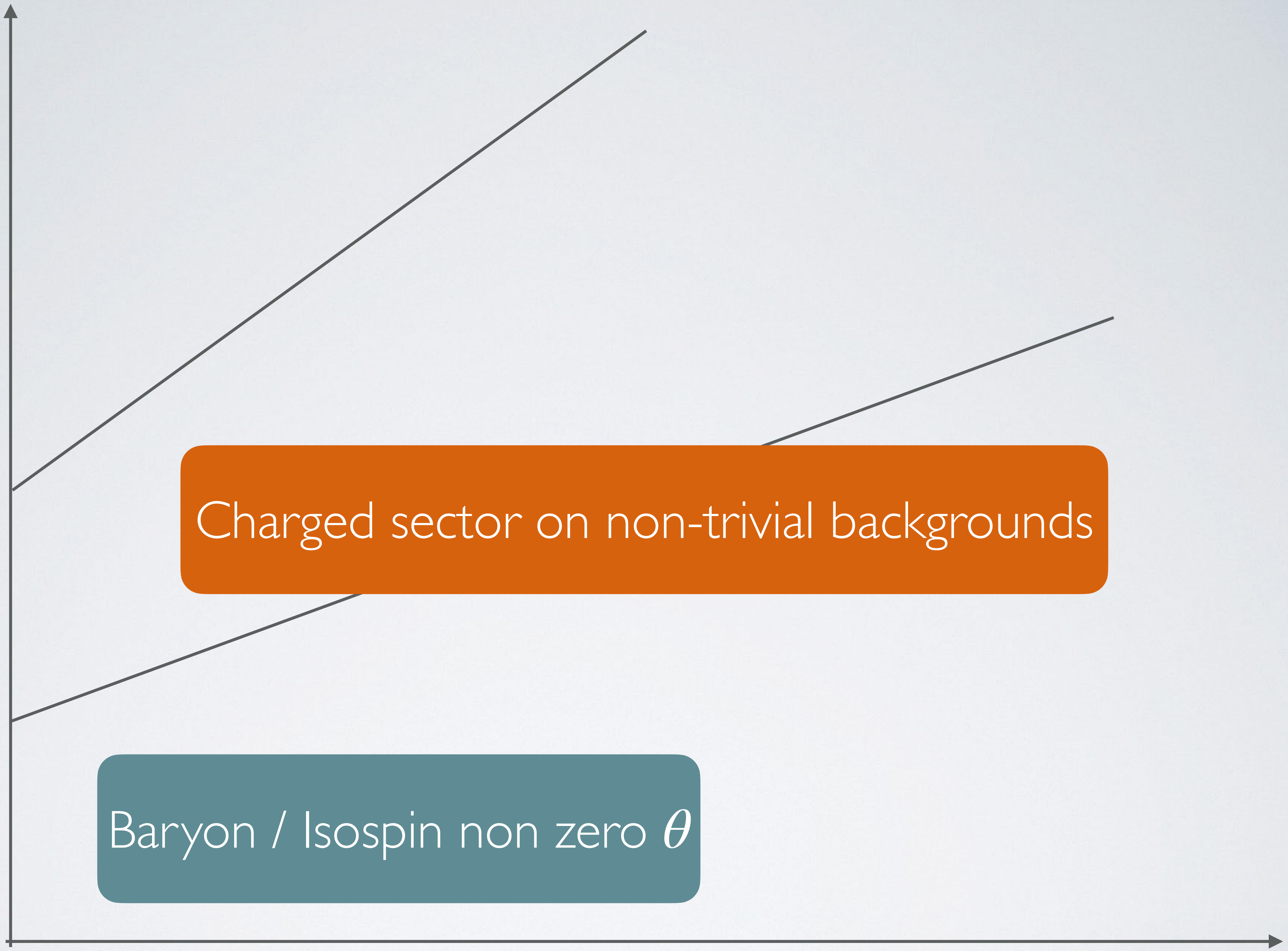
Sannino, Tuominen, PRD 71 (2005) 051901

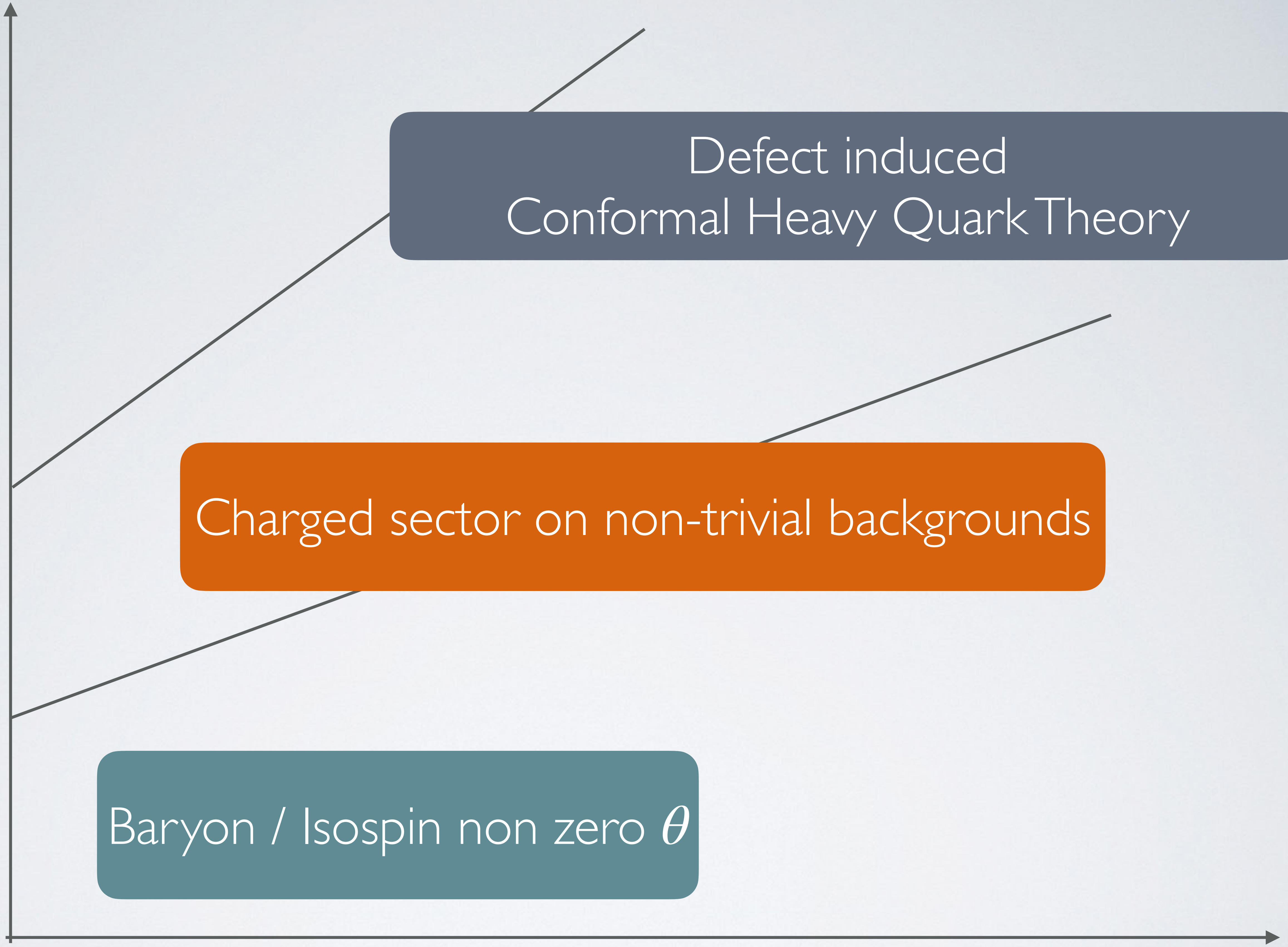
Dietrich, Sannino, PRD 75 (2007) 085018

D.K. Hong, Hsu, Sannino, PLB 516 (2001) 362


New tools via quasi explored symmetries/limits


Baryon / Isospin non zero θ


Charged sector on non-trivial backgrounds
Baryon / Isospin non zero θ


Defect induced
Conformal Heavy Quark Theory
Charged sector on non-trivial backgrounds
Baryon / Isospin non zero θ

Executive summary

θ-angle vs μ phase diagram for 2 & 3 colors QCD for different flavours

New way to access near conformal (dilaton/quark mass) information

$$\Delta_Q = \Delta_Q^* + \left(\frac{m_\sigma}{4\pi\nu}\right)^2 Q^{\frac{\Delta}{3}} B_1 + \left(\frac{m_\pi(\theta)}{4\pi\nu}\right)^4 Q^{\frac{2}{3}(1-\gamma)} B_2 + \mathcal{O}\left(m_\sigma^4, m_\pi^8, m_\sigma^2 m_\pi^4\right)$$

Di Vecchia, Sannino, Eur. Phys.J.Plus 129 (2014) 262

Orlando, Reffert, Sannino, PRD101(2020) 6, 065018: *PRD*103(2021)10, 105026

Bersini, D'Alise, Sannino, Torres, JHEP 11(2022)080; PR107(2023)12,12; 2310.04083

Bersini, D'Alise, Gambardella, Sannino, 2401.08457, published in PRD

Short trailer

Defect induced
Conformal Heavy Quark Theory

2406.09758

Defect Induced Heavy Meson Dynamics in The QCD Conformal Window

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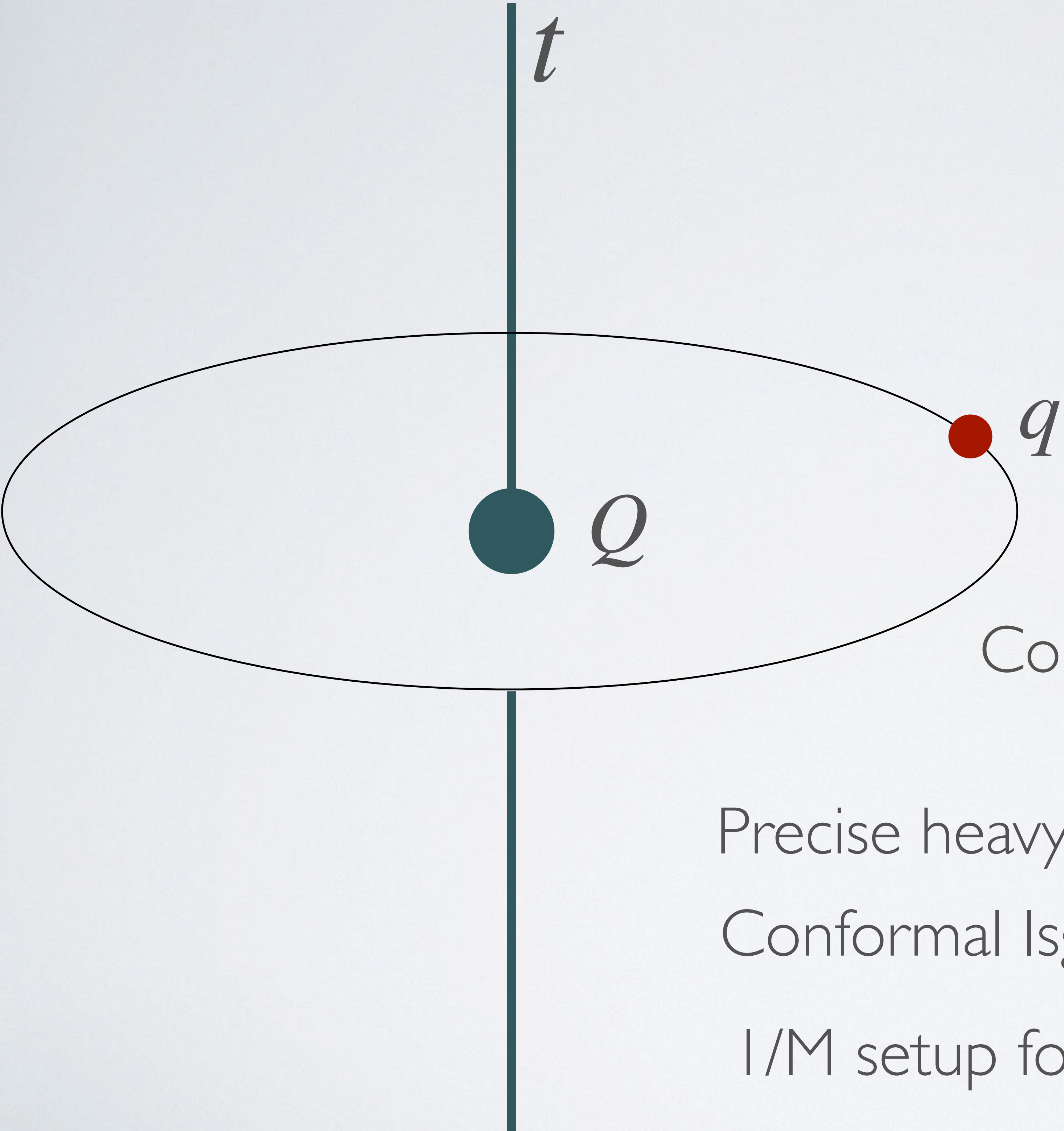
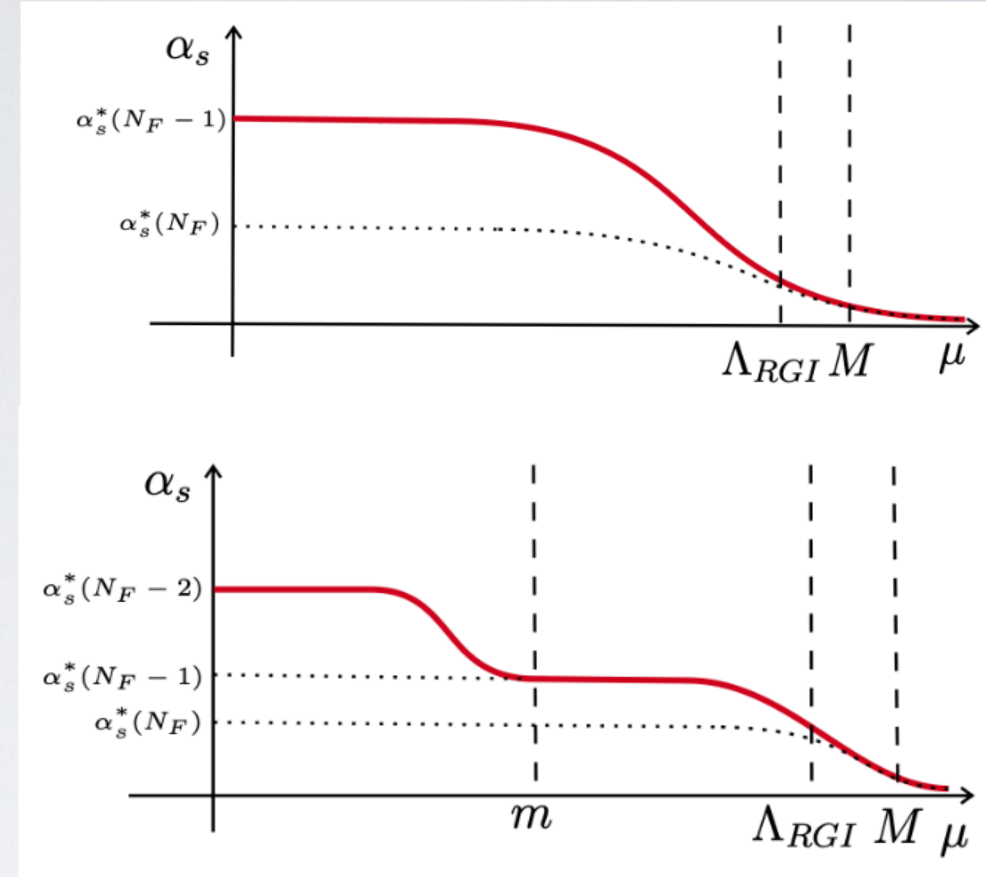
2406.09758

Abstract

Upon introducing an heavy quark in the perturbative regime of the QCD conformal window we precisely determine the associated heavy meson spectrum and wave functions in terms of the number of light flavours and mass. We then compute the conformal Isgur-Wise function which is a central quantity in heavy quark physics. We further determine the impact of the residual low energy confining dynamics on the heavy meson spectrum. As a working framework, we adapt the heavy quark effective theory to the perturbative conformal window dynamics. Our work lays the foundations to systematically go beyond the infinite mass defect approximation in conformal field theories.

Wilson line defect / Massive heavy quark





Conformal brown muck

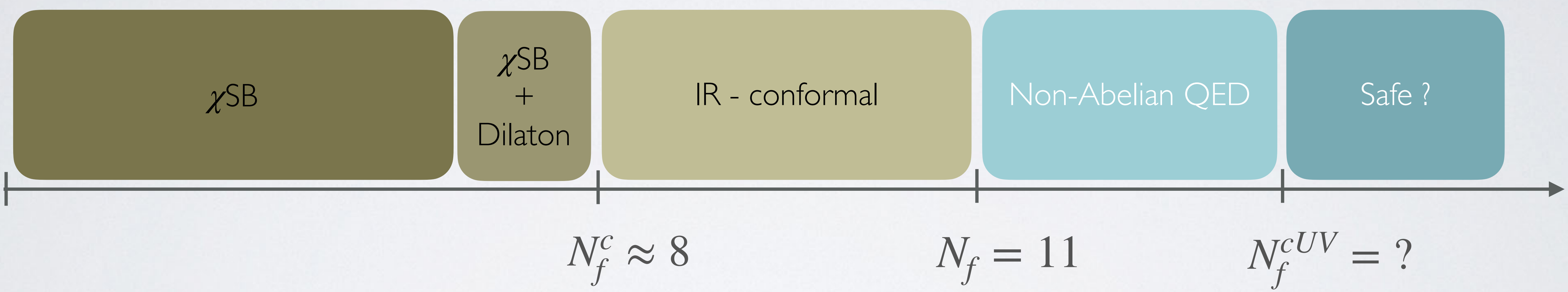
Precise heavy meson spectrum

Conformal Isgur-Wise function

1/M setup for Dynamical Defects

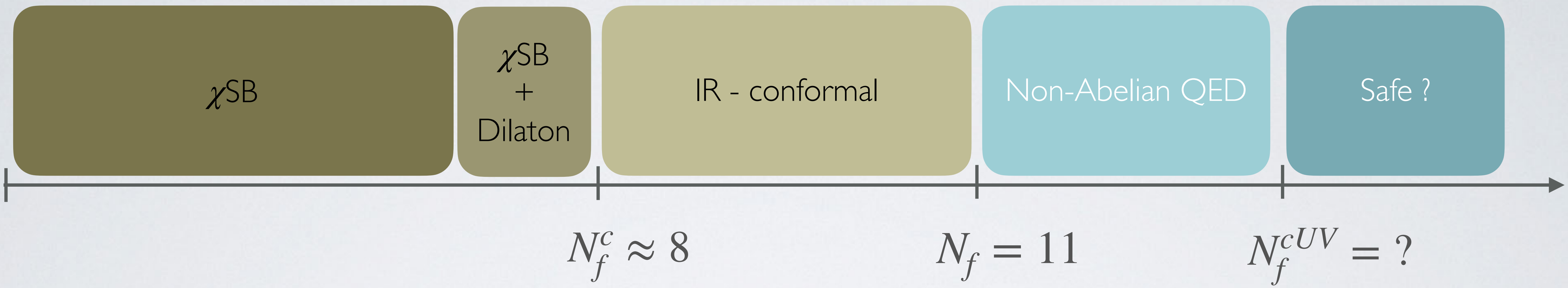
Back to the executive summary

2 color QCD



Sannino, *Conformal window of Sp(2N) and SO(N) Gauge theories*, Phys. Rev. D 79 (2009) 096007, arXiv 0902.3494

2 color QCD



BSM/LHC

Fund. (Partial) Composite Higgs

Flavour physics template

Technicolor

g-2, Mw, ….

Dark matter & Inflation

GB (Asymmetric) DM

SIMP (# changing operators)

Composite inflation

Gravitational Waves

Lattice

No sign problem/QCD Isospin

Conformal window

Large number of flavours

Spectroscopy

Cacciapaglia, Pica, Sannino, 2002.04914, *Phys. Rept. 877 (2020) 1-70*

Fixed charge, θ-angle

&

(near) conformal dynamics

Di Vecchia, Sannino, Eur. Phys.J.Plus 129 (2014) 262

Orlando, Reffert, Sannino, PRD101(2020) 6, 065018: *PRD*103(2021)10, 105026

Bersini, D'Alise, Sannino, Torres, JHEP 11(2022)080; PR107(2023)12,12; 2310.04083

Bersini, D'Alise, Gambardella, Sannino, 2401.08457, PRD

Why fixed charge?

Squeezed matter

Matter in extreme conditions

Early universe

Dark/Bright Asymmetries

As passepartout

Large charge sectors

Infinite order results

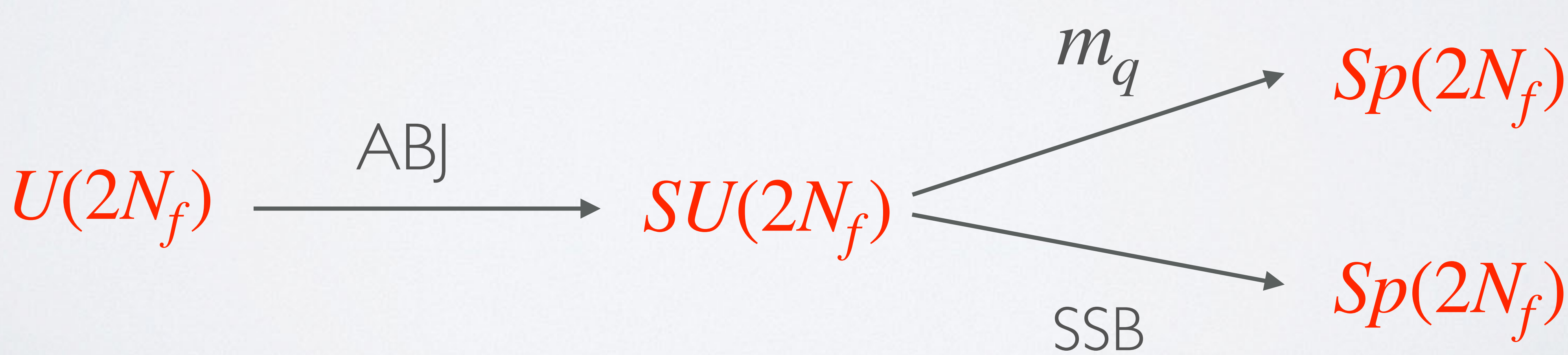
Nonpertubative results

Precise study of CFT breaking

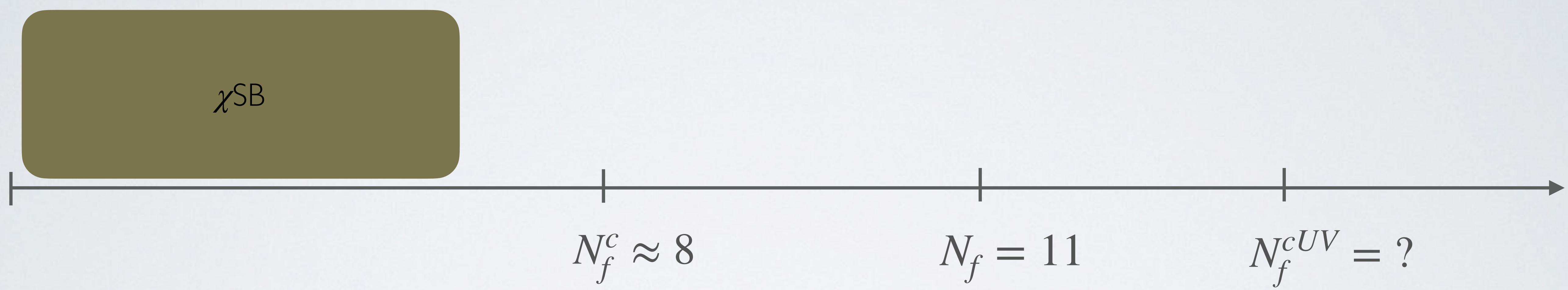
2/Sp(2N) color QCD

$$\mathscr{L} = -\frac{1}{4g^2}\vec{G}_{\mu\nu}\cdot\vec{G}^{\mu\nu} + i\bar{\mathcal{Q}}\bar{\sigma}^{\nu}\left[\partial_{\nu} - i\vec{G}_{\nu}\cdot\frac{\vec{\tau}}{2}\right]\mathcal{Q} - \frac{1}{2}m_q\mathcal{Q}^T\tau_2 E\mathcal{Q} + \mathrm{h.c.}\,.$$

$$\mathcal{Q} = \begin{pmatrix} q_L \\ i\sigma_2\tau_2 q_R^* \end{pmatrix} \qquad E = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}\otimes 1_{N_f}$$



χ SB with fixed charge and θ-angle



Baryon charged chiral lagrangian

$$\mathcal{L}_{\text{eff}} = \nu^2 Tr\{\partial_\mu \Sigma \partial^\mu \Sigma^\dagger\} + m_\pi^2 \nu^2 Tr\{M\Sigma + M^\dagger \Sigma^\dagger\}$$

$$\Sigma \to u \Sigma u^T \,, \quad u \in SU(2N_f)$$

Democratic mass matrix

$$M = -i\sigma_2 \otimes 1_{N_f} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \otimes 1_{N_f}$$

Baryon chemical potential

$$\partial_\mu \mapsto D_\mu = \partial_\mu - i\mu\delta_\mu^0 B \,, \qquad B \equiv \begin{pmatrix} 1/2 & 0 \\ 0 & -1/2 \end{pmatrix} \otimes 1_{N_f}$$

Kogut, Stephanov, Toublan, Verbaarschot, Zhitnitsky NPB 582 (2000) 477-513

* Lenaghan, Sannino, Splittorff, PRD 65 (2002) 054002

** Fist paper where chemical potential was used to investigate near CFT dynamics*

θ - angle

Allowed topological term

$$q(x) = \frac{g^2}{64\pi^2}\epsilon^{\mu\nu\rho\sigma}F^a_{\mu\nu}F^a_{\rho\sigma}$$

At the effective lagrangian level

$$\mathscr{L}_{q(x)} = \frac{i}{4}q(x)Tr\{\log\Sigma - \log\Sigma^\dagger\} - \theta q(x) + \frac{q(x)^2}{4a\nu^2}$$

Reproduces underlying axial anomaly $\partial_\mu J^\mu_5 = 4N_f\, q(x)$

θ - angle

$q(x)$: auxiliary field which integrated out yields

$$\mathcal{L}_\theta = \nu^2 Tr\{\partial_\mu \Sigma \partial^\mu \Sigma^\dagger\} + 4\mu\nu^2 Tr\{B\Sigma^\dagger \partial_0 \Sigma\} + m_\pi^2 \nu^2 Tr\{M\Sigma + M^\dagger \Sigma^\dagger\}$$

$$+2\mu^2\nu^2 \left[Tr\{\Sigma B^T \Sigma^\dagger B\} + Tr\{BB\}\right] - a\nu^2 \left(\theta - \frac{i}{4} Tr\{\log\Sigma - \log\Sigma^\dagger\}\right)^2$$

Vacuum

For $\theta = 0$ the homogenous ground state ansatz

$$\Sigma_c = \begin{pmatrix} 0 & 1_{N_f} \\ -1_{N_f} & 0 \end{pmatrix} \cos\varphi + i \begin{pmatrix} \mathscr{I} & 0 \\ 0 & \mathscr{I} \end{pmatrix} \sin\varphi \quad \text{where} \quad \mathscr{I} = \begin{pmatrix} 0 & -1_{N_f/2} \\ 1_{N_f/2} & 0 \end{pmatrix}$$

Diquark and chiral condensate compete

Witten variables α_i to account for θ-angle

$$\Sigma_0 = U(\alpha_i)\Sigma_c\,, \qquad U(\alpha_i) \equiv \text{diag}\{e^{-i\alpha_1}, \ldots, e^{-i\alpha_{N_f}}, e^{-i\alpha_1}, \ldots, e^{-i\alpha_{N_f}}\}$$

Each phase is an axial transformation for each left-right quark pair

Vacuum structure

Evaluate the lagrangian on the vacuum ansatz

$$\mathscr{L}_\theta[\Sigma_0] = \nu^2 \left[4m_\pi^2 X \cos\varphi + 2\mu^2 N_f \sin^2\varphi - a\bar{\theta}^2\right]$$

$$\bar{\theta} = \theta - \sum_i^{N_f} \alpha_i\,, \qquad X = \sum_i^{N_f} \cos\alpha_i$$

The EoM reads

$$\sin\varphi \left(N_f \cos\varphi - \frac{m_\pi^2}{\mu^2} X \right) = 0$$

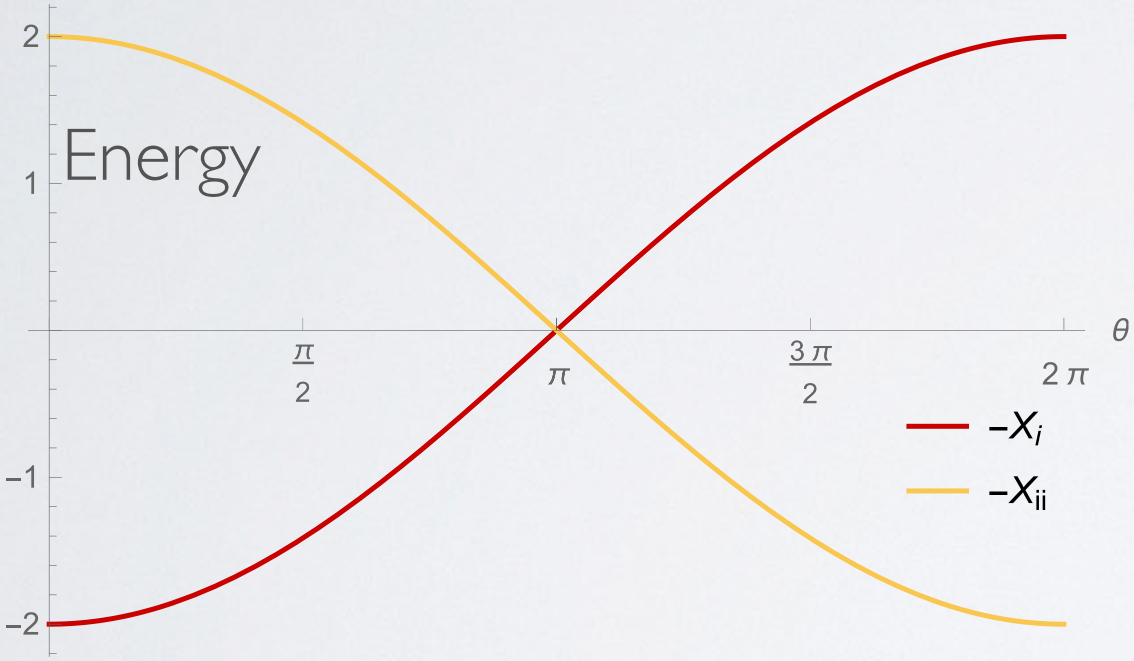
$$2m_\pi^2\, \sin\alpha_i\, \cos\varphi = a\,\bar{\theta}\,, \quad i = 1,..,N_f$$

$N_f = 2$

Normal

The solutions cross at $\theta = \pi$

where SSB of CP occurs

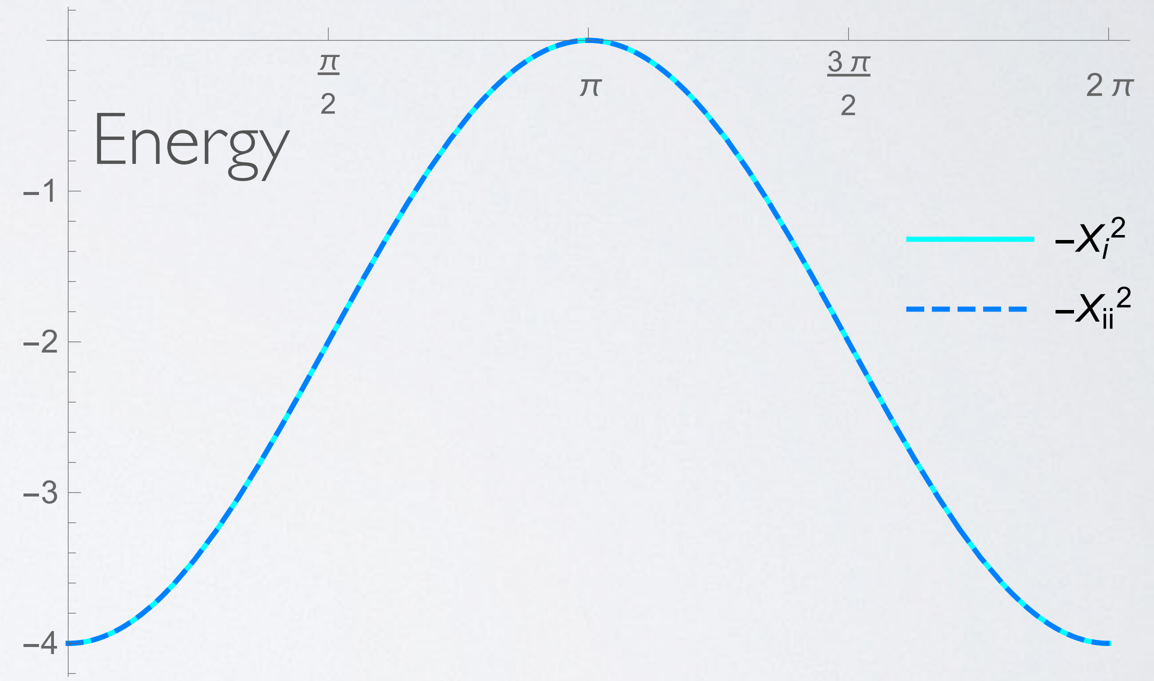


$$\bar{\theta} = \frac{2m_\pi^2}{a}\sin\frac{\theta}{2} \underset{\theta=\pi}{=} \frac{2m_\pi^2}{a} + \mathcal{O}\left(\frac{m_\pi^6}{a^3}\right)$$

Superfluid

Energy is an analytic function of θ

No SSB of CP occurs



$$\bar{\theta} = \frac{m_\pi^4}{a\mu^2}\sin\theta \underset{\theta=\pi}{=} 0$$

NB, you can also think of the breaking of CP as explicit ($\bar{\theta} \neq 0$)

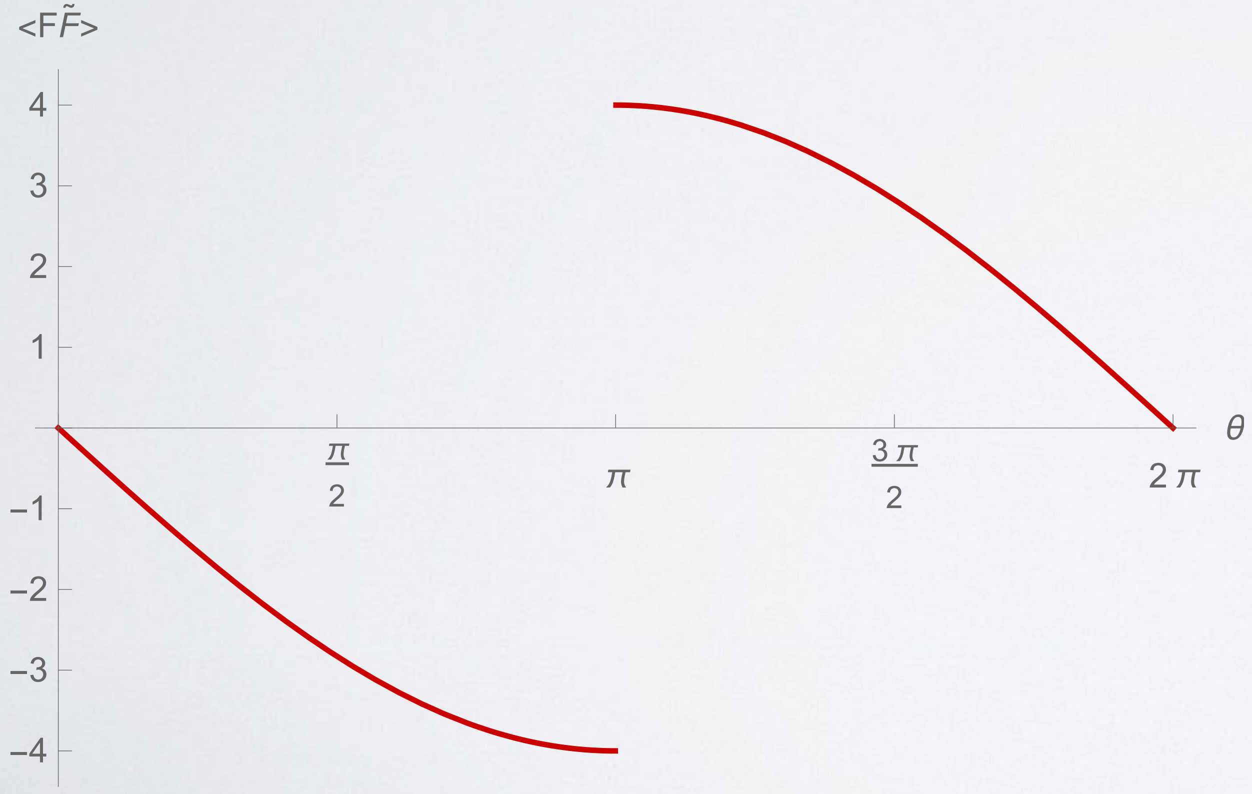
CP is intact

CP - Order parameter

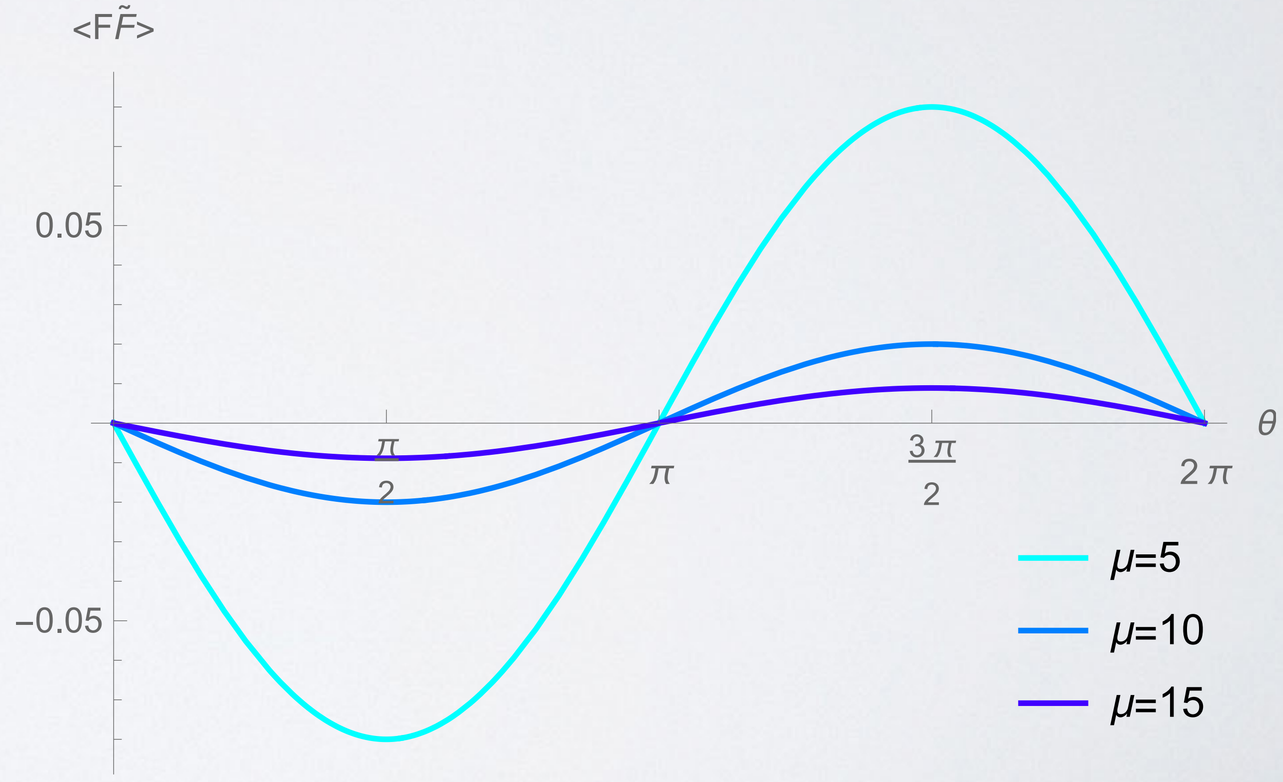
$N_f = 2$

$$\langle F\tilde{F}\rangle \propto -\frac{\partial E}{\partial \theta}$$

Normal



Superfluid

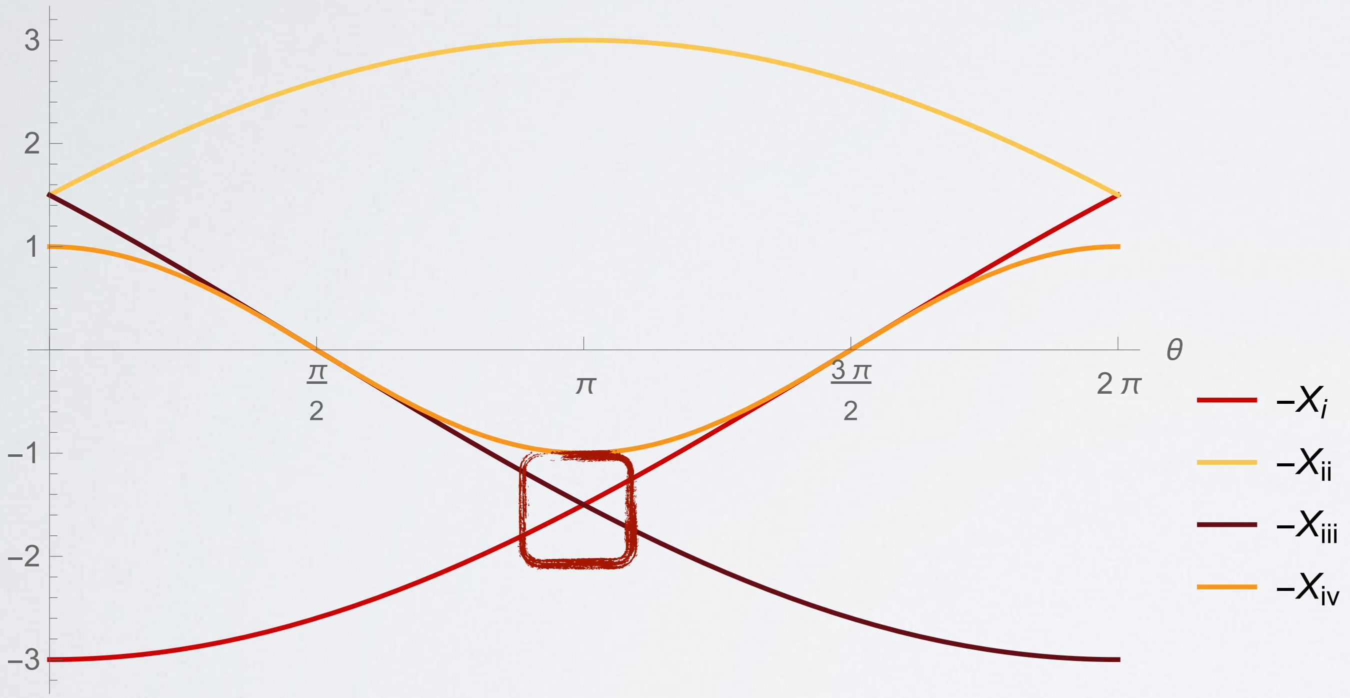


$N_f = 3$

Normal

Ground state given by i. & iii.

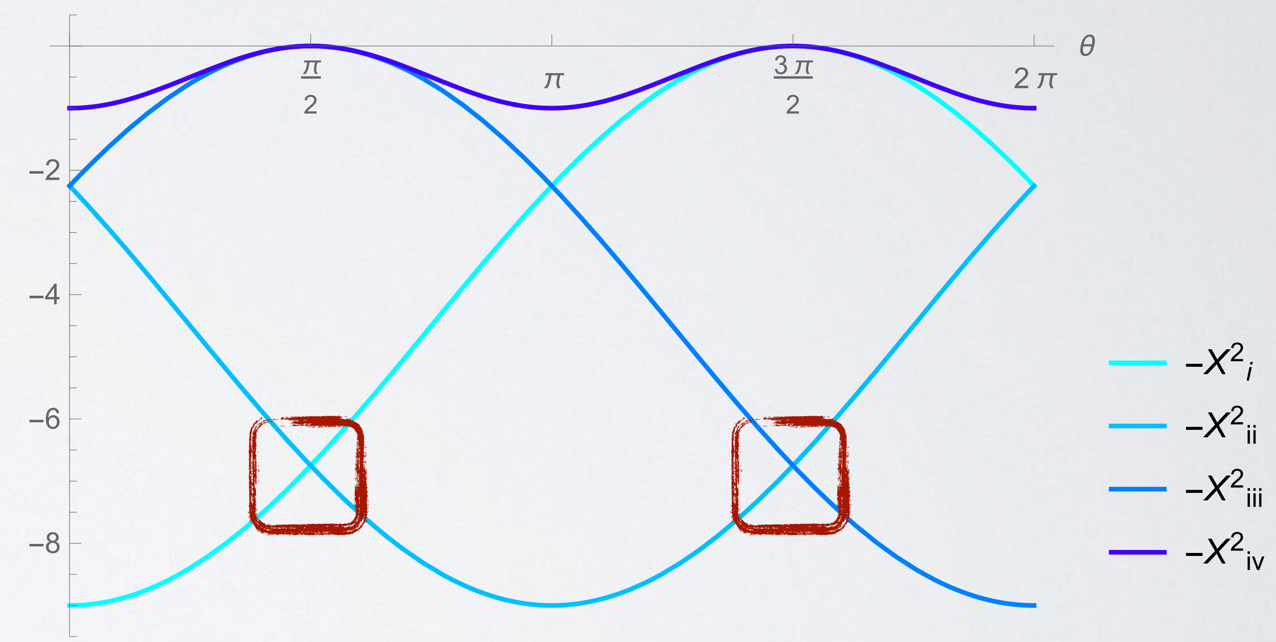
CP - SSB for $\theta = \pi$



Superfluid

2 novel first-order phase transitions at

$\theta = \dfrac{\pi}{2}$ and $\theta = \dfrac{3\pi}{2}$.



Metastable vacua can be long-lived and we estimated their decay rate

Generalized for odd/even N_f

For odd N_f there is a subtle meta-stable phase at intermediate μ as function of θ

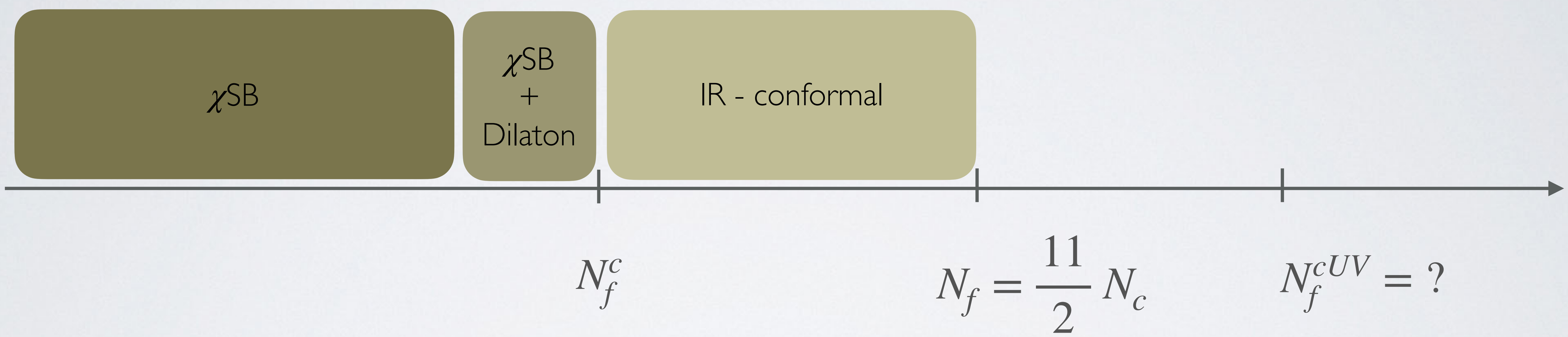
3 colors QCD

Similar analysis but for isospin chemical potential

Axion at non-zero μ similar to 2 colors

Nearly completed

Near conformality



Orlando, Reffert, Sannino, PRD101(2020) 6, 065018: *PRD*103(2021)10, 105026

Bersini, D'Alise, Gambardella, Sannino, 2401.08457

Near conformality toolbox

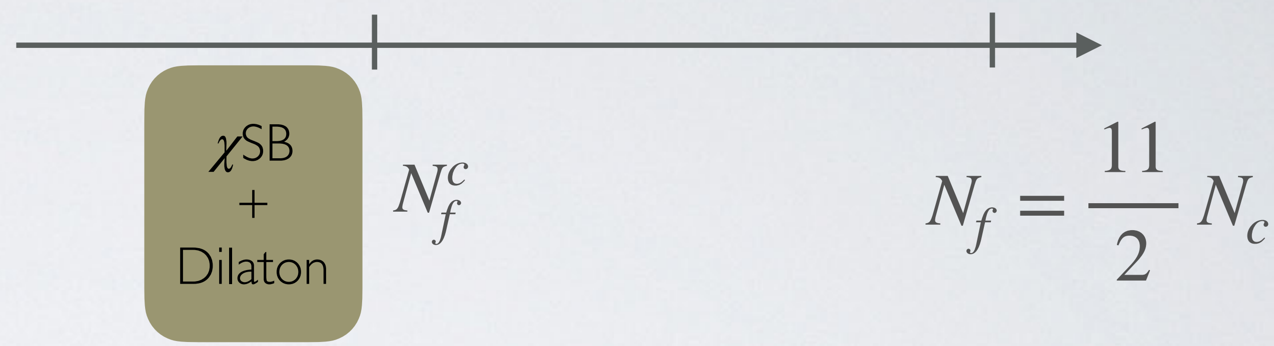
Dress the χ theory with a dilaton

Conformality breaking via dilaton and/or quark (pion) masses

Orlando, Reffert, Sannino, PRD101(2020) 6, 065018: *PRD*103(2021)10, 105026

Bersini, D'Alise, Gambardella, Sannino, 2401.08457

Fixed charge near conformality toolbox



Add a nonzero charge sector Q (Isospin in QCD)

Extend the theory to be in a non-dynamical gravity background (Cylinder)

Compute the ground state energy on the cylinder

Hellerman, Orlando, Reffert, Watanabe, JHEP 12 (2015), 071

Badel, Cuomo, Monin, Rattazzi, JHEP 11 (2019), 110

Dictionary

For a conformal field theory

Δ_Q^* Lowest-lying operator with charge Q

Map it on the cylinder $\mathbb{R} \times \mathbb{S}_3 \rightarrow$ $R = \frac{6}{r^2}$

State - operator correspondence

$$\Delta_Q^* = r\, E_Q \qquad E_Q = \mu Q - \mathscr{L}$$

Advantage: semiclassical computations

What do we learn?

$$\Delta_Q \equiv r\, E_Q = \Delta_Q^* + \text{near CFT terms}$$

r is the cylinder radius

Δ_Q^* is the CFT contribution which is r independent

Near CFT terms are geometry and CFT breaking dependent

Result

$$\Delta_Q = \Delta_Q^* + \left(\frac{m_\sigma}{4\pi\nu}\right)^2 Q^{\frac{\Delta}{3}} B_1 + \left(\frac{m_\pi(\theta)}{4\pi\nu}\right)^4 Q^{\frac{2}{3}(1-\gamma)} B_2 + \mathcal{O}\left(m_\sigma^4, m_\pi^8, m_\sigma^2 m_\pi^4\right)$$

Δ is the conformal dim of the (dilaton) dynamics

$3 - \gamma$ is the dim of the quark mass operator

B_1 depends on Δ and geometry (up to subleading Q terms)

B_2 depends on γ and geometry (up to subleading Q terms)

Bersini, D'Alise, Gambardella, Sannino, 2401.08457 PRD

Message

$$\Delta_Q = \Delta_Q^{*} + \left(\frac{m_\sigma}{4\pi\nu}\right)^2 Q^{\frac{\Delta}{3}} B_1 + \left(\frac{m_\pi(\theta)}{4\pi\nu}\right)^4 Q^{\frac{2}{3}(1-\gamma)} B_2 + \mathcal{O}\left(m_\sigma^4, m_\pi^8, m_\sigma^2 m_\pi^4\right)$$

Dilaton mass corrections dominate on the pion mass

Actually $m_\sigma \sim m_\pi^2 \sim m_q$ i.e. conformal breaking counts m_q & m_σ

Novel way to disentangle dilaton properties

Bersini, D'Alise, Gambardella, Sannino, 2401.08457

Corrections

$$\Delta_Q = \Delta_Q^* + \left(\frac{m_\sigma}{4\pi\nu}\right)^2 Q^{\frac{\Delta}{3}} B_1 + \left(\frac{m_\pi(\theta)}{4\pi\nu}\right)^4 Q^{\frac{2}{3}(1-\gamma)} B_2 + \mathcal{O}\left(m_\sigma^4, m_\pi^8, m_\sigma^2 m_\pi^4\right)$$

$$\Delta_Q^* = c_{4/3} Q^{4/3} + c_{2/3} Q^{2/3} + \mathcal{O}\left(Q^0\right)$$

$$c_{4/3} = \frac{3}{8}\left(\frac{2\Lambda^2}{\pi N_f \nu^2}\right)^{2/3}, \quad c_{2/3} = \frac{1}{4f^2}\left(\frac{2\pi^2}{N_f \nu^2 \Lambda^4}\right)^{1/3}$$

$$B_1 = \frac{c_{2/3} 2^{9-2\Delta} 3^{\frac{\Delta}{2}-1} (\pi\nu r)^{4-\Delta} \left(c_{4/3} N_f\right)^{1-\frac{\Delta}{2}}}{(\Delta-4)\Delta} \left(1 - \frac{\Delta\, c_{2/3}}{4c_{4/3}} Q^{-2/3} + \mathcal{O}\left(Q^{-4/3}\right)\right),$$

$$B_2 = -\, 3^{4-\gamma} 2^{4\gamma-3} \pi^{2\gamma+2} c_{4/3}^{\gamma-4} N_f^{\gamma-1} (\nu r)^{2(\gamma+1)} \left(1 + \frac{(\gamma-4) c_{2/3}}{2c_{4/3}} Q^{-2/3} + \mathcal{O}\left(Q^{-4/3}\right)\right)$$

Geometry

Charge expansion

Bersini, D'Alise, Gambardella, Sannino, 2401.08457

Summary

Outstanding templates for SM physics ad Beyond

Rich phase diagram in the $\mu - \theta$ plane vs N_f

2 color, N_f odd 2 *novel first order phase transitions at* $\theta = \pi/2, 3\pi/2$

Novel way to extract near conformal (dilaton) information

Wide applications across (astro) particle physics and cosmology

thank you

One step closer to the real world