Lessons about walking theories from complex CFTs

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Based on [1807.11512] and [1808.04380] with V. Gorbenko and S. Rychkov and [2005.07708] with V. Gorbenko

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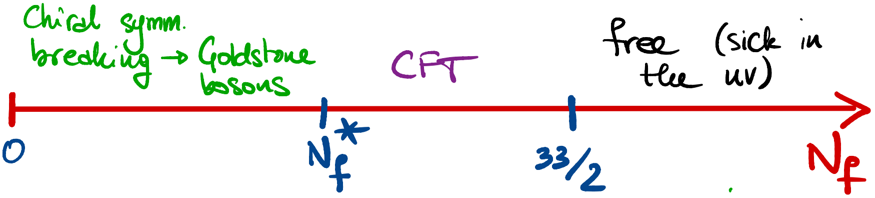
Conformal windows

QCD$_4$, $SU(3)$ gauge group, N_f massless fundamental quarks.



Conformal window: $N_f^* < N_f < 33/2$, **Banks-Zaks** fixed point. $N_f^* \sim 8 - 10$.
Similar behavior in adjoint QCD$_4$, QED$_3$, . . .

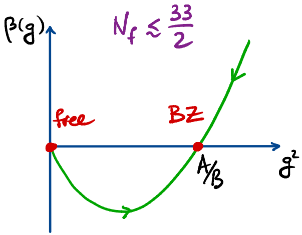
How do fixed points disappear?

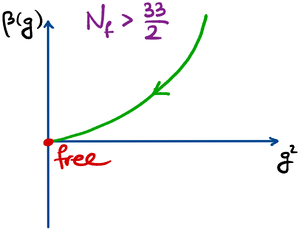
The easy end

Upper end of conformal window: $N_f = 33/2$

$$\beta(g) \equiv \frac{dg}{d\log E} = -Ag^3 + Bg^5 + \ldots$$

$$A = c\left(\frac{33}{2} - N_f\right) \text{ with } c > 0\,, \qquad B > 0$$



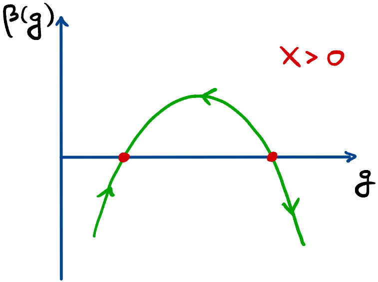


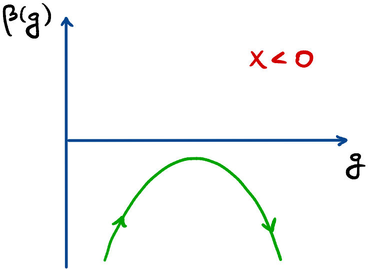
BZ merges with the free theory at $N_f = 33/2$.

Lower end: the minimal scenario

Most generic is saddle node bifurcation. [Kaplan, Lee, Son, Stephanov '09]

$$\beta(g) \equiv \frac{dg}{d\log E} = x - (g - g_0)^2 + \dots \qquad x \sim N_f - N_f^*$$

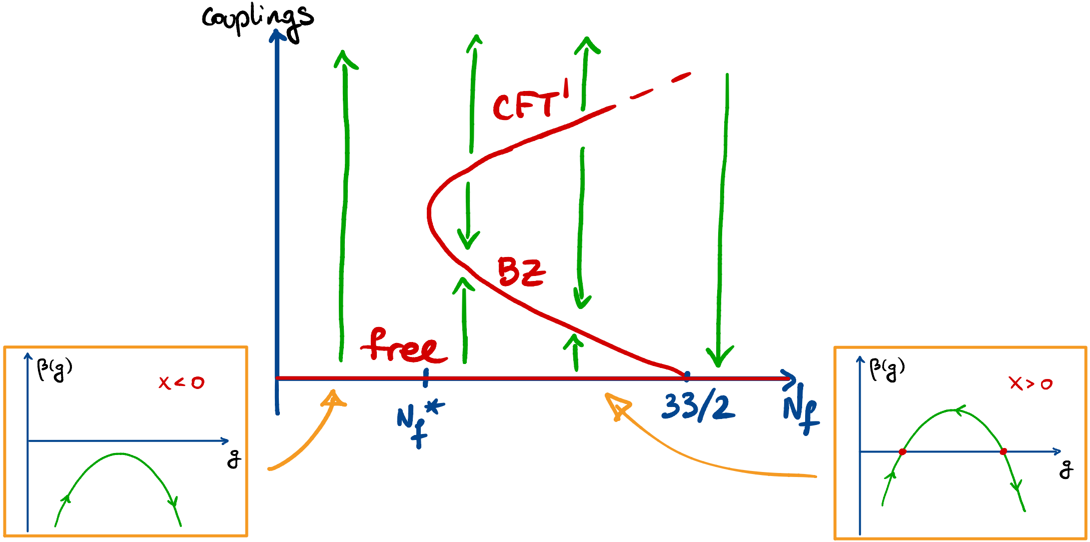




At $x = 0$, have (classically) marginal operator.

[Assumptions: β function is analytic.]

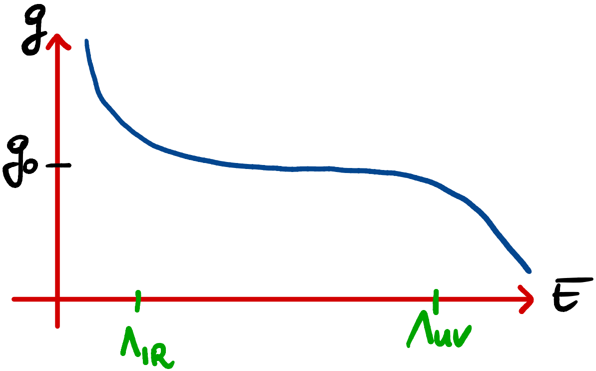
The minimal scenario



Walking behavior

$$\beta_g = x - (g - g_0)^2 + \dots$$

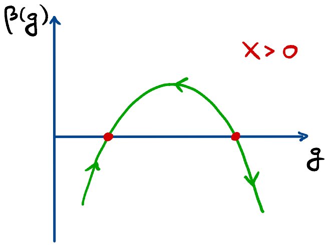
Just outside the conformal window $x \lesssim 0$, **large separation of scales**

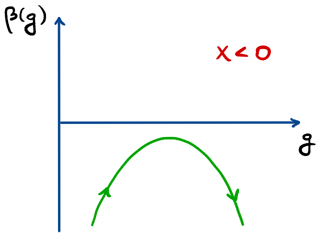


$$\frac{\Lambda_{\mathrm{UV}}}{\Lambda_{\mathrm{IR}}} = \exp\left(\frac{\pi}{\sqrt{|x|}}\right)$$

Complex CFTs

$$\beta(g) = x - (g - g_0)^2 + \dots$$

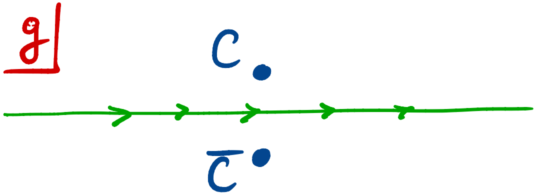




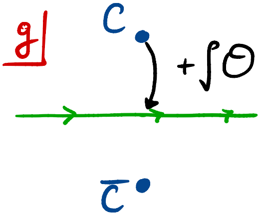
- $x > 0$, real zeros $\rightarrow$ CFTs.
- $x < 0$, complex zeros $g_* = g_0 \pm i\sqrt{|x|}$ $\rightarrow$ **complex CFTs** [Gorbenko, Rychkov, BZ '18]. **Non-unitary** but well defined.

Walking from complex CFTs

Walking = passing through two complex CFTs!



Conformal perturbation theory to describe the walking region as a perturbation of the complex CFT.



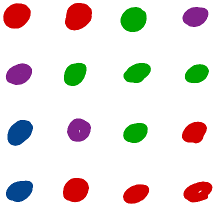
Particle physics from boiling water

The same phenomena happen in some statistical models.

[Gorbenko, Rychkov, BZ '18] [Gorbenko, BZ '20]

E.g. Q states **Potts model** in 2d: S_Q symmetry.

- $Q > 4$: 1^{st} order phase transition
- $0 < Q \leq 4$: 2^{nd} order phase transition



$Q \in \mathbb{R}_+$ is well defined here.

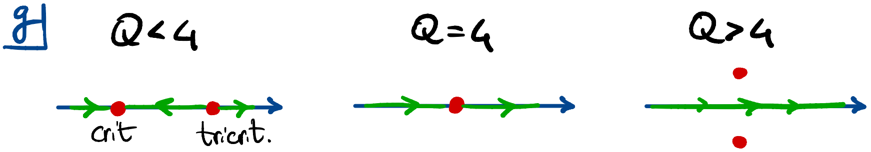
Phase transition is **weakly first order** for $Q \gtrsim 4$

Q	5	6	7	8	9	10
ξ/a	2512.2	158.9	48.1	23.9	14.9	10.6

Potts model and complex CFTs

Conformal window as in QCD_4!

Two fixed points (critical and tricritical) become complex for $Q > 4$



Exact results in $2d$ $\to$ we can predict e.g. [Gorbenko, Rychkov, BZ '18]:

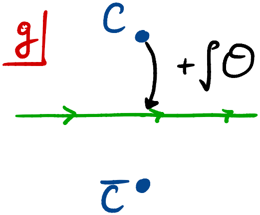
$$\xi/a \sim \exp\left(\frac{\pi^2}{\sqrt{Q-4}}\right)$$

Agrees with exact lattice results!

Should we expect a light dilaton?

Light dilaton expected by some in walking theories e.g. [Appelquist, Bai '10]. Let us revisit this question

Walking behavior: **explicit** breaking of conformal symmetry!



Conformal symmetry broken weakly $\rightarrow$ particle masses are small. But their ratio will generically be $\sim O(1)$!

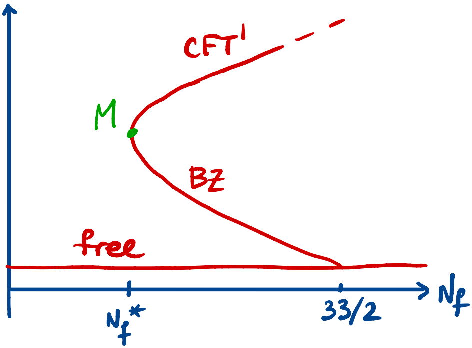
Thanks to integrability, can explicitly check there is no light dilaton in $Q = 4 + \epsilon$ 2d Potts model [Delfino, Cardy '00].

Walking vs moduli space

Light dilaton requires **moduli space**, to allow *spontaneous* symmetry breaking. This question is independent of walking!

2d Potts model does not have one. I do not know of any interacting, finite # d.o.f., non-SUSY theory that has one.

Does QCD at N_f^* have a moduli space? To my knowledge, no evidence of this.



Further developments

- Same phenomenon in the 2d $O(n)$ model
  [Gorbenko, BZ '20]
- Can be found at weak coupling
  [Benini, Iossa, Serone '19] [Azatov, Vanvlasselaer '20], [Jepsen, Klebanov, Popov '20], [Antipin, Bersini, Sannino, Wang, Zhang '20] [Jepsen, Popov '21]^2, ...
- Holography
  [Faedo, Hoyos, Mateos, Subils '19, '21]
- Numerical lattice results in 2d
  [Ma, He '18] [Haldar, Tavakol, Ma, Scaffidi '23] [Jacobsen, Weise '24]
- Suspected of playing a role in the deconfined quantum critical point in $2+1$ d
  [Song et al. '23], ...

Conclusions

- Minimal scenario of QCD conformal window is through saddle node bifurcation
- Saddle node bifurcation implies existence of complex CFTs
- Complex CFTs control the regime of walking theories
- Walking theory: *explicitly* broken conformal symmetry
- Walking by itself does not give a light dilaton
- Light dilaton requires a moduli space, no evidence of that in QCD