AdS/Yang Mills and the dilaton at the edge of the conformal window

Nick Evans
University of Southampton

Southampton Theory Astrophysics Gravity Research Centre

Work with Johanna Erdmenger, Kostas Rigatos and Werner Porod: 1907.09489 [hep-th]
2009.10737 [hep-ph]
2010.10279 [hep-ph]
2012.00032 [hep-ph]

Work with JE, WP and Yang Liu: 2304.09190 [hep-th]
2404.14480 [hep-ph]

Work with Matt Ward: 2304.10816 [hep-ph]
Work with Anja Alfano – to be published

AdS/YM Programme

The D3/probe D7 system in holography is a remarkably simple calculator of quark physics in a strongly coupled gauge theory…

It is not tied to supersymmetry or conformality and ties chiral symmetry breaking to the running of the anomalous dimension of qq…

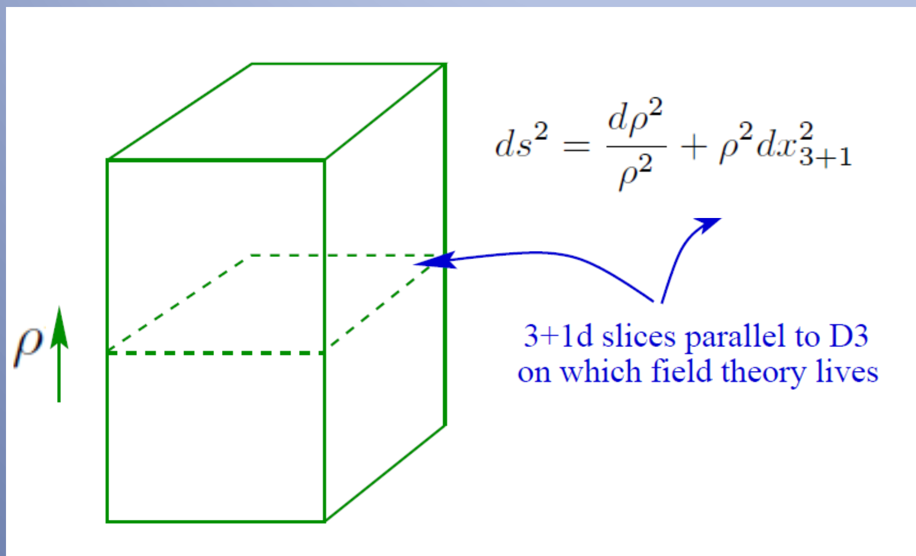
We've constructed simple toy models of ANY non-supersymmetric gauge + fermion system….

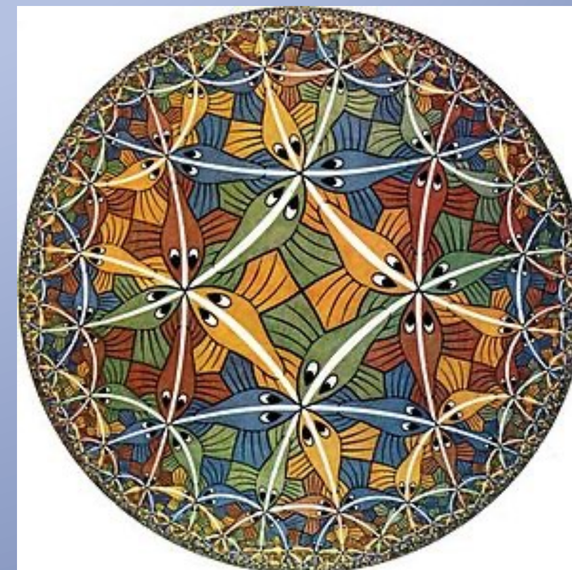
What does it say about Nf dependence of QCD and dilatons (and caveats)?

Composite higgs models… multi-scale theories…

How Does AdS/CFT Work 1

A weak strong duality that at least works for N=4 SYM and its deformations…





Dilatations

$$\int d^4x \, \partial^\mu \phi \partial_\mu \phi, \qquad x \to e^{-\alpha} x, \qquad \phi \to e^{\alpha} \phi$$

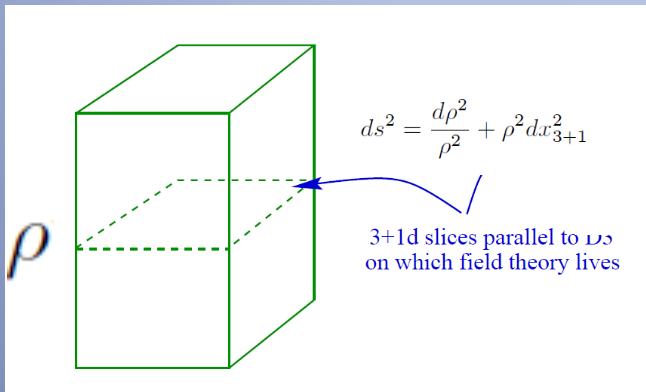
Become spacetime symmetry of AdS

$$\rho \to e^{\alpha} \rho$$

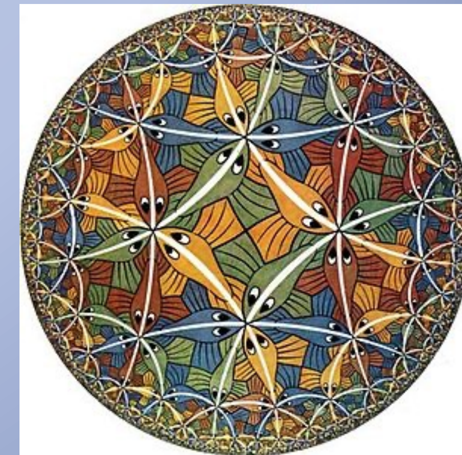
ρ is a continuous mass dimension

$\to$ RG Scale

How Does AdS/CFT Work 2



$$\sqrt{-Detg} = Det\left[- \begin{pmatrix} -\rho^2 & 0 & 0 & 0 & 0 \\ 0 & \rho^2 & 0 & 0 & 0 \\ 0 & 0 & \rho^2 & 0 & 0 \\ 0 & 0 & 0 & \rho^2 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{\rho^2} \end{pmatrix} \right]^{1/2} = \rho^3$$



Operators and sources appear as fields in the bulk

Eg

$$\int d^4x \; m \; \bar{\psi}\psi$$

m is the quark mass
c is the quark condensate

A field for the mass/condensate:

$$S = \int d^4x \int d\rho \; \frac{1}{2}\rho^3 (\partial_\rho L)^2$$

$$\partial_\rho \left[\rho^3 \partial_\rho L\right] = 0$$

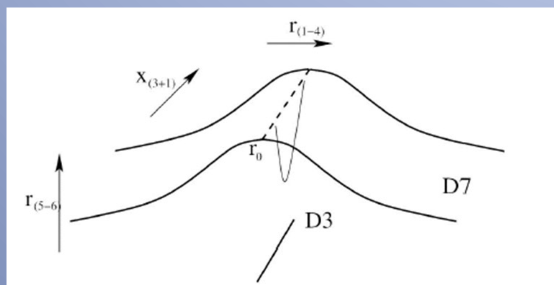
$$L = m + \frac{c}{\rho^2}$$

AdS/CFT Contains Non-SUSY Theories

Eg Witten black holes = finite T theories

hep-ph/9802150

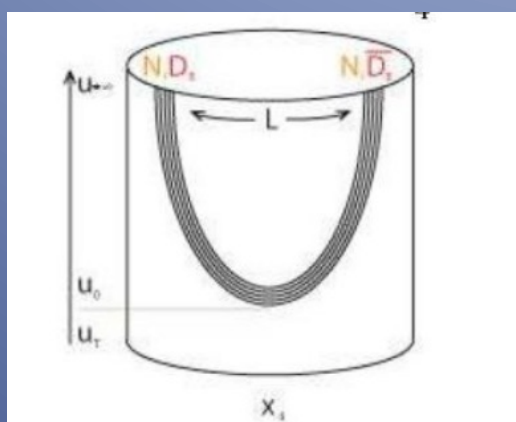
Top/down hep-th/0306018



Top down models that describe dynamical chiral symmetry breaking exist….

Magnetic catalysis is the most controlled case… (Johnson, Filev)

Sakai-Sugimoto model (D4/D8)



They all look a bit baroque…

Probe limit DBI Action captures key elements

The running of anomalous dimensions underlies all these models…

Running Dimensions in Gauge Theory

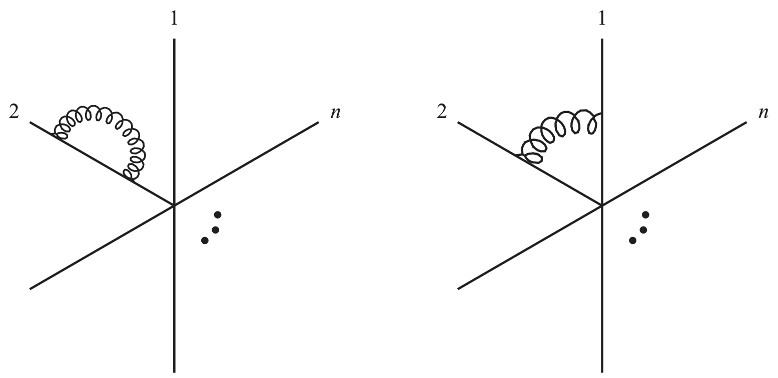


FIG. 1: Diagrams at one loop order contributing to the anomalous dimension of a gauge invariant scalar operator with n quark legs.

$$\mathcal{O} = \tilde{\mathcal{O}}\mu^d,$$

Z_{0n}

$$d = \frac{1}{\mathcal{O}}\ \mu\frac{d\mathcal{O}}{d\mu}$$

$$\gamma_{\mathcal{O}} = -\frac{1}{Z_{\mathcal{O}}}\ \mu\frac{dZ_{\mathcal{O}}}{d\mu}$$

Wave function renormalization of n-legs

$$Z_\psi = 1 - C_2(R)\ \xi\ \frac{\alpha}{4\pi}\ \frac{1}{\epsilon}$$

ξ is gauge parameter

Vertex factor (upto constant Cn)

$$Z_{\mathcal{O}n} = \left(1 + C_n(3+\xi)\ \frac{\alpha}{4\pi}\ \frac{1}{\epsilon}\right) Z_\psi^{n/2}$$

$$C_n = nC_2(R)/2.$$

For ξ independence

$$Z_{\mathcal{O}n} = 1 + \frac{3n}{2}C_2(R)\ \frac{\alpha}{4\pi}\ \frac{1}{\epsilon}$$

$$\gamma_{\mathcal{O}n}(\mu) = -n\ \frac{\alpha(\mu)}{\pi}$$

Running Dimensions in Holography

Raul Alvares, NE, Keun-Young arXiv:1204.2474 [hep-ph]; Matti Jarvinen, Elias Kiritsis arXiv:1112.1261 [hep-ph]

Holographically we can change the dimension of our operator by adding a mass term

$$\partial_\rho[\rho^3\partial_\rho L] - \rho\Delta m^2 L = 0$$

$$L = \frac{m_{FP}}{\rho^\gamma} + \frac{c_{FP}}{\rho^{2-\gamma}}, \qquad \gamma(\gamma-2) = \Delta m^2$$

$\Delta m^2 = -1$ corresponds to $\gamma = 1$ and is special – the Breitenlohner Freedman bound instability…

So we can include a running coupling by a ρ dependent mass squared for the scalar.

Top down derivation: many string constructions eg probe D7 branes in D3 backgrounds are examples of this…

Very complex geometries describe the gauge theory glue-dynamics… a single quark in that background is described by a DBI field such as this with the running of the mass determined by the glue-dynamics…

Dynamic AdS/YM

Timo Alho, NE, KimmoTuominen
1307.4896

$$S = \int d^4x\, d\rho\, \mathrm{Tr}\, \rho^3 \left[\frac{1}{\rho^2 + |X|^2} |DX|^2 + \frac{\Delta m^2}{\rho^2} |X|^2 \right.$$

$$X = L(\rho)\, e^{2i\pi^a T^a}.$$

$$ds^2 = \frac{d\rho^2}{(\rho^2 + |X|^2)} + (\rho^2 + |X|^2) dx^2,$$

$|X| = L$ is now the dynamical field whose solution will determine the condensate as a function of m - the phase is the pion.

We use the top-down IR boundary condition on mass-shell: $X'(\rho=X) = 0$

X enters into the AdS metric to cut off the radial scale at the value of m or the condensate – no hard wall

The gauge DYNAMICS is input through a guess for Δm

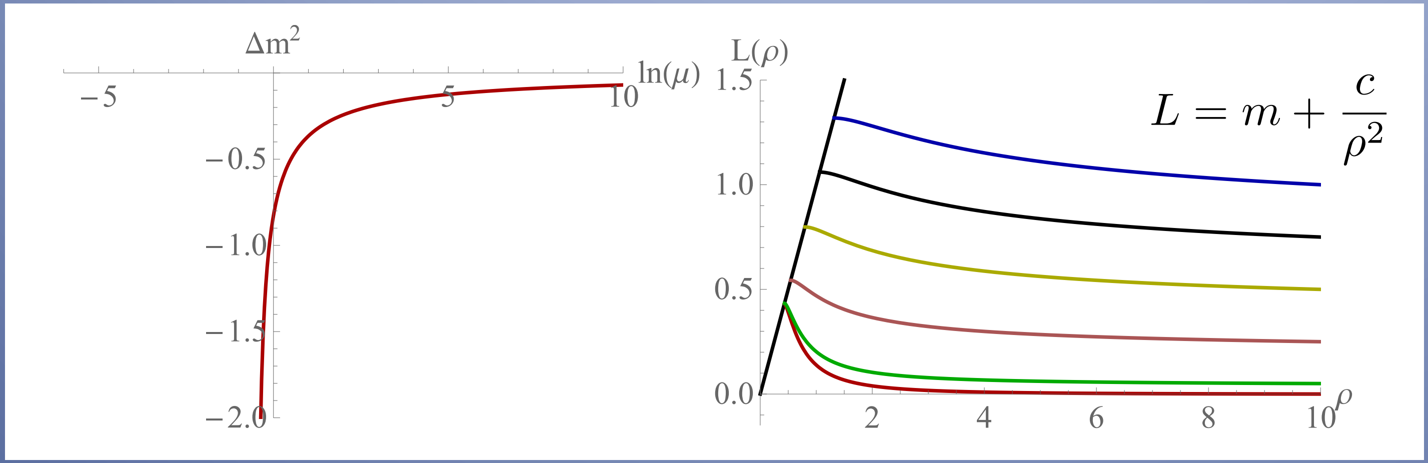
$$\Delta m^2 = -2\gamma = -\frac{3(N_c^2 - 1)}{2N_c \pi} \alpha$$

The only free parameters are Nc, Nf, m, Λ

Formation of the Chiral Condensate

We solve for the vacuum configuration of L

$$\partial_\rho[\rho^3\partial_\rho L] - \rho\Delta m^2 L = 0\,.$$



Read off m and qq in the UV…

Δm^2 from QCD

Shoot out with

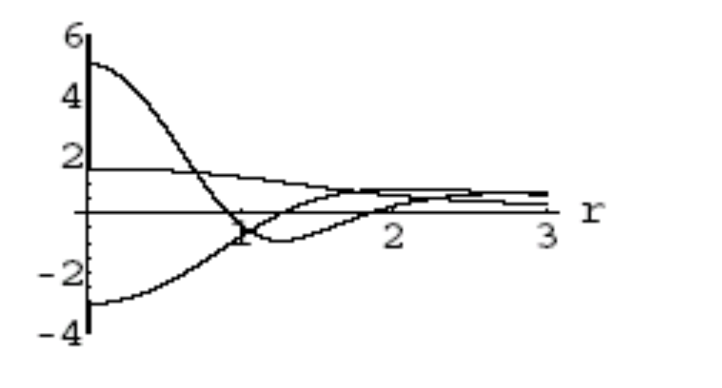
$L'(\rho = L) = 0$

Meson Fluctuations

$$S = \int d^4x\, d\rho\, \mathrm{Tr}\, \rho^3 \left[\frac{1}{\rho^2 + |X|^2} |DX|^2 + \frac{\Delta m^2}{\rho^2} |X|^2 + \frac{1}{2\kappa^2} (F_V^2 + F_A^2) \right]$$

$$L = L_0 + \delta(\rho) e^{ikx} \qquad k^2 = -M^2$$

$$\partial_\rho(\rho^3 \delta') - \Delta m^2 \rho \delta - \rho L_0 \delta \left. \frac{\partial \Delta m^2}{\partial L} \right|_{L_0} + M^2 R^4 \frac{\rho^3}{(L_0^2 + \rho^2)^2} \delta = 0 \,.$$



The source free solutions pick out particular mass states… the σ and its radial excited states…

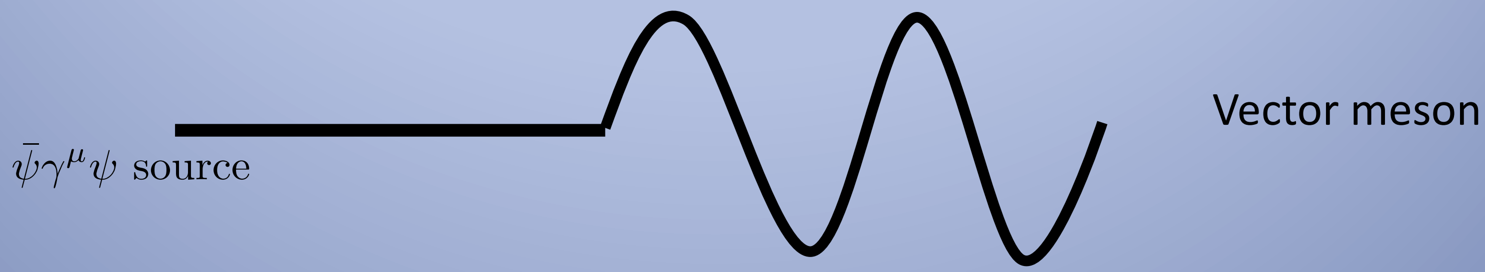
The gauge fields let us also study the operators and states

$$\bar{q} \gamma^\mu q \to \rho \text{ meson}$$

$$\bar{q} \gamma^\mu \gamma^5 q \to \text{a meson}$$

Decay Constants (a la. AdS/QCD - hep-ph/0501128 [hep-ph])

Decay constants are determined by allowing a source to couple to a physical state



F_V^2

Note not $F_V\, m_V$

Now we need to fix the normalizations of the holographic linear perturbations…

For the physical states we canonically normalize the kinetic terms…

For the source solutions we fix κ and the norms so that we match perturbative results for eg Π_{VV} in the UV…

$$N_V^2 = N_A^2 = \frac{g_5^2\ d(R)\ N_f(R)}{48\pi^2}$$

Baryons

cf Brodsky, de Teramond hep-th/0501022 [hep-th]

Plus our 1907.09489 [hep-th]

In D3/D7 system some quark-gaugino-quark tri-fermion states are described by world volume fermions on the D7 – it does not seem unreasonable to include three quark states in this way therefore.

$$S_{1/2} = \int d^5x \ \rho^3 \ \bar{\Psi} \left(\not{D}_{\mathrm{AAdS}} - m \right) \Psi \, .$$

The four component fermion satisfies the second order equation

$$\left(\partial_\rho^2 + \mathcal{P}_1 \partial_\rho + \frac{M_B^2}{r^4} + \mathcal{P}_2 \frac{1}{r^4} - \frac{m^2}{r^2} - \mathcal{P}_3 \frac{m}{r^3} \ \gamma^\rho \right) \psi = 0 \, ,$$

where M_B is the baryon mass and the pre-factors are given by

$$\begin{aligned}
\mathcal{P}_1 &= \frac{6}{r^2} \left(\rho + L_0 \ \partial_\rho L_0 \right) \ , \\
\mathcal{P}_2 &= 2 \left((\rho^2 + L_0^2) L \partial_\rho^2 L_0 + (\rho^2 + 3L_0^2)(\partial_\rho L_0)^2 + 4\rho L_0 \partial_\rho L_0 + 3\rho^2 + L_0^2 \right) \ , \\
\mathcal{P}_3 &= \left(\rho + L_0 \ \partial_\rho L_0 \right) \ .
\end{aligned}$$

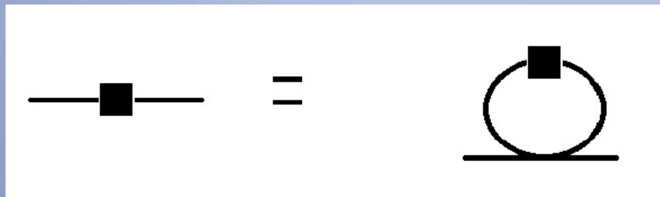
$$\begin{aligned}
\psi_+ &\sim \mathcal{J} \sqrt{\rho} + \mathcal{O} \frac{M_B}{6} \rho^{-11/2} \, , \\
\psi_- &\sim \mathcal{J} \frac{M_B}{4} \frac{1}{\sqrt{\rho}} + \mathcal{O} \rho^{-9/2} \, .
\end{aligned}$$

IR boundary conditions

$$\begin{aligned}
\psi_+(\rho = L_{IR}) &= 1, \qquad \partial_\rho \psi_+(\rho = L_{IR}) = 0 \, , \\
\psi_-(\rho = L_{IR}) &= 0, \qquad \partial_\rho \psi_-(\rho = L_{IR}) = \frac{1}{L_{IR}} \, .
\end{aligned}$$

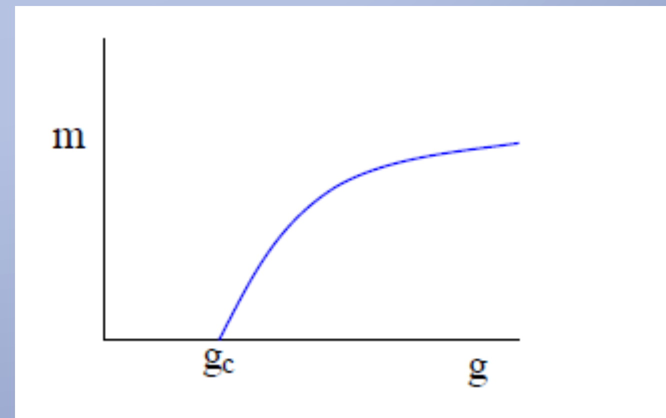
NJL Operators

$$\mathcal{L} = \bar{\psi}_L \partial\!\!\!/ \psi_L + \bar{\psi}_R \partial\!\!\!/ \psi_R + \frac{g^2}{\Lambda_{UV}^2} \bar{\psi}_L \psi_R \bar{\psi}_R \psi_L$$



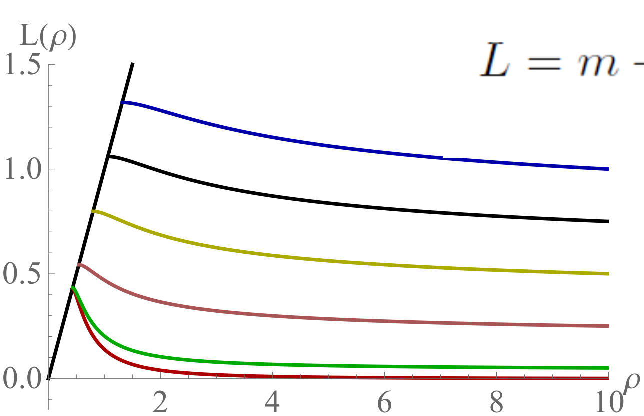
$$-im = -\frac{g^2}{\Lambda_{UV}^2} \int \frac{k^2 dk^2}{16\pi^2} \frac{Tr(k\!\!\!/ + m)}{k^2 + m^2}$$

$$1 = \frac{g^2}{4\pi^2} \left(1 - \frac{m^2}{\Lambda_{UV}^2} \log\left[(\Lambda_{UV}^2 + m^2)/m^2\right]\right)$$



Witten's Holographic Prescription

1601.02824 [hep-th]



$$L = m + \frac{\sigma}{\rho^2}$$

$$\frac{g^2}{\Lambda_{UV}^2} \bar{\psi}_L \psi_R \bar{\psi}_R \psi_L \to \frac{g^2}{\Lambda_{UV}^2} \langle \bar{\psi}_L \psi_R \rangle \bar{\psi}_R \psi_L$$

$$m = \frac{g^2}{\Lambda_{UV}^2} \sigma$$

QCD Dynamics – Nc=3, Nf=2, m_q=0

$$\mu \frac{d\alpha}{d\mu} = -b_0 \alpha^2, \qquad b_0 = \frac{1}{6\pi}(11 N_c - 2N_F),$$

$$\gamma = \frac{3C_2}{2\pi}\alpha = \frac{3(N_c^2-1)}{4N_c\pi}\alpha\,.$$

Observables (MeV)	QCD	AdS/SU(3) 2 F 2 $\bar{F}$	Deviation
M_ρ	775	775*	fitted
M_A	1230	1183	- 4%
M_S	500/990	973	+64%/-2%
M_B	938	1451	+43%
f_π	93	55.6	-50%
f_ρ	345	321	- 7%
f_A	433	368	-16%
$M_{\rho,n=1}$	1465	1678	+14%
$M_{A,n=1}$	1655	1922	+19%
$M_{S,n=1}$	990 /1200-1500	2009	+64%/+35%
$M_{B,n=1}$	1440	2406	+50%

Table 1: The predictions for masses and decay constants (in MeV) for $N_f = 2$ massless QCD. The ρ-meson mass has been used to set the scale (indicated by the *).

Scale fixed by V-meson

Pattern sensible

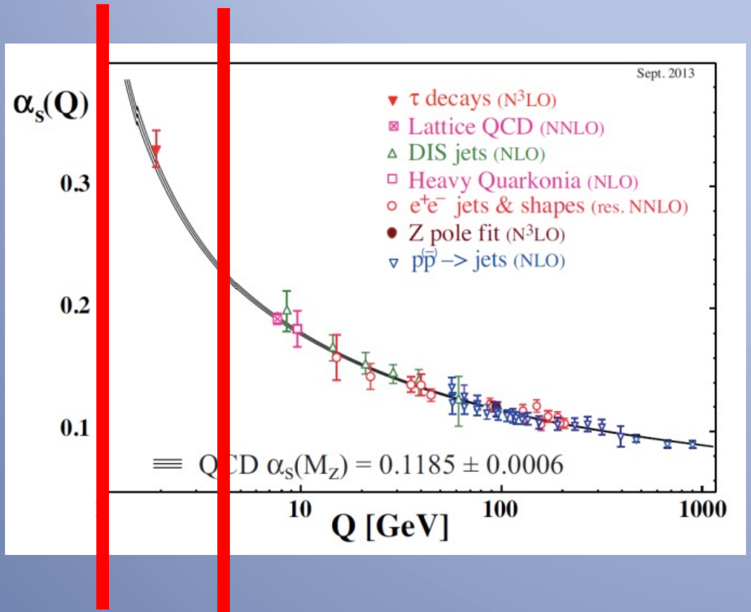
Pion decay constant needs a mass term

Baryon mass high

Radial excitations scale wrongly – no string physics included

2010.10279 [hep-ph]

Perfecting with HDOs



The weakly coupled gravity dual should only live between the red lines… probably we need HDOs at the UV scale to include matching effects… and stringy effects in the gravity model….

$$\frac{g_S^2}{\Lambda_{UV}^2}|\bar{q}q|^2\,, \qquad \frac{g_V^2}{\Lambda_{UV}^2}|\bar{q}\gamma^\mu q|^2\,, \qquad \frac{g_A^2}{\Lambda_{UV}^2}|\bar{q}\gamma^\mu\gamma_5 q|^2\,, \qquad \frac{g_B^2}{\Lambda_{UV}^5}|qqq|^2\,,$$

Observables (MeV)	QCD	Dynamic AdS/QCD	HDO coupling
M_V	775	775	sets scale
M_A	1230	1230	fitted by $g_A^2 = 5.76149$
M_S	500/990	597	prediction $+20\%/-40\%$
M_B	938	938	fitted by $g_B^2 = 25.1558$
f_π	93	93	fitted by $g_S^2 = 4.58981$
f_V	345	345	fitted by $g_V^2 = 4.64807$
f_A	433	444	prediction $+2.5\%$
$M_{V,n=1}$	1465	1532	prediction $+4.5\%$
$M_{A,n=1}$	1655	1789	prediction $+8\%$
$M_{S,n=1}$	990/1200-1500	1449	prediction $+46\%/0\%$
$M_{B,n=1}$	1440	1529	prediction $+6\%$

Table 2: The spectum and the decay constants for two-flavour QCD with HDOs from fig. 7 used to improve the spectrum.

Pretty good… but we've lost some predictivity….

Proton/neutron mass still unconvincing

2304.10816 [hep-ph]

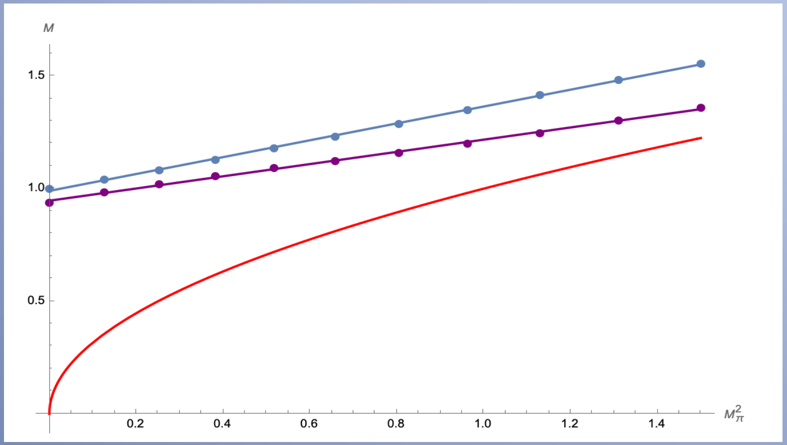
Add in the anomalous dimension for the qqq operator…

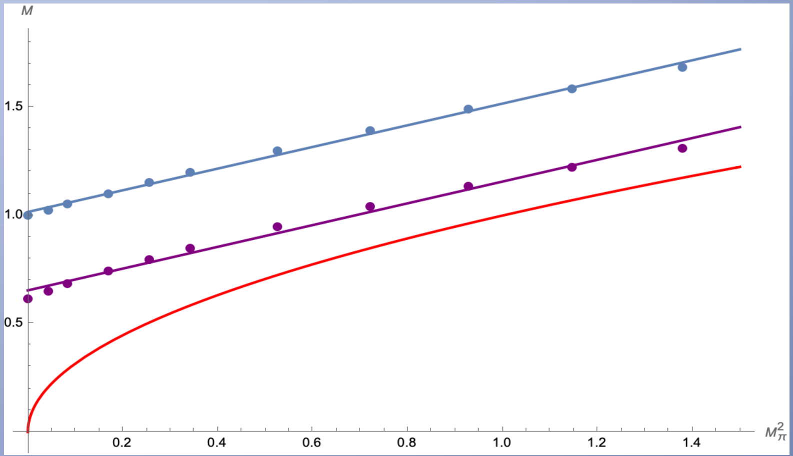
$$\Delta m_\psi = \gamma = -\frac{3}{\pi}\alpha$$

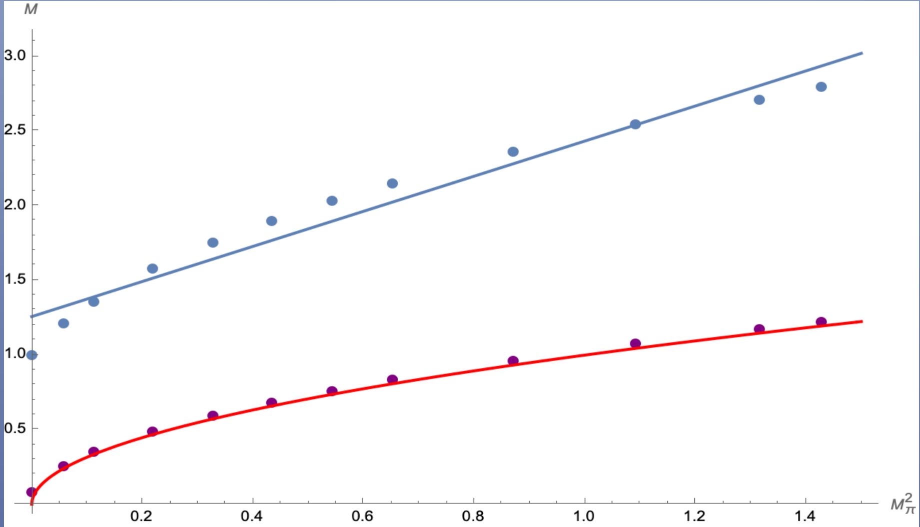
$$M_B \; = \; 1.40 M_\rho \; = \; 1.08 \;\; \mathrm{GeV}$$

SU(3) with Nf=3,7,11 using two loop beta function – $m\pi$, $m\rho$, $m\sigma$ vs $m\pi$

Plots by Anja Alfano







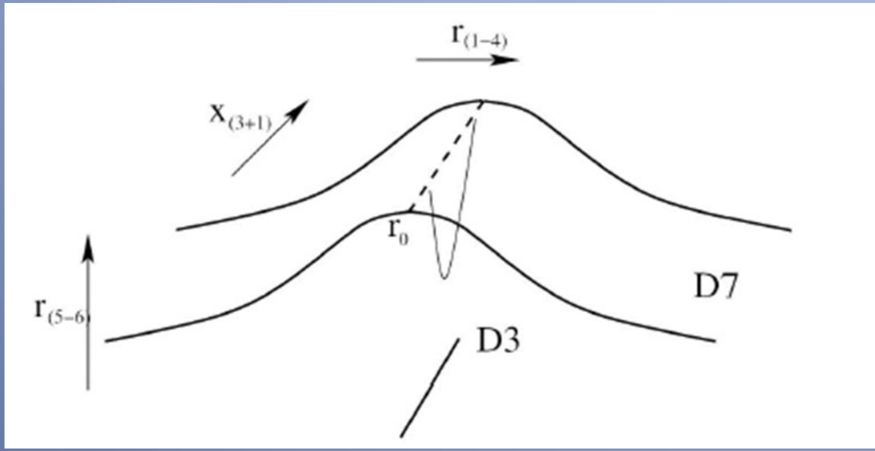
Rho mass at zero quark mass used to set scale

$$\partial_\rho(\rho^3\delta') - \Delta m^2\rho\delta - \rho L_0\delta \left.\frac{\partial\Delta m^2}{\partial L}\right|_{L_0} + M^2R^4\frac{\rho^3}{(L_0^2+\rho^2)^2}\delta = 0\,.$$

In the limit where the gradient of the running vanishes the pion and sigma equations are analytically identical

In the probe models the quark physics knows nothing of the geometry at smaller r… where the quarks are decoupled and running is that of pure YMs…

If I fluctuate the brane I just move the YMs running region to lower r though still invisible to the fluctuation.

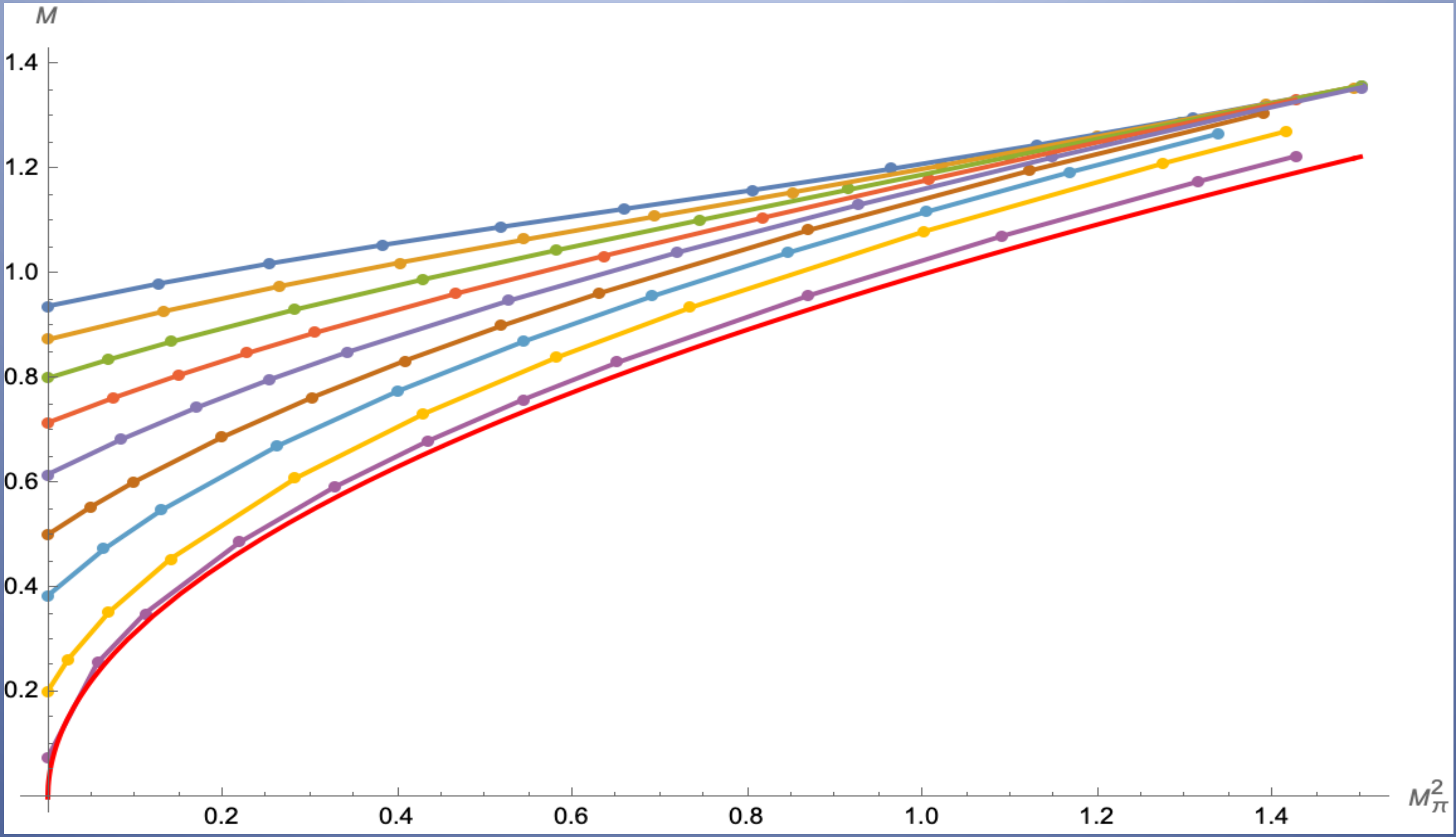


… where the pure glue running is very non-conformal…

Mixing with glueballs?

In Kiritsis & Jarvinen model their fields all extend to r=0 and so they don't see a dilaton in the same limit…. who is decoupling correctly?

SU(3) with Nf=3…11 - mσ vs mπ



$\mathrm{Sp}(2N_c)$ gauge theory with 2 Dirac fundamentals

$$\psi_i = \begin{pmatrix} U_L^C \\ D_L^C \\ D_R \\ U_R \end{pmatrix},$$

$$X = \begin{pmatrix} 0 & L_0 & 0 & 0 \\ -L_0 & 0 & 0 & 0 \\ 0 & 0 & 0 & L_0 \\ 0 & 0 & -L_0 & 0 \end{pmatrix}.$$

$$M = \begin{pmatrix} 0 & m_1 & 0 & 0 \\ -m_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & m_2 \\ 0 & 0 & -m_2 & 0 \end{pmatrix}$$

SU(2)L preserving vacuum. (anti-symmetric in flavour)

U(4) -> Sp(4). With 5 (6 - anomaly) (pseudo-)Goldstones of which pi (2,2) is ready to be made into a composite higgs

$$X_f = \begin{pmatrix} 0 & \sigma - Q_5 + iS - i\pi_5 & Q_2 - \pi_2 + i\pi_1 - iQ_1 & -Q_4 + \pi_4 + iQ_3 - i\pi_3 \\ -\sigma + Q_5 + i\pi_5 - iS & 0 & Q_4 + \pi_4 + iQ_3 + i\pi_3 & Q_2 + \pi_2 + iQ_1 + i\pi_1 \\ \pi_2 - Q_2 + iQ_1 - i\pi_1 & -Q_4 - \pi_4 - iQ_3 - i\pi_3 & 0 & \sigma + Q_5 + iS + i\pi_5 \\ Q_4 - \pi_4 + i\pi_3 - iQ_3 & -Q_2 - \pi_2 - iQ_1 - i\pi_1 & -\sigma - Q_5 - iS - i\pi_5 & 0 \end{pmatrix}.$$

NJL operators favour technicolour breaking...

$$\mathcal{L} = \frac{g_s^2}{\Lambda_{UV}^2}(\bar{\Psi}_L U_R \bar{U}_R \Psi_L + \bar{\Psi}_L D_R \bar{D}_R \Psi_L),$$

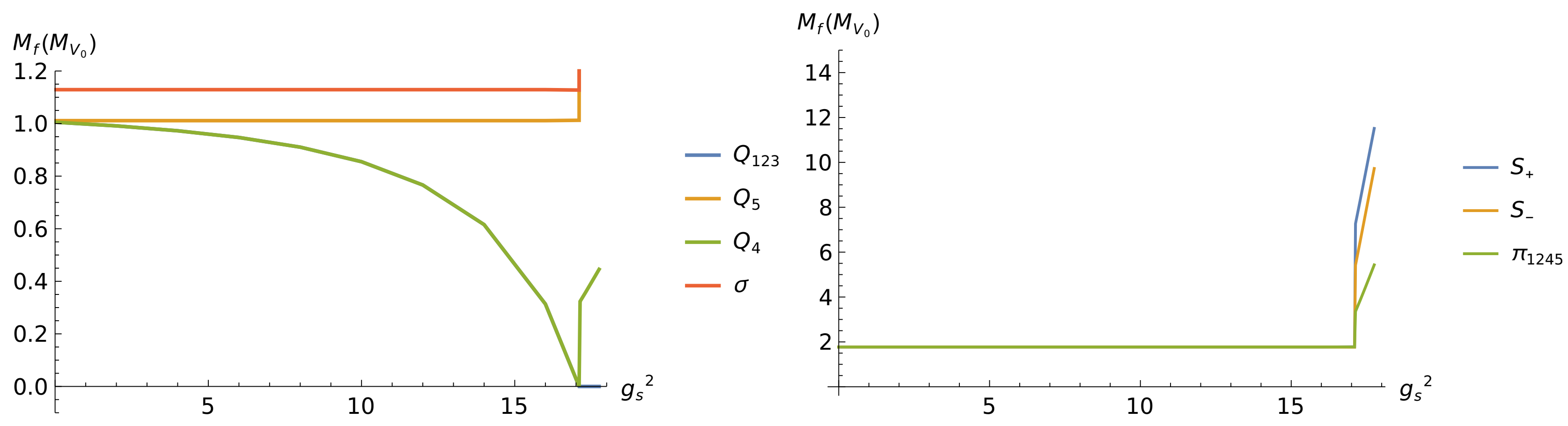
$$X_{Q_0} = \begin{pmatrix} 0 & 0 & 0 & -Q_0 \\ 0 & 0 & Q_0 & 0 \\ 0 & -Q_0 & 0 & 0 \\ Q_0 & 0 & 0 & 0 \end{pmatrix}.$$

2304.09190 [hep-th] 2404.14480 [hep-ph]

A holographic model has 12 real scalars (X) and 16 U(4) gauge fields in the bulk…

You need a non-abelian DBI – X is a flavour matrix with all terms having a flavour trace in the action…

We can see the "rotation" from composite higgs to technicolour as the NJL operators go through their critical value



(a) $M_f^2,\ f = Q_{123}, Q_4, Q_5, \sigma$ (b) $M_f^2,\ f = \pi_{1245}, S_+, S_-$

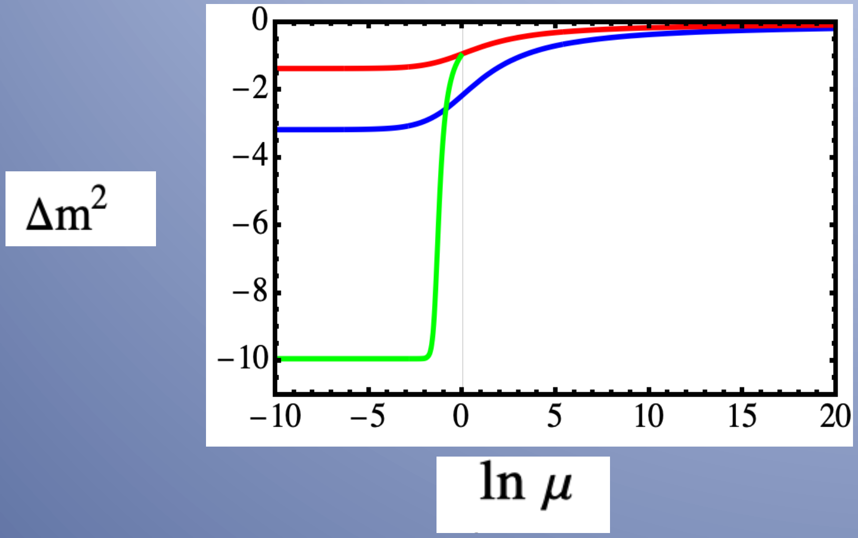
The rotation is of course very sharp (here a 10 TeV cut off for the NJL)

Multiple Mass Scales

2012.00032 [hep-ph]. + Anja Alfano

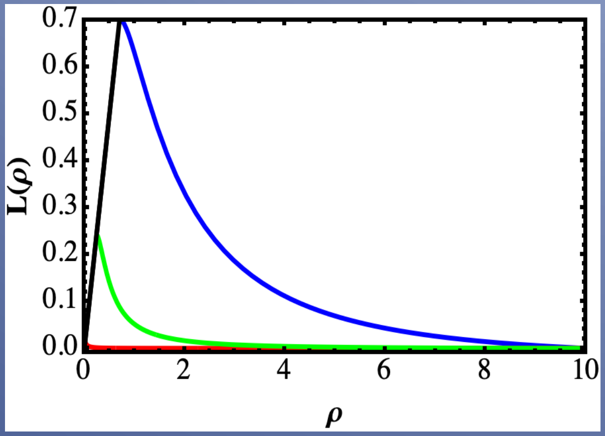
In theories with two different representations of fermions the BF bound violation point (γ=1) can be very separated

Eg SU(5) with one two-index symmetric rep (15) + Nf=15 fundamentals



15 condenses first… then we decouple them from the running…

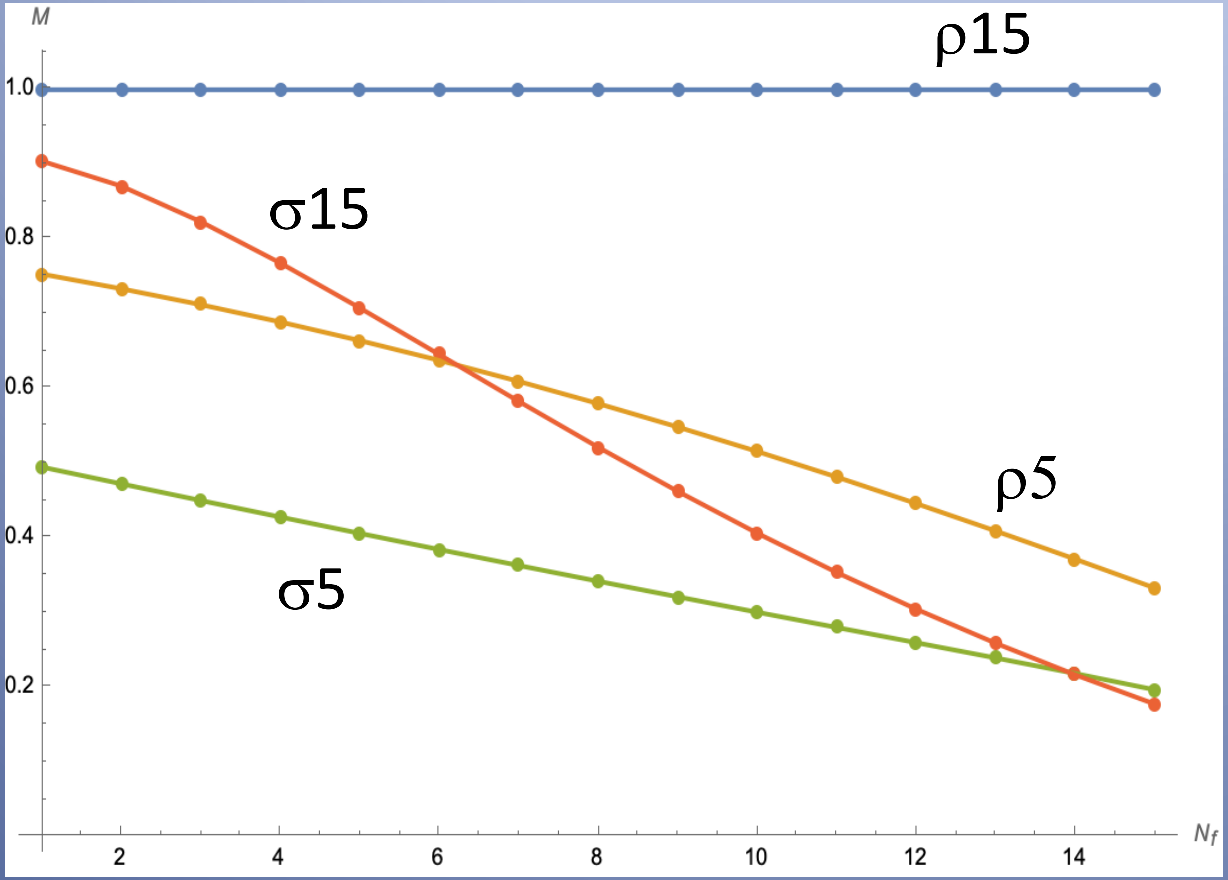
There's a factor of 15 between the scales…



But in the holographic model the walking at the high scale reduces the 15s IR mass and the gap is only 3ish…

(It lives with light BF bound violation)

The spectrum against Nf of the 5



Thermal theory should have chiral symmetry breaking in only one sector…

Are these theories too walking to study on the lattice?

Confinement is below the 5 scale?

Summary

The D3/probe D7 system in holography is a remarkably simple calculator of quark physics in a strongly coupled gauge theory and ties chiral symmetry breaking to the running of the anomalous dimension of qq…

We've constructed simple toy models of ANY non-supersymmetric gauge + fermion system….

Model contains a dilaton that becomes degenerate with the pion in the walking limit… mass halves at Nf=8

Sp(2Nc) composite higgs models… spectrum computations as rotate CH to TC…

Multi-scale theories… two reps can have ~3 difference in chiral symmetry breaking scale – splits from confinement…