Holographic light dilaton at the conformal edge

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Dilaton Dynamics - from Theory to Applications
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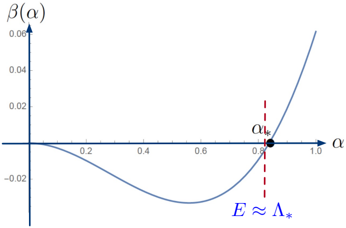
1. Introduction

Near conformal dynamics

- For certain sets of N_c and N_f, some gauge theories might flow into an IR-fixed point (Banks-Zaks theory, 1982):

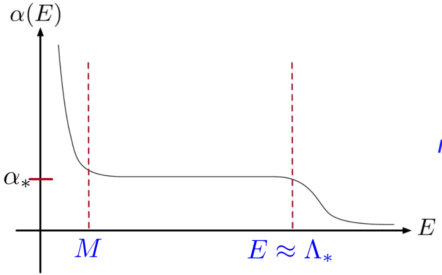


$$\alpha_* = \alpha(\Lambda_*)$$

- Near the IR fixed point ($E \approx \Lambda_*$), the theory is approximately conformal. If the scale symmetry is broken spontaneously at Λ_{SB} near IR fixed point, the theory will rest very close to the conformal edge, since $\beta(\alpha) \approx 0$, until fermions are decoupled.

Near conformal dynamics

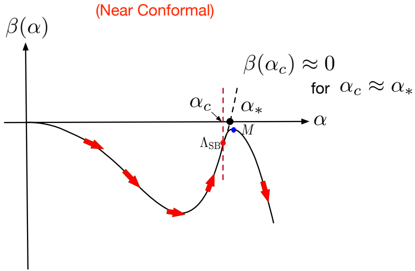
- Walking behavior of gauge coupling:



$$m = \Lambda_* e^{-\int \frac{d\alpha}{\beta(\alpha)}}$$

Near conformal dynamics

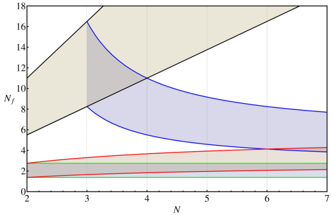
- The near conformal dynamics may be realized in a deformed BZ theory, having the dynamical generation of fermion mass.

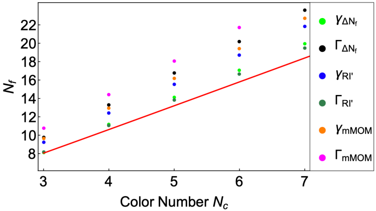


- The theory can be slightly deformed or $\alpha_c \approx \alpha_*$ in the large n_f limit or introducing additional interactions (DKH 2018).

Near Conformal Window

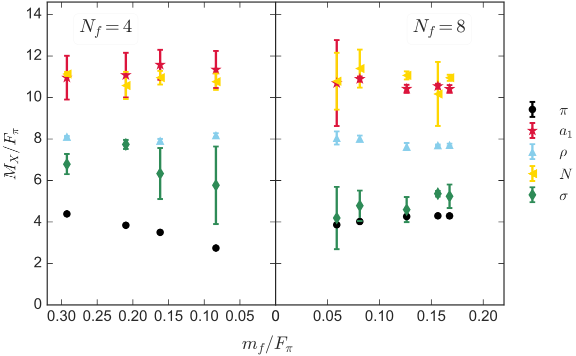
- Conformal windows from the beta-function of $SU(N)$ (Rytov+Sannino 2007) or from the anomalous dimensions of $\bar{\psi}\psi$ (Kim+DKH+Lee, PRD '20)





Spectrum Near Conformal Window

- $SU(3)$ with $N_f = 4, 8$ (LSD collaboration 2019)



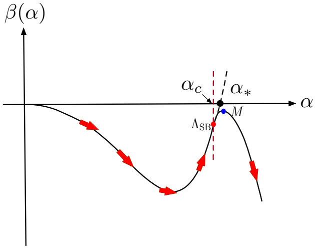
2. Light dilaton at conformal edge

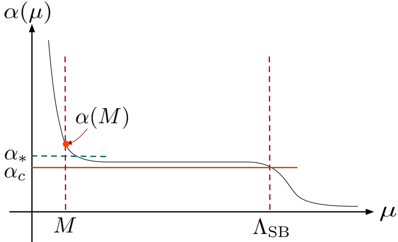
Near conformal dynamics

- How the IR scale M is related to the intrinsic scale of the deformed BZ theory?
- We may deform it by breaking chiral symmetry spontaneously, having the critical coupling for the chiral symmetry breaking $\alpha_c \approx \alpha_*$.

Near conformal dynamics

- Because $\beta(\alpha_c) \approx 0$, one expects the dynamical mass $M \ll \Lambda_{\rm SB}$, very different from QCD, where $M \sim \Lambda_{\rm SB}$.



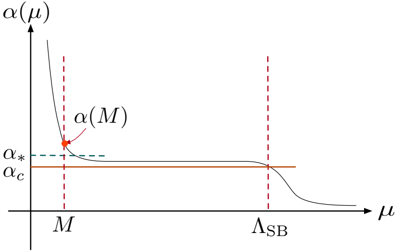


Miransky-BKT scaling

- The dynamical mass M of χSB is argued to be given by the Miransky-BKT Scaling (cf. complex CFT):

$$M(\alpha) = \Lambda_{\rm SB}(\alpha_c)\exp\left(-\frac{\pi}{\sqrt{\alpha-\alpha_c}}\right) \quad (\alpha > \alpha_c)$$

- The theory is almost scale-invariant for $M < E < \Lambda_{\rm SB}$, exhibiting walking dynamics, since $\beta(\alpha) \approx 0$.



Miransky-BKT scaling

- In the walking region we have approximate scale invariance and ladder approximation is good. The BS equation for the scalar bound-state then becomes

$$\left[P^2 + \partial^2 + \frac{\alpha/\alpha_c}{r^2}\right]\chi_P(x) = 0.$$

- Since the potential is singular, we need to regularize it:

$$V(r) = \begin{cases} -\frac{\alpha/\alpha_c}{r^2} & \text{if } r \geq a, \\ -\frac{\alpha/\alpha_c}{a^2} & \text{if } r \leq a. \end{cases}$$

Very light dilaton

- For bound states to be the cutoff-independent, we require the coupling to depend on the cutoff. (DKH+Rajeev '90)

$$\alpha(a) = \alpha_c + \frac{\pi^2}{[\ln(a\mu)]^2}\,.$$

- The non-perturbative beta function is then

$$\beta^{\mathrm{np}}(\alpha) = a\frac{\partial}{\partial a}\alpha(a) = -\frac{2}{\pi}(\alpha-\alpha_c)^{3/2}$$

- The gap equation has a nontrivial solution with this beta function for $\alpha \geq \alpha_c$. (Bardeen et al '86):

$$M \simeq \Lambda(\alpha)\exp\left[\int_{\alpha_0}^{\alpha}\frac{d\alpha}{\beta^{\mathrm{np}}(\alpha)}\right] = \Lambda_{UV}\, e^{-\frac{\pi}{\sqrt{\alpha-\alpha_c}}}\,.$$

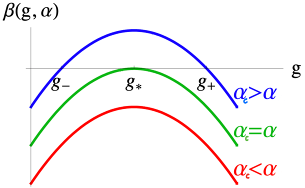
Very light dilaton

- Non-perturbative renormalization requires a new scale.
- In the walking region $\gamma_{\bar{\psi}\psi} \simeq 1$ new marginal operator emerges and therefore generates the new scale, $M \ll \Lambda_{UV}$ (DKH+Rajeev '90):

$$\frac{\lambda}{\Lambda_{UV}^2} \left(\bar{\psi}\psi\right)^2 .$$

Complex CFT

- Suppose the beta-function of the coupling of the marginal four-Fermi operator is given as (work under progress, DKH+Im+Lee)



$$\beta(\lambda) = -\left(\lambda - \lambda_*\right)^2 - \alpha + \lambda_*^2$$

Marginal deformation of CFT by four-Fermi operator

$$(\lambda = g,\, \alpha_c = \lambda_*^2, \alpha = \alpha_*)$$

- Conformality is lost when the UV fixed point collides with the IR fixed point. (Kaplan-Lee-Son-Stephanov, '09)

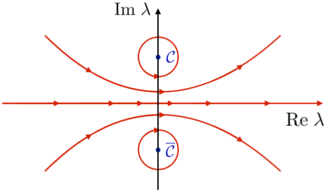
Complex CFT

- The walking dynamics is complex CFT. (V. Gorbenko, S. Rychkov, B. Zan 2018)

$$M = \Lambda \exp\left[-\oint_C \frac{d\lambda}{\beta(\lambda)}\right] = \Lambda \exp^{-\frac{\pi}{\sqrt{\alpha-\alpha_c}}}$$

Near conformal window a new marginal operator rises

whose coupling λ



Very light dilaton

- When χSB occurs at $\alpha = \alpha_c$ or at $\Lambda_{\rm SB}$, generating massless pions, the scale symmetry is also spontaneously broken.

$$0 \neq 3 \langle \bar{\psi}\psi \rangle = \langle [D, \bar{\psi}\psi] \rangle$$

- In the chirally broken phase therefore we should also have light dilaton, associated the spontaneously broken scale symmetry,

$$\langle 0 | D_\mu(x) | D(p) \rangle = -i f p_\mu e^{-ip\cdot x} \,,$$

where the dilatation current $D_\mu = x^\nu \theta_{\mu\nu}$, if the scale anomaly is small $|\langle \theta^\mu_\mu \rangle| \sim M^4 \ll \Lambda^4_{\rm SB}$, which is the salient feature of near conformal window, unlike QCD.

Very light dilaton

- Consider WT identity:

$$0 = \int_x \partial^\mu \langle 0| \mathrm{T} D_\mu(x) \theta^\nu_\nu(y) |0\rangle = \langle 0|[D, \theta^\nu_\nu]|0\rangle + \int_x \langle 0| \mathrm{T} \partial^\mu D_\mu(x) \theta^\nu_\nu |0\rangle$$

- Partially conserved dilatation current (PCDC) hypothesis:

$$f^2 m_D^2 = -4 \langle \theta^\mu_\mu \rangle \approx -16\, \mathcal{E}_{\rm vac} \sim M^4 \sim m^4_{\rm dyn}\,.$$

- The scale anomaly is given by the dynamical mass at IR, $m^4_{\rm dyn}$ (Gusynin+Miransky '89; 2302.08112 and to appear)

Very light dilaton

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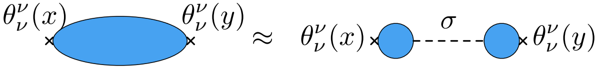

$$f^2 m_D^2 = -4 \left\langle \theta^\mu_\mu \right\rangle \approx -16\, \mathcal{E}_{\mathrm{vac}} \sim M^4 \sim m_{\mathrm{dyn}}^4 \,.$$

- The scale anomaly is given by the dynamical mass at IR, m_{dyn}^4 (Gusynin+Miransky '89; 2302.08112 and to appear)

PCDC and Very light dilaton

- Very light dilaton from quasi-conformal UV sector ($f \sim \Lambda_{SB}$):

$$m_D^2 = -\frac{4\,\langle\theta^\nu_\nu\rangle}{f^2} \sim \frac{M^4}{f^2} \ll M^2\,.$$

- By Miransky scaling, the dynamical mass or the IR scale is

$$M = \Lambda_{\rm SB}(\alpha_c)\exp\left(-\frac{\pi}{\sqrt{\alpha-\alpha_c}}\right)\,.$$

- The dilaton mass $m_D \sim \frac{M^2}{f} \ll M$ if $f \gg M$.
- By the holography we find (Cruz Hojas+DKH+Im+Jarvinen, JHEP '23)

$$m_D = c_1 M\cdot\sqrt{\nu} \quad \text{or} \quad f \sim \frac{M}{\sqrt{\nu}}$$

3. Holographic light dilaton

2302.08112

Holographic dilaton

- Consider a holographic dual of near conformal gauge theory:

$$S = S_{\rm g}[g_{\mu\nu}, \phi] + S_{\rm m}[g_{\mu\nu}, \phi, X] \ ,$$

where ϕ and X are dual to ${\rm Tr}\left(G_{\mu\nu}^2\right)$ and $q\bar{q}$, respectively.

$$S_{\rm g} = \frac{1}{2\kappa^2}\int d^5x\sqrt{-g}\left[R - g^{MN}\partial_M\phi\partial_N\phi + V(\phi)\right] + S_{\rm GH}$$

$$ds^2 = e^{2A(r)}\left(dr^2 - dt^2 + d\mathbf{x}^2\right), \quad \left(r_{\rm UV} < r < r_{\rm IR}\right),$$

$$S_{\rm m} = -\frac{1}{2\kappa^2 N_c}\int d^5x\sqrt{-g}\ {\rm Tr}\left[g^{MN}\partial_M X^\dagger\partial_N X - m_X(r)^2 X^\dagger X\right]$$

Holographic dilaton

- Near the conformal edge, the scaling dimension of $q\bar{q}$ is $\Delta_{\rm IR} = 2 \pm i\nu$ $(\nu \ll 0)$. The 5d mass then violates the BF bound,
$$m_X^2 = -4 - \nu^2 < -4\,.$$
- The background solution is then
$$X(r) = m_q r + \sigma r^3\ , \qquad (r < r_{\rm uv})$$
$$X(r) = X_0 \left(\frac{r}{r_{\rm uv}}\right)^2 \sin\left(\nu \log \frac{r}{r_{\rm uv}} + \alpha\right)\ , \quad (r_{\rm uv} < r < r_{\rm ir})$$

Holographic dilaton

- For the UV boundary conditions, we match two solutions smoothly to get

$$\tan\alpha = \nu\frac{\sigma r_{\rm uv}^3 + m_q r_{\rm uv}}{\sigma r_{\rm uv}^3 - m_q r_{\rm uv}},$$
$$X_0 = \frac{\sigma r_{\rm uv}^3 + m_q r_{\rm uv}}{\sin\alpha},$$

where we see that the phase $\alpha \sim \mathcal{O}(\nu)$ for $m_q \approx 0$..

- For the IR boundary conditions, we impose

$$\mathcal{A}X(r_{\rm ir}) + \mathcal{B}r_{\rm ir}X'(r_{\rm ir}) = 0\ ,$$

Holographic dilaton

- We now consider small fluctuations. Among them relevant ones are

$$\psi = \frac{1}{6}\left(h^{\mu}_{\mu} - \frac{\partial^{\mu}\partial^{\nu}}{\partial^2} h_{\mu\nu} \right)$$

and for the scalars

$$\phi(z,x) = \bar{\phi}(z) + \varphi(z,x) \; , \qquad X_{ij}(z,x) = \delta_{ij}\bar{X}(z) + \delta_{ij}\chi(z,x) \; .$$

Holographic dilaton

- The relevant gauge invariant combination is

$$\xi = \psi - \frac{A'}{\bar{X}'}\chi$$

- Solving the equations of motion, we get in the probe approximation

$$\xi(r) = \frac{C_1 \Re[J_{i\nu}(\omega r)] + C_2 \Re[Y_{i\nu}(\omega r)]}{2\sin(\nu \ln \frac{r}{r_{\rm uv}} + \alpha) + \nu \cos(\nu \ln \frac{r}{r_{\rm uv}} + \alpha)} \; .$$

Holographic dilaton

- From the UV and IR boundary conditions that parametrize the theory near the conformal edge one finds

$$\omega^2 r_{\rm ir}^2 = 4\frac{\frac{\mathcal{A}}{\mathcal{B}}t_\beta + \nu\left(t_\beta\frac{1+\frac{\nu}{2}t_{\beta-\alpha}}{t_{\beta-\alpha}-\frac{\nu}{2}} - 1\right)}{2t_\beta + \nu + (t_\beta+\nu)\left(\frac{\mathcal{A}}{\mathcal{B}} + \nu\frac{1+\frac{\nu}{2}t_{\beta-\alpha}}{t_{\beta-\alpha}-\frac{\nu}{2}}\right)}$$

where β is defined as

$$\frac{r_{\rm ir}}{r_{\rm uv}} \equiv e^{(\pi-\beta)/\nu}\,.$$

Holographic spectrum

- We find that a light dilaton exists if $\mathcal{A} = 0$, namely for Neunmann IR boundary condition:

$$\omega = r_{\rm ir}^{-1}\sqrt{\nu}\,.$$

- The dilaton can be parametrically lighter than all other hadrons which are $\mathcal{O}\left(z_{\rm ir}^{-1}\right)$. (2302.08112 and to appear)

Conclusion

- Near conformal dynamics shows the Miransky-BKT scaling

$$M(\alpha) = \Lambda_{\rm SB}(\alpha_c)\exp\left(-\frac{\pi}{\sqrt{\alpha-\alpha_c}}\right) \quad (\alpha > \alpha_c)$$

- The marginal four-Fermi interaction derives the BZ theory into a complex CFT (DKH+Im+Lee to appear).

$$\frac{\lambda}{\Lambda_{\rm SB}^2}\left(\bar{\psi}\psi\right)^2; \quad \beta(\lambda) = -(\lambda-\lambda_*)^2 - \alpha - \alpha_c$$

- The holographic analysis shows (2302.08112 and to appear), since $\langle\theta^\mu_\mu\rangle \sim M^4$ (Gusynin+Miransky '89)

$$m_D = c_1 M\cdot\sqrt{\nu} \quad \text{or} \quad f \sim M\cdot\nu^{-1/2}$$