Holographic light dilatons near quantum phase transitions

Javier Subils
June 27, Higgs Centre for Theoretical Physics, Edinburgh

Based on [**1909.04008**], [**2106.01802**] in collaboration with *A. Faedo, C. Hoyos* and *D. Mateos* and [**2406.04974**] with *A. Faedo, C. Hoyos, M. Piai* and *R. Rodgers*.

Utrecht University

Motivation

Our current understanding of the Universe is pledged with hierarchies,

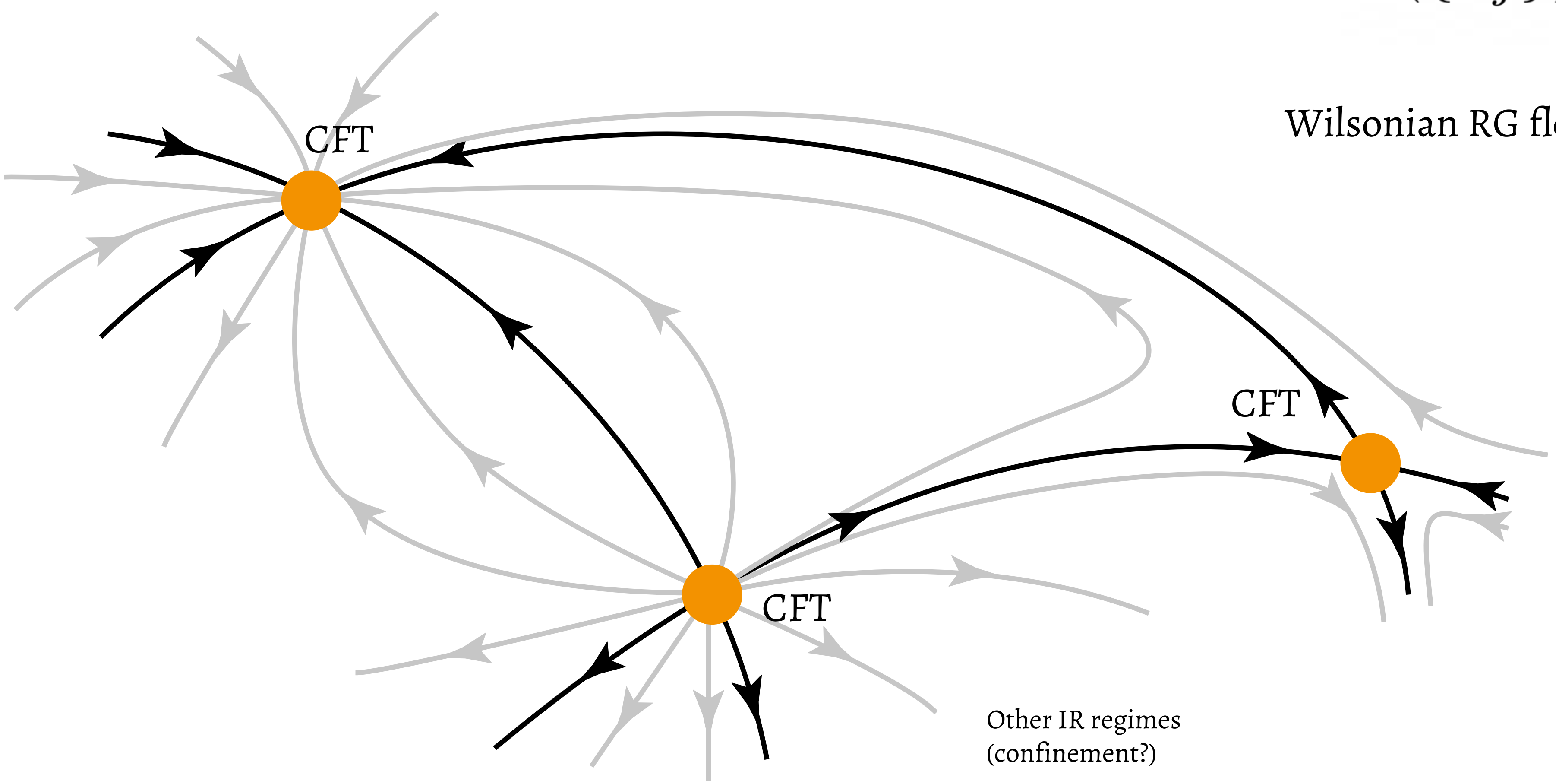
- The electron mass is 6 orders of magnitude smaller than the top-quark mass. Neutrino masses are even smaller by several orders of magnitude.
- The mixings in the quark sector, i.e. the entries in the CKM matrix, span 3 orders of magnitude.
- The Higgs mass is 17 orders of magnitude smaller than the Planck mass.
- The dark energy density driving the accelerated expansion of the Universe differs from the Planck scale by 123 orders of magnitude.

What kind of Wilsonian dynamics may give rise to such multiple hierarchies?

Motivation

$$\beta_i(\{\lambda_j\}) = \frac{\partial \lambda_i}{\partial \log \mu}$$

Wilsonian RG flow ~ dynamical system
in the space of theories



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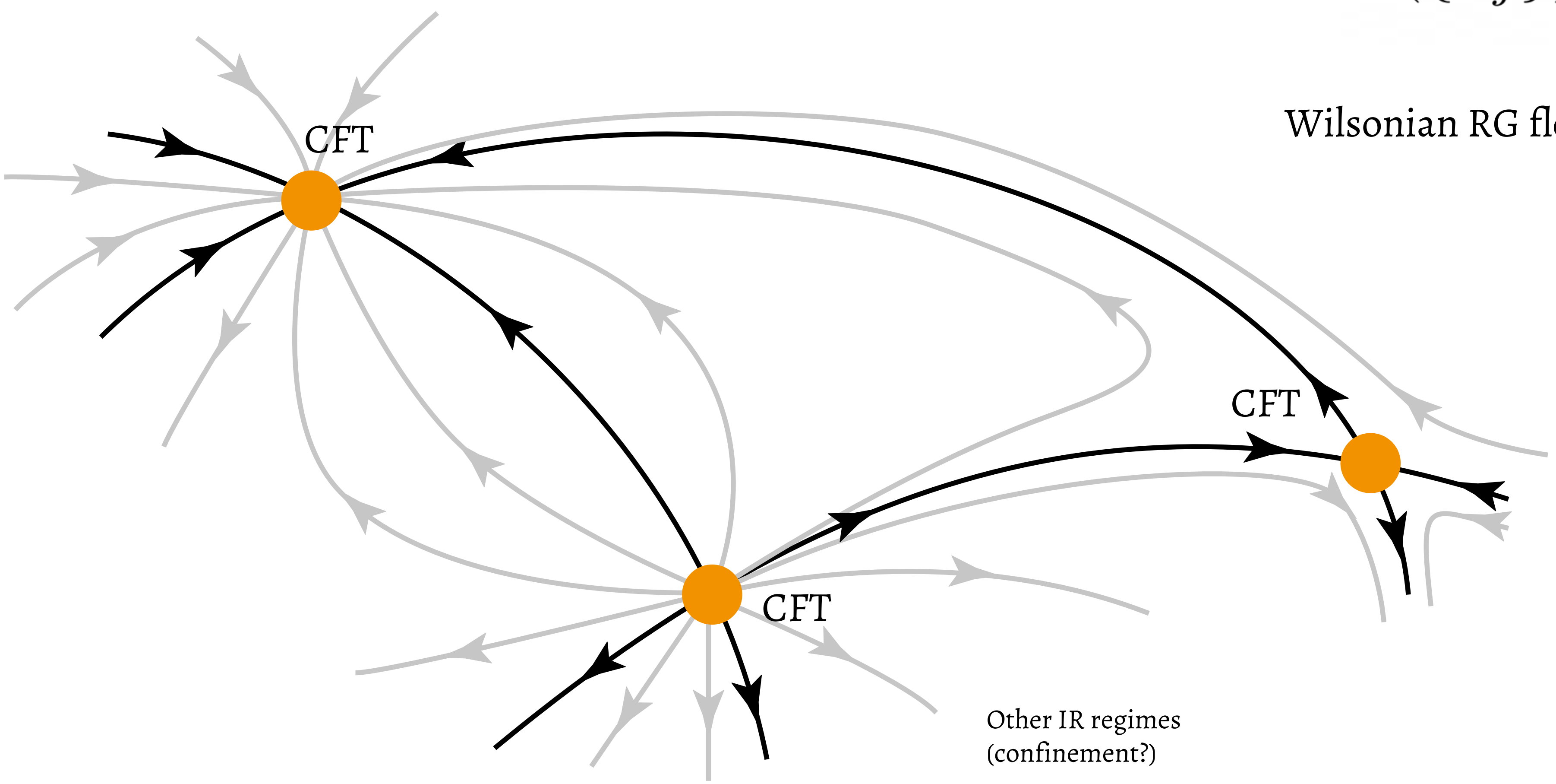


The question: can the proximity of a fixed point foster the appearance of a light state?

Contents

- **Fixed Point Annihilation (FPA)** as a mechanism by which large hierarchies can be produced.
    - Strongly – coupled, holographic realization. Holographic complex CFTs.

- Can this mechanism lead to a light dilaton, in a confining theory?
    - *Spoiler alert: generically not, but...*

- Light dilatons from quantum phase transitions in confining theories.

Fixed Point Annihilation

Fixed Point Annihilation

Fixed Point Annihilation (FPA) is a mechanism by which large hierarchies can be produced.

$$\beta(g) = (\alpha - \alpha_*) - (g - g_*)^2 + O(g - g_*)^3$$

Vanishes at two points:

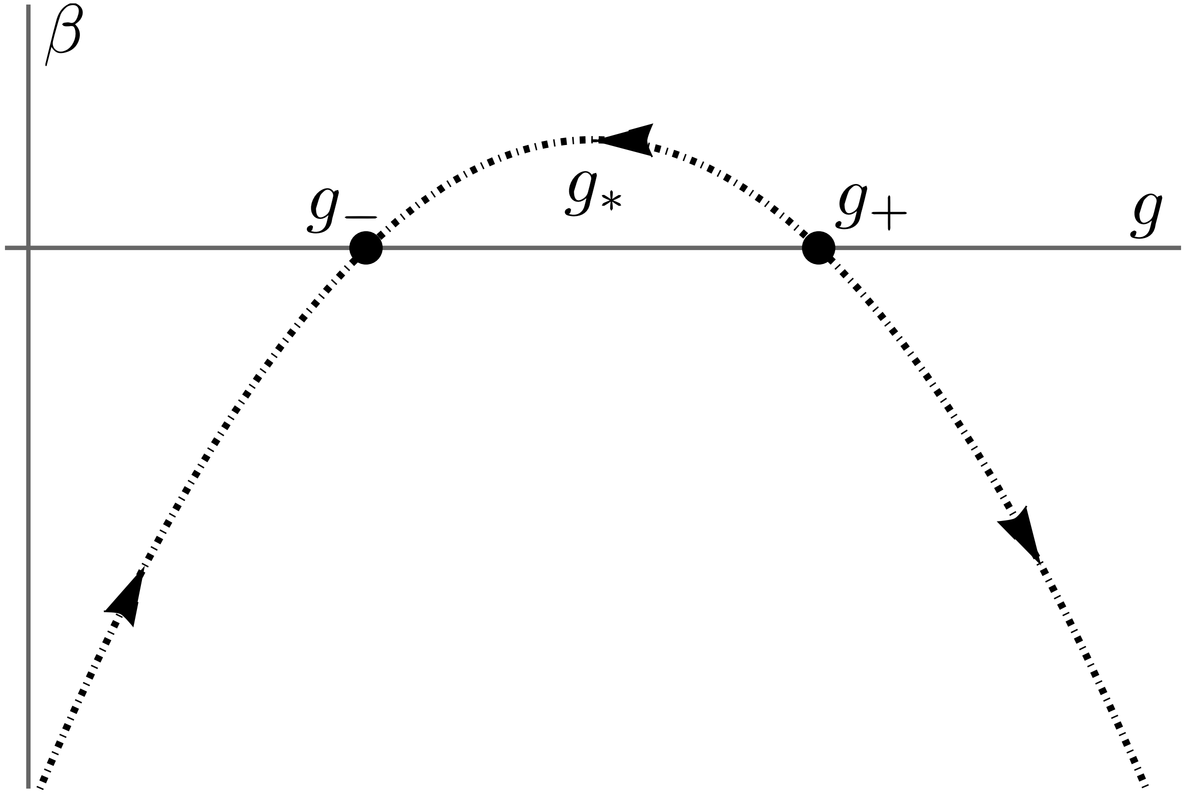
$$g_\pm = g_* \pm \sqrt{\alpha - \alpha_*}$$

We want to understand what happens as the critical value of the parameter is reached.

[*Kaplan, Lee, Son, Stephanov;* **0905.4752**].

Fixed Point Annihilation

$$g_\pm = g_* \pm \sqrt{\alpha - \alpha_*}$$



$$\alpha > \alpha_*$$

Two real fixed points.
A flow between them.

[*Kaplan, Lee, Son, Stephanov;* **0905.4752**].

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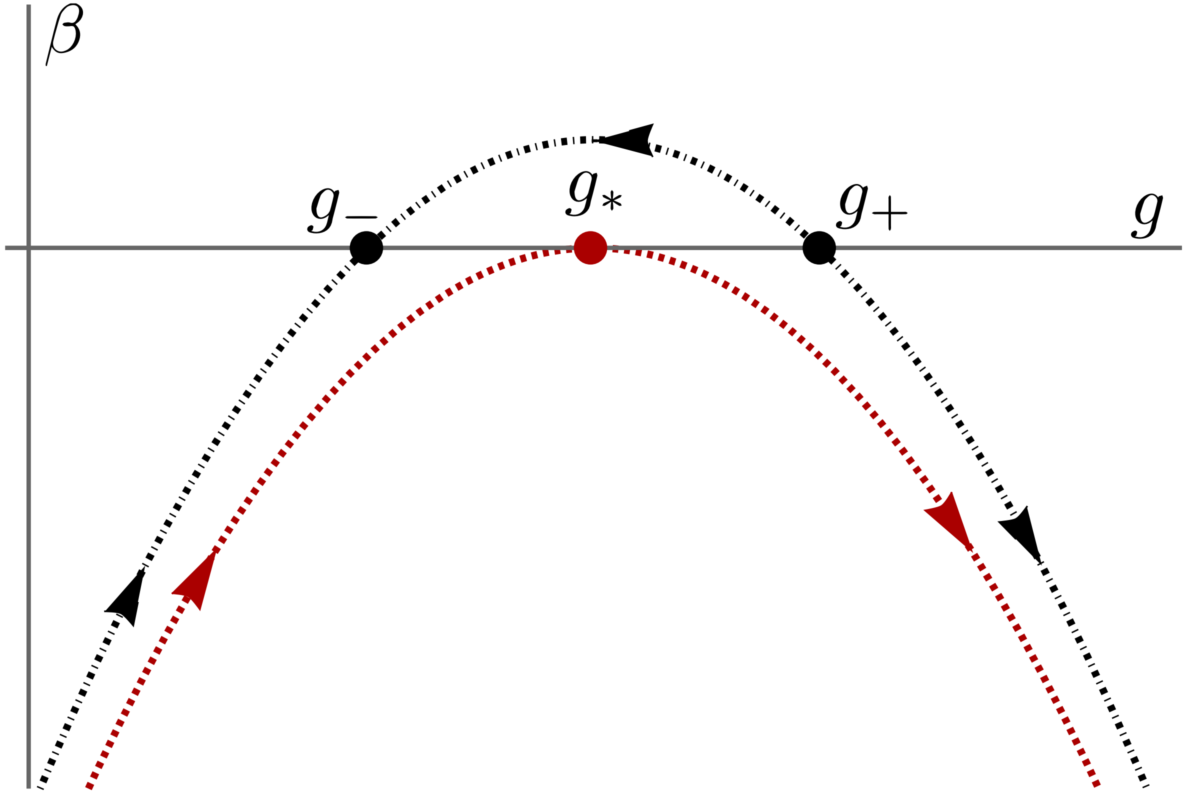

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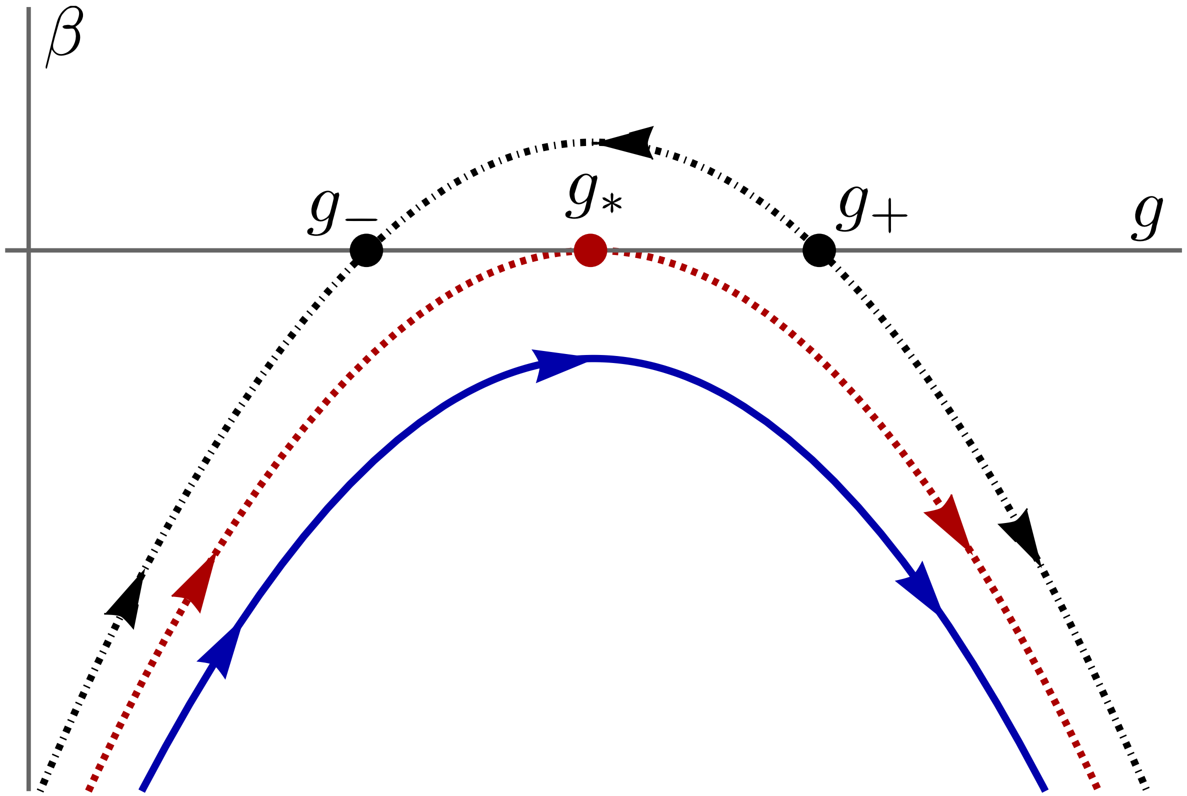

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$$\alpha < \alpha_*$$

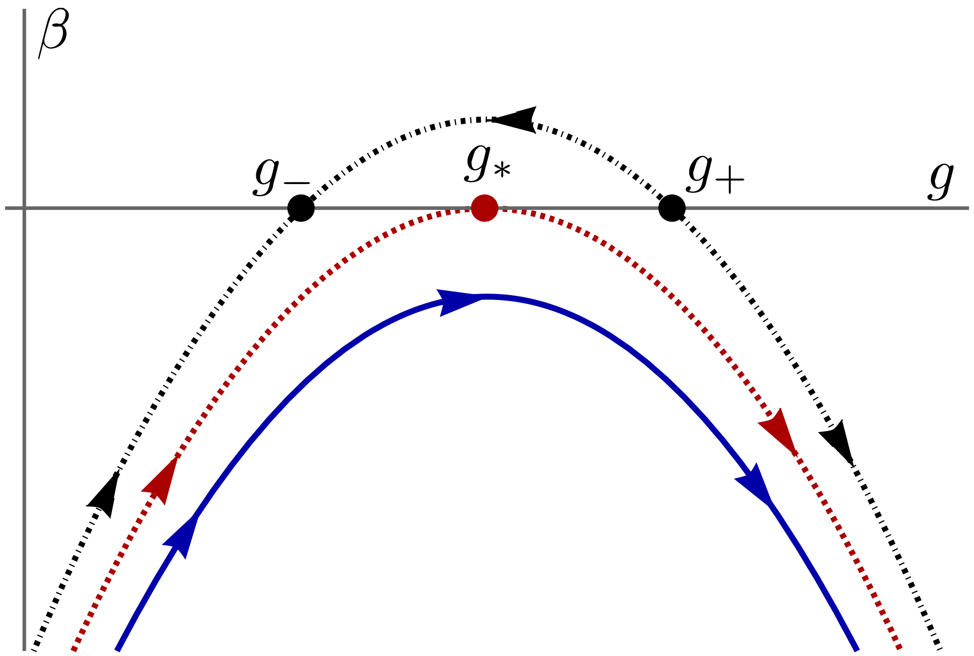
Two complex fixed points

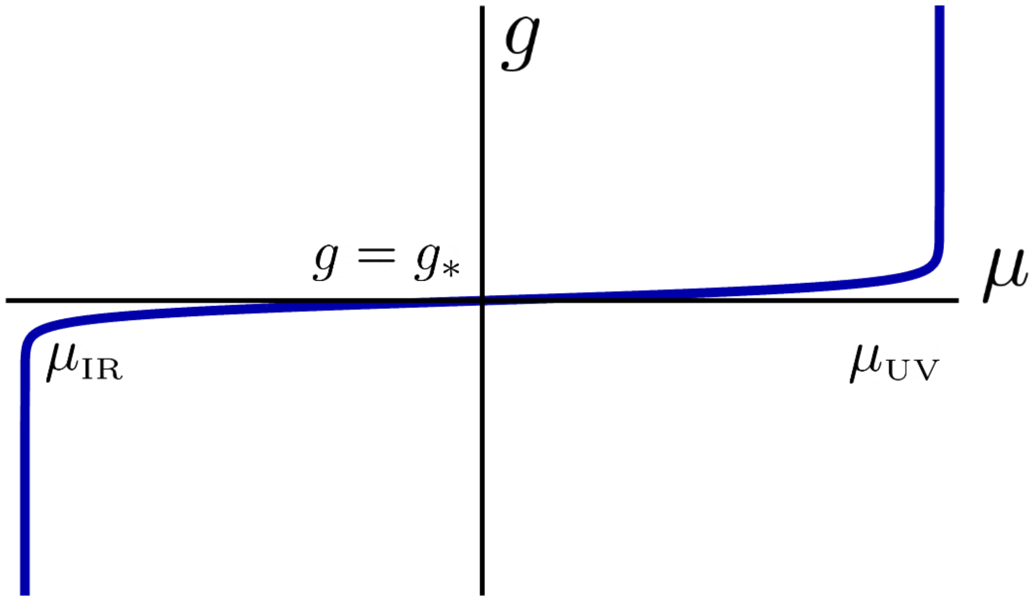
Miranski, BKT scaling:

$$\frac{\mu_{\rm UV}}{\mu_{\rm IR}} = \exp\left(\frac{2\pi}{\sqrt{\alpha_* - \alpha}}\right)$$

[Kaplan, Lee, Son, Stephanov; 0905.4752].

Fixed Point Annihilation





$$\frac{\mu_{\mathrm{UV}}}{\mu_{\mathrm{IR}}} = \exp\left(\frac{2\pi}{\sqrt{\alpha_* - \alpha}}\right)$$

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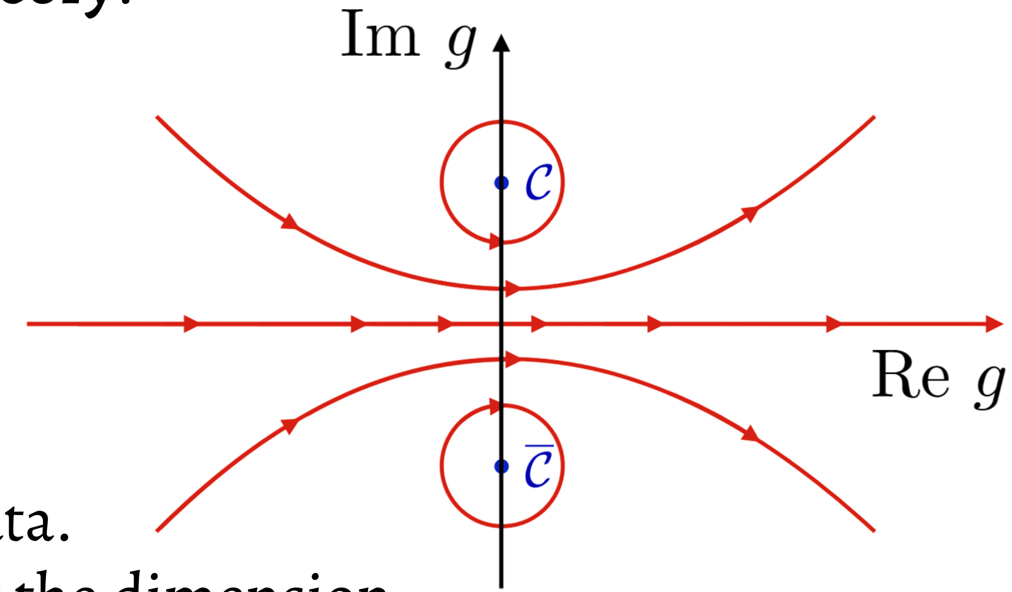
Complex CFTs

It was postulated that we can think of the theory at each Complex Fixed Point as a **Complex Conformal Field Theory**.



- Non-unitary.
- They come in pairs.
- Scaling dimensions in both theories are related by complex conjugation.

Properties of the real flow in terms of cCFT data.
For instance, we can express the scaling using the dimension of the operator associated to g at the cCFT.

$$\frac{\mu_{\mathrm{UV}}}{\mu_{\mathrm{IR}}} = \exp\left(\frac{2\pi}{|\mathrm{Im}\Delta|}\right)$$

[*Gorbenko, Rychkov, Zan;* ***1807.11512*** *&* ***1808.04380***].

Holographic realization

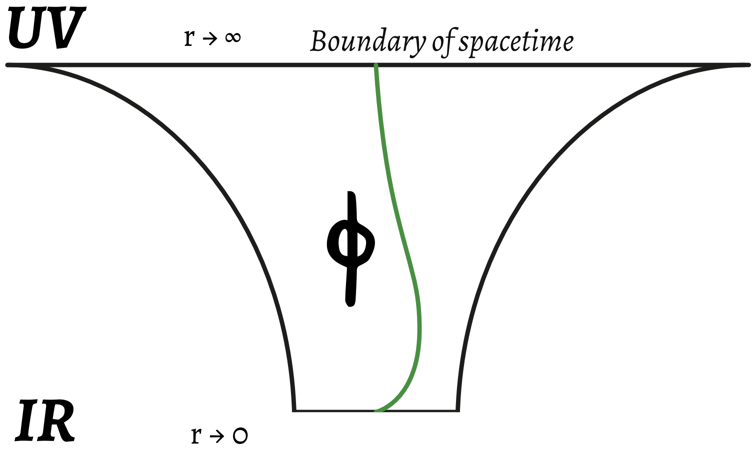
Intermezzo: **Holographic BKT transitions**

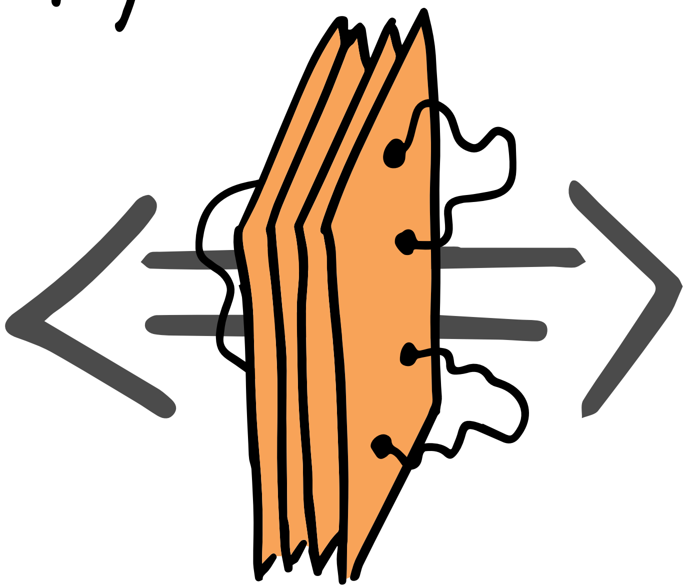
$$S = \frac{1}{2\kappa_5^2}\int d^5x\sqrt{-G}\left(R - 2\Lambda + \dots + (\partial\phi)^2 + V(\phi)\right)$$

$$\phi \simeq \frac{\phi_s}{r^{4-\Delta}} + \frac{\phi_v}{r^{\Delta}} + \dots$$

STRING THEORY

$$S = -\frac{1}{4}\int d^4x\sqrt{-\gamma}\left(F_{\mu\nu}^{a}F^{a\mu\nu} + \dots + M\,\mathcal{O}_\Delta\right)$$





Source and vacuum expectation values related to the fall-off of the scalar field near the boundary.

Scaling dimension related to the mass. For us, $\Delta = 3$ always.

Scalar field dual to a coupling of the QFT.

Intermezzo: **Holographic BKT transitions**

Historically, the holographic dual of BKT transitions has been identified with models where a field violates the BF bound.

- This is already stated in [*Kaplan, Lee, Son, Stephanov;* ***0905.4752***].

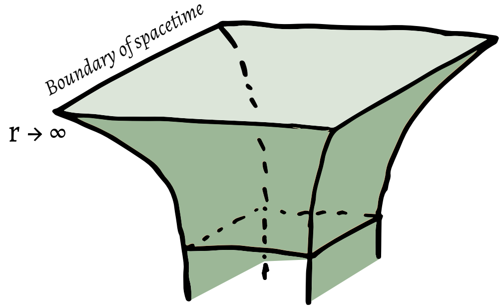
- First instance in a top-down model [*Jensen, Karch, Son, Thompson,* ***1002.3159***] in the D3/D5 system with magnetic field
  (shortly after, realized that it is destroyed at finite T, [*Evans, Gebauer, Kim & Magou;* ***1003.2694***]).

- Quickly followed by [*Jensen* ***1006.3066***], ABJM with flavour and flavored (1, 1) little string theory.

Besides, we propose another mechanism.

Holographic complex CFTs

$$S = \frac{1}{2\kappa^2}\int d^5x\sqrt{-g}\left(R - \frac{1}{2}(\partial\phi)^2 - V(\phi)\right)$$



Our proposal:
- At the zeros of the beta function, we have a CFT.

[*Faedo, Hoyos, Mateos, **JS**; 1909.04008*].

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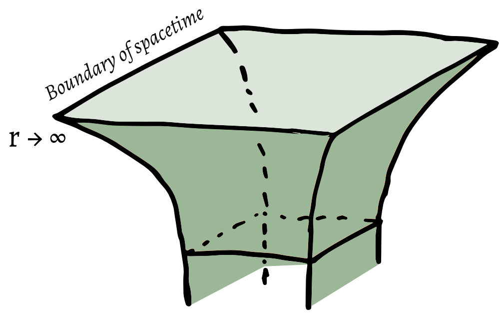

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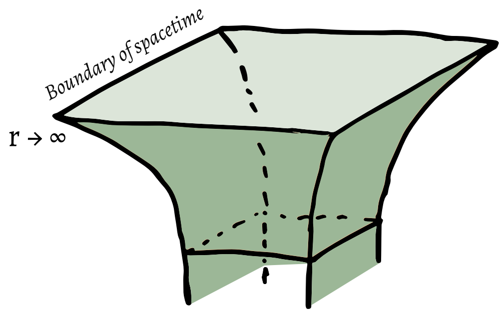

Our proposal:

- At the zeros of the beta function, we have a CFT.
- In the gravity side, AdS solution.
- AdS solutions found at the extrema of the potential.

We choose a potential such that:

$$V'(\phi) \;\propto\; \phi\,(\phi - \phi_0)\,(\phi - \overline{\phi}_0)$$

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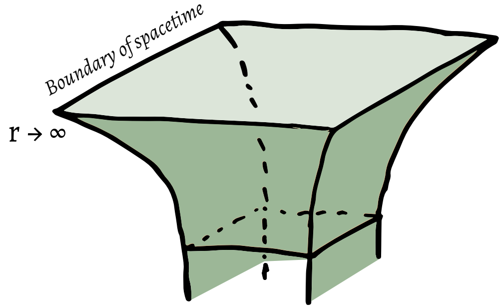

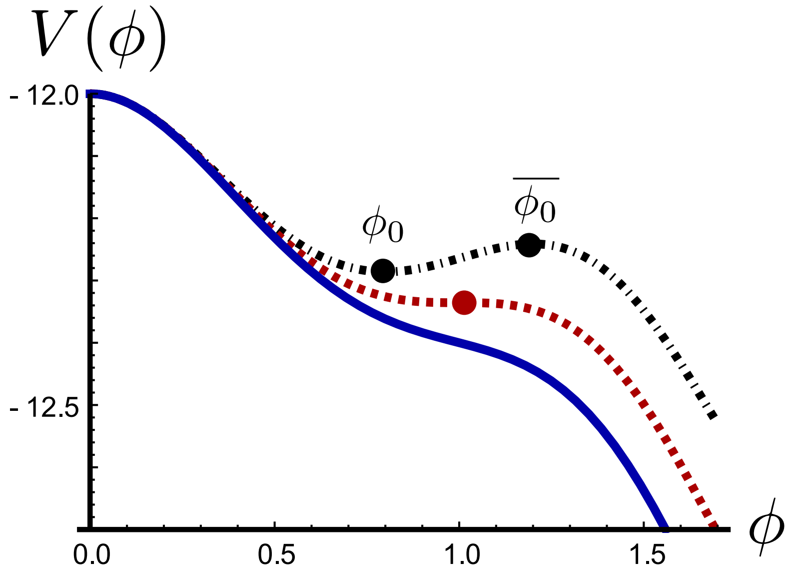

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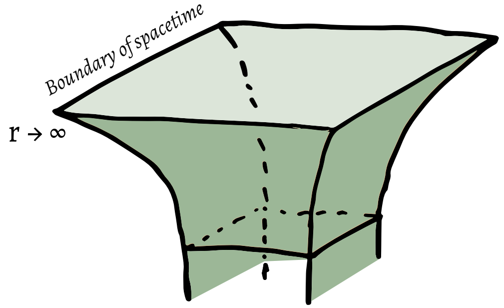

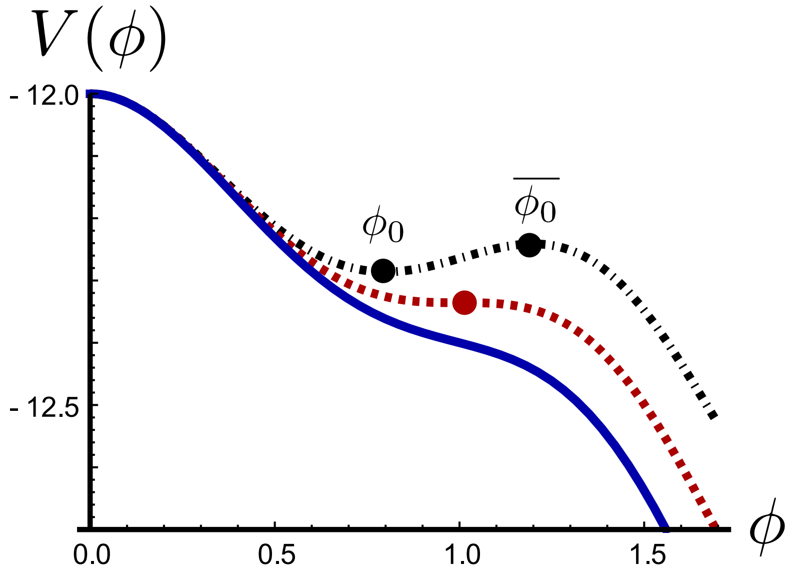

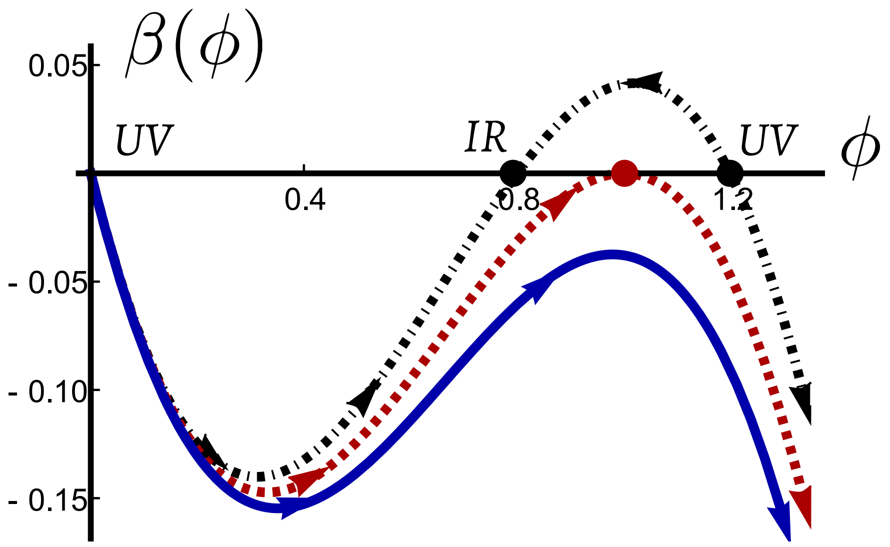

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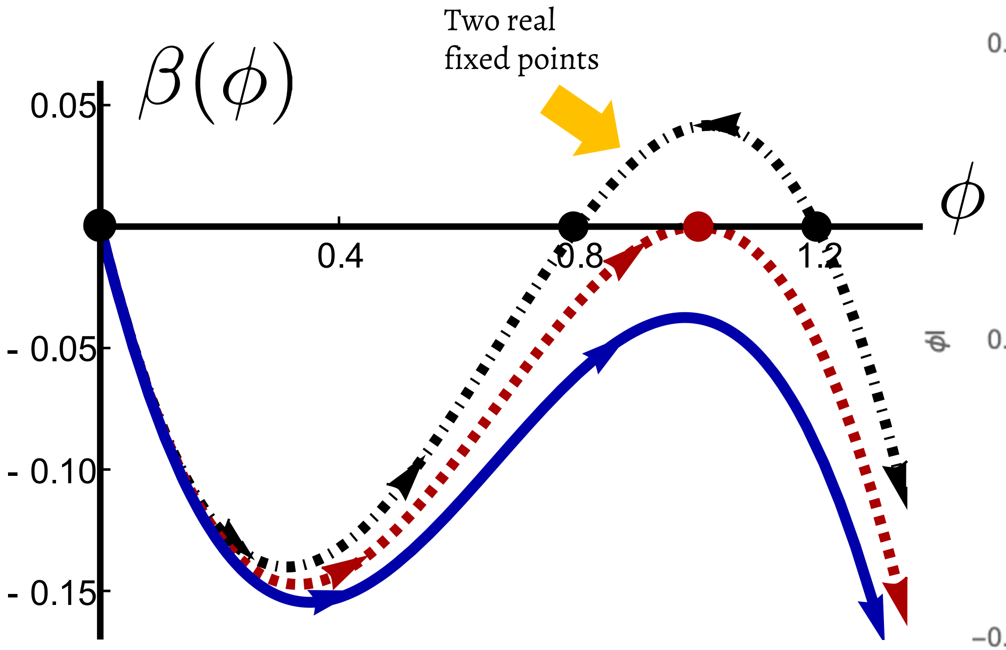

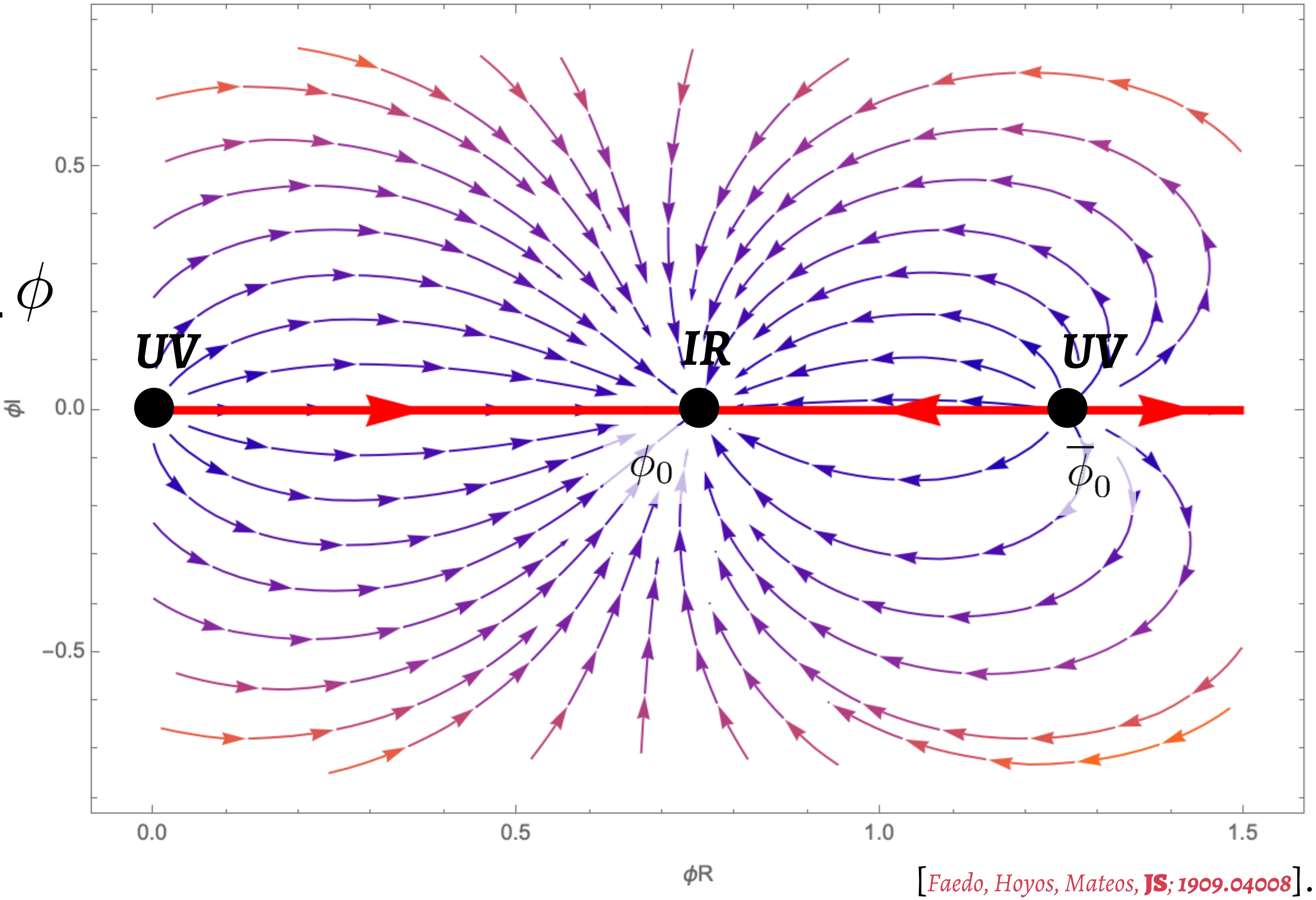

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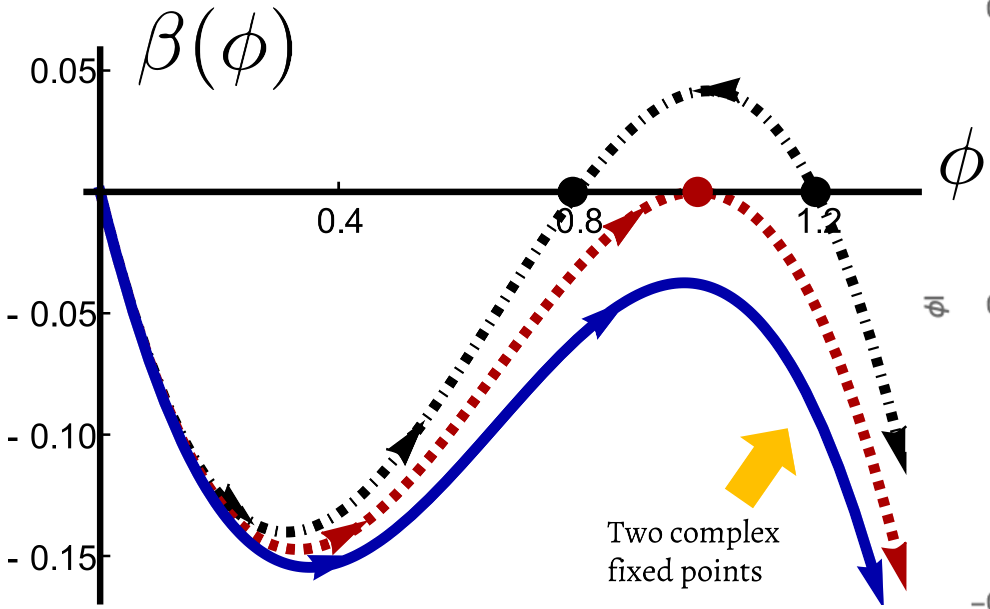

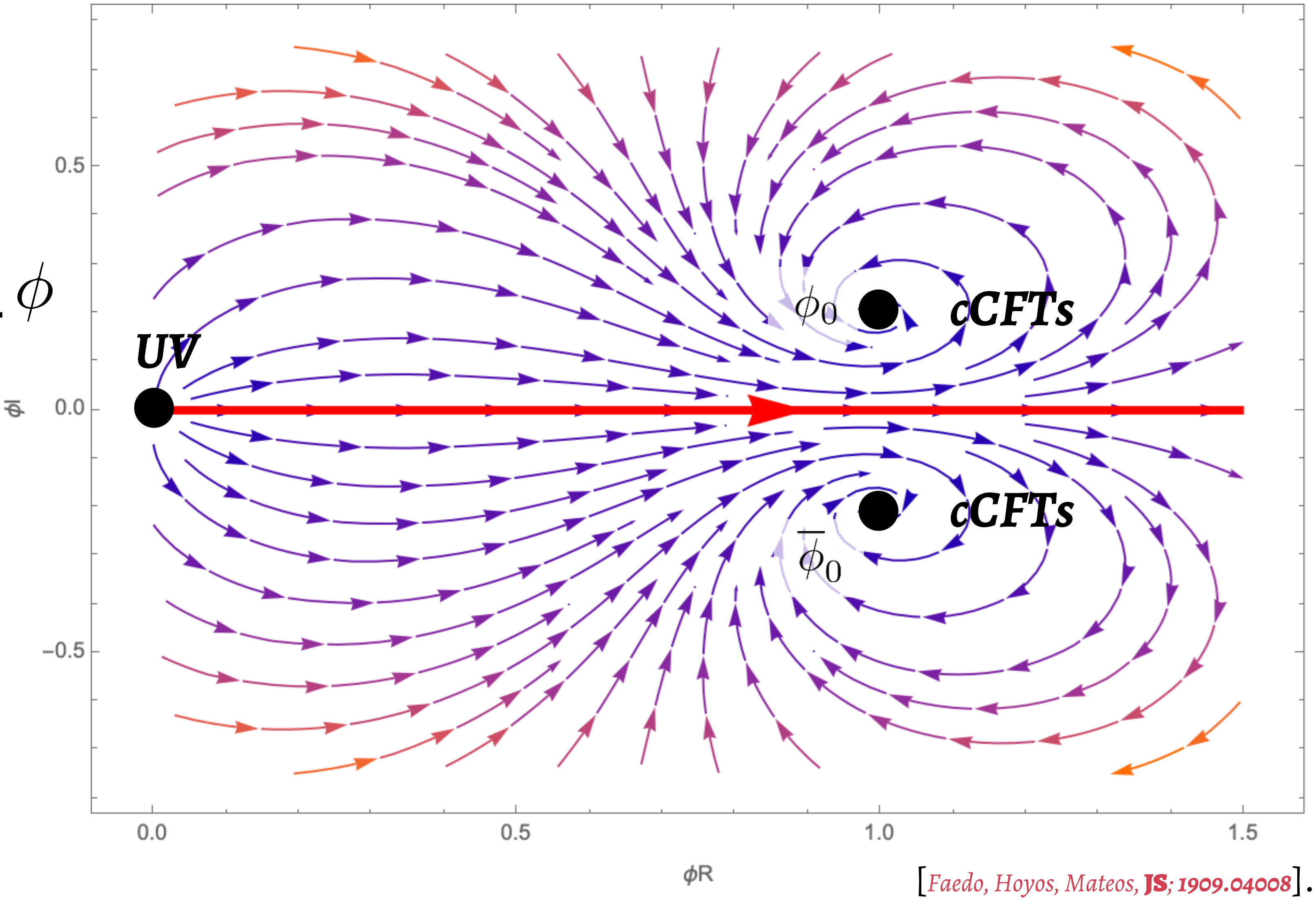

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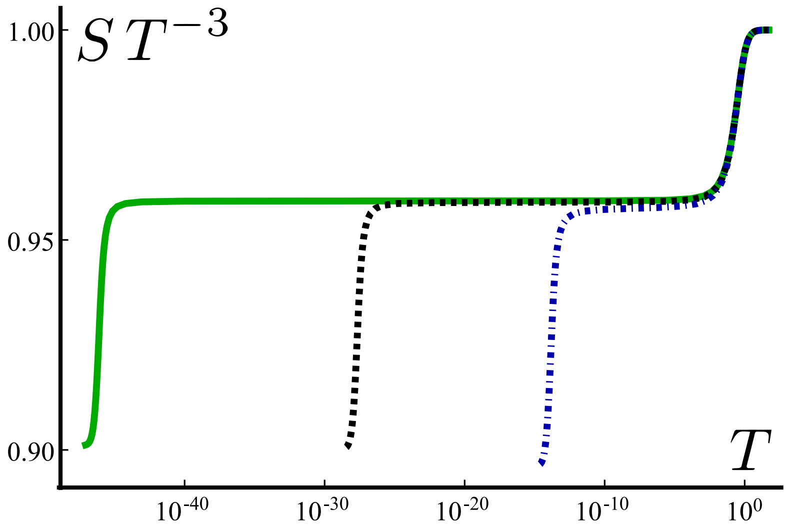

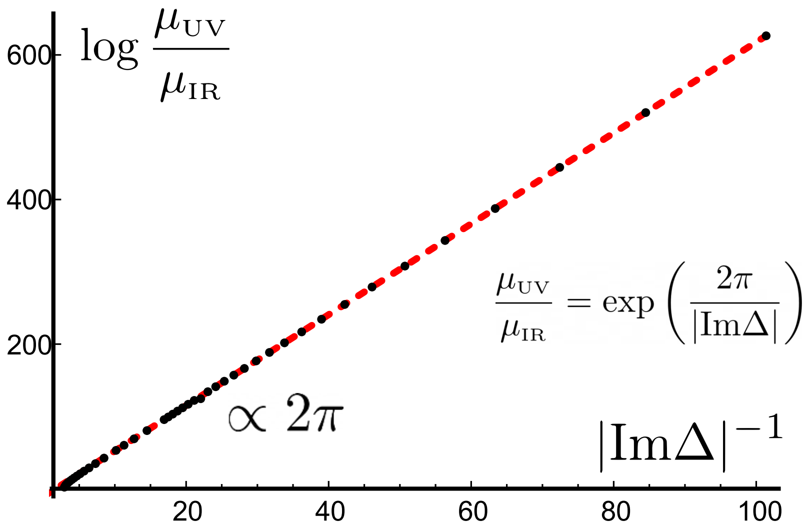

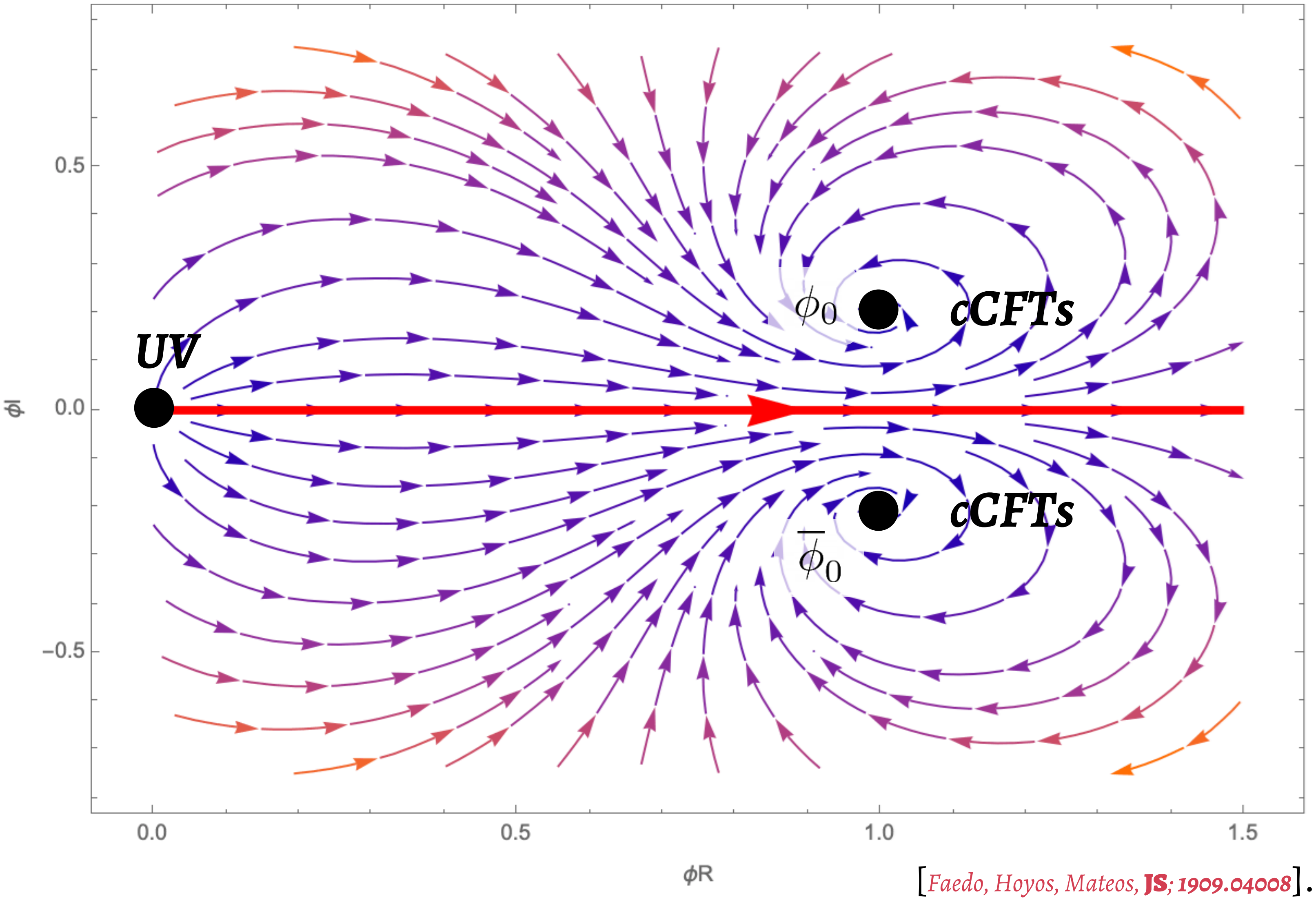

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Light dilatons from cCFTs?

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$$S = \frac{1}{2\kappa^2} \int d^5x \sqrt{-g} \left(R - \frac{1}{2}(\partial\phi)^2 - V(\phi) \right)$$

We now want to consider a *confining* (gapped) theory.
Again, with an appropriate engineering of the potential.

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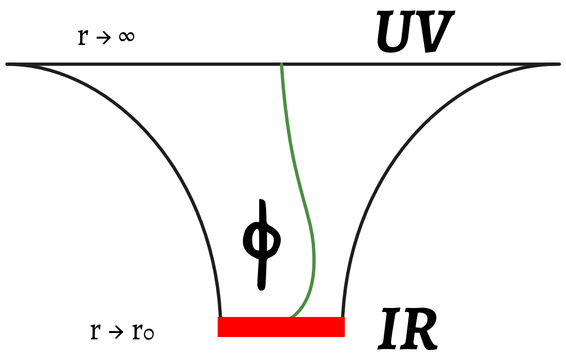
$$V'(\phi) \;\propto\; \frac{M^2}{\alpha} \left(\frac{\phi^2}{\phi_c^2} - 1 \right) \left(\frac{\phi^2}{\phi_c^{*2}} - 1 \right) \sinh(\alpha\phi)$$

The extra piece, with a convenient choice of α, causes confinement (gap).
A singularity cuts off the geometry (IR scale).



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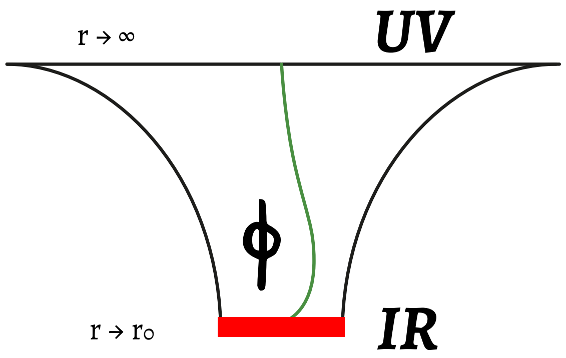
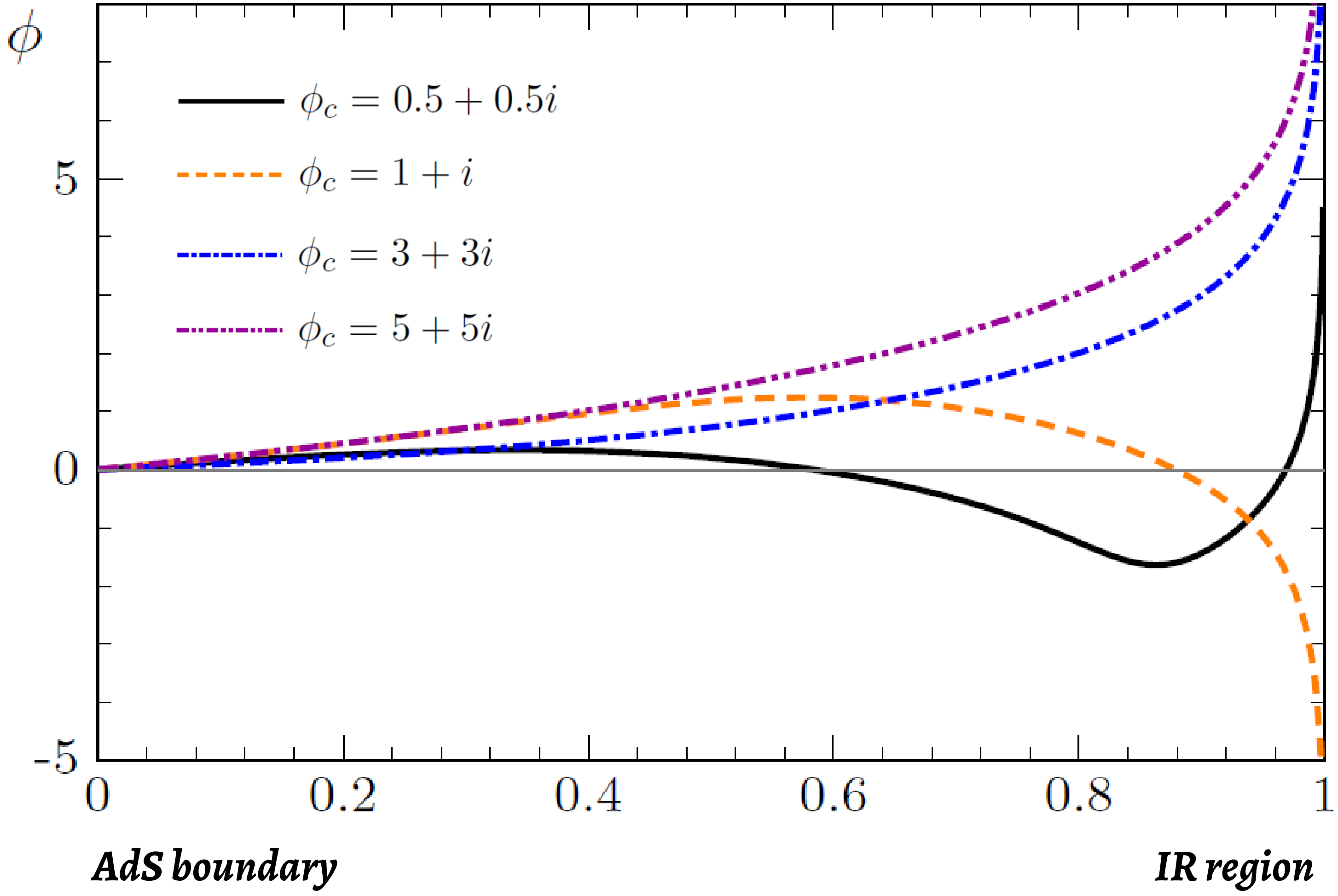
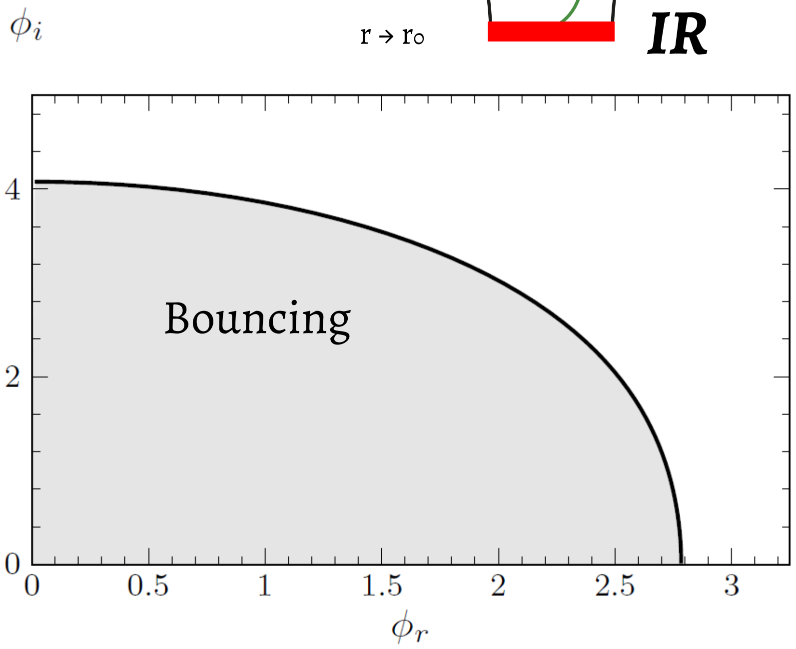
A singularity cuts off the geometry (IR scale). We fix α to one of such values.

Light dilatons from cCFTs?





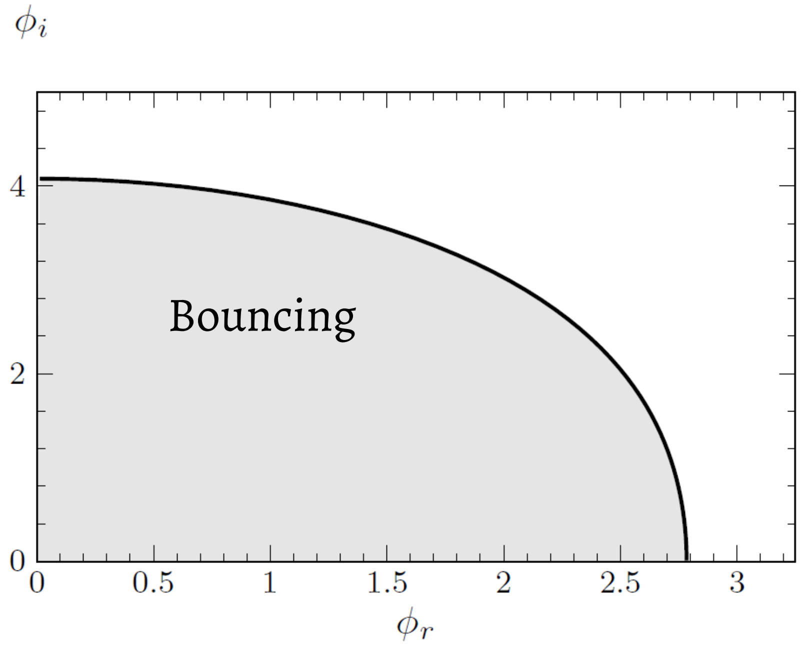


Spectrum of scalar particles

$$g_{\mu\nu} \rightarrow g_{\mu\nu} + \delta g_{\mu\nu}$$

$$\Phi \rightarrow \Phi + \delta\Phi$$

The spectrum of particles of the theory corresponds to linear perturbations of the fields in the gravity side.



[*Witten;* ***hep-th/9803131***].
[*Brower, Mathur, Tan;* ***hep-th/0003115***], [*Wen, Yang;* ***hep-th/0404152***], [*Kuperstein, Sonnenschein;* ***hep-th/0411009***].
[*Bianchi, Prisco, Mueck;* ***hep-th/0310129, hep-th/0507285 & hep-th/0612224***], [*Elander;* ***0912.1600***], [*Elander, Piai;* ***1010.1964***].

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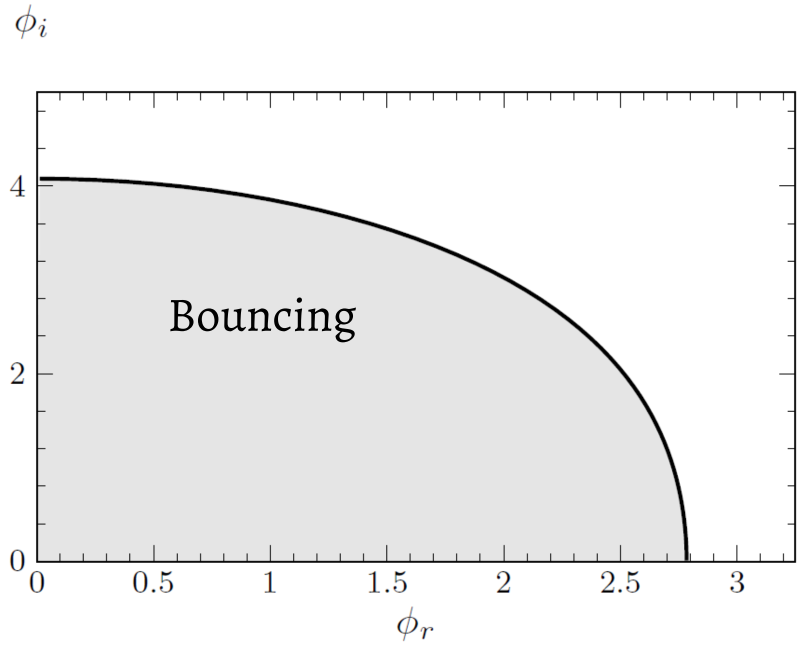
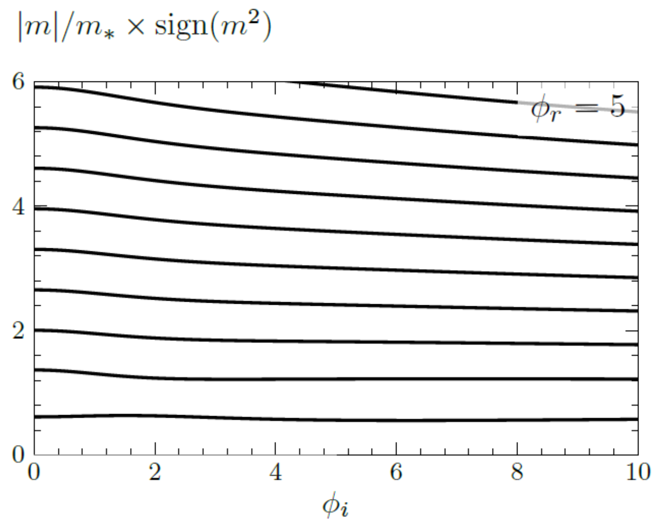
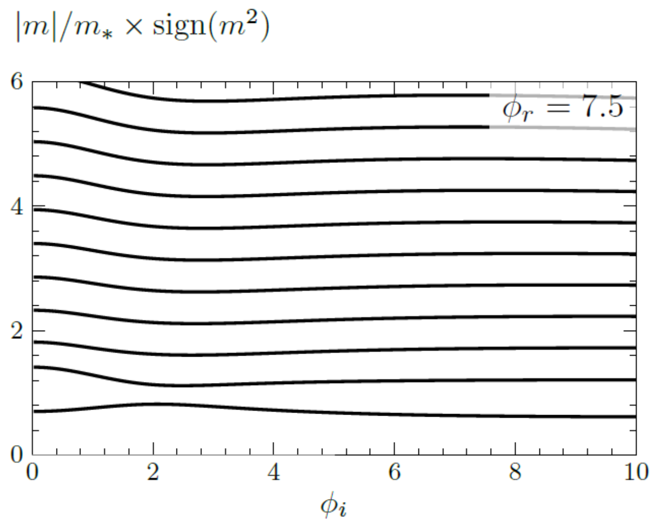
The spectrum of particles of the theory corresponds to linear perturbations of the fields in the gravity side.
Nothing surprising happens for big values of the real part of ϕ_c.







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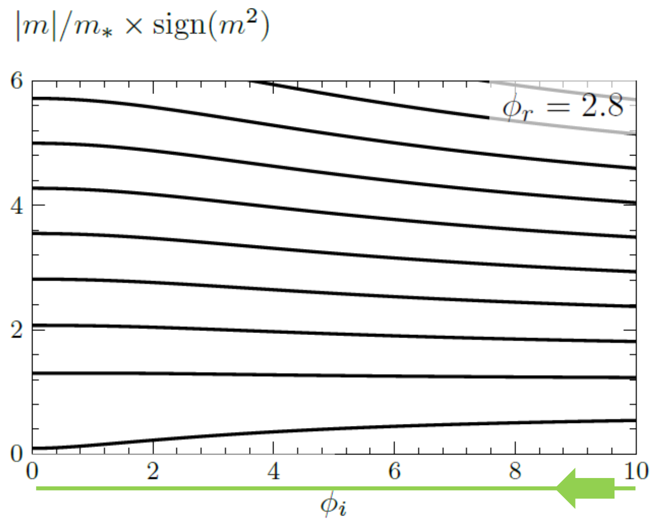
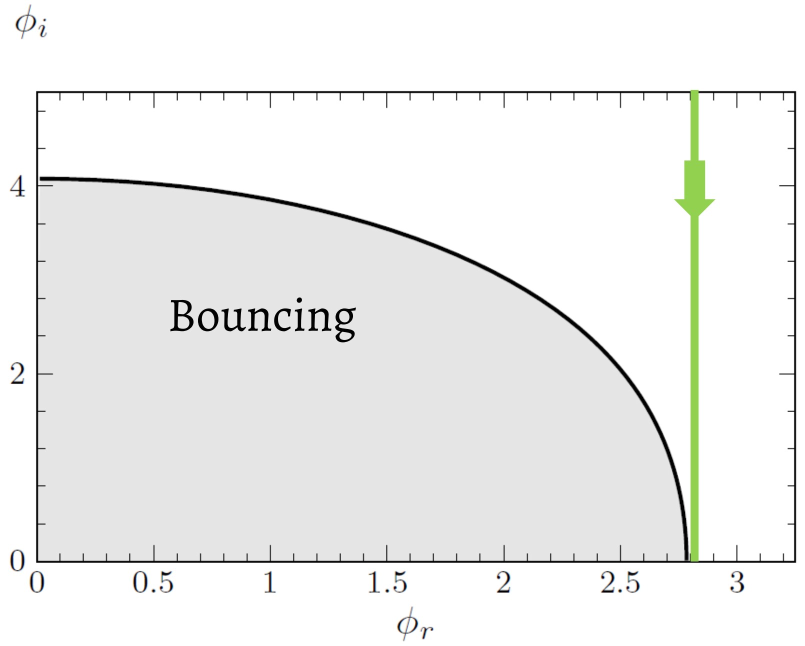
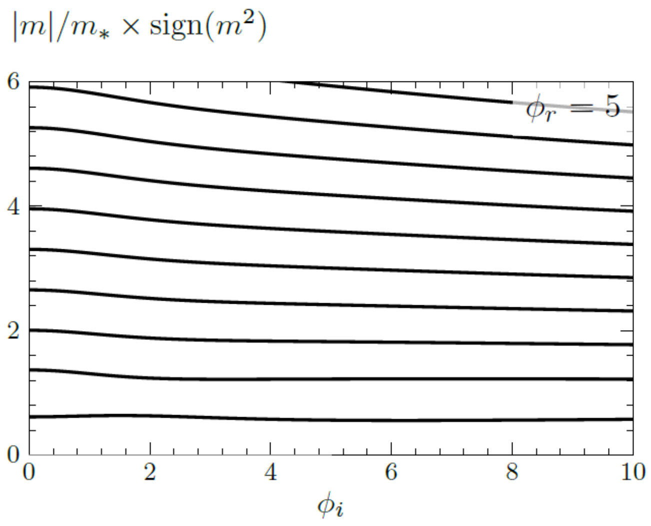
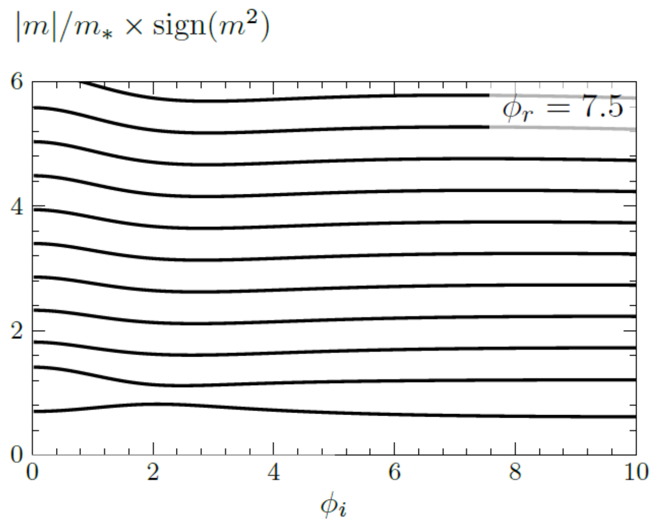


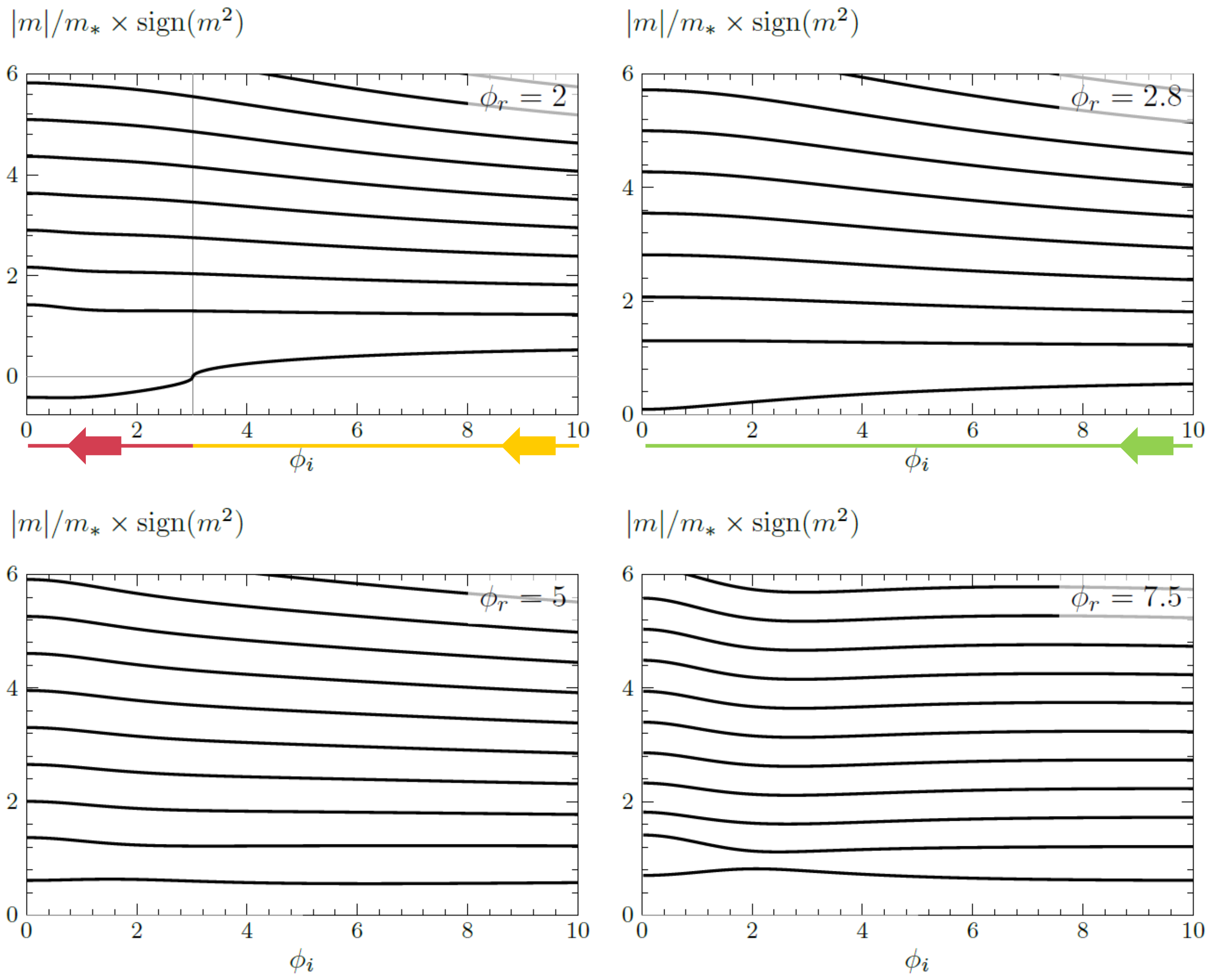


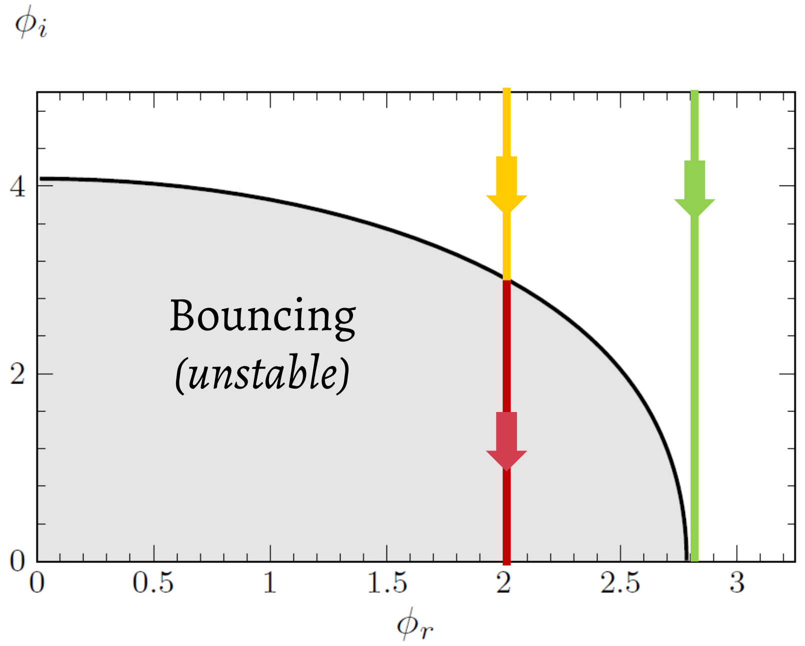




A light state is found in the vicinity of the boundary of the bouncing region.

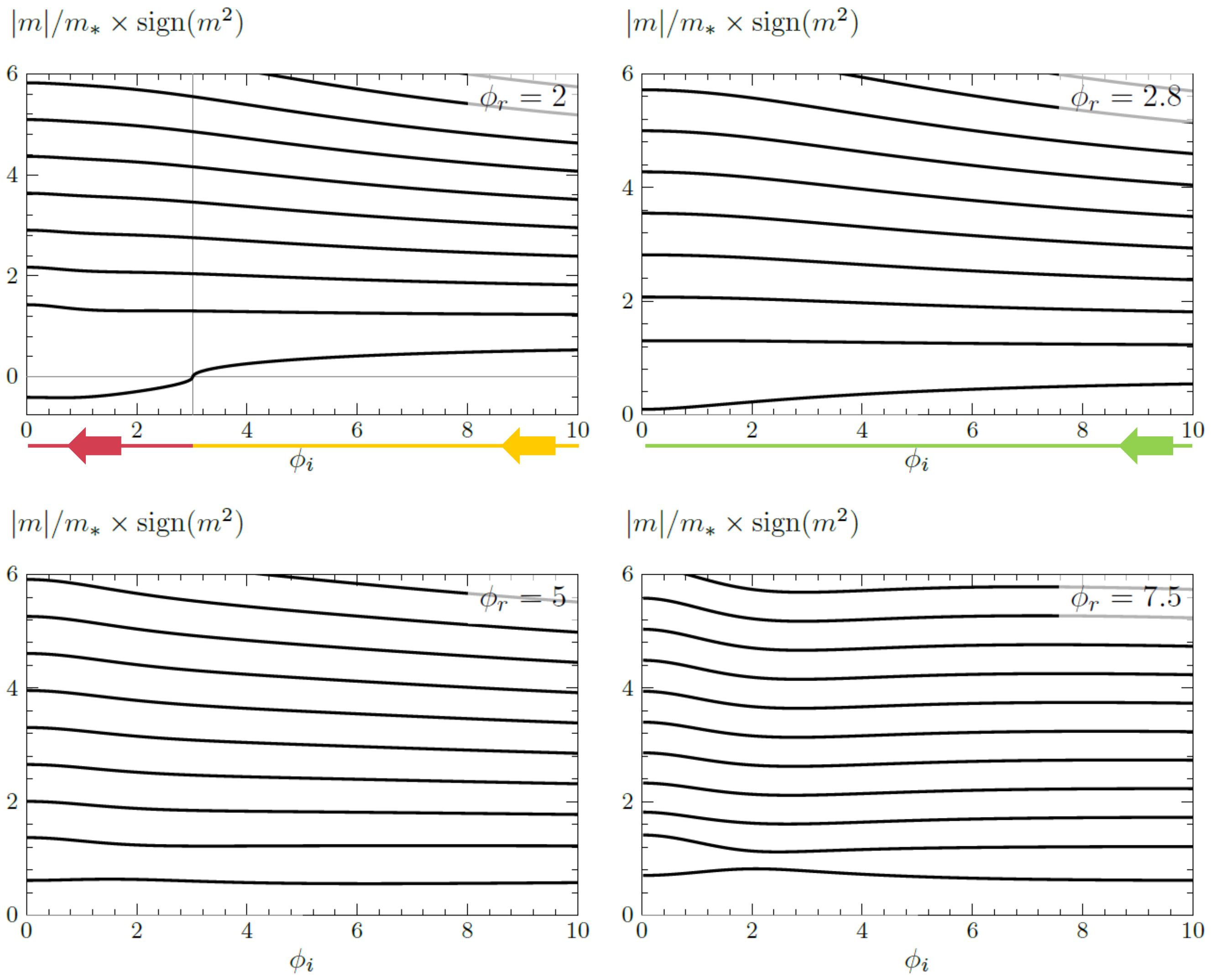
Spectrum of scalar particles

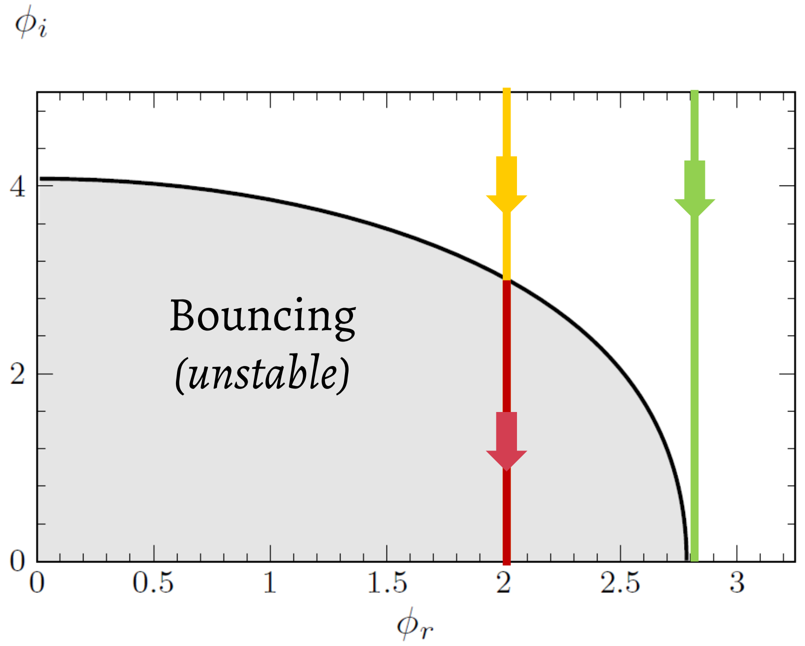




An instability is found for the bouncing solutions (a tachyon signals that the ground state is unstable).

Spectrum of scalar particles





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Light state does not seem a consequence of BKT scaling. (Quantum) phase transition?

Light dilatons from (quantum) phase transitions

[*Faedo, Hoyos, Piai, Rodgers,* **JS**; **2406.04974**].

Phase transitions in confining theories

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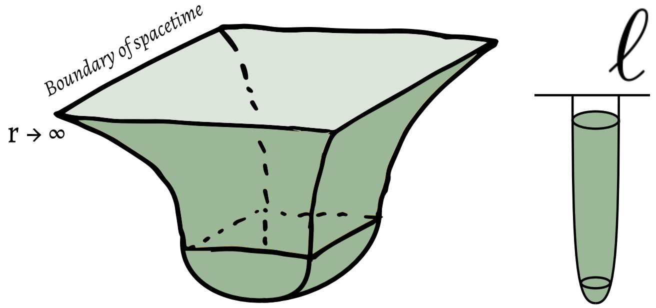

Confinement is obtained by forcing one of the gauge directions to collapse smoothly. This introduces the confining scale ℓ.

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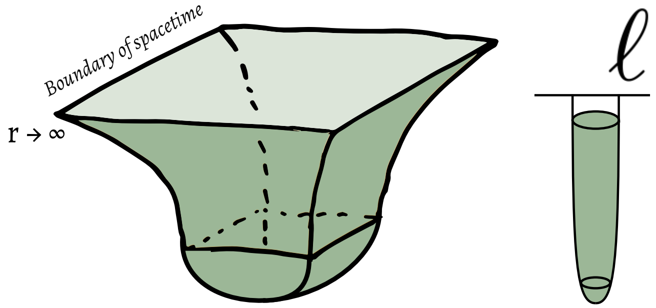

$$V(\phi) = \mathrm{P}[\phi_{\mathrm{M}}](\phi)$$

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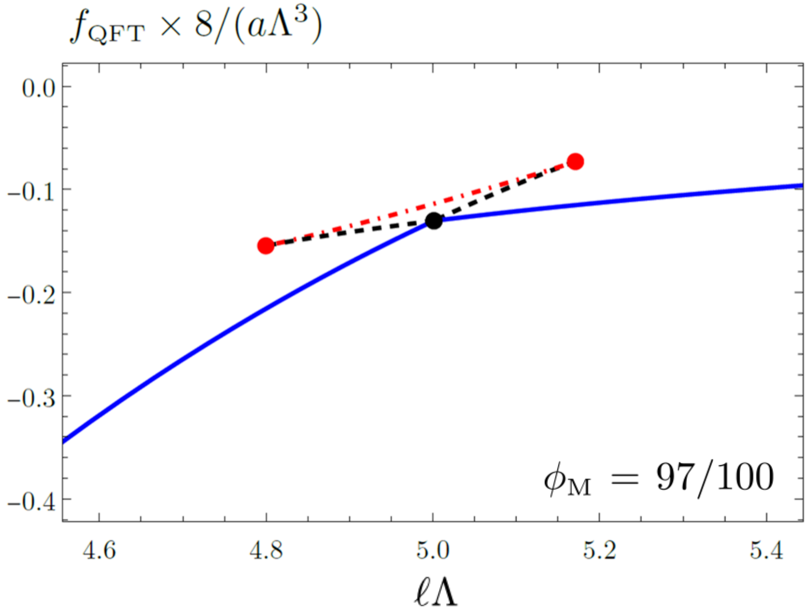
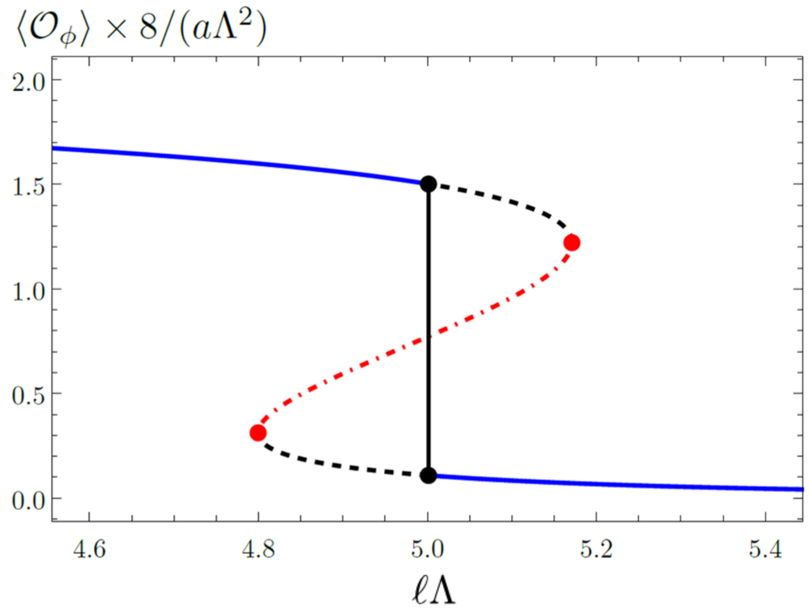
Now we choose a (simple, polynomial) potential that reproduces the expected phase structure in terms of the parameters ϕ_{M} and ℓ.

[*Bea, Mateos;* ***1805.01806***].

Phase transitions in confining theories

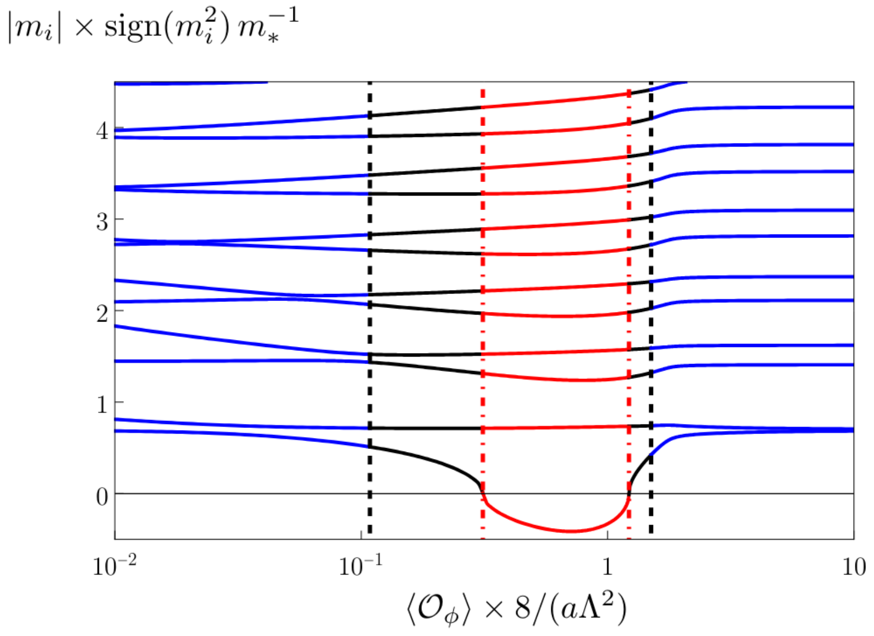


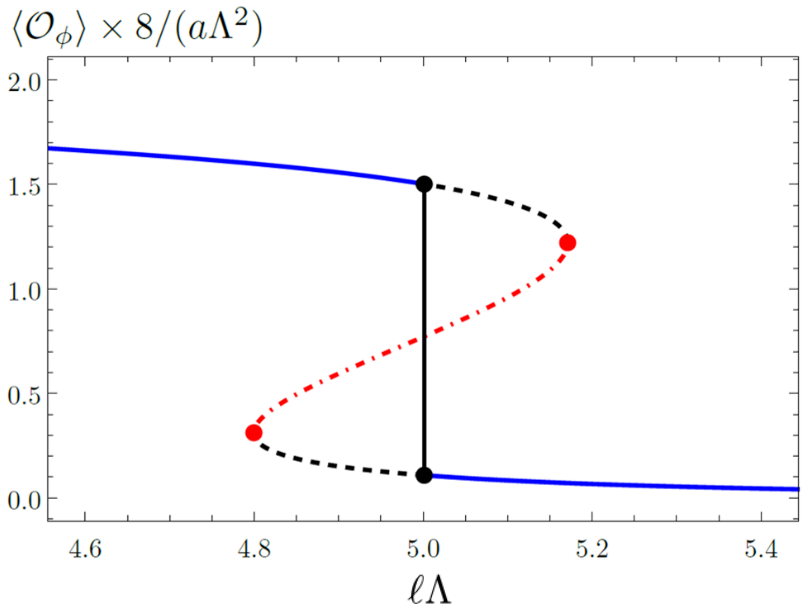


Confinement is obtained by forcing one of the gauge directions to collapse smoothly. This introduces the confining scale ℓ.

Now we choose a (simple, polynomial) potential that reproduces the expected phase structure in terms of the parameters $\phi_{\rm M}$ and ℓ.

Phase transitions in confining theories





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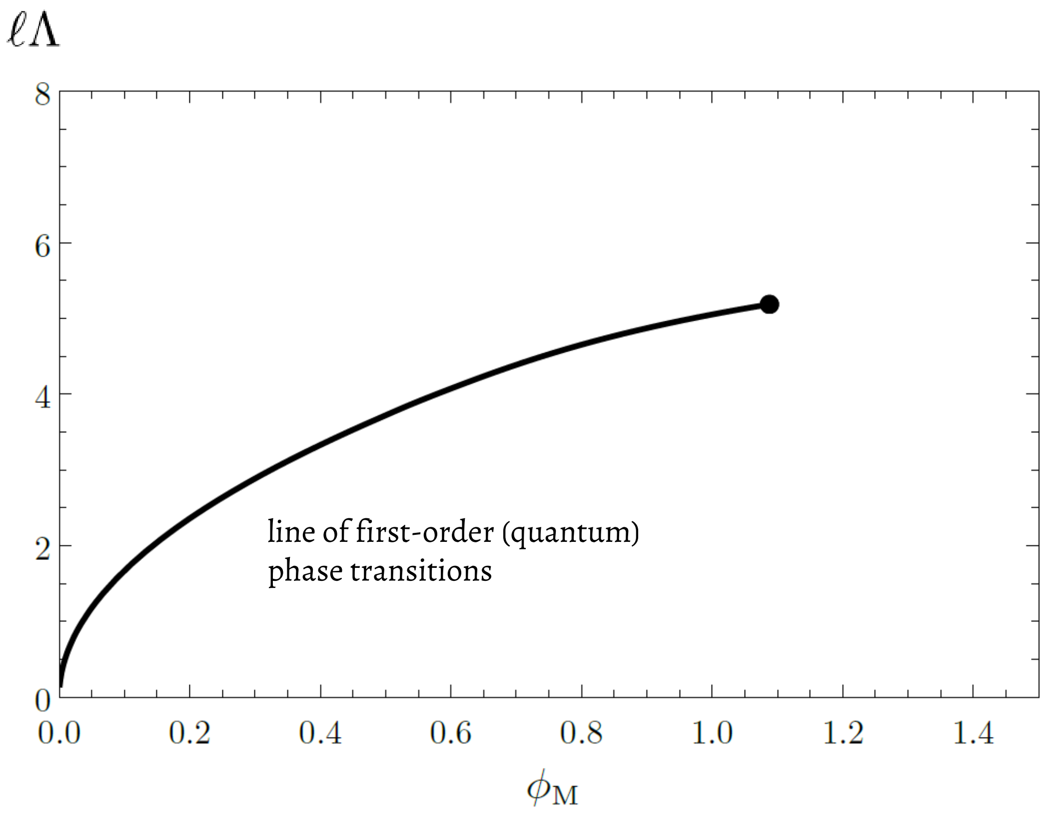
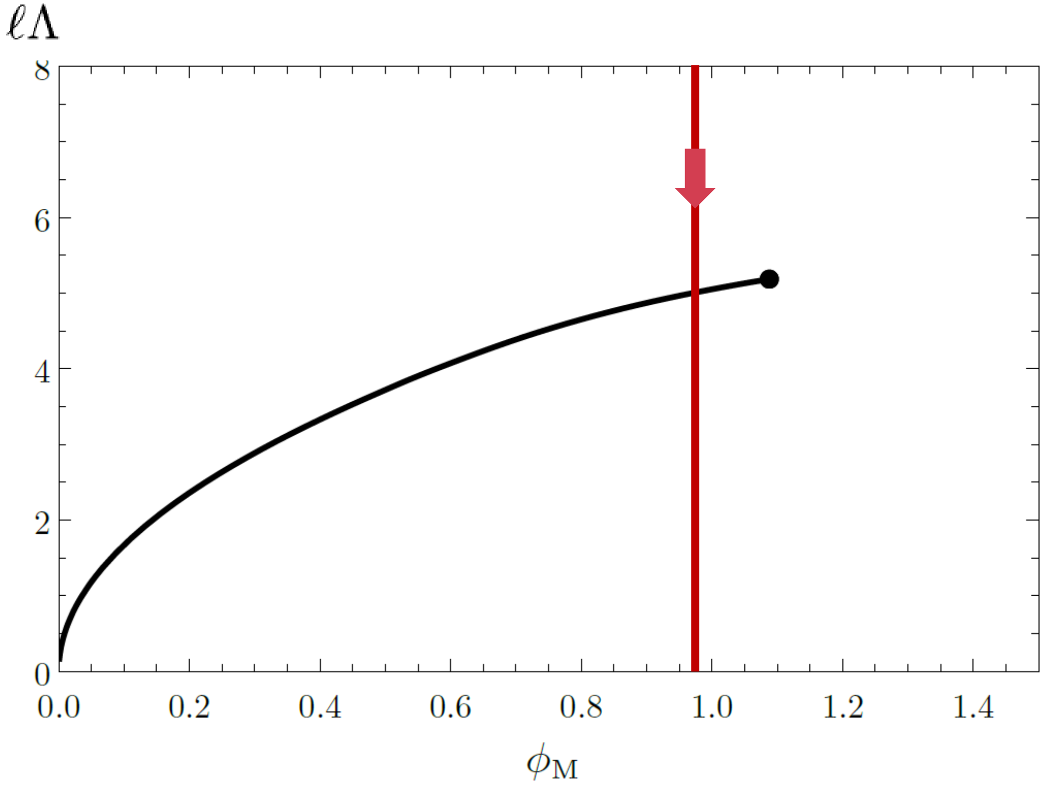
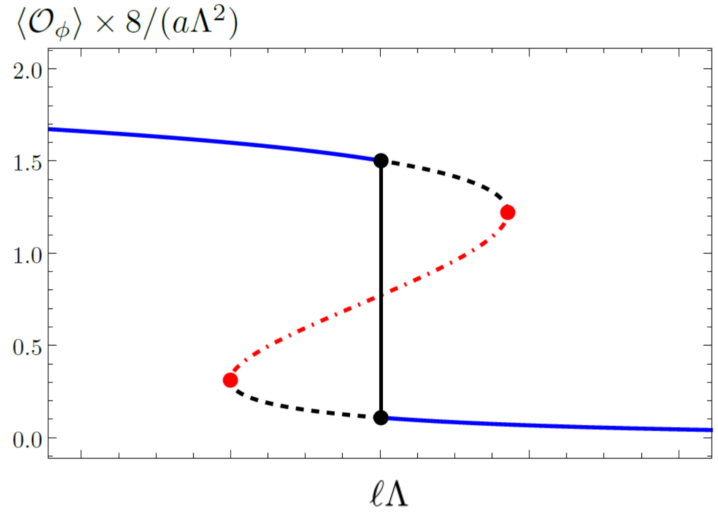
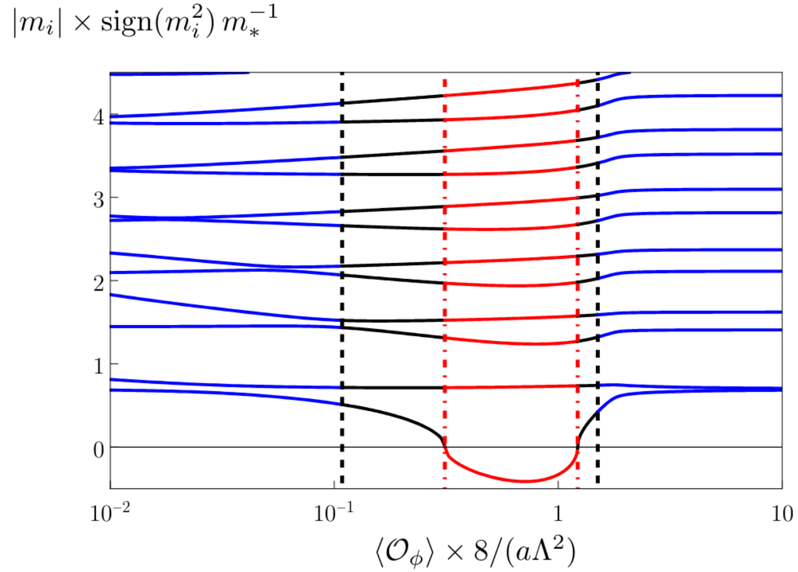


Exploring the parameters space, we discover a critical point, where a second-order (quantum) phase transition takes place.

Phase transitions in confining theories

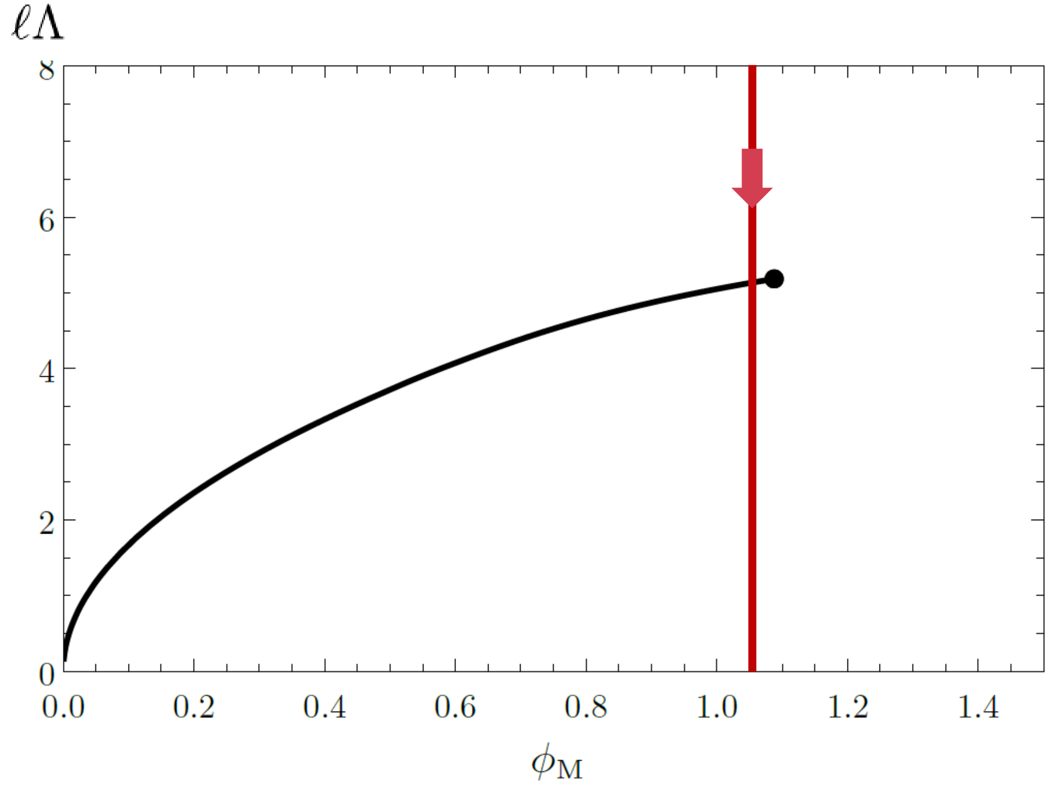


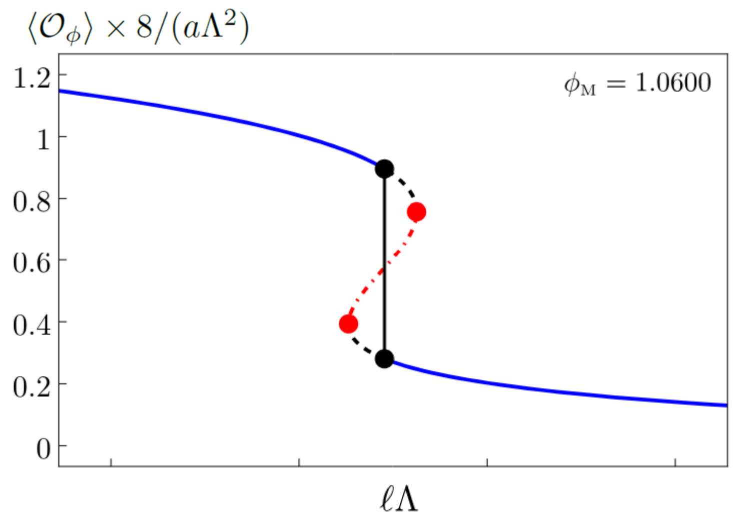


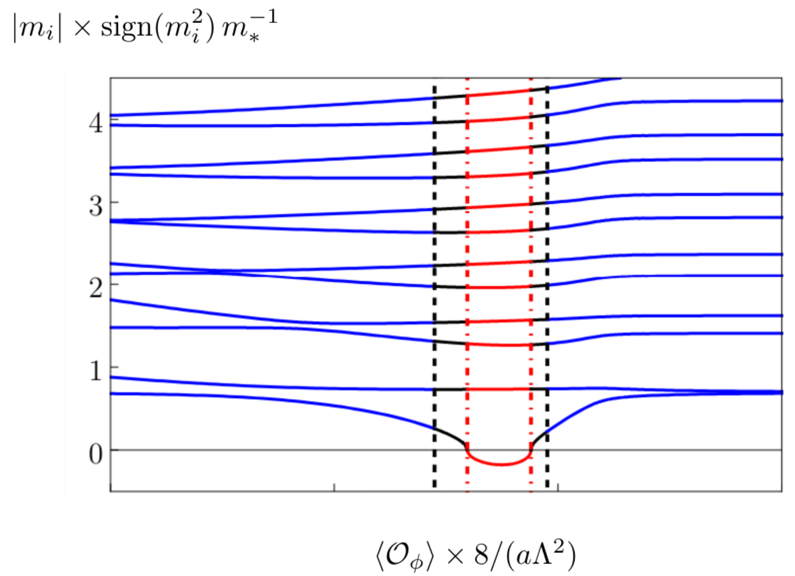


What happens as we come close to the critical point?

Phase transitions in confining theories







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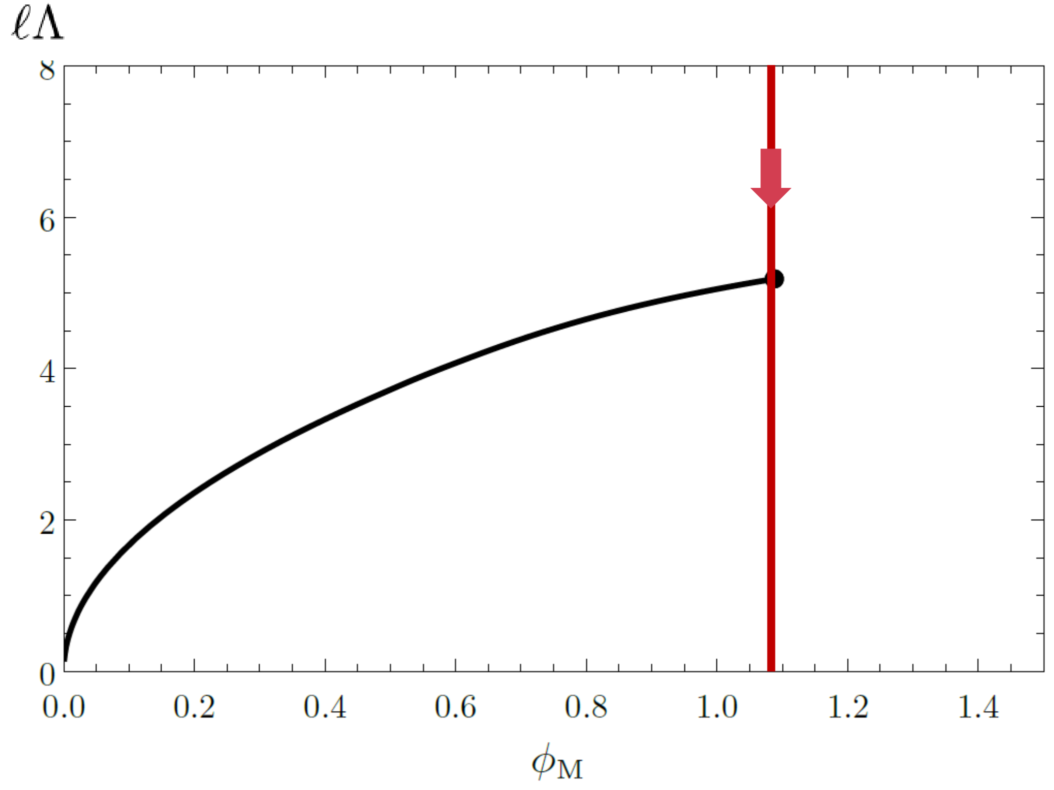

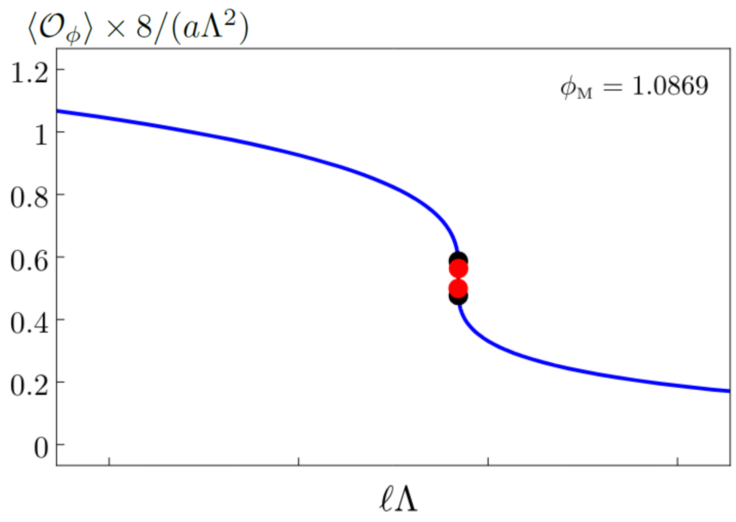

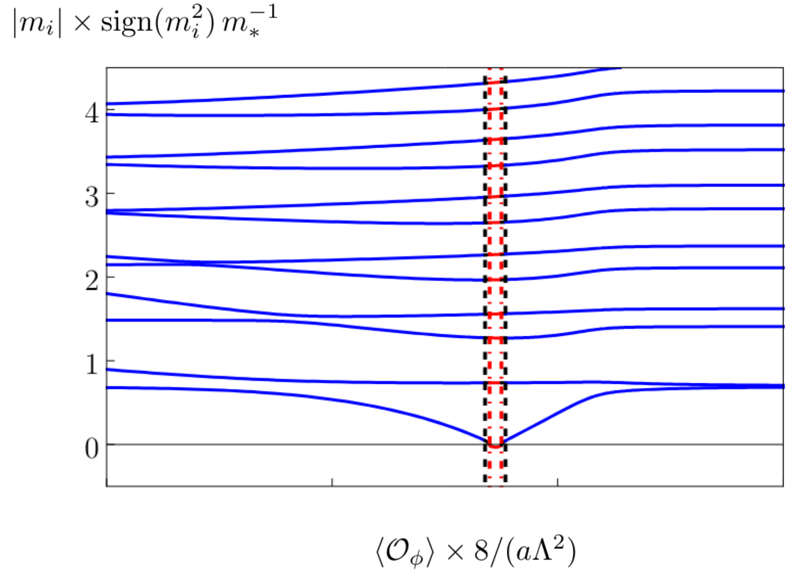

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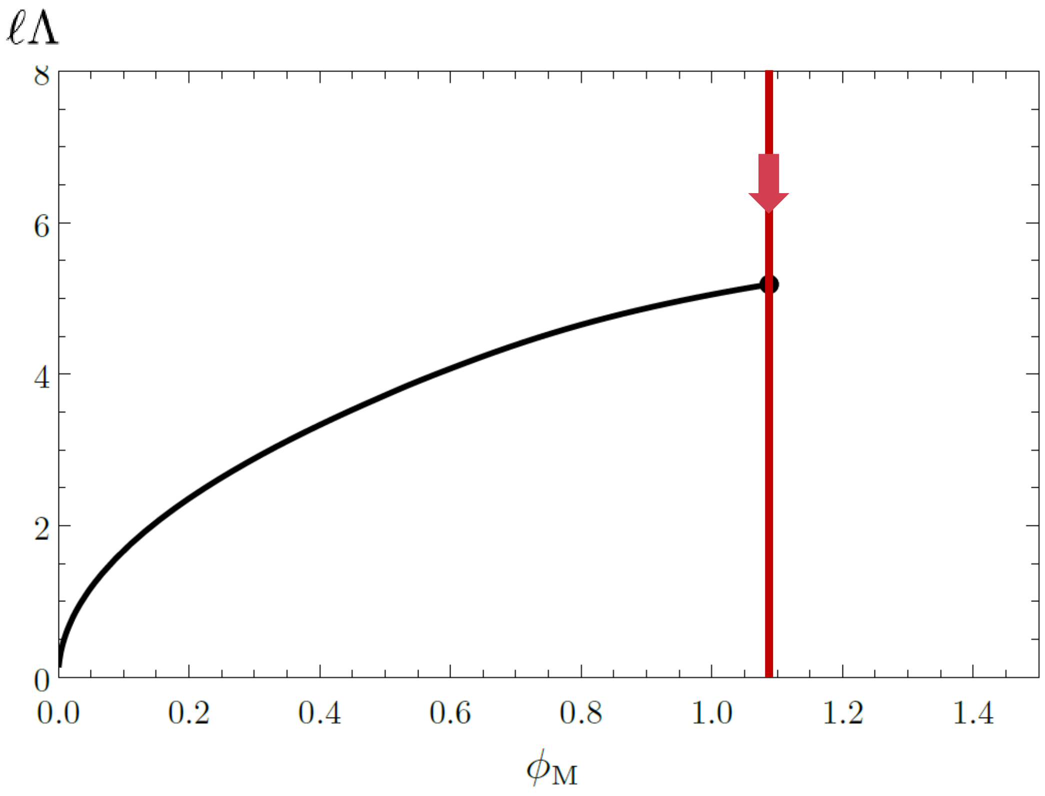

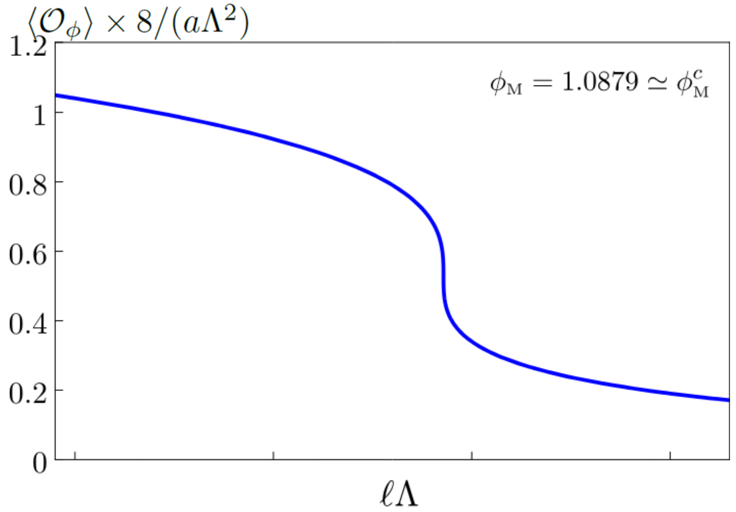

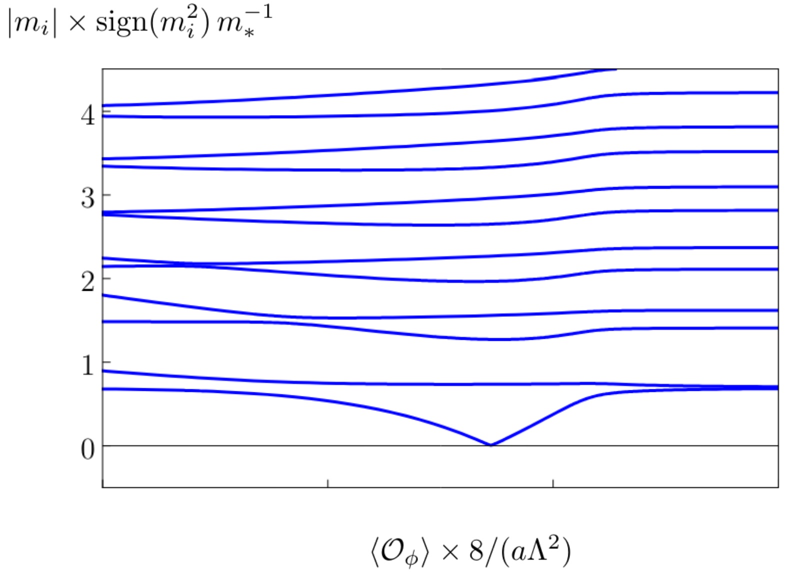

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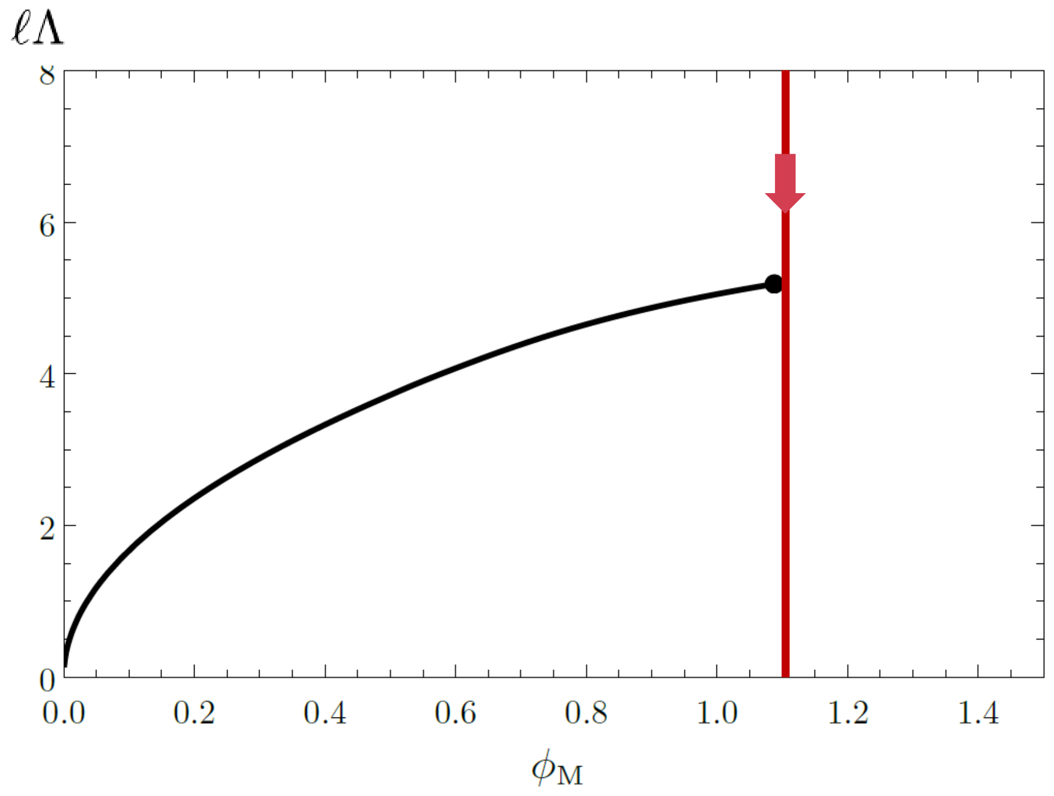
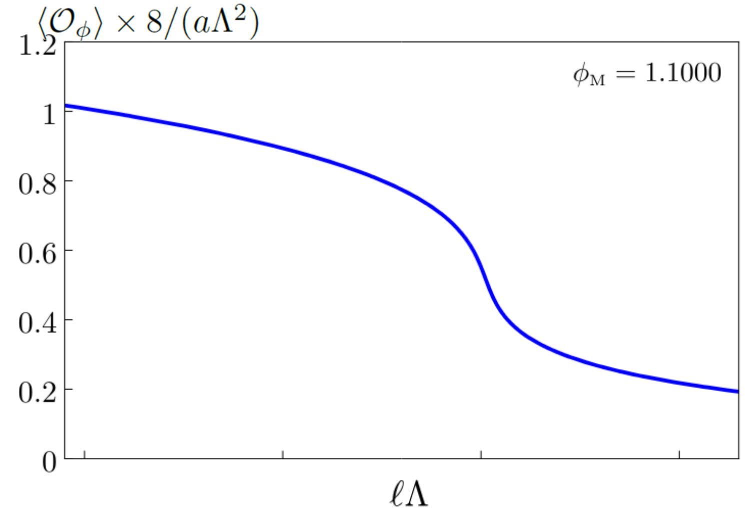
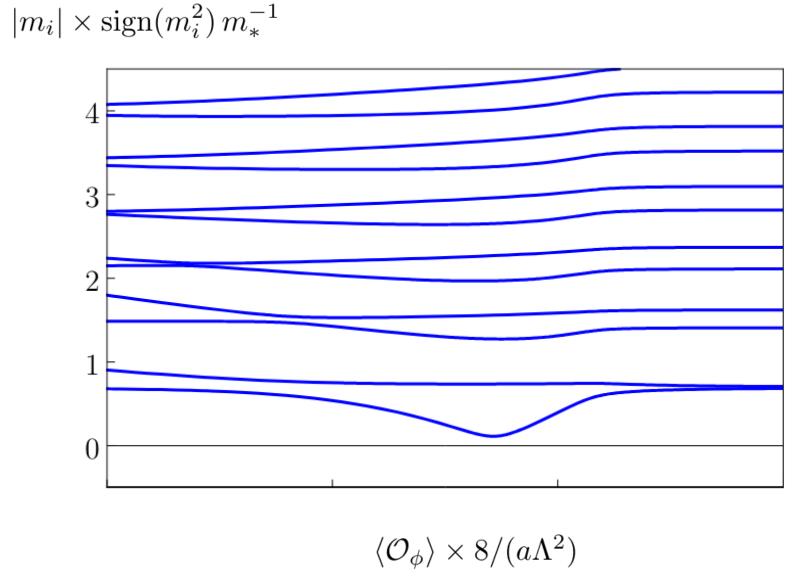






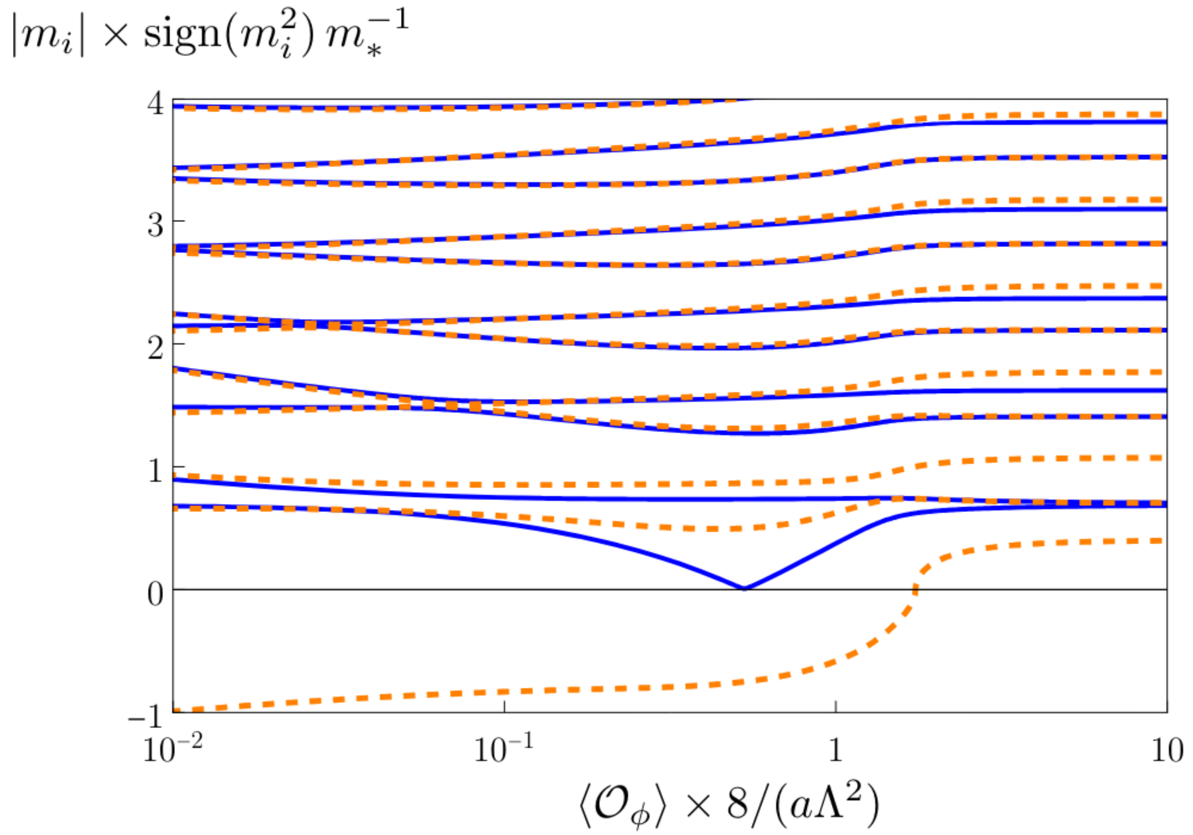
What happens as we come close to the critical point?

Is this light state a dilaton?

Yes, the light state is missed when the perturbations of the metric are ignored.

$$\cancel{g_{\mu\nu} \rightarrow g_{\mu\nu} + \delta g_{\mu\nu}}$$

$$\Phi \rightarrow \Phi + \delta\Phi$$



[*Elander, Piai, Roughley;* ***2004.05656***].

Conclusion

We proposed a holographic dual of complex CFTs, that does not involve a field violating the BF bound.

We investigated the appearance of light states in the vicinity of cCFTs. Our results suggest that the emergence of such state is not related to Miranski (BKT) scaling.

Rather, the light state appears in the boundaries of stability of the vacuum of the theory.

We showed this happening explicitly in a concrete, controllable model.

Outlook

Understand the missing phase in the first model, and see what causes the instability and in what regions is the light state achieved in a stable vacuum.

Find holographic cCFTs in string theory models.

Following the intuition developed, find a light state in a trustworthy (string theory) holographic construction.

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Thanks.