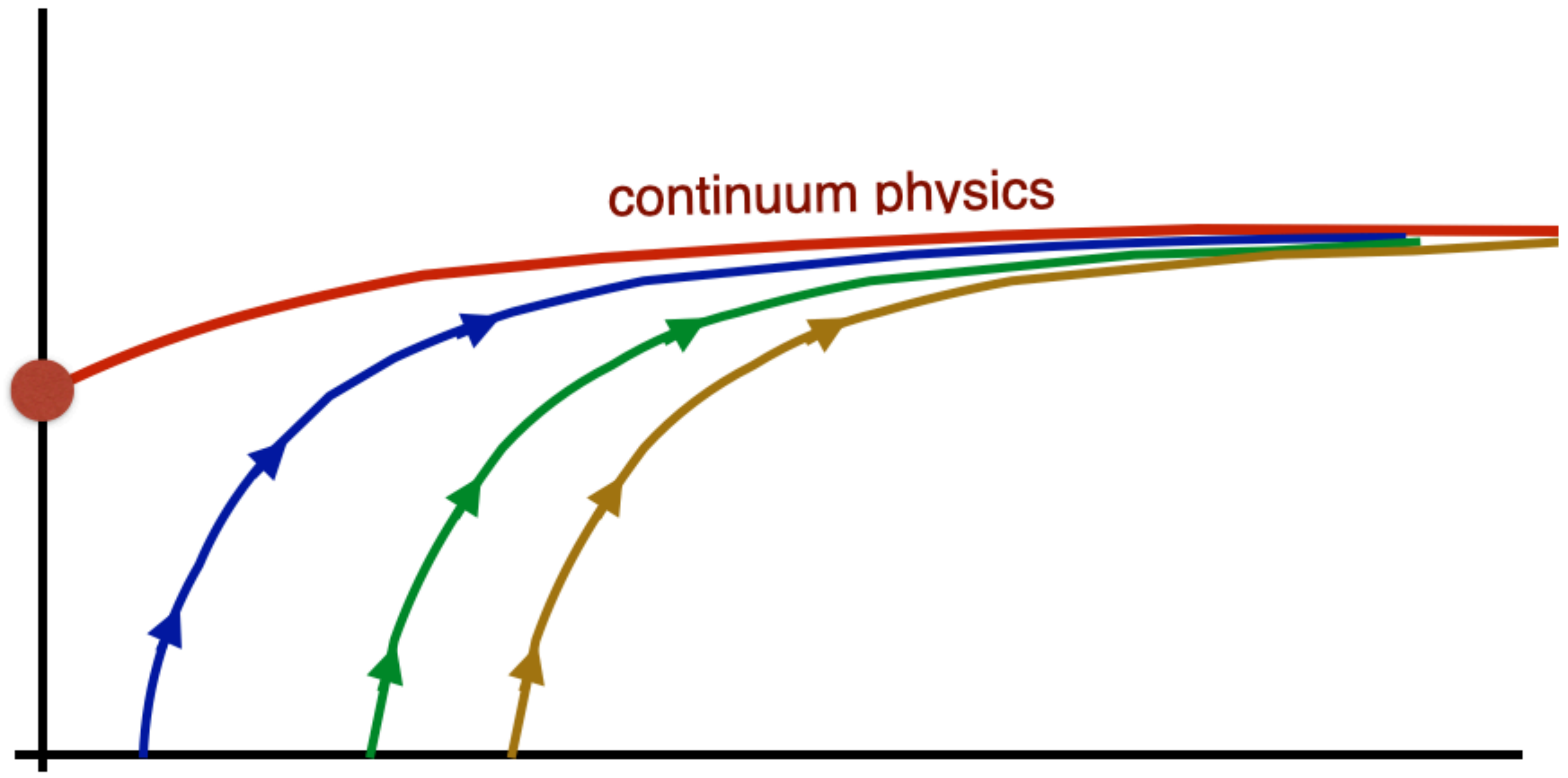
The phase structure and infrared properties of SU(3) gauge with 8 fundamental fermions

Anna Hasenfratz
University of Colorado Boulder

Dilaton Dynamics - from Theory to Applications
Higgs Centre Workshop
June 28 2024



Preview: SU(3) gauge + 8 fundamental flavors

8 Dirac flavors are special:

anomaly free → could support

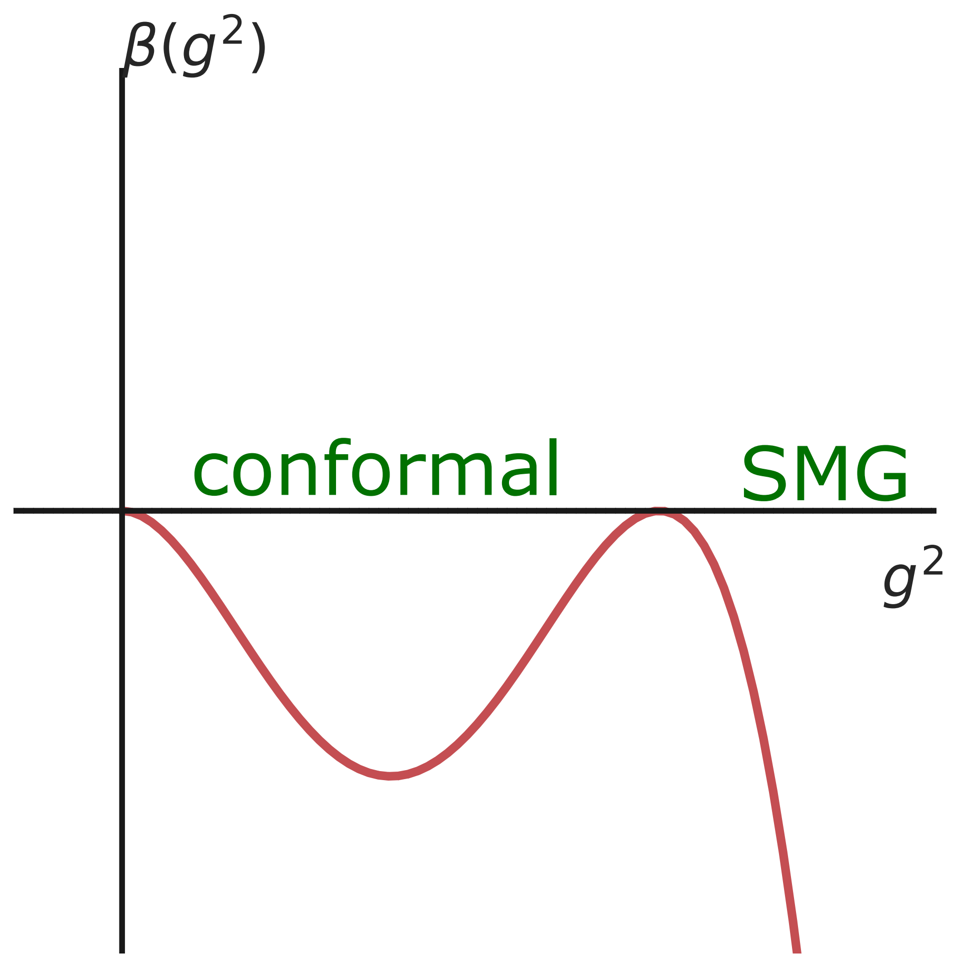
Symmetric Mass Generation

- confining but chirally symmetric
- bound states are gapped, ∄ Goldstones

Details later

Numerical simulations of SU(3) gauge + $N_f = 8$ show (represented with staggered fermions)

- weak coupling phase appears conformal
- strong coupling phase ($\not{S}^4$) with SMG properties
- continuous phase transition → ∃ continuum limit
- could be 'walking': opening of the conformal window



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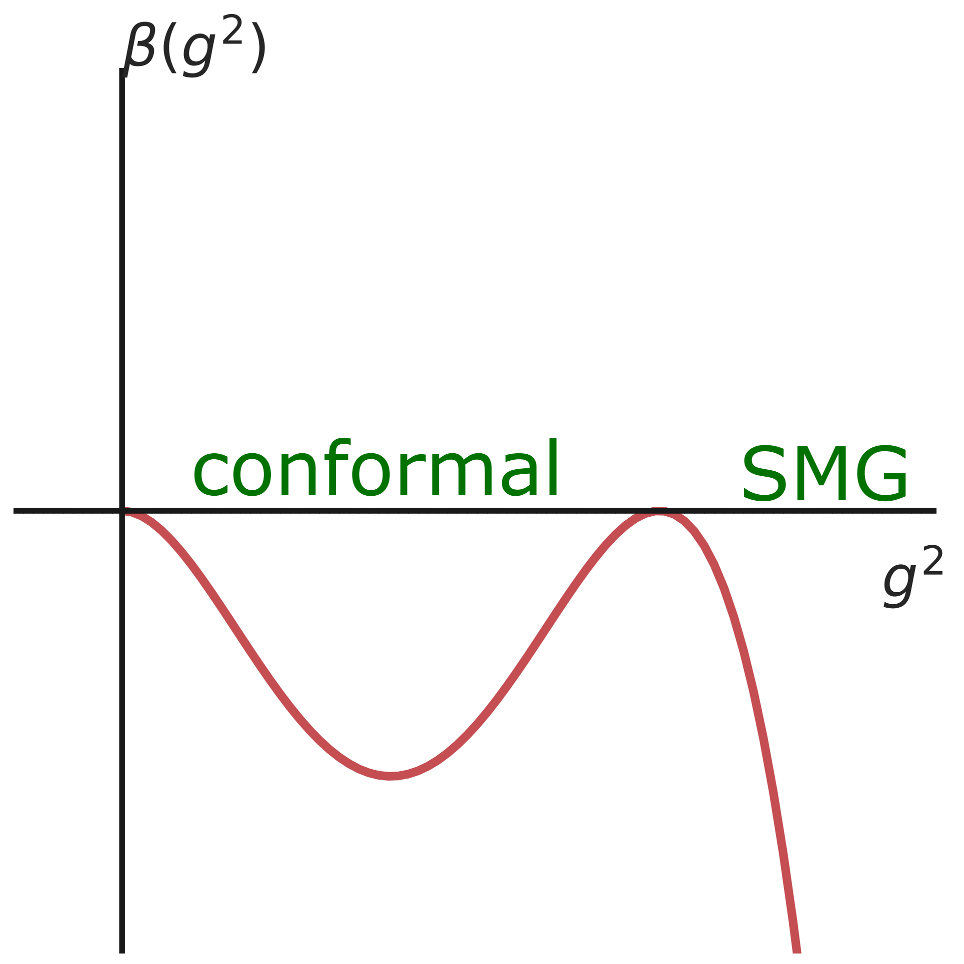


Why would the conformal sill be at integer N_f ?

➡ Systems with 8 flavors/2 sets of staggered fields are special

Is SMG useful for model building/dilaton dynamics?

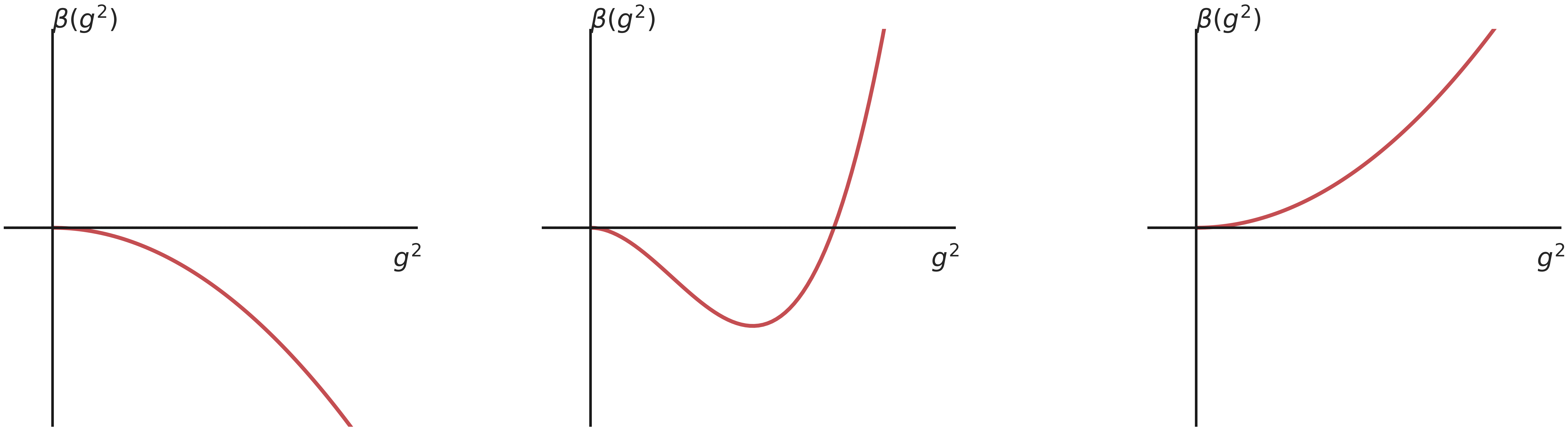
➡ Let's discuss

Phases of gauge-fermion systems

Hypothetical

SU(3) gauge + N_f (fundamental) *massless* fermions

small N_f — Confining | $N^* < N_f < N^{IF}$ — Conformal | $N_f > N^{IF}$ — Infrared free



Perturbatively: the IR fixed point emerges at $g_0^2 = \infty$ at $N_f = N^*$, moves to $g_0^2 = 0$ as $N_f \to N^{IF}$

Nonperturbatively: the IR fixed point could emerge at finite g_*^2 if $\beta(g) \sim (\alpha - \alpha_*) - (g - g_*)^2$

Conformality lost at IR-UV fixed point merger

Kaplan et al PRD80,125005 (2009)
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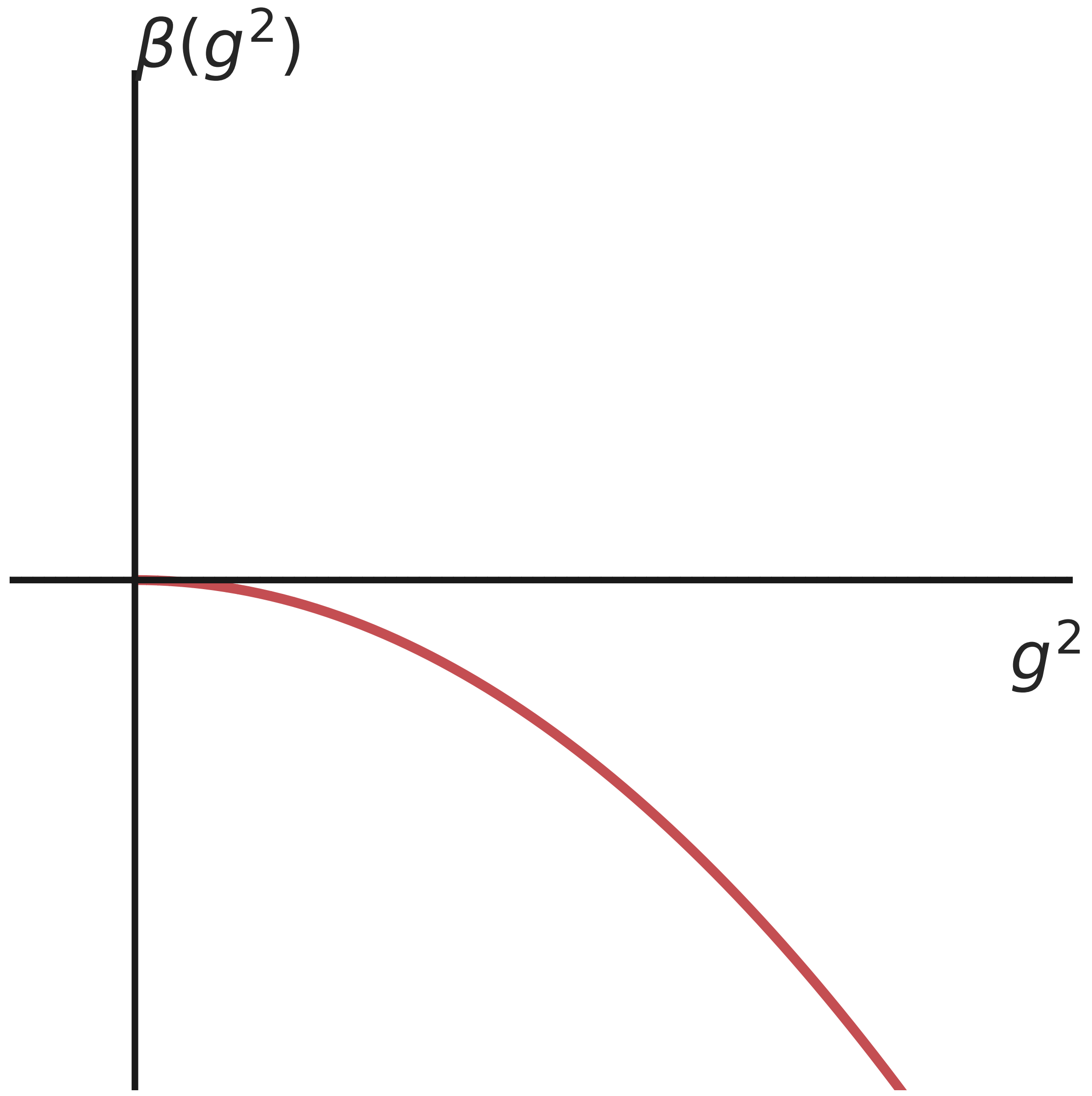

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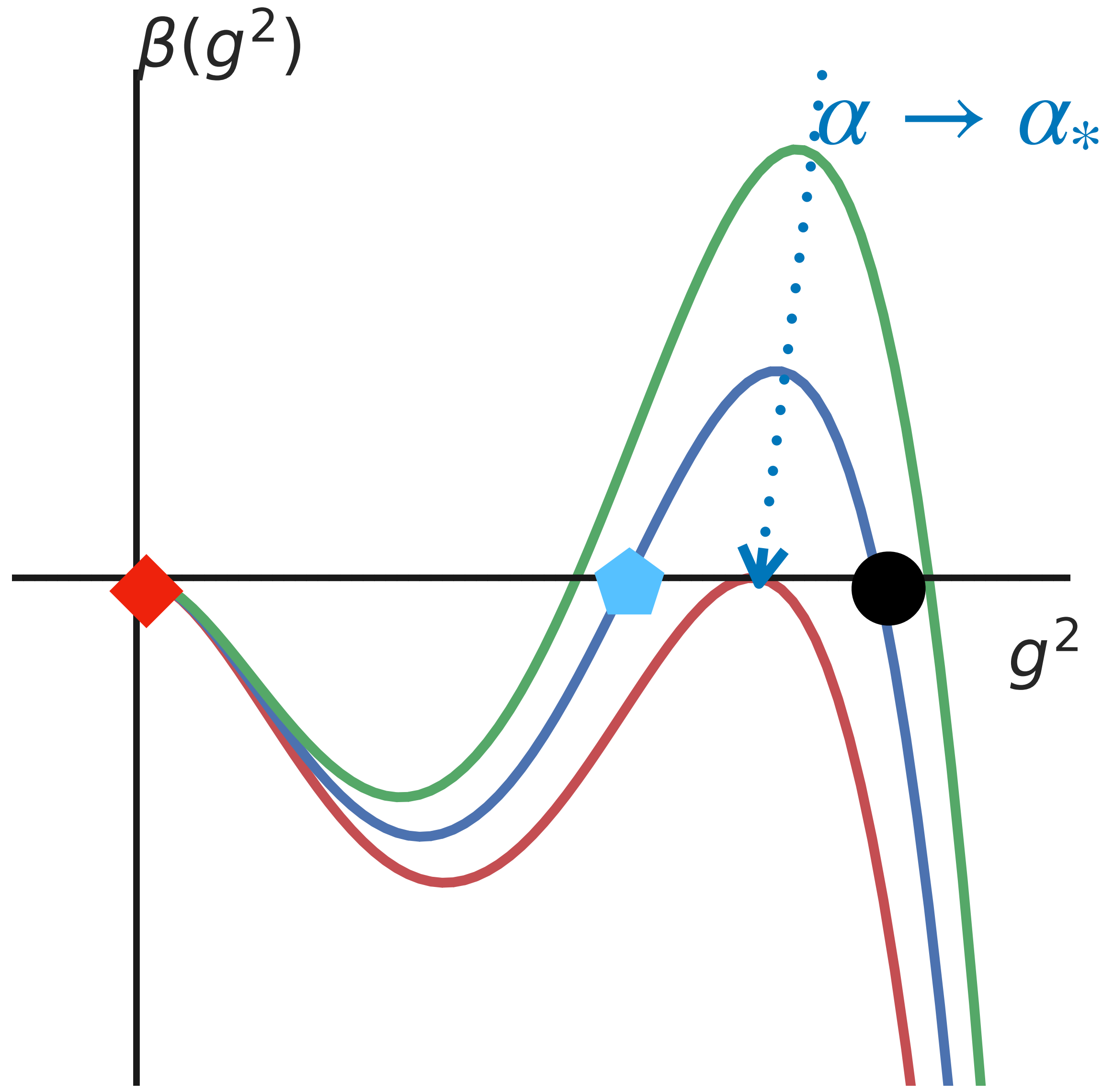
Conformal



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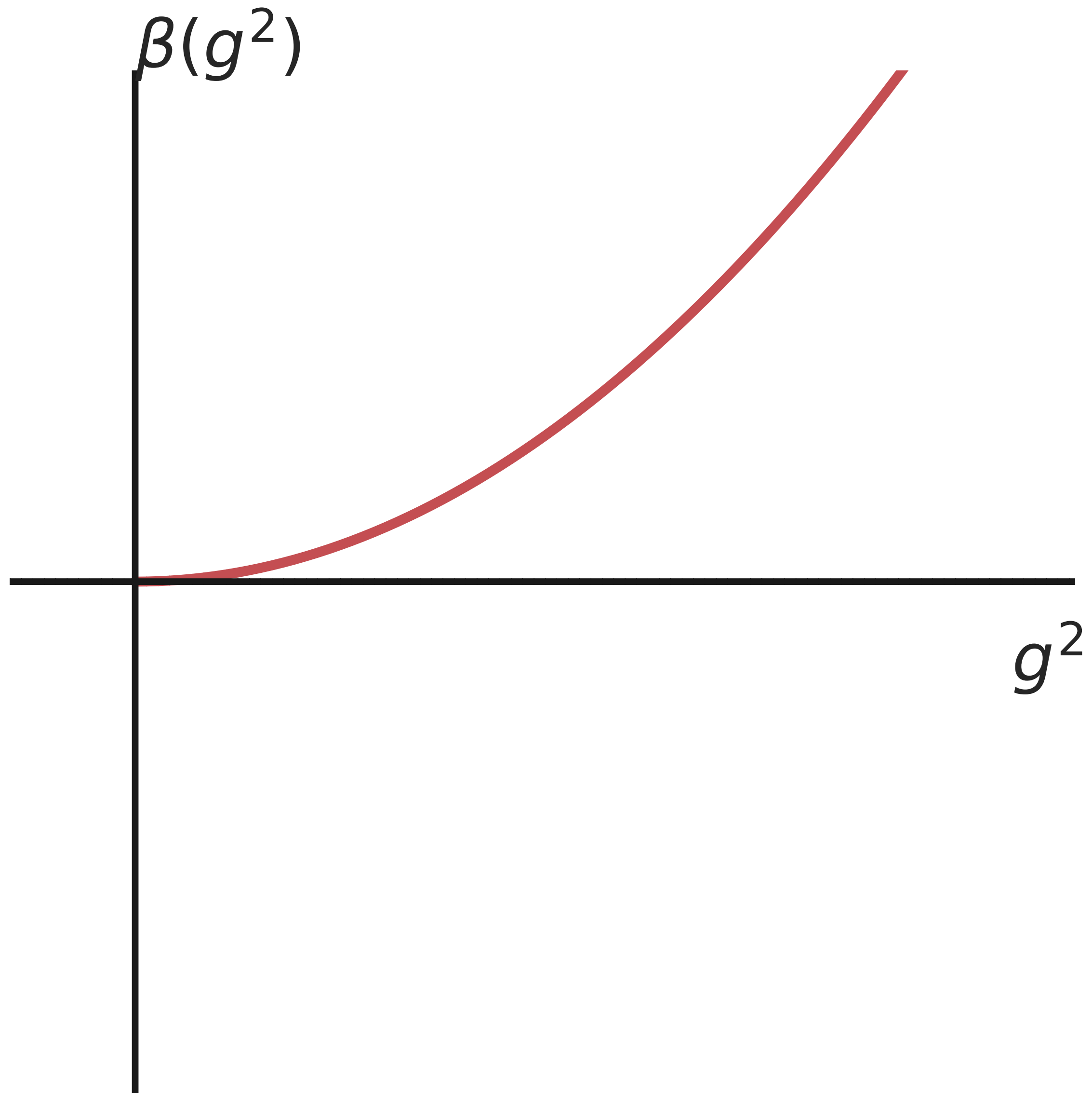

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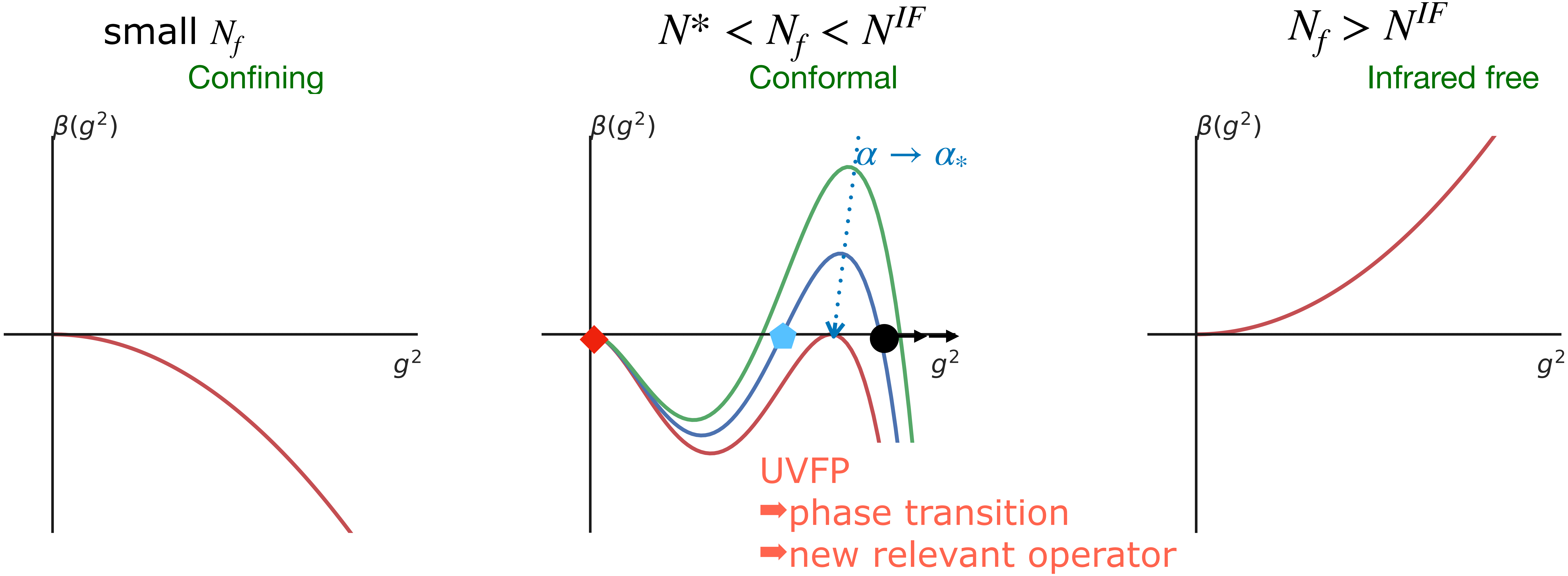

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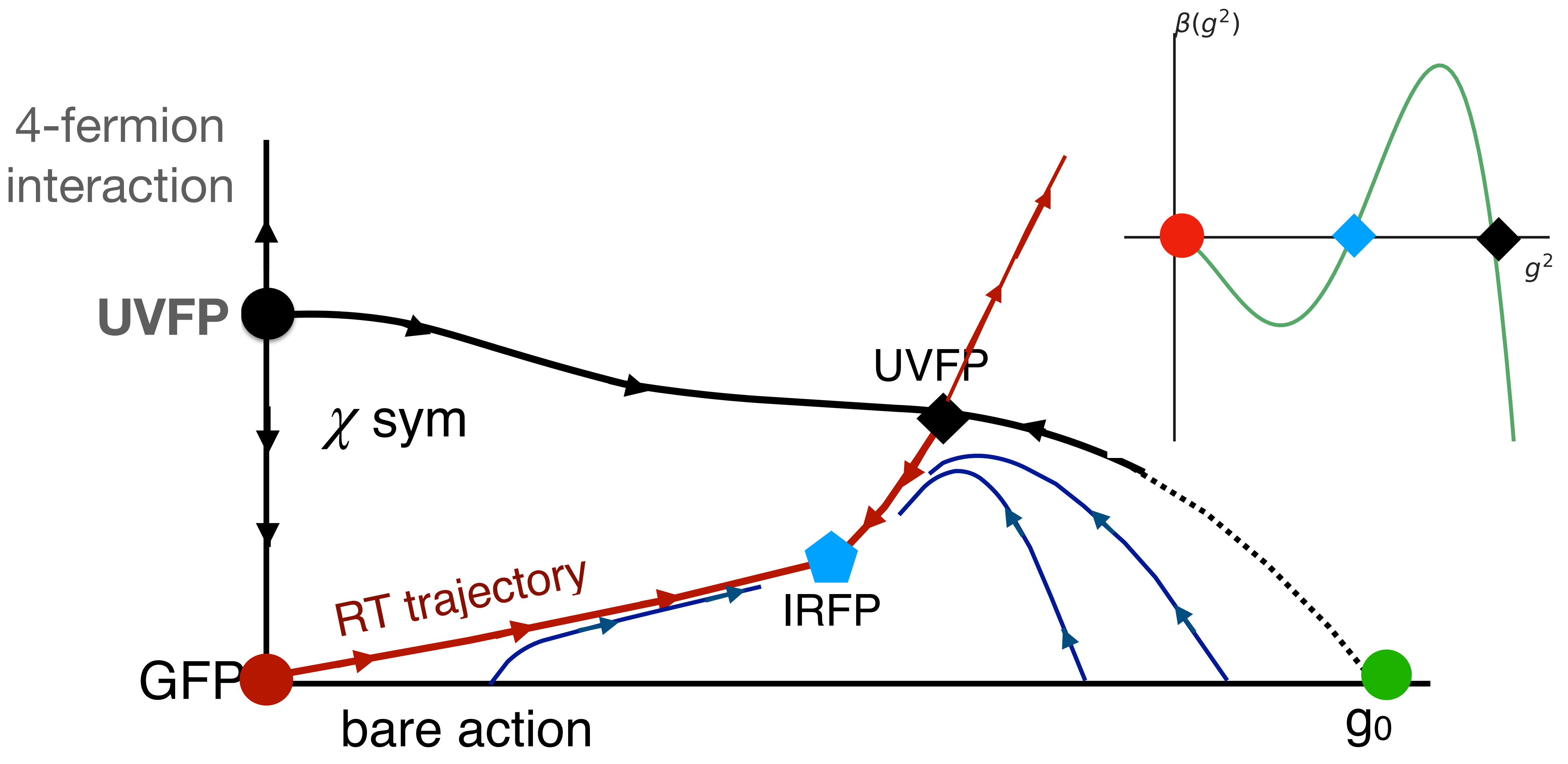
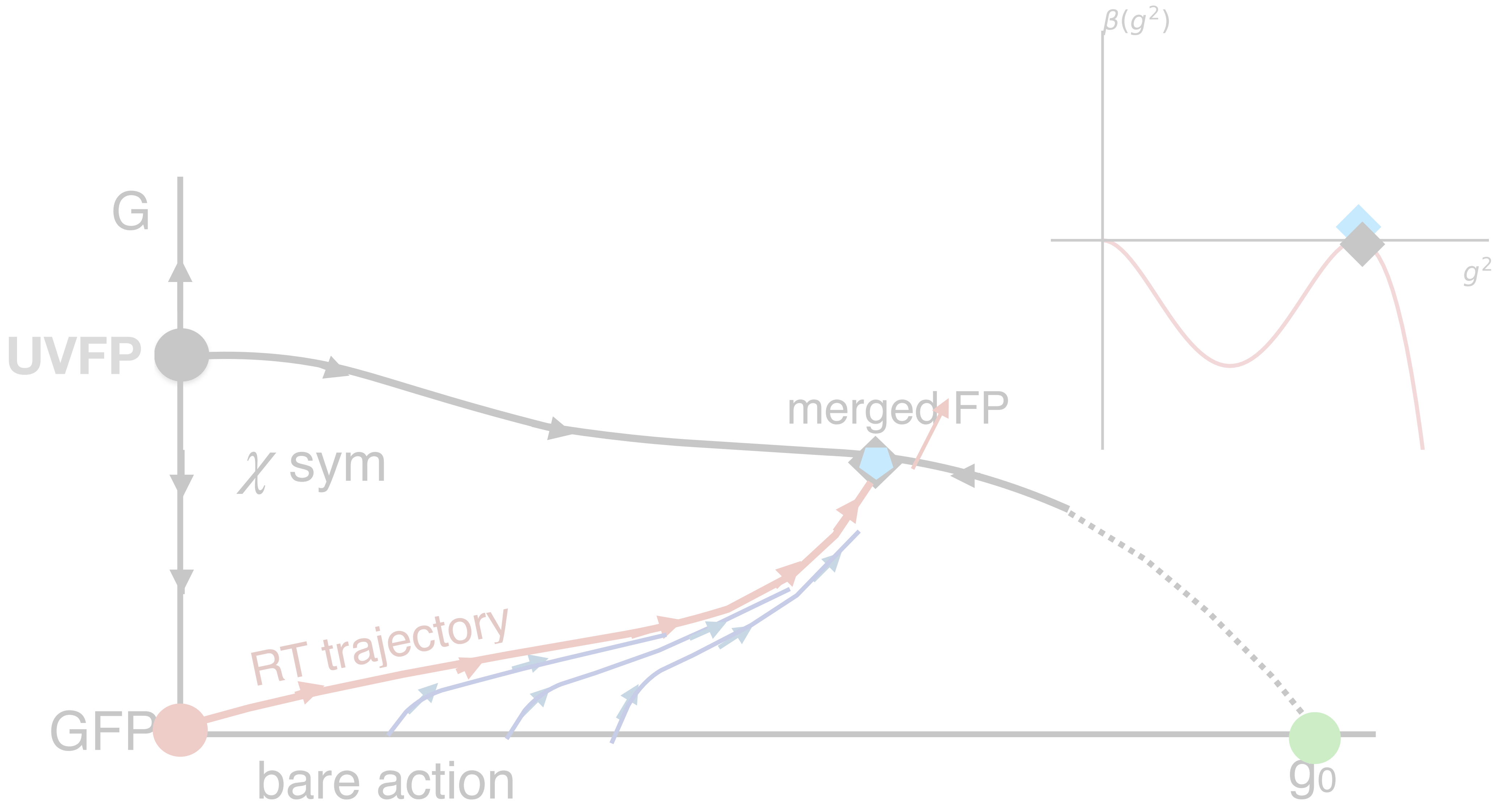
New relevant operator



Conformal: distinct IRFP / UVFP:

- Universality on the lattice extends from GFP to IRFP only
- The strongly coupled phase not universal
- Depends on the lattice details



The FPs merge at the conformal sill: "walking scaling" (BKT) phase transition

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- We can study the new phase within the gauge-fermion system

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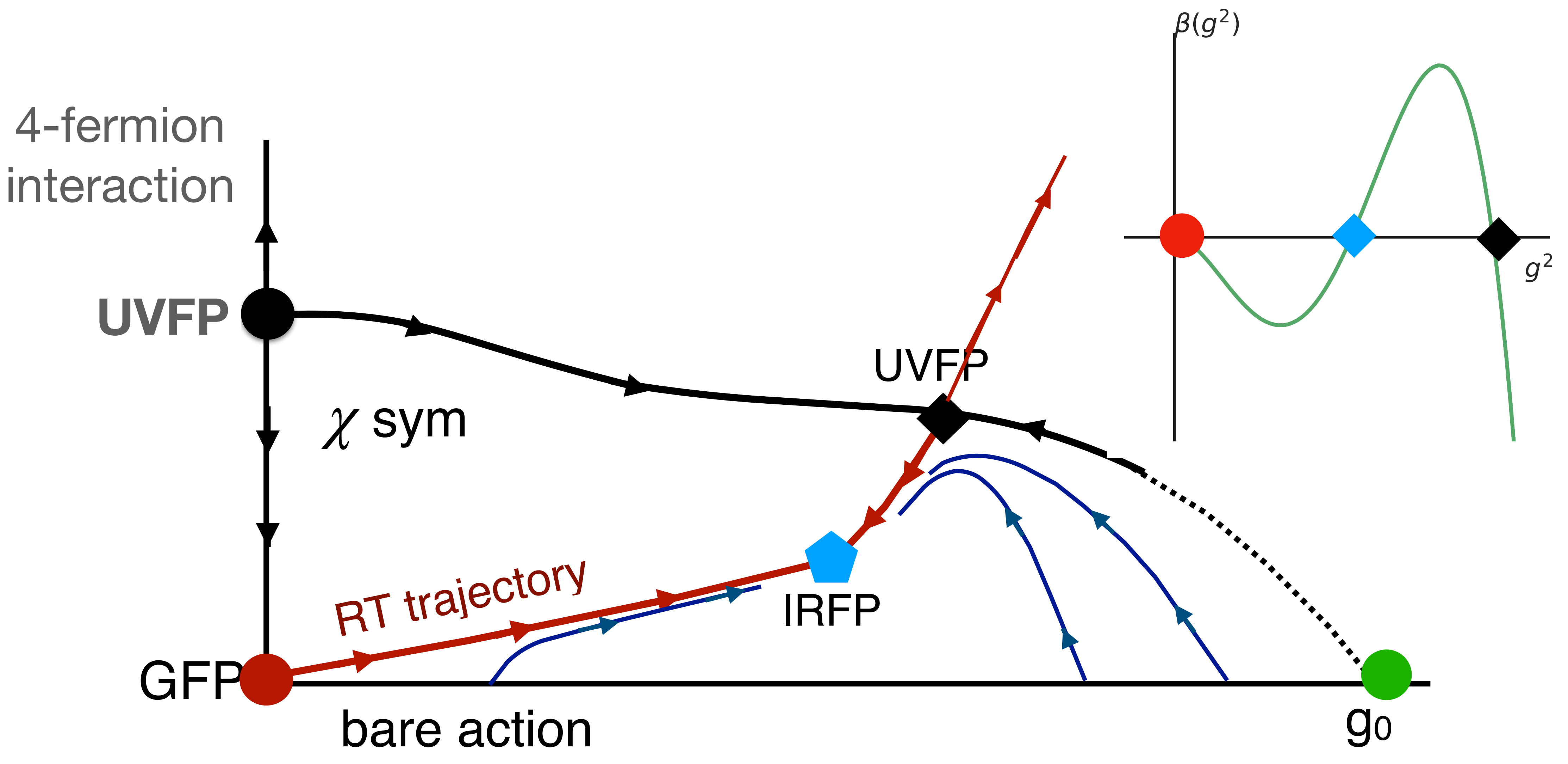

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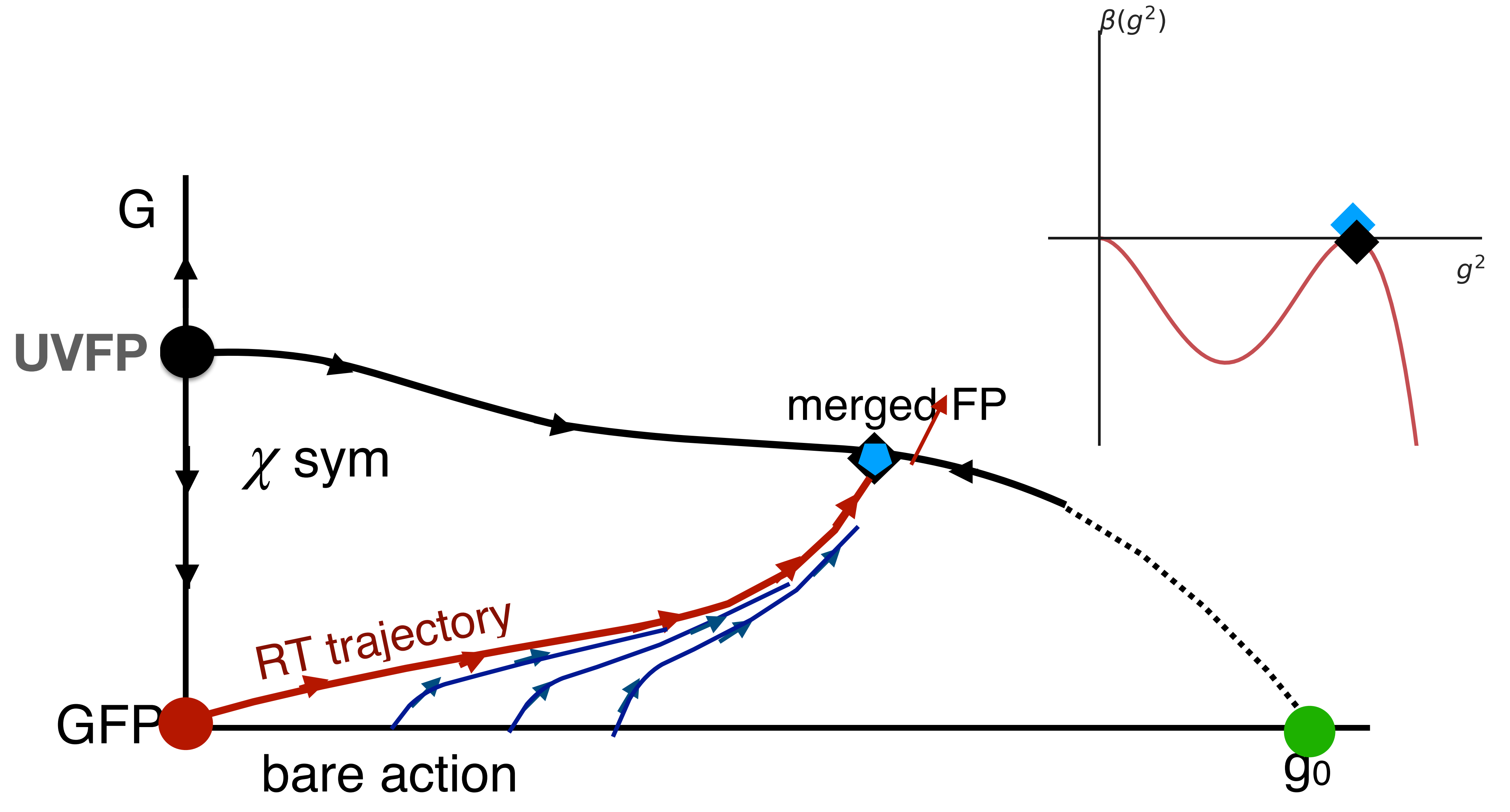

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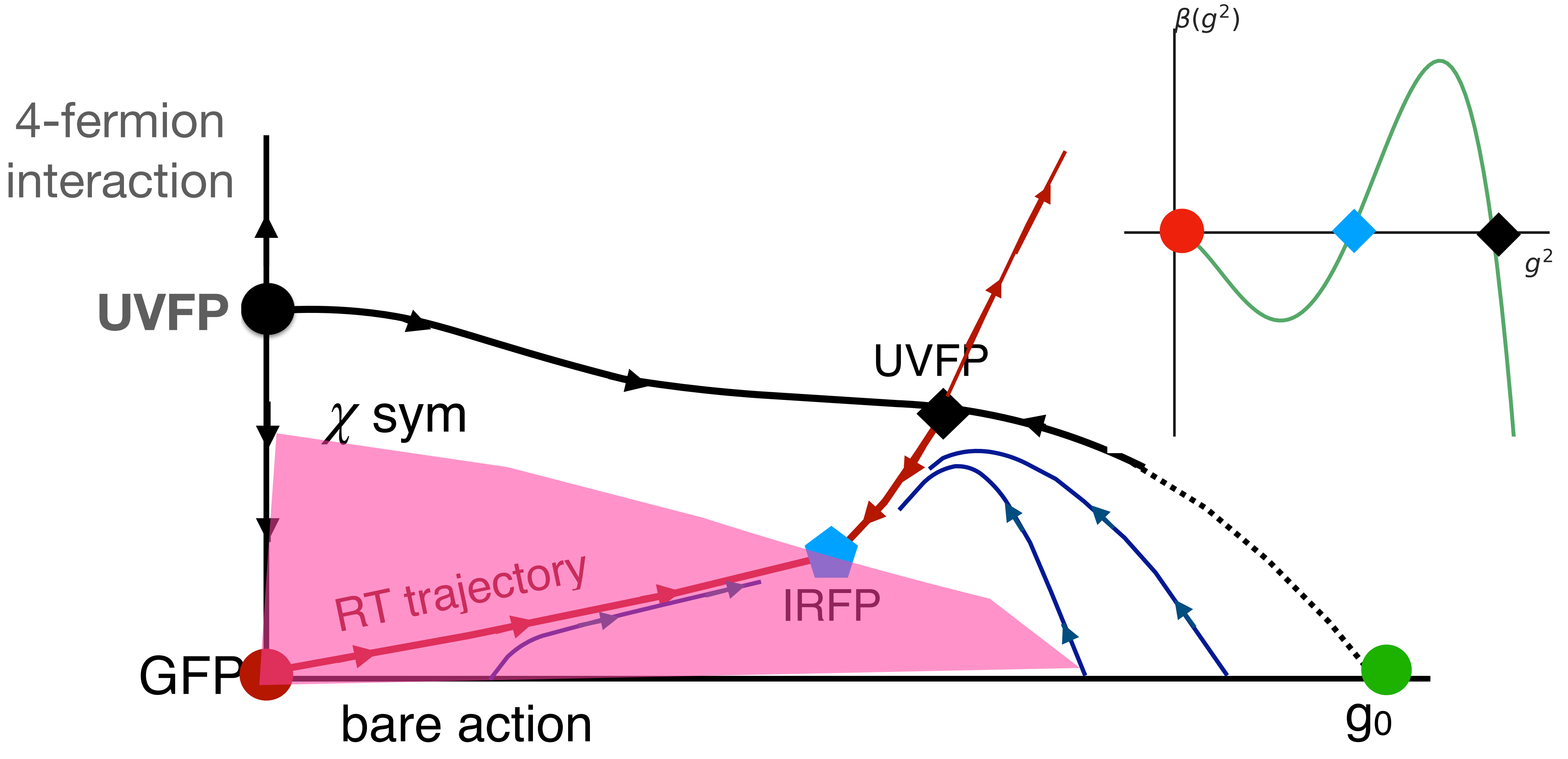

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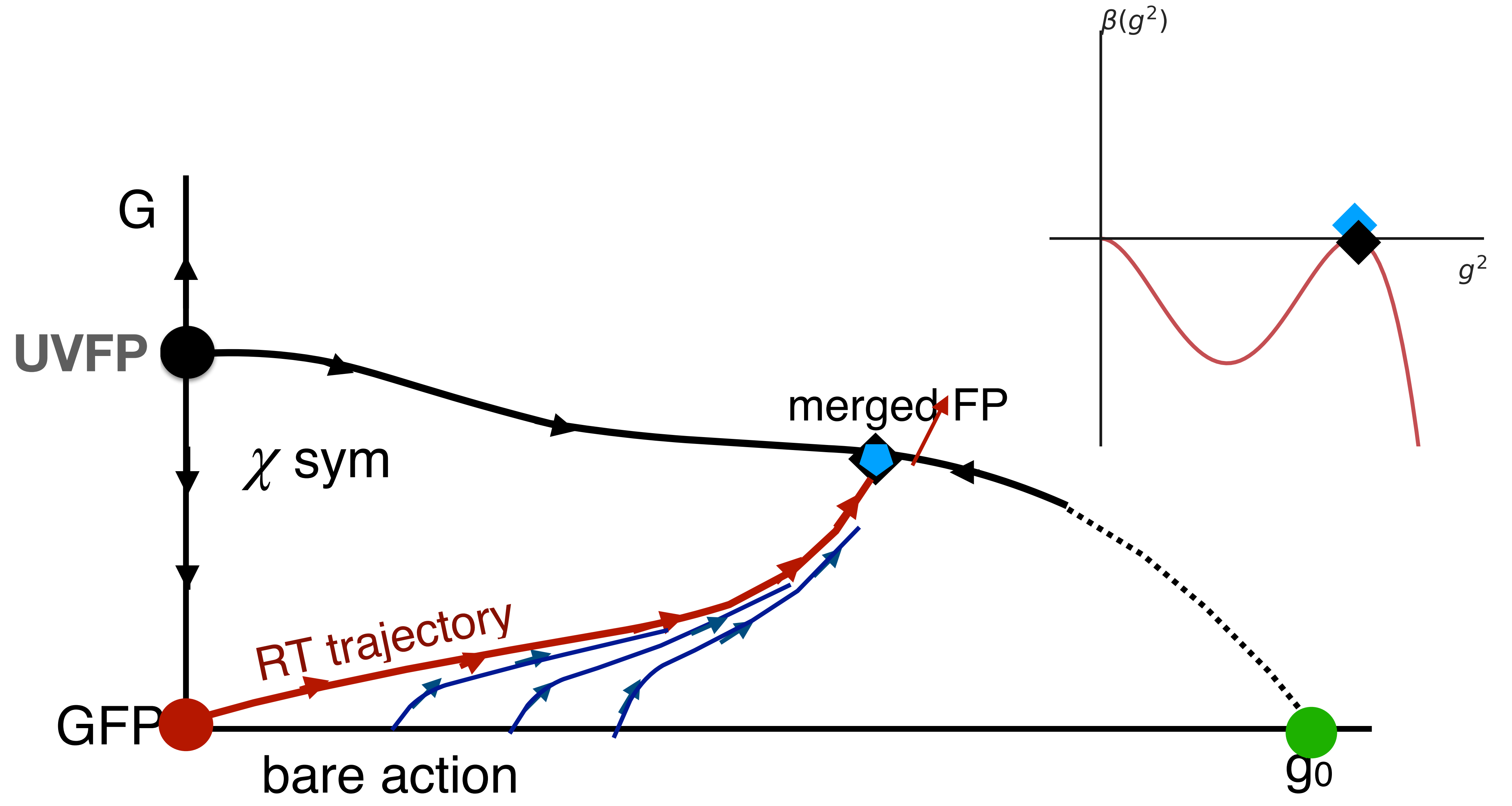

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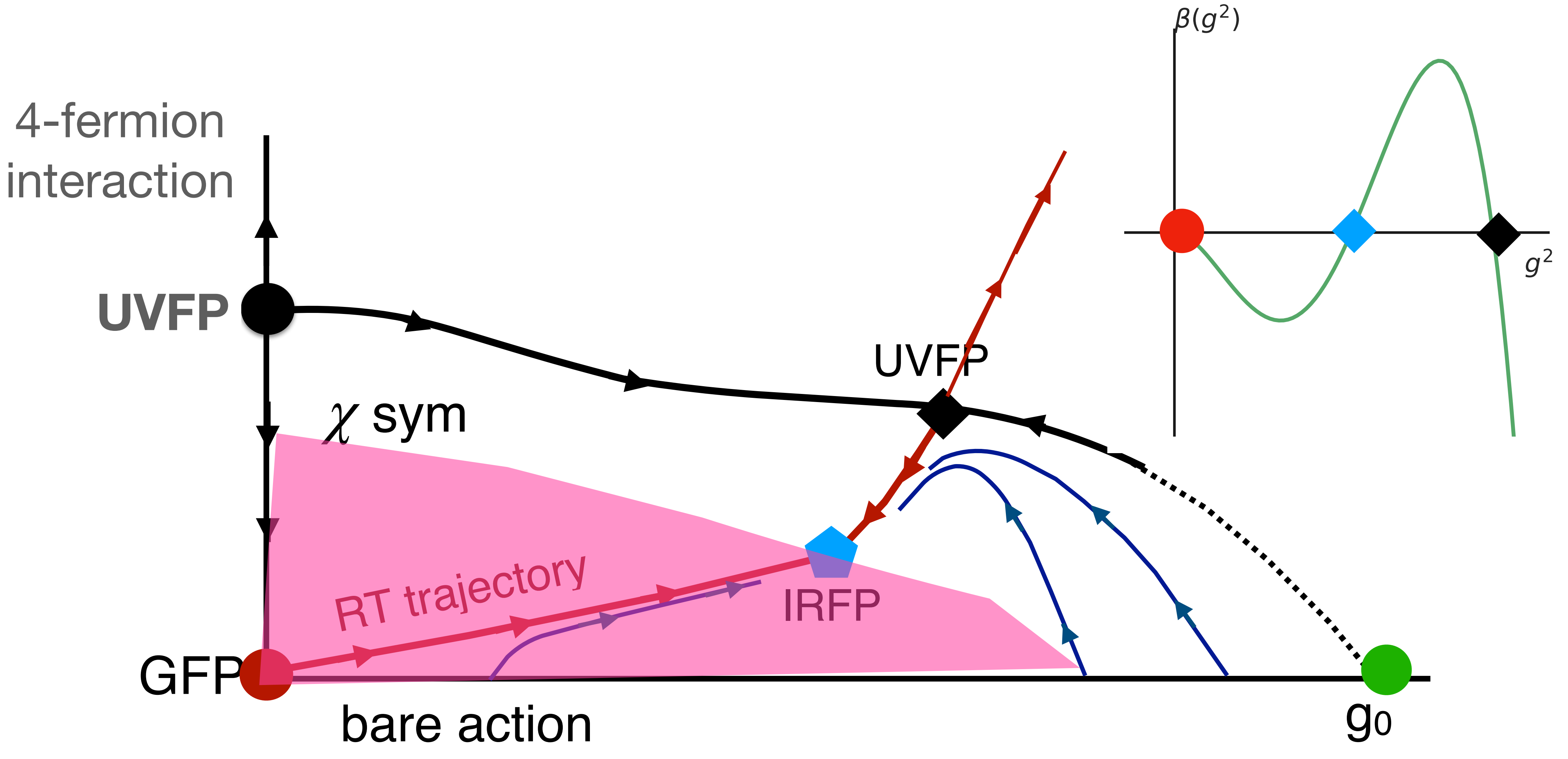

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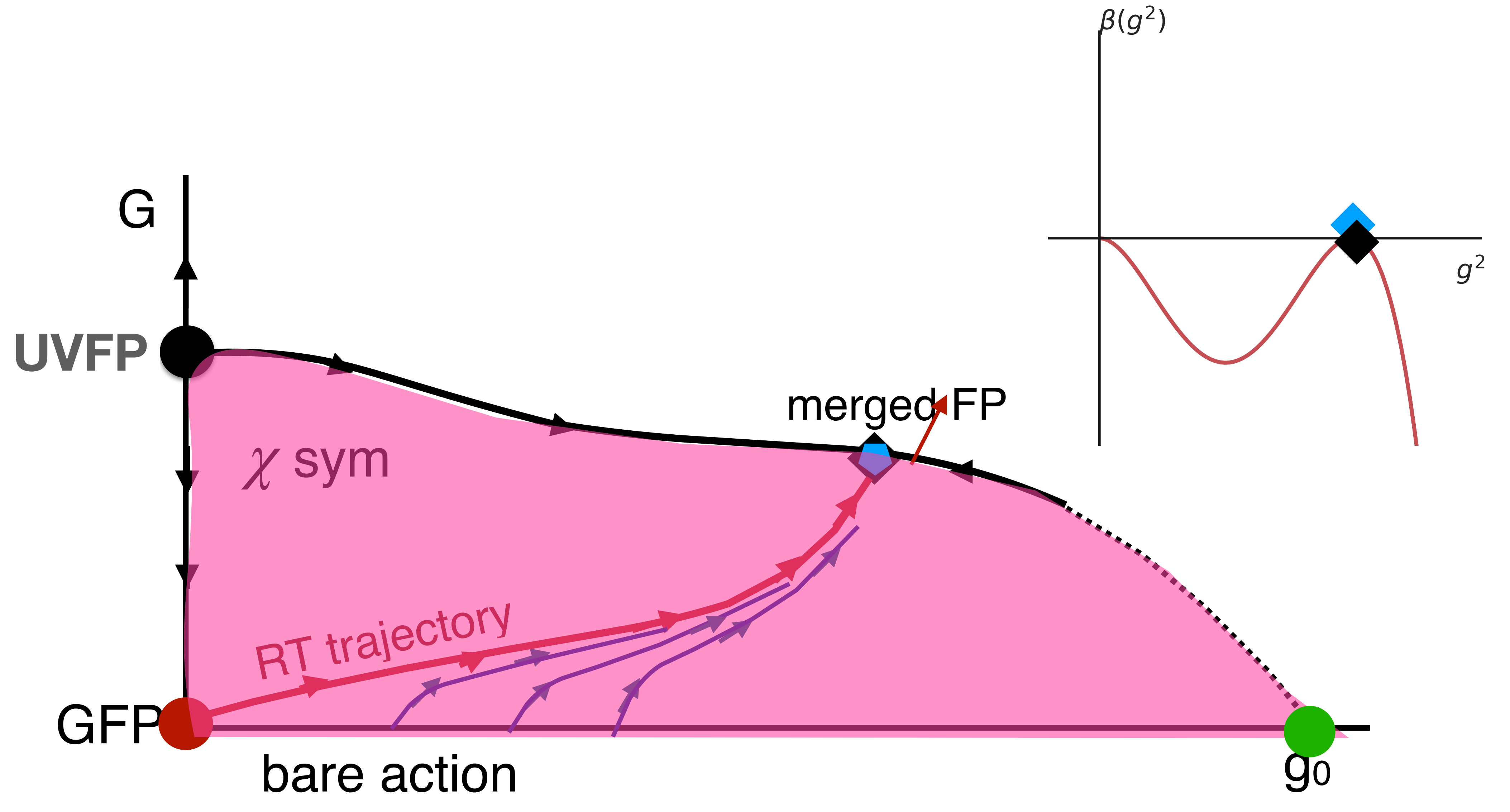

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Where is the conformal sill?

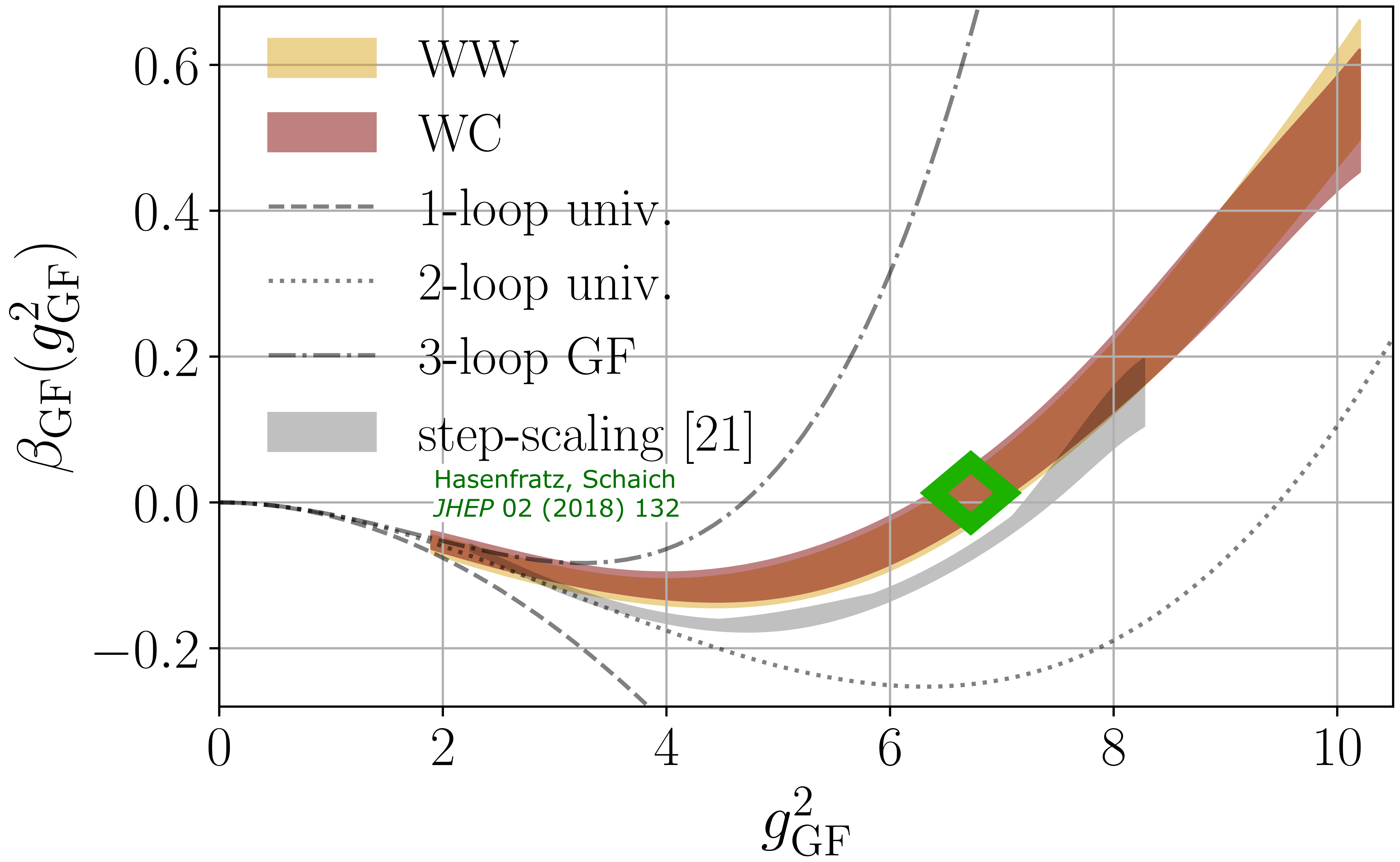
Brief and (slightly) biased review

Where is the conformal sill?

After~15 years of lattice studies:

- $N_f = 12$ looks conformal
- $N_f = 10$ looks conformal
- $N_f = 8$ could go either way

Peterson, Hasenfratz, 2402.1803



RG β function in the gradient flow scheme

- SU(3) gauge
- staggered fermions

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Hasenfratz.,Neil, Shamir, Svetitsky, Witzel,
Phys.Rev.D 108 (2023) 7

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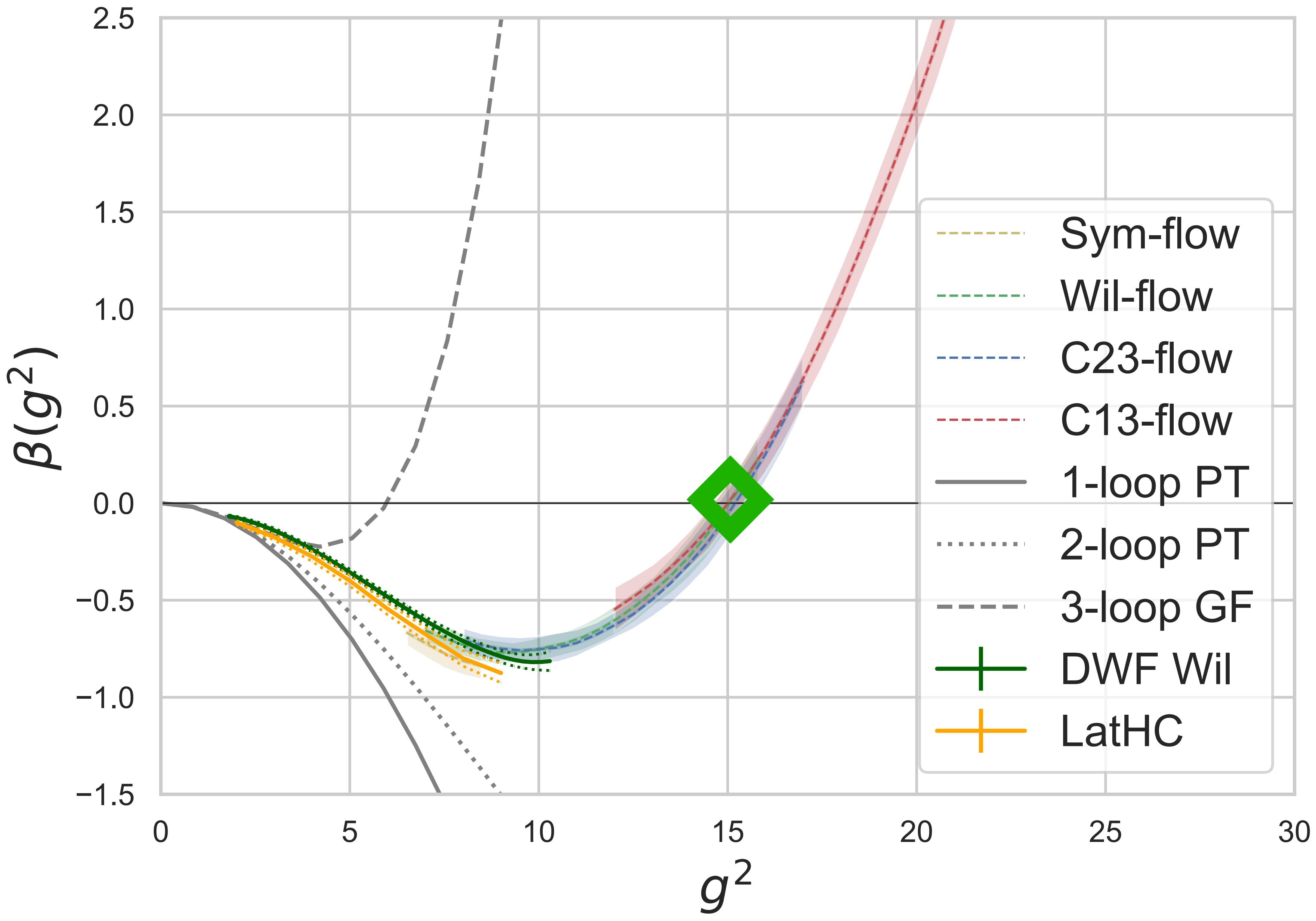

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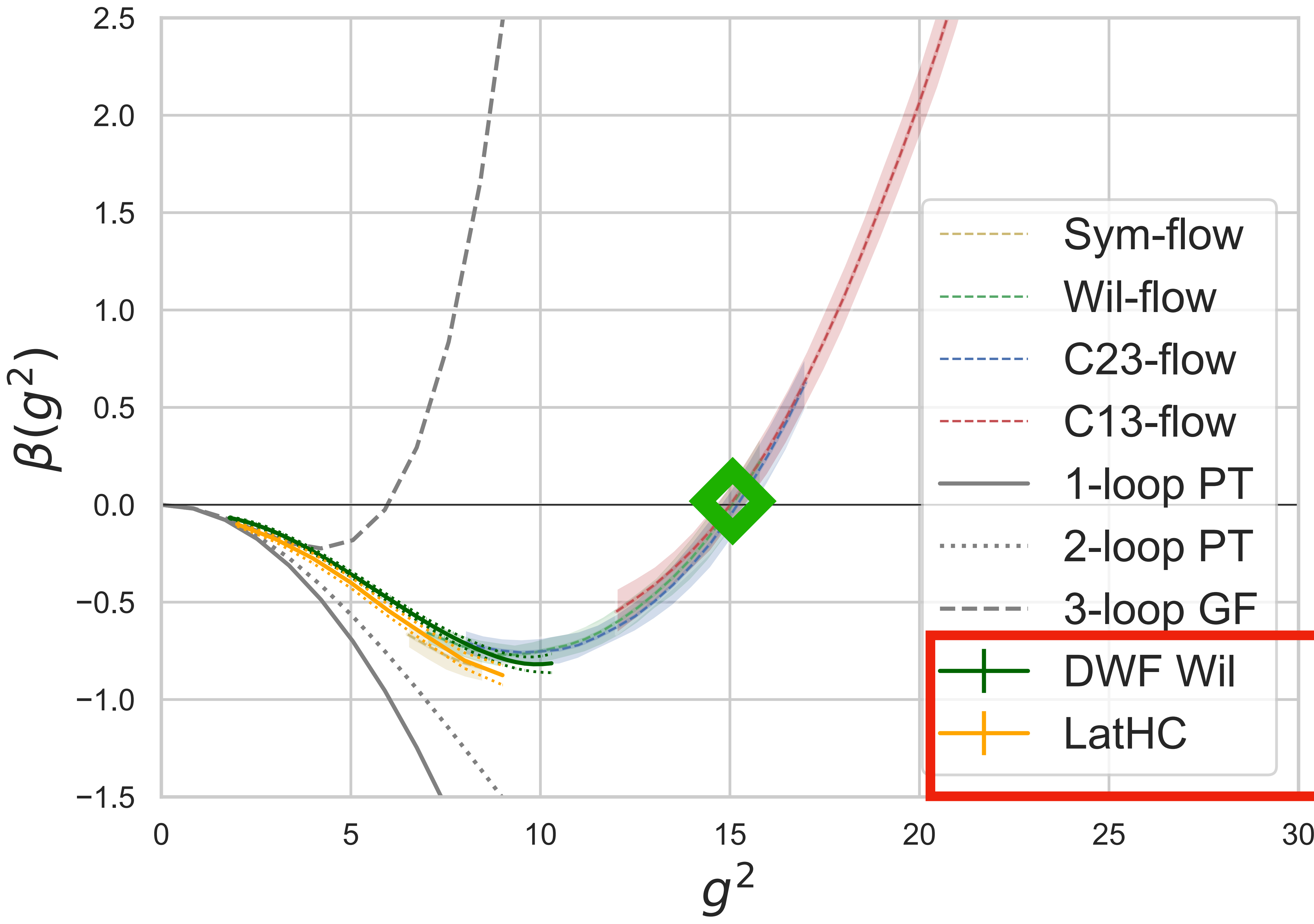

Prior works:

Domain wall f.
Staggered f.

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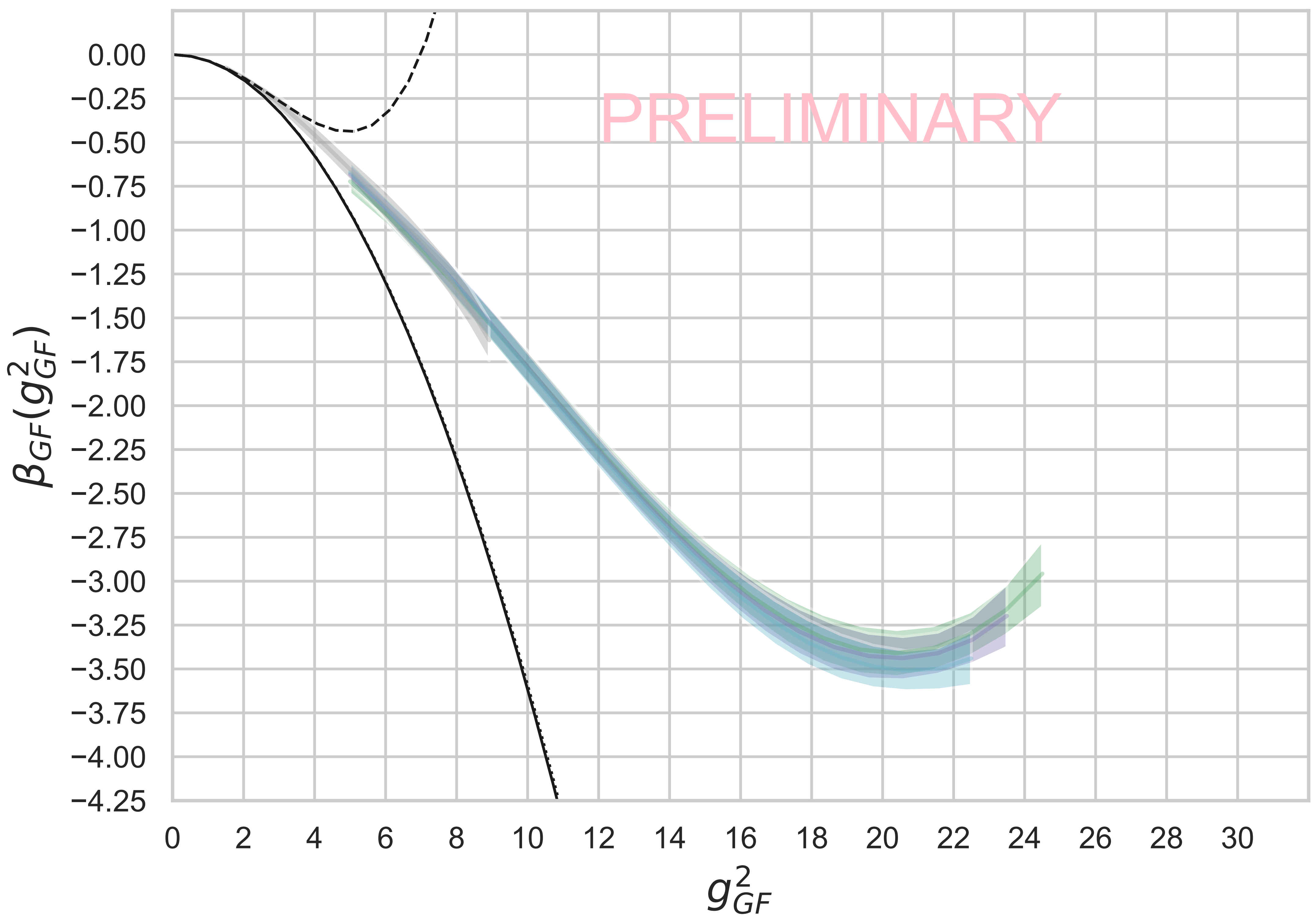

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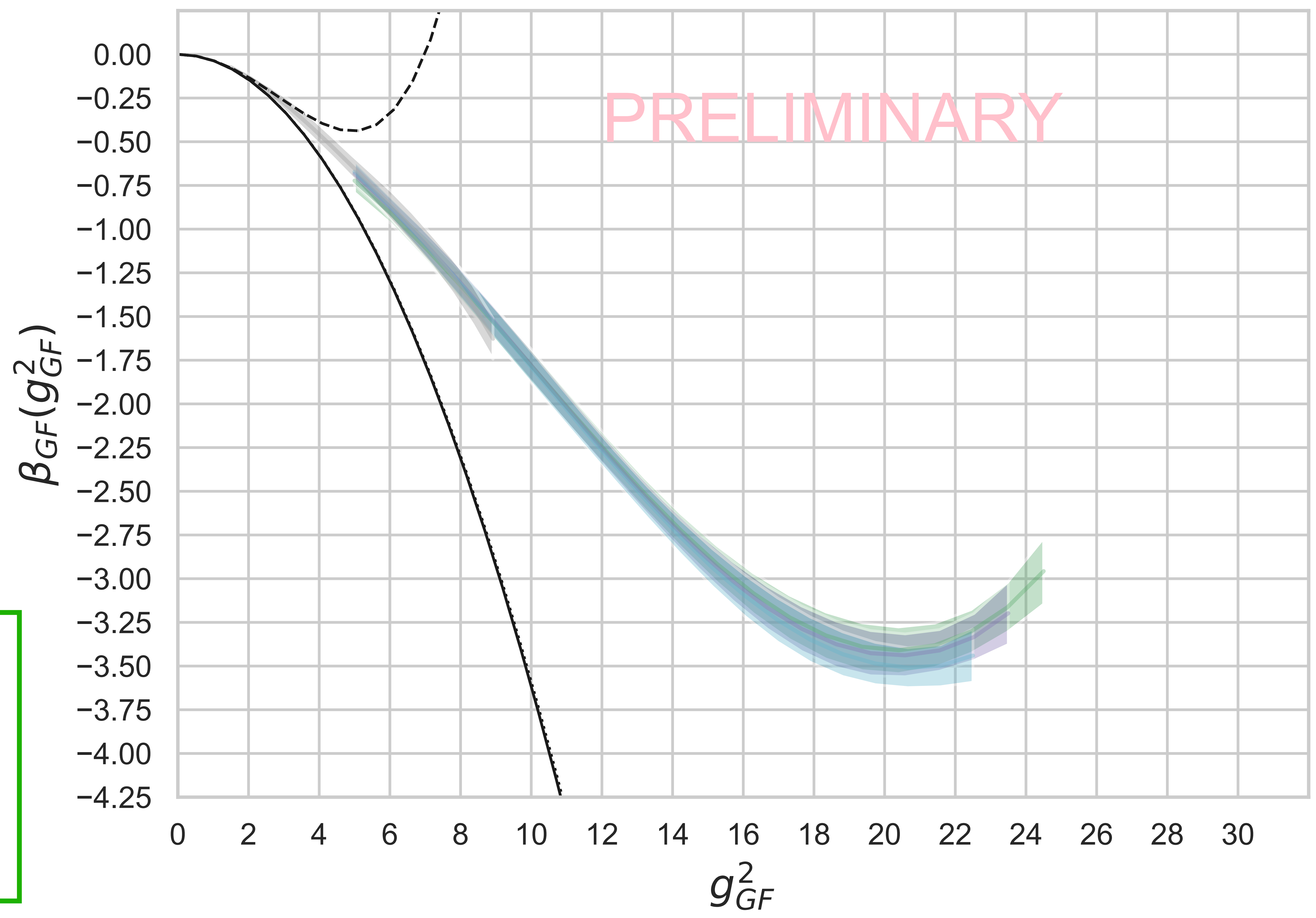
Nonperturbative cutoff effects due to instantons make the predicted $|\beta(g^2)|$ too large



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“But Wikipedia says $N_f = 8$ is chirally broken!”

SU(3) $N_f = 8$ has been long favored as composite Higgs model:

- thought to be chirally broken
- expected (hoped) to be slowly walking with light scalar (Higgs)

This might not be the case:

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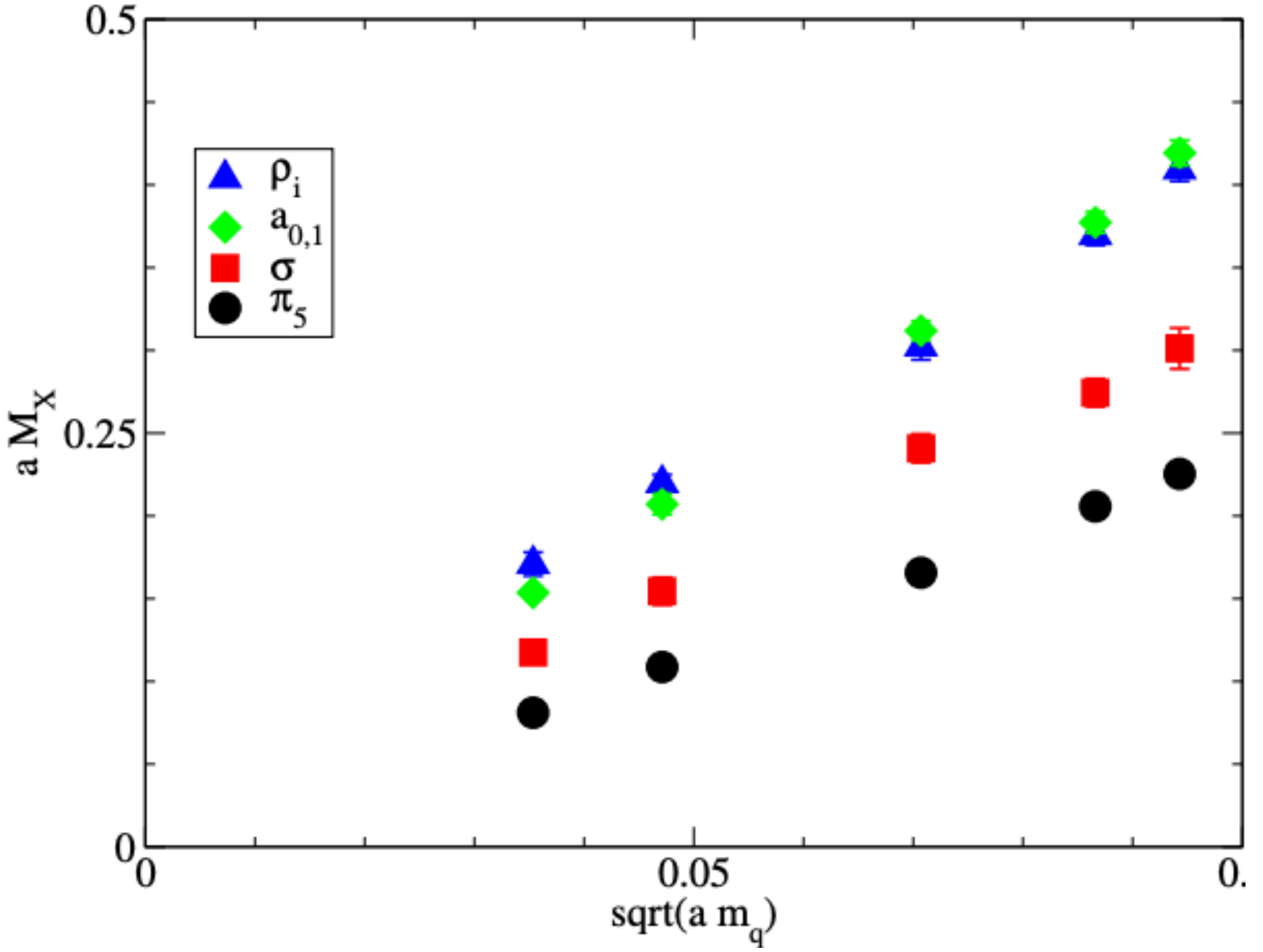
LSD Collaboration 2306.06095, *Phys.Rev.D* 108 (2023) 9



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- staggered fermions but different lattice action

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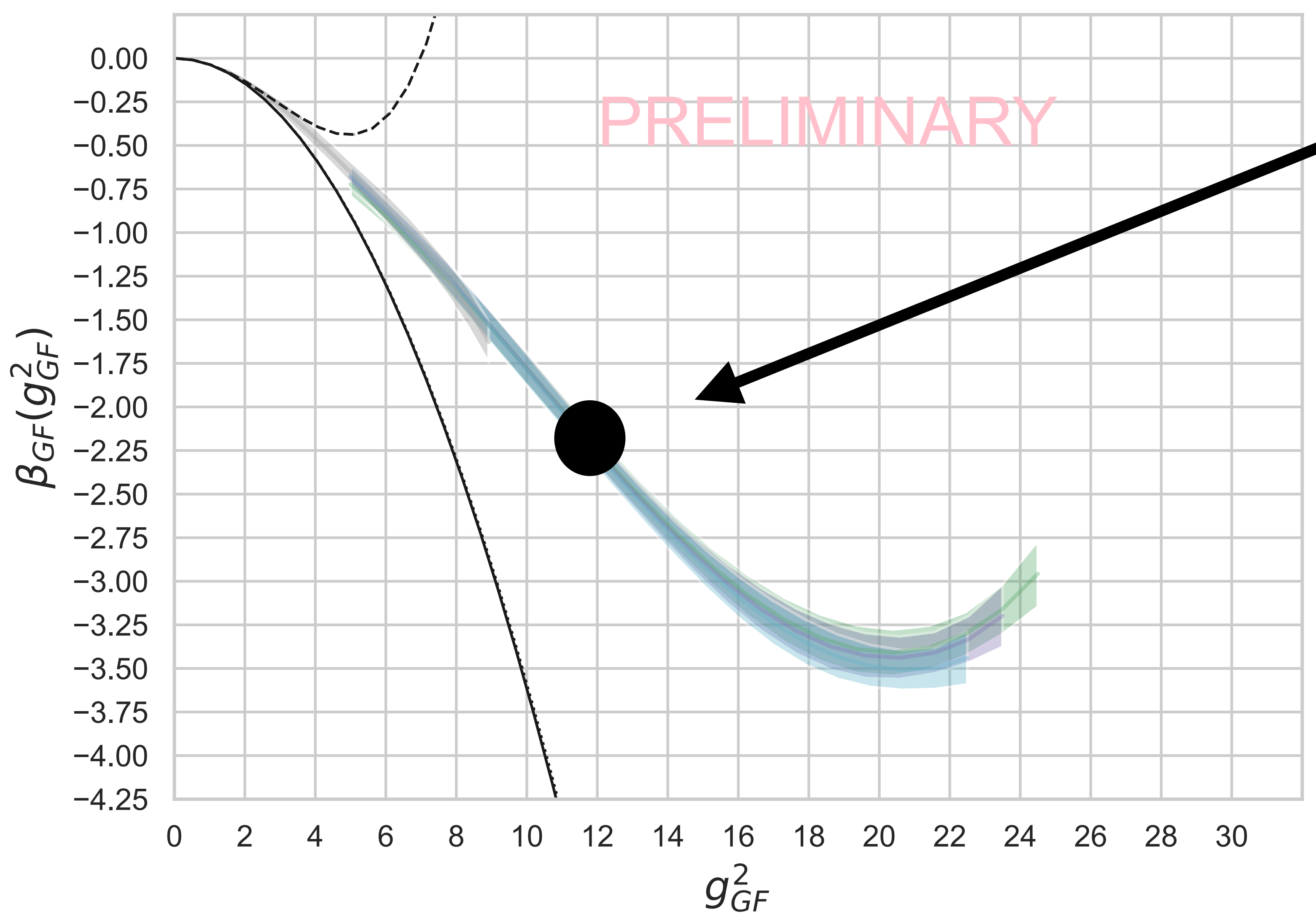
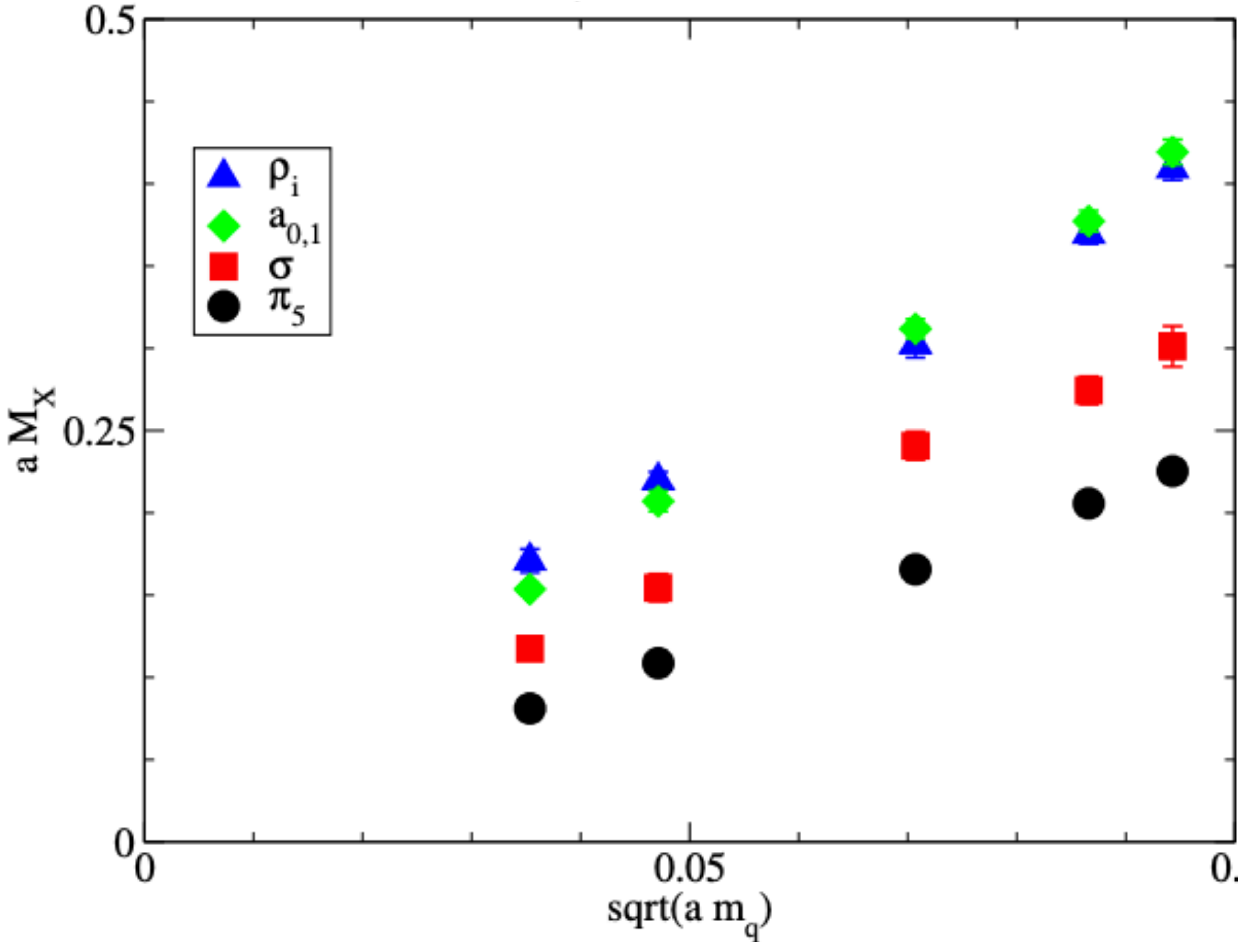
These simulations were done at relatively weak coupling





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LSD Coll. arXiv: 1506.08791

A.H. *Phys.Rev.D* 106 (2022) 1, 014513
A.H., Peterson, in prep
LSD, in progress

Mass generation

$SU(N_c)$ gauge + N_f massless fermions:

- $N_f < N^*$: confinement and spontaneous chiral symmetry breaking
  - massless Goldstones + massive composite states
- $N_f > N^*, N_f < N_{IR}$: conformal
  - massless states with hyperscaling

Could there be a mechanism that leads to
confinement and gapped spectrum but preserves chiral symmetry?
(Symmetric mass generation)

➡ all hadronic states are massive

➡ such phase could show up in the strong coupling of conformal systems

Only if the system is **free of all ’t Hooft anomalies**

Symmetric mass generation (SMG)

Fidowski, Kitaev (2010) d=1+1
Y-Z You, C. XU (2015),Razamat, Tong (2020)

Recent developments in CM show SMG phases:

- ➡ Even in vector-like systems there are discrete (e.g. time reversal) symmetries that can be anomalous.
- ➡ Only systems with multiples of 8 Dirac or 16 Majorana fermions are anomaly free (D=3, 4)
- ➡ When these anomalies cancel there can be symmetric mass generation
- ➡ but 4-fermion interactions is needed to trigger it

Symmetric mass generation (SMG)

A.H., Neuhaus, 1986
Chandrasekharan (2012)...
Catterall (2020),(2023),...

On the lattice:

➡ Staggered fermions are Kaehler-Dirac fermions distributed in a 2^4 hypercube
- massless fermions have exact U(1) symmetry
- on spherical lattice they exhibit Z_4 mixed anomaly, cancelled with 2 sets of fields
- (on a torus U(1) symmetry remains)

➡ 2 sets of staggered fermions are anomaly free

➡ 4-fermion interaction is needed to trigger SMG:
can come from gauge interaction or Yukawa

SU(3) gauge + 8 fundamental flavors

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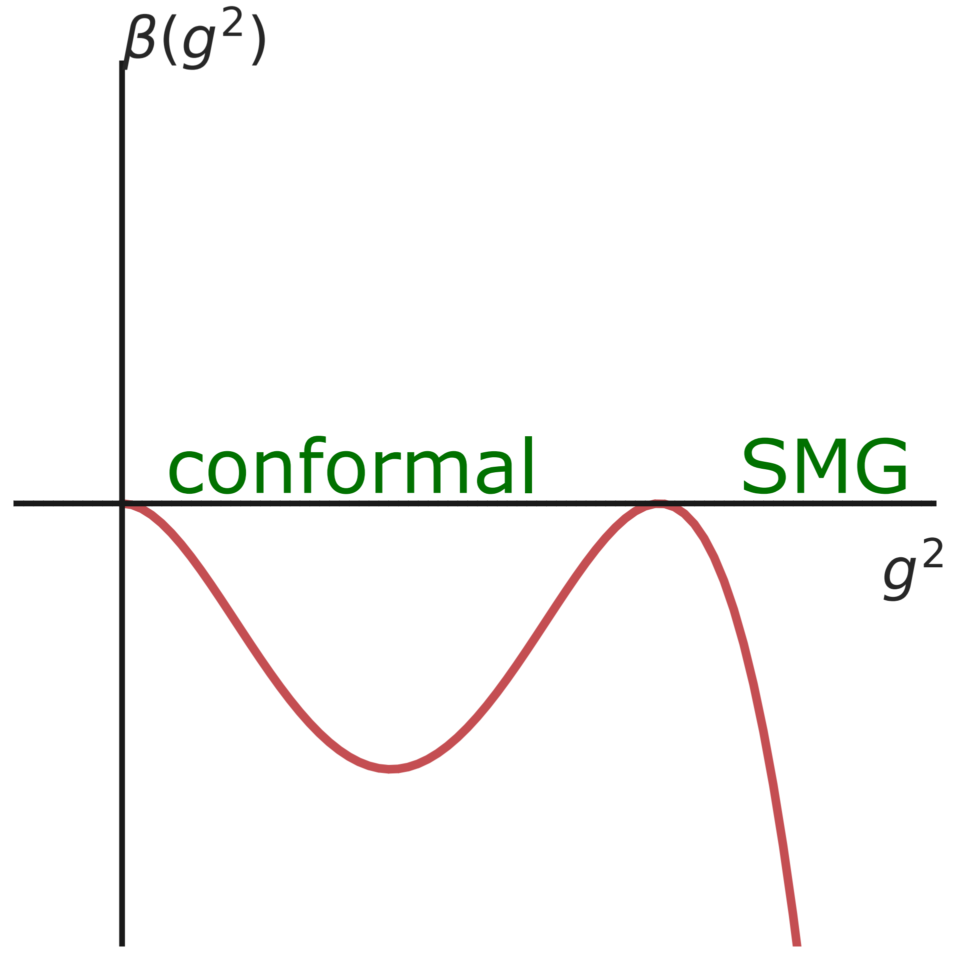

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Some numerical details

Digress: Not all lattice actions are equal….

Large cutoff effects can trigger unphysical bulk phase transitions :
need improved action

➡ **Pauli-Villars improvement:** AH, Shamir, Svetitsky, PRD104, 074509 (2021)

Add heavy PV bosons

-same interaction as fermions but with bosonic statistics

- $S_{eff} < 0$, β increases

  - in the IR the heavy flavors decouple, do not change physics

  - equivalently: range of effective gauge action is $\sim exp(-2am_{PV})$

- Add many PV bosons reduce the lattice fluctuations

PV improvement was essential in the β function results with $N_f = 12$, 10, 8

Meson spectrum

Zero momentum correlators $C(t) = \sum_{\bar{x},\bar{y}} \langle O_S(\bar{x}, t=0) O_S(\bar{y}, t) \rangle$

"PS/S states" :	spin ⊗ taste	in terms of 1-component fields	
pseudoscalar :	$P1 = \gamma_5 \otimes \gamma_5$:	$\mathcal{O}_S = \bar{q}(\bar{x})\, q(\bar{x})\, (-1)^{x_1+x_2+x_3}$	parity partners
scalar :	$S1 = \gamma_0\gamma_5 \otimes \gamma_0\gamma_5$:	$\mathcal{O}_S = \bar{q}(\bar{x})\, q(\bar{x})$	
pseudoscalar :	$P2 = \gamma_5 \otimes \gamma_i\gamma_5$:	$\mathcal{O}_S = \bar{q}(\bar{x}) U_i(\bar{x}) q(\bar{x}+i)(-1)^{x_1+x_2+x_3}$	parity partners
scalar :	$S2 = \gamma_0\gamma_5 \otimes \gamma_0\gamma_i\gamma_5$:	$\mathcal{O}_S = \bar{q}(\bar{x}) U_i(\bar{x}) q(\bar{x}+i)$	

- all four operators couple to scalar and pseudoscalar, but mostly to one only
- P1 is the lightest state
- Simulations done at $am_f = 0.0 - 0.10$ on $16^3 \times 32$ and $24^3 \times 64$ volumes

Correlators are parity doubled:

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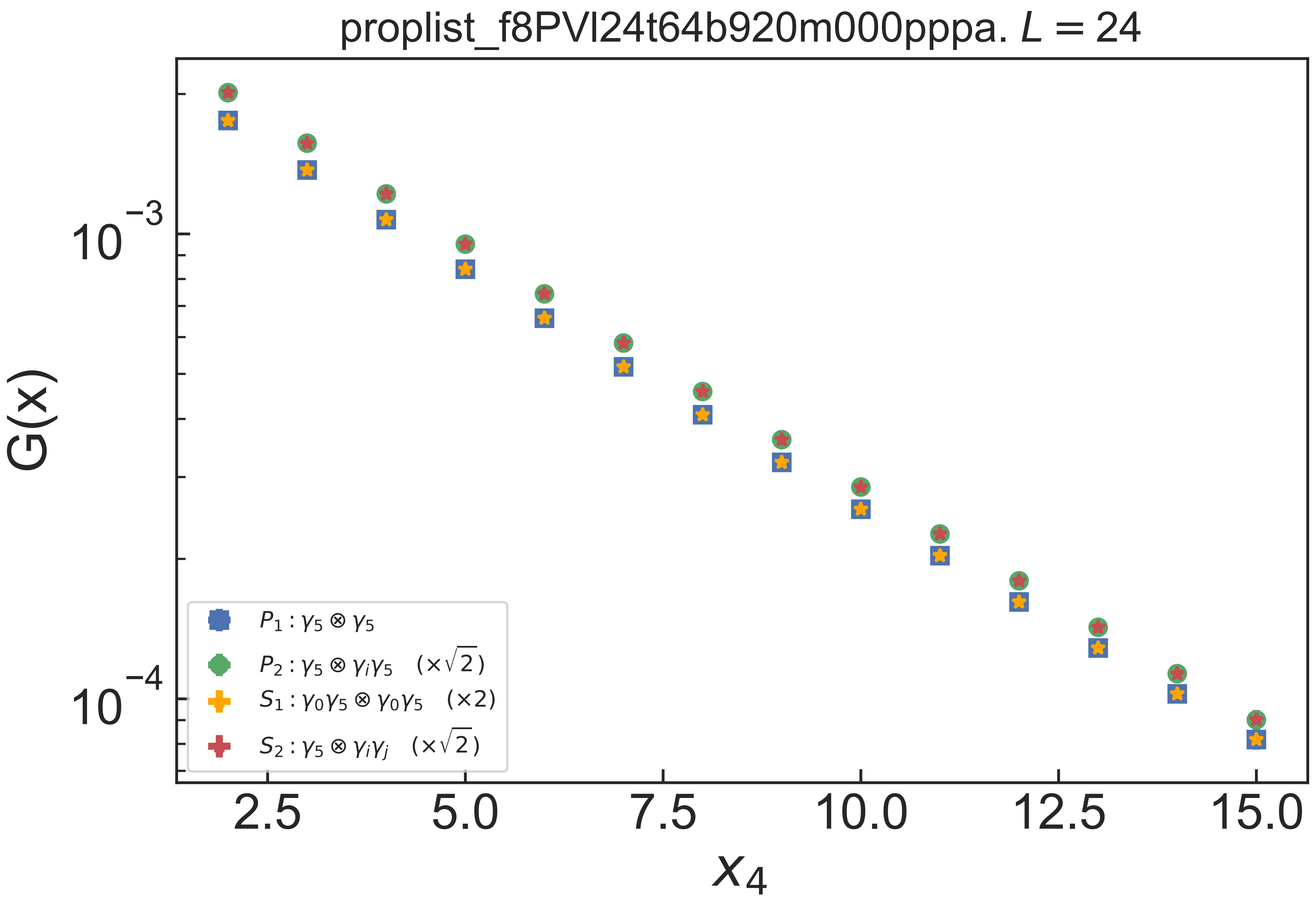
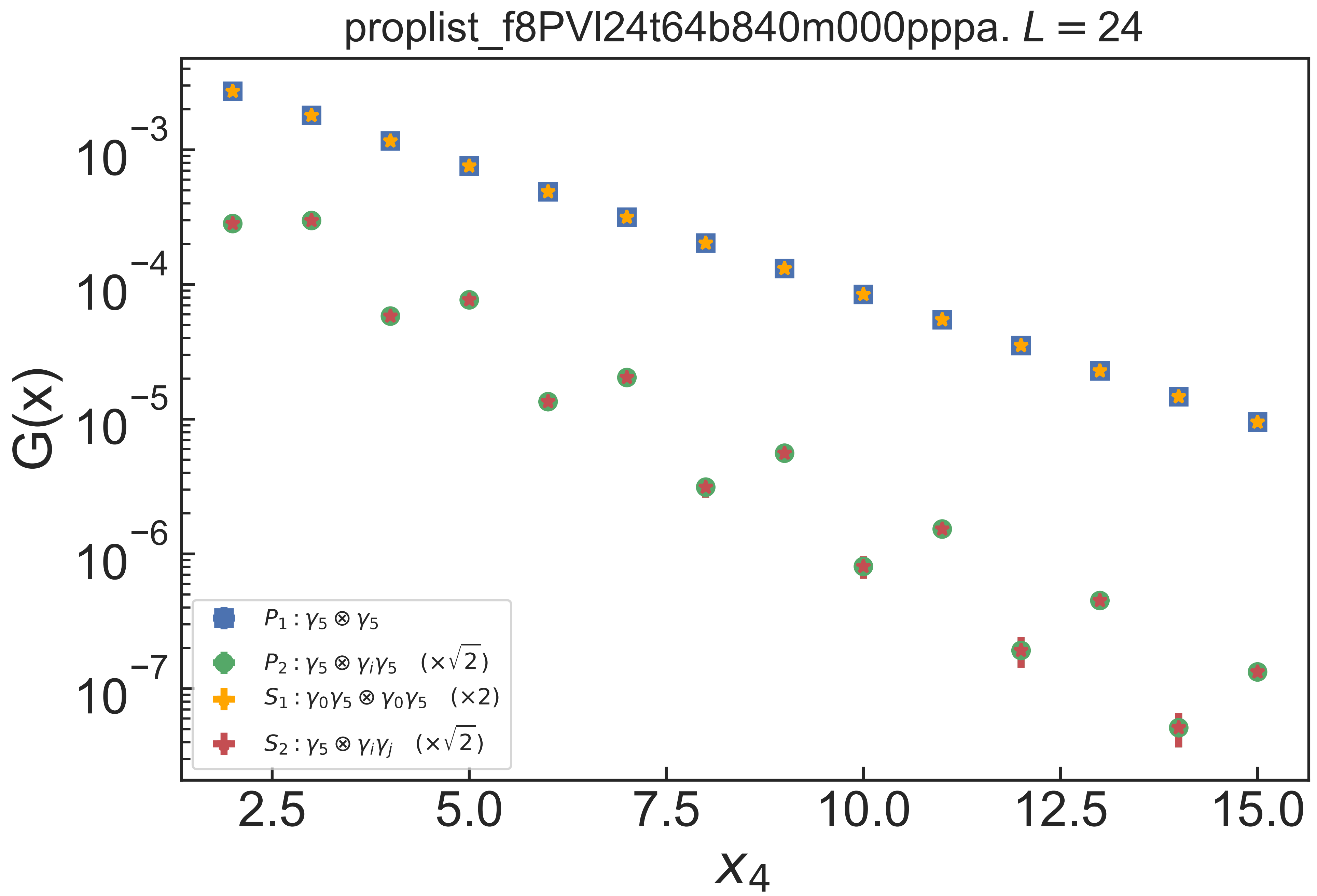
pseudo scalar and scalar correlators





Weak coupling phase

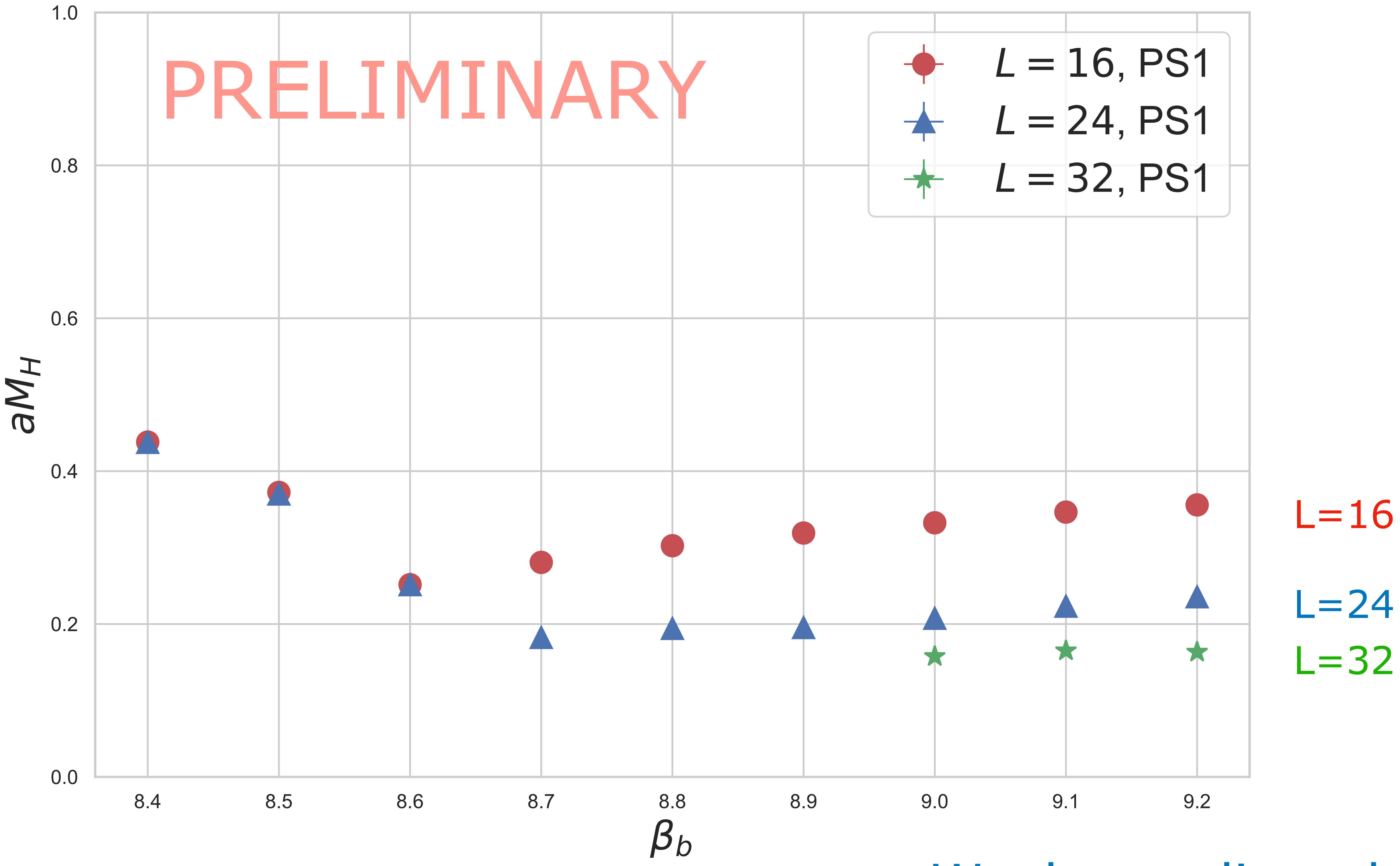
- chirally symmetric (P = S)
- P1,P2, S1,S2 are nearly degenerate
  (taste symmetry / breaking)

$\not{S}^4$ phase

- chirally symmetric (P = S)
- true for all other parity pairs
- P1-P2, S1-S2 are broken

Spectrum is gapped in $\cancel{S^4}$:

P1 meson mass vs β



S4 phase :

- M_P independent of the volume
- mesons and baryons are massive in the infinite volume chiral limit

Weak coupling phase :

- $M_H \propto 1/L$ (conformal)
- volume-squeezed

Spectrum is gapped in $\not{S}^4$:

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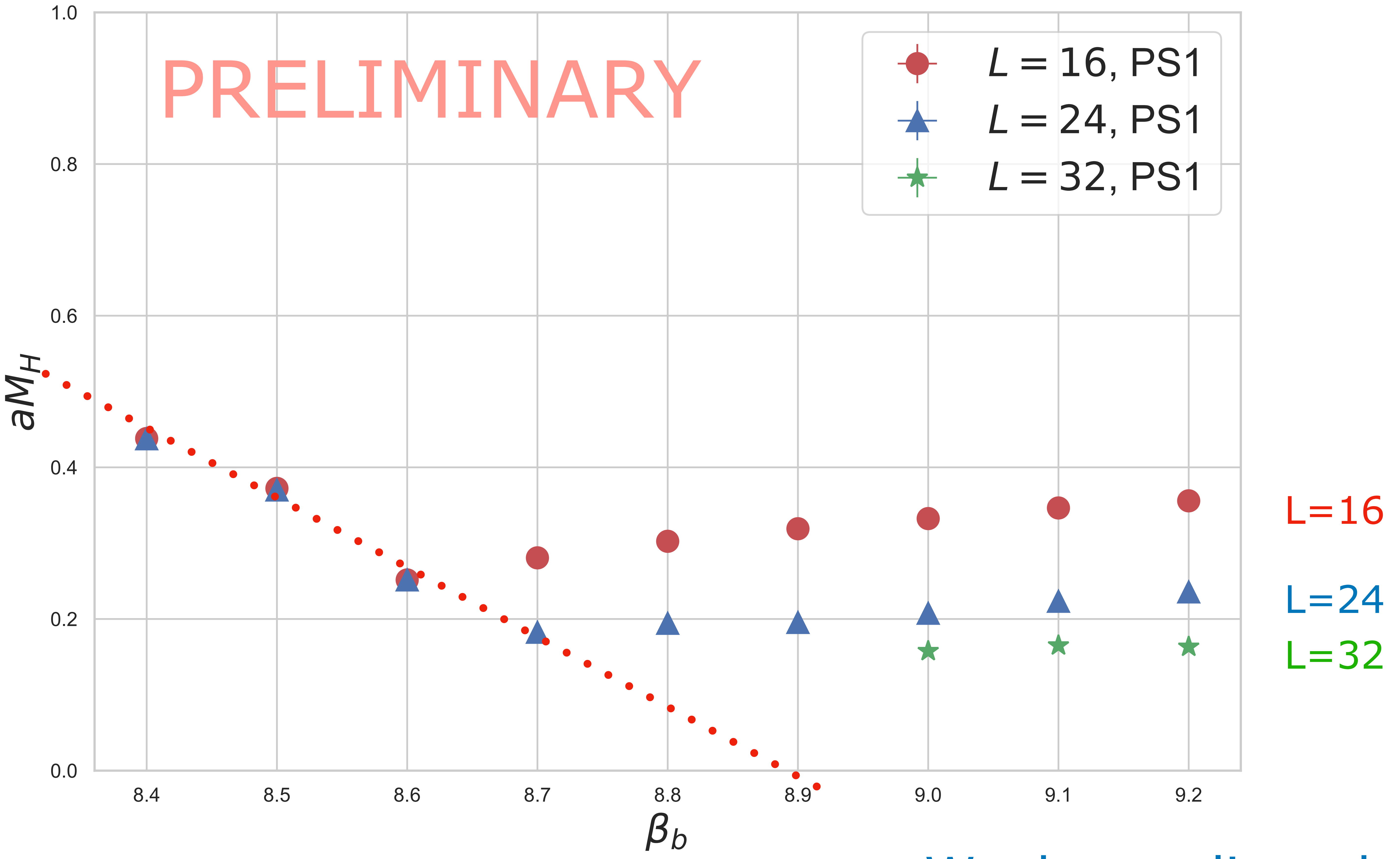

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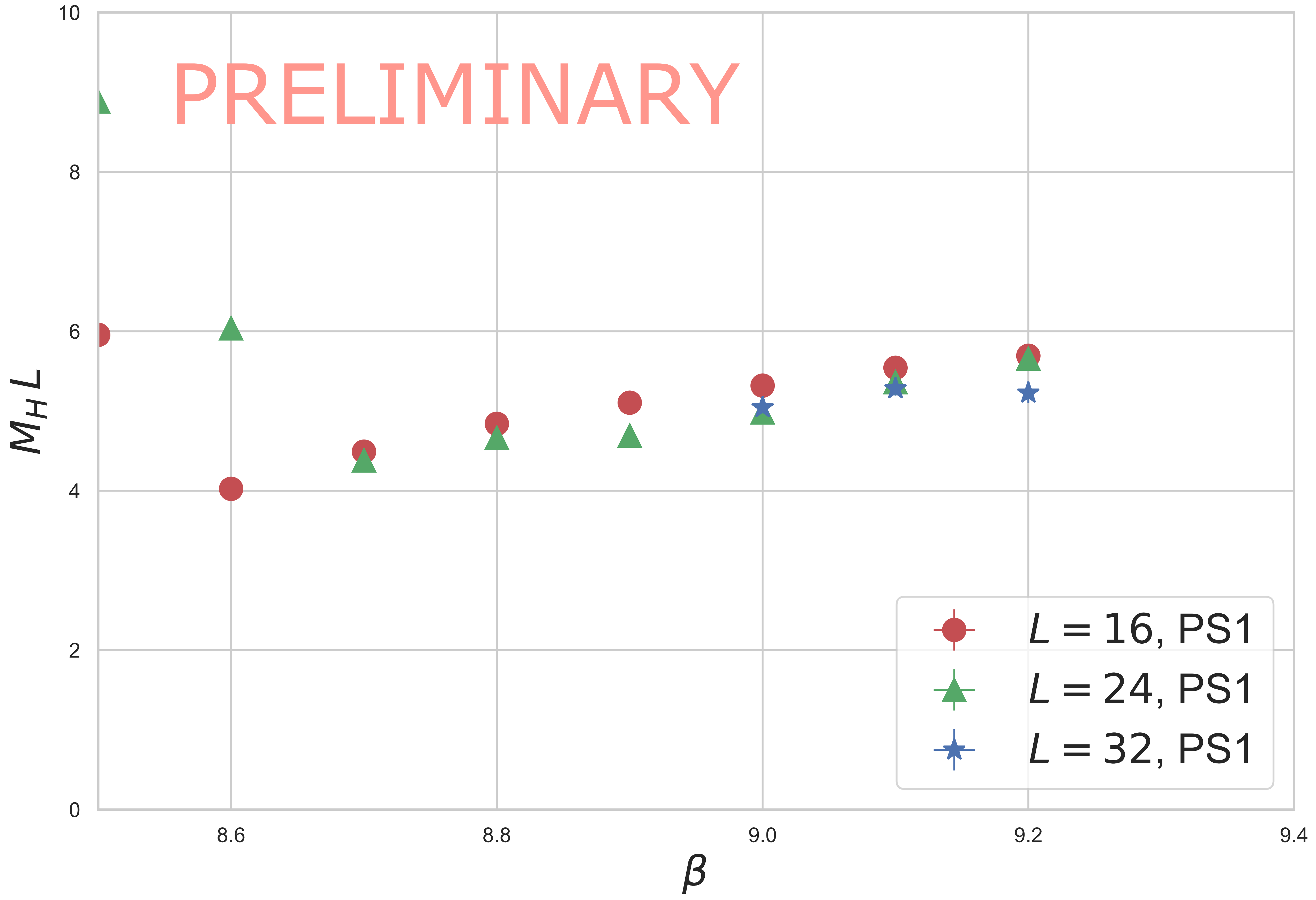
Spectrum is conformal in the weak coupling:

P1 meson mass vs β



Weak coupling phase :

- $M_H \propto 1/L$ (volume squeezed conformal)
- $M_H L$ could serve as a running coupling $\to \beta$ function

Order of the phase transition / FSS :

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Finite size scaling/curve collapse analysis:

At the critical point $\beta \to \beta_*$ RG scaling predicts $g^2_{GF}(\beta, L) = f\left(L/\xi\right)$

- ξ: correlation length
- $f(x = L/\xi)$ unique curve, independent of L

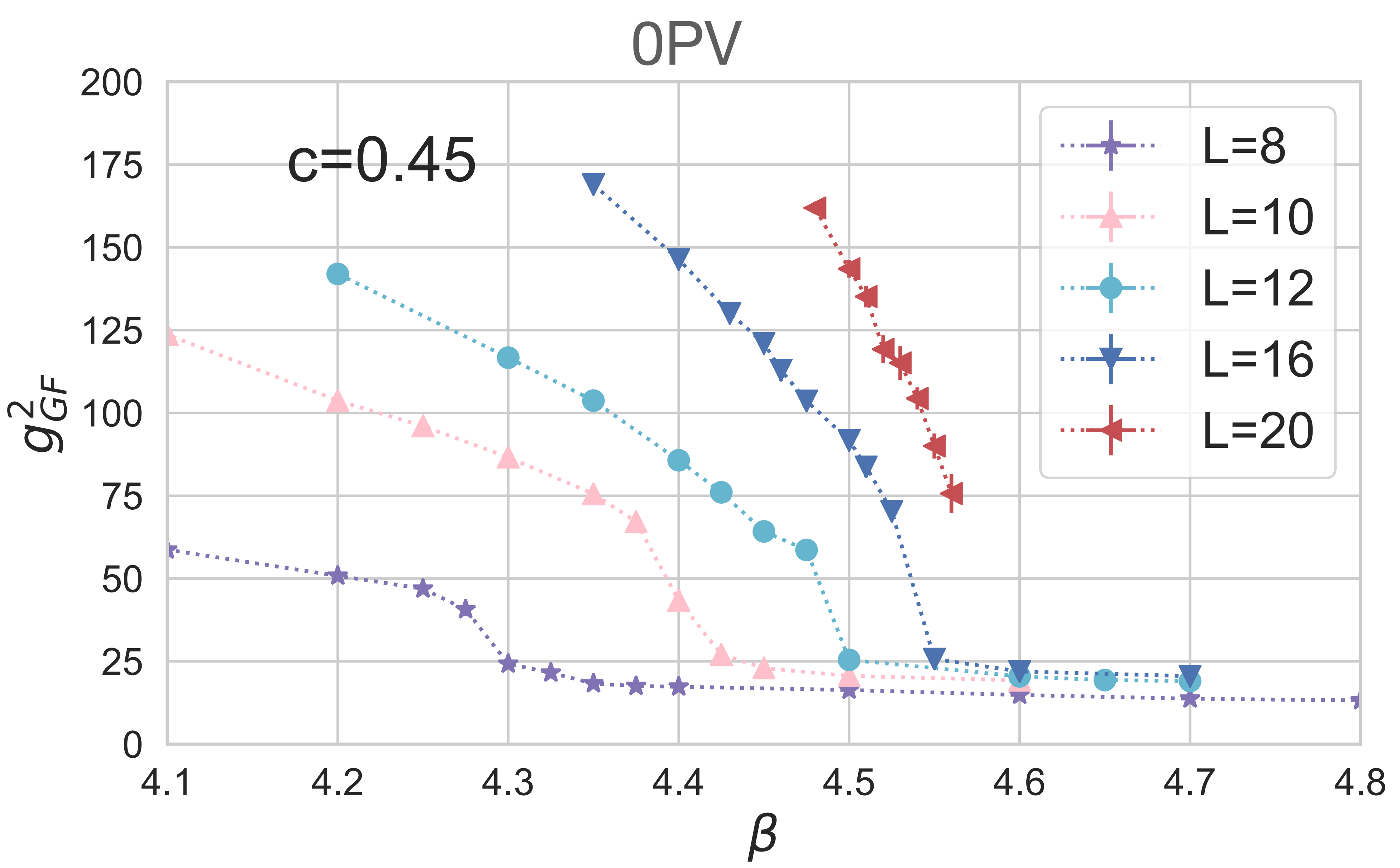
- 2nd order scaling: $\xi \propto |\beta/\beta_* - 1|^{-\nu}$, $x = L\,|\beta/\beta_* - 1|^{\nu}$

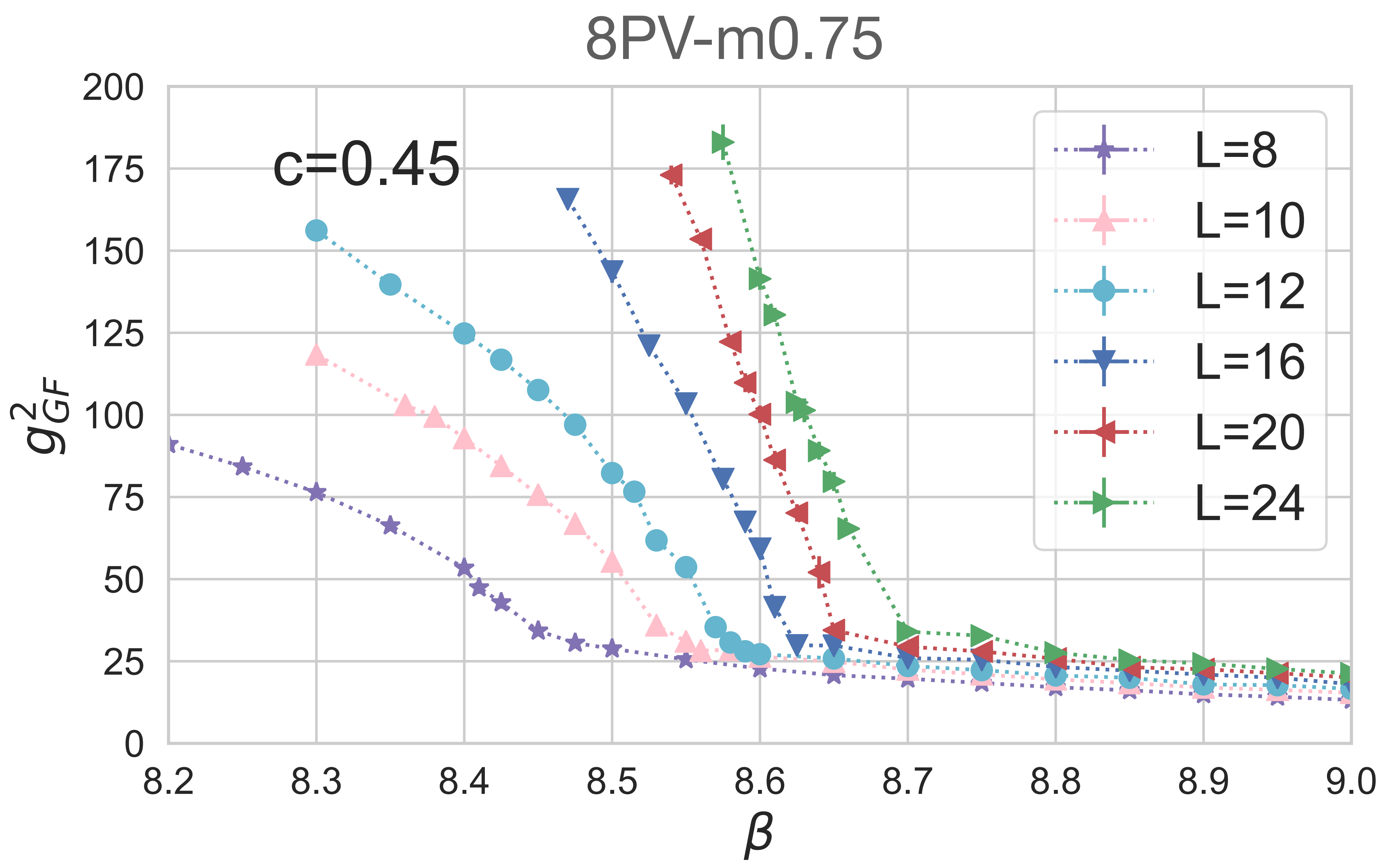
- 1st order scaling: like 2nd order but $\nu = 1/d = 0.25$

- BKT or walking scaling: if $\beta(g^2) \sim (g^2 - g_*^2)^2 \;\to\; \xi \propto e^{\zeta/|\beta/\beta_* - 1|}$, $x = L\,e^{-\zeta|\beta/\beta_* - 1|^{-1}}$

Find the exponents by standard curve-collapse analysis ;

Phase transition with GF coupling g^2_{GF}





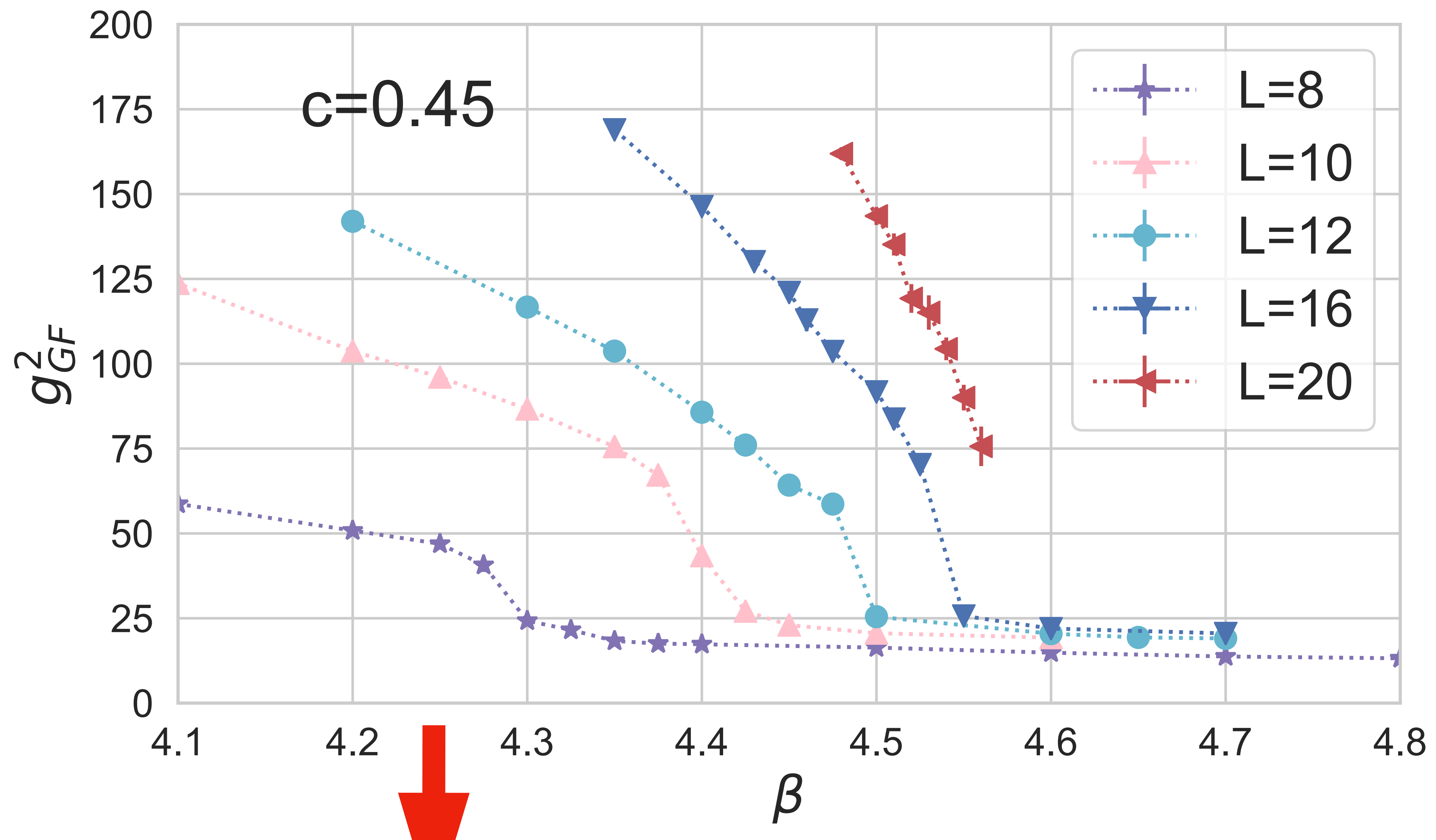
Observable:

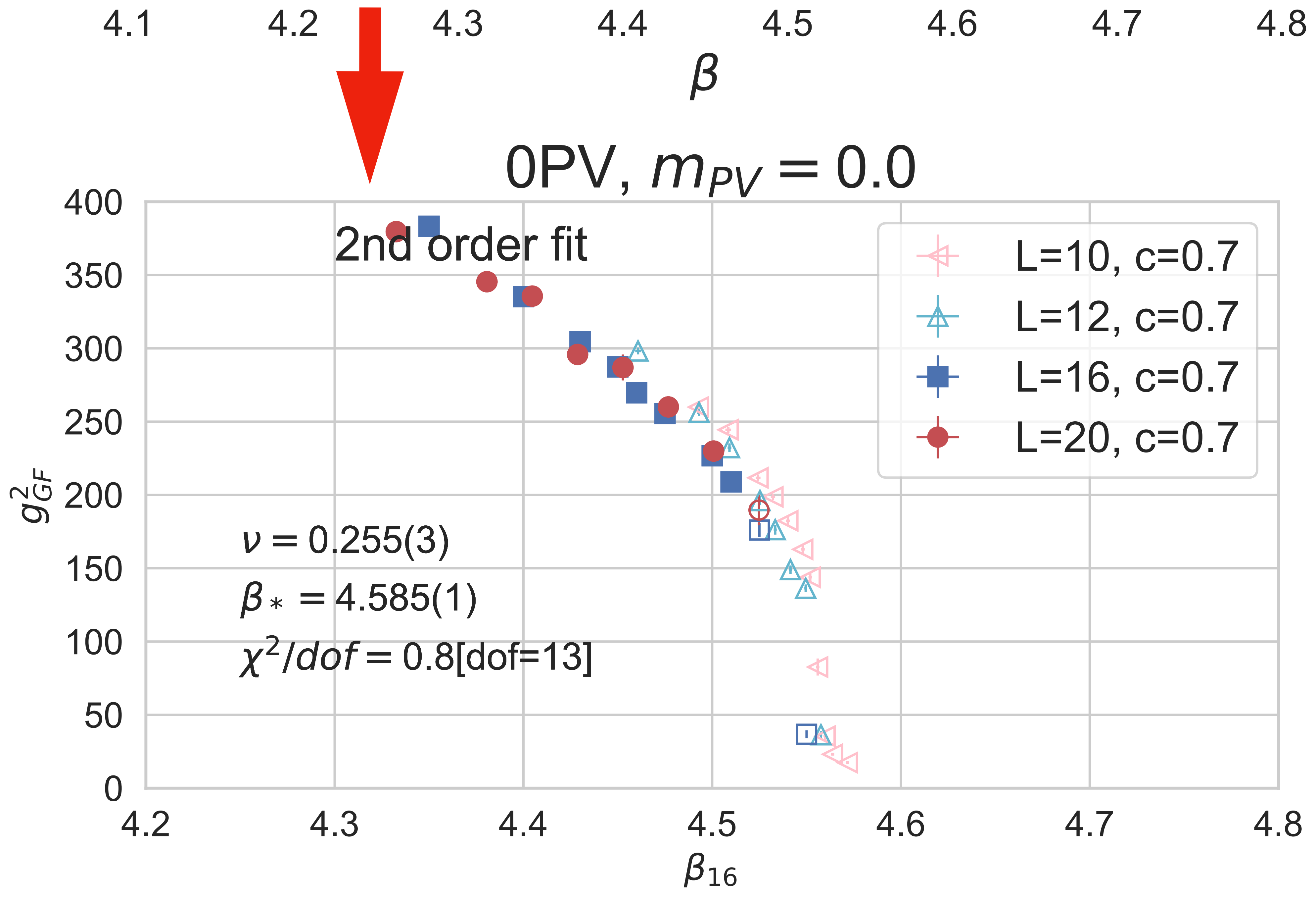
Finite volume gradient flow (GF) coupling:

$$g^2_{GF}(\beta, L; t) = \mathcal{N} t^2 \langle E(t) \rangle_{\beta,L} \;, \quad t/L^2 = c/8, \text{ fixed}$$

- mimics RG blocked observables
- good signal for the phase transition
- good observable for finite size scaling

Curve collapse -original action, no PV





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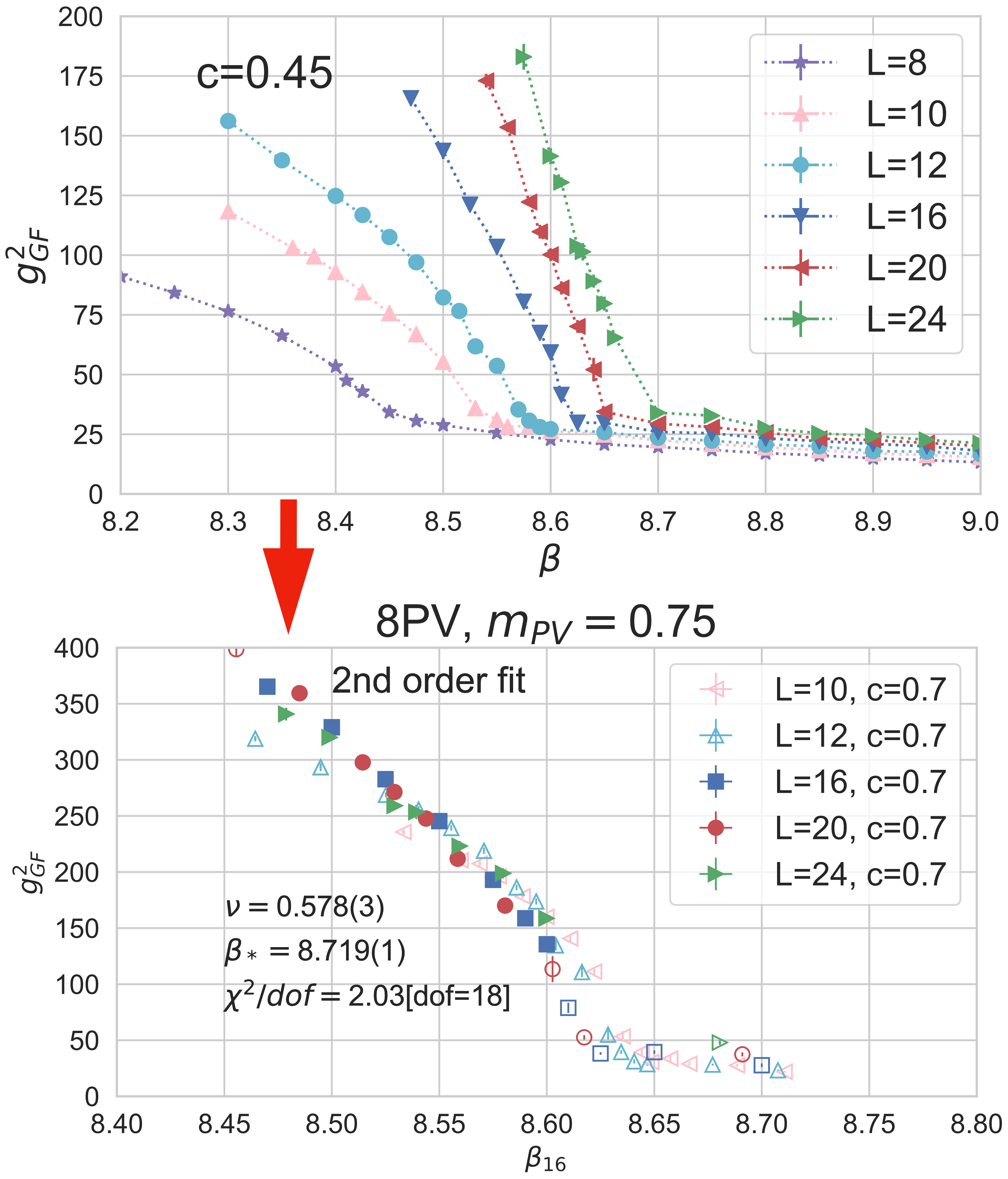
- Good χ^2/dof ,

  $\nu \approx 0.255$

  — consistent with first order transition
- Robust fit:
  - adding/removing volumes does not change the results

Curve collapse - action with **PV**



Second order curve collapse fit:

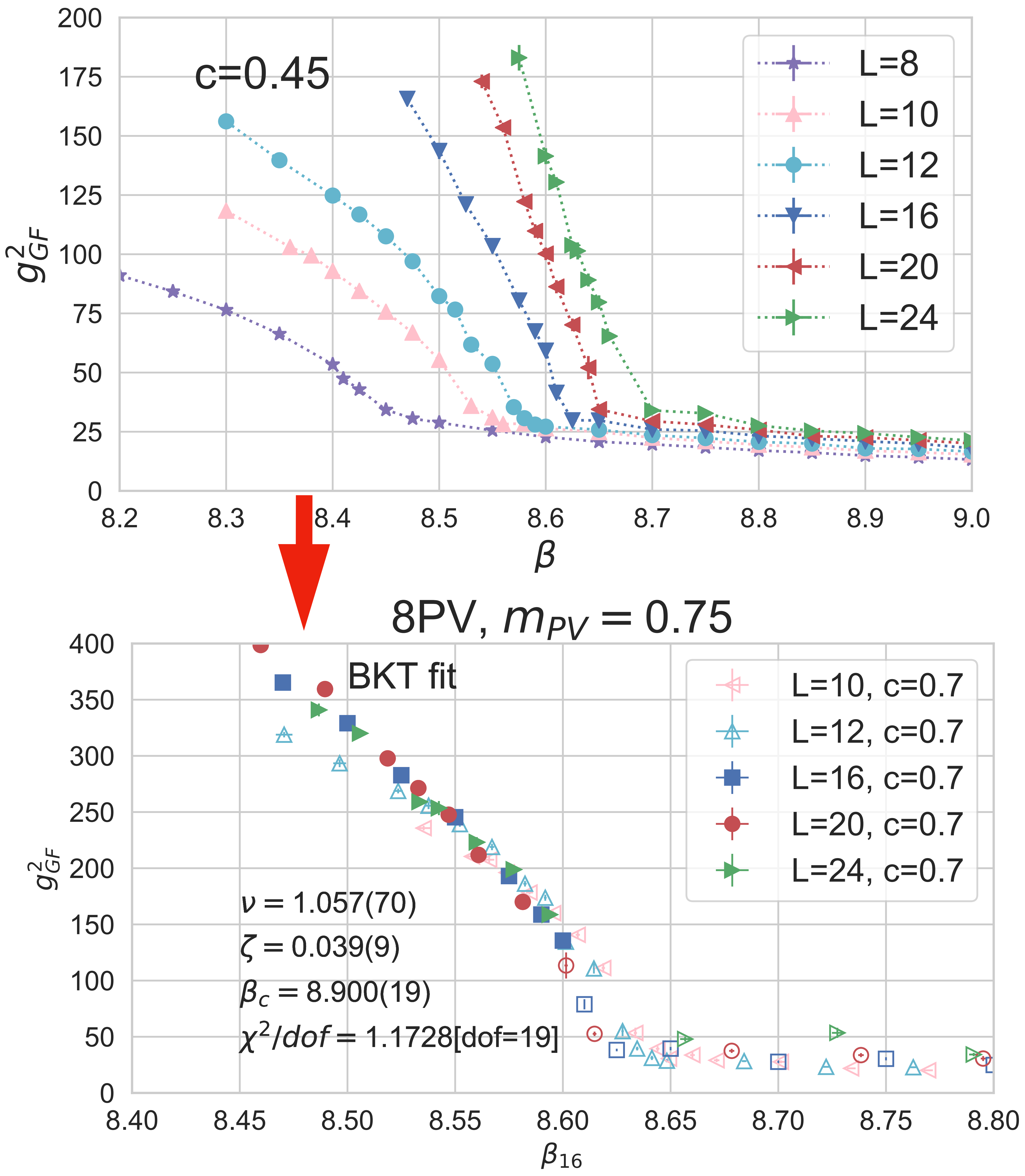
- Acceptable χ^2/dof

  $\nu \approx 0.58 \rightarrow 0.63$

Phase transition is

- NOT consistent with first order transition
- Could be 2nd order transition

Curve collapse - action with **PV**



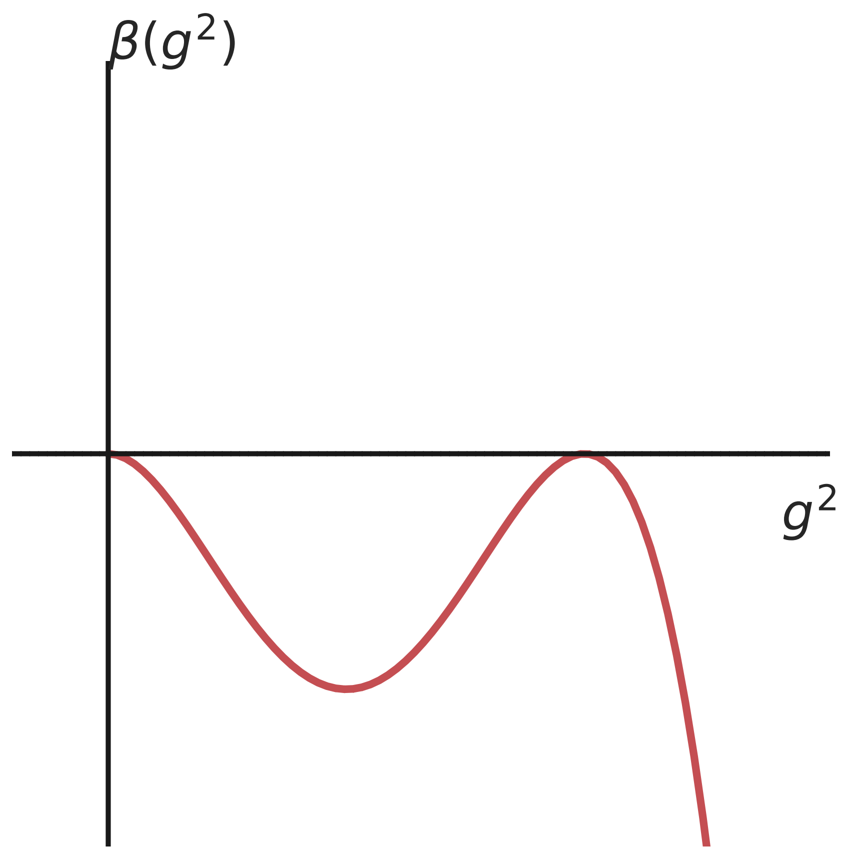
Walking scaling curve collapse fit:

- Good χ^2/dof ,

Summary: $N_f = 8$ ($N_s = 2$ staggered) is special

- The strong coupling phase ($\not{S}^4$):
  - Shows symmetric mass generation:
    - Chirally symmetric and confining
    - gapped

- Lattice simulations with PV improved actions show a smooth phase transition
- Finite size scaling
  - is not consistent with 1st order transition
  - could be 2nd order
  - consistent with "walking scaling" transition



Why would $N_f = 8$ be the conformal sill?

Because it is special: anomaly free, supports SMG

BUT : this picture needs further verification!

EXTRA SLIDES

Staggered fermions

are Kaehler-Dirac fermions distributed in a 2^4 hypercube

Becher, Joos 1982

$$S_f = \frac{1}{2}\sum_{n,\mu}(\bar{\chi}_n \alpha_\mu(n) U_\mu(n)\chi_{n+\mu} + cc) + m_f \sum_n \bar{\chi}_n \chi_n \;, \qquad \alpha_\mu(n) = (-1)^{n_0+\ldots n_{\mu-1}}$$

χ : 1-component fermion

U(1) chiral symmetry: $\chi(x) \to e^{i\alpha\epsilon(x)}\chi(x), \quad \epsilon(x) = (-1)^{x_0+\ldots+x_{D-1}}$

shift symmetry: $\chi(x) \to \xi(x)\chi(x+\mu), \quad \xi(x) = (-1)^{x_{\mu+1}+\ldots+x_{D-1}}$

1 set of staggered fermions ≡ 4 Dirac flavors in flat space, $g_0^2 = 0$

2 sets of massless staggered fermions ≡ 4 sets of reduced staggered

≡ 16 Weyl fermions

Massless staggered fermions suffer from Z_4 mixed anomaly - cancelled when 2 staggered species are present

—> 2 staggered species could exhibit symmetric mass generation

Catterall et al
PRD104,014503 (2021)
PRD107,014501 (2022)

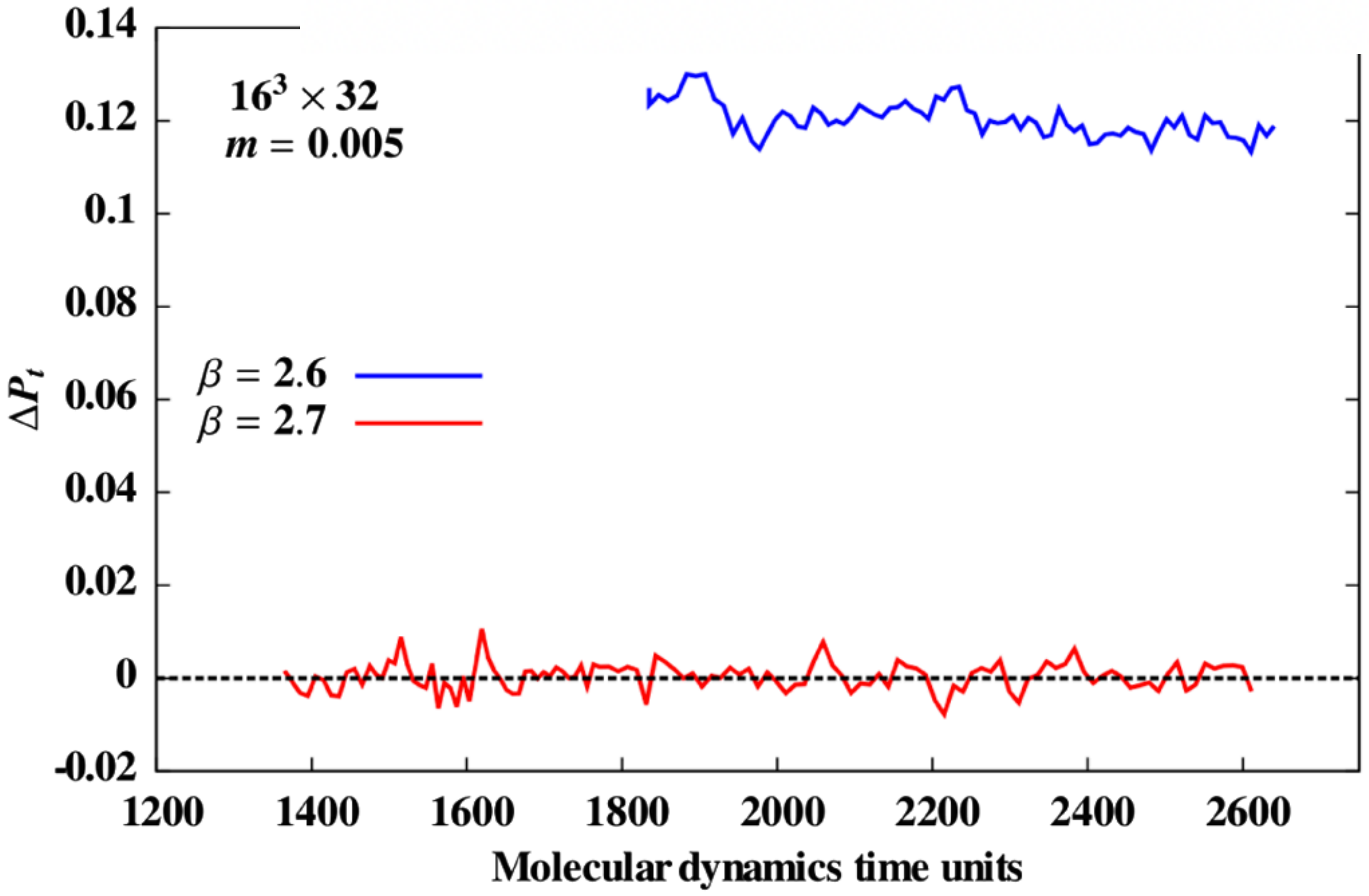
S4 phase

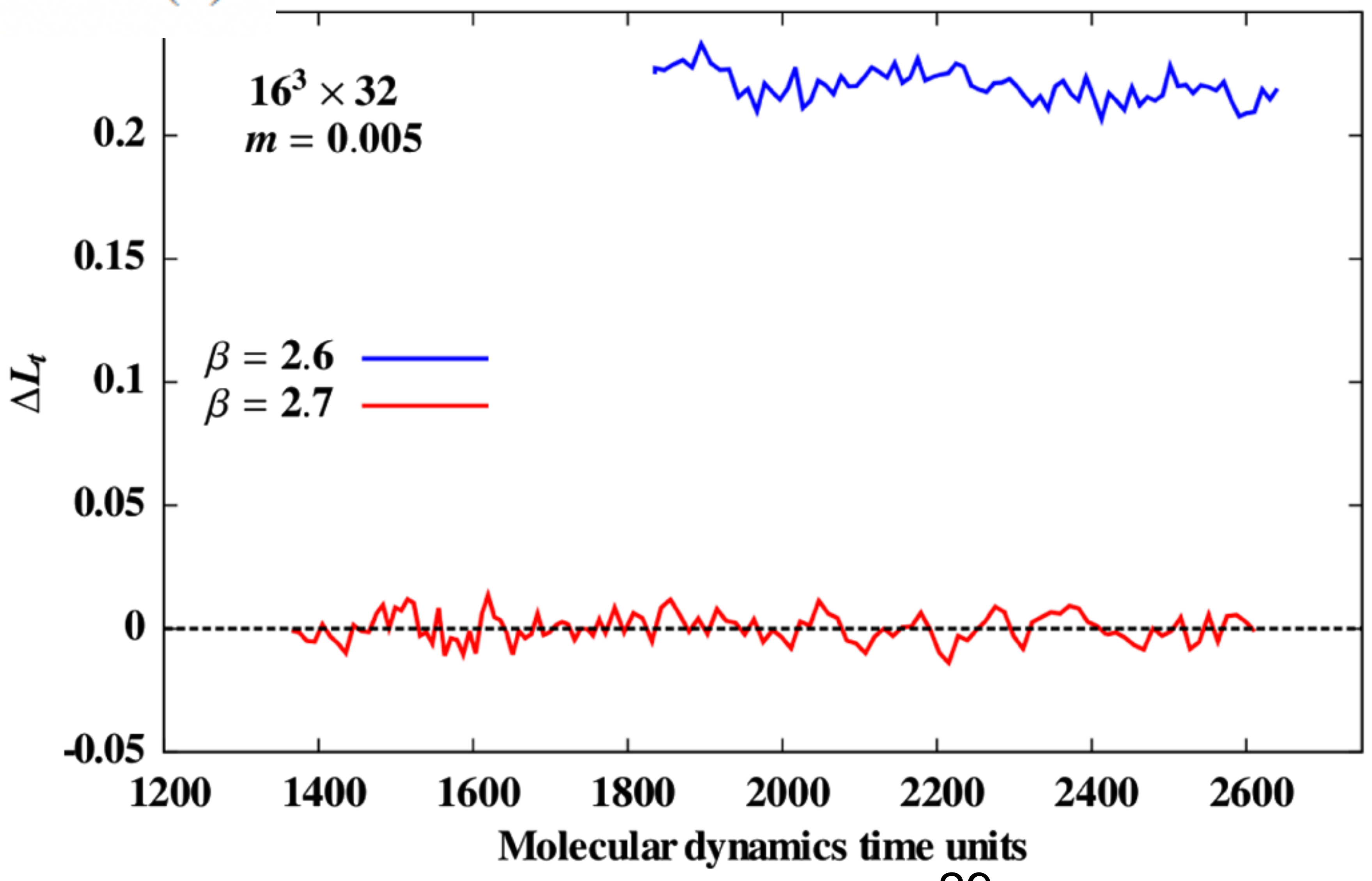
Cheng et al, PRD85, 094509

- Breaks single site translational symmetry
- Confining, all hadrons are heavy in the chiral limit
- Chirally symmetric
- Has a local order parameter that measures staggered symmetry breaking

$$\Delta P_\mu = \langle \mathrm{Re\,Tr}\Box_n - \mathrm{Re\,Tr}\Box_{n+\mu}\rangle_{n_\mu \mathrm{even}}, \quad (3)$$

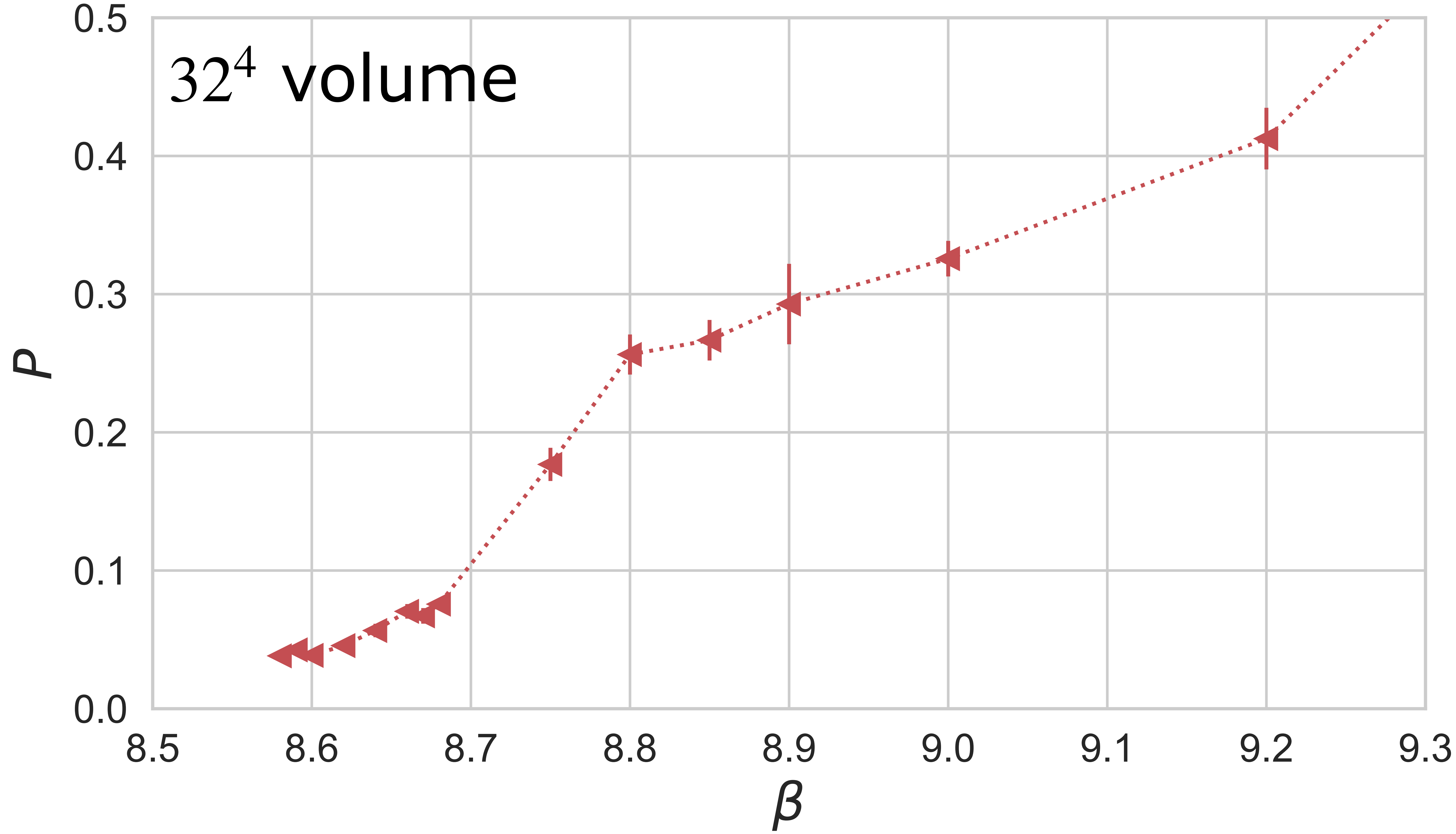
$$\Delta L_\mu = \langle \alpha_\mu(n)\bar{\chi}(n)U_\mu(n)\chi(n+\mu) - \alpha_\mu(n+\mu)\bar{\chi}(n+\mu)U_\mu(n+\mu)\chi(n+2\mu)\rangle_{n_\mu \mathrm{even}}, \quad (4)$$



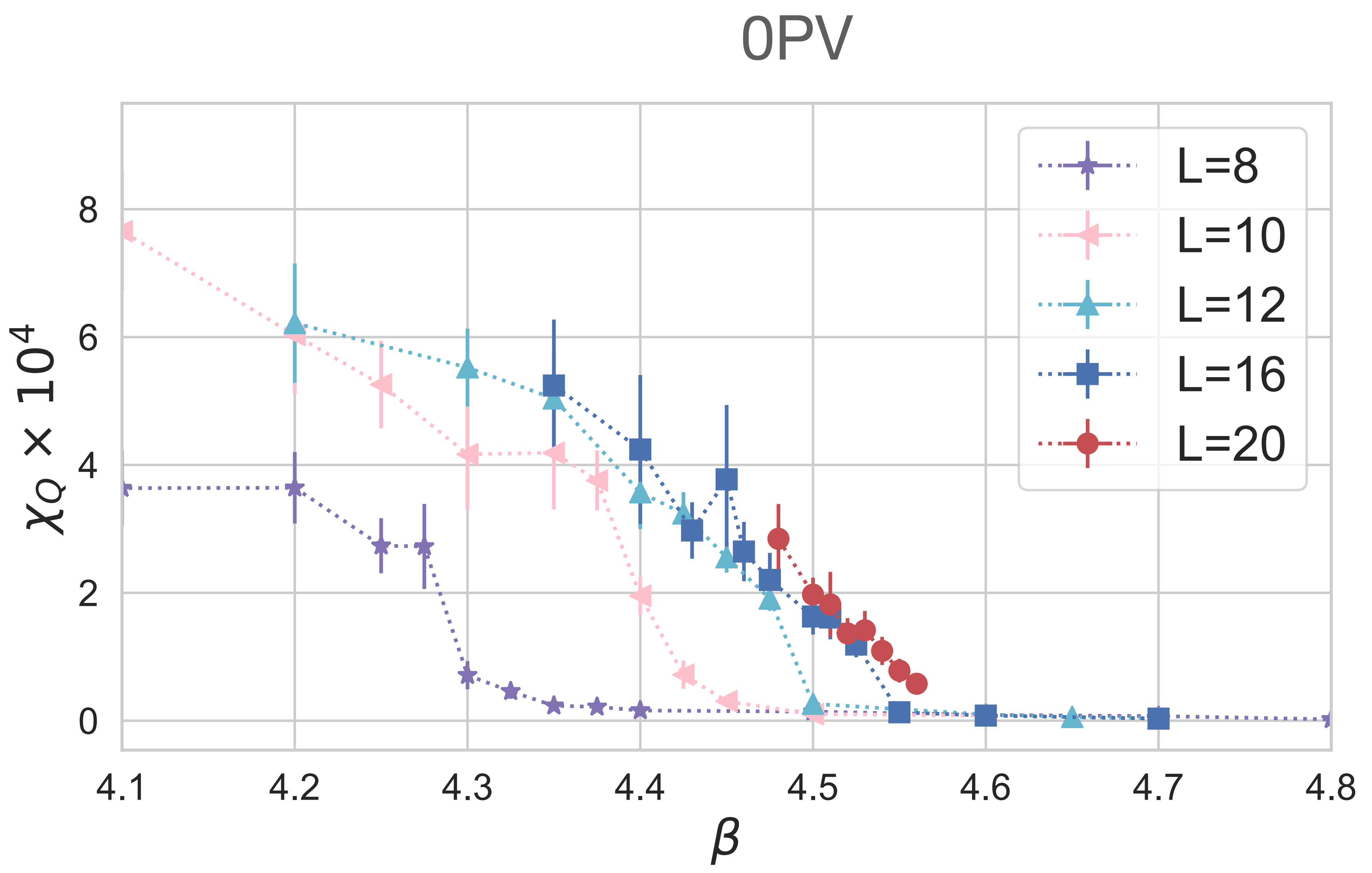


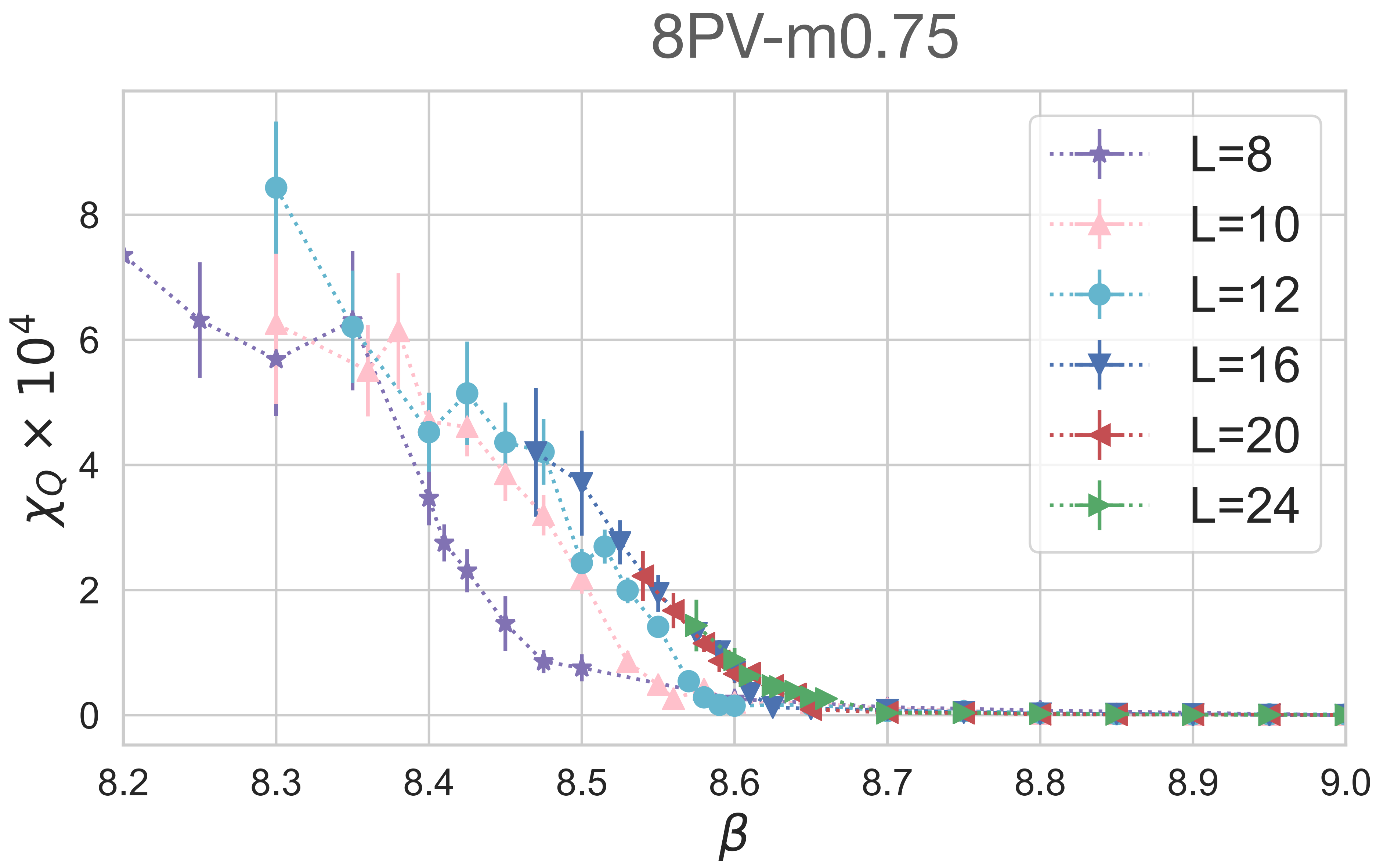
Properties of the $\not{S}^4$ (SMG) phase

Confining: Polyakov loop drops to zero



S4 phase - topological susceptibility





calculated with GF at large flow time

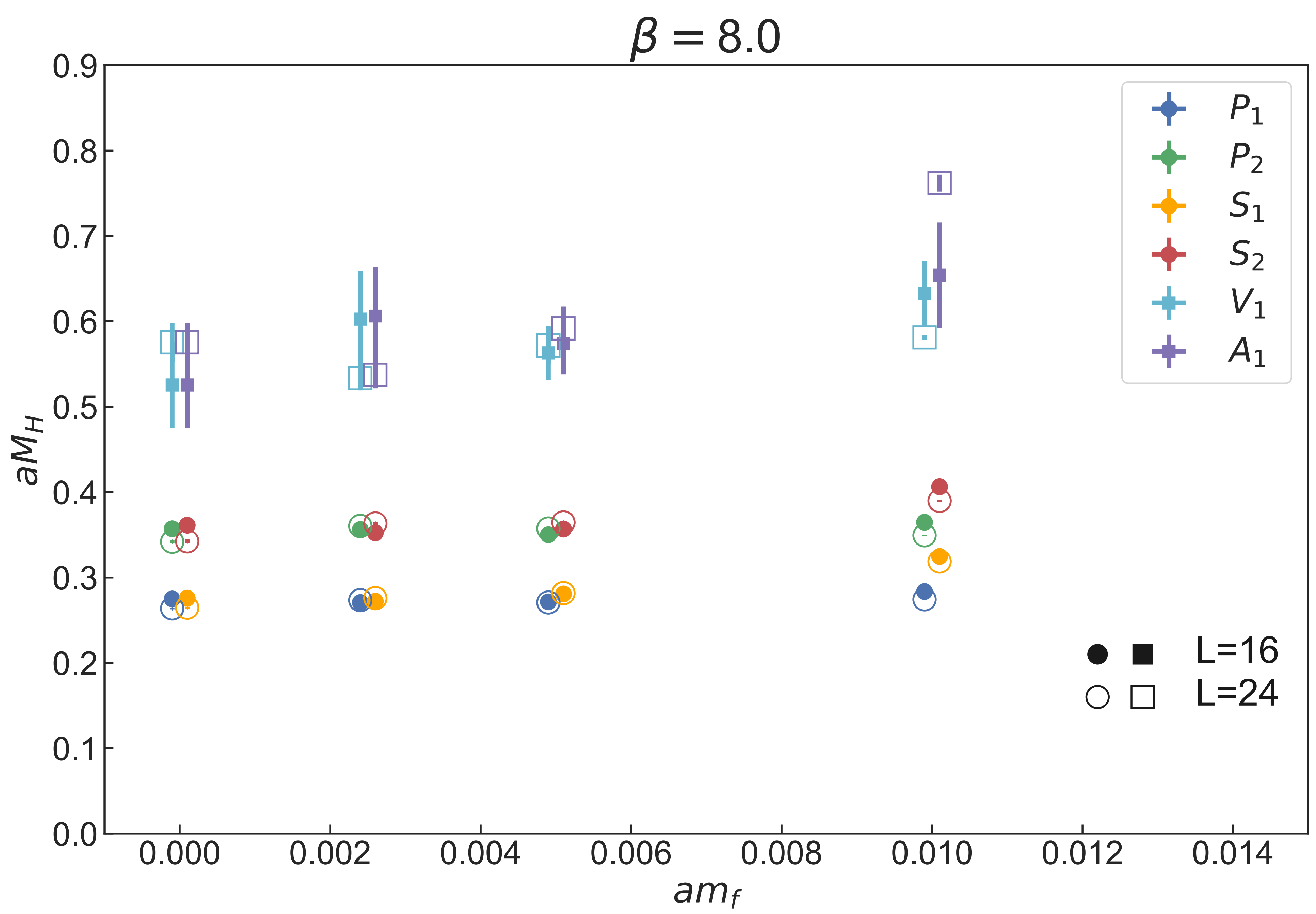
- Both in conformal and chirally broken systems topology is suppressed **in the chiral limit**

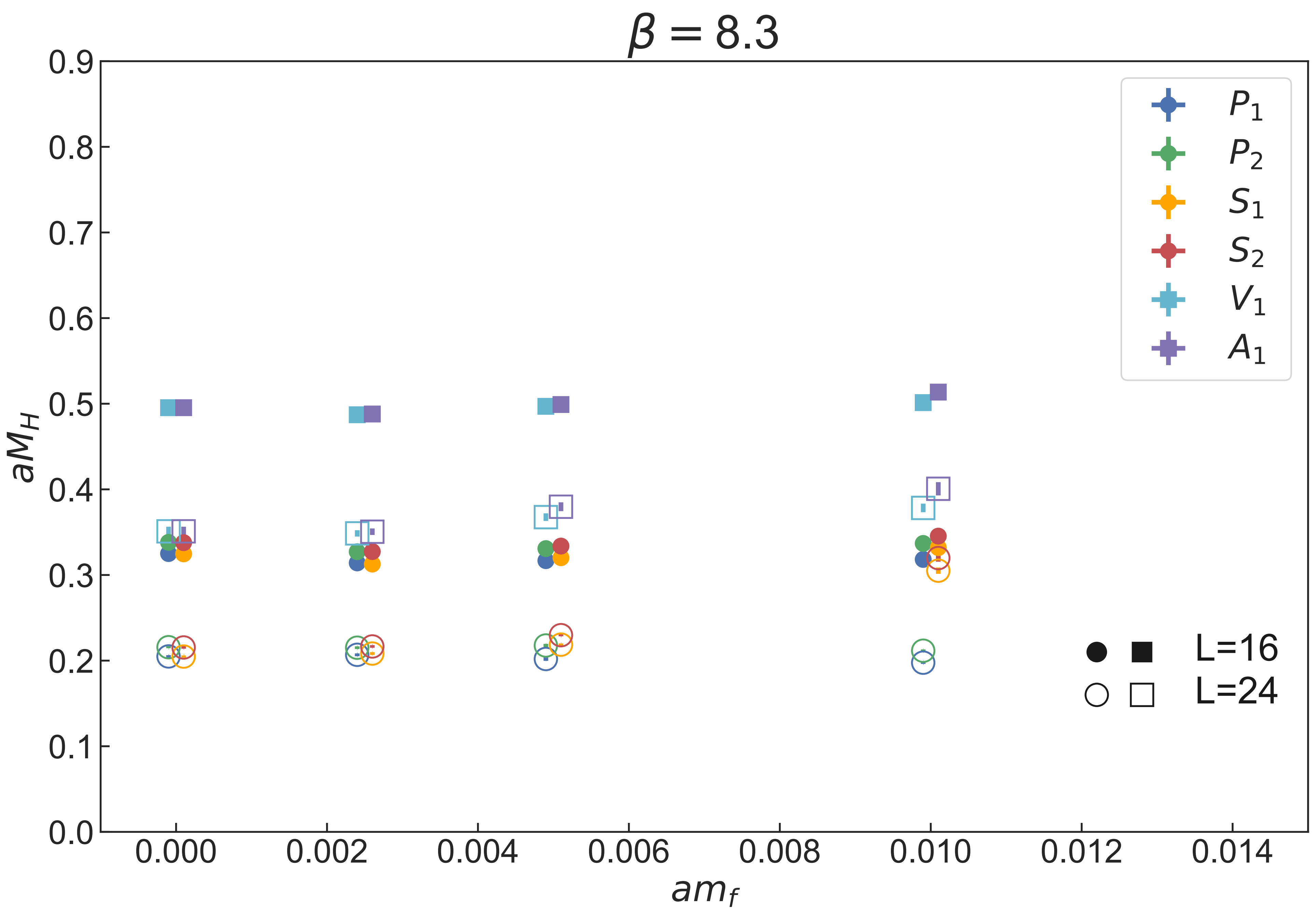
$$\chi_Q = \langle Q^2 \rangle / V = 0$$

- The new strongly coupled phase is full with unpaired instantons
  - how do they avoid the index theorem? (possibly surface modes (?))

Fermion mass dependence :

Meson masses, volume dependence





S4 phase :

- independent of the fermion mass and volume
- mesons are **massive in the infinite volume chiral limit**

Weak coupling phase :

- $M_H \propto 1/L$ (conformal)
- volume-squeezed for $am_f \lesssim 0.01$

Numerical details

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A.H.,C.Peterson, in prep

- Simulations with 2 staggered fermions :
  - no PV improvement
  - 16 PV bosons, $am_{PV} = 0.75$ (heavy PV bosons)
- Volumes up to 40^4
- Finite volume gradient flow (GF) coupling:

$$g^2_{GF}(\beta, L; t) = \mathcal{N} t^2 \langle E(t) \rangle_{\beta,L} \qquad t/L^2 = c/8, \text{ fixed}$$

- mimics RG blocked observables
- good signal for the phase transition
- good observable for finite size scaling