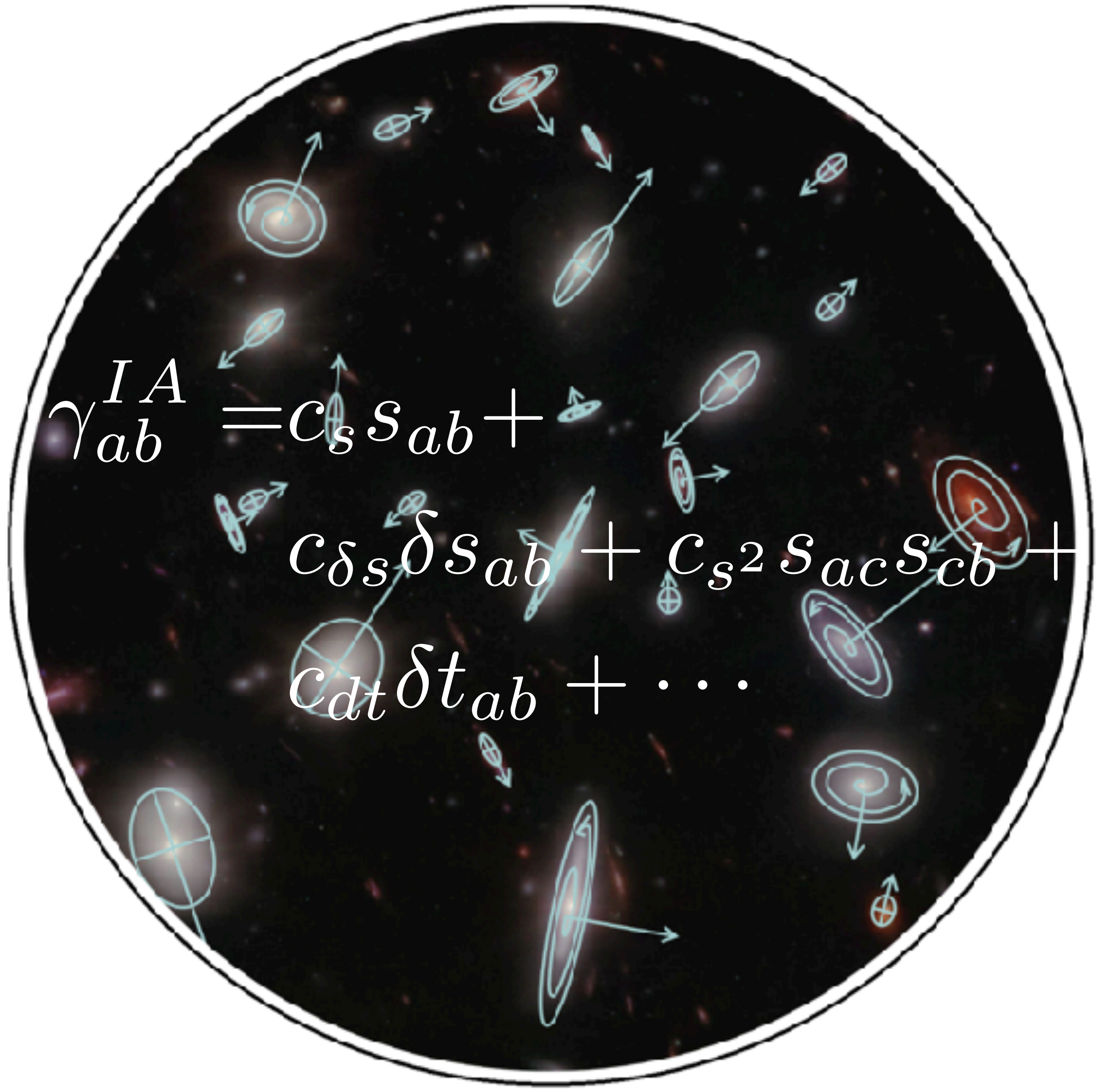
A Lagrangian Theory of Galaxy Shape Statistics



Adapted from Lamman et al 23

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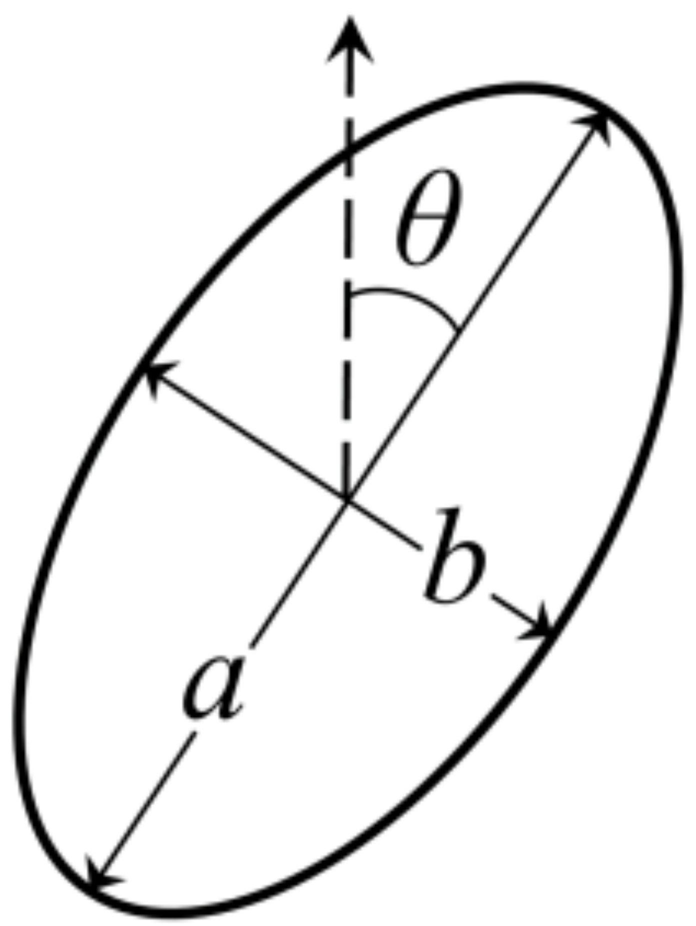
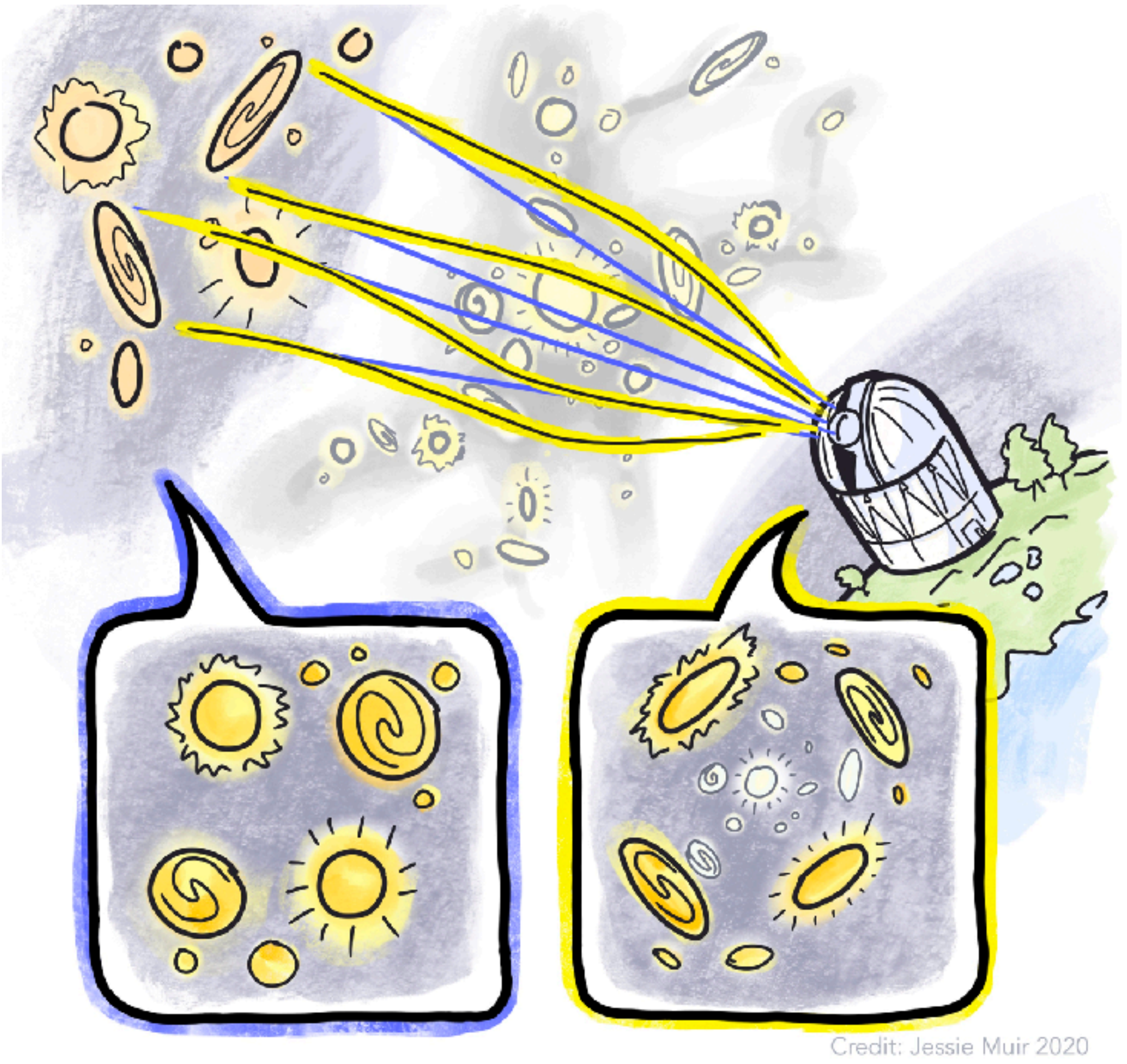
Based on arXiv:2309.16761 w/ Stephen Chen

Cosmic shear surveys measure galaxy shapes

Practically — treat every galaxy as an ellipse.





Cosmic shear signal is teased out from correlations of these shapes

In practice galaxies are 3D objects, every galaxy has an associated ellipsoid — shape described by a 3x3 symmetric traceless matrix.

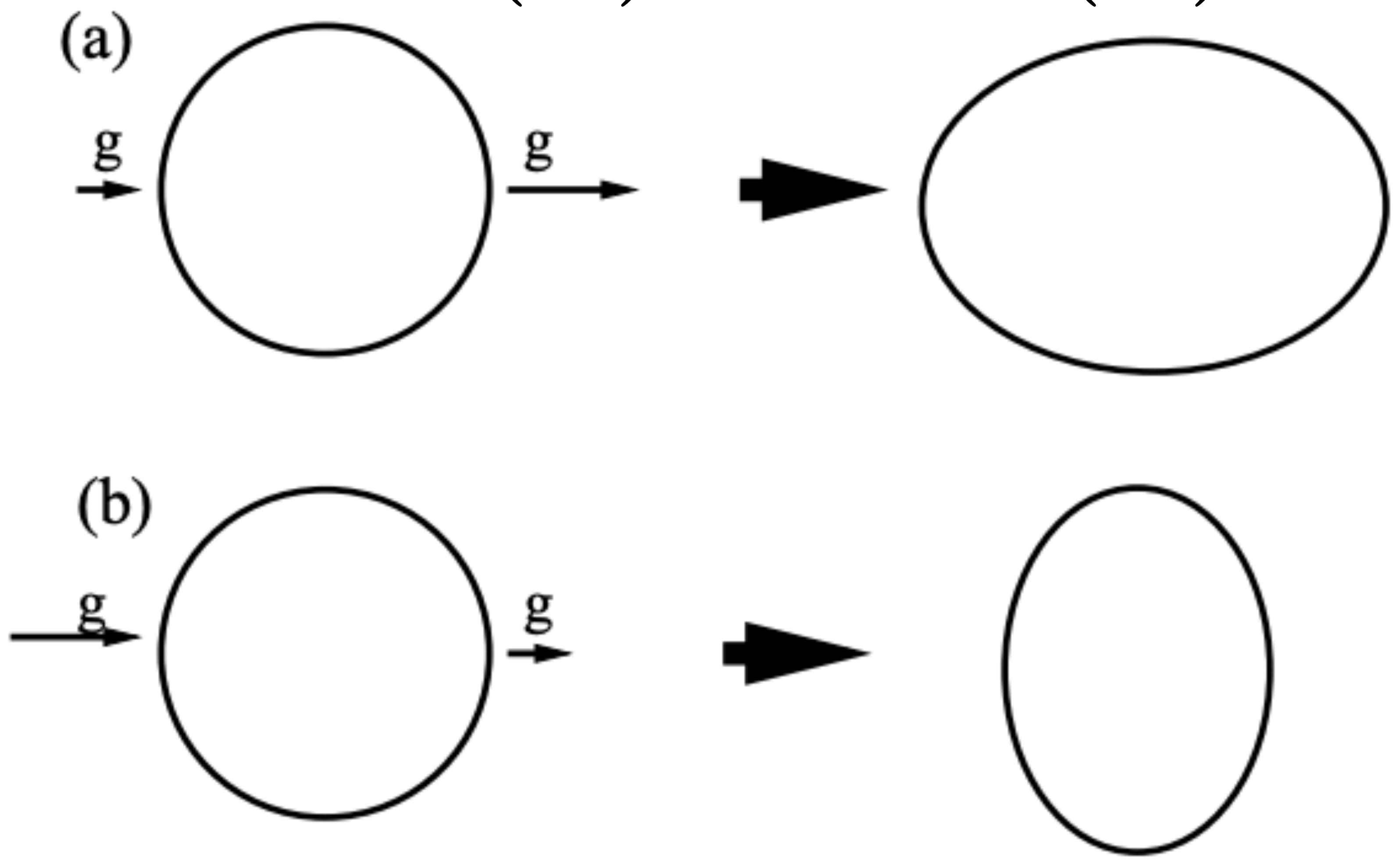
Can also go to higher order distortions (e.g. flexion — 2309.08653)

Extended bodies respond to tidal fields

$$\frac{2}{3}\frac{\partial_i\partial_j\Phi(\boldsymbol{x},a)}{\mathcal{H}^2(a)\Omega_m(a)} = \frac{\delta^K_{ij}}{3}\underbrace{\delta(\boldsymbol{x},a)}_{\text{Matter density contrast}} + \underbrace{s_{ij}(\boldsymbol{x},a)}_{\text{Traceless tidal field strength}}$$

Linear tidal alignment

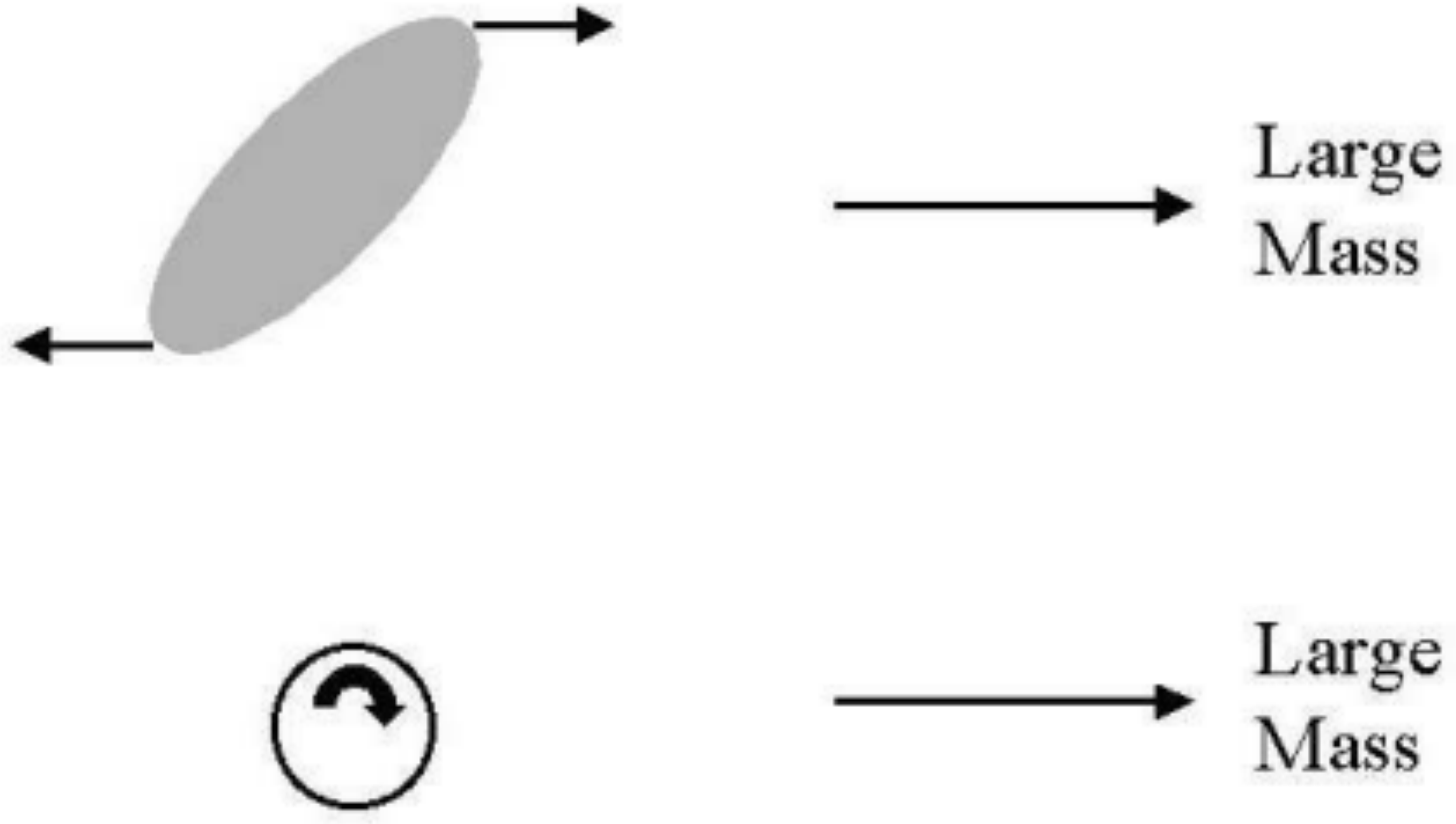
$$M_{ab}(\boldsymbol{q}) \propto s_{ab}(\boldsymbol{q})$$



Catelan, Kamionkowski & Blandford 2000

Tidal torquing

$$M_{ab}(\boldsymbol{q}) \propto L_a(\boldsymbol{q})L_b(\boldsymbol{q}) \propto (s_{ac}s_{cb})(\boldsymbol{q})$$

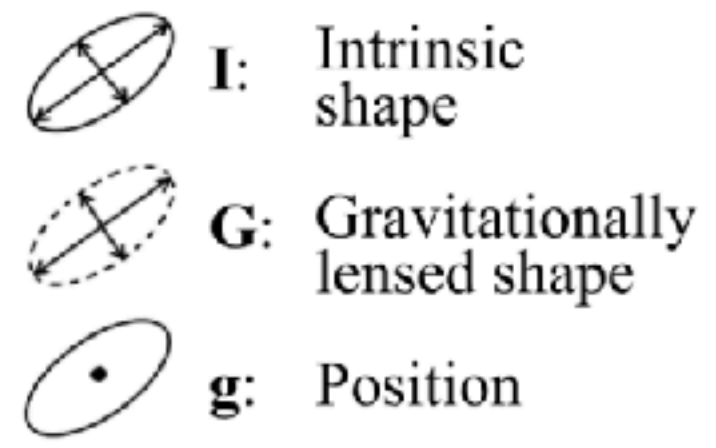


$$L_a \sim \epsilon_{abc}(\partial_b\partial_d\Phi(\boldsymbol{q}))\tilde{M}_{dc}(\boldsymbol{q})$$

Correlations between shapes and tides mimic weak lensing

$$\gamma^{\rm obs} = \gamma^G + \gamma^I$$

$$\langle\gamma^{\rm obs}\gamma^{\rm obs}\rangle = \langle\gamma^G\gamma^G\rangle + \underbrace{\langle\gamma^G\gamma^I\rangle + \langle\gamma^I\gamma^G\rangle}_{\text{Matter — shape correlations}} + \underbrace{\langle\gamma^I\gamma^I\rangle}_{\text{Shape — shape correlations}}$$



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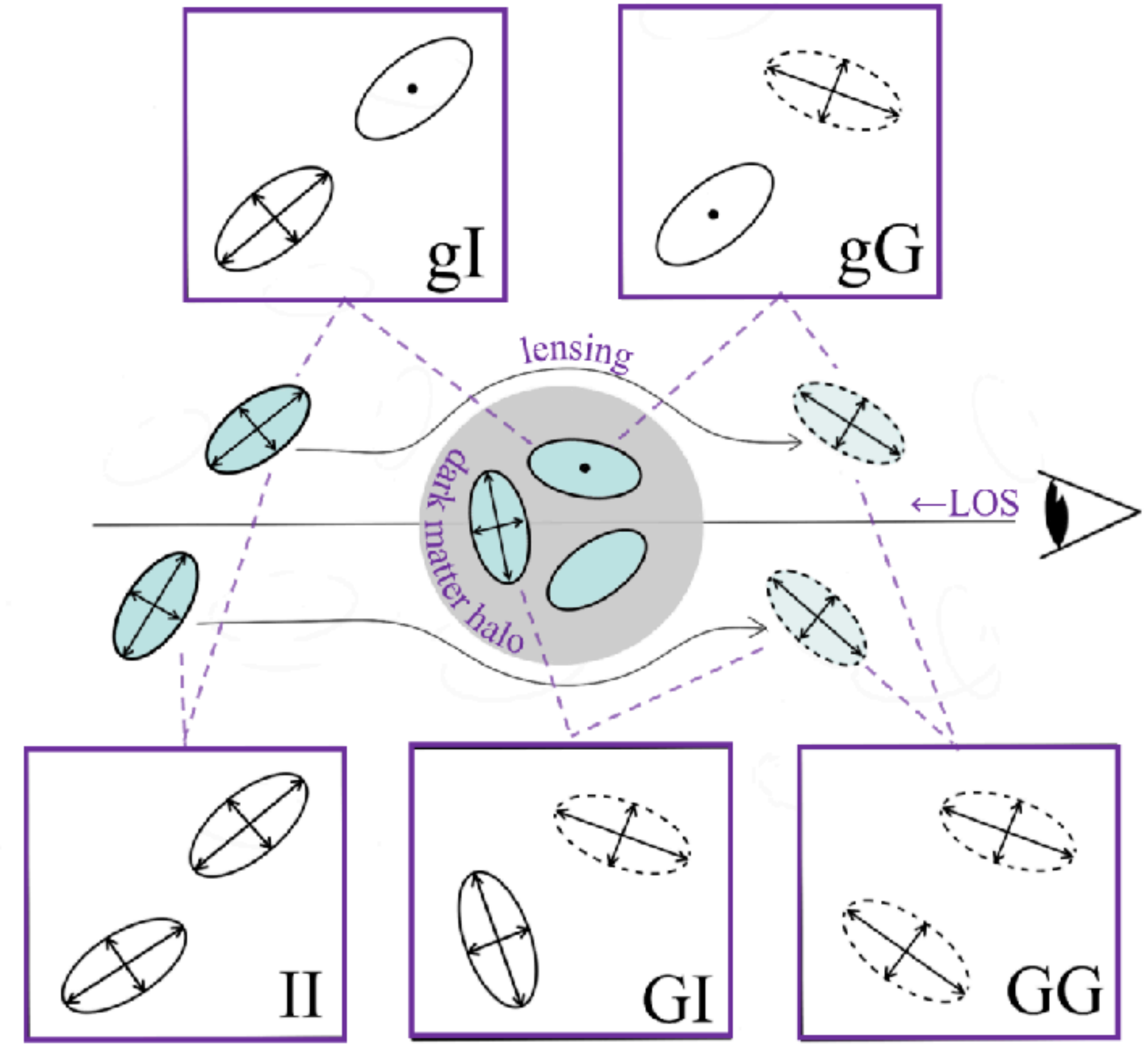
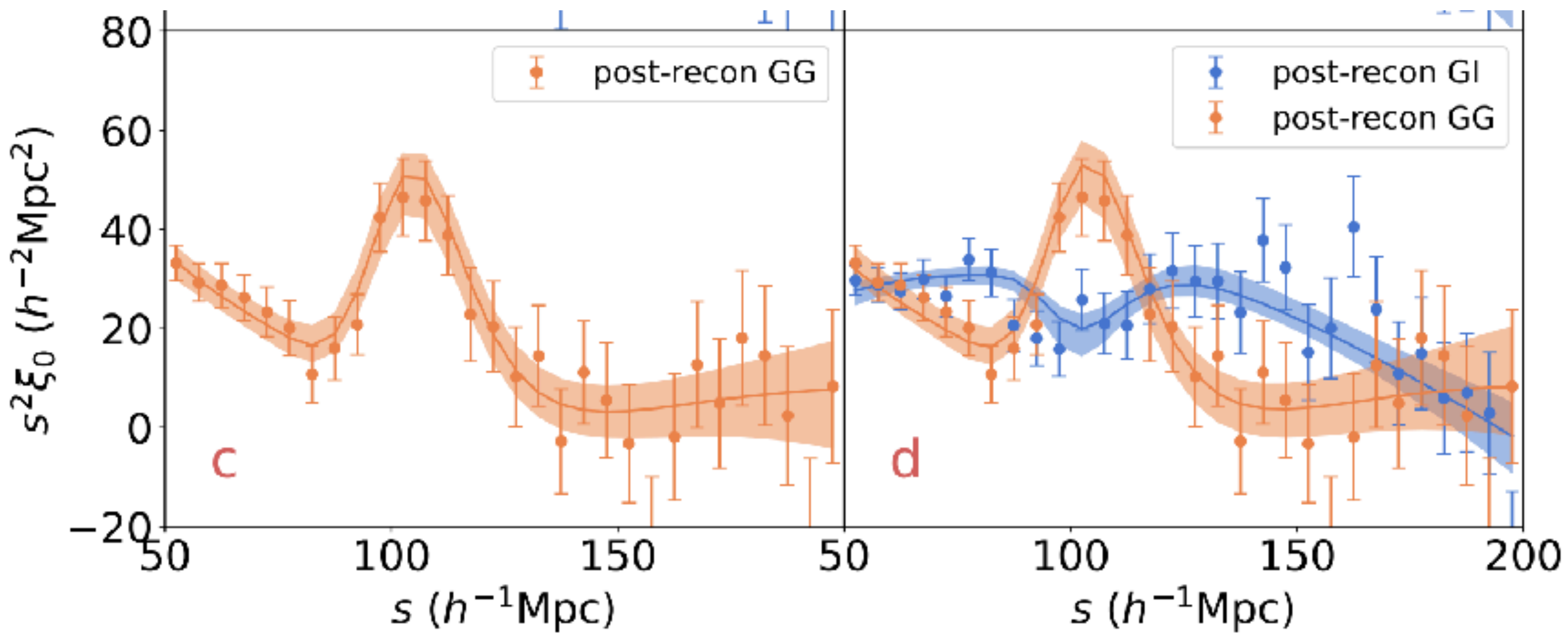


Not properly including this contaminant can lead to *severe* biases in the stage-IV era (Krause et al 2015)

Recent results also point to its potential as a complementary *signal,* from BAO measurements (Xu et al 2023, Nature Astronomy) to inflationary physics (Schmidt, Chisari, Dvorkin 15)



Intrinsic alignments are flourishing!

Lots of recent interest in intrinsic alignments, as they become more important for shear surveys and also a new probe for spectroscopic surveys!

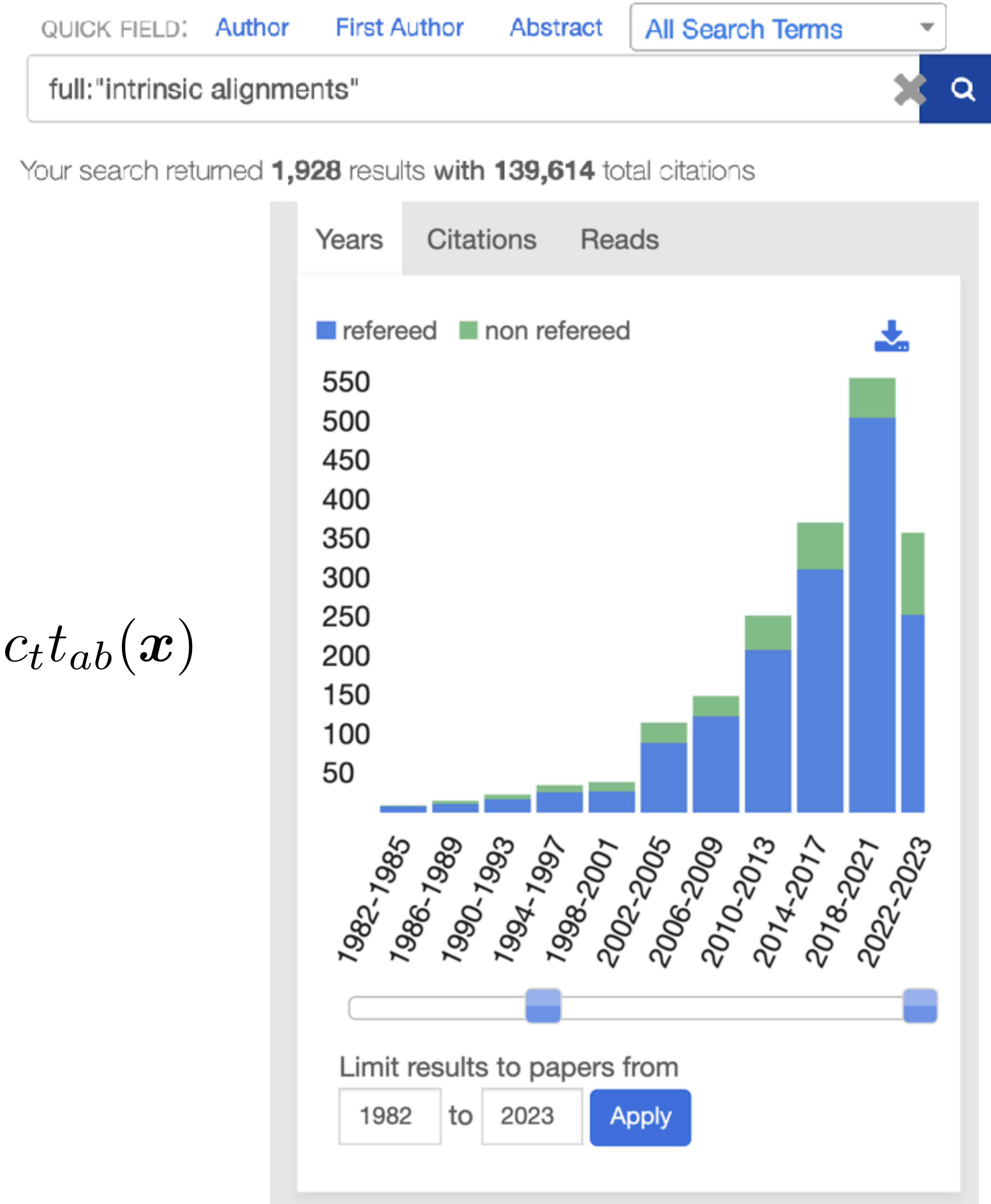
Models until recently were constructed in very piecemeal ways —separately including linear or tidal torquing contributions.

Arguments for a general symmetries-based expansion were carried out in <u>Blazek et al. 2017</u> — defining what is now known as the "TATT" model

$$\gamma^I_{ab}(\boldsymbol{x}) \approx c_s s_{ab}(\boldsymbol{x}) + c_{\delta s}[\delta s_{ab}](\boldsymbol{x}) + c_{s^2}[s_{ac}s_{cb} - \frac{1}{3}\delta_{ab}s^2](\boldsymbol{x}) + c_t t_{ab}(\boldsymbol{x})$$

Terms up to s² (but not t) implemented in public codes. Used as the DES fiducial model for Y3.

For the model to be well-defined, additional contributions up to cubic order are needed! Vlah et al 2019 extended this theory to cubic order for *Eulerian* bias. No public codes of this Eulerian IA EFT exist.



Formulating the Lagrangian theory of IA

For galaxy bias, we study the density of *protohalos* and assume their number density is conserved to find

$$1 + \delta_g(\boldsymbol{x}, \tau) = \int d^3\boldsymbol{q}\, (1 + \delta_g(\boldsymbol{q}))\, \delta^D(\boldsymbol{x} - \boldsymbol{q} - \boldsymbol{\Psi}(\boldsymbol{q}, \tau))$$

Consider, now, the moment of inertia of a Lagrangian patch. Define the *shape density* as

$$M_{ab}(\boldsymbol{q}) = (1 + \delta_g(\boldsymbol{q}))I_{ab}(\boldsymbol{q}), \quad I_{ab}(\boldsymbol{q}) = \int d^3\mathbf{s}\ \rho(\mathbf{s}, \boldsymbol{q})\ \mathbf{s}_a \mathbf{s}_b$$

Volume elements at q are sent to q+Ψ(q), leading to a late-time shape density of

$$M_{ab}(\boldsymbol{x}) = \int d^3\boldsymbol{q}\ \delta_D(\boldsymbol{x} - \boldsymbol{q} - \Psi(\boldsymbol{q}))\ R_{ac}(\boldsymbol{q})R_{db}(\boldsymbol{q})M_{cd}(\boldsymbol{q}),\ \ R_{ab}(\boldsymbol{q}) = \delta_{ab} + \nabla_a \Psi_b.$$

Starting from even spherically symmetric configurations will lead to sheared shape densities! Expressing LPT displacements in terms of standard Eulerian operators we find

$1^{\rm st}$ order: s_{ij}

$2^{\rm nd}$ order: $s^2_{ij},\ \delta s_{ij}, t_{ij}$

$3^{\rm rd}$ order: $L^{(3)}_{ij},\ (st)_{ij},\ \delta t_{ij},\ s^3_{ij},\ s^2 s_{ij}, \delta^2 s_{ij},\ \delta s^2_{ij}$

Contributions in red are terms that have been previously included in cosmic shear surveys (TATT model)

The structure of LPT calculations I

The power spectrum of two Lagrangian fields in LPT is given by

$$P(\mathbf{k}) = \int d^3\boldsymbol{q}\ e^{i\mathbf{k}\cdot\boldsymbol{q}} \langle e^{i\mathbf{k}\cdot\Delta(\boldsymbol{q}_1,\boldsymbol{q}_2)} F_\alpha(\boldsymbol{q}_1) F_\beta(\boldsymbol{q}_2)\rangle_{\boldsymbol{q}=\boldsymbol{q}_2-\boldsymbol{q}_1}$$

We can define a generating function for these correlations

$$M(\boldsymbol{\alpha}) = \left\langle \exp\left[i\mathbf{k}\cdot\Delta + \sum_a \alpha_1^a O^a(\boldsymbol{q}_1) + \alpha_2^a O^a(\boldsymbol{q}_2)\right] \right\rangle .$$

Using the cumulant expansion, tracer-matter and tracer-cross correlations simplify significantly, e.g.

$$\left.\frac{\partial M}{\partial \alpha_2^a}\right|_{\boldsymbol{\alpha}=0} = M(\mathbf{0})\Big(ik_i\langle \Delta_i O_2^a\rangle - \frac{1}{2}k_i k_j \langle \Delta_i \Delta_j O_2^a\rangle + ...\Big) \longrightarrow \quad P_{mT} = \sum_a b_a \int d^3\boldsymbol{q}\, e^{i\mathbf{k}\cdot\boldsymbol{q}}\ \left.\frac{\partial M}{\partial \alpha^a}\right|_{\boldsymbol{\alpha}=0}$$

The structure of LPT calculations II

We can categorize the entire structure of IA correlations using this generating function approach.
Building the entire model is then a systematic exercise of enumerating all possible contributions and implementing them…

$$\begin{aligned}\text{“GI”} &\ni \langle \Delta_i \gamma_{ab} \rangle, \langle \Delta_i \Delta_j \gamma_{ab} \rangle \\ \text{“gI”} &\ni \langle \gamma_{ab} \delta_g \rangle, \langle \Delta_i \delta_g \rangle \langle \Delta_j \gamma_{ab} \rangle, \langle \Delta_i \gamma_{ab} \delta_g \rangle, \\ \text{“II”} &\ni \langle \gamma_{ab} \gamma_{cd} \rangle, \langle \Delta_i \gamma_{ab} \rangle \langle \Delta_j \gamma_{cd} \rangle, \langle \Delta_i \gamma_{ab} \gamma_{cd} \rangle \end{aligned}\Bigg\}$$

Tensor correlators of ranks 3, 4, 5

Techniques from high order multipole expansions in E&M

We introduced an orthogonal basis of “traceless symmetric tensor Legendre polynomials” which allows us to compute Lagrangian correlators and power spectra. As an example:

$$\langle O_{ab}(\mathbf{k}) | \Psi_i(\mathbf{k}') \rangle = F_3(k) \mathcal{Q}^3_{iab}(\hat{k}) + F_1(k) \mathcal{Q}^1_{iab}(\hat{k})$$

$$\mathcal{Q}^1_{iab}(\hat{k}) = \hat{k}_i \delta_{ab} - \frac{3}{2} \hat{k}_{(a} \delta_{b)i}$$

$$\mathcal{Q}^3_{iab}(\hat{k}) = \frac{1}{2} \left(5 \hat{k}_i \hat{k}_a \hat{k}_b - (\hat{k}_i \delta_{ab} + \hat{k}_{(a} \delta_{b)i} \right)$$

The structure of LPT calculations III

How to numerically evaluate these contributions efficiently? Consider cubic shape x density operator as an example:

$$\langle \delta t_{ab}(\mathbf{k}) | \delta(\mathbf{k}') \rangle' = \frac{-4}{7} P(k) \int_{\mathbf{p}} \left(\frac{(\mathbf{k}-\mathbf{p})_a (\mathbf{k}-\mathbf{p})_b}{(\mathbf{k}-\mathbf{p})^2} - \frac{1}{3}\delta_{ab} \right) \left(1 - \frac{(\mathbf{k}\cdot\mathbf{p})^2}{k^2 p^2} \right) P(p)$$

First, dot tensor Legendre polynomial $P_2^{ab}(\hat{k})$ to extract the corresponding scalar function.

Like in *velocileptors*, the scalar correlators are decomposed into 'generalized correlation functions'.

$$\xi_n^\ell[F](q) = \int_0^\infty \frac{dk}{2\pi^2} k^{2+n} j_\ell(kq) F(k)$$

Gives

$$A_{\delta t}(k) = -\frac{8}{735} \int dq q \left[42k^2 \xi_0^0(q) j_0(kq) - 56k\xi_1^1(q) j_1(kq) + 5(-7k^2 \xi_0^2(q) + 5\xi_2^2(q)) j_2(kq) \right.$$
$$\left. + 42k\xi_1^3(q) j_3(kq) - 18\xi_2^4(q) j_4(kq) \right] + \frac{8}{105}\sigma^2$$

Then every term is evaluating an Hankel transform which can be efficiently done numerically. This decomposition also Fourier transforms into itself, making Fourier / configuration-space calculations very simple!

Every one-loop term (contributing to GG, GI, gI) is coded up *spinosaurus.*

One last ingredient… shape stochasticity

The discrete sampling of the galaxy density field introduces stochasticity in the bias expansion — *shot noise*

The analogous effect for galaxy shapes is the well-known *shape noise*. But the tensor structure of correlators induces some non-trivial considerations. This is best seen in the *helicity* decomposition

$$M_{ab}(\mathbf{k}) = \sum_{m=-2}^{m=2} M_m(\mathbf{k}) Y_{2,ab}^m(\hat{k})$$

Which implies there is only one fundamental correlator (m=0) in scalar x shape and three (m={0, 1, 2}) in shape x shape. We thus find *four* total stochastic correlators to O(k^2). Three for II and one for gI:

$$\langle \epsilon_{ab}\epsilon_{cd} \rangle \ni \{\mathcal{Q}^0_{abcd}, k^2 \mathcal{Q}^0_{abcd}, k^2 \mathcal{Q}^2_{abcd}\} \longrightarrow$$

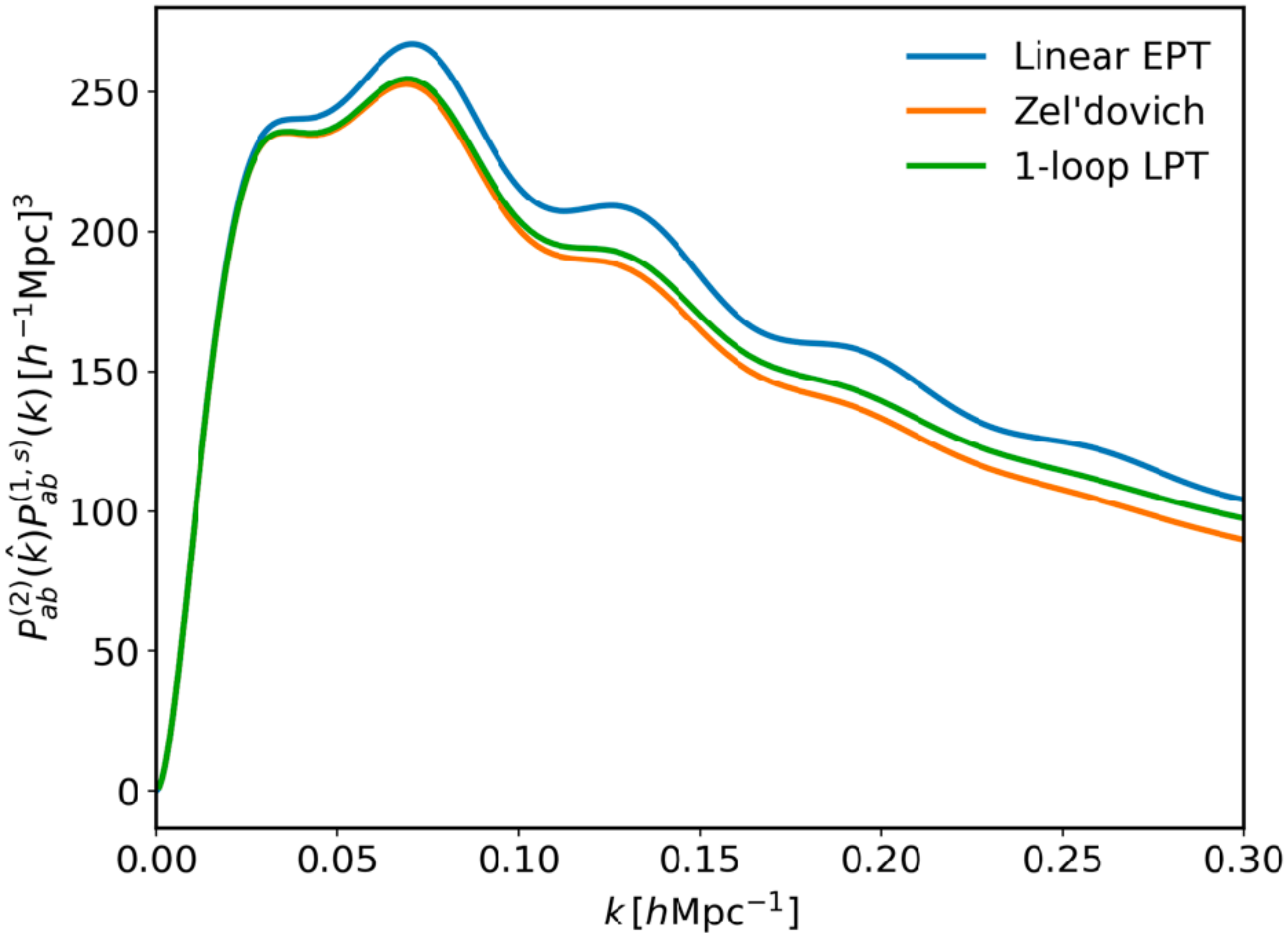
Two k^2 terms but they show up in different ways in the helicity spectra -> Constrainable!

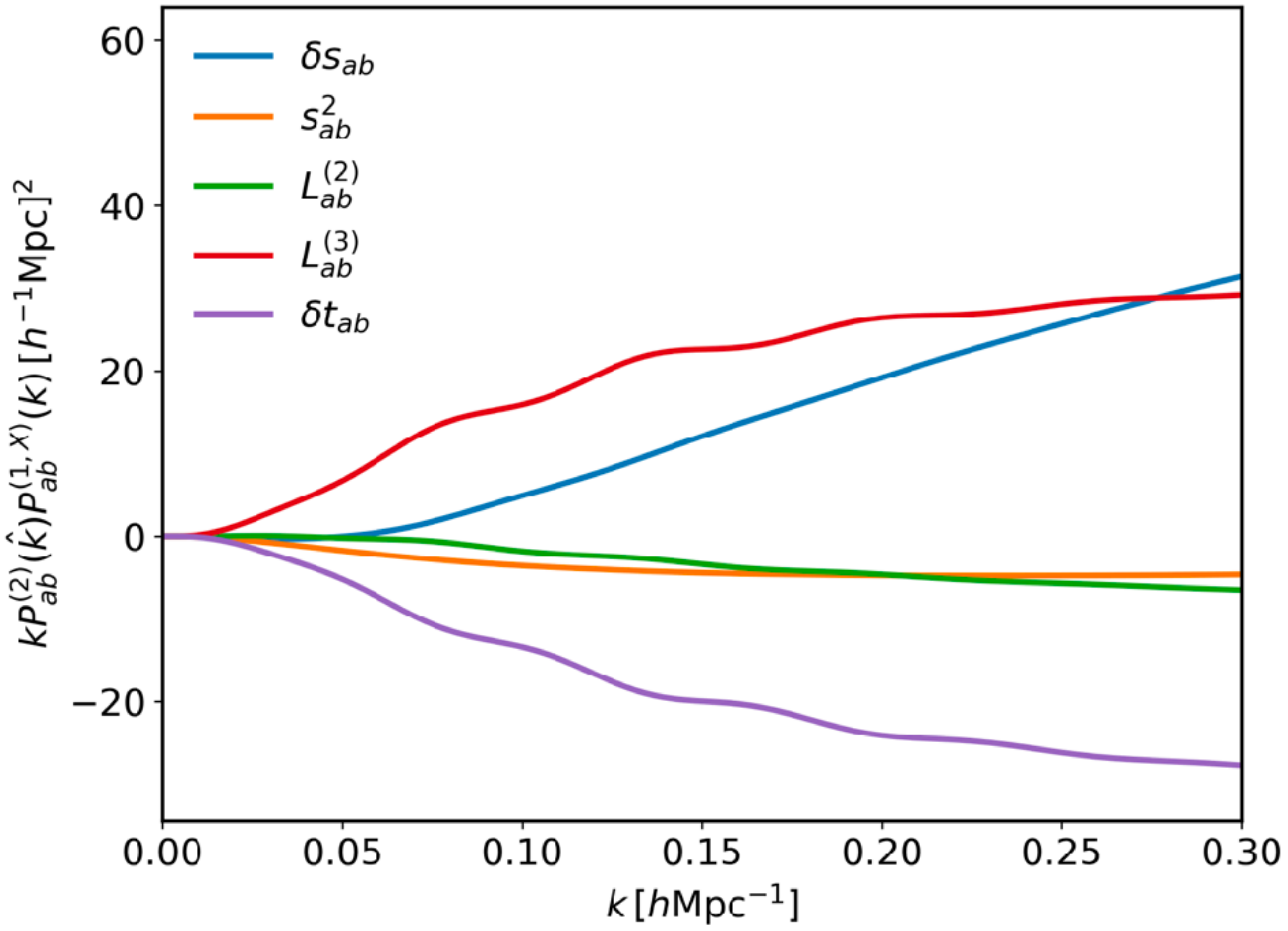
$$\langle \epsilon_{ab}\epsilon \rangle \sim k^2 \left(\hat{k}_a \hat{k}_b - \frac{1}{3}\delta_{ab} \right) \longrightarrow$$

In gI the first stochasticity scales as k^2

Results for GI I - the spectra

Despite there being 7 new operators at cubic order, their contributions to spectra are degenerate at 1-loop. Only two new third order parameters are needed. Similar case in scalar galaxy bias where only one cubic galaxy bias parameter is needed.





Results for GI II - broadening of the BAO dip

Results for Shape-Shape I: Advection mixes modes

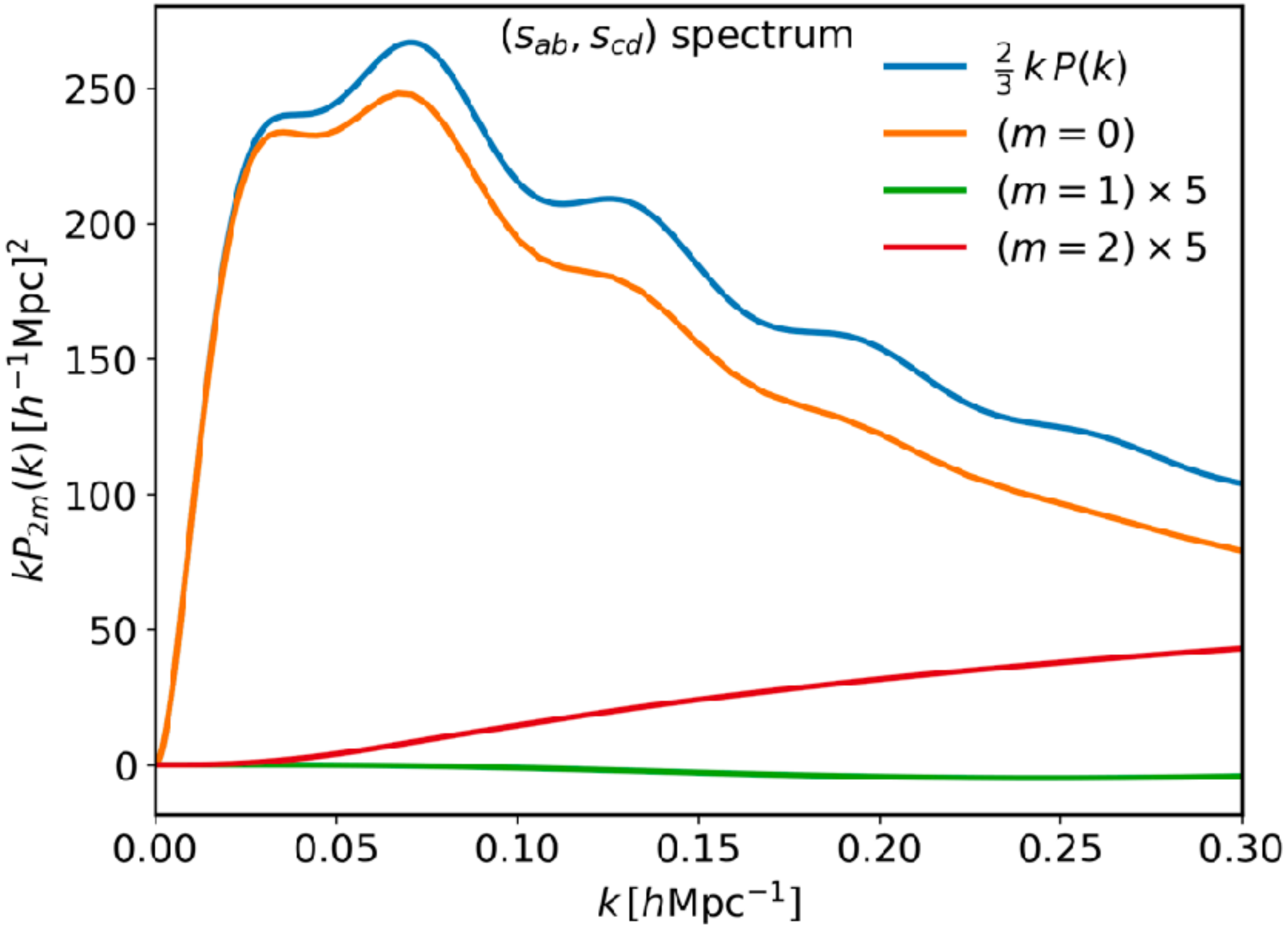
We project shape-shape spectra onto the helicity basis — full 3D structure of these shape correlations.

Lagrangian displacements produce m=1 helicities even in the linear alignment model. This is analogous to CMB lensing producing B-modes. But very very small!

The relation between the projected shape decompositions and the helicities are (Vlah, Chisari, Schmidt 20):

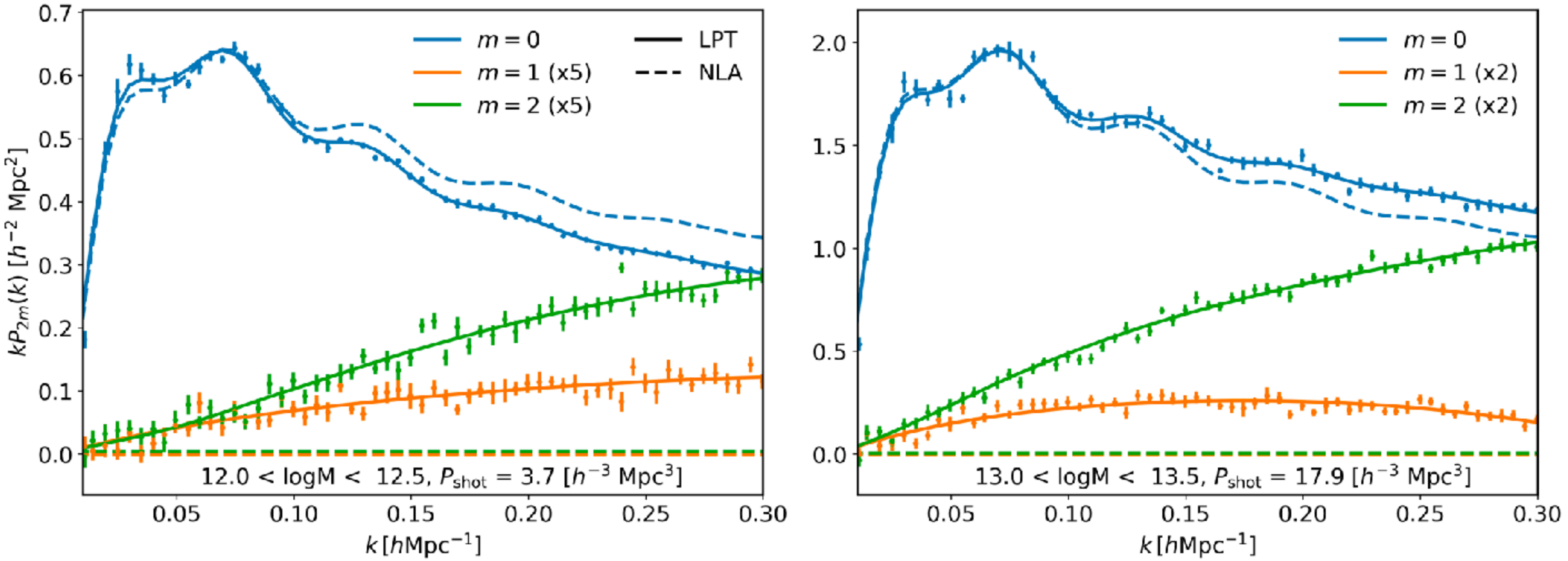
$$C_{EE}(\ell) = \frac{1}{8}\int d\chi \frac{W_g^2(\chi)}{\chi^2}\left(3P_{20}(\ell/\chi) + P_{22}(\ell/\chi)\right)$$

$$C_{BB}(\ell) = \int d\chi \frac{W_g^2(\chi)}{2\chi^2} P_{21}(\ell/\chi)$$



Results for Shape-Shape II: Fitting to Nbody

Shape-shape power spectra multipoles measured in N-body sims, combined volume of V=27 (Gpc/h)³



Comparison with NLA — predicts the higher helicity spectra should be exactly zero. Even the m=0 spectrum cannot be fit past k~0.1 h/Mpc.

Full model fits very well — we also explore reduced parameterizations in the paper which have fewer free parameters but work less well. Large hits to chi2.

Non-Poisson + scale-dependent shape noise is very significant in halos, especially most massive bin…

Wrapping up

We've developed the theory to produce Lagrangian Perturbation Theory predictions of the statistics of galaxy shapes.

These calculations are directly relevant for modelling intrinsic alignments as well as shape—density statistics in spectroscopic surveys.

Developing and implementing the full one loop model, we've learned:

1. **Despite there being seven cubic fields, only two new free parameters are required to describe these terms.**

2. **The full model dictated by symmetries (10 parameters) fits 3D structure of halo shape correlations exquisitely well.**

3. **In this Lagrangian picture we generate B-mode contamination in the cosmic shear spectrum even in the simplest theory. This is analogous to CMB lensing producing B-modes in CMB maps.**

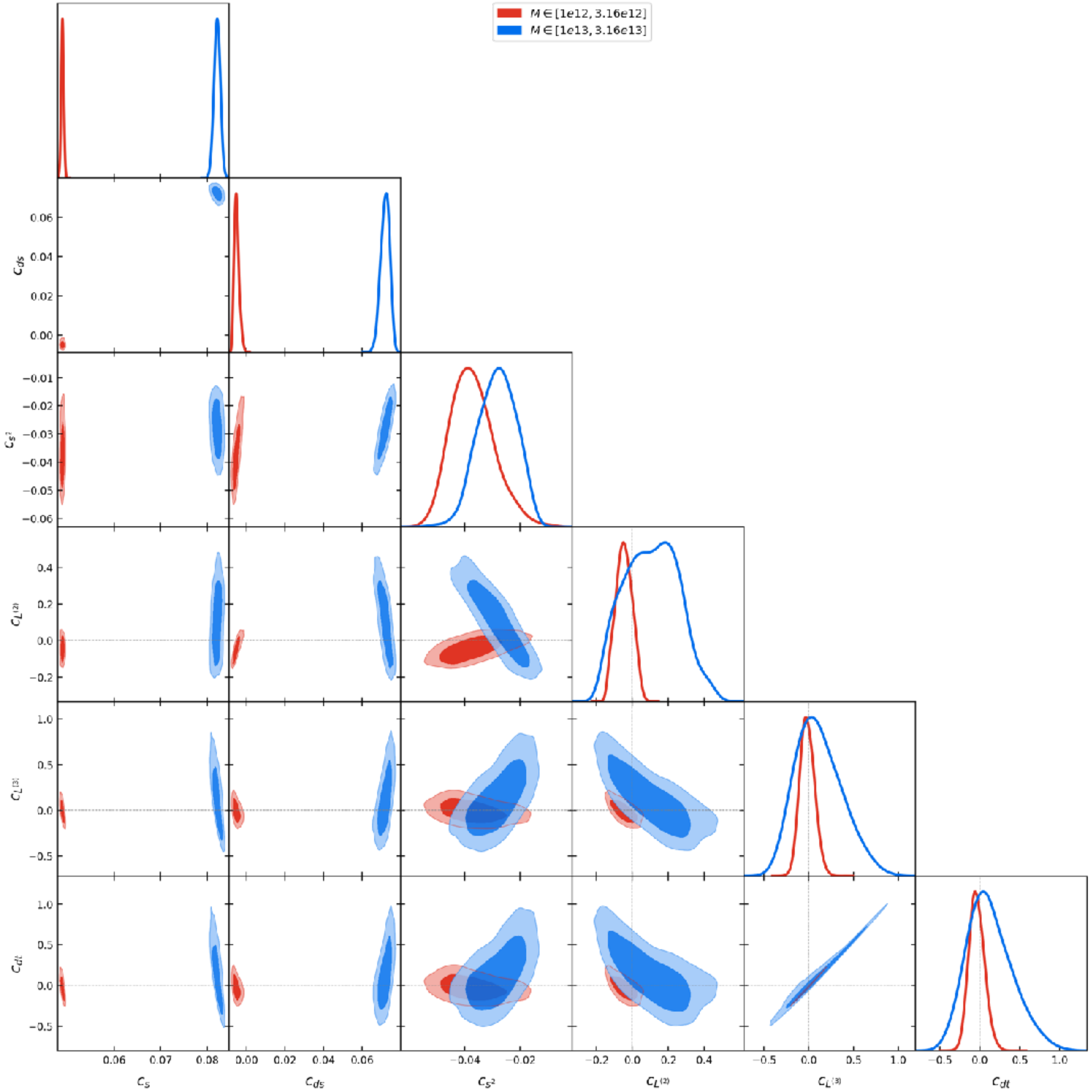
4. **The NLA model completely fails to capture the structure of these 3D correlations.**

5. **The code to generate these predictions (and reproduce all figures in our paper) is publicly available in a convenient package, `spinosaurus` (github.com/sfschen/spinosaurus)**

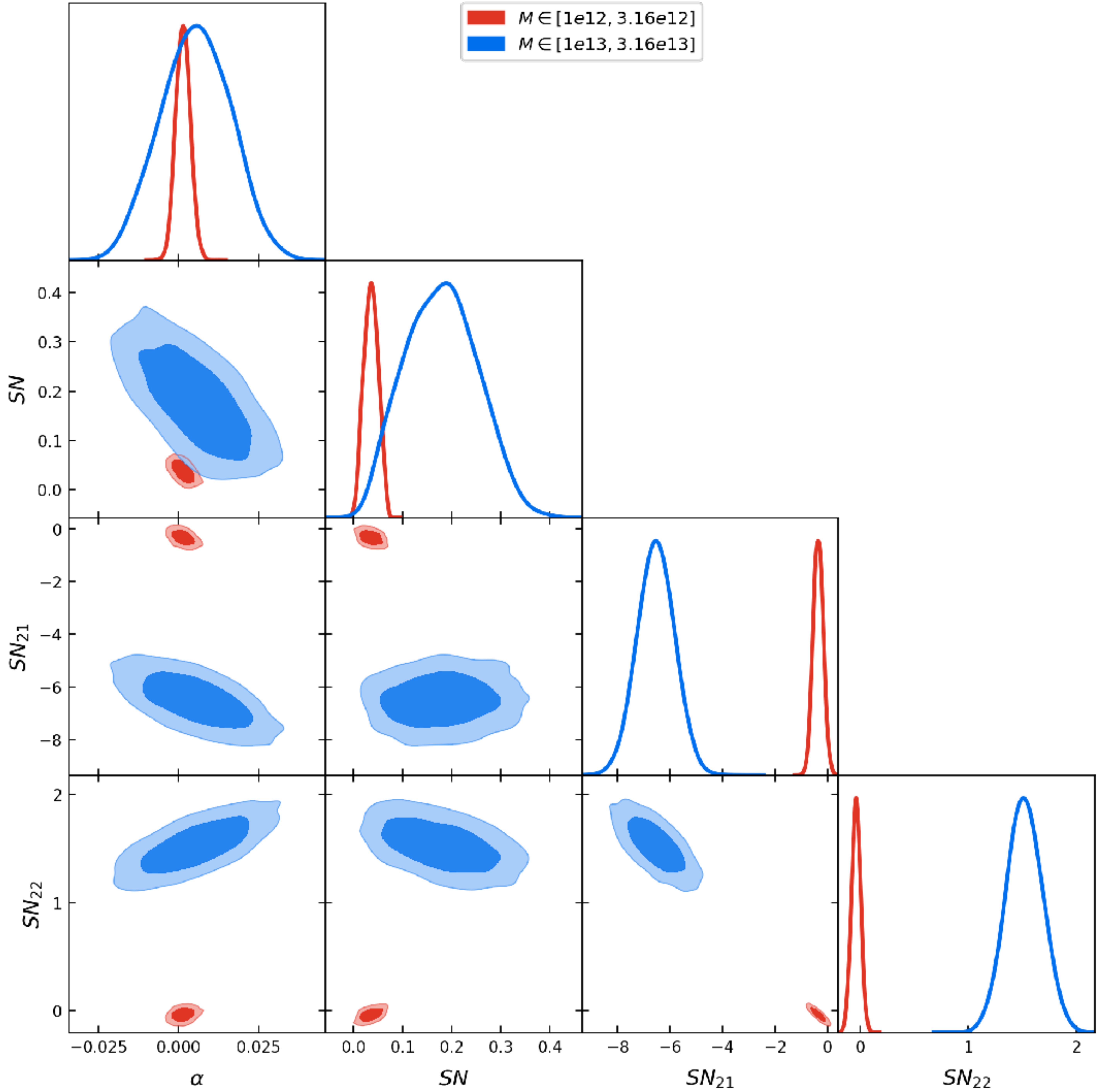
Extra slides

Cumulant expansion of G.F.

$$\frac{M(\boldsymbol{\alpha})}{M(\mathbf{0})} = \exp\left\{\Big[\sum_a(\alpha_1^a+\alpha_2^a)ik_i\langle\Delta_i O_1^a\rangle + \sum_{a,b}\alpha_1^a\alpha_2^b\langle O_1^a O_2^b\rangle\Big]\right\}$$
$$\exp\left\{\Big[-\frac{1}{2}\sum_a(\alpha_1^a+\alpha_2^a)k_ik_j\langle\Delta_i\Delta_j O_1^a\rangle_c + \sum_{a,b}i\alpha_1^a\alpha_2^b k_i\langle\Delta_i O_1^a O_2^b\rangle_c + \ldots\Big]\right\} \quad (3.6)$$



$M \in [1e12, 3.16e12]$
$M \in [1e13, 3.16e13]$
c_{ds}
c_{s^2}
$c_{L^{(2)}}$
$c_{L^{(3)}}$
c_{dt}
c_s



$M \in [1e12, 3.16e12]$
$M \in [1e13, 3.16e13]$
α
SN
SN_{21}
SN_{22}

Symmetry imposes *all* shape-shape correlators are a weighted sum of these curves: