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The Renormalization Group for Large-Scale Structure (RGforLSS)

Henrique Rubira (TUM)

In collaboration with Fabian Schmidt and Charalampos Nikolis

2307.15031,
2404.16929,
2405.21002

Edinburgh, June 2024

Henrique Rubira

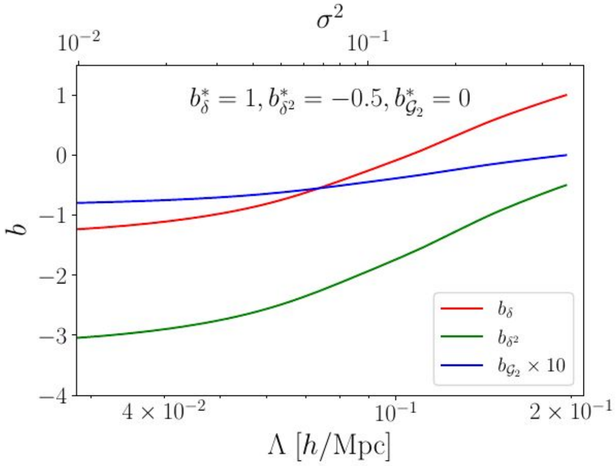
Message to take home

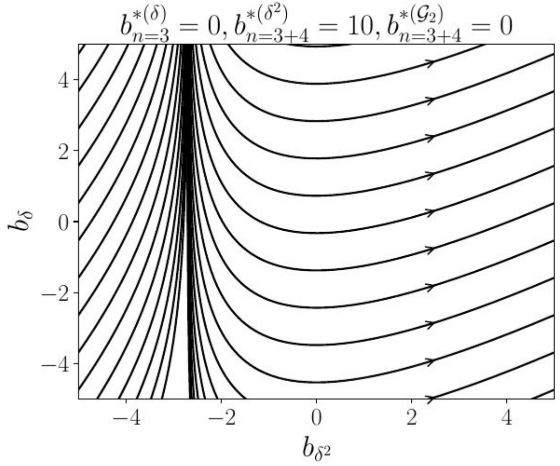
We derive the Callan-Symanzik equation for the galaxy bias and stochastic parameters (including PNG)

$$\frac{db_\delta}{d\Lambda} = -\left[\frac{68}{21}b_{\delta^2} + 3b^*_{\delta^3} - \frac{4}{3}b^*_{\mathcal{G}_2\delta}\right]\frac{d\sigma^2_\Lambda}{d\Lambda}\,,$$

$$\frac{db_{\delta^2}}{d\Lambda} = -\left[\frac{8126}{2205}b_{\delta^2} + \frac{17}{7}b^*_{\delta^3} - \frac{376}{105}b^*_{\mathcal{G}_2\delta} + b^{*(\delta^2)}_{n=4}\right]\frac{d\sigma^2_\Lambda}{d\Lambda}\,,$$

$$\frac{db_{\mathcal{G}_2}}{d\Lambda} = -\left[\frac{254}{2205}b_{\delta^2} + \frac{116}{105}b^*_{\mathcal{G}_2\delta} + b^{*(\mathcal{G}_2)}_{n=4}\right]\frac{d\sigma^2_\Lambda}{d\Lambda}\,.$$



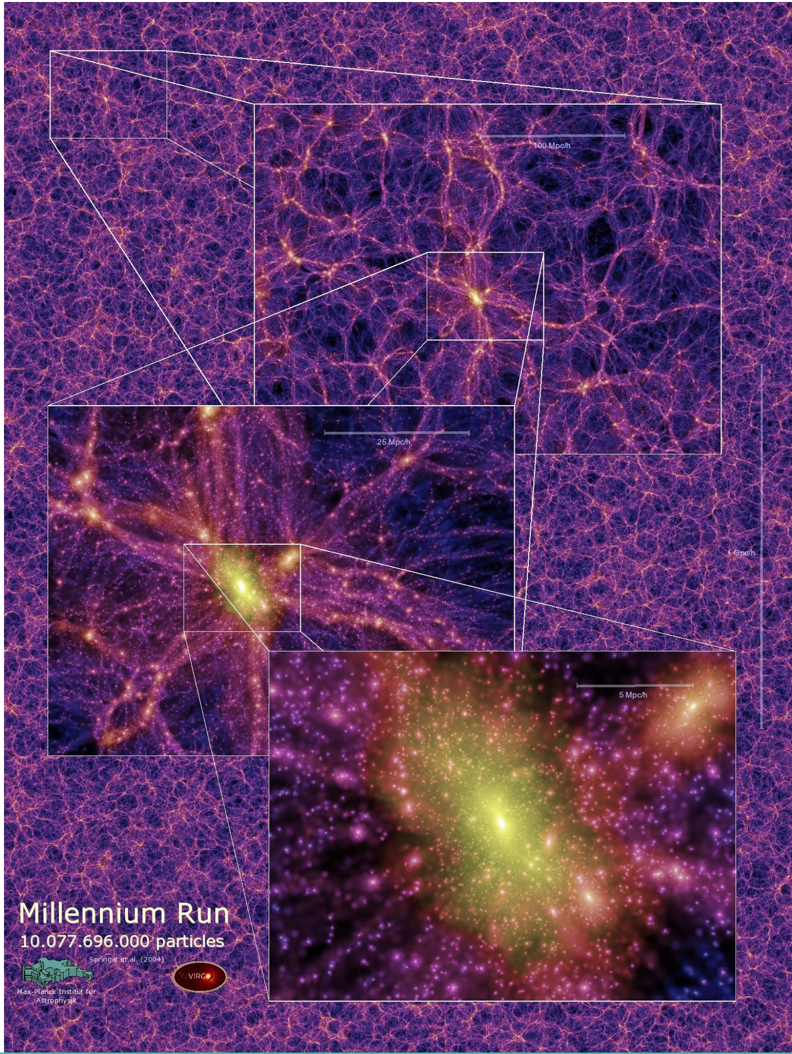


Many things to explore:

- systematic construction of operator basis and their priors,
- systematic renormalization of n-point functions,
- extra cross-checks,
- more information from galaxy clustering (to be investigated)

Part I - Preamble(s)

Motivation:
Scales in structure formation



QFT101

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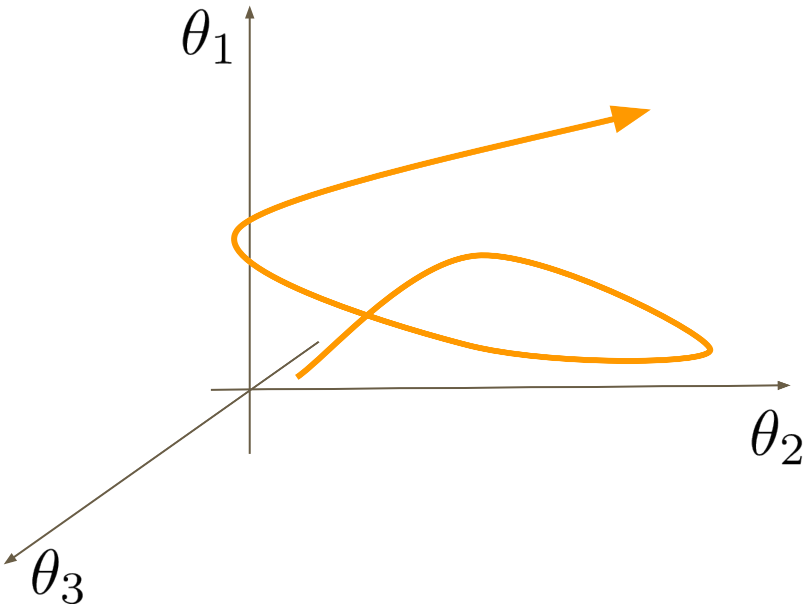
Renormalization group: coupling constants evolve with the cutoff ("flow").

Observables don't depend on the cutoff!



Callan-Symanzik equation:

$$\frac{\partial g}{\partial \ln \mu} = \beta(g)$$

QED: $\beta(e) = \frac{e^3}{12\pi^2}$

QCD: $\beta(g) = -\left(11 - \frac{n_s}{6} - \frac{2n_f}{3}\right)\frac{g^3}{16\pi^2}$

The galaxy bias expansion

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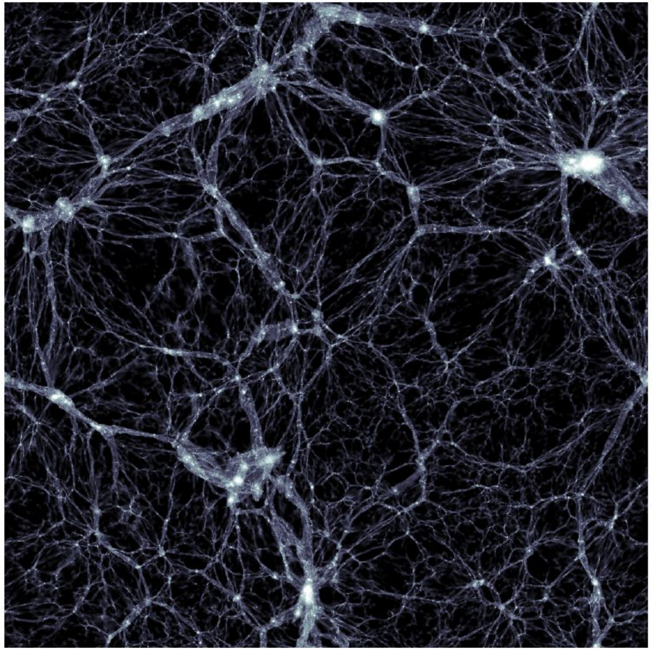
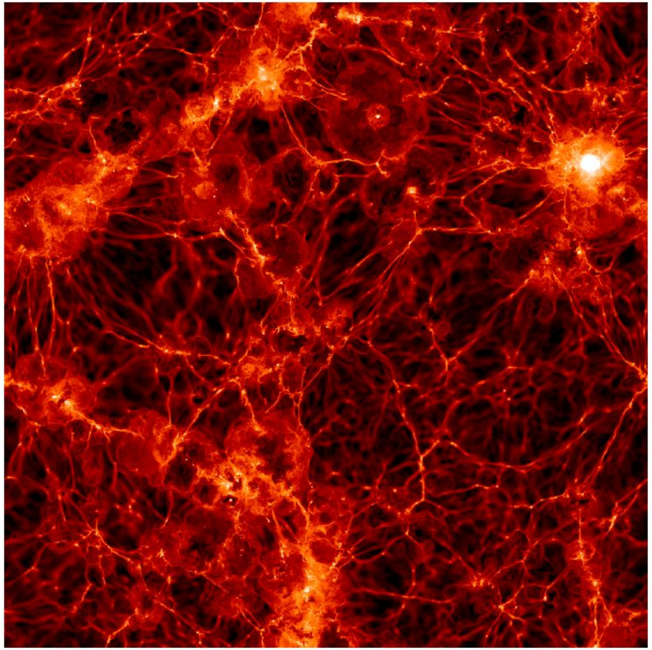


(a) dark matter



(b) baryons

From Illustris simulation, Haiden, Steinhauser, Vogelsberger, Genel, Springel, Torrey, Hernquist, 15

$$\delta_g(\boldsymbol{x},\tau) \equiv \frac{n_g(\boldsymbol{x},\tau)}{\bar{n}_g(\tau)} - 1 = \sum_O \left[b_O(\tau) + c_{\epsilon,O}(\tau)\epsilon(\boldsymbol{x},\tau)\right] O(\boldsymbol{x},\tau) + \epsilon(\boldsymbol{x},\tau)$$

The galaxy bias expansion

Henrique Rubira

Bias and stochastic parameters

Stochastic field

$$\delta_g(\boldsymbol{x},\tau) \equiv \frac{n_g(\boldsymbol{x},\tau)}{\bar{n}_g(\tau)} - 1 = \sum_O \left[b_O(\tau) + c_{\epsilon,O}(\tau)\epsilon(\boldsymbol{x},\tau)\right] O(\boldsymbol{x},\tau) + \epsilon(\boldsymbol{x},\tau)$$

Operators:

First order:	$\delta\,;$
Second order:	$\delta^2,\ \mathcal{G}_2\,;$
Third order:	$\delta^3,\ \delta\,\mathcal{G}_2,\ \Gamma_3,\ \mathcal{G}_3\,;$

Bias review: Desjacques, Jeong, Schmidt

Part II - Renormalization in LSS

Renormalizing the bias parameters

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$$\delta_g(\boldsymbol{x},\tau) \equiv \frac{n_g(\boldsymbol{x},\tau)}{\bar{n}_g(\tau)} - 1 = \sum_O \left[b_O(\tau) + c_{\epsilon,O}(\tau)\epsilon(\boldsymbol{x},\tau)\right] O(\boldsymbol{x},\tau) + \epsilon(\boldsymbol{x},\tau)$$

$$O[\delta](\boldsymbol{k}) = \int_{\boldsymbol{p}_1,\ldots,\boldsymbol{p}_n} \delta_{\rm D}(\boldsymbol{k} - \boldsymbol{p}_{1\ldots n}) S_O(\boldsymbol{p}_1,\ldots \boldsymbol{p}_n)\delta(\boldsymbol{p}_1)\cdots\delta(\boldsymbol{p}_n)$$

First order:	$\delta\,;$
Second order:	$\delta^2,\ \mathcal{G}_2\,;$
Third order:	$\delta^3,\ \delta\,\mathcal{G}_2,\ \Gamma_3,\ \mathcal{G}_3\,;$

Contribution from arbitrarily small scales!

Renormalizing the bias parameters

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$$\delta_g(\boldsymbol{x},\tau) \equiv \frac{n_g(\boldsymbol{x},\tau)}{\bar{n}_g(\tau)} - 1 = \sum_O \left[b_O^{\Lambda}(\tau) + c_{\epsilon,O}^{\Lambda}(\tau)\epsilon^{\Lambda}(\boldsymbol{x},\tau)\right] O^{\Lambda}(\boldsymbol{x},\tau) + \epsilon^{\Lambda}(\boldsymbol{x},\tau)$$

\+ counter-terms (Λ)

$$O[\delta](\boldsymbol{k}) = \int_{\boldsymbol{p}_1,\ldots,\boldsymbol{p}_n}^{\Lambda} \delta_{\rm D}(\boldsymbol{k} - \boldsymbol{p}_{1\ldots n}) S_O(\boldsymbol{p}_1,\ldots \boldsymbol{p}_n)\delta(\boldsymbol{p}_1)\cdots\delta(\boldsymbol{p}_n)$$

Notation:

$[\![O]\!] = O^{\Lambda}$ +counter-terms (Λ)

How to determine the renormalization condition?

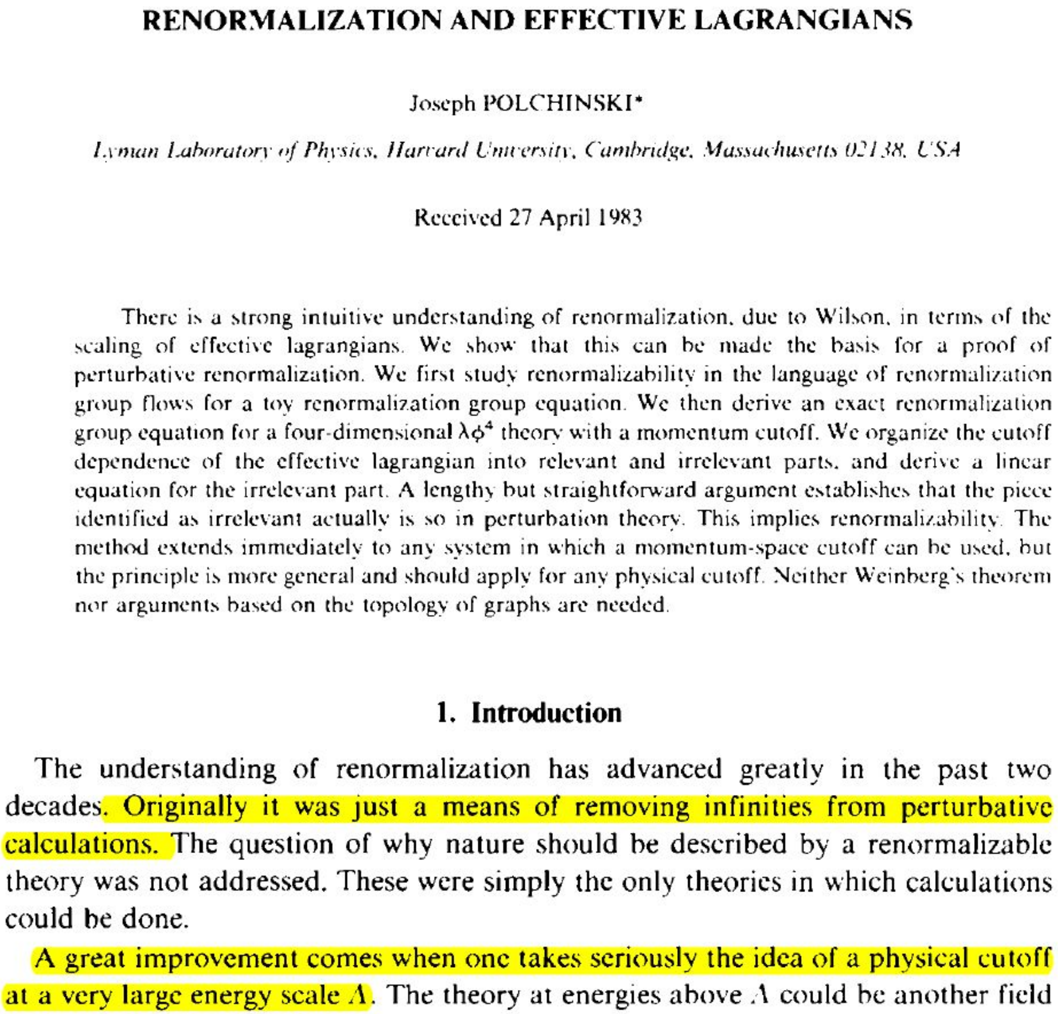
First order: $\delta\,;$

Second order: $\delta^2,\ \mathcal{G}_2\,;$

Third order: $\delta^3,\ \delta\,\mathcal{G}_2,\ \Gamma_3,\ \mathcal{G}_3\,;$

Contribution from arbitrarily small scales!

Main motivation


RENORMALIZATION AND EFFECTIVE LAGRANGIANS

Joseph POLCHINSKI*

Lyman Laboratory of Physics, Harvard University, Cambridge, Massachusetts 02138, USA

Received 27 April 1983

There is a strong intuitive understanding of renormalization, due to Wilson, in terms of the scaling of effective lagrangians. We show that this can be made the basis for a proof of perturbative renormalization. We first study renormalizability in the language of renormalization group flows for a toy renormalization group equation. We then derive an exact renormalization group equation for a four-dimensional $\lambda\phi^4$ theory with a momentum cutoff. We organize the cutoff dependence of the effective lagrangian into relevant and irrelevant parts, and derive a linear equation for the irrelevant part. A lengthy but straightforward argument establishes that the piece identified as irrelevant actually is so in perturbation theory. This implies renormalizability. The method extends immediately to any system in which a momentum-space cutoff can be used, but the principle is more general and should apply for any physical cutoff. Neither Weinberg's theorem nor arguments based on the topology of graphs are needed.

1. Introduction

The understanding of renormalization has advanced greatly in the past two decades. Originally it was just a means of removing infinities from perturbative calculations. The question of why nature should be described by a renormalizable theory was not addressed. These were simply the only theories in which calculations could be done.

A great improvement comes when one takes seriously the idea of a physical cutoff at a very large energy scale Λ. The theory at energies above Λ could be another field

Motivation (for different tastes)

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Extend the renormalization picture constructing the Wilson-Polchinski renormalization group that describe the evolution of the finite-scale bias parameters with the cutoff.

Lattice person: "At field level you smooth out over your cutoff and those bias parameters have to be defined at a fixed scale!"

HEP person: "Everything is an EFTs and RG-flow is the next thing to do. "

Cosmo-MCMC person: "How can we be sure we are not messing up with the priors in my EFT analysis? Maybe extract more information..."

EFT-negationist person: "You have a bunch of free parameters. How can you trust them?"

Part III - The Wilson-Polchinski path integral approach

Warning (and apologies in advance):
next 2 slides will be technical, they are just there to trigger interest

The bias partition function (based on Carroll, Leichenauer, Pollack, 13)

$$\mathcal{Z}[J_\Lambda] = \int \mathcal{D}\delta_\Lambda^{(1)} \mathcal{P}[\delta_\Lambda^{(1)}] \exp\left(\int_{\boldsymbol{k}} J_\Lambda(\boldsymbol{k}) \left[\sum_O b_O^\Lambda O[\delta_\Lambda^{(1)}](-\boldsymbol{k}) \right] + \frac{1}{2} P_\epsilon^\Lambda \int_{\boldsymbol{k}} J_\Lambda(\boldsymbol{k}) J_\Lambda(-\boldsymbol{k}) + \mathcal{O}[J_\Lambda^2 \delta_\Lambda^{(1)},\ J_\Lambda^3] \right)$$

Path-integral over linear-smoothed density, normalized

Single-current term

Double-current term captures stochasticity source

See Cabass, Schmidt 19

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Path-integral over linear-smoothed density, normalized

Single-current term

Double-current term captures stochasticity source

N-point correlators evaluated as:

$$\left. \frac{\partial \mathcal{Z}}{\partial J_\Lambda \dots \partial J_\Lambda} \right|_{J_\Lambda = 0}$$

See Cabass, Schmidt 19

The shell expansion (Wilson formalism)

Henrique Rubira

Consider a very thin shell with width: $\Lambda = \Lambda' - \lambda$

$$\delta^{(1)}_{\Lambda'}(\boldsymbol{k}) = \delta^{(1)}_{\Lambda}(\boldsymbol{k}) + \delta^{(1)}_{\text{shell}}(\boldsymbol{k})$$

Idea: Integrate out the shell!

$$\mathcal{Z}[J_\Lambda] = \int \mathcal{D}\delta^{(1)}_\Lambda \mathcal{P}[\delta^{(1)}_\Lambda] \exp\Bigg(\int_{\boldsymbol{k}} J_\Lambda(\boldsymbol{k}) \left[\sum_O b_O^{\Lambda'} O[\delta^{(1)}_\Lambda](-\boldsymbol{k}) \right] + \frac{1}{2} P_\epsilon^{\Lambda'} \int_{\boldsymbol{k}} J_\Lambda(\boldsymbol{k}) J_\Lambda(-\boldsymbol{k}) + \mathcal{O}[J_\Lambda^2 \delta^{(1)}_\Lambda,\ J_\Lambda^3] \Bigg)$$

$$\times \left(1 + \int_{\boldsymbol{k}} J_\Lambda(\boldsymbol{k}) \left[\sum_O b_O^{\Lambda'} \left(\mathcal{S}^1_O[\delta^{(1)}_\Lambda](-\boldsymbol{k}) + \mathcal{S}^2_O[\delta^{(1)}_\Lambda](-\boldsymbol{k}) + \ldots \right) \right] + \frac{1}{2} \int_{\boldsymbol{k},\boldsymbol{k}'} J_\Lambda(\boldsymbol{k}) J_\Lambda(\boldsymbol{k}') \sum_{O,O'} b_O^{\Lambda'} b_{O'}^{\Lambda'} \left[\mathcal{S}^{11}_{OO'}[\delta^{(1)}_\Lambda](\boldsymbol{k},\boldsymbol{k}') + \ldots \right] + \mathcal{O}[J_\Lambda^2 \delta^{(1)}_\Lambda,\ J_\Lambda^3] \right)$$

The running of the bias/stochastic operators is done connecting both cutoff

What appears after integrating out the shell

The shell expansion (Wilson formalism)

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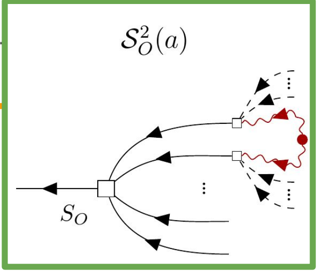
$$\times \left(1 + \int_{\boldsymbol{k}} J_\Lambda(\boldsymbol{k}) \left[\sum_O b_O^{\Lambda'} \left(\mathcal{S}^1_O[\delta^{(1)}_\Lambda](-\boldsymbol{k}) + \mathcal{S}^2_O[\delta^{(1)}_\Lambda](-\boldsymbol{k}) + \ldots\right)\right] + \frac{1}{2} \int_{\boldsymbol{k},\boldsymbol{k}'} J_\Lambda(\boldsymbol{k}) J_\Lambda(\boldsymbol{k}') \sum_{O,O'} b_O^{\Lambda'} b_{O'}^{\Lambda'} \left[\mathcal{S}^{11}_{OO'}[\delta^{(1)}_\Lambda](\boldsymbol{k},\boldsymbol{k}') + \ldots\right] \right.$$

The running of the bias/stochastic operators is done connecting both cutoff

What appears after integrating out the shell

Bias corrections



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Idea: Integrate out the shell!

$$\mathcal{Z}[J_\Lambda] = \int \mathcal{D}\delta^{(1)}_\Lambda \mathcal{P}[\delta^{(1)}_\Lambda] \exp\left(\int_{\boldsymbol{k}} J_\Lambda(\boldsymbol{k}) \left[\sum_O b_O^{\Lambda'} O[\delta^{(1)}_\Lambda](-\boldsymbol{k}) \right] + \frac{1}{2} P_\epsilon^{\Lambda'} \int_{\boldsymbol{k}} J_\Lambda(\boldsymbol{k}) J_\Lambda(-\boldsymbol{k}) + \mathcal{O}[J_\Lambda^2 \delta^{(1)}_\Lambda,\ J_\Lambda^3] \right)$$

$$\times \left(1 + \int_{\boldsymbol{k}} J_\Lambda(\boldsymbol{k}) \left[\sum_O b_O^{\Lambda'} \left(\mathcal{S}^1_O[\delta^{(1)}_\Lambda](-\boldsymbol{k}) + \mathcal{S}^2_O[\delta^{(1)}_\Lambda](-\boldsymbol{k}) + \ldots \right) \right] + \frac{1}{2} \int_{\boldsymbol{k},\boldsymbol{k}'} J_\Lambda(\boldsymbol{k}) J_\Lambda(\boldsymbol{k}') \sum_{O,O'} b_O^{\Lambda'} b_{O'}^{\Lambda'} \left[\mathcal{S}^{11}_{OO'}[\delta^{(1)}_\Lambda](\boldsymbol{k},\boldsymbol{k}') + \ldots \right] \right)$$

The running of the bias/stochastic operators is done connecting both cutoff

What appears after integrating out the shell

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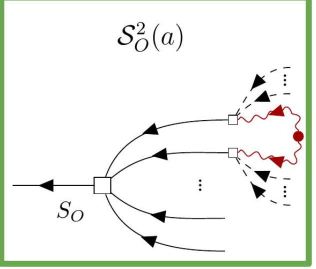
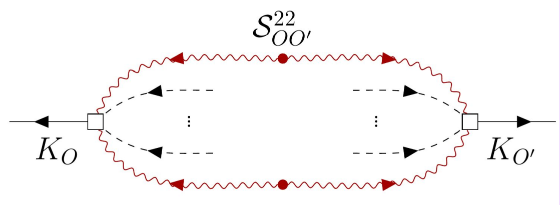


Stochastic corrections



Example...

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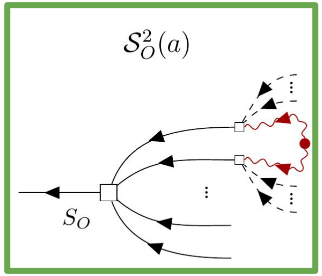
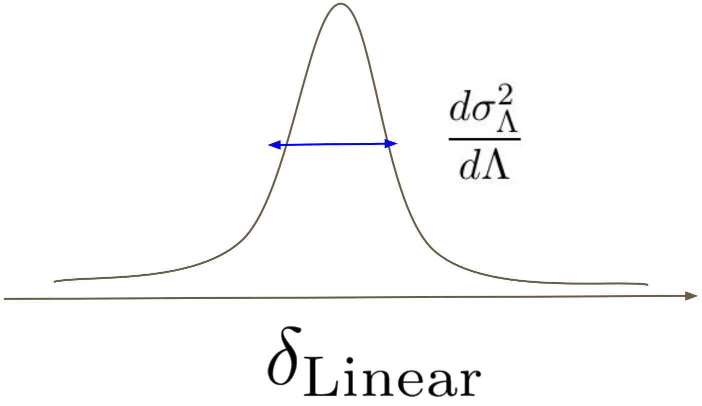


Correction to those operators!

$$\mathcal{S}^2_{\delta^2}[\delta^{(1)}_\Lambda](\boldsymbol{k}) = \left[\frac{68}{21}\delta^{(1+2)}(\boldsymbol{k}) + \frac{8126}{2205}\left[\delta^2(\boldsymbol{k})\right]^{(2)} + \frac{254}{2205}\mathcal{G}_2^{(2)}(\boldsymbol{k})\right]\int \frac{p^2 dp}{2\pi^2} P_{\rm shell}(p)$$

$$+ \text{higher derivative (h.d.)} + \mathcal{O}\left[\left(\delta^{(1)}_\Lambda\right)^3\right],$$

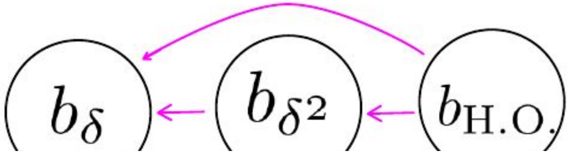
$$\int_{\boldsymbol{p}} P_{\rm shell}(p) = \int_\Lambda^{\Lambda+\lambda} \frac{p^2 dp}{2\pi^2} P_{\rm L}(p) = \frac{d\sigma^2_\Lambda}{d\Lambda}\bigg|_\Lambda \lambda + \mathcal{O}(\lambda^2),$$



Results

$$\frac{d}{d\Lambda} b_O(\Lambda) = -\frac{d\sigma_\Lambda^2}{d\Lambda} \sum_{O'} s^O_{O'}\, b_{O'}(\Lambda)\,,$$

J^1



$s^O_{O'}$	δ	δ^2	$\mathcal{G}_2$	δ^3	$\mathcal{G}_3$	Γ_3	$\delta\mathcal{G}_2$
$\mathbb{1}$	-	-	-	-	-	-	-
δ	-	$68/21$	-	3	-	-	$-4/3$
δ^2	-	$8126/2205$	-	$68/7$	-	-	$-376/105$
$\mathcal{G}_2$	-	$254/2205$	-	-	-	-	$116/105$

Solutions

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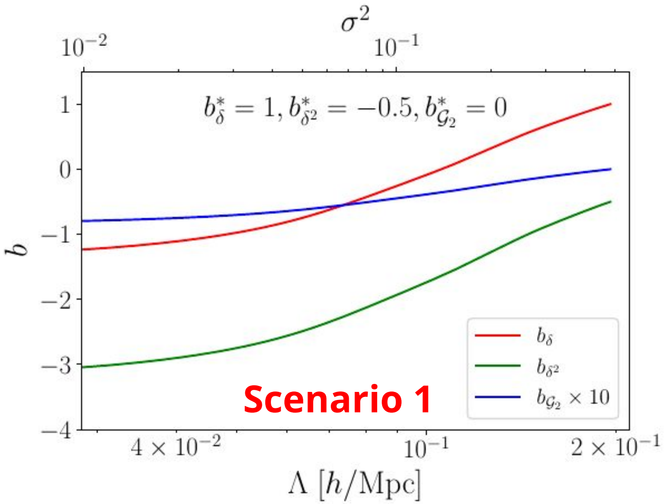
Wilson-Polchinski RG-equations

$$\frac{db_\delta}{d\Lambda} = -\left[\frac{68}{21}b_{\delta^2} + 3b^*_{\delta^3} - \frac{4}{3}b^*_{\mathcal{G}_2\delta}\right]\frac{d\sigma^2_\Lambda}{d\Lambda},$$

$$\frac{db_{\delta^2}}{d\Lambda} = -\left[\frac{8126}{2205}b_{\delta^2} + \frac{17}{7}b^*_{\delta^3} - \frac{376}{105}b^*_{\mathcal{G}_2\delta} + b^{*(\delta^2)}_{n=4}\right]\frac{d\sigma^2_\Lambda}{d\Lambda},$$

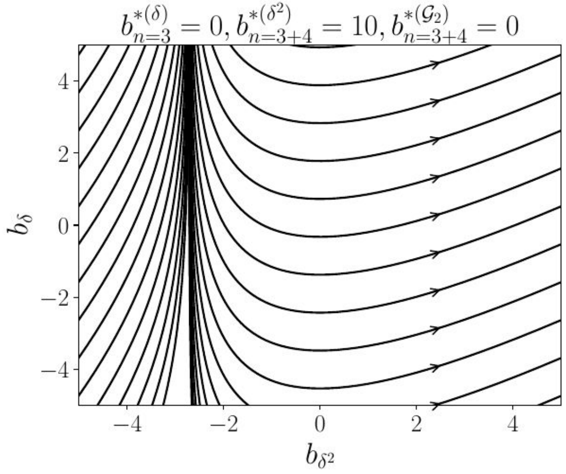
$$\frac{db_{\mathcal{G}_2}}{d\Lambda} = -\left[\frac{254}{2205}b_{\delta^2} + \frac{116}{105}b^*_{\mathcal{G}_2\delta} + b^{*(\mathcal{G}_2)}_{n=4}\right]\frac{d\sigma^2_\Lambda}{d\Lambda}.$$



Scenario 1

Notice that:

- **Bias parameter that are zero, may be sourced;**
- **Bias parameters may change sign!**



Part IV - PNGs (2405.21002)

w/ Charalampos Nikolis
(master's student at MPA)

PNGs

Interaction kernel

$$\delta^{(1)}(\boldsymbol{k}) = \delta_{\rm G}^{(1)}(\boldsymbol{k}) + f_{\rm NL} \int_{\boldsymbol{p}_1,\boldsymbol{p}_2} \hat{\delta}_{\rm D}(\boldsymbol{k} - \boldsymbol{p}_{12}) K_{\rm NL}(\boldsymbol{p}_1, \boldsymbol{p}_2) \frac{M(|\boldsymbol{p}_1 + \boldsymbol{p}_2|)}{M(p_1)M(p_2)} \delta_{\rm G}^{(1)}(\boldsymbol{p}_1)\delta_{\rm G}^{(1)}(\boldsymbol{p}_2) + \mathcal{O}[f_{\rm NL}^2, g_{\rm NL}],$$

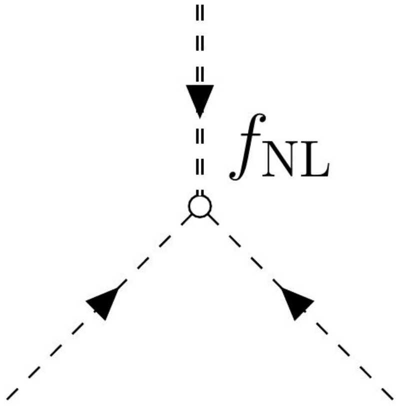
PNGs

Interaction kernel

$$\delta^{(1)}(\boldsymbol{k}) = \delta_{\rm G}^{(1)}(\boldsymbol{k}) + f_{\rm NL} \int_{\boldsymbol{p}_1,\boldsymbol{p}_2} \hat{\delta}_{\rm D}(\boldsymbol{k} - \boldsymbol{p}_{12}) K_{\rm NL}(\boldsymbol{p}_1, \boldsymbol{p}_2) \frac{M(|\boldsymbol{p}_1 + \boldsymbol{p}_2|)}{M(p_1)M(p_2)} \delta_{\rm G}^{(1)}(\boldsymbol{p}_1)\delta_{\rm G}^{(1)}(\boldsymbol{p}_2) + \mathcal{O}[f_{\rm NL}^2, g_{\rm NL}],$$

Cubic vertex interaction (like in QFT)

$$\mathcal{Z}[J_{m,\Lambda}, J_{g,\Lambda}] = \int \mathcal{D}\delta_\Lambda^{(1)} \mathcal{P}[\delta_\Lambda^{(1)}] \exp\left\{ S_{\rm int} + \int_{\boldsymbol{k}} J_{m,\Lambda}(\boldsymbol{k})\delta[\delta_\Lambda^{(1)}](-\boldsymbol{k}) + \int_{\boldsymbol{k}} J_{g,\Lambda}(\boldsymbol{k})\delta_g[\delta_\Lambda^{(1)}](-\boldsymbol{k}) \right\}$$



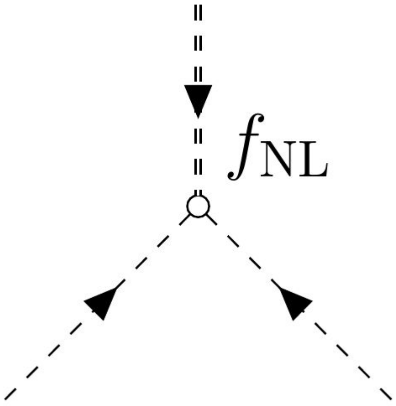
PNGs

Interaction kernel

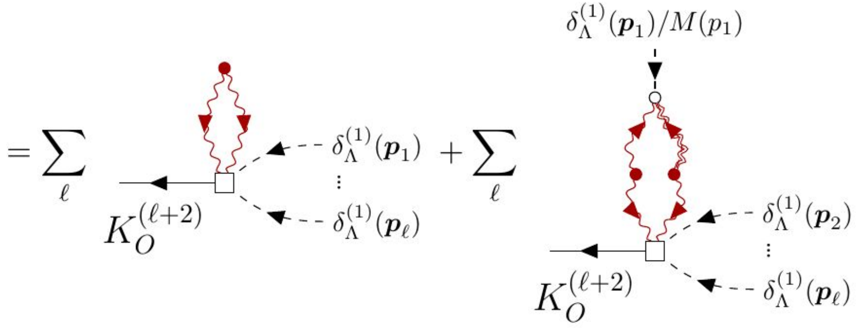
$$\delta^{(1)}(\boldsymbol{k}) = \delta_{\mathrm{G}}^{(1)}(\boldsymbol{k}) + f_{\mathrm{NL}}\int_{\boldsymbol{p}_1,\boldsymbol{p}_2} \hat{\delta}_{\mathrm{D}}(\boldsymbol{k}-\boldsymbol{p}_{12})\, K_{\mathrm{NL}}(\boldsymbol{p}_1,\boldsymbol{p}_2)\frac{M(|\boldsymbol{p}_1+\boldsymbol{p}_2|)}{M(p_1)M(p_2)}\delta_{\mathrm{G}}^{(1)}(\boldsymbol{p}_1)\delta_{\mathrm{G}}^{(1)}(\boldsymbol{p}_2) + \mathcal{O}[f_{\mathrm{NL}}^2, g_{\mathrm{NL}}],$$

Cubic vertex interaction (like in QFT)

$$\mathcal{Z}[J_{m,\Lambda}, J_{g,\Lambda}] = \int \mathcal{D}\delta_\Lambda^{(1)}\mathcal{P}[\delta_\Lambda^{(1)}]\exp\left\{S_{\mathrm{int}} + \int_{\boldsymbol{k}} J_{m,\Lambda}(\boldsymbol{k})\delta[\delta_\Lambda^{(1)}](-\boldsymbol{k}) + \int_{\boldsymbol{k}} J_{g,\Lambda}(\boldsymbol{k})\delta_g[\delta_\Lambda^{(1)}](-\boldsymbol{k})\right\}$$



$$\mathcal{S}_O(\boldsymbol{k}) \equiv [\mathcal{S}_O^2]_{\mathrm{free}}(\boldsymbol{k}) + [\mathcal{S}_O]_{\mathrm{int}}(\boldsymbol{k})$$



PNGs

Spin-0

$$\text{First order:} \quad \delta,\ \Psi;$$
$$\text{Second order:} \quad \delta^2,\ \mathcal{G}_2,\ \delta\Psi;$$
$$\text{Third order:} \quad \delta^3,\ \delta^2\Psi,\ \delta\mathcal{G}_2,\ \Psi\mathcal{G}_2,\ \Gamma_3,\ \mathcal{G}_3$$

Spin-2

$$\text{First order:} \quad \delta;$$
$$\text{Second order:} \quad \delta^2,\ \mathcal{G}_2,\ \mathrm{Tr}\left[\Psi\Pi^{[1]}\right];$$
$$\text{Third order:} \quad \delta^3,\ \delta\mathcal{G}_2,\ \delta\ \mathrm{Tr}\left[\Psi\Pi^{[1]}\right],\ \Gamma_3,\ \mathcal{G}_3,\ \mathrm{Tr}\left[\Psi\Pi^{[2]}\right]$$

PNGs

Free term

New interaction

$$\frac{db_\delta}{d\Lambda} = -\left[\frac{68}{21} b_{\delta^2}(\Lambda) + b_{n=3}^{*\{\delta\}\mathrm{G}}\right] \frac{d\sigma_\Lambda^2}{d\Lambda}$$

$$- a_0 f_{\mathrm{NL}} \left[-\frac{13}{21} b_\Psi + \frac{13}{21} b_{\Psi\delta} + b_{n=3}^{*\{\delta\}\mathrm{NG}}\right] \left(\frac{H_0}{\Lambda}\right)^2 \frac{3\,\Omega_m}{2\,T(\Lambda)} \frac{d\sigma_\Lambda^2}{d\Lambda}\,;$$

PNGs

Free term

New interaction

$$\frac{db_\delta}{d\Lambda} = -\left[\frac{68}{21} b_{\delta^2}(\Lambda) + b_{n=3}^{*\{\delta\}\mathrm{G}}\right] \frac{d\sigma_\Lambda^2}{d\Lambda}$$

$$- a_0 f_{\mathrm{NL}} \left[-\frac{13}{21} b_\Psi + \frac{13}{21} b_{\Psi\delta} + b_{n=3}^{*\{\delta\}\mathrm{NG}}\right] \left(\frac{H_0}{\Lambda}\right)^2 \frac{3\,\Omega_m}{2\,T(\Lambda)} \frac{d\sigma_\Lambda^2}{d\Lambda} ;$$

Now a coupled set of ODEs

$$\frac{db_\Psi}{d\Lambda} = -a_0 f_{\mathrm{NL}} b_{n=3}^{*\{\Psi\}\mathrm{NG}} \frac{d\sigma_\Lambda^2}{d\Lambda} - 4 a_0 f_{\mathrm{NL}} b_{\delta^2} \frac{d\sigma_\Lambda^2}{d\Lambda} ,$$

$$\frac{db_{\Psi\delta}}{d\Lambda} = -a_0 f_{\mathrm{NL}} \left[\frac{272}{21} b_{\delta^2} + b_{n=3+4}^{*\{\Psi\delta\}\mathrm{G}} + b_{n=3+4}^{*\{\Psi\delta\}\mathrm{NG}}\right] \frac{d\sigma_\Lambda^2}{d\Lambda} ,$$

PNGs

Free term

New interaction

$$\frac{db_\delta}{d\Lambda} = -\left[\frac{68}{21} b_{\delta^2}(\Lambda) + b^{*\{\delta\}\mathrm{G}}_{n=3}\right]\frac{d\sigma^2_\Lambda}{d\Lambda} - a_0 f_{\rm NL}\left[-\frac{13}{21} b_\Psi + \frac{13}{21} b_{\Psi\delta} + b^{*\{\delta\}\mathrm{NG}}_{n=3}\right]\left(\frac{H_0}{\Lambda}\right)^2 \frac{3\,\Omega_m}{2\,T(\Lambda)}\frac{d\sigma^2_\Lambda}{d\Lambda};$$

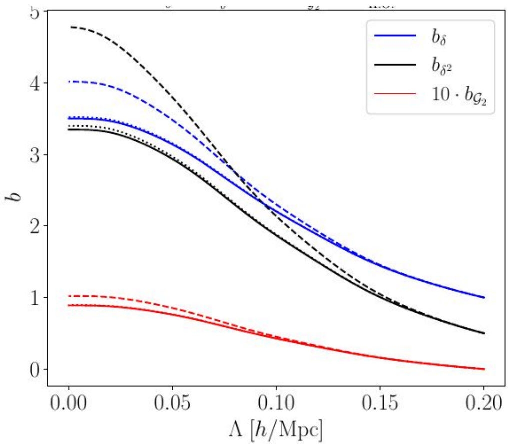
Now a coupled set of ODEs

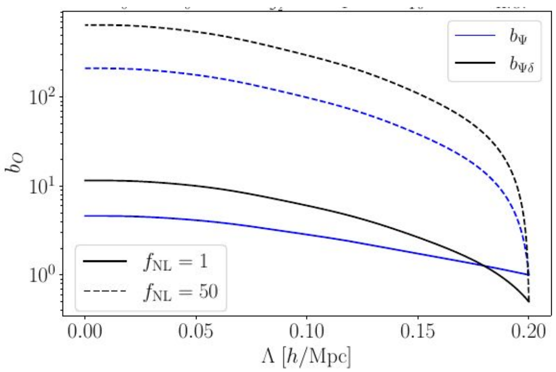
$$\frac{db_\Psi}{d\Lambda} = -a_0 f_{\rm NL} b^{*\{\Psi\}\mathrm{NG}}_{n=3}\frac{d\sigma^2_\Lambda}{d\Lambda} - 4 a_0 f_{\rm NL} b_{\delta^2}\frac{d\sigma^2_\Lambda}{d\Lambda},$$

$$\frac{db_{\Psi\delta}}{d\Lambda} = -a_0 f_{\rm NL}\left[\frac{272}{21} b_{\delta^2} + b^{*\{\Psi\delta\}\mathrm{G}}_{n=3+4} + b^{*\{\Psi\delta\}\mathrm{NG}}_{n=3+4}\right]\frac{d\sigma^2_\Lambda}{d\Lambda},$$

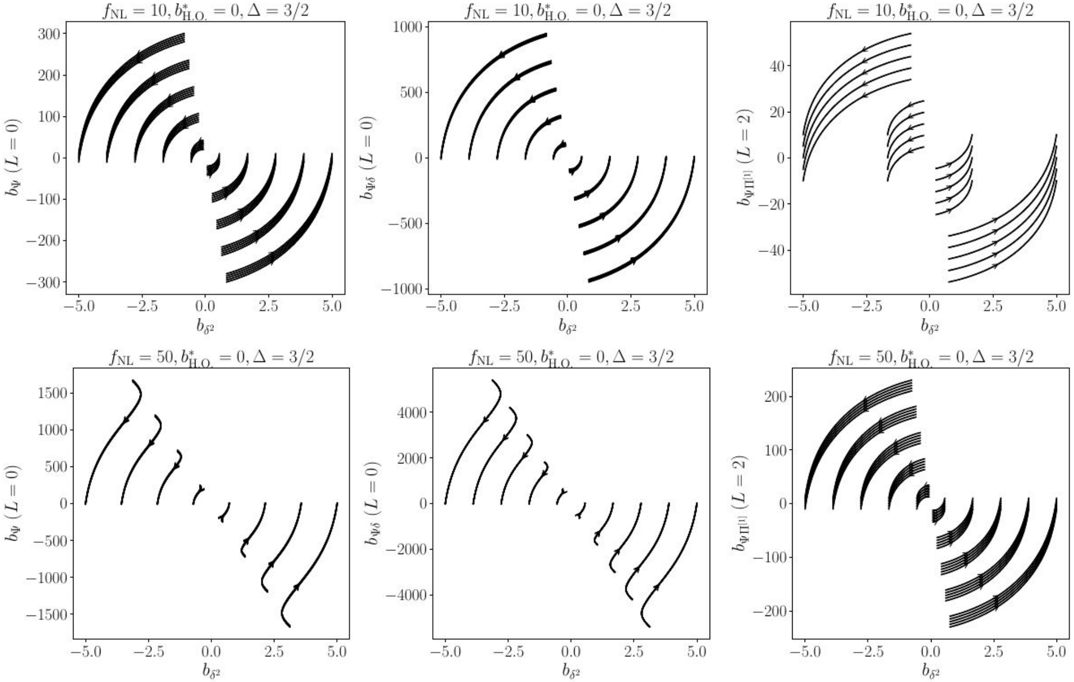
$s^O_{O'}$	δ^2	δ^3	$\delta\mathcal{G}_2$	Ψ	$\Psi\delta$	$\Psi\delta^2$	$\Psi\mathcal{G}_2$	$\mathrm{Tr}\,\Psi\Pi^{[1]}$	$\delta\,\mathrm{Tr}\,\Psi\Pi^{[1]}$	$\mathrm{Tr}\,\Psi\Pi^{[2]}$
δ	68/21	3	−4/3	−13/21	13/21	2	−4/3	34/21	1	34/21
δ^2	8126/2205	68/7	−376/105	43/135	478/135	47/21	−31/21	124/315	178/105	14347/6027
$\mathcal{G}_2$	254/2205	-	116/105	−1699/13230	79/2205	-	−1/21	−661/4410	4/35	−241/735
Ψ	4	-	-	-	-	1	-	-	-	-
$\delta\Psi$	272/21	12	−8/3	-	-	68/21	-	-	-	-
$\mathrm{Tr}\,\Psi\Pi^{[1]}$	64/105	-	16/15	-	-	-	-	-	8/105	58/305

PNGs





Fuzzy phase space



Part V - Stochasticity in LSS (2404.16929)

Stochasticity

$$\delta_g(\boldsymbol{x},\tau) \equiv \frac{n_g(\boldsymbol{x},\tau)}{\bar{n}_g(\tau)} - 1 = \sum_O \left[b_O(\tau) + c_{\epsilon,O}(\tau)\epsilon(\boldsymbol{x},\tau)\right] O(\boldsymbol{x},\tau) + \epsilon(\boldsymbol{x},\tau)$$

Properties of the noise:

The 'shot-noise terms'

$$\langle \epsilon(\boldsymbol{k}_1)\epsilon(\boldsymbol{k}_2)\rangle = \hat{\delta}_{\rm D}(\boldsymbol{k}_{12}) P_{\epsilon,\mathbb{1}} \,,$$

$$\langle \epsilon(\boldsymbol{k}_1)\epsilon(\boldsymbol{k}_2)\epsilon(\boldsymbol{k}_3)\rangle = \hat{\delta}_{\rm D}(\boldsymbol{k}_{123}) B_{\epsilon,\mathbb{1}} \,,$$

$$\langle \epsilon(\boldsymbol{k}_1)\ldots\epsilon(\boldsymbol{k}_m)\rangle = \hat{\delta}_{\rm D}(\boldsymbol{k}_{1\ldots m}) C^{(m)}_{\epsilon,\mathbb{1}} \,.$$

Linearly does not correlate with O's

$$\langle \epsilon(\boldsymbol{k}_1) O(\boldsymbol{k}_2) O'(\boldsymbol{k}_3)\ldots\rangle = 0$$

$$\langle \epsilon(\boldsymbol{k}_1)\ldots\epsilon(\boldsymbol{k}_m) O(\boldsymbol{k}_{m+1})\rangle = \hat{\delta}_{\rm D}(\boldsymbol{k}_{1\ldots m}) C^{(m)}_{\epsilon,O}\, O(\boldsymbol{k}_{m+1})$$

Henrique Rubira

Stochasticity

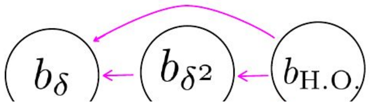
Coupled to higher powers of J

$$\mathcal{Z}[J_\Lambda] = \int \mathcal{D}\delta^{(1)}_\Lambda \mathcal{P}[\delta^{(1)}_\Lambda] \exp\left(\sum_m \left\{ \frac{1}{m!} \int_{\boldsymbol{x}} \left[(J_\Lambda(\boldsymbol{x}))^m \sum_O C_O^{(m)}(\Lambda') O[\delta^{(1)}_\Lambda](\boldsymbol{x}) \right] + \zeta^{(m)}[J_\Lambda, \delta^{(1)}_\Lambda] \right\} \right)$$

Shell corrections

$$\frac{d}{d\Lambda} b_O(\Lambda) = -\frac{d\sigma^2_\Lambda}{d\Lambda} \sum_{O'} s^O_{O'}\, b_{O'}(\Lambda)\,,$$

J^1



Henrique Rubira

Stochasticity

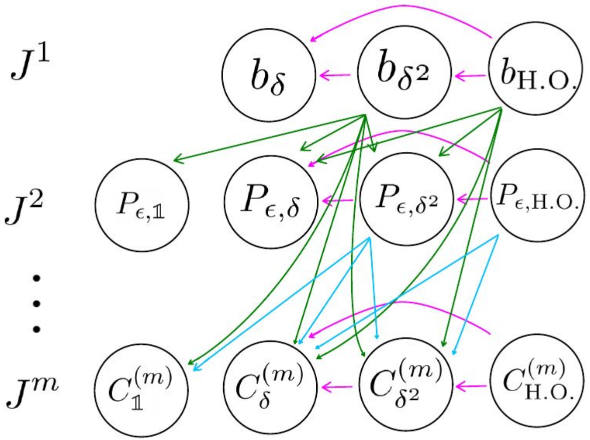
Coupled to higher powers of J

$$\mathcal{Z}[J_\Lambda] = \int \mathcal{D}\delta_\Lambda^{(1)} \mathcal{P}[\delta_\Lambda^{(1)}] \exp\left(\sum_m \left\{ \frac{1}{m!} \int_{\boldsymbol{x}} \left[(J_\Lambda(\boldsymbol{x}))^m \sum_O C_O^{(m)}(\Lambda') O[\delta_\Lambda^{(1)}](\boldsymbol{x}) \right] + \zeta^{(m)}[J_\Lambda, \delta_\Lambda^{(1)}] \right\} \right)$$

Shell corrections

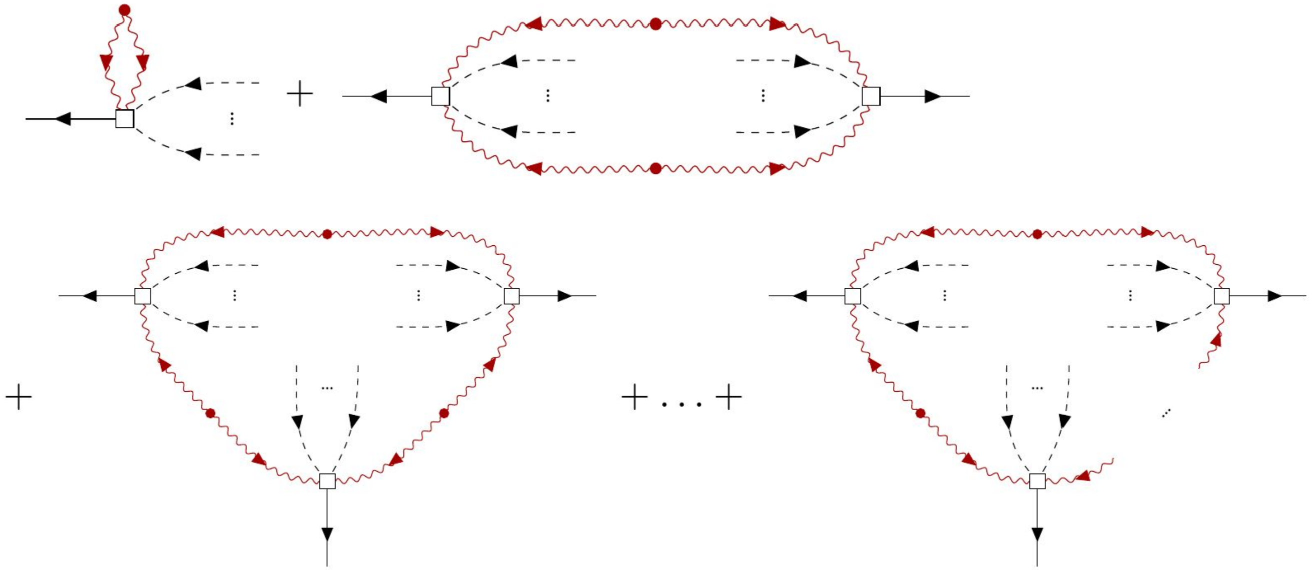
$$\frac{d}{d\Lambda} b_O(\Lambda) = -\frac{d\sigma_\Lambda^2}{d\Lambda} \sum_{O'} s_{O'}^O \, b_{O'}(\Lambda)\,,$$

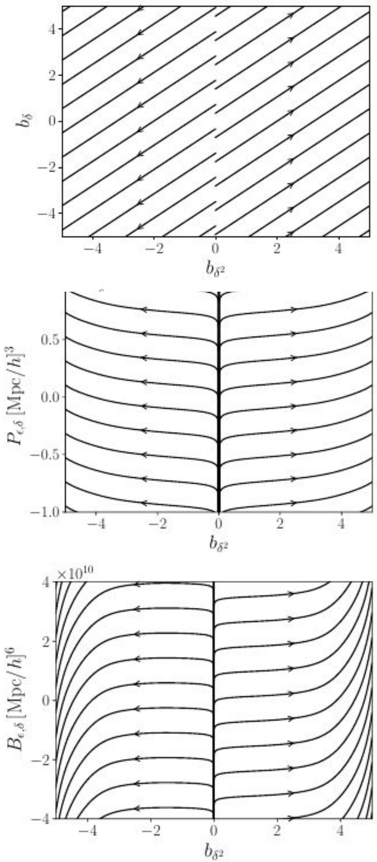
$$\frac{d}{d\Lambda} C_O^{(m)}(\Lambda) \propto -[P_{\rm L}(\Lambda)]^{p-1} \frac{d\sigma_\Lambda^2}{d\Lambda} \sum_{O_1, O_2, \ldots O_m} s^O_{O_1 O_2 \ldots O_m} C_{O_1}^{(i_1)}(\Lambda) \ldots C_{O_p}^{(i_p)}(\Lambda)$$



Stochasticity

$$\frac{d}{d\Lambda} C_O^{(m)}(\Lambda) \propto -\left[P_{\mathrm{L}}(\Lambda)\right]^{p-1} \frac{d\sigma_\Lambda^2}{d\Lambda} \sum_{O_1, O_2, \ldots O_m} s^O_{O_1 O_2 \ldots O_m} C^{(i_1)}_{O_1}(\Lambda) \ldots C^{(i_p)}_{O_p}(\Lambda)$$





Part IV - Final remarks

How to relate the renormalization schemes?

N-point function renormalized bias
(Assassi, Baumann, Green, Zaldarriaga)

Finite cutoff bias
(This work)

How to connect both?

$$[\![O']\!](\boldsymbol{k}') \longleftrightarrow O'[\delta_\Lambda^{(1)}](\boldsymbol{k}')$$

Henrique Rubira

How to relate the renormalization schemes?

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Finite cutoff bias
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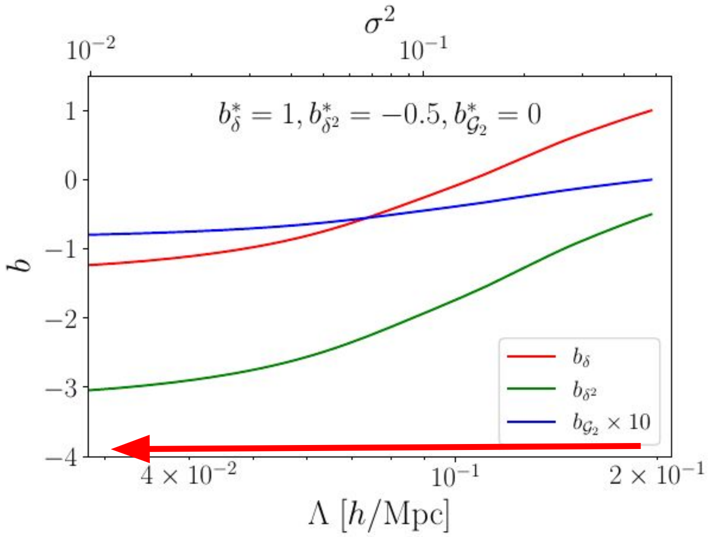
How to connect both?

$$[\![O']\!](\boldsymbol{k}') \longleftrightarrow O'[\delta_\Lambda^{(1)}](\boldsymbol{k}')$$



Solution: Run the bias towards

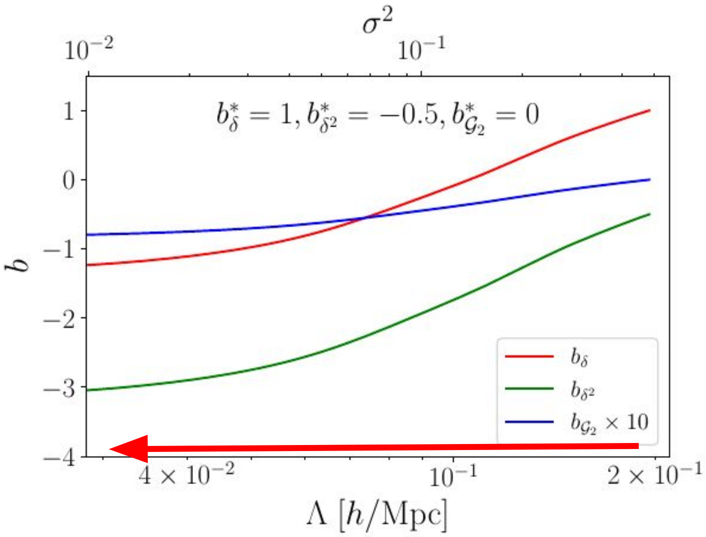
$$\Lambda \to 0$$

How to relate the renormalization schemes?

N-point function renormalized bias
(Assassi, Baumann, Green, Zaldarriaga)

Finite cutoff bias
(This work)

$$\lim_{\Lambda\to 0;\ k/\Lambda\ \text{fixed}} \langle O[\delta_\Lambda^{(1)}](\boldsymbol{k})[\![O']\!](\boldsymbol{k}')\rangle = \lim_{\Lambda\to 0;\ k/\Lambda\ \text{fixed}} \langle O[\delta_\Lambda^{(1)}](\boldsymbol{k})O'[\delta_\Lambda^{(1)}](\boldsymbol{k}')\rangle$$



Solution: Run the bias towards

$$\Lambda \to 0$$

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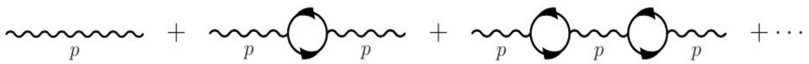
Logs in QFT

Logs in QFT: Arise when we have a hierarchy of scales $$\lim_{E\to\infty} \Gamma(E,m) = E^d \Gamma(1, \frac{m}{E}) \times O\left[\ln\left(\frac{E}{m}\right)\right]$$

Approach 1) Resum diagrams by hand (if you can)



$$\tilde{V}(p^2) = \frac{e_R^2}{p^2}\left[1 + \frac{e_R^2}{12\pi^2}\ln\frac{p^2}{p_0^2} + \left(\frac{e_R^2}{12\pi^2}\ln\frac{p^2}{p_0^2}\right)^2 + \cdots\right] = \frac{1}{p^2}\left[\frac{e_R^2}{1 - \frac{e_R^2}{12\pi^2}\ln\frac{p^2}{p_0^2}}\right]$$

$$e_{\text{eff}}^2(p^2) = \frac{e_R^2}{1 - \frac{e_R^2}{12\pi^2}\ln\frac{p^2}{p_0^2}}$$

Extracted from Schwartz's and Weinberg's books

Approach 2) Direct from the RGE

$$p_0^2 \frac{d}{dp_0^2}\tilde{V}(p^2) = 0$$

$$p_0^2 \frac{de_{\text{eff}}}{dp_0^2} = \frac{e_{\text{eff}}^3}{24\pi^2}$$

$$e_{\text{eff}}^2(p^2) = \frac{e_R^2}{1 - \frac{e_R^2}{12\pi^2}\ln\frac{p^2}{p_0^2}}$$

Why you should care

Henrique Rubira

- Additional cross-check for EFT inference;
- Systematic renormalization of bias and stochastic parameters (including PNG);
- Completely absorb cutoff dependence in the counter-terms keeping also sub-leading contributions;
- Systematic renormalization of n-point functions. Self-consistent renormalization for P(k), B(k1,k2,k3), ...
- Priors for EFT analysis in $\Lambda \to 0$


Thanks a lot!

Why just not taking $\Lambda \to \infty$?

$$\delta^{(1)}_{\Lambda'}(\boldsymbol{k}) = \delta^{(1)}_{\Lambda}(\boldsymbol{k}) + \delta^{(1)}_{\rm shell}(\boldsymbol{k}) \qquad \Lambda = \Lambda' - \lambda$$

$$\mathcal{Z}[J_\Lambda] = \int \mathcal{D}\delta^{(1)}_\Lambda \mathcal{P}[\delta^{(1)}_\Lambda] \exp\left(\int_{\boldsymbol{k}} J_\Lambda(\boldsymbol{k}) \left[\sum_O b_O^{\Lambda'} O[\delta^{(1)}_\Lambda](-\boldsymbol{k}) \right] \right.$$

Continuum of the theory is determined by taking $\Lambda' \to \infty$

This determines local terms to be added to the action that will cancel out UV dependence (the counter-terms)

In Wilson-Polchinski we integrate modes up to the cutoff of the theory $k_{\rm NL}$

Renormalization scale $\Lambda_* < k_{\rm NL}$

The n-point function renormalized bias
(Assassi, Baumann, Green, Zaldarriaga, 2014)

Henrique Rubira

Intuition: Define the bias parameter of order "n" as the large-scale limit of "n+1"-point functions

Example 1: Define the linear bias in the large-scale limit of P(k):

$$b_\delta = \lim_{k \to 0} \frac{\langle \delta_g \delta \rangle}{\langle \delta \delta \rangle}$$

Example 2: Define the 2nd-order bias parameters in the large-scale limit of B(k1,k2,k3)

More formally:

$$\langle \delta^{(1)}(\boldsymbol{k}_1) \cdots \delta^{(1)}(\boldsymbol{k}_m) [\![O]\!](\boldsymbol{k}) \rangle \xrightarrow{k_i \to 0} \langle \delta^{(1)}(\boldsymbol{k}_1) \cdots \delta^{(1)}(\boldsymbol{k}_m) O[\delta](\boldsymbol{k}) \rangle_{\mathrm{LO}}$$

Example:

$$[\![\delta^2]\!] = \delta^2 - \sigma_\infty^2 \left(1 + \frac{68}{21}\delta + \frac{8126}{2205}\delta^2 + \frac{254}{2205}\mathcal{G}_2 \right)$$

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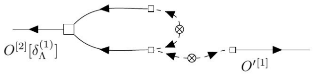
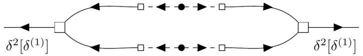
Differences between renormalization schemes

	N-point renormalization	Finite Λ
In practice, one has to:	Subtract all UV-dep part	Nothing to subtract. Bias params run… and we know how
Potential to:	Error-prone, missing finite contributions	Extra sanity-checks
$\langle(\delta)^{(1)}(\delta^2)^{(3)}\rangle$ 	Completely removed by c.t., but missing sub-leading $k^2 P_{\rm L}(k)\int_{\boldsymbol{p}} p^{-2} P_{\rm L}(p)$	Finite: $4 b_\delta^\Lambda b_{\delta^2}^\Lambda \int_p F_2(\boldsymbol{p},\boldsymbol{k}-\boldsymbol{p}) P_{\rm L}^\Lambda(p) P_{\rm L}^\Lambda(k)$
$\langle(\delta^2)^{(2)}(\delta^2)^{(2)}\rangle$ 	Subtracted to the stochastic term, but missing sub-leading $k^2 \int_{\boldsymbol{p}} p^{-2}[P_{\rm L}(p)]^2$	Finite and contributes to the stochastic running: $2\,(b_{\delta^2}^\Lambda)^2 \int_{\boldsymbol{p}} P_{\rm L}^\Lambda(p) P_{\rm L}^\Lambda(\lvert\boldsymbol{k}-\boldsymbol{p}\rvert)$

Logs in LSS

$$\Delta^2_{1-loop} = \left(\frac{k}{k_{NL}}\right)^{3+n} + \left(\frac{k}{k_{NL}}\right)^{2(3+n)} \left[\alpha(n) + \tilde{\alpha}(n) \ln\left(\frac{k}{k_{NL}}\right)\right]$$

n	-2	-3/2	-1	-1/2	0	1/2	1	3/2	2	5/2	3
α_{13}	$\frac{5\pi^2}{112}$	$\frac{992\pi}{6,615}$	$\dots$	$-\frac{416\pi}{8,085}$	$-\frac{\pi^2}{336}$	$\dots$	$\dots$	$\dots$	$-\frac{\pi^2}{168}$	$\dots$	$\dots$
α_{22}	$\frac{75\pi^2}{784}$	-0.232	$\dots$	$.698$	$\frac{29\pi^2}{784}$	$\dots$	$\dots$	$\dots$	$\frac{\pi^2}{392}$	$\dots$	$\dots$
$\tilde{\alpha}_{13}$	0	0	$\frac{61}{315}$	0	0	0	$-\frac{4}{105}$	0	0	0	$\frac{20}{1,323}$
$\tilde{\alpha}_{22}$	0	0	0	0	0	$-\frac{9}{98}$	0	$\frac{31}{16,464}$	0	$-\frac{359}{26,880}$	0
α	1.38	.239	$\dots$	.537	.336	$\dots$	$\dots$	$\dots$	$-.0336$	$\dots$	$\dots$
$\tilde{\alpha}$	0	0	.194	0	0	$-.0918$	.0381	$-.00188$	0	$-.0134$	.0151

Pajer+Zaldarriaga, 2013

The shell expansion (Wilson formalism)

Henrique Rubira

Consider a very thin shell with width: $\Lambda = \Lambda' - \lambda$

$$\delta^{(1)}_{\Lambda'}(\boldsymbol{k}) = \delta^{(1)}_{\Lambda}(\boldsymbol{k}) + \delta^{(1)}_{\text{shell}}(\boldsymbol{k})$$

Idea: Integrate out the shell!

$$\mathcal{Z}[J_\Lambda] = \int \mathcal{D}\delta^{(1)}_\Lambda \mathcal{P}[\delta^{(1)}_\Lambda] \int \mathcal{D}\delta^{(1)}_{\text{shell}} \mathcal{P}[\delta^{(1)}_{\text{shell}}] \tag{2.7}$$
$$\times \exp\left(\int_{\boldsymbol{k}} J_\Lambda(\boldsymbol{k}) \left[\sum_O b_O^{\Lambda'} O[\delta^{(1)}_\Lambda + \delta^{(1)}_{\text{shell}}](-\boldsymbol{k}) \right] + \frac{1}{2} P_\epsilon^{\Lambda'} \int_{\boldsymbol{k}} J_\Lambda(\boldsymbol{k}) J_\Lambda(-\boldsymbol{k}) + \mathcal{O}[J_\Lambda^2 \delta^{(1)}_\Lambda,\ J_\Lambda^3] \right)$$

1) Expand the operators in terms of the number of shell fields and integrate those out!

$$O^{(n)}[\delta^{(1)}_\Lambda + \delta^{(1)}_{\text{shell}}] = O^{(n)}[\delta^{(1)}_\Lambda] + O^{(n),(1)_{\text{shell}}}[\delta^{(1)}_\Lambda, \delta^{(1)}_{\text{shell}}] + O^{(n),(2)_{\text{shell}}}[\delta^{(1)}_\Lambda, \delta^{(1)}_{\text{shell}}] \tag{2.8}$$
$$+ \ldots + O^{(n),(n-1)_{\text{shell}}}[\delta^{(1)}_\Lambda, \delta^{(1)}_{\text{shell}}] + O^{(n)}[\delta^{(1)}_{\text{shell}}],$$

2) Integrate the shells

$$\mathcal{S}^2_O[\delta^{(1)}_\Lambda] = \sum_{n\geq 2} \int \mathcal{D}\delta^{(1)}_{\text{shell}} \mathcal{P}[\delta^{(1)}_{\text{shell}}]\, O^{(n),(2)_{\text{shell}}}[\delta^{(1)}_\Lambda, \delta^{(1)}_{\text{shell}}](\boldsymbol{k})$$

$$\mathcal{S}^{11}_{OO'}[\delta^{(1)}_\Lambda](\boldsymbol{k}, \boldsymbol{k}') = \sum_{n,n'\geq 1} \int \mathcal{D}\delta^{(1)}_{\text{shell}} \mathcal{P}[\delta^{(1)}_{\text{shell}}]\, O^{(n),(1)_{\text{shell}}}[\delta^{(1)}_\Lambda, \delta^{(1)}_{\text{shell}}](\boldsymbol{k})\, O'^{(n'),(1)_{\text{shell}}}[\delta^{(1)}_\Lambda, \delta^{(1)}_{\text{shell}}](\boldsymbol{k}')$$

Solutions

Henrique Rubira

1) Neglect fourth-order+ bias;
2) Neglect ***the running*** of fourth-order+ bias;
3) Ansatz for higher-order bias running.

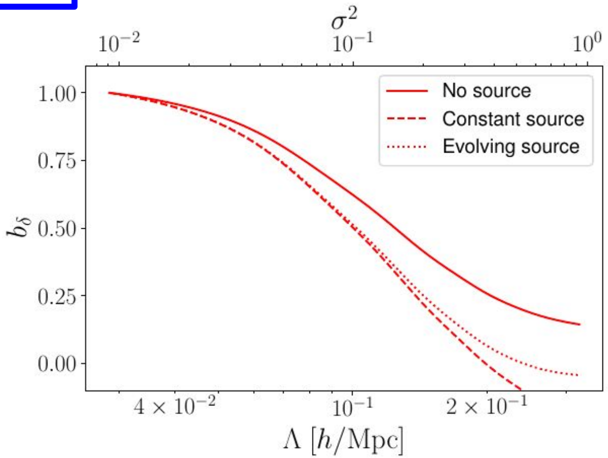
$$b^{(O)}_{n=3+4}(\sigma^2) = b^{*(O)}_{n=3+4}\, e^{-c^{(O)}(\sigma^2-\sigma^2_*)}$$

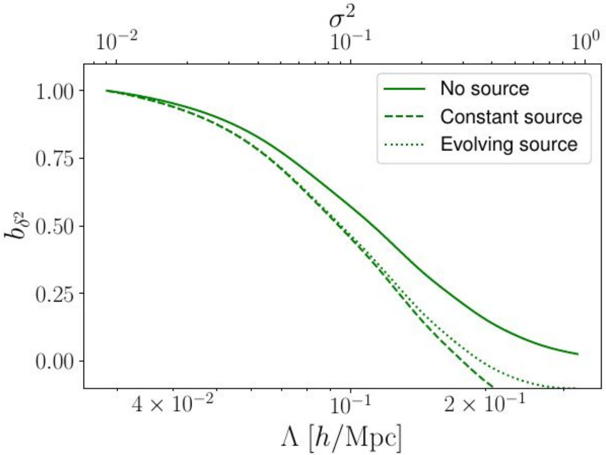
Wilson-Polchinski RG-equations

$$\frac{db_\delta}{d\Lambda} = -\left[\frac{68}{21}b_{\delta^2} + 3b^*_{\delta^3} - \frac{4}{3}b^*_{\mathcal{G}_2\delta}\right]\frac{d\sigma^2_\Lambda}{d\Lambda}\,,$$
$$\frac{db_{\delta^2}}{d\Lambda} = -\left[\frac{8126}{2205}b_{\delta^2} + \frac{17}{7}b^*_{\delta^3} - \frac{376}{105}b^*_{\mathcal{G}_2\delta} + b^{*(\delta^2)}_{n=4}\right]\frac{d\sigma^2_\Lambda}{d\Lambda}\,,$$
$$\frac{db_{\mathcal{G}_2}}{d\Lambda} = -\left[\frac{254}{2205}b_{\delta^2} + \frac{116}{105}b^*_{\mathcal{G}_2\delta} + b^{*(\mathcal{G}_2)}_{n=4}\right]\frac{d\sigma^2_\Lambda}{d\Lambda}\,.$$

Conclusions:

1) **Neglecting source affects result;**
2) **Evolving source does not affect the result!**





EFTofLSS via Wilson Polchinski

(based on Carroll, Leichenauer, Pollack, 13)

$$Z[J] = \int \mathcal{D}\phi_{\rm in}\ \exp\left(S_0[\phi_{\rm in}] + J_i \phi^i[\phi_{\rm in}]\right)$$

with

$$S_0[\phi_{\rm in}] = -\frac{1}{2}\phi^i [P(\Lambda)^{-1}]_{ij}\phi^j$$

We use

$$\left.\frac{\partial \mathcal{Z}}{\partial J_\Lambda \ldots \partial J_\Lambda}\right|_{J_\Lambda = 0}$$

Since

$$\langle \phi^{i_1} \cdots \phi^{i_n} \rangle = \int \mathcal{D}\phi_{\rm in}\ \phi^{i_1}[\phi_{\rm in}] \cdots \phi^{i_n}[\phi_{\rm in}] e^{S_0[\phi_{\rm in}]}$$

$$\phi^i_{\rm SPT} \equiv K^i_{{\rm SPT}\,j}\,\phi^j_{\rm in} + \frac{1}{2} K^i_{{\rm SPT}\,jk}\,\phi^j_{\rm in}\phi^k_{\rm in} + \cdots$$

Advantages:

- Path integral formulation
- Systematic generation EFT structure (coefficients are closed under RG flow)
- Keeps small (yet-perturbative) modes in the theory

$$\frac{d}{d\Lambda} K^{j_1\cdots j_n}_{i_1\cdots i_m} = -\frac{1}{2}\left(\frac{dP^{ij}}{d\Lambda} K^{j_1\cdots j_n}_{ij i_1\cdots i_m} + \frac{dP^{ij}}{d\Lambda}\sum_{k=0}^{m}\sum_{l=0}^{n}\binom{m}{k}\binom{n}{l} K^{j_1\cdots j_l}_{i i_1 \cdots i_k} K^{j_{l+1}\cdots j_n}_{j i_{k+1}\cdots i_m}\right)$$

Historical overview and frameworks

Henrique Rubira

- **Dim Reg, scale transformations and applications to QED**: Stueckelberg, Petermann, Gell-Mann, Low ~1953
- **RG in condensed matter**: Kadanoff, 1966
- **RG in the continuum, derivation of RG equations and critical phenomena**: Callan and Symanzik 1970, Kenneth Wilson, 1970/71 (Nobel Prize 1982)
- **RG via path integrals**: Polchinski, 1984

Framework 1 (a la Wilson/Polchinski):

$$\Lambda \frac{d}{d\Lambda} Z[J] = 0$$

Sliding cutoff,
integrate out modes between cutoffs

$$\Lambda \to \Lambda'$$

Framework 2:
Sliding renormalization conditions (e.g. Dim Reg), no UV regulator

$$\frac{\partial g}{\partial \ln \mu} = \beta(g)$$

- More practical for computations