The Minimum Energy Principle
Linking protohalo shapes to anisotropic infall

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Based on 2303.02142 and 2405.20207 (with Ravi Sheth and Giulia Despali)

Theoretical Modeling of LSS Workshop
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Why analytical (non-PT) methods?

- Analytical understanding
- Try to extract information from the very small scales
- They often simplifies numerical/data analysis
- Awe at seeing abstract maths at work!

Outline

1. Peaks of the energy overdensity field

A (better) proxy for protohalo centers

2. The minimum energy principle

A handle on protohalo shapes

3. The role of energy shear

What determines the space orientation

1. Peaks of the energy overdensity

(the right place)

MM, R.Sheth (2021)

Motivation for energy peaks

- Protohalo often identified with peaks of mean matter overdensity $\delta_R(\mathbf{x})$ (à la BBKS)
- Peaks of $\delta_R(\mathbf{x})$ are mathematically ill-behaved. Must tweak the filter to avoid divergences
- Different point of view: spheres more energetically bound collapse sooner
- Halos of mass M originate from peaks of the mean energy overdensity field $\epsilon_R(\mathbf{x})$
- Peaks of $\epsilon_R(\mathbf{x})$ are physically sounder, mathematically well behaved and numerically more stable. Also closer to reality!

Motivation for energy peaks

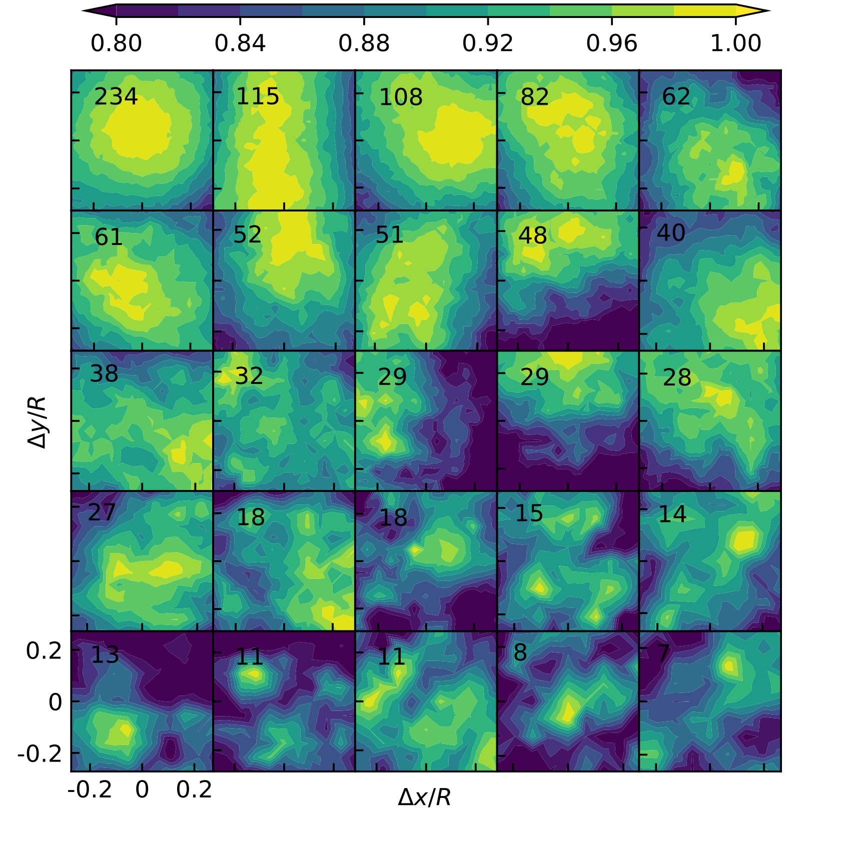
- Inertial radius of volume V: $R_I^2 \equiv \frac{5}{3M}\int_V \mathrm{d}^3 r \rho(\mathbf{r},t)|\mathbf{r}-\mathbf{r}_{\rm cm}|^2$
- Time derivative: $\dot{R}_I R_I \propto \int_V \mathrm{d}^3 r \rho(\mathbf{r},t)(\mathbf{r}-\mathbf{r}_{\rm cm})\cdot(\dot{\mathbf{r}}-\dot{\mathbf{r}}_{\rm cm})$
- Decoupling from Hubble flow governed by the mean energy overdensity

$$\epsilon \equiv \frac{5}{VR_I^2}\int_V \mathrm{d}^3 r\,(\mathbf{r}-\mathbf{r}_{\rm cm})\cdot(\nabla\phi-\nabla\phi_{\rm cm})$$

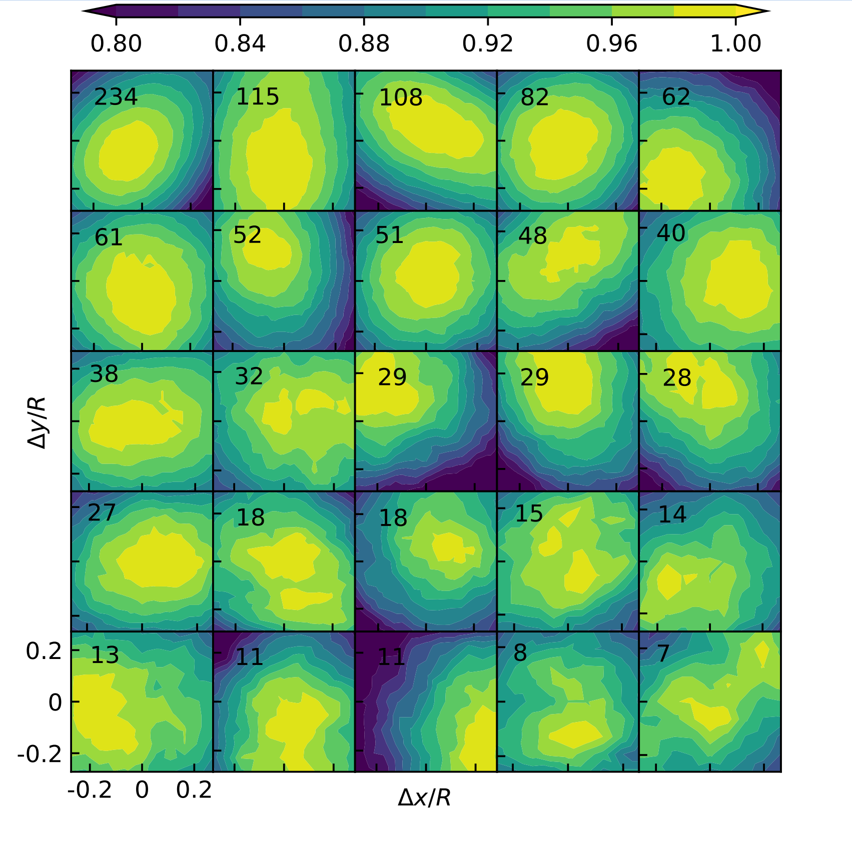
  Using ZA ($\dot{\mathbf{r}} \simeq H\mathbf{r}-\dot{D}\nabla\phi$) gives $\dot{R}_I \simeq R_I(H-\dot{D}\epsilon/3)$
- In spherical collapse, $\dot{R} \simeq R(H-\dot{D}\delta_R/3)$. R_I evolves like a SC solution with mean matter overdensity δ_R replaced by ϵ . Then look for peaks of ϵ !

Testing spherical energy peak ansatz

Mean matter overdensity field δ_R



Mean energy overdensity field ϵ_R



- Energy peaks are a better proxy for protohalo centers!

2. The Minimum Energy Principle

(the right shape)

MM, R. Sheth (2023)

Shape of maximal ϵ

- So far we used spheres. But also for **arbitrary shapes** the total initial energy is:

$$E = -\frac{5}{3}\frac{4\pi G\bar{\rho}}{3}R_I^2\epsilon$$

- Deforming a sphere of maximal ϵ gives even higher ϵ (lower E). The inertial radius R_I of the deformed region collapses even faster

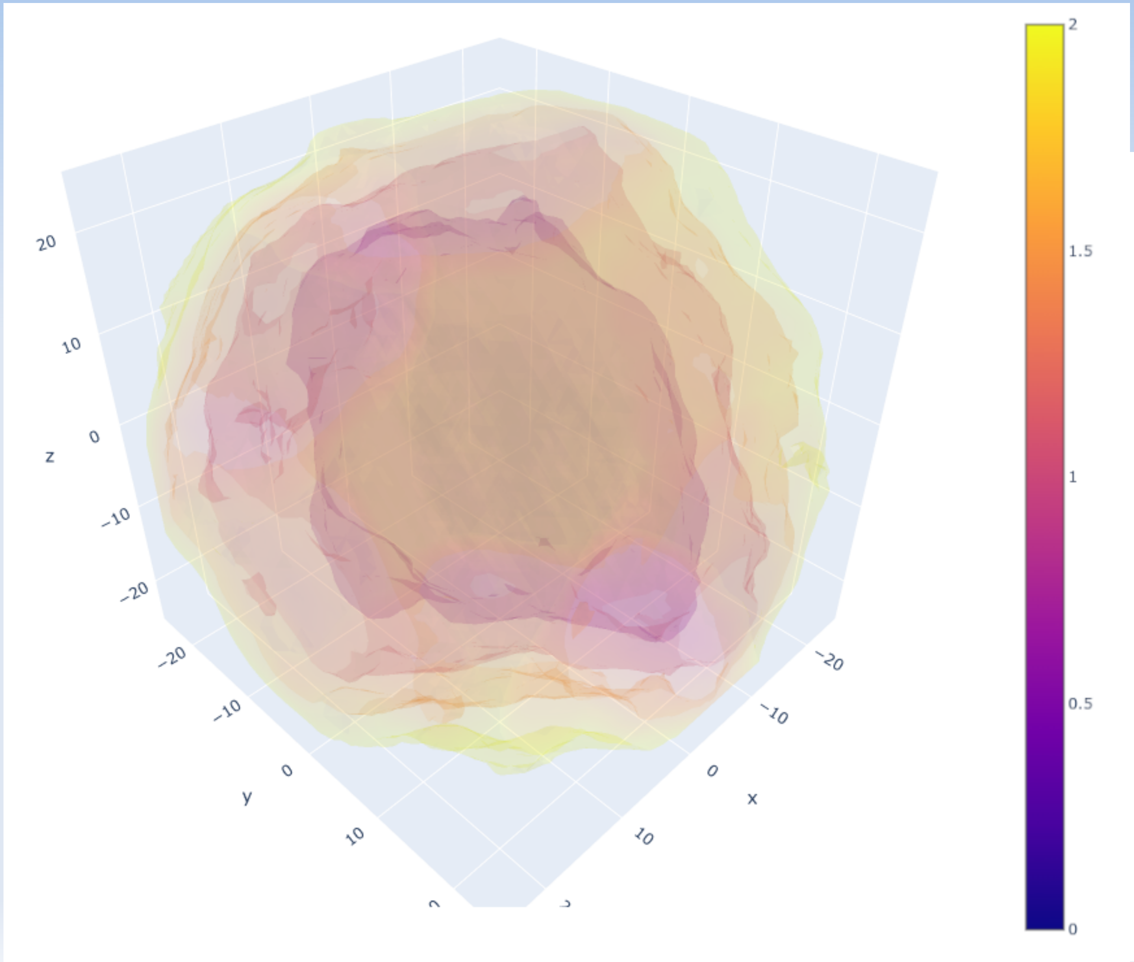
- Shapes of maximal ϵ (minimal E) are most energetically favored

- Boundaries of regions of maximal ϵ are isosurfaces of infall potential

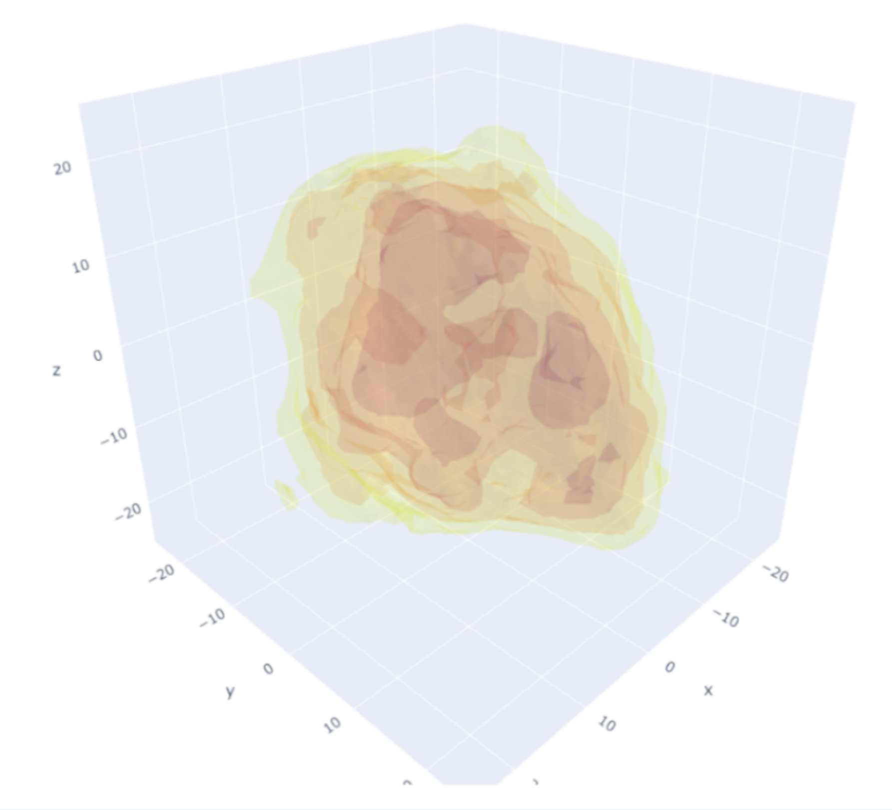
$$\mathcal{V}(\mathbf{r}) \equiv (\mathbf{r} - \mathbf{r}_{\rm cm}) \cdot \left[\nabla\phi - \nabla\phi_{\rm cm} - \frac{\epsilon}{3}(\mathbf{r} - \mathbf{r}_{\rm cm})\right]$$

- The equipotential surface is non-spherical if $\nabla\phi$ is anisotropic. Longest axis in the direction of maximum compression (orthogonal to the filament)

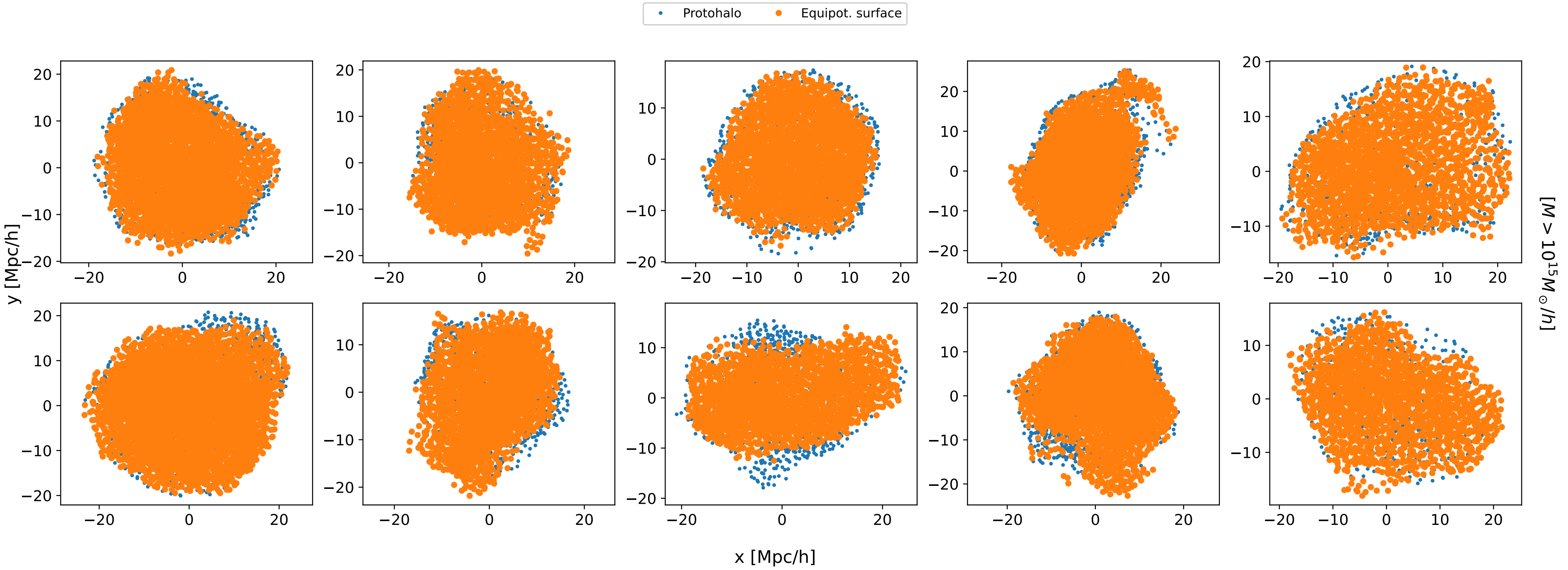
Equipotential surfaces



Two sets of surfaces of constant $\mathcal{V}$ around a protohalo center

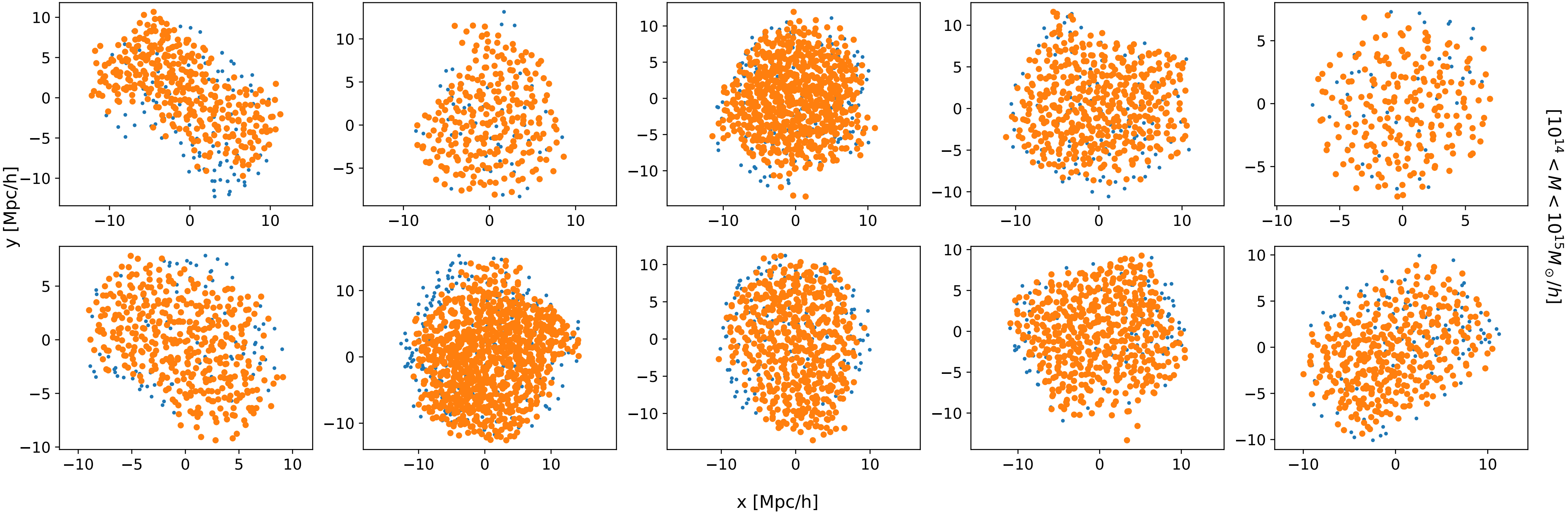


Protohaloes vs equipotential surfaces

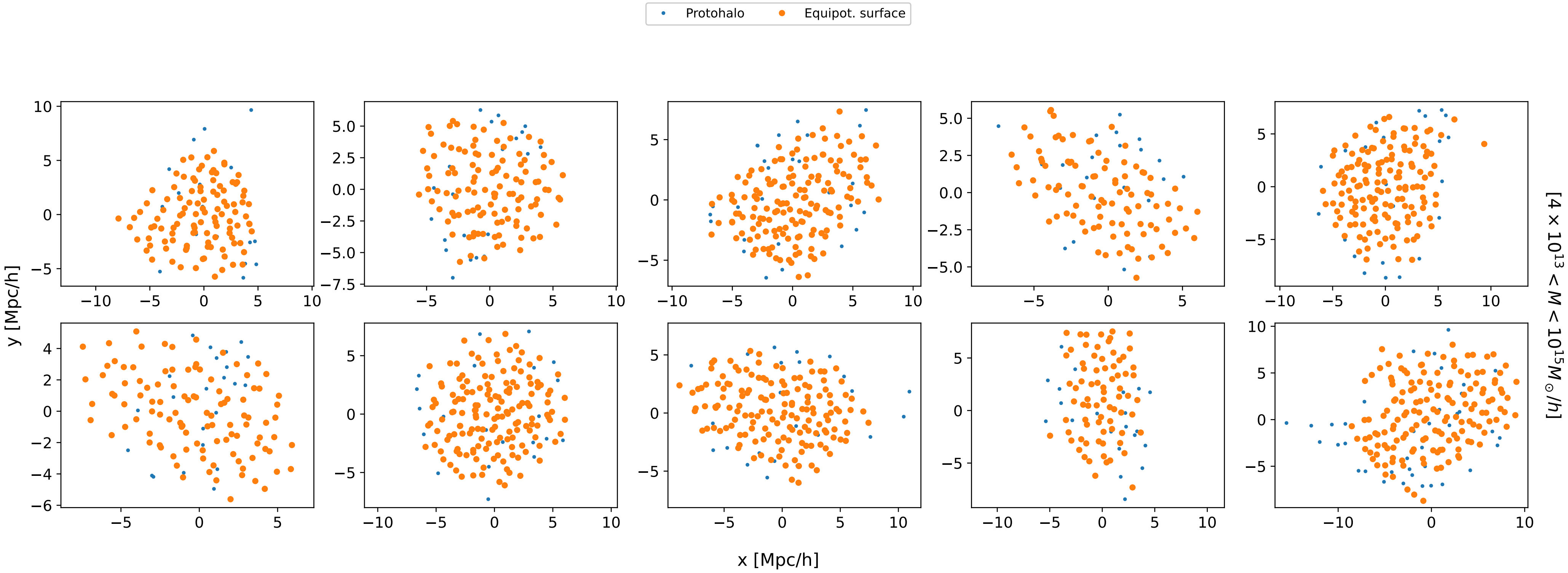


see also Nikakhtar’s talk

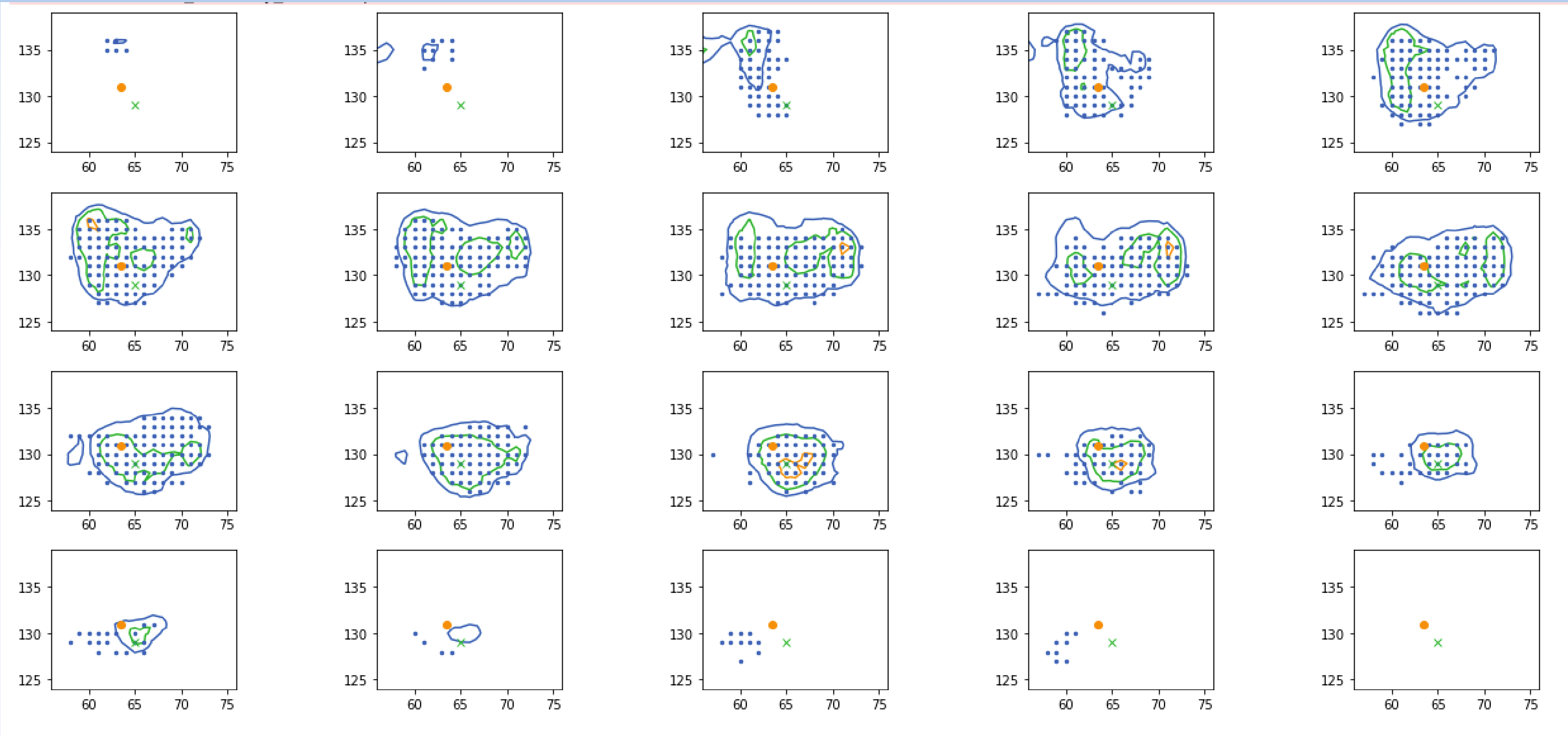
Protohaloes vs equipotential surfaces



Protohaloes vs equipotential surfaces

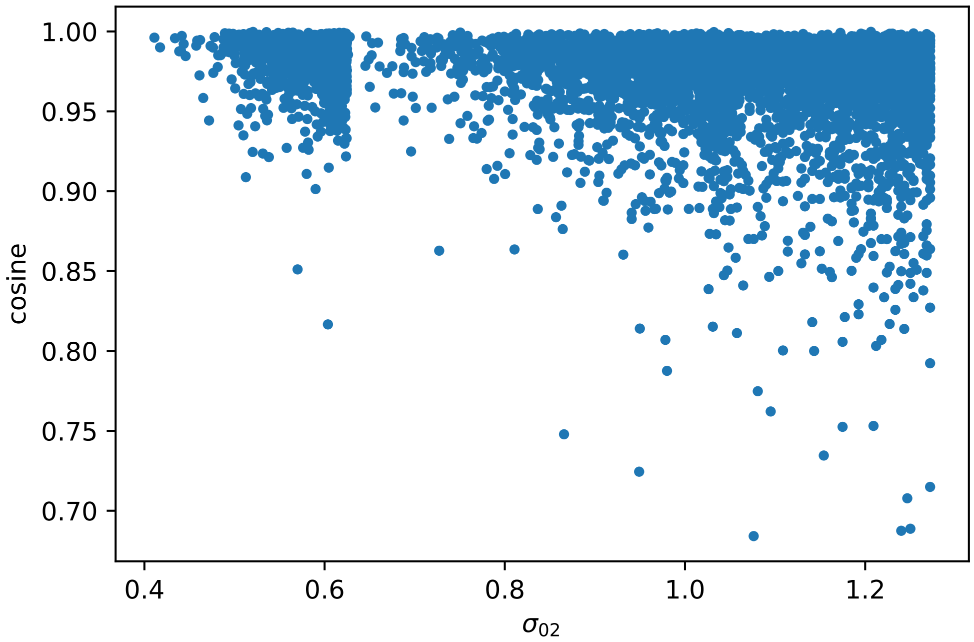


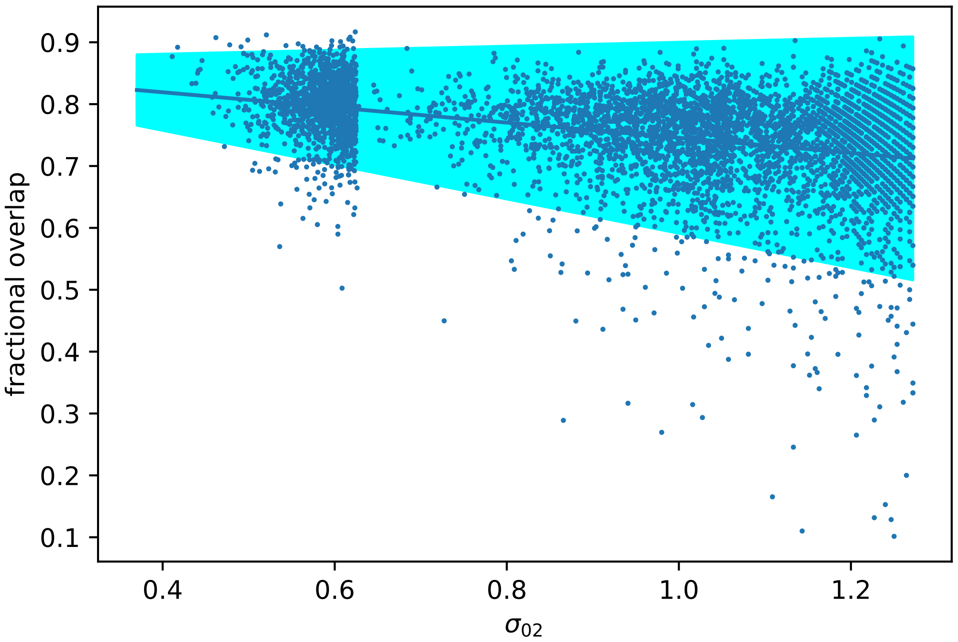
Protohaloes vs equipotential surfaces



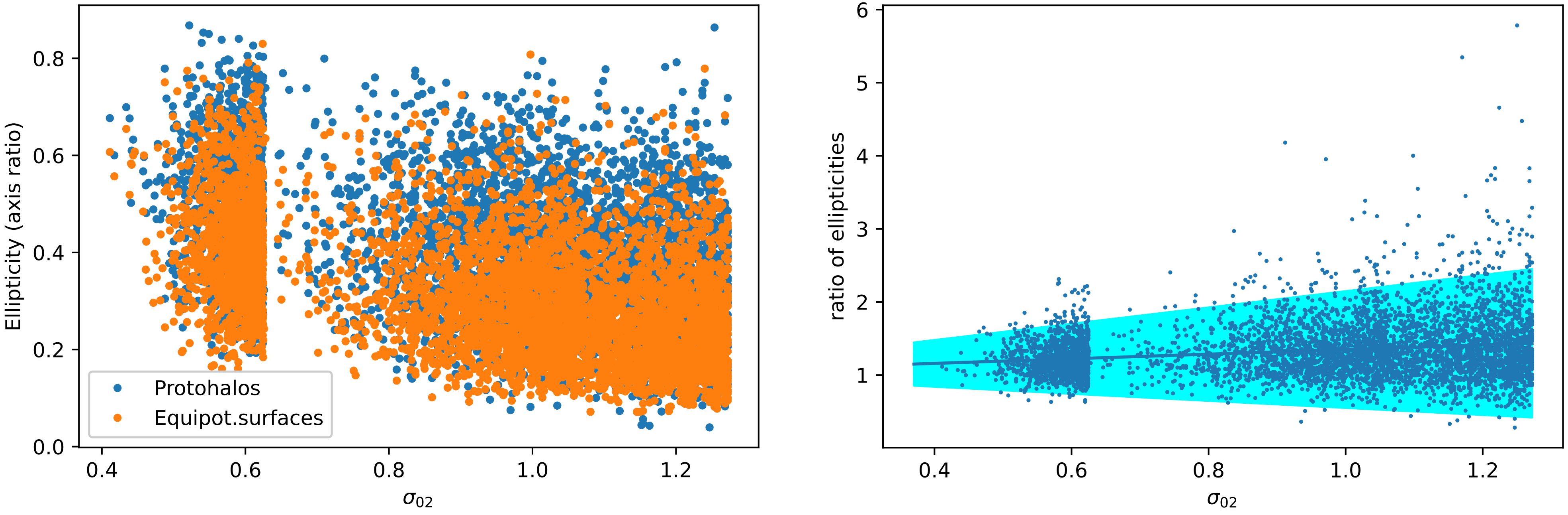
Slices in the x-y plane of a protohalo of ~ 10^{12} solar masses, with isocontours

Protohaloes vs equipotential surfaces





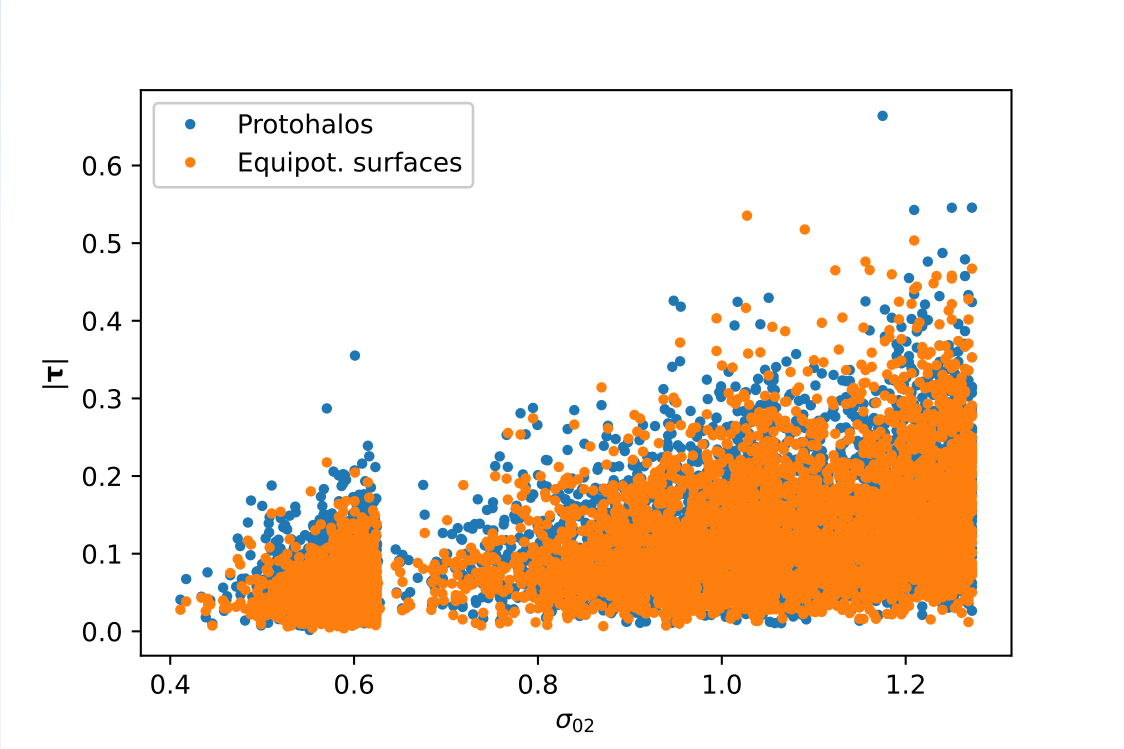
Ellipticities



Torques

$$\tau_i = -\frac{MR_I^2}{5}\varepsilon_{ijk}\epsilon_{jk}$$

u_{jk} = energy ovedensity tensor.
No approx here!



3. Energy shear vs deformation tensor

(the right orientation)

MM, G.Despali, R.Sheth (2024)

Shape of maximal ϵ

- Many works (e.g. White, Porciani, Van de Weygaert, Hahn, Feldbrugge...) studied the (mis-)alignment of protohalo axes with eigenvectors of mean deformation tensor $\langle \nabla_i \nabla_j \phi \rangle_V$

- However, like for the inertial radius, the evolution of the inertia tensor is governed by the mean energy overdensity tensor (or energy shear)

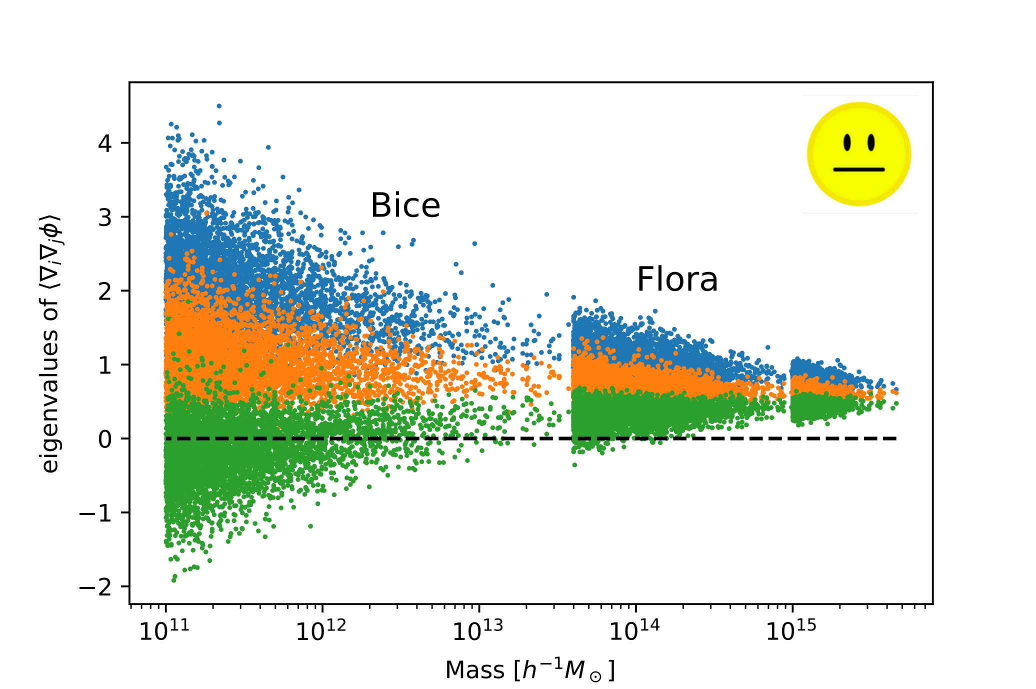
$$u_{ij} \equiv \frac{5}{M R_I^2} \int_V \mathrm{d}^3 r \rho(\mathbf{r}, t)(r_i - r_{\mathrm{cm},i})(\nabla_j \phi - [\nabla_j \phi]_{\mathrm{cm}})$$

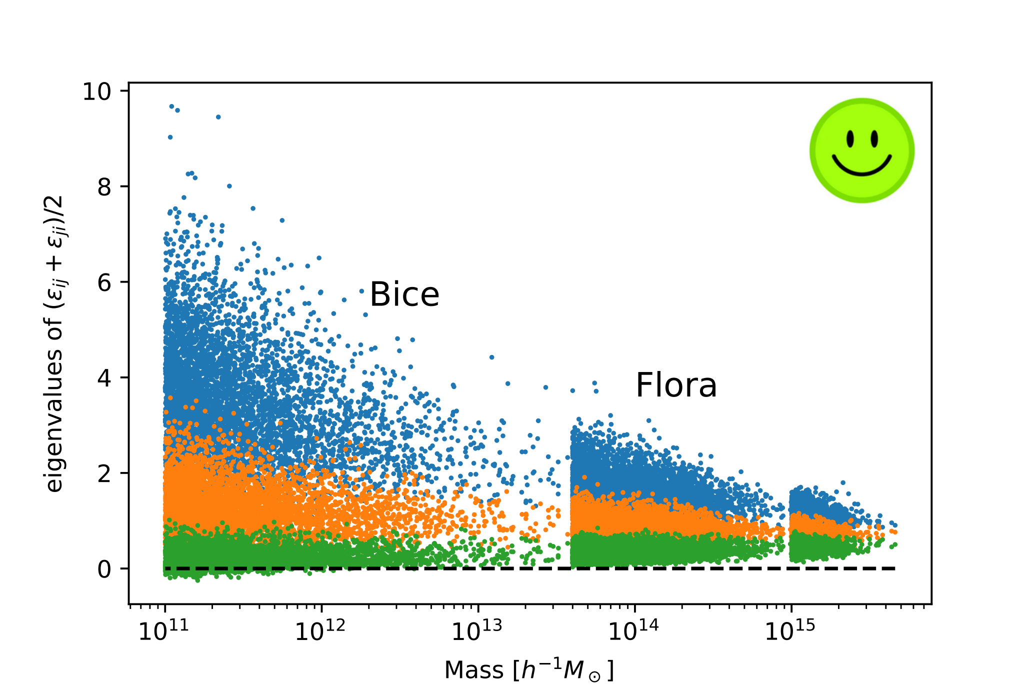
- The (symmetric part of) u_{ij} must be positive definite for all three axes to collapse and form caustics (see Feldbrugge's talk)

- The energy shear squeezes matter into the halo. The longest axis will align with the direction of maximum compression (largest eigenvalue of u_{ij})

- See also Matsubara's and Kokron's talk

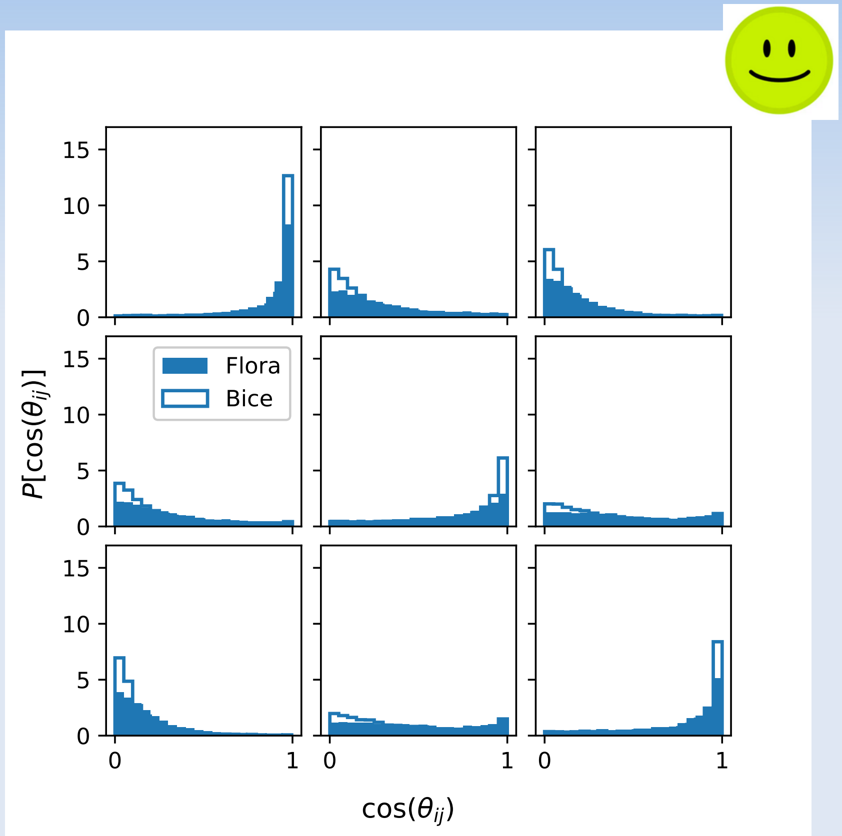
Positivity of eigenvalues

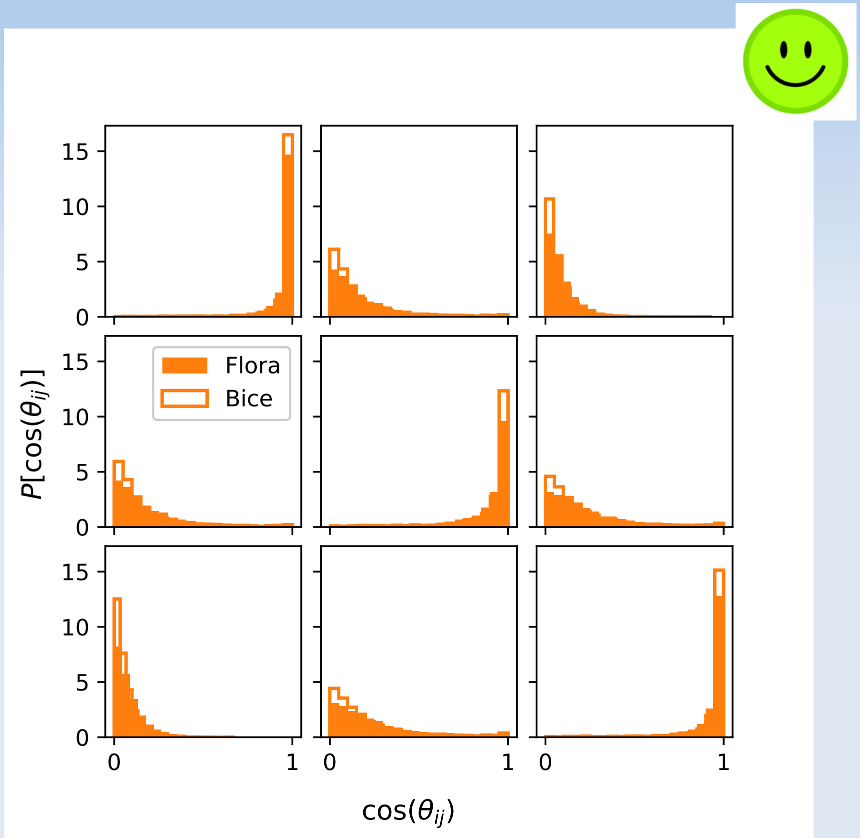




Eigenvalues of deformation tensor (left) and energy shear (right) of protohaloes

Shear-shape alignment

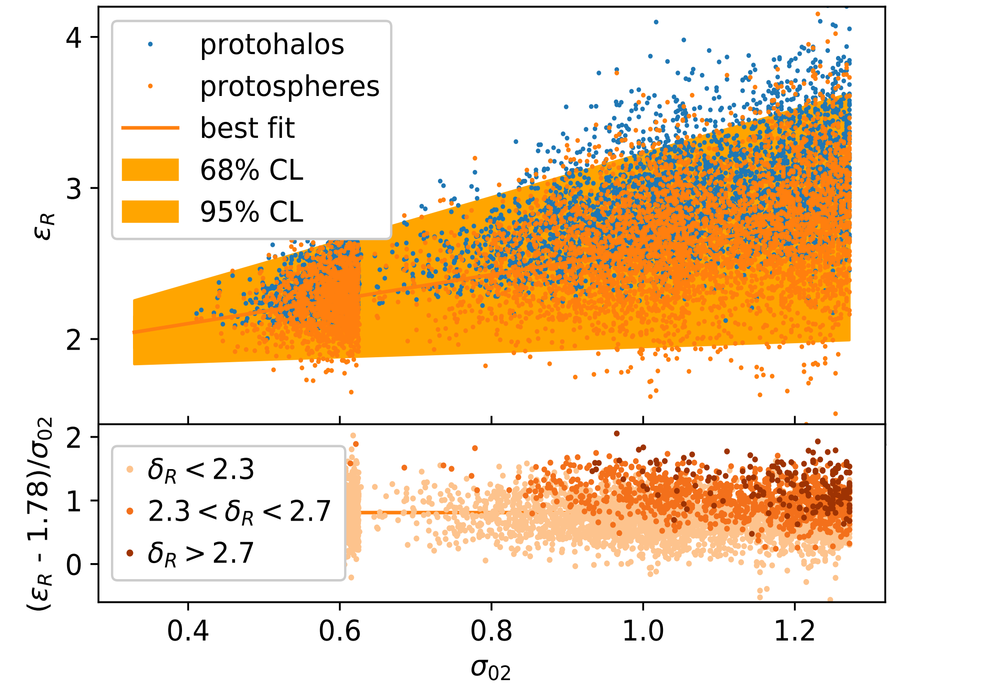




Axes alignment with eigenvectors of deformation tensor (left) and energy shear (right)

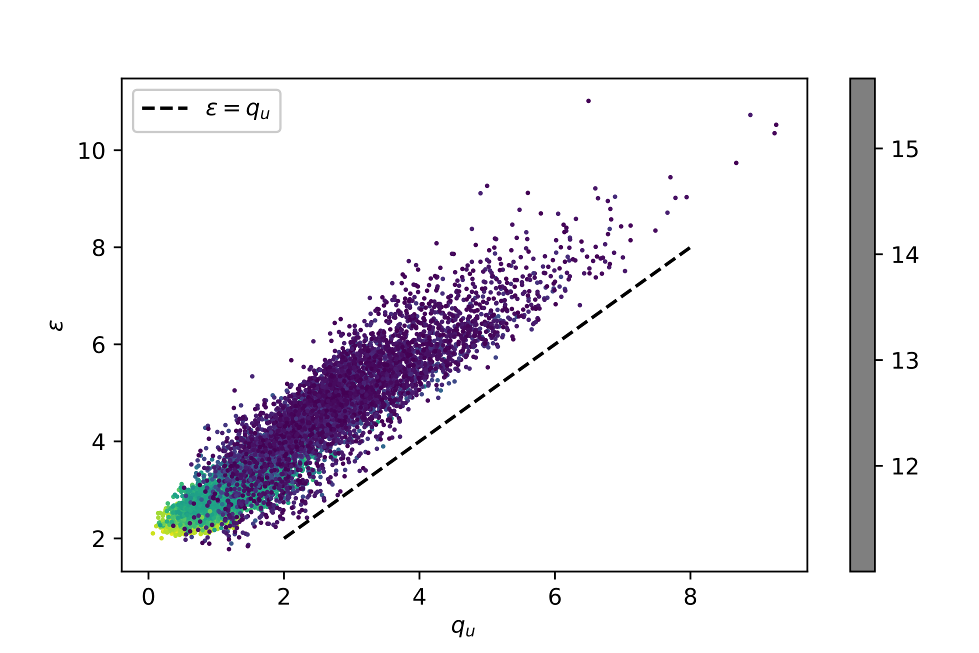
Does this affect collapse?

- Could some of this phenomenology of eigenvalues explain some of the scatter in the value of ϵ measured in protohaloes? (~collapse time)



Positivity of eigenvalues

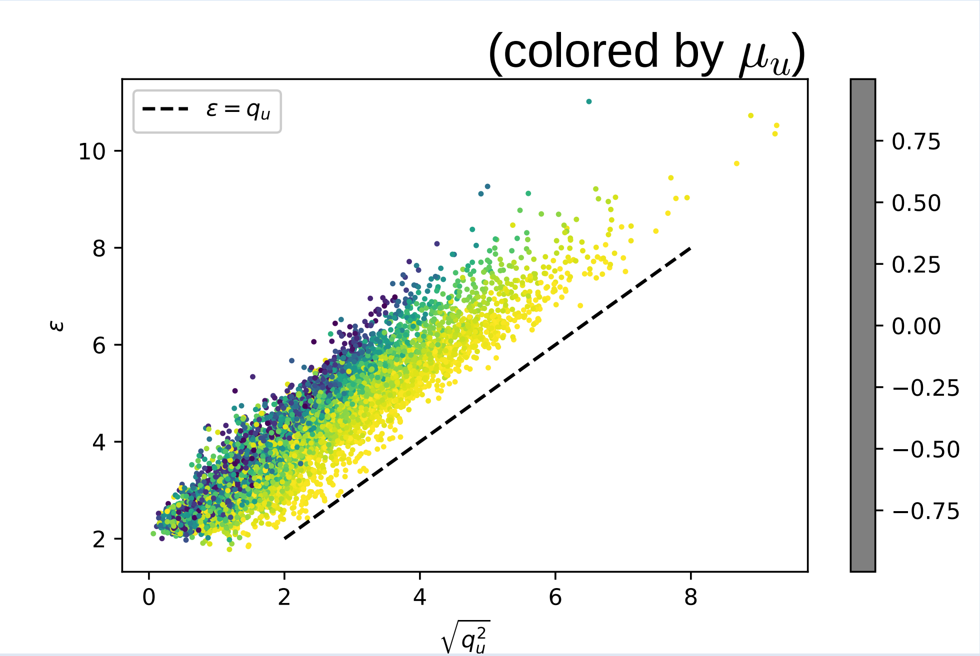
- For positive definite matrices, $\epsilon^2 \equiv (\sum \lambda_i)^2 > \sum_{i\neq j}(\lambda_i - \lambda_j)^2/2 \equiv q_u^2$

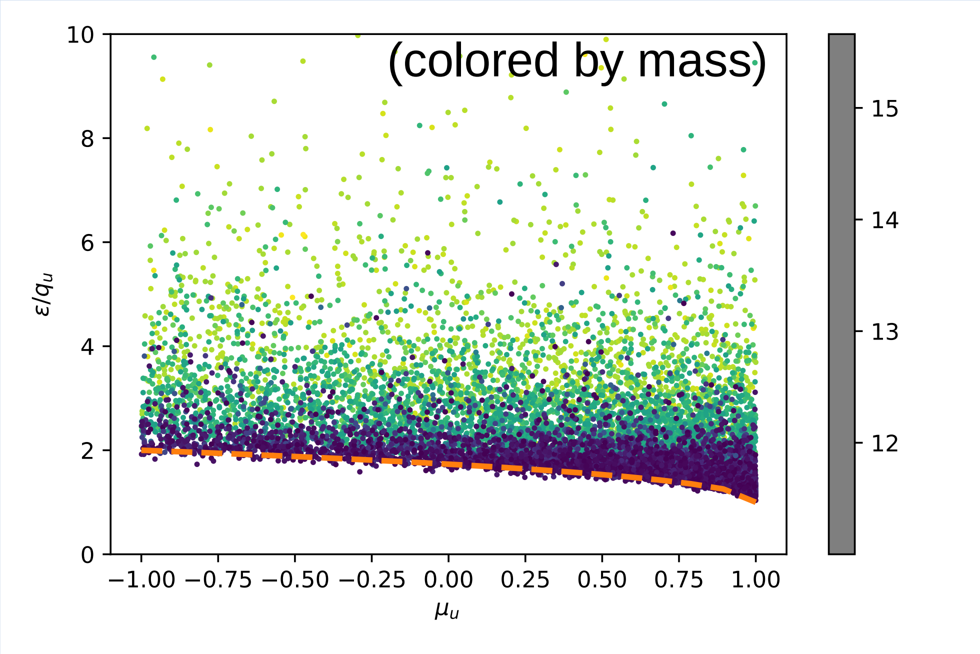


- Positivity alone introduces a correlation between trace (ϵ) and traceless shear, also clearly measured

Positivity of eigenvalues

- The third rotational invariant also matters: $\mu_u \equiv \prod_i (3\lambda_i - \epsilon)/2q_u^3$
- For positive definite matrices, $\epsilon > 2q_u \cos[\arccos(-\mu_u)/3]$





MM, R.Sheth (in prep)

Conclusions

- Protohaloes are peaks of the initial energy overdensity field. Not densest but most energetically bound initial regions, having fastest collapse times.
- Peaks in ϵ_R create convergent matter flows. Final high mean density results dynamically, not put in "by hand"
- Energy density peaks are better behaved, and better proxies for protohalo centers
- Non-spherical shapes of maximal ϵ are equipotential surfaces
- Excellent description of individual protohalo shape
- The energy overdensity tensor (or energy shear) dictates the orientation of the shape (better that the deformation tensor)
- VERY easy to measure (only needs particle positions and velocities)
- Positivity of the eigenvalues induces correlations of ϵ with the traceless shear

Open questions and outlook

- Can we predict critical value ϵ_c ? PT on top of SC? Toy models of spherical virialization (in progress)?
- Relation with halo finder? SO, ellipsoidal, FOF? Energy-based?
- Angular momentum (TTT)?
- Assembly bias and secondary halo properties (e.g. accretion rate, spin…). See Nikakhtar's talk
- Final shear/shape alignments? See Kokron's talk
- Voids? Cosmic web? Skeleton/merger trees? Caustics? See Feldbrugge's talk
- …

Tapadh leibh!!

(= thank you to someone older, more senior, or a group of people)