Institute for Astronomy

ROE

Extending Hybrid Models of structure formation

Exploring the information content of bias approaches

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web: https://bacco.dipc.org

Structure formation? Use N-body simulations

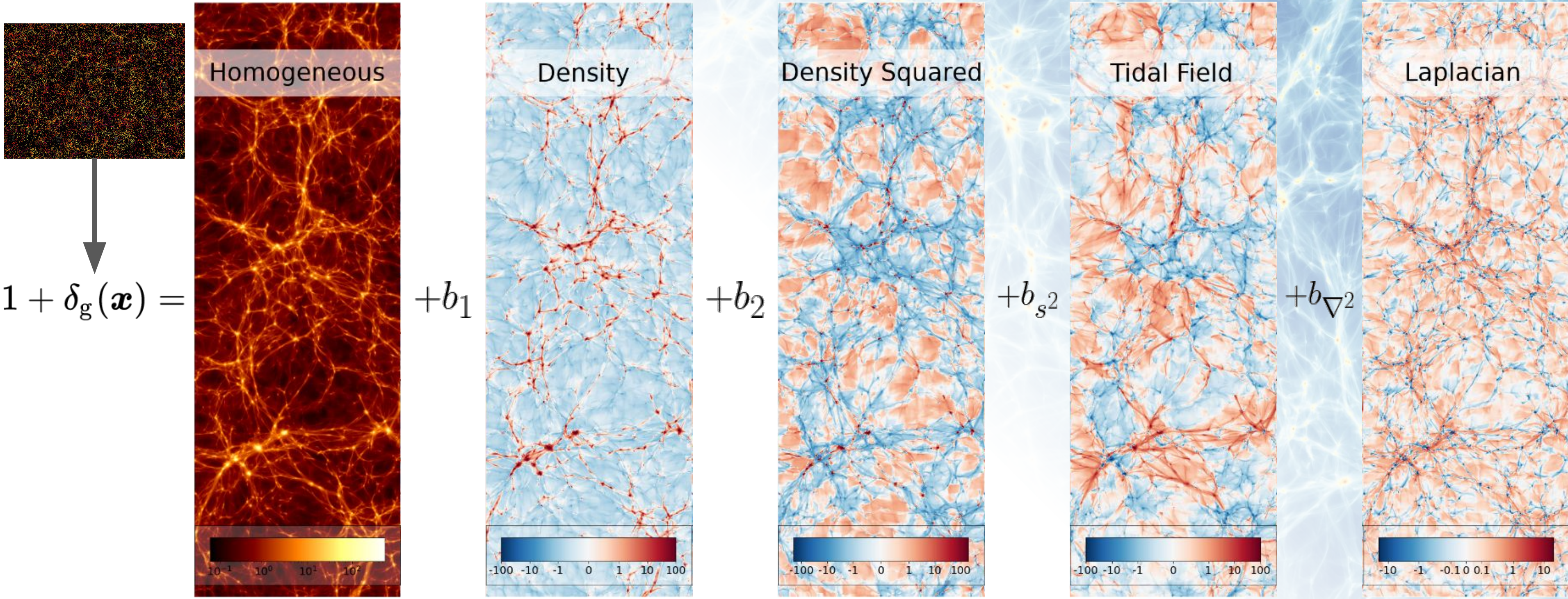
$$1 + \delta_{\rm g}(\boldsymbol{x}) = \int {\rm d}^3 q \, w(\boldsymbol{q}) \, \delta_{\rm D}(\boldsymbol{x} - \boldsymbol{q} - \boldsymbol{\psi}(\boldsymbol{q}))$$

$$w(\boldsymbol{q}) = 1 + b_1^{\rm L} \delta(\boldsymbol{q}) + b_2^{\rm L} \left[\delta^2(\boldsymbol{q}) - \langle \delta^2 \rangle\right] + b_{s^2}^{\rm L} \left[s^2(\boldsymbol{q}) - \langle s^2 \rangle\right] + b_{\nabla^2\delta}^{\rm L} \nabla^2 \delta(\boldsymbol{q})$$

N-body displacements

See Modi, Chen & White (2020)

Model



$$1 + \delta_{\mathrm{g}}(\boldsymbol{x}) = [\text{Homogeneous}] + b_1 [\text{Density}] + b_2 [\text{Density Squared}] + b_{s^2} [\text{Tidal Field}] + b_{\nabla^2} [\text{Laplacian}]$$

Zennaro, Angulo, **MPI** et al.
(2020)

First extension: Redshift Space

Advection + Velocity assignment

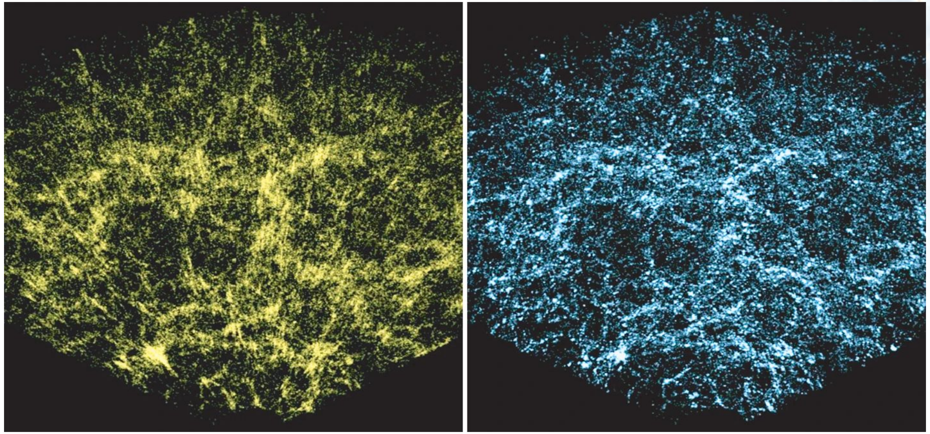
$$1 + \delta_g^s(x) = \int d^3q\, w(q)\, \delta_D\left(x - q - \psi^s(q)\right)$$

$$\psi^s(q_z) = \psi(q_z) + \frac{1}{aH(z)} v_{tr}(q_z)$$

Matter particles velocities?

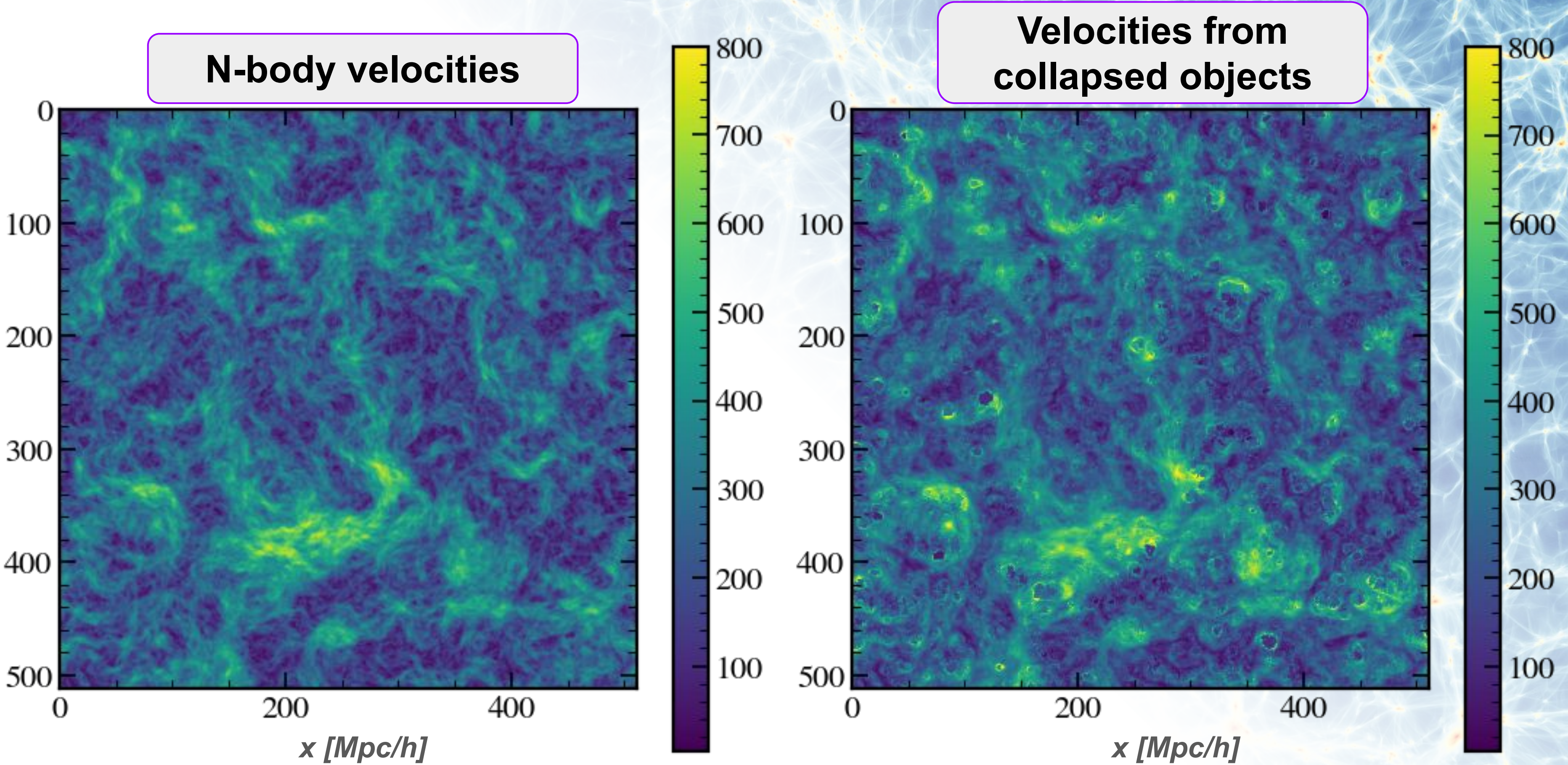
N-body displacements

Redshift Space Distortions Real space



Bianchini (2018)

First step: averaging over masked velocities

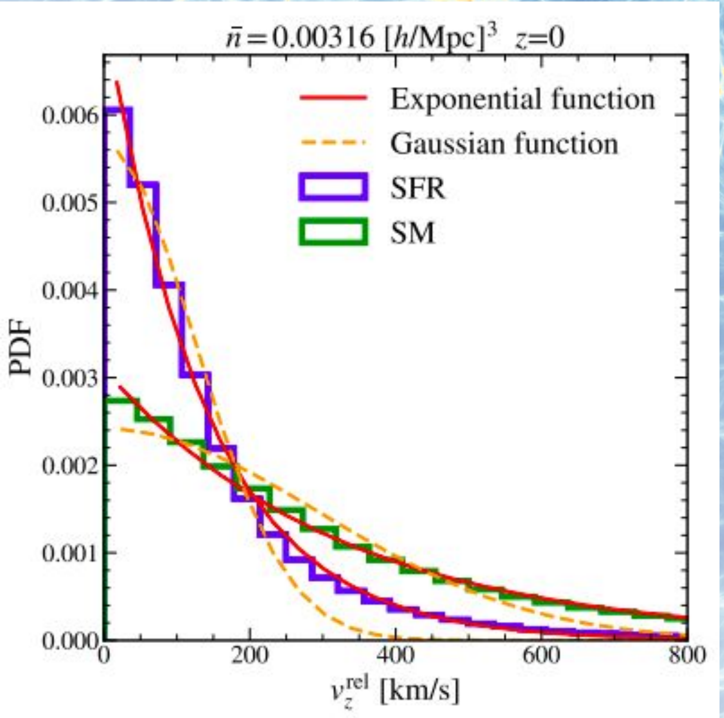


Second step: include the missing velocity scatter

$$p(v_z) = (1 - f_{\rm sat})\delta_{\rm D}(v_z) + f_{\rm sat}\exp(-\lambda v_z)$$

$$\delta_{\rm tr}^{\rm FoG} = \delta_{\rm tr} *_z ((1 - f_{\rm sat})\delta_{\rm D}(s_z) + f_{\rm sat}\exp(-\lambda_{\rm FoG} s_z))$$

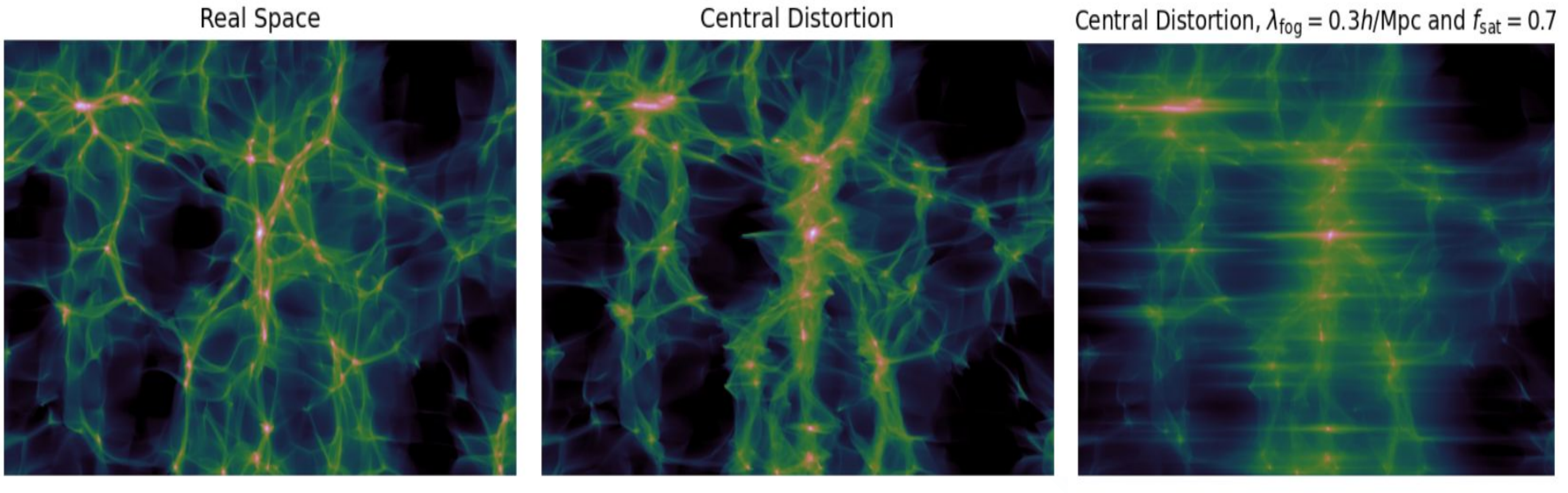
$$P_{\rm tr}^{\rm FoG}(k,\mu) = P_{\rm tr}(k,\mu)\left((1 - f_{\rm sat}) + f_{\rm sat}\frac{\lambda_{\rm FoG}^2}{\lambda_{\rm FoG}^2 + k^2\mu^2}\right)^2$$



Pellejero-Ibañez et al. (2022)

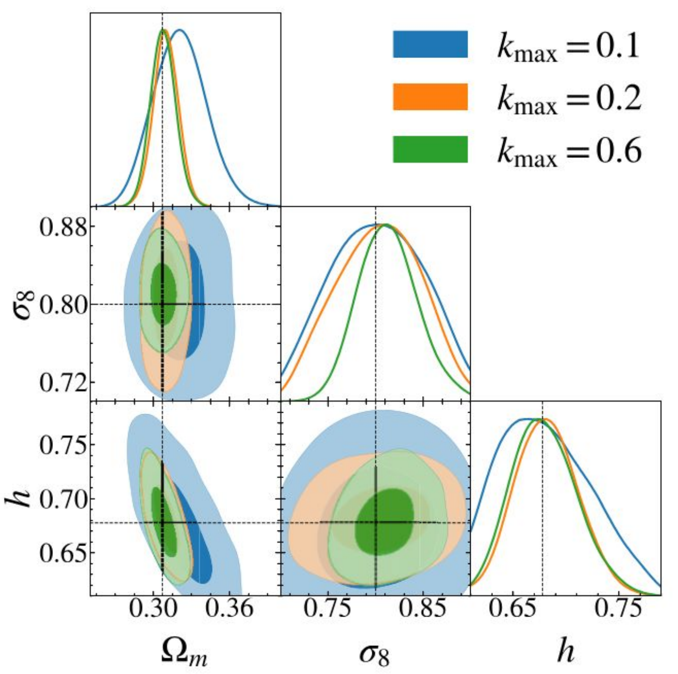
In a nutshell

Advection + Velocity assignment + velocity dispersion (FoG effect)

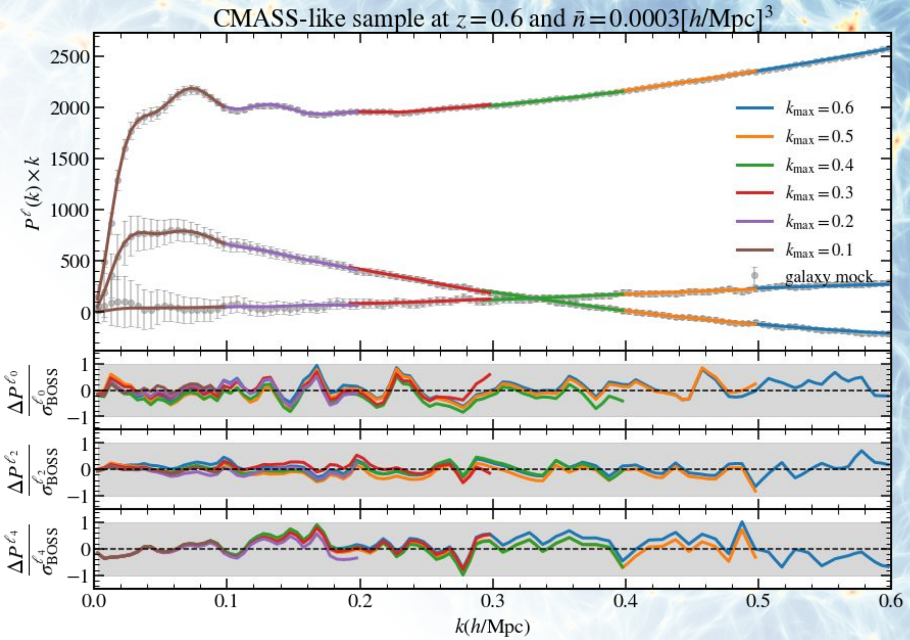


Pellejero-Ibañez et al. (2021)

Does it work?



Hybrid model



Pellejero-Ibañez et al. (2023)

Second extension: Gaussian Lagrangian galaxy bias

Weighting scheme?

$$1 + \delta_{\mathrm{g}}(\boldsymbol{x}) = \int \mathrm{d}^3 q\, w(\boldsymbol{q})\, \delta_{\mathrm{D}}(\boldsymbol{x} - \boldsymbol{q} - \boldsymbol{\psi}(\boldsymbol{q}))$$

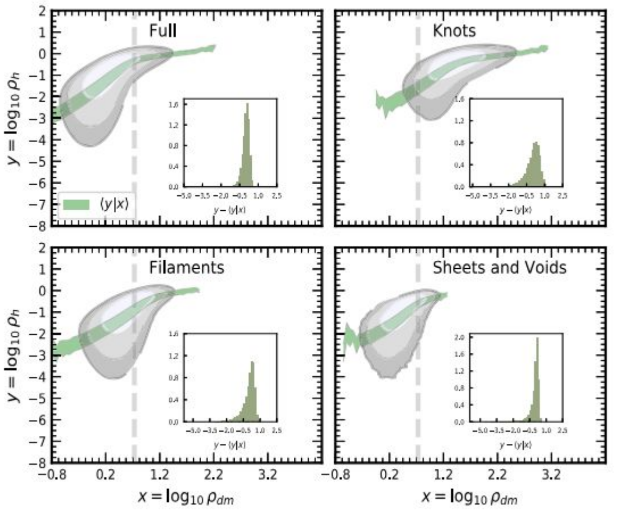
$$\begin{aligned} w(\boldsymbol{q}) = 1 &+ b_1^{\mathrm{L}} \delta(\boldsymbol{q}) + b_2^{\mathrm{L}} \left[\delta^2(\boldsymbol{q}) - \langle \delta^2 \rangle\right] \\ &+ b_{s^2}^{\mathrm{L}} \left[s^2(\boldsymbol{q}) - \langle s^2 \rangle\right] + b_{\nabla^2\delta}^{\mathrm{L}} \nabla^2 \delta(\boldsymbol{q}) \end{aligned}$$

N-body weighting?

N-body displacements

Weighting scheme?

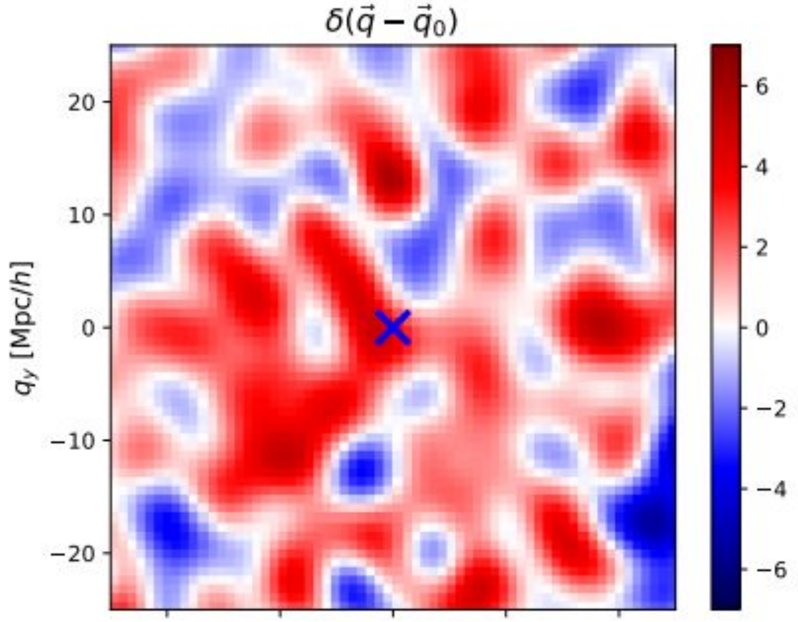
- Important points:
    - **Measure bias function** / Understand through **probability theory**
    - Formulate math independently of perturbation theory
    - Can we use this beyond perturbative regime?



Problems with stochasticity

Balaguera-Antolinez et al. (2019)

Prediction problem



Given some knowledge about the Lagrangian environment

$$\vec{x} = \mathrm{Functional}(\delta(\vec{\mathrm{q}} - \vec{\mathrm{q}}_0))$$
$$\text{e.g.} \quad \vec{x} = \delta_{\mathrm{smoothed}}(\vec{q}_0)$$

How does the probability of "g" depend on the knowledge of $\vec{x}$?

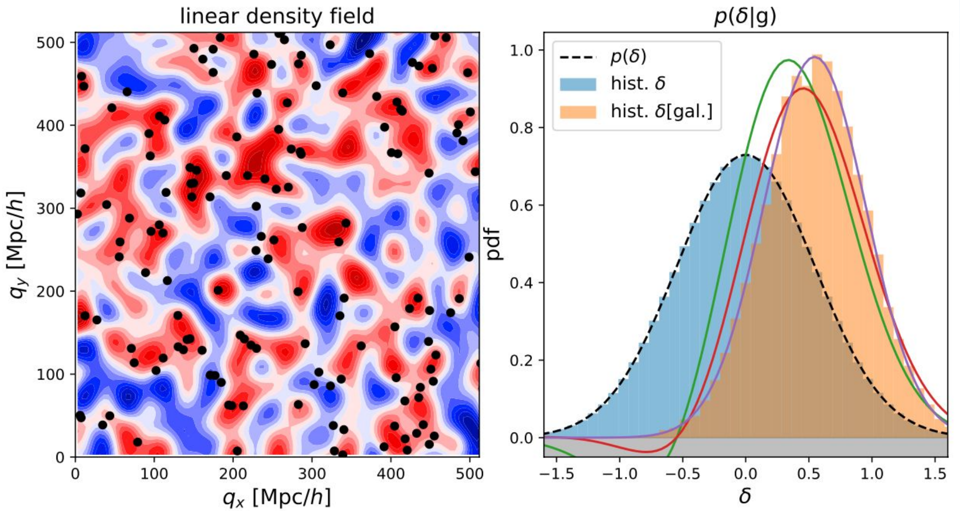
$$f(\delta) = \frac{p(g|\delta)}{p(g)}$$

How does the **probability** of forming a galaxy depend on aspects of the linear density field? This definition is a **simplification** that avoids dealing with "stochastic terms". E.g. a coarse grained galaxy density would read

$$1 + \delta_g = f(\delta) + \text{stochastic terms}$$

Galaxy environment distribution

$$p(\delta|g)$$

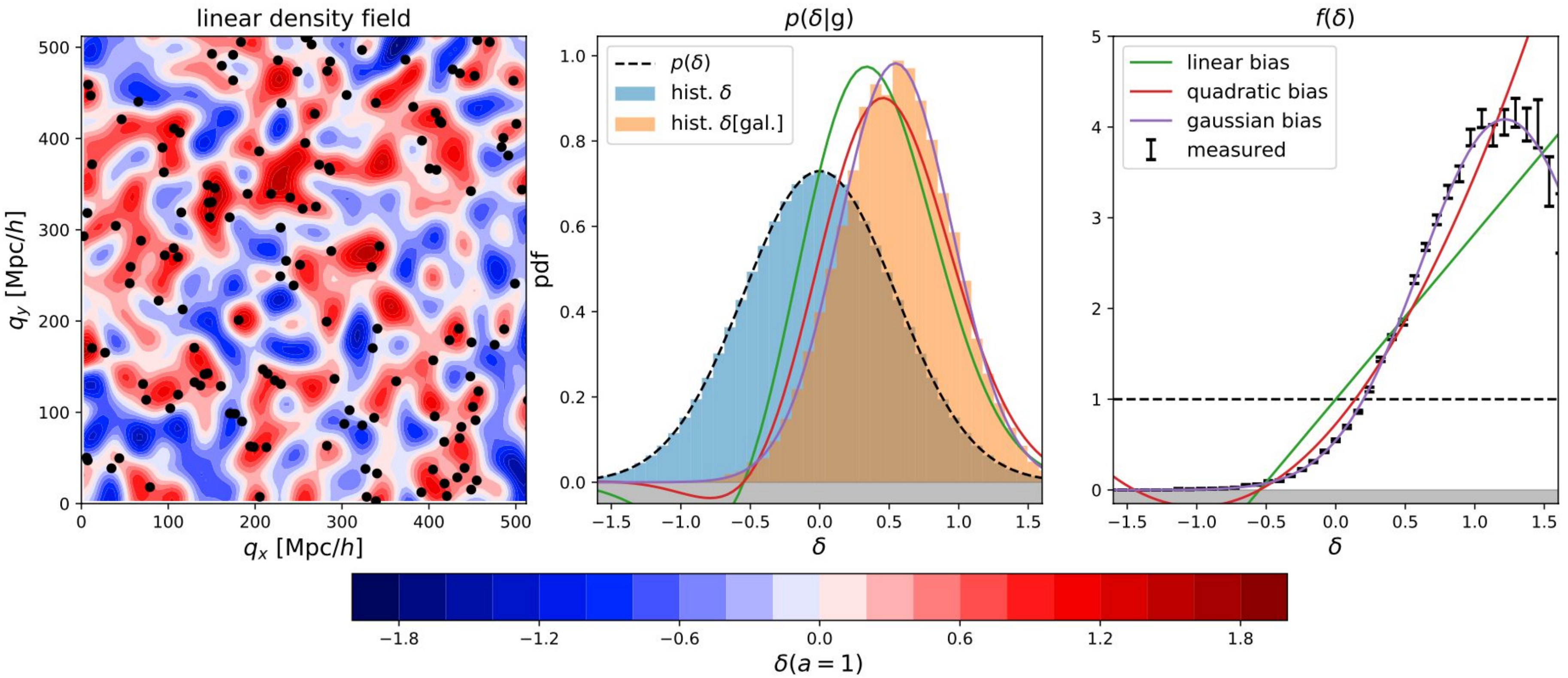


$$p(\delta|g) = \frac{p(\delta \cap g)}{p(g)} = \frac{p(g|\delta)p(\delta)}{p(g)} = f(\delta)p(\delta)$$

$$\Rightarrow f(\delta) = \frac{p(\delta|g)}{p(\delta)}$$

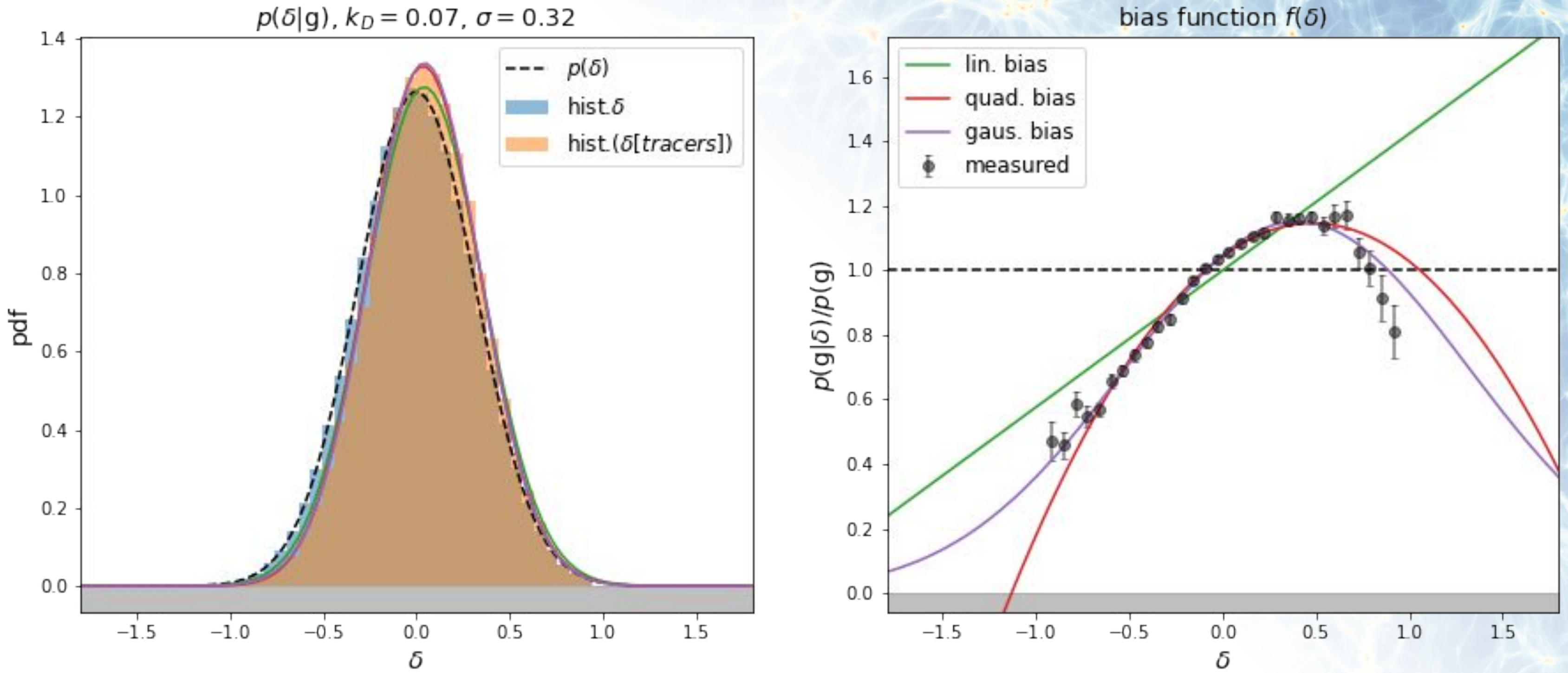
Stuecker, MPI, et al (2024)

Measuring the bias function



Stuecker, MPI, et al (2024)

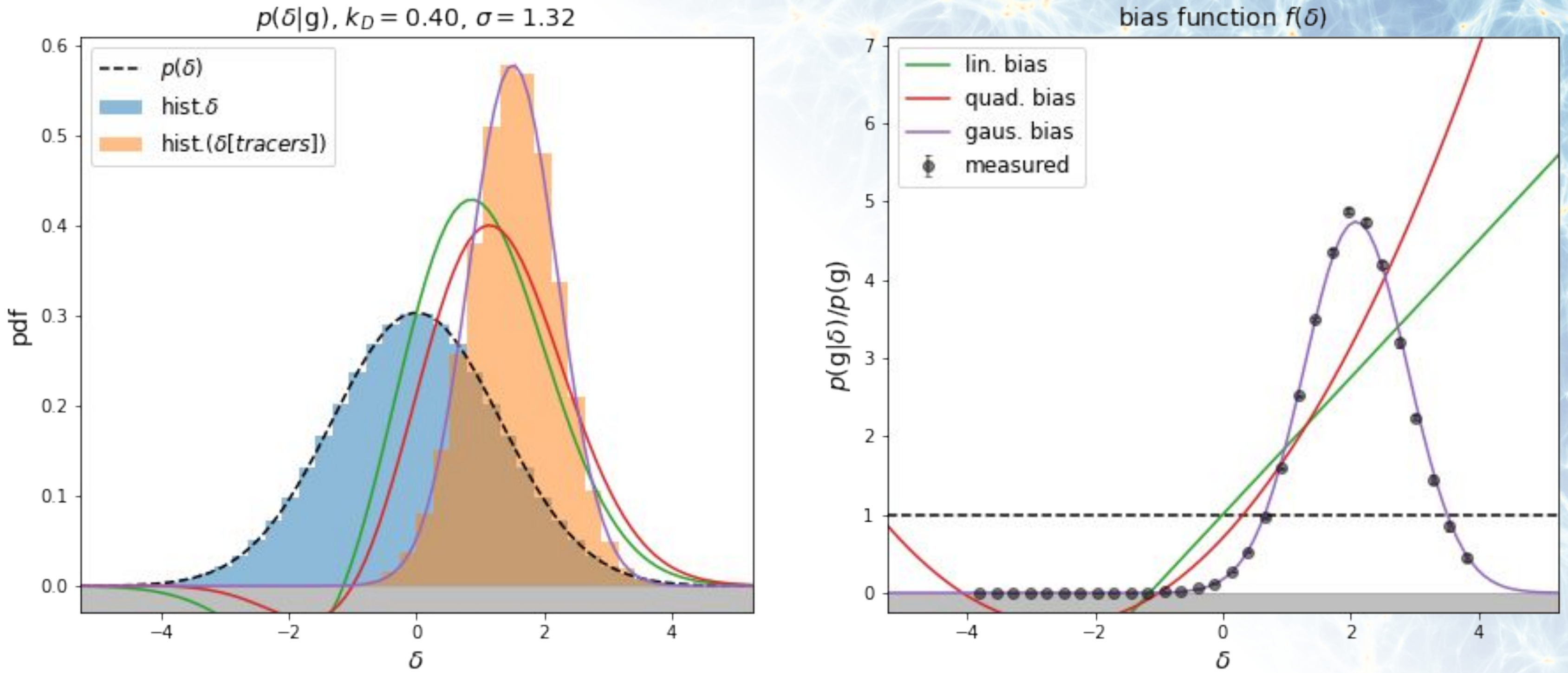
Measuring the bias function: Haloes



M = 5e13 [Msun/h]

Stuecker, MPI, et al (2024)

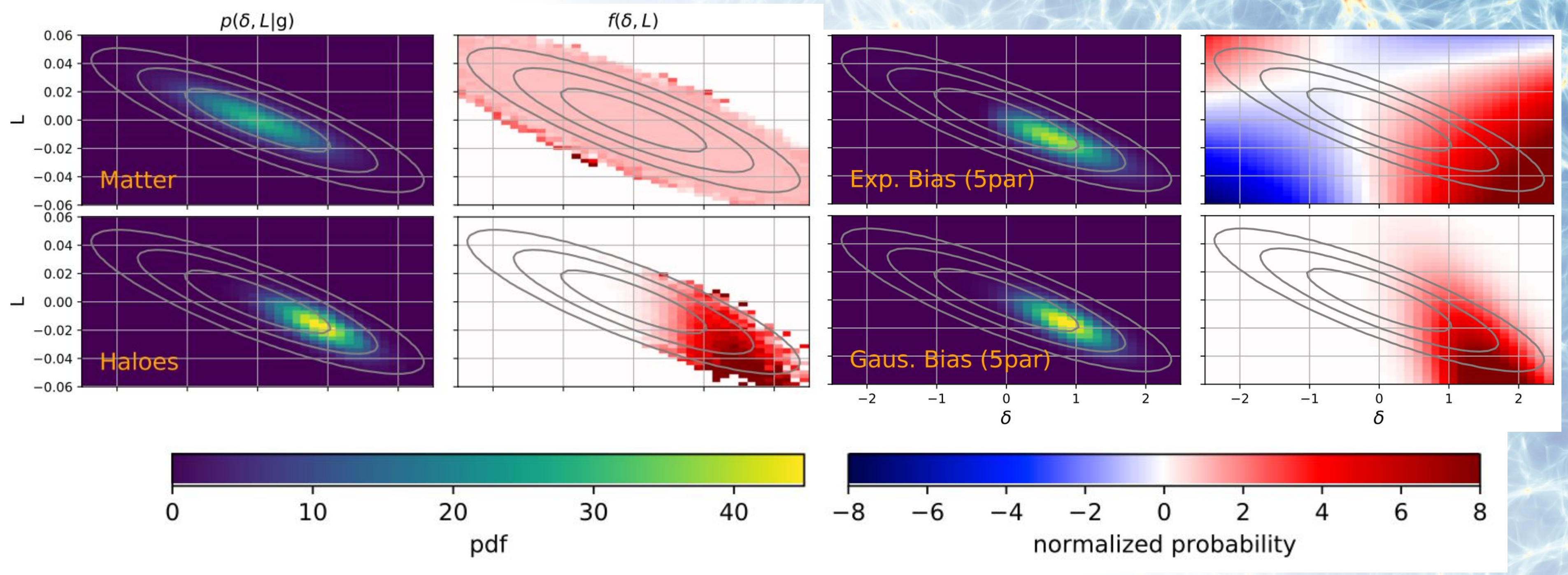
Measuring the bias function: Haloes



M = 5e13 [Msun/h]

Stuecker, MPI, et al (2024)

Measuring the bias function: Multivariate case



M = 2e14[Msun/h],
kd=0.2[h/Mpc]

Stuecker, MPI, et al (2024)

Notes

- **Lagrangian bias function** of haloes is very well approximated by a **Gaussian**

- This is so, even far **beyond perturbative scales** e.g. at $k_d = 0.4\ h\ \mathrm{Mpc}^{-1}$

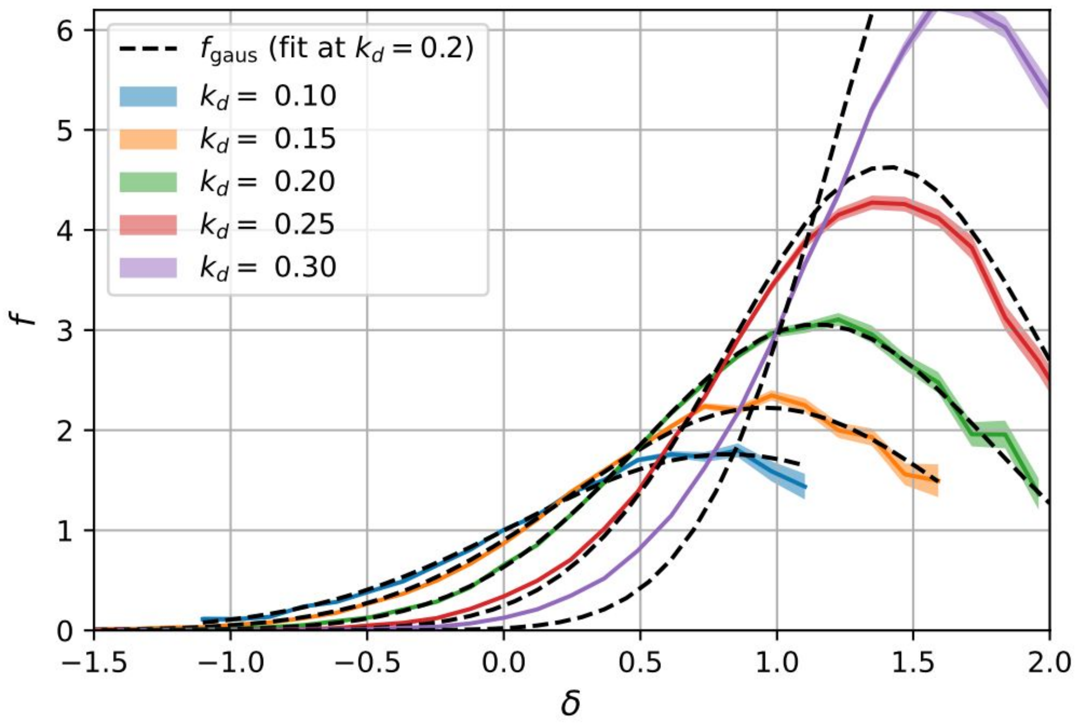
- Difference to expansion models is most extreme for highly biased objects and small scales

However…

- If we fitted a Gaussian at one scale, is it valid on larger scales?

=> Have to understand **scale dependence** of bias function

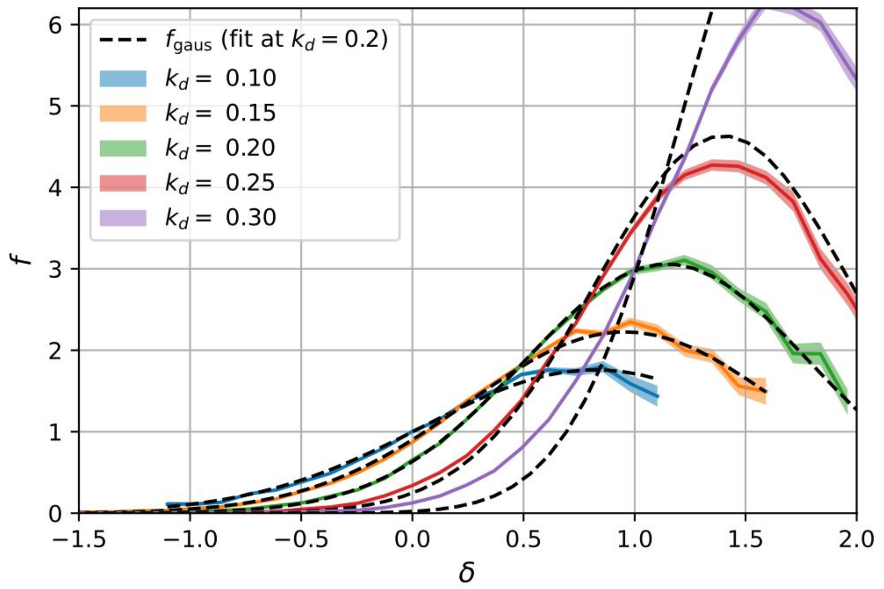
Scale dependency of the bias function

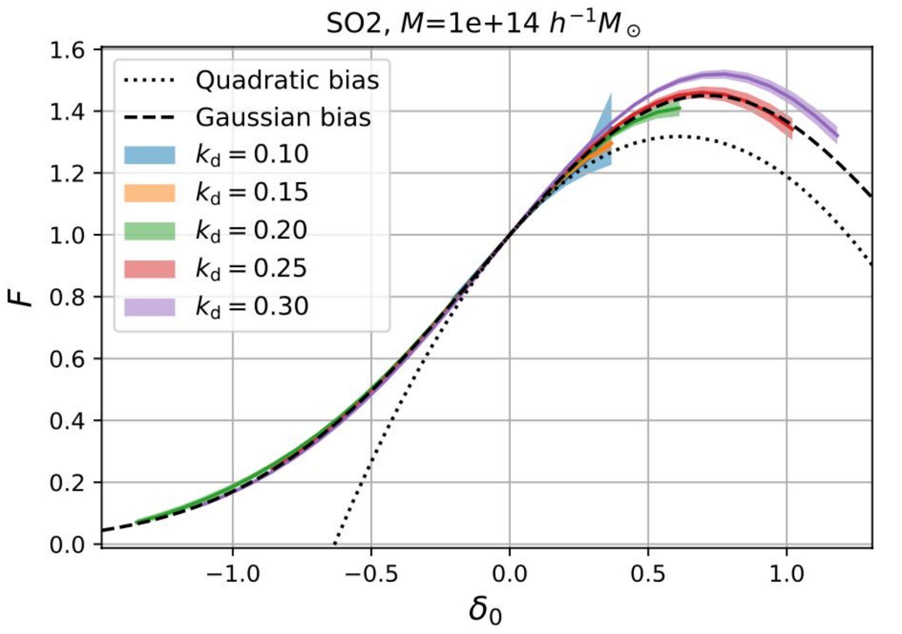


Stuecker, MPI, et al (2024)

Scale dependency of the bias function

$$F(\delta_0) = \langle f(\delta + \delta_0) \rangle$$





Stuecker, MPI, et al (2024)

Conclusions

Gaussian Lagrangian Bias

- Never worse than quadratic bias
- Can describe full fields with scale independency up to $k = 0.2\ h\ \mathrm{Mpc}^{-1}$
- May describe $p(\delta|g)$ to even much smaller scales
- Important property $f > 0$ => Can be used with probability theory

Thank you!

Based on Peak Background Split

$$b_N = \frac{1}{n_{g,0}} \frac{\partial^N n_g}{\partial \delta^N}$$

$$\frac{n_g(\delta)}{n_{g,0}} = \left\langle \frac{p(g|\delta)}{p(g)} \right\rangle = \left\langle \frac{p(\delta|g)}{p(\delta)} \right\rangle$$

$$b_N = \left\langle \left(\frac{p(\delta|g)}{p(\delta)} \right)^{(N)} \right\rangle$$

Expectation value

Expectation value

$$b_N = \int \left(\frac{p(\delta|g)}{p(\delta)} \right)^{(N)} p(\delta) d\delta = (-1)^N \int p(\delta|g) \frac{p^{(N)}(\delta)}{p(\delta)} d\delta = (-1)^N \left\langle \frac{p^{(N)}}{p} \right\rangle_{\text{gal}}$$

Integration by parts

Bias measurements

For a finite number of tracers

$$b_N = (-1)^N \frac{1}{N_{\rm gal}} \sum_{\rm galaxies} \frac{p^{(N)}(\delta_i)}{p(\delta_i)}$$

The bias parameters are expected values of derivatives w.r.t. the density at the galaxy positions. This is independent of the bias function. We can measure the bias parameters without assuming any bias function. Since the probability of the density is a Gaussian in Lagrangian space:

$$\frac{p^{(N)}}{p} = (-1)^N \frac{H_N(\delta/\sigma)}{\sigma^N}$$

$$b_N = \left\langle \frac{H_N(\delta/\sigma)}{\sigma^N} \right\rangle_{\rm gal}$$

(3) Relating models & measurements

$$\{\mu_b, \sigma_b, N_b\} \longrightarrow \{b_0, b_1, b_2\}$$ **Gaussian bias**

$$\{c_0, c_1, c_2\} \longrightarrow \{b_0, b_1, b_2\}$$ **Expansion bias**

To match the model to an observed distribution it has to lead to the same bias parameters

$$\left\langle \frac{\partial^N f_{\text{model}}}{\partial \delta^N} \right\rangle \equiv b_N$$

Specific modelling

- **Expansion bias**

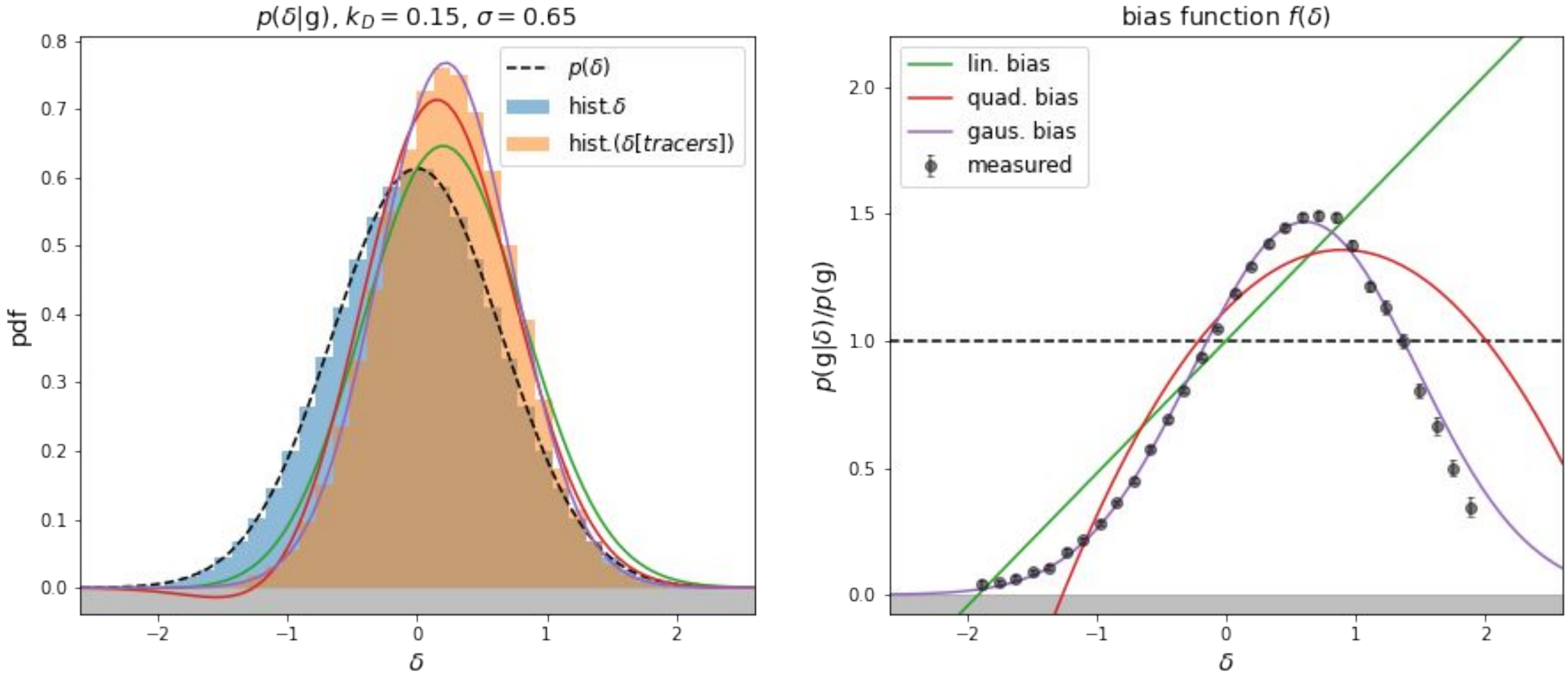
$$\left.\begin{aligned} 1 &= b_0 = c_0 + c_1 \langle \delta \rangle + c_2 \langle \delta^2 \rangle = c_0 + c_2 \sigma^2 \\ b_1 &= c_1 + 2c_2 \langle \delta \rangle = c_1 \\ b_2 &= 2c_2 \end{aligned}\right\} \quad f_{\exp}(\delta) = 1 + b_1 \delta + \tfrac{1}{2} b_2 (\delta^2 - \sigma^2)$$

- **Gaussian bias**

$$\begin{cases} 1 = b_0 = N_g \sqrt{2\pi}\sigma_g \\ b_1 = \frac{\mu_g}{\sigma^2} \\ b_2 = \frac{\mu_g^2 - \sigma^2 + \sigma_g^2}{\sigma^4} \end{cases} \Longrightarrow \left\{\begin{aligned} \sigma_b^2 &= \frac{1}{b_1^2 - b_2} - \sigma^2 \\ \mu_b &= \frac{b_1}{b_1^2 - b_2} \\ N_b &= \frac{\exp\left(\frac{b_1^2}{2\left(b_1^2 - b_2\right)}\right)}{\sqrt{1 - b_1^2 \sigma^2 + b_2 \sigma^2}} \end{aligned}\right\} \quad f_{\text{gaus}}(\delta) = N_b \exp\left(-\frac{(\delta - \mu_b)^2}{2\sigma_b^2}\right)$$

Measuring the bias function

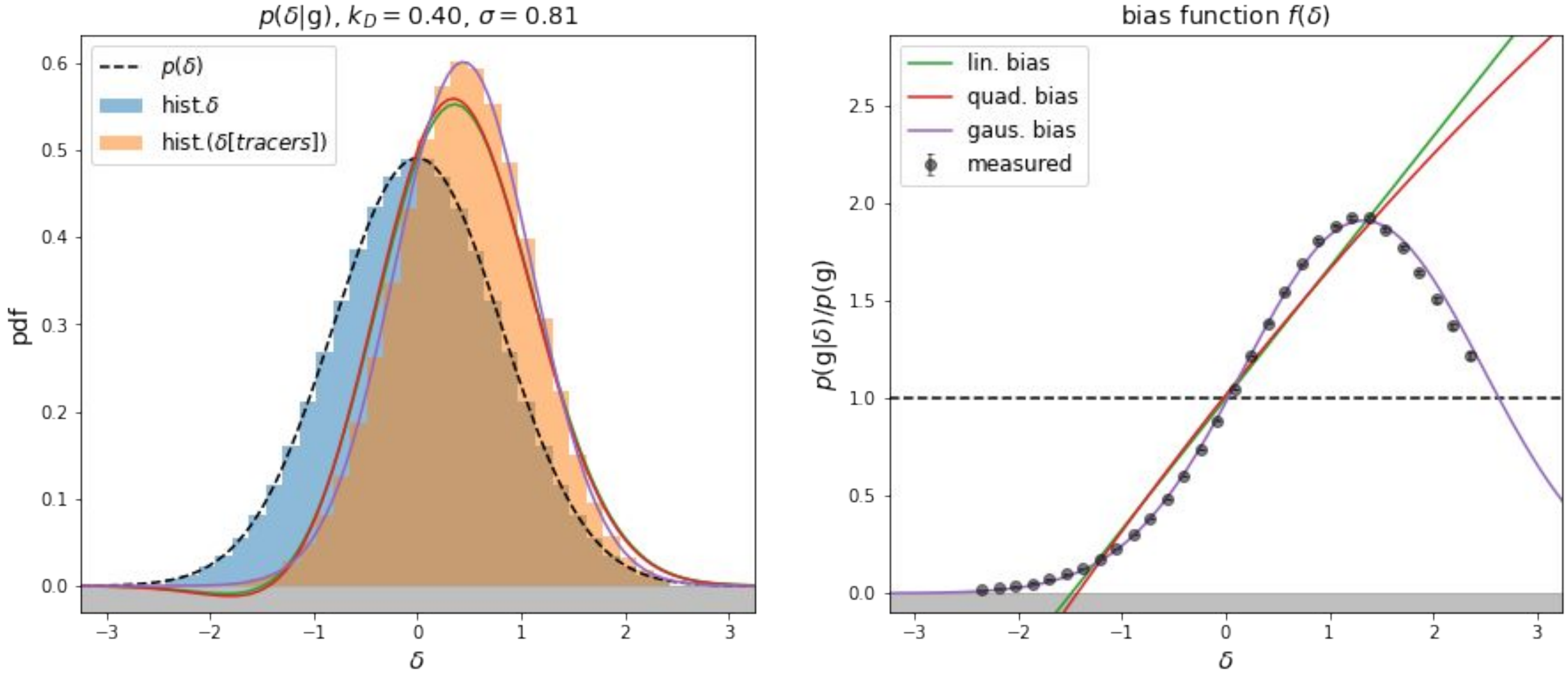
$$f(\delta) = \frac{p(\delta|g)}{p(\delta)}$$



M = 5e13

Stellar Mass sel. galaxies

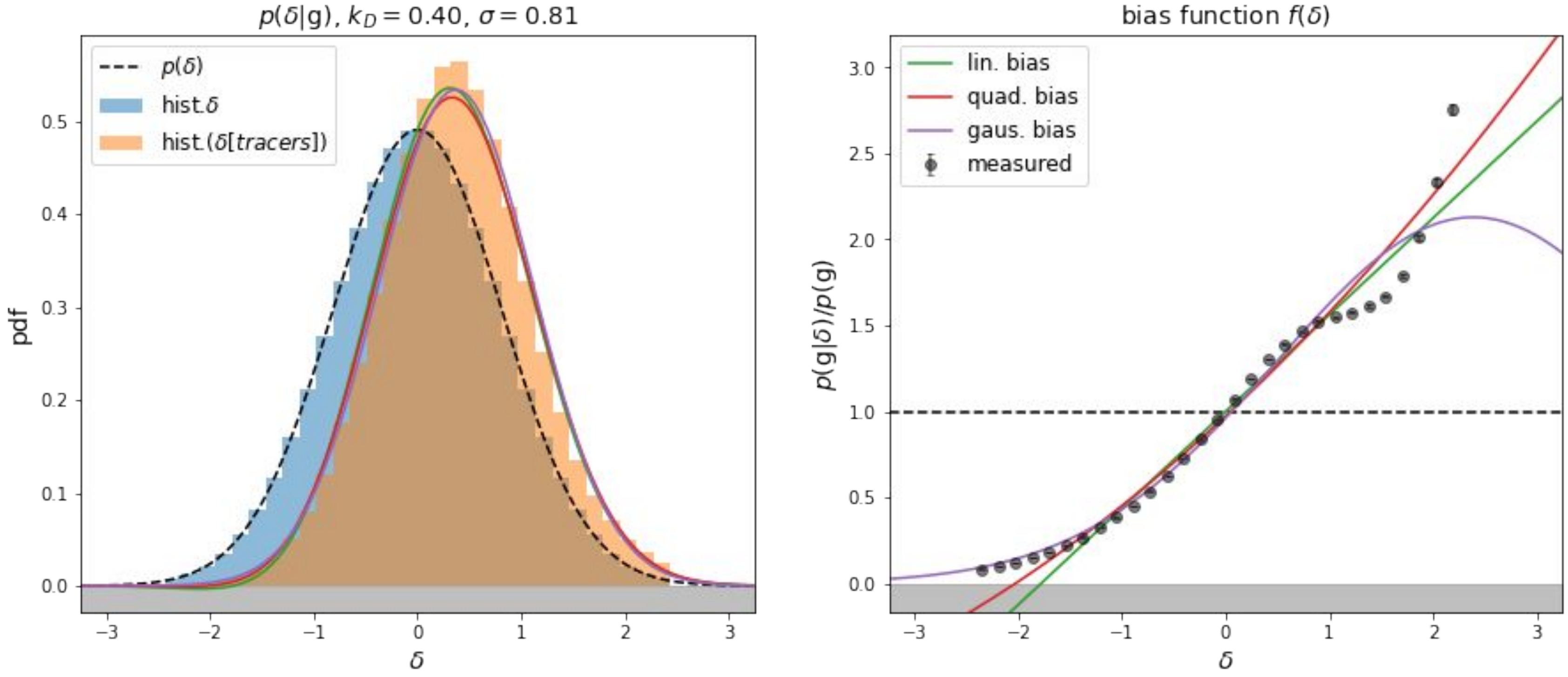
$$f(\delta) = \frac{p(\delta|g)}{p(\delta)}$$



M = 5e13

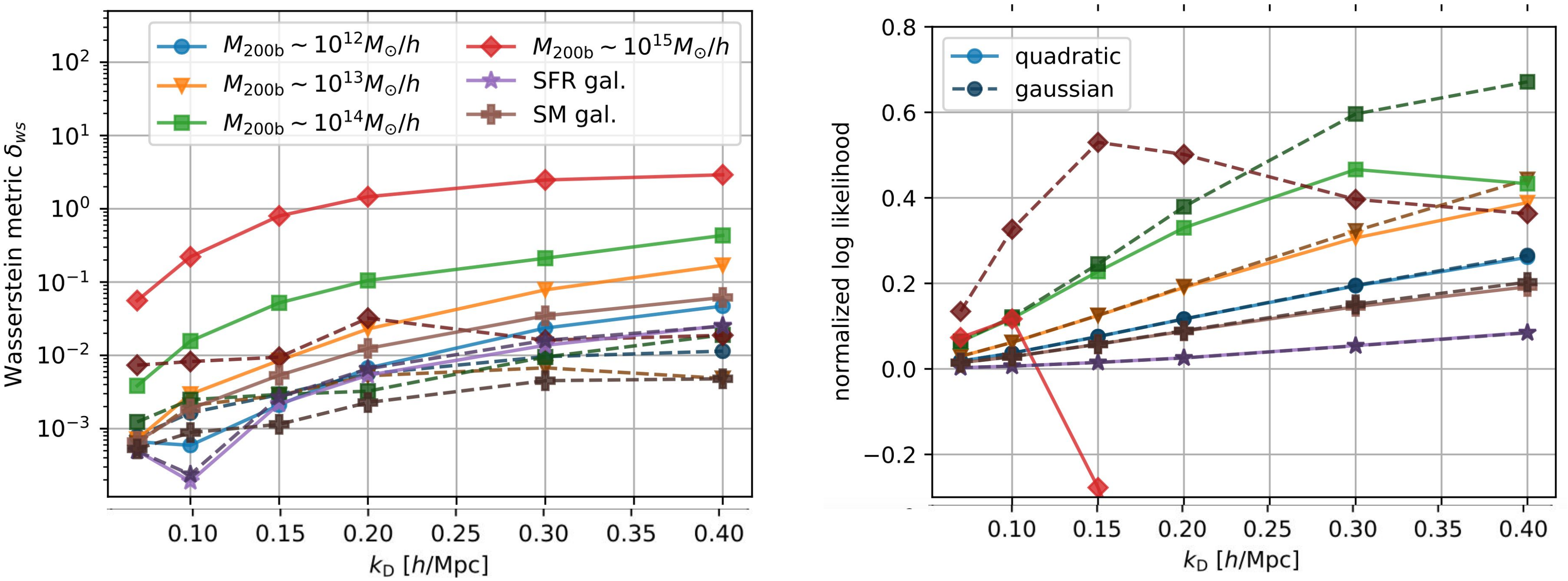
SFR sel. galaxies

$$f(\delta) = \frac{p(\delta|g)}{p(\delta)}$$



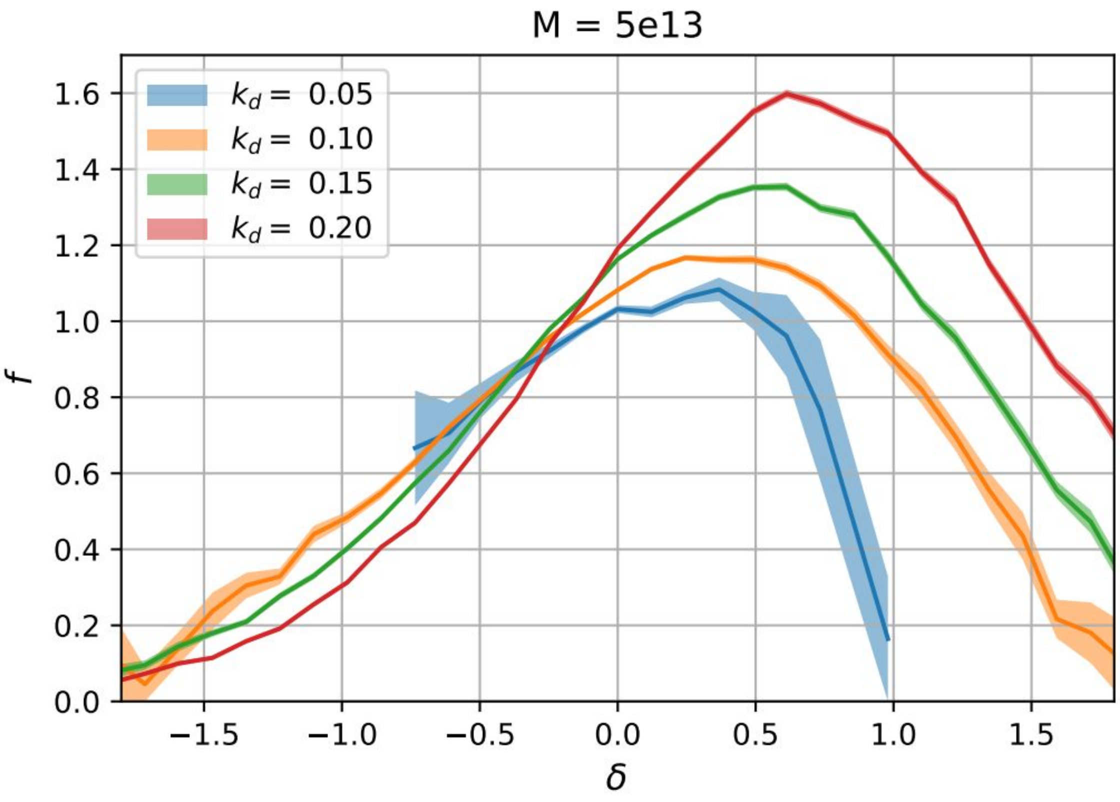
M = 5e13

Metrics



$M_{200b} \sim 10^{12} M_\odot/h$
$M_{200b} \sim 10^{13} M_\odot/h$
$M_{200b} \sim 10^{14} M_\odot/h$
$M_{200b} \sim 10^{15} M_\odot/h$
SFR gal.
SM gal.
Wasserstein metric δ_{ws}
k_D [h/Mpc]
quadratic
gaussian
normalized log likelihood
k_D [h/Mpc]

The scale dependence of the bias function



The scale dependence of the bias function

$$f_l(\delta_l) = \int f(\delta_s) p(\delta_s|\delta_l) \mathrm{d}\delta_s$$

Assumption:
Peak-Background split

$$f(\delta_s, \delta_l) = f(\delta_s)$$

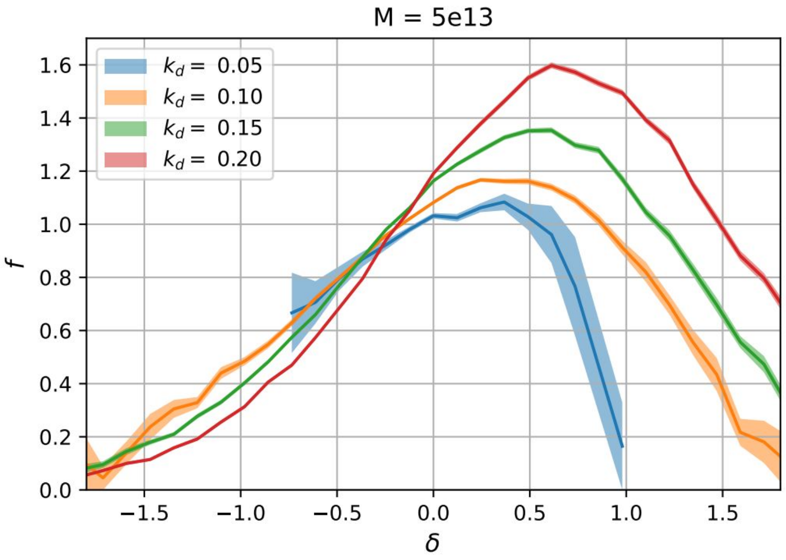
The scale dependence of the bias function

$$f_l(\delta_l) = \int f(\delta_s) p(\delta_s|\delta_l) \mathrm{d}\delta_s$$

- Corresponds to a convolution with a Gaussian!

Assumption:
Peak-Background split

$$f(\delta_s, \delta_l) = f(\delta_s)$$



The scale dependence of the bias function

$$f_l(\delta_l) = \int f(\delta_s) p(\delta_s | \delta_l) \mathrm{d}\delta_s$$

Assumption:
Peak-Background split

$$f(\delta_s, \delta_l) = f(\delta_s)$$

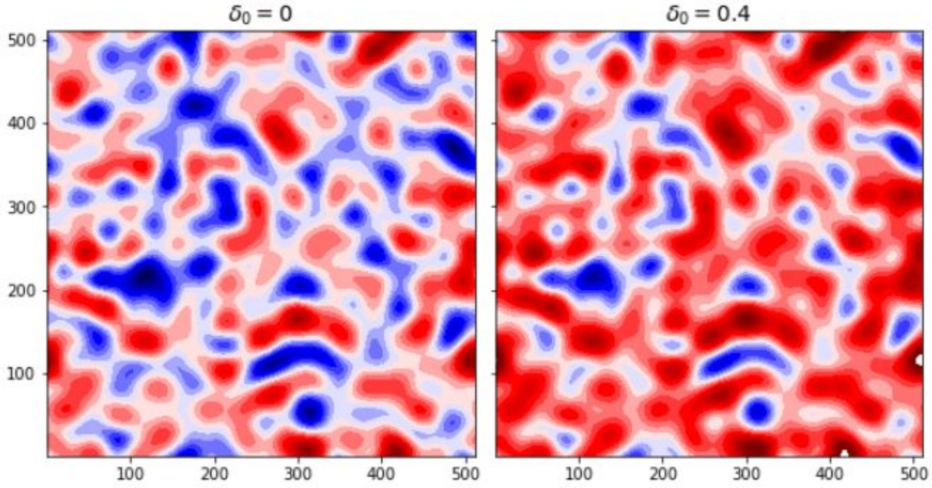
- Limit of large scales + sharp k-filter:

$$\begin{aligned} F(\delta_0) &= \int f(\delta) p(\delta - \delta_0) \mathrm{d}\delta \\ &= \int f(\delta + \delta_0) p(\delta) \mathrm{d}\delta \\ &= \langle f(\delta + \delta_0) \rangle \end{aligned}$$

- Call F “**renormalized** bias function”

Separate universe experiments

$$F(\delta_0) = \frac{n_g(\delta_0)}{n_{g,0}}$$



$$F(\delta_0) = \langle f(\delta + \delta_0) \rangle$$

$$F(\delta_0) = \langle f(\delta + \delta_0) \rangle$$

Renormalization of expansion

find f so that:

$$F_{\text{quad}}(\delta_0) = 1 + b_1\delta_0 + \frac{1}{2}b_2\delta_0^2$$

$$F(\delta_0) = \langle f(\delta + \delta_0) \rangle$$

Renormalization of expansion

find f so that:

$$F_{\text{quad}}(\delta_0) = 1 + b_1\delta_0 + \frac{1}{2}b_2\delta_0^2$$

Ansatz:

$$f_{\text{quad}}(\delta) = c_0 + c_1\delta + \frac{1}{2}c_2\delta^2$$

$$F_{\text{quad}}(\delta_0) = \left\langle c_0 + c_1(\delta + \delta_0) + \frac{1}{2}c_2(\delta^2 + 2\delta\delta_0 + \delta_0^2) \right\rangle$$

$$= c_0 + c_1\delta_0 + \frac{1}{2}c_2\sigma^2 + \frac{1}{2}c_2\delta_0^2$$

$$f_{\text{quad}}(\delta) = 1 + b_1\delta + \frac{1}{2}b_2(\delta^2 - \sigma^2)$$

Renormalization of Gaussian Bias

find f so that:

$$\log F_{\text{gaus}} = \beta_1 \delta_0 + \frac{1}{2}\beta_2 \delta_0^2$$

(I call this a 2nd order **"cumulant expansion"**)

Renormalization of Gaussian Bias

find f so that:

$$\log F_{\rm gaus} = \beta_1 \delta_0 + \frac{1}{2}\beta_2 \delta_0^2$$

(I call this a 2nd order **"cumulant expansion"**)

Find:

$$f_{\rm gaus} = \frac{\exp\left(-\frac{\beta_1^2}{2\beta_2}\right)}{\sqrt{1+\beta_2\sigma^2}} \exp\left(\frac{\beta_2\left(\frac{\beta_1}{\beta_2}+\delta\right)^2}{2(1+\beta_2\sigma^2)}\right)$$

$$\beta_1 = b_1$$

$$\beta_2 = b_2 - b_1^2$$

Important result: **Gaussian** bias has **simple renormalized form**!

-> As easy to use as second order expansion