fkPT - Large scale structure formation in the presence of additional scales

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Theoretical Modeling of Large Scale Structure of the Universe
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with Hernan Noriega, Arka Banerjee ++

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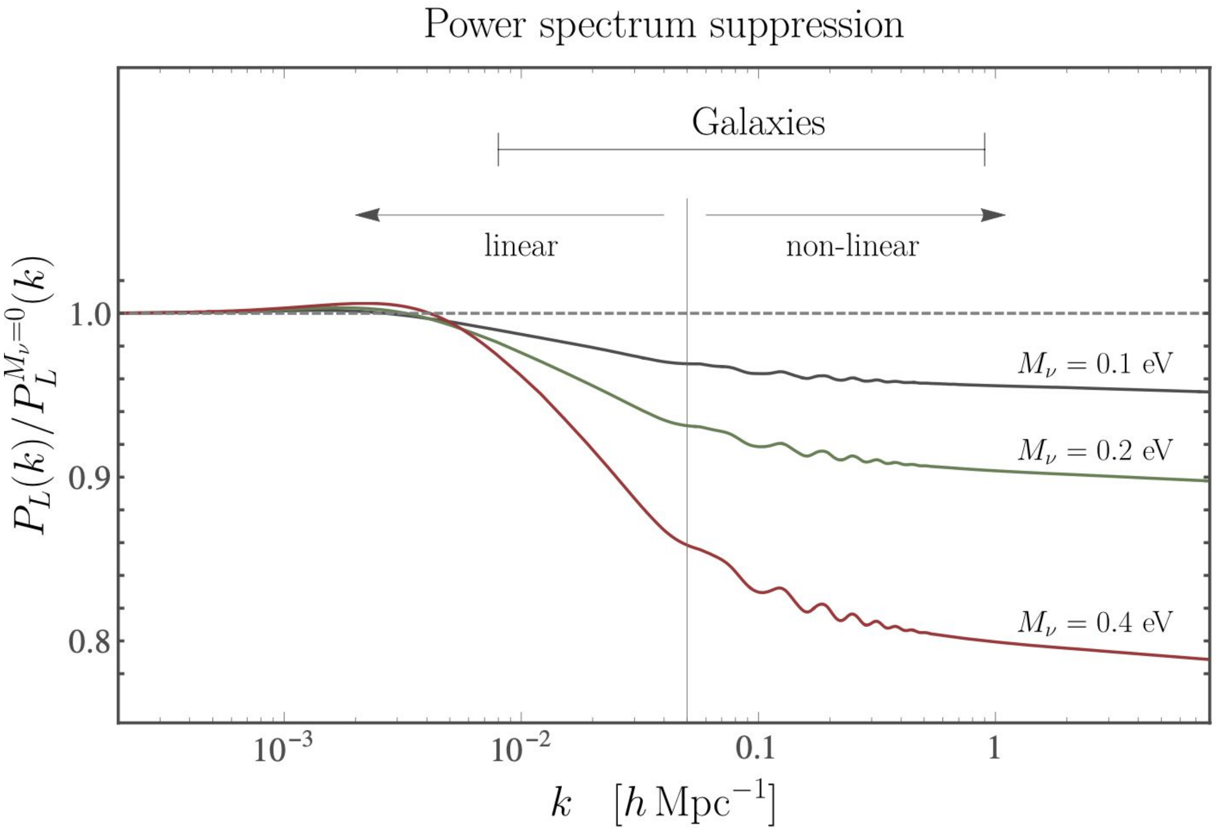
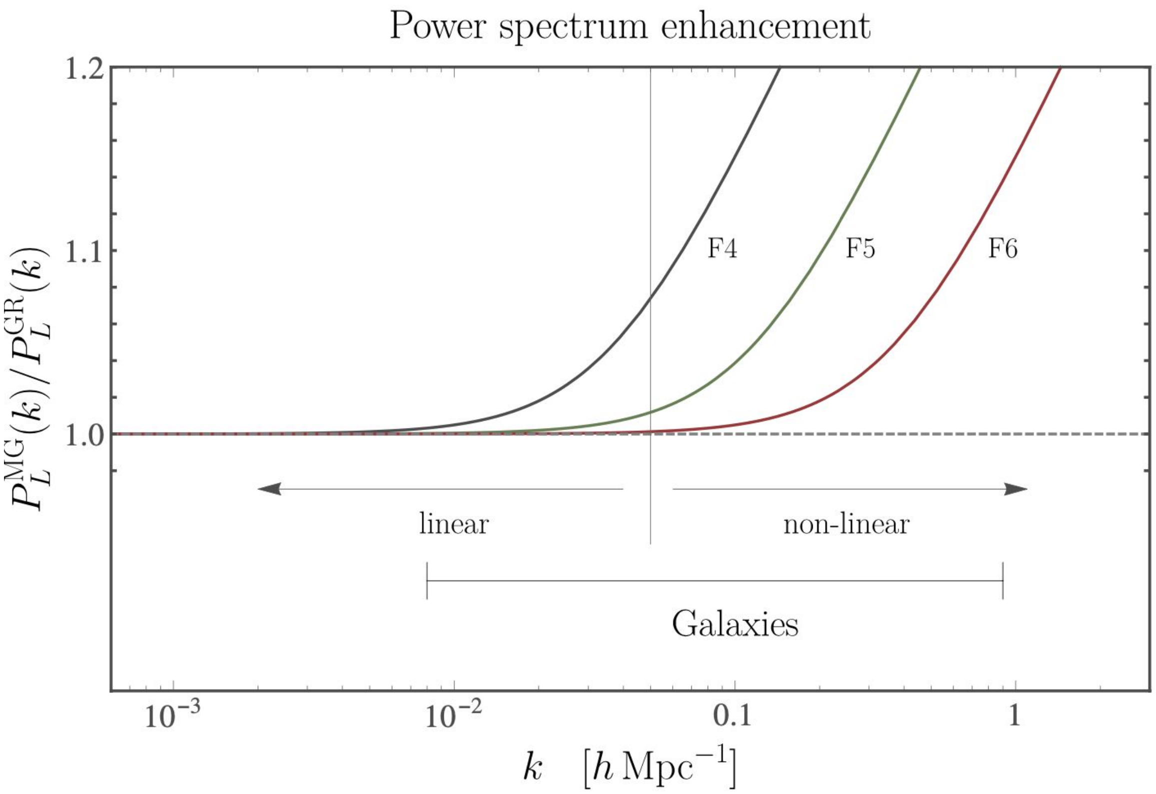
Motivation 1: massive neutrinos



Motivation 2: modified gravity



Lagrangian perturbation theory

Initial to final coordinates map $\boldsymbol{x}(\boldsymbol{q},t) = \boldsymbol{q} + \boldsymbol{\Psi}(\boldsymbol{q},t)$.

Geodesic equation:

$$\hat{\mathcal{T}}\,\boldsymbol{\Psi}(\boldsymbol{q},t) \equiv \left(\frac{d^2}{dt^2} + 2H\frac{d}{dt}\right)\boldsymbol{\Psi}(\boldsymbol{q},t) = \boldsymbol{\nabla}_{\mathbf{x}}\Phi(\boldsymbol{x},t)\Big|_{\boldsymbol{x}=\boldsymbol{q}+\boldsymbol{\Psi}}$$

Modified Poisson equation:

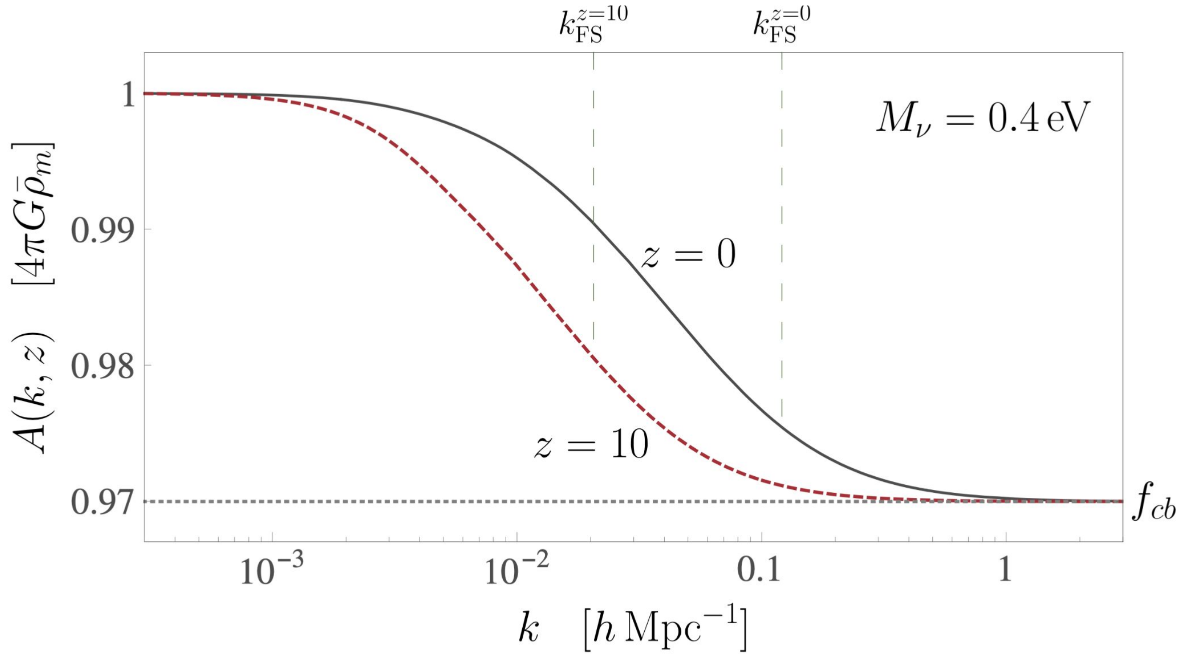
$$\frac{1}{a^2}\nabla^2_{\mathbf{x}}\Phi(\boldsymbol{x},t) = 4\pi G\bar{\rho}_m\delta(\boldsymbol{x},t) + S(\boldsymbol{x},t)$$

Neutrinos : $\delta = f_{cb}\delta_{cb}, \quad S = 4\pi G\bar{\rho}_m f_\nu \delta_\nu \quad f_\nu = \frac{\Omega_\nu}{\Omega_m},\ f_{cb} = \frac{\Omega_{cb}}{\Omega_m}$

Modified Gravity : $\delta = \delta_m, \quad S = -\frac{1}{2a^2}\nabla^2_{\mathbf{x}}\phi_{\rm MG}.$ e.g. DGP, f(R),..., Horndeski

Scale dependent source: neutrinos

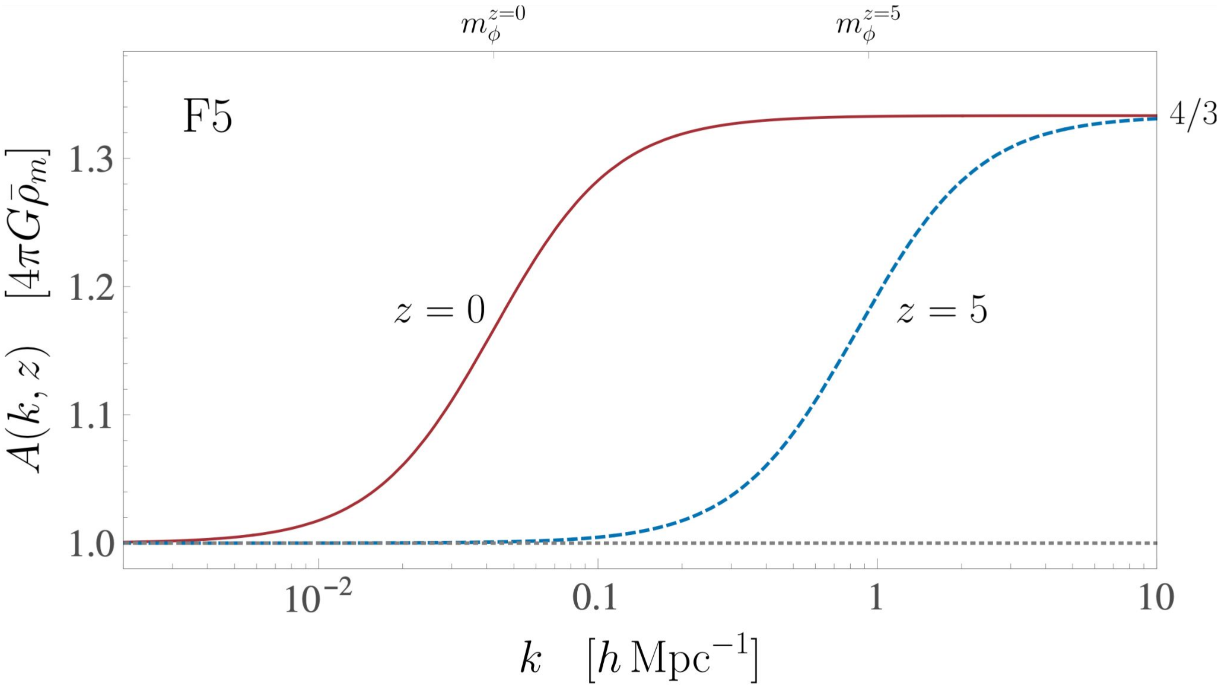
$$-\frac{k^2}{a^2}\Phi(\boldsymbol{k}) = 4\pi G\bar{\rho}_m(f_{cb}\delta_{cb}(\boldsymbol{k}) + f_\nu\delta_\nu(\boldsymbol{k})) = A(k,t)\delta_{cb}(\boldsymbol{k})$$



$$\delta_\nu(\boldsymbol{k}) \approx \frac{T_\nu(k)}{T_{cb}(k)}\delta_{cb}(\boldsymbol{k})$$

Scale dependent source: MG

$$-\frac{k^2}{a^2}\Phi(\boldsymbol{k}) = 4\pi G\bar{\rho}_m\delta_m(\boldsymbol{k}) + S(\boldsymbol{k},t) = A(k,t)\delta_m(\boldsymbol{k})$$



The equation of motion becomes

$$\Big[\boldsymbol{\nabla}_{\mathbf{x}} \cdot \hat{\mathcal{T}}\boldsymbol{\Psi}(\boldsymbol{q})\Big](\boldsymbol{k}) = -4\pi G\bar{\rho}_m\tilde{\delta}(\boldsymbol{k}) - \tilde{S}(\boldsymbol{k}).$$

$\Big[(\cdots)\Big](\boldsymbol{k})$ indicates the Fourier transform of $(\cdots)(\boldsymbol{q})$: $\int d^3q\, e^{-i\boldsymbol{k}\cdot\boldsymbol{q}}(\cdots)$

A tilde means the q-Fourier Transform of a function that takes arguments in Eulerian space:

$$\begin{aligned}\tilde{S}(\boldsymbol{k}) &\equiv \int d^3q\, e^{-i\boldsymbol{k}\cdot\boldsymbol{q}}\, S(\boldsymbol{x}) = \int d^3q\, e^{-i\boldsymbol{k}\cdot\boldsymbol{q}} S(\boldsymbol{q}+\Psi) \\ &= \int d^3q\, e^{-i\boldsymbol{k}\cdot\boldsymbol{q}}\Big(S(\boldsymbol{q}) + \Psi_i(\boldsymbol{q})S_{,i}(\boldsymbol{q}) + \frac{1}{2}\Psi_i(\boldsymbol{q})\Psi_j(\boldsymbol{q})S_{,ij}(\boldsymbol{q}) + \cdots\Big) \\ &= S(\boldsymbol{k}) \;+\; \int_{\boldsymbol{k}_{12}=\boldsymbol{k}} ik_1^i S(\boldsymbol{k}_1)\Psi_i(\boldsymbol{k}_2) \;-\; \frac{1}{2}\int_{\boldsymbol{k}_{123}=\boldsymbol{k}} k_1^i k_1^j S(\boldsymbol{k}_1)\Psi_i(\boldsymbol{k}_2)\Psi_j(\boldsymbol{k}_3) + \cdots\end{aligned}$$

Notation: $\displaystyle\int_{\boldsymbol{k}_{1\cdots n}=\boldsymbol{k}} = \int \frac{d^3k_1\cdots d^3k_n}{(2\pi)^{3n}}(2\pi)^3\delta_{\rm D}(\boldsymbol{k}_1+\cdots+\boldsymbol{k}_n-\boldsymbol{k})$

Equation of motion

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$$\left[\nabla_{\mathbf{x}} \cdot \hat{\mathcal{T}}\Psi(\boldsymbol{q})\right](\boldsymbol{k}) = -4\pi G\bar{\rho}_m\tilde{\delta}(\boldsymbol{k}) - \tilde{S}(\boldsymbol{k}) = -A(k)\tilde{\delta}(\boldsymbol{k}) + \text{FL}.$$

FL (Frame Lagging) terms appear when transforming from q–FT to x–FT

LCDM → $A_0 = 4\pi G\bar{\rho}$

$$\begin{aligned}\left(\hat{\mathcal{T}} - A(k)\right)[\Psi_{i,i}](\boldsymbol{k}) &= [\Psi_{i,j}\hat{\mathcal{T}}\Psi_{j,i}](\boldsymbol{k}) - \frac{A(k)}{2}[\Psi_{i,j}\Psi_{j,i}](\boldsymbol{k}) - \frac{A(k)}{2}[(\Psi_{l,l})^2](\boldsymbol{k}) \\ &- [\Psi_{i,k}\Psi_{k,j}\hat{\mathcal{T}}\Psi_{j,i}](\boldsymbol{k}) + \frac{A(k)}{6}[(\Psi_{l,l})^3](\boldsymbol{k}) + \frac{A(k)}{2}[\Psi_{l,l}\Psi_{i,j}\Psi_{j,i}](\boldsymbol{k}) + \frac{A(k)}{3}[\Psi_{i,k}\Psi_{k,j}\Psi_{j,i}](\boldsymbol{k}) \\ &- \int_{\boldsymbol{k}_{12}=\boldsymbol{k}} K^{\text{FL}}_{ki}(\boldsymbol{k}_1,\boldsymbol{k}_2)\Psi_k(\boldsymbol{k}_1)\Psi_i(\boldsymbol{k}_2) - \int_{\boldsymbol{k}_{123}=\boldsymbol{k}} K^{\text{FL}}_{kij}(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3)\Psi_k(\boldsymbol{k}_1)\Psi_i(\boldsymbol{k}_2)\Psi_j(\boldsymbol{k}_3)\end{aligned}$$

with $\quad K^{\text{FL}} \propto A(k) - A(k_1)$

LPT kernels

$$\Psi_i^{(n)}(\boldsymbol{k},t) = \frac{i}{n!}\int_{\boldsymbol{k}_{1\cdots n}=\boldsymbol{k}} L_i^{(n)}(\boldsymbol{k}_1,\cdots,\boldsymbol{k}_n;t)\delta^{(1)}(\boldsymbol{k}_1,t)\cdots\delta^{(1)}(\boldsymbol{k}_n,t)$$

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AA & A.Banerjee 2020

$$L_i^{(1)}(\boldsymbol{k},t) = \frac{k_i}{k^2},$$

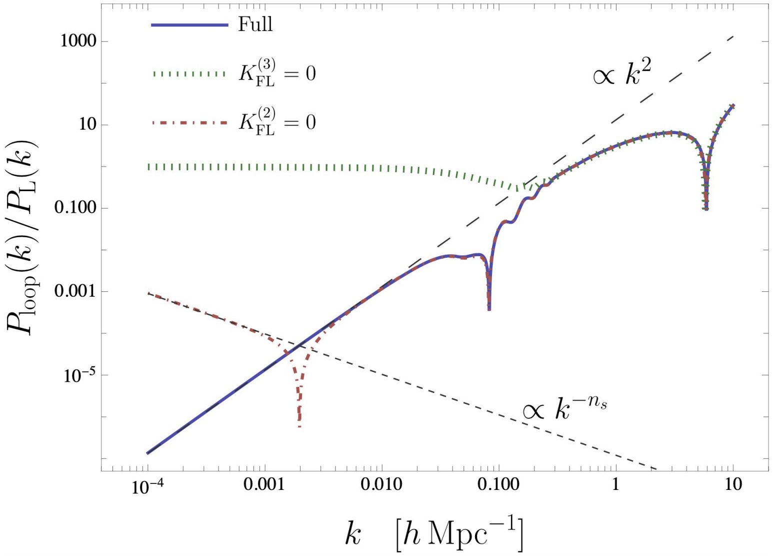
$$L_i^{(2)}(\boldsymbol{k}_1,\boldsymbol{k}_2,t) = \frac{3}{7}\frac{k_i}{k^2}\left(\mathcal{A}^{(2)} - \mathcal{B}^{(2)}\frac{(\boldsymbol{k}_1\cdot\boldsymbol{k}_2)^2}{k_1^2k_2^2}\right),$$

$$L_i^{(3)}(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3) = \frac{k^i}{k^2}\left\{\frac{5}{7}\left(\mathcal{A}^{(3)} - \mathcal{B}^{(3)}\frac{(\boldsymbol{k}_2\cdot\boldsymbol{k}_3)^2}{k_2^2k_2^3}\right)\left(1-\frac{(\boldsymbol{k}_1\cdot\boldsymbol{k}_{23})^2}{k_1^2k_{23}^2}\right)\right.$$
$$\left. -\frac{1}{3}\left(\mathcal{C}^{(3)} - 3\mathcal{D}^{(3)}\frac{(\boldsymbol{k}_2\cdot\boldsymbol{k}_3)^2}{k_2^2k_3^2} + 2\mathcal{E}^{(3)}\frac{(\boldsymbol{k}_1\cdot\boldsymbol{k}_2)(\boldsymbol{k}_2\cdot\boldsymbol{k}_3)(\boldsymbol{k}_3\cdot\boldsymbol{k}_1)}{k_1^2k_2^2k_3^2}\right)\right\},$$

with *A*, *B*,... scale and time dependent function.

From LPT to EPT $\quad L^{(n)}, \dot{L}^{(n)} \longrightarrow F_n, G_n$

$$F_n, G_n(\boldsymbol{k}_1, \ldots, \boldsymbol{k}_n) \neq F_n, G_n(\lambda\boldsymbol{k}_1, \ldots, \lambda\boldsymbol{k}_n)$$



$$P_{\rm loop}(k \to 0)\big|_{K_{\rm FL}^{(2)}=0} \sim \frac{9}{98} \int_{p \gg k} \frac{dp}{4\pi^2} p^2 P_L^2(p) \left[\mathcal{A}(-\boldsymbol{p}, \boldsymbol{p})\big|_{\text{No FL}} - \mathcal{B}(-\boldsymbol{p}, \boldsymbol{p})\right]^2,$$

$$P_{\rm loop}(k \to 0)\big|_{K_{\rm FL}^{(3)}=0} \propto P_L(k) \int_{p \gg k} \frac{dp}{4\pi^2} p^2 P_L(p),$$

Velocity field

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At linear order:

$$\theta^{(1)}(\boldsymbol{k}) = \frac{f(k)}{f_0}\delta^{(1)}(\boldsymbol{k}).$$

where *f(k,t)* is the scale dependent growth function, and f_0 is its large scales value ($k << k_m$)

$$f(k,t) = \frac{d \ln D_+(k,t)}{d \ln a(t)}, \qquad f_0(t) = f(k \to 0, t).$$

The dipole in the second order density kernel F_2 arises from expanding $\delta(\mathbf{x} + \boldsymbol{\Psi})$ to lowest order

$$\delta^{(2)}(\boldsymbol{x}) \ni \delta^{(1)}(\boldsymbol{x} + \Psi) - \delta^{(1)}(\boldsymbol{x}) = \partial_i \left[\nabla^{-2}\delta^{(1)}(\boldsymbol{x})\right] \partial_i \delta^{(1)}(\boldsymbol{x})$$

In Fourier space

$$\frac{\hat{\boldsymbol{k}}_1 \cdot \hat{\boldsymbol{k}}_2}{2} \left(\frac{k_2}{k_1} + \frac{k_1}{k_2} \right) \in F_2(\boldsymbol{k}_1, \boldsymbol{k}_2)$$

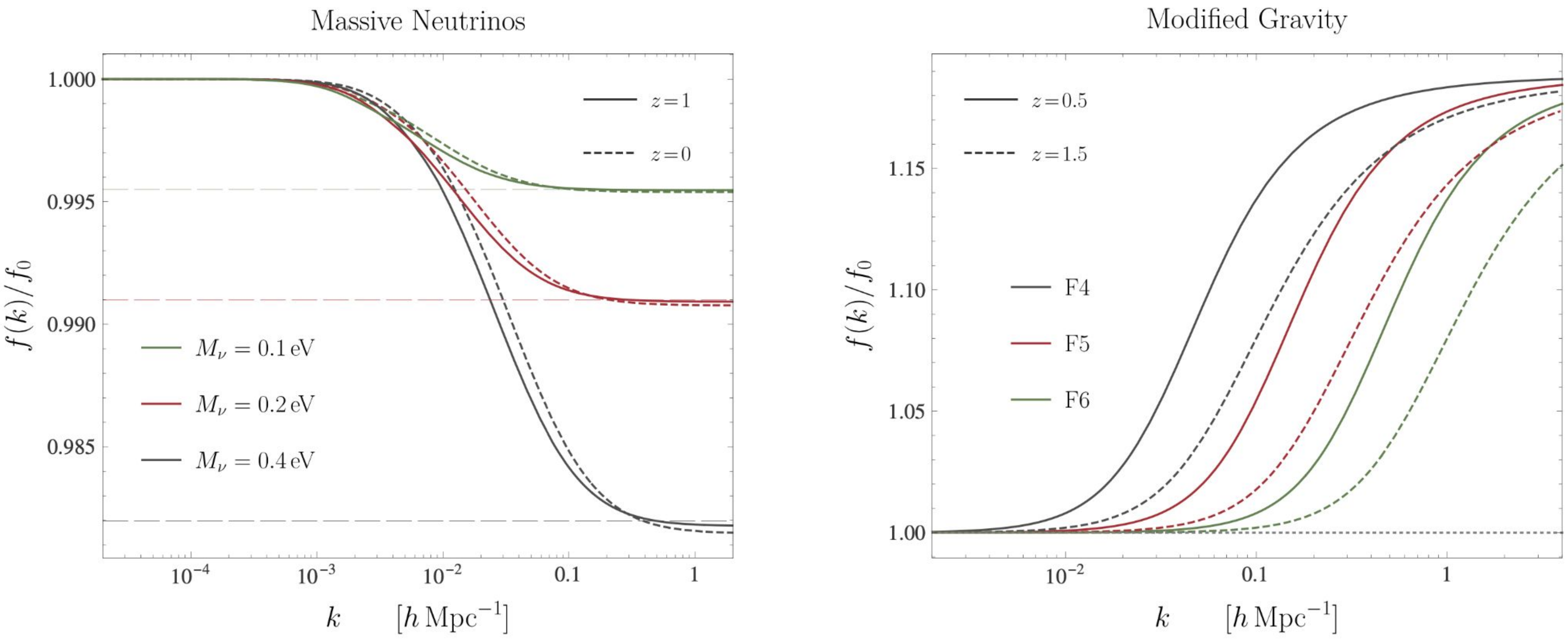
Similarly, for the velocity field

$$\theta^{(2)}(\boldsymbol{x}) \ni \theta^{(1)}(\boldsymbol{x} + \Psi) - \theta^{(1)}(\boldsymbol{x}) = \partial_i \left[\nabla^{-2}\delta^{(1)}(\boldsymbol{x})\right] \partial_i \theta^{(1)}(\boldsymbol{x})$$

Using $\theta^{(1)}(\boldsymbol{k}) = \frac{f(k)}{f_0} \delta^{(1)}(\boldsymbol{k})$

$$\frac{\hat{\boldsymbol{k}}_1 \cdot \hat{\boldsymbol{k}}_2}{2} \left(\frac{f(k_2)}{f_0} \frac{k_2}{k_1} + \frac{f(k_1)}{f_0} \frac{k_1}{k_2} \right) \in G_2(\boldsymbol{k}_1, \boldsymbol{k}_2).$$

Growth rate



Massive Neutrinos
$f(k)/f_0$
1.000
0.995
0.990
0.985
$z=1$
$z=0$
$M_\nu = 0.1\,\mathrm{eV}$
$M_\nu = 0.2\,\mathrm{eV}$
$M_\nu = 0.4\,\mathrm{eV}$
10^{-4}
10^{-3}
10^{-2}
0.1
1
$k \quad [h\,\mathrm{Mpc}^{-1}]$
Modified Gravity
$f(k)/f_0$
1.15
1.10
1.05
1.00
$z=0.5$
$z=1.5$
F4
F5
F6
10^{-2}
0.1
1
$k \quad [h\,\mathrm{Mpc}^{-1}]$

Full PT model

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$$P(k,\mu) = \sum_{m=0}^{\infty}\sum_{n=0}^{m} \mu^{2n} f_0^m I_{nm}(k)$$

With functions

$$I_{mn}(k) = \int_{\boldsymbol{p}} \mathcal{I}_{mn}(k, p, \hat{k}\cdot\hat{p})$$

For each volume element of the integration we need to solve a system of differential equations to find the second and third order functions *A*, *B*, ...

Hence, these integrals are computationally expensive, precluding the use of efficient parameter sampling algorithms for estimation of cosmological parameters.

fkPT

However, the primary *new* contributions to the loop corrections stem from the growth rates `f(k)`, instead of the computationally expensive *A, B*... functions.

Define the fk-kernels

$$F_2^{\mathrm{fk}}(\boldsymbol{k}_1, \boldsymbol{k}_2) = F_2(\boldsymbol{k}_1, \boldsymbol{k}_2)\Big|_{\mathcal{A}=\mathcal{B}=\mathcal{A}^{\Lambda\mathrm{CDM}}},$$
$$G_2^{\mathrm{fk}}(\boldsymbol{k}_1, \boldsymbol{k}_2) = G_2(\boldsymbol{k}_1, \boldsymbol{k}_2)\Big|_{\mathcal{A}=\mathcal{B}=\mathcal{A}^{\Lambda\mathrm{CDM}}},$$

and similar for higher orders.

A.A.++ 2021, Noriega++ 2022 & Rodríguez-Meza++ 2023

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For example, one of the $I_{mn}(k)$ functions is

$$P^{22}_{\theta\theta}(k) = 2\int_{\boldsymbol{p}} [G_2(\boldsymbol{p}, \boldsymbol{k}-\boldsymbol{p})]^2 P_L(p) P_L(\boldsymbol{k}-\boldsymbol{p}),$$

with

$$G_2(\boldsymbol{k}_1, \boldsymbol{k}_2) = \frac{3\left[f(k_1)+f(k_2)\right]\mathcal{A} + 3\dot{\mathcal{A}}/H}{14 f_0} + \frac{\hat{\boldsymbol{k}}_1\cdot\hat{\boldsymbol{k}}_2}{2}\left(\frac{f(k_2)}{f_0}\frac{k_2}{k_1} + \frac{f(k_1)}{f_0}\frac{k_1}{k_2}\right) + \left(\hat{\boldsymbol{k}}_1\cdot\hat{\boldsymbol{k}}_2\right)^2\left(\frac{f(k_1)+f(k_2)}{2f_0} - \frac{3\left[f(k_1)+f(k_2)\right]\mathcal{B} + 3\dot{\mathcal{B}}/H}{14 f_0}\right)$$

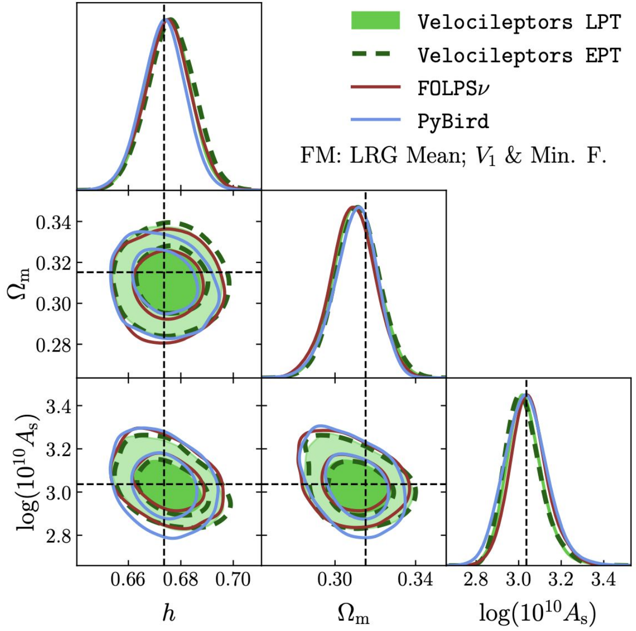
fk-kernels: Replace $A(k_1,k_2)$ by its large scale value A^{LS}~1.01. Also for B.
For EdS $A=B=1$

H. Noriega, AA,.. 2022

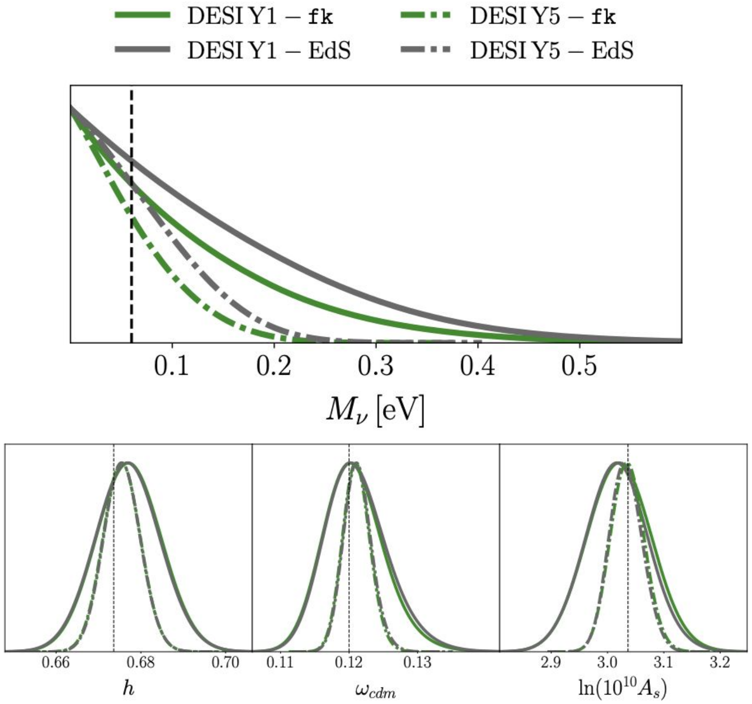
github.com/henoriega/FOLPS-nu < 0.2 s

github.com/cosmodesi/folpsax < 0.02 s

Arnaud de Mattia

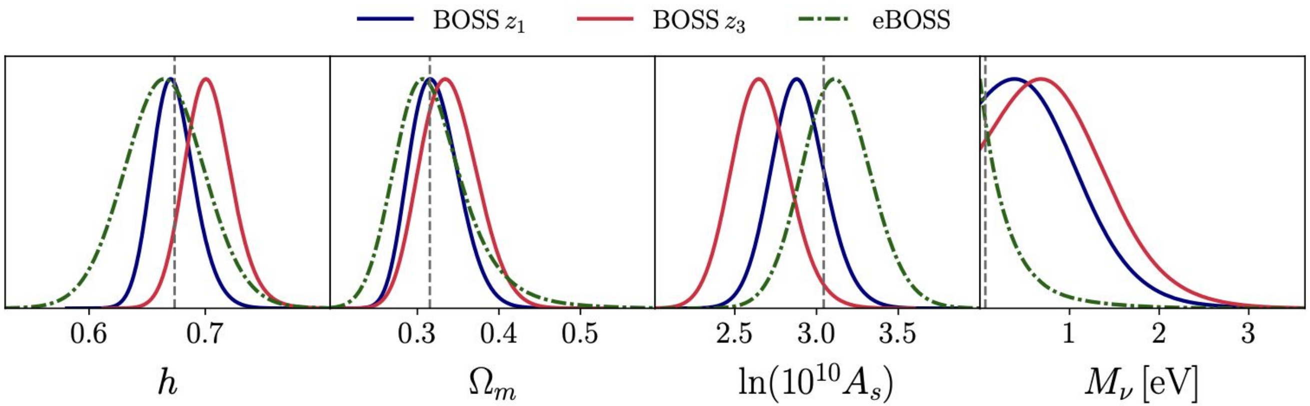


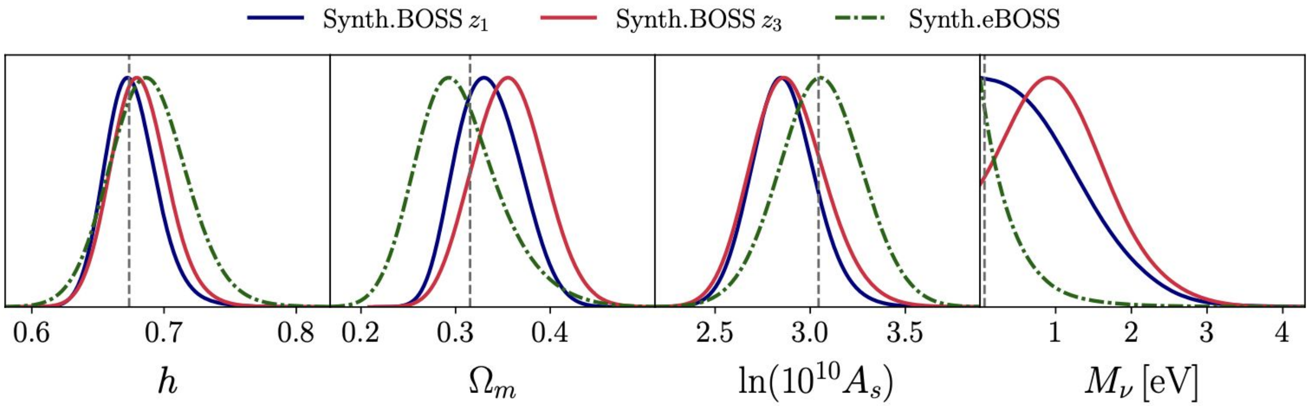
M. Maus et al 2024



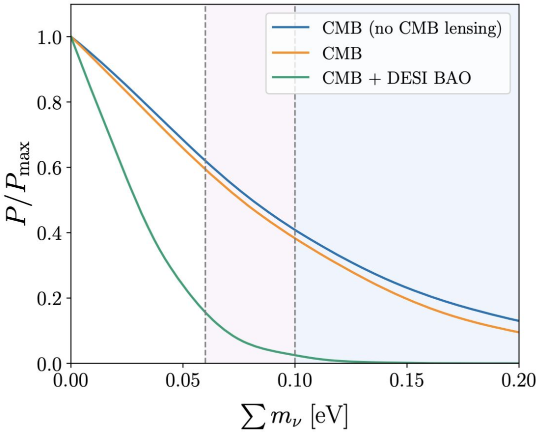
H. Noriega et al 2024

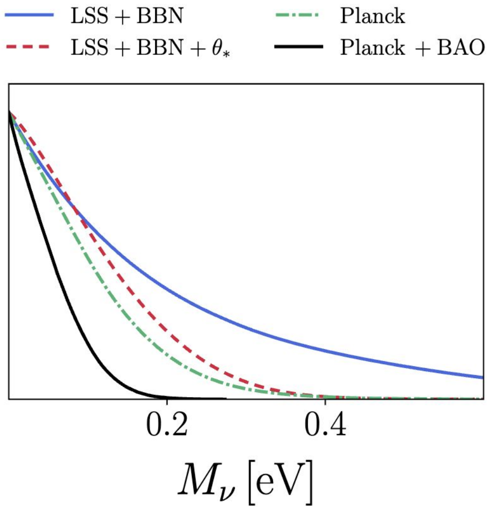
DR12 BOSS LRGs and eBOSS QSOs

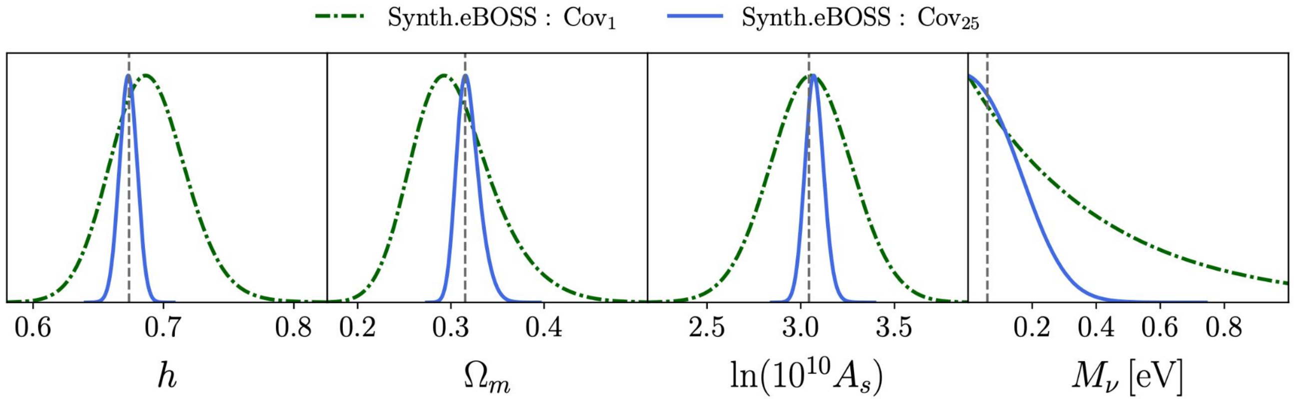




DESI 2024: cosmo params







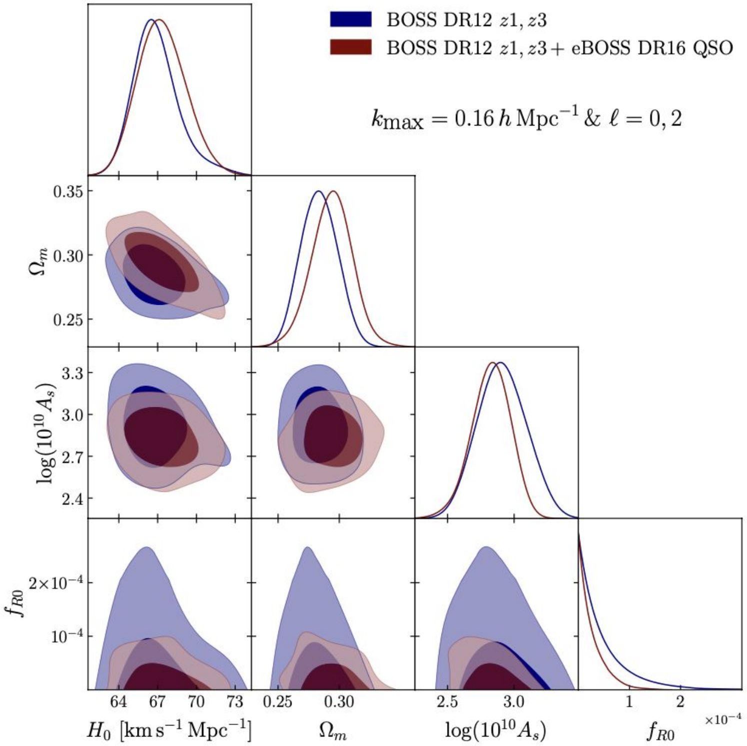
Modified gravity

DR12 BOSS LRGs and eBOSS QSO

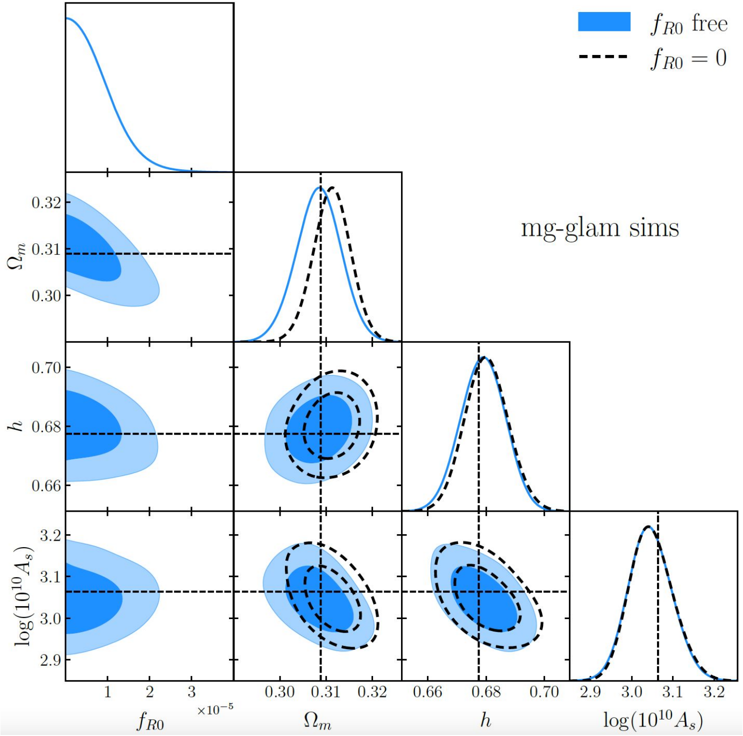
github.com/alejandroaviles/fkpt

BOSS DR12 z1,z3 $|f_{R0}| < 3.29 \times 10^{-5}$

+ eBOSS DR16 QSOs $|f_{R0}| < 3.13 \times 10^{-5}$

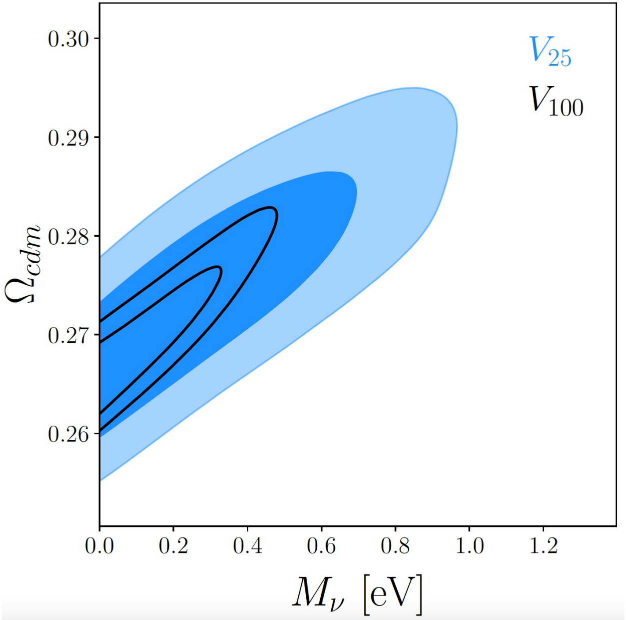


Modified gravity

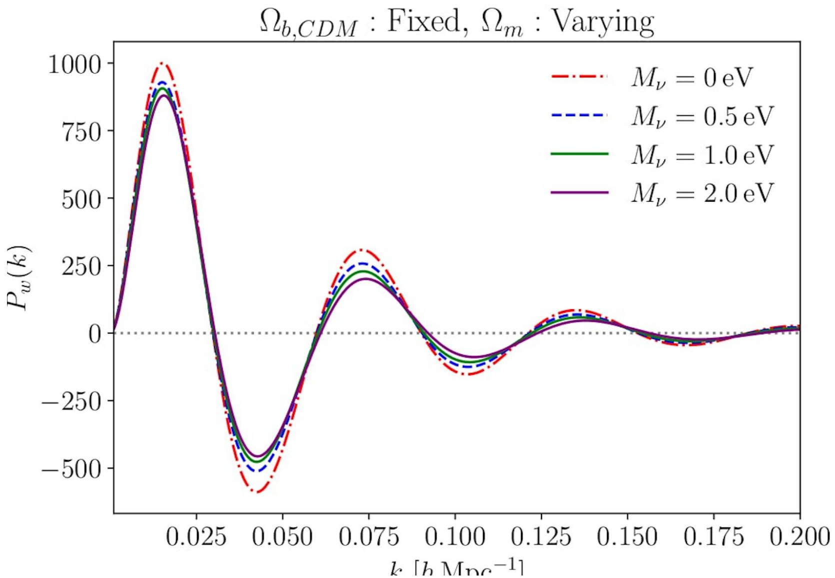


- The power spectrum is enhanced on scales above $k \propto f_{R0}^{-1}$.
- f_{R0} doesn't affect the background expansion.
- Notice the degeneracy between Ω_m and f_{R0}

Neutrinos



V_{25}
V_{100}
0.30
0.29
0.28
0.27
0.26
Ω_{cdm}
0.0
0.2
0.4
0.6
0.8
1.0
1.2
M_ν [eV]



$\Omega_{b,CDM}$: Fixed, Ω_m : Varying
$M_\nu = 0$ eV
$M_\nu = 0.5$ eV
$M_\nu = 1.0$ eV
$M_\nu = 2.0$ eV
1000
750
500
250
0
−250
−500
$P_w(k)$
0.025
0.050
0.075
0.100
0.125
0.150
0.175
0.200

- We may search for other methods to constrain the neutrino mass.

- An alternative would be to probe the suppression of the power spectrum, as was initially proposed.

- Despite the significantly reduced constraining power, it can be more robust.

Broadband vs wiggles

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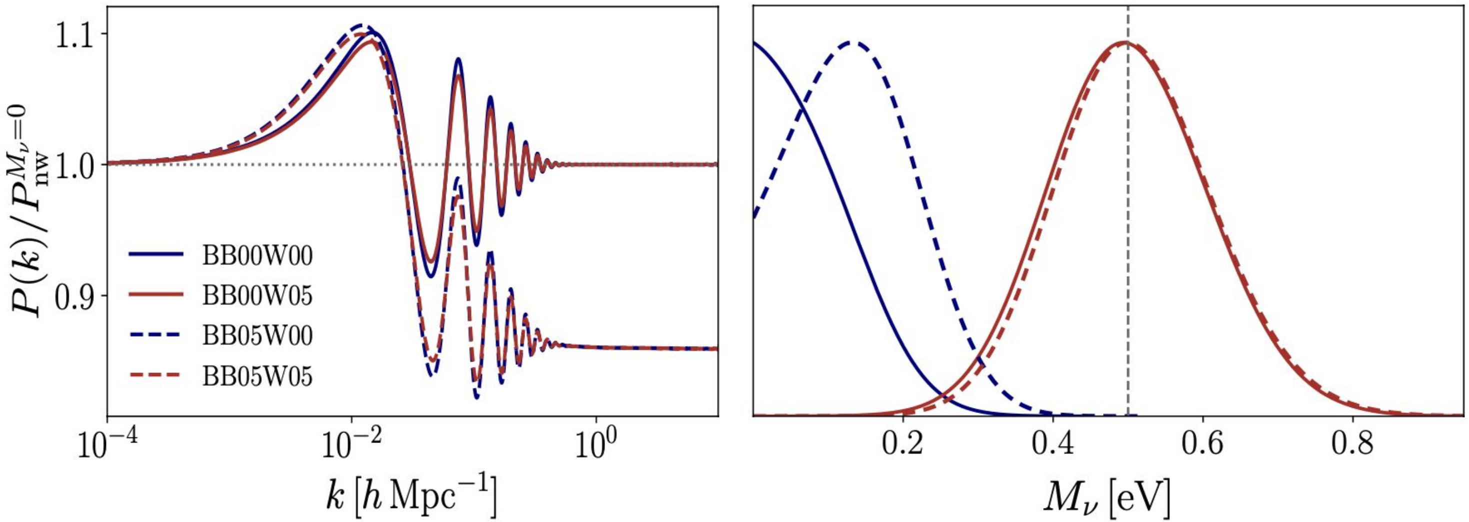
Construct $P_L = P_{\rm w} + P_{\rm nw}$ for $M_\nu = 0.0$ eV and $M_\nu = 0.5$ eV

Decompose the two linear power spectra into broadband + wiggles and mix the pieces to obtain four models. Then, we evolve them non linearly and generate four synthetic data:

- BB00W00 - Broadband of the model with mass $M_\nu = 0.0$ eV, wiggles of the model with mass $M_\nu = 0.0$ eV
- BB00W05 - Broadband of the model with mass $M_\nu = 0.0$ eV, wiggles of the model with mass $M_\nu = 0.5$ eV
- BB05W00 - Broadband of the model with mass $M_\nu = 0.5$ eV, wiggles of the model with mass $M_\nu = 0.0$ eV
- BB05W05 - Broadband of the model with mass $M_\nu = 0.5$ eV, wiggles of the model with mass $M_\nu = 0.5$ eV

Broadband vs wiggles



Most of the information about neutrino mass comes from the suppression of wiggles, rather than from the suppression of broadband.

Thank you!

See Hernan’s talk in Neutrinos from Home 2024 in youtube: