From BCJ double copy to inflationary cosmology

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This talk...

- Not (quite) a talk about amplitudes, but it relates to subjects discussed by JJ and Chia-Hsien.
- Very low-tech.
- Work in progress.

Amplitudes and beyond

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Amplitudes for Astrophysicists I: Known Knowns

Daniel J. Burger, Raúl Carballo-Rubio, Nathan Moynihan, Jeff Murugan, Amanda Weltman

(Submitted on 17 Apr 2017 (v1), last revised 15 May 2017 (this version, v3))

The use of quantum field theory to understand astrophysical phenomena is not new. However, for the most part, the methods used are those that have been developed decades ago. The intervening years have seen some remarkable developments in computational quantum field theoretic tools. In particle physics, this technology has facilitated calculations that, even ten years ago would have seemed laughably difficult. It is remarkable, then, that most of these new techniques have remained firmly within the domain of high energy physics. We would like to change this. As alluded to in the title, this is the first in a series of papers aimed at showcasing the use of modern on-shell methods in the context of astrophysics and cosmology. In this first article, we use the old problem of the bending of light by a compact object as an anchor to pedagogically develop these new computational tools. Once developed, we then illustrate their power and utility with an application to the scattering of gravitational waves.

Comments: 50 pages (sorry!) but lots of figures, some worked examples and a glossary. v2: now with feynman diagrams and added references

Subjects: **High Energy Astrophysical Phenomena (astro-ph.HE)**; General Relativity and Quantum Cosmology (gr-qc); High Energy Physics - Theory (hep-th)

Cite as: arXiv:1704.05067 **[astro-ph.HE]**

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Cosmology: Introduction

- Cosmology aims to describe the origin and evolution of the universe.

- First approached with general relativity, trough Friedmann-Robertson-Walker solution

$$ds^2 = dt^2 + a(t)^2 \left[\frac{dr^2}{1 - \kappa r^2} + r^2 d\Omega^2 \right],$$

  a isotropic homogeneous expanding solution.

- Solution leads to Big bang (cosmological singularity). Cosmic microwave background is evidence in favour of hot early universe. Yet, it fails to explain observational data (classic problems: singularity, flatness, horizon).

Cosmology: Chaotic Inflation

- To solve some of these problems, Guth presents inflationary model. Exponential expansion in a supercooled false vacuum state. It also assumes thermal equilibrium in the beginning.
- Problems of older inflation versions are solved by Linde's chaotic inflation. This doesn't require thermal equilibrium, and it can be realized by potentials as simple as ϕ^2.

  If $V(\phi) \sim \phi^2$, the Einstein equation is

  $$H^2 + \frac{k}{a^2} = \frac{1}{6}(\dot{\phi}^2 + m^2\phi^2)$$

  Thus, there is inflation even with the simplest model

  $$\frac{1}{\sqrt{-g}}\mathcal{L} = \frac{1}{2}R - \frac{1}{2}\partial\phi^2 - \frac{1}{2}m^2\phi^2.$$

  But this predicts larger than experimental wave scattering

Cosmology: α-attractors

Its simplest modification is

$$\frac{1}{\sqrt{-g}}\mathcal{L}_T = \frac{1}{2}R - \frac{1}{2}\frac{\partial\phi^2}{\left(1-\frac{\phi^2}{6\alpha}\right)^2} - \frac{1}{2}m^2\phi^2.$$

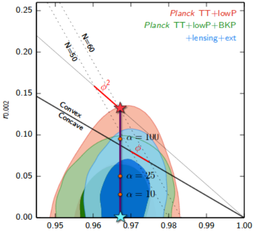
This is not canonically normalized. Instead, using the change of basis

$$\phi = \sqrt{6\alpha}\tanh\frac{\varphi}{\sqrt{6\alpha}},$$

The potential is given by

$$V = 3\alpha m^2\tanh^2\frac{\varphi}{\sqrt{6\alpha}}$$

This is called α-attractor T-model, and it shows good agreement with experimental observations.



Cosmology: Supergravity models

There is a large class of general inflationary models in $\mathcal{N}=1$ supergravity based on two chiral multiplets, the inflaton multiplet Φ and the goldstino multiplet S. They have Kähler and superpotential

$$K = K(\Phi, \bar{\Phi}, S, \bar{S}), \qquad W = Sf(\Phi).$$

The inflaton superfield is

$$\Phi = \phi + ia + \sqrt{2}\theta\xi + \theta^2 F^{\Phi},$$

while the Goldstino superfield is

$$S = s + \sqrt{2}\theta G + \theta^2 F^S,$$

It is straightforward to obtain the potential

$$V = e^K \left\{ \left(\frac{\partial^2 K}{\partial\Phi\partial\bar{\Phi}} \right)^{-1} D_\Phi W D_{\bar{\Phi}} \bar{W} - 3|W|^2 \right\}$$

with

$$D_\Phi W = \frac{\partial W}{\partial \Phi} + \frac{\partial K(\Phi + \bar{\Phi})}{\partial \Phi} W$$

Cosmology: Nilpotent superfields

It is possible to obtain the model with

$$V = m^2\phi^2$$

from a supergravity with potentials

$$\mathcal{K} = S\bar{S} - \frac{1}{2}(\Phi - \bar{\Phi})^2 - \zeta(S\bar{S})^2 + \frac{\gamma}{2}S\bar{S}(\Phi - \bar{\Phi})^2,$$
$$W = Sf(\Phi)$$

To guarantee that the moduli will be stable during the inflation phase, it was necessary to add

$$\mathcal{K}_{\text{stabiliser}} = -\zeta(S\bar{S})^2 + \frac{\gamma}{2}S\bar{S}(\Phi - \bar{\Phi})^2.$$

It could as well be stabilised by considering a nilpotent superfield

$$S^2 = 0$$

This implies

$$S = \frac{G^2}{2F} + \sqrt{2}\theta G + \theta^2 F.$$

Lagrangian from Nilpotent superfields

The most general Lagrangian without derivatives is

$$\mathcal{L} = \int \mathrm{d}^4\theta S\bar{S} + \int \mathrm{d}^2\theta f S + \int \mathrm{d}^2\bar{\theta} f \bar{S}$$

and substituting $S = \frac{G^2}{2F} + \sqrt{2}\theta G + \theta^2 F$, we obtain

$$\mathcal{L} = i\partial_\mu \bar{G}\bar{\sigma}^\mu G + \bar{F}F + \frac{G^2}{2\bar{F}}\partial^2\left(\frac{G^2}{2F}\right) + (fF + c.c.)$$

after integrating F, we obtain

$$\mathcal{L} = -f^2 + i\partial_\mu \bar{G}\bar{\sigma}^\mu G + \frac{1}{4f^2}\bar{G}^2\partial^2 G^2 - \frac{1}{16f^6}G^2\bar{G}^2\partial^2 G^2\partial^2\bar{G}^2$$

Making the substitutions $2f^2 = \kappa^{-2}$, ${v_a}^b \equiv i\lambda\sigma^b\partial_a\bar{\lambda}$ and

$$\begin{aligned} G_\alpha(\lambda,\bar{\lambda}) =& \lambda_\alpha + i\frac{\kappa^2}{2}(\sigma^a\bar{\lambda})_\alpha(\partial_a\lambda^2)(1+\kappa^2\langle\bar{v}\rangle) \\ &+\frac{\kappa^4}{2}\lambda_\alpha(\langle v\bar{v}\rangle - \frac{1}{2}\langle\bar{v}^2\rangle - \langle\bar{v}\rangle^2 + \frac{1}{2}\partial^a\lambda^2\partial_a\bar{\lambda}^2 + \frac{3}{4}\bar{\lambda}^2\Box\lambda^2) \\ &-\kappa^6\lambda_\alpha(\langle v\bar{v}^2\rangle + \frac{1}{2}\langle v\bar{v}\rangle\langle\bar{v}\rangle - \frac{1}{2}\langle v\rangle\langle\bar{v}^2\rangle - \\ &\qquad -\frac{1}{4}\langle v\rangle\langle\bar{v}\rangle^2 + \frac{3}{4}\langle\bar{v}\rangle\partial^a\lambda^2\partial_a\bar{\lambda}^2), \end{aligned}$$

we have

$$\begin{aligned} S[\lambda,\bar{\lambda}] = -\frac{1}{2}\int \mathrm{d}^4x\ &\langle v+\bar{v}\rangle + 2\kappa^2\left(\langle v\rangle\langle\bar{v}\rangle - \langle v\bar{v}\rangle\right) \\ &+\kappa^4\left(\langle v^2\bar{v}\rangle - \langle v\rangle\langle v\bar{v}\rangle - \frac{1}{2}\langle v^2\rangle\langle\bar{v}\rangle + \frac{1}{2}\langle v\rangle^2\langle\bar{v}\rangle + \text{c.c.}\right). \end{aligned}$$

Finally, defining ${\Xi_a}^b = \kappa^2{(v+\bar{v})_a}^b$, this is

$$S_{\mathrm{VA}}[\lambda,\bar{\lambda}] = \frac{1}{2\kappa^2}\int \mathrm{d}^4x(1-\det\Xi),$$

the Volkov-Akulov action

Volkov-Akulov and amplitudes

- The Volkov-Akulov model is an effective field theory that describes the interaction of goldstinos (Goldstone fermions from the spontaneous breaking of (super)symmetry).
- There is a renewed interest in amplitudes for Volkov-Akulov theories as a consequence of progress with soft theorems, recursion relations, and a unified web of theories (see Chia-Hsien's talk) For example, one of the theories that have been understood as double copies recently is

$$M_{\text{BI}} = A_{\text{NLSM}} \times A_{\text{YM}}.$$

A (4-dimensional) supersymmetric version of this yields

$$M_{\text{DBI-VA}} = A_{\text{NLSM}} \times A_{\text{SYM}},$$

or restricting to only fermion states, we can have the relation

$$M_{\text{VA}} = A_{\text{NLSM}} \times A_{\text{SYM}|_{\text{fermions}}},$$

BCJ double copy: Minimal review

We can, schematically write the Yang-Mills scattering amplitude as

$$\mathcal{A}_{\mathrm{YM}} \sim \sum_{\text{cubic graphs}} \frac{n_i c_i}{\prod_j p_j^2}.$$

The color factors satisfy

$$c_i + c_j + c_k = 0.$$

This is simply the Jacobi identity. It is possible to make the numerators (kinematic information) obey analogous relations.

$$n_i + n_j + n_k = 0.$$

This is called colour-kinematics duality. As a byproduct, we have

$$\mathcal{M}_{\mathrm{Gravity}} \sim \sum_{\text{cubic graphs}} \frac{n_i \tilde{n}_i}{\prod_j p_j^2}.$$

We will simply write

$$M_{\mathrm{Gravity}} = A_{\mathrm{YM}} \times \tilde{A}_{\mathrm{YM}},$$

BCJ double copy: Action games

- The Lagrangian for Yang-Mills takes the form

$$\mathcal{L}_{\mathrm{YM}} = \frac{1}{2} A^{a\mu} \partial^2 A^a_\mu - g f^{abc} \partial_\mu A^a_\nu A^{b\mu} A^{c\nu} - \frac{1}{4} g^2 f^{abe} f^{ecd} A^a_\mu A^b_\nu A^{c\mu} A^{d\nu}.$$

  This satisfies colour-kinematics duality only up to four points.

- For higher points, is possible to add terms (identically zero) to the Lagrangian to ensure that the duality is satisfied. For example, to five points

$$\mathcal{L}'_5 = -\frac{1}{2} g^3 f^{abf} f^{fcg} f^{gde} \times \left(\partial_{[\mu} A^a_{\nu]} A^b_\rho A^{c\mu} + \partial_{[\mu} A^b_{\nu]} A^c_\rho A^{a\mu} + \partial_{[\mu} A^c_{\nu]} A^a_\rho A^{b\mu} \right) \frac{1}{\Box} A^{d\nu} A^{e\rho}$$

- In [Bern,Dennen,Huang,Kiermaier '10] this was verified through 6 points. [Tolotti, Weinzierl '13] sistematize the idea.

BCJ double copy: Action games

- Action is not cubic. It can become cubic by considering an auxiliary field B

$$\mathcal{L}_{\rm YM} = \frac{1}{2}A^{a\mu}\partial^2 A^a_\mu + B^{a\mu\nu\rho}\partial^2 B^a_{\mu\nu\rho} - g f^{abc}\left(\partial_\mu A^a_\nu - \partial^\rho B^a_{\rho\mu\nu}\right) A^{b\mu}A^{c\nu}.$$

where the equation of motion for the auxiliary field $B^a_{\mu\nu\rho}$ becomes

$$\partial^2 B^a_{\mu\nu\rho} = \frac{g}{2} f^{abc}\partial_\mu(A^b_\nu A^c_\rho).$$

- In momentum space, we have

$$S_{\rm YM} = \int d^4k_1 d^4k_2 \delta^4(k_1+k_2) k_2^2 [A_\mu(k_1)A^\mu(k_2) - 2B^{\mu\nu\rho}(k_1)B_{\mu\nu\rho}(k_2)]$$
$$+ \int d^4k_1 d^4k_2 d^4k_3 \delta^4(k_1+k_2+k_3)$$
$$P_6\Big([k_{1\,\mu}A_\nu(k_1) + k_1^\rho B_{\rho\mu\nu}(k_1)]A^\mu(k_2)A^\nu(k_3)\Big).$$

Since gravity has no colour, we encode antisymmetry and cyclic information by introducing the operator P_6 i.e.

$P_6\{k_{1\,\alpha_2}\eta_{\alpha_1\alpha_3}\} \equiv \eta_{\alpha_1\alpha_3}(k_1-k_3)_{\alpha_2} + \eta_{\alpha_1\alpha_2}(k_2-k_1)_{\alpha_3} + \eta_{\alpha_2\alpha_3}(k_3-k_2)_{\alpha_1}.$

BCJ double copy: Action games

- We can implement the identification

$$H_{\mu\nu}(k) = A_\mu(k)\tilde{A}_\nu(k).$$

Use a double copy procedure to build the gravity Lagrangian. [Bern,Dennen,Huang,Kiermaier '10]

$$S_{\text{gravity}} = S_{\text{kin}} + S_{\text{int}},$$

where

$$S_{\text{kin}} = \frac{1}{4}\int d^4k_1 d^4k_2 \delta^4(k_1+k_2) \times [A_\mu(k_1)A^\mu(k_2) - 2B^{\mu\nu\rho}(k_1)B_{\mu\nu\rho}(k_2)] \times [\tilde{A}_\sigma(k_1)\tilde{A}^\sigma(k_2) - 2\tilde{B}^{\sigma\tau\lambda}(k_1)\tilde{B}_{\sigma\tau\lambda}(k_2)],$$

while the interaction part of the Lagrangian is

$$S_{\text{int}} = \int d^4k_1 d^4k_2 d^4k_3 \delta^4(k_1+k_2+k_3) \times P_6\Big([k_{1\mu}A_\nu(k_1) + k_1^\rho B_{\rho\mu\nu}(k_1)]A^\mu(k_2)A^\nu(k_3)\Big) \times P_6\Big([k_{1\sigma}\tilde{A}_\tau(k_1) + k_1^\lambda \tilde{B}_{\lambda\sigma\tau}(k_1)]\tilde{A}^\sigma(k_2)\tilde{A}^\tau(k_3)\Big)$$

Shameless advertisement

- Using this double copied action, we have perturbatively computed classical solutions for gravity (GR + dilaton + 2-form)

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High Energy Physics - Theory

Perturbative spacetimes from Yang-Mills theory

A. Luna, R. Monteiro, I. Nicholson, A. Ochirov, D. O'Connell, N. Westerberg, C. D. White

(Submitted on 22 Nov 2016 (v1), last revised 30 Jan 2017 (this version, v2))

The double copy relates scattering amplitudes in gauge and gravity theories. In this paper, we expand the scope of the double copy to construct spacetime metrics through a systematic perturbative expansion. The perturbative procedure is based on direct calculation in Yang-Mills theory, followed by squaring the numerator of certain perturbative diagrams as specified by the double-copy algorithm. The simplest spherically symmetric, stationary spacetime from the point of view of this procedure is a particular member of the Janis-Newman-Winicour family of naked singularities. Our work paves the way for applications of the double copy to physically interesting problems such as perturbative black-hole scattering.

Comments:	v2: 20 pages + refs, minor improvements
Subjects:	**High Energy Physics - Theory (hep-th)**; General Relativity and Quantum Cosmology (gr-qc)
Journal reference:	JHEP 1704 (2017) 069
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Report number:	Edinburgh 2016/17, QMUL-PH-16-20
Cite as:	**arXiv:1611.07508 [hep-th]**
	(or **arXiv:1611.07508v2 [hep-th]** for this version)

NLSM: The non-linear sigma model

- The NLSM is the effective model that describes the low energy behaviour of Goldstone bosons associated with $SU(N) \times SU(N) \to SU(N)$ symmetry breaking.
- The Lagrangian is given by

$$\mathcal{L} = \frac{F^2}{4} \mathrm{Tr}(\partial_\mu U \partial^\mu U^\dagger)$$

where F is the Decay constant. U is defined (in Cayley parametrization) as

$$U = 1 + 2 \sum_{n=1}^{\infty} \left(\frac{1}{2F} \phi \right)^n, \qquad \phi = \sqrt{2} \phi^a T^a$$

T^a are the generators of the $SU(N)$ Lie algebra.

- It satisfies flavour decomposition analogous to Yang Mills. [Kampf, Novotny, Trnka '12, '13] The flavour ordered vertices are

$$V_{2n+1} = 0,$$

$$V_{2n+2} = \left(1 \frac{1}{2F^2}\right)^n \left(\sum_{i=0}^{n} p_{2i+1} \right)^2 = \left(1 \frac{1}{2F^2}\right)^n \left(\sum_{i=0}^{n} p_{2i+2} \right)^2$$

NLSM: A CK dual action

- It is possible to write an action for the NLSM that is already cubic, and generates colour-kinematics satisfying numerators. [Cheung, Shen '16] This is given by

$$S = \int Z^{a\,\mu}\partial^2 X^a_\mu + \frac{1}{2}Y^a\partial^2 Y^a - f^{abc}\left(Z^{a\,\mu}Z^{b\,\nu}X^c_{\mu\nu} + Z^{a\,\mu}(Y^b\overleftrightarrow{\partial}_\mu Y^c)\right)$$

  where $X_{\mu\nu} = \partial_\mu X_\nu - \partial_\nu X_\mu$ and $\overleftrightarrow{\partial} = \frac{1}{2}(\overrightarrow{\partial} - \overleftarrow{\partial})$ See also Chia-Hsien's talk.

- Then, the tree amplitudes of the NLSM are equal to the tree amplitudes

$$A(\ldots, Y_i, \ldots, Y_j, \ldots)$$

  where the ellipses denote all other external particles taken to be longitudinally polarized Z-states for which $\epsilon_\mu = ik_\mu$

Double copy: Special Galileon

- It is then possible to obtain an action for the special galileon by performing a double copy of the action [Cheung, Shen '16]

$$S_{\mathrm{SG}} = \int Z^{\mu\bar{\mu}}\partial^2 X^{\mu\bar{\mu}} + \frac{1}{2}Y\partial^2 Y \\ + 2\left(Z^{\mu\bar{\mu}}Z^{\nu\bar{\nu}}X_{\mu\nu\bar{\mu}\bar{\nu}} + Z^{\mu\bar{\mu}}(Y\overleftrightarrow{\partial}_{\mu}\overleftrightarrow{\partial}_{\bar{\mu}}Y)\right)$$

  where

$$X_{\mu\nu\bar{\mu}\bar{\nu}} = \partial_\mu\partial_{\bar{\mu}}X_{\nu\bar{\nu}} + \partial_\nu\partial_{\bar{\nu}}X_{\mu\bar{\mu}} - \partial_\mu\partial_{\bar{\nu}}X_{\nu\bar{\mu}} - \partial_\nu\partial_{\bar{\mu}}X_{\mu\bar{\nu}}$$

- Then, the tree amplitudes of the special Galileon are equal to the tree amplitudes

$$A(\ldots, Y_i, \ldots, Y_j, \ldots)$$

  where the ellipses denote all other external particles taken to be longitudinally polarized Z-states for which $\epsilon_{\mu\bar{\mu}} = ik_\mu k_{\bar{\mu}}$.

Recap and Outlook

- There is a family of (inflationary) cosmological models called α-attractors, which has fermionic action VA.
- It is possible to understand VA as NLSM times fermions in SYM
- A simpler Lagrangian formulation of NLSM [Cheung, Shen '16] is well suited for double copy.
- Can I build a "double copied" action for Volkov-Akulov?
- If nothing else, good laboratory for action double copy asymmetric
- Stepping stone for Einstein-VA
- Different approach: Use amplitudes inspired methods to explore other physics

Thank you