Exploring Reggeon bound states in strongly coupled $\mathcal{N}=4$ super Yang-Mills theory

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based on work with Martin Sprenger [to appear]

Motivation

- Amplitudes in planar $\mathcal{N} = 4$ SYM:

$$A_{n,\text{MHV}} = A_{n,\text{MHV}}^{\text{tree}} \cdot e^{A_{\text{BDS},n} + R_n(u_i)}$$

- Goal: Solve n-point remainder function $R_n(u_i)$ at finite coupling

- study special kinematic regime, the multi-Regge limit
  $\rightarrow$ remainder function physically described by effective particles, the so-called Reggeons

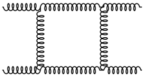
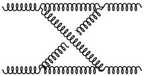
- all-loop expression for 6-point remainder function in MRL is known

- How can we generalise the results to n points?
  $\rightarrow$ begin in the strong coupling limit

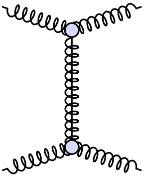
Reggeisation

Example: 4-particle scattering ($s \gg -t$)

- leading contributions to the one-loop amplitude

 $+$  $\sim \alpha_s^2 \log s \frac{s}{-t}$

- limit $s \to \infty$: $\alpha_s^2 \log s \sim \mathcal{O}(1)$, for n loops: $\alpha_s^n \log^{n-1} s \sim \mathcal{O}(1)$
- for leading logarithmic approximation: resum ∞ number of diagrams



$$A_{2\to 2} \sim \frac{s^{1+\omega(t)}}{-t}$$

$\to$ Reggeon-exchange

- every logarithmic order contains information about all loop orders

Multi-Regge kinematics

- R_n only depends on reduced set of variables
- $3n - 15$ independent dual conformal cross-ratios $u_i = \frac{x_{i,j}^2 x_{k,l}^2}{x_{i,k}^2 x_{j,l}^2}$

$$x_{i,j}^2 = (k_{i+1} + \cdots + k_j)^2$$

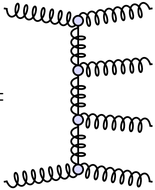
- MRL defined such that

$$u_{1,s} \to 1, \;\; u_{2,s} \to 0, \;\; u_{3,s} \to 0$$

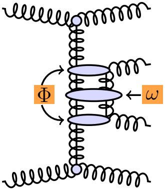
$$\text{s.t. } \tilde{u}_{2,s} = \frac{u_{2,s}}{1-u_{1,s}} = \text{ finite and } \tilde{u}_{3,s} = \frac{u_{3,s}}{1-u_{1,s}} = \text{ finite}$$

6-point remainder function in the MRL

$$e^{R_6+i\delta_6}\big|_{\mathrm{MRL}} =$$



$$+$$



- finite-coupling expression for the 6-point remainder function:

  [Bartels,Lipatov,Sabio Vera; Lipatov,Prygarin; Fadin,Lipatov; Dixon,Duhr,Pennington; Dixon,Drummond,von Hippel,Pennington; Caron-Huot; Basso,Caron-Huot,Sever; Dixon,von Hippel,McLeod]

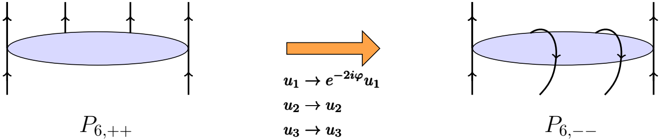
$$e^{R_6+i\delta_6}\big|_{\mathrm{MRL}} = \cos\pi\omega_{ab} +$$

$$i\frac{\lambda}{2}\sum_n (-1)^n \left(\frac{w}{w^*}\right)^{\frac{n}{2}} \int \frac{d\nu}{\nu^2+\frac{n^2}{4}} |w|^{2i\nu}\, \Phi_{\mathrm{Reg}}(\nu,n) \left(-(1-u_1)\sqrt{\tilde{u}_2\tilde{u}_3}\right)^{-\omega(\nu,n)}$$

- expect similar relations for $n > 6$

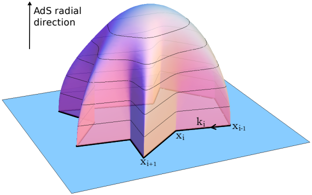
Kinematic regions

- Unitarity requires amplitudes to have cuts
- analytic continuation in kinematic variables can lead to cut contributions
- through the study of different kinematic regions we study the analytical properties of the amplitude
- example: 6-point amplitude



- remainder function trivial in $P_{6,++}$ but non-trivial in $P_{6,--}$

Gluon scattering amplitudes at strong coupling



[Figure from 1002.2459]

[Alday,Maldacena; Alday,Maldacena,Sever,Vieira; Alday,Gaiotto,Maldacena]

- via AdS/CFT correspondence
- $\mathcal{A} \sim e^{-\frac{\sqrt{\lambda}}{2\pi}\text{Area}(k_i)}$
- $\text{Area} = A_{\text{div}} + \Delta(u_i) + A_{\text{per}}(m_{a,s}, \phi_{a,s}) + A_{\text{free}}$
- $R_n := -\frac{\sqrt{\lambda}}{2\pi}\left(\text{Area} - A_{\text{div}}\right)$

$$\log \mathrm{Y}_{a,s}(\theta) = -|m_{a,s}|\cosh\theta + C_{a,s} + \sum_{a',s'}\int_{\mathbb{R}} d\theta' \mathcal{K}^{a,a'}_{s,s'}(\theta,\theta')\log\left(1+\mathrm{Y}_{a',s'}(\theta')\right)$$

TBA

$$A_{\text{free}} = \sum_{a,s}\frac{|m_{a,s}|}{2\pi}\int_{\mathbb{R}} d\theta \cosh\theta \log\left(1+\mathrm{Y}_{a,s}(\theta)\right)$$

with $a = 1, 2, 3$ and $s = 1, 2, \ldots, n-5$

The Y-system in the MRL

[Bartels,Schomerus,Sprenger]

- $3n-15$ Y-system parameters: $m_{a,s}=|m_{a,s}|e^{i\phi_s}$, $C_{a,s}$

- to reproduce MRL in kinematics:

$$|m_{a,s}|\to\infty,\ \phi_s\to(1-s)\frac{\pi}{4}\ \text{and}\ C_{a,s}=\text{const.}$$

$$\int_{\mathbb{R}} d\theta' K(\theta-\theta')\log(1+\mathrm{Y}(\theta'))\cong\int_{\mathbb{R}} d\theta' K(\theta-\theta')\log\left(1+e^{-|m|\cosh\theta'}\right)\xrightarrow{|m|\to\infty}0$$

MRL $\Rightarrow$ $$\mathrm{Y}_{a,s}(\theta)\cong e^{-|m_{a,s}|\cosh\theta+C_{a,s}}$$

- $R_n\to 0$ in this limit

From the TBA to the Bethe ansatz

- Physical region: Amplitude in MRL fully described by BDS ansatz $\rightarrow$ continue to other kinematic regions
- analytic continuation in cross-ratios $\rightarrow$ analytic continuation in Y-system parameters [Dorey,Tateo]
- $\mathrm{Y}(\theta) = -1$ is a pole of the Y-system integrand $\rightarrow$ solutions crossing the integration axis can lead to new contributions in the remainder function

$$\log \mathrm{Y}'_{a,s}(\theta) = -\left|m_{a,s}\right|' \cosh(\theta) + C'_{a,s} + \sum_{a',s'} \int_{\mathbb{R}} d\theta' \mathcal{K}^{a,a'}_{s,s'}(\theta,\theta') \log\left(1 + \mathrm{Y}'_{a',s'}(\theta')\right)$$

$$-2\pi i \mathcal{K}^{a,a'}_{s,s'}(\theta,\theta') =: \partial_{\theta'} S^{a,a'}_{s,s'}(\theta,\theta')$$

$\theta_{\pm}$ ... crossing solutions in Y_{a_0,s_0}

$$A'_{\text{free}} = \sum_{a,s} \frac{|m_{a,s}|'}{2\pi} \int_{\mathbb{R}} d\theta \cosh\theta \log\left(1 + \mathrm{Y}'_{a,s}(\theta)\right)$$

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$$+ \log \frac{S^{a,a_0}_{s,s_0}(\theta,\theta_-)}{S^{a,a_0}_{s,s_0}(\theta,\theta_+)}$$

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MRL

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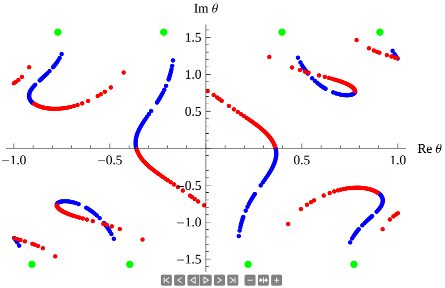
From the TBA to the Bethe ansatz

[Bartels,Kotanski,Schomerus; Bartels,Kotanski,Schomerus,Sprenger]

6-point case: one pair of crossing solutions in $\mathrm{Y}_3(\theta)$ for $P_{6,++} \to P_{6,--}$



Solutions of $\mathrm{Y}_3(\theta) = -1$ along continuation

$$\log \mathrm{Y}_3'(\theta) = -|m|' \cosh(\theta) + C' + \log\left(\frac{S^{3,3}(\theta - \theta_-)}{S^{3,3}(\theta - \theta_+)}\right)$$

The 6-point remainder function

- endpoint conditions $\leftrightarrow$ Bethe ansatz equations:

$$-1 = \mathrm{Y}_3'(\theta_\pm) = e^{-|m|' \cosh(\theta_\pm) + C'} \left(\frac{S^{3,3}(\theta_\pm - \theta_-)}{S^{3,3}(\theta_\pm - \theta_+)} \right)$$

- can uniquely solve for endpoints of crossing solutions $\theta_\pm = \pm \frac{i\pi}{4}$
- remainder function at strong coupling:

$$e^{R_6 + i\delta_6}|_{\mathrm{MRL}} \sim \left(-(1-u_1)\sqrt{\tilde{u}_2 \tilde{u}_3} \right)^{\frac{\sqrt{\lambda}}{2\pi} e_2}$$

BFKL eigenvalue
$e_2 = -\sqrt{2} + \log(1+\sqrt{2})$

- compare to field theory result:

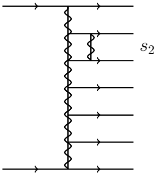
$$e^{R_6 + i\delta_6}|_{\mathrm{MRL}} = \cos \pi\omega_{ab} +$$

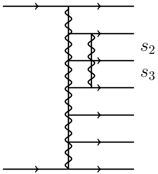
$$i\frac{\lambda}{2} \sum_n (-1)^n \left(\frac{w}{w^*} \right)^{\frac{n}{2}} \int \frac{d\nu}{\nu^2 + \frac{n^2}{4}} |w|^{2i\nu} \Phi_{\mathrm{Reg}}(\nu, n) \left(-(1-u_1)\sqrt{\tilde{u}_2 \tilde{u}_3} \right)^{-\omega(\nu, n)}$$

The 9-point remainder function

- analytic continuations include numerical computations that get more and more difficult for higher n
- new approach to the n-point remainder function
  $\rightarrow$ study spectrum of BA equations without carrying out continuation
- get essential information – BFKL eigenvalues – from BA

2-Reggeon bound states:





$\ldots$

$R_{9,--+++}(u_{a,s}) = R_{6,--}(u_{1,1}, u_{2,1}, u_{3,1})$

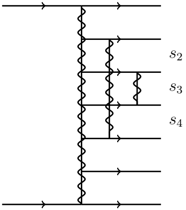
$R_{9,---++}(u_{a,s}) = R_{9,--+++}(u_{a,s}) + R_{9,+--++}(u_{a,s})$

all BFKL eigenvalues e_2

The 3-Reggeon bound state

- for $n \geq 8$ a 3-Reggeon bound state contributes, $e_3 = 2 \cdot e_2$?
- large n: multiple-Reggeon bound states with BFKL eigenvalues $k \cdot e_2$? main question
- require multiple analytic continuations $\rightarrow$ new method very useful

Regge-cut with e_3



remainder function:

$$e^{R_{9,\text{new}}}\Big|_{\text{MRL}} \sim$$

$$((1-u_{1,1})(1-u_{1,3}))^{\frac{\sqrt{\lambda}}{2\pi}e_2} \cdot (1-u_{1,2})^{\frac{\sqrt{\lambda}}{2\pi}2 \cdot e_2}$$

- found only small number of configurations are admissible, all of which have BFKL eigenvalue $2 \cdot e_2$

Conclusions

- remainder function in the MRL described by Reggeon bound states
- strong coupling: TBA equations $\rightarrow$ Bethe ansatz
- new approach to n-point remainder function without use of numerics
- all essential information about the BFKL eigenvalues is stored in BA
- results strongly support integrability proposal that all BFKL eigenvalues are $k \cdot e_2$

Outlook:

- all studied configurations for 9-point amplitude give $2 \cdot e_2$, can we formally proof that this is always the case?
- check our proposal by carrying out the analytic continuation for the 8-point amplitude