Non-perturbative effects in Twisted Holography

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Simons Collaboration on Celestial Holography

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PERIMETER INSTITUTE

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In this talk:

$$\boxed{\text{2d chiral algebra} \iff \text{B-model on } SL(2,\mathbb{C})}$$

Twisting Supersymmetric QFTs

Twisting SQFT is the procedure of passing to a cohomology of a supercharge:

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\begin{aligned}
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  - **topological** twist [Witten, . . .]
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- Extra math structure [Gwilliam, Saberi, Williams, . . .]
  - ∞-dim symmetry algebras
  - λ-brackets

Chiral algebra subsector

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  - Take cohomology wrt
    $$\boldsymbol{Q} \equiv Q^1_- + \tilde{S}^{2\dot{-}}$$
- The chiral algebra of $\mathcal{N} = 4$ SYM is a $\boldsymbol{u(N)}$ **gauged $\boldsymbol{\beta\gamma}$ system**:

$$X^a_b(z) Y^c_d(0) \sim \delta^a_d \delta^c_b \frac{1}{N} \frac{1}{z}$$

$$Q_{\text{BRST}} \sim N \oint \mathrm{Tr}\left(c[X, Y] + \frac{1}{2} b[c, c] \right)$$

Twisted Holography

Protected subsector of $\text{AdS}_5/\text{CFT}_4$: [Costello, Gaiotto '18]

$\mathcal{N}=4$ SYM with $U(N)$ $\approx$ type IIB on $\text{AdS}_5 \times S^5$

"$Q+S$" twist [Beem et al. '13] ↓

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- SFT of B-model is the **Kodaira-Spencer/BCOV theory**
- D1-branes are **holomorphic curves** in $SL(2, \mathbb{C})$

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B-model on $\mathbb{C}^3 + N$ D1-branes $\longrightarrow$ B-model on $SL(2, \mathbb{C}) \approx$ AdS$_3 \times S^3$

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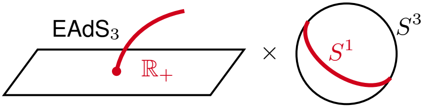
- Non-conformal vacua $\longleftrightarrow$ "Multicenter" asymptotically $SL(2, \mathbb{C})$ geometries

Giant Gravitons

Determinant operator in the chiral algebra

$$\det(m + Z(u;z)), \qquad Z(u;z) = X(z) + uY(z), \qquad m \in \mathbb{C}$$

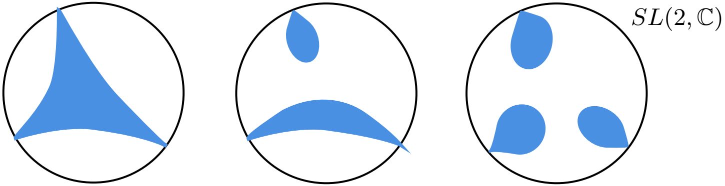
is dual to a D1-brane wrapping $\mathbb{C}^* \cong \mathbb{R}_+ \times S^1$ in $SL(2,\mathbb{C}) \cong \text{EAdS}_3 \times S^3$



- z = position at the boundary of AdS$_3$
- u controls orientation of $S^1 \subset S^3$
- m controls size of $S^1 \subset S^3$

Giant Gravitons

Many possible brane configurations with the same boundary behaviour



We will match saddles ρ^* of correlation functions of determinants with brane configurations

- m_i, u_i, z_i control boundary behaviour
- Saddles ρ will control the shape in the bulk

Determinant correlation functions

[Jiang, Komatsu, Vescovi '19]

- Fermionize determinants

$$\det(m + Z(u;z)) = \int \mathrm{d}\bar{\psi}\mathrm{d}\psi \; e^{\bar{\psi}(m+Z(u,z))\psi}, \qquad \bar{\psi}_I, \psi^I, \quad I = 1, \ldots, N$$

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- Rewrite correlators using **auxiliary bosonic variables** ρ^i_j for $i \neq j$, $\rho^i_i \equiv m_i$

$$\Big\langle \prod_i^k \det(m_i + Z(u_i; z_i)) \Big\rangle \sim \int \mathrm{d}\rho \; e^{N\, S[\rho]}$$

with action

$$S[\rho] = \frac{1}{2} \sum_{i \neq j} \frac{z_i - z_j}{u_i - u_j} \rho^i_j \rho^j_i + \log\det\rho$$

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- **Saddle point equations** in the matrix form:

$$[\zeta, \rho] + [\mu, \rho^{-1}] = 0$$

  where

$$\zeta = \mathrm{diag}(z_i), \quad \mu = \mathrm{diag}(u_i), \quad \rho^i_i = m_i$$

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$$D(a) = a\zeta\mu + \rho^{-1}\mu - \zeta\rho$$

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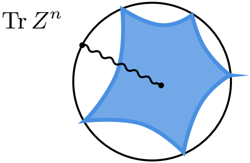
- The matrices are defined so that:
  - They commute when ρ satisfies the saddle point equations
  - They satisfy

$$aD(a) - B(a)C(a) = 1$$

  - S_ρ has expected boundary behavior

Holographic checks

- Correlation functions of determinants with a **single trace** operator
  [Jiang, Komatsu, Vescovi '19]



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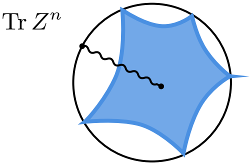

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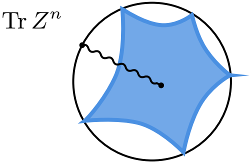

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- BRST-closed modifications of determinants $\longleftrightarrow$ excitations of the brane

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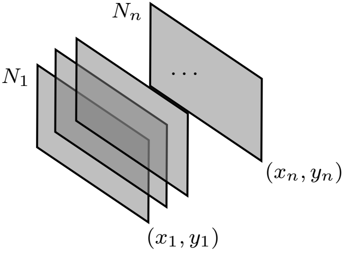
Coulomb branch of $\mathcal{N}=4$ SYM $\longleftrightarrow$ multi-center solutions

- Backreact stack of **non-coincident** branes
- Dual Calabi-Yau geometries are **deformations of $SL(2,\mathbb{C})$**

$$z_I - z_{I'} = +\frac{N_i/N}{(x-x_i)(y-y_i)}$$

For standard $SL(2,\mathbb{C})$ geometry:

$$z_0 - z_\infty = \frac{1}{xy}$$



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  [KB, Gaiotto, Kulp, Williams, Wu, Yu '23]

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- Holomorphic twist of 4d $\mathcal{N} = 4$ SYM $\leftrightarrow$ non-commutative BCOV on $\mathbb{C}^5$
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