The double copy in twistor space

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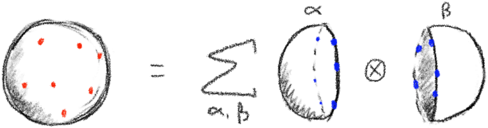
Based on an upcoming paper with T. Adamo

What is the double copy?

$$\text{gravity} = \text{gauge} \otimes \text{gauge}$$

- The original double copy relation was discovered by Kawai-Lewellen-Tye (KLT) in 1986, relating closed and open string amplitudes

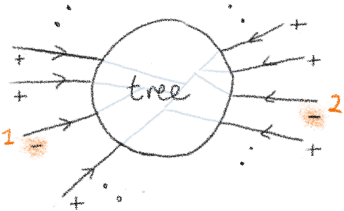


- Taking the field theory limit this amounts

$$\mathcal{M}^{\text{GR}}_{\text{tree}}(1,\ldots,n) = \sum_{\alpha,\beta} \mathcal{A}^{\text{YM}}_{\text{tree}}(\alpha) \underbrace{S[\alpha|\beta]}_{\otimes} \mathcal{A}^{\text{YM}}_{\text{tree}}(\beta)$$

- Exists in many guises: colour-kinematics duality (BCJ), CHY, classical (Kerr-Schild), etc.

Tree-level amplitudes: maximally helicity violating (MHV)



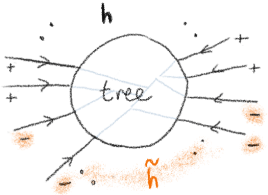
- Surprisingly beautiful structures in tree-level amplitudes were first seen with the Parke-Taylor (PT) ['86] formula, at *all multiplicity*

$$\mathcal{A}^{\rm YM}_{\rm tree}\underbrace{(1^-2^-3^+\dots n^+)}_{\rm MHV} = \delta^4(\cdots)\frac{\langle 12\rangle^4}{\langle 12\rangle\langle 23\rangle\cdots\langle n1\rangle}, \quad p_i^{\alpha\dot\alpha} = |i\rangle^{\dot\alpha}[i|^{\alpha}$$

- In gravity, there is a corresponding expression (Hodges formula) ['12]

$$\mathcal{M}^{\rm GR}_{\rm tree}(1^-2^-3^+\dots n^+) = \delta^4(\cdots)\langle 12\rangle^8 {\rm det}'({\rm H})$$

Tree-level amplitudes: N^{d-1}MHV from twistor space



- At N^{d-1}MHV level (with $d+1$ negative helicity particles), we have [Witten:'04; Roiban-Spradlin-Volovich:'04]

$$\mathcal{A}^{\mathrm{YM}}_{n,d}[\rho] = \int \mathrm{d}\mu_d \, |\tilde{\mathbf{g}}|^4 \, \mathrm{PT}_n[\rho] \prod_{i \in \mathbf{g}} a_i \prod_{j \in \tilde{\mathbf{g}}} \bar{a}_j$$

where $\mathrm{PT}_n[\rho] = \frac{1}{(\rho(1)\rho(2))\ldots(\rho(n)\rho(1))}$ and $\mathcal{A}_{n,d} = \sum_\rho \mathrm{Tr}[T^\rho] \mathcal{A}_{n,d}[\rho]$

- And for gravity the CS formula [Cachazo-Skinner:'12]

$$\mathcal{M}^{\mathrm{GR}}_{n,d} = \int \mathrm{d}\mu_d \, |\tilde{\mathbf{h}}|^8 \, \det{}'(\mathbb{H}) \, \det{}'(\mathbb{H}^\vee) \prod_{i \in \mathbf{h}} h_i \prod_{j \in \tilde{\mathbf{h}}} \bar{h}_j$$

$$\mathsf{MHV} \to \mathsf{N}^{d-1}\mathsf{MHV}$$

$$\frac{\langle 12\rangle^4}{\langle 12\rangle\langle 23\rangle\cdots\langle n1\rangle} \quad \to \quad \frac{|\tilde{\mathbf{g}}|^4}{(12)(23)\cdots(n1)}$$

$$\langle 12\rangle^8 \mathrm{det}'(\mathrm{H}) \quad \to \quad |\tilde{\mathbf{h}}|^8\, \mathrm{det}'(\mathbb{H})\, \mathrm{det}'(\mathbb{H}^\vee)$$

and then integrate these over

$$\mathrm{d}\mu_d \prod_{\substack{i\in \mathrm{pos.helicity} \\ \mathbf{g}/\mathbf{h}}} \xi_i(Z) \prod_{\substack{j\in \mathrm{neg.helicity} \\ \tilde{\mathbf{g}}/\tilde{\mathbf{h}}}} \bar{\xi}_i(Z)$$

Tree-level amplitudes: CHY [Cachazo, He, Yuan: '13]

- This formulation rests on evaluating integrands on the support of solutions $\sigma_1, \ldots, \sigma_n \in \mathbb{CP}^1$ to the scattering equations for

$$P(\sigma) = \sum_{i=1}^{n} \frac{k_i}{\sigma - \sigma_i}, \qquad k_i \cdot P(\sigma_i) = \sum_{j \neq i} \frac{k_i \cdot k_j}{\sigma_i - \sigma_j} = 0$$

- A generic scattering amplitude is

$$\mathcal{A}_n^{\text{tree}} = \sum_{\substack{\text{solutions} \\ k_i \cdot P(\sigma_i) = 0}} \frac{1}{J(\sigma_i, k_i)} \; \mathcal{I}_n(\sigma_i, k_i, \epsilon_i)$$

The CHY double copy

- The theory is captured in the form of the integrand

$$\text{Gravity:} \quad \mathcal{I}_n^{\mathrm{GR}} = \mathrm{Pf}'(\Phi)\,\mathrm{Pf}'(\tilde{\Phi}),$$

$$\text{Yang-Mills:} \quad \mathcal{I}_n^{\mathrm{YM}}[\alpha] = \mathrm{PT}_n(\alpha)\,\mathrm{Pf}'(\Phi),$$

$$\Big(\text{Bi-adjoint scalar:} \quad \mathcal{I}_n^{\mathrm{BAS}}[\alpha|\beta] = \mathrm{PT}_n(\alpha)\,\mathrm{PT}_n(\beta)\Big)$$

- Equivalently

$$\mathcal{I}_n^{\mathrm{GR}} = \mathcal{I}_n^{\mathrm{YM}}[\alpha]\,\underbrace{\frac{1}{\mathcal{I}_n^{\mathrm{BAS}}[\alpha|\beta]}}_{\otimes}\,\tilde{\mathcal{I}}_n^{\mathrm{YM}}[\beta]$$

CHY $\xleftrightarrow{\text{helicity grading}}$ $\mathbb{PT}$ formulae

CHY $\xrightarrow{\text{integrands}}$

$$\mathcal{I}_n^{\mathrm{GR}} = \mathcal{I}_n^{\mathrm{YM}} \otimes \mathcal{I}_n^{\mathrm{YM}}$$

Double copy manifested

$\mathbb{PT}$ formulae $\xrightarrow{\text{integrands?}}$

$$\det{}'(\mathbb{H})\det{}'(\mathbb{H}^{\vee}) \stackrel{?}{=} \mathrm{PT}[\alpha] \otimes \mathrm{PT}[\beta]$$

????

$\xrightarrow{\text{??}}$

New amplitudes relations?

Double copy on curved spacetimes?

CHY $\longleftrightarrow$ (helicity grading) $\mathbb{PT}$ formulae

integrands $\downarrow$

$$\mathcal{I}_n^{\mathrm{GR}} = \mathcal{I}_n^{\mathrm{YM}} \otimes \mathcal{I}_n^{\mathrm{YM}}$$

Double copy manifested

$\downarrow$ integrands?

$$\det{}'(\mathbb{H})\det{}'(\mathbb{H}^\vee) \stackrel{?}{=} \mathrm{PT}[\alpha] \otimes \mathrm{PT}[\beta]$$

????

$\downarrow$??

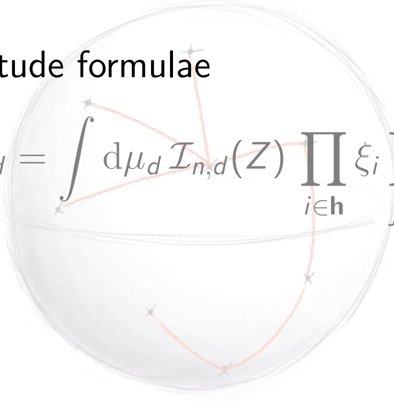
New amplitudes relations?

Double copy on curved spacetimes?

This talk

- Tree-level amplitude formulae
- Applications of graph theory
- Double copy in twistor space
- Conclusion and outlook

Tree-level amplitude formulae



$$\mathcal{M}_{n,d} = \int \mathrm{d}\mu_d \, \mathcal{I}_{n,d}(Z) \prod_{i \in \mathbf{h}} \xi_i \prod_{j \in \tilde{\mathbf{h}}} \bar{\xi}_j$$

Map moduli integrals

$$\mathcal{M}_{n,d} = \int \mathrm{d}\mu_d \, \mathcal{I}_{n,d}(Z) \prod_{i \in \mathbf{h}} \xi_i \prod_{j \in \tilde{\mathbf{h}}} \bar{\xi}_j$$

- For N^{d-1}MHV we consider maps of degree d

$$\mathcal{Z} : \mathbb{CP}^1 \to \mathbb{PT}, \qquad \mathbb{PT} \overset{\text{open}}{\subset} \mathbb{CP}^3$$

with coordinates on $\mathbb{CP}^1$ given by $\sigma = (\sigma^0, \sigma^1) \sim r(\sigma^0, \sigma^1)$, and

$$\mathcal{Z}(r\sigma) = r^d \mathcal{Z}(\sigma), \quad \mathcal{Z}((u,1)) = U_d u^d + U_{d-1} u^{d-1} + \ldots + U_0$$

- Each map has $4(d+1)$ degrees of freedom up to proj. scalings
- The integration measure of the moduli space of these maps and n marked points on $\mathbb{CP}^1$

$$\mathrm{d}\mu_d := \frac{\mathrm{d}^{4(d+1)} U}{\mathrm{vol}\, \mathbb{C}^* \times \mathrm{SL}(2, \mathbb{C})} \prod_{i=1}^{n} (\sigma_i \, \mathrm{d}\sigma_i)$$

External states - twistor representatives

$$\mathcal{M}_{n,d} = \int \mathrm{d}\mu_d \, \mathcal{I}_{n,d}(Z) \prod_{i \in \mathbf{h}} \xi_i \prod_{j \in \tilde{\mathbf{h}}} \bar{\xi}_j$$

- The Penrose transform: equates solutions of the zero-rest-mass equations on spacetime to cohomology classes on twistor space

$$\phi^h(Z) \in H^{0,1}(\mathbb{PT}, \mathcal{O}(2h-2)), \qquad h = \text{helicity}$$

- The twistor wavefunctions for momentum eigenstates take the form

$$\phi_i^h(Z) = \int_{\mathbb{C}^*} \frac{\mathrm{d}t_i}{t_i^{2h-1}} \, \bar{\delta}^2(\kappa_i - t_i \, \lambda) \, e^{i t_i [\mu \, \tilde{\kappa}_i]}$$

where $Z = (\mu^{\dot{\alpha}}, \lambda_\alpha)$ and $k_i^{\alpha\dot{\alpha}} = \tilde{\kappa}_i^{\alpha} \kappa_i^{\dot{\alpha}}$

- For our theories of interest

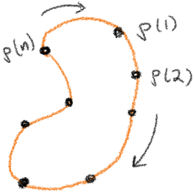
$$\text{GR:} \quad \frac{1}{t_i^3}, \; t^5; \qquad \text{YM:} \quad \frac{1}{t_i}, \; t_i^3; \qquad \text{BAS:} \quad t_i$$

Integrands in twistor space

$$\mathcal{M}_{n,d} = \int \mathrm{d}\mu_d \, \mathcal{I}_{n,d}(Z) \prod_{i \in \mathbf{h}} \xi_i \prod_{j \in \tilde{\mathbf{h}}} \bar{\xi}_j$$

Vandermonde: $$|\mathbf{A}| = \prod_{\{i,j\} \in \mathbf{A}} (i\,j), \qquad (i\,j) = \epsilon_{ab} \sigma_i^a \sigma_j^b$$

Yang-Mills: $$\frac{|\tilde{\mathbf{g}}|^4}{(\rho(1)\rho(2)) \cdots (\rho(n-1)\rho(n))(\rho(n)\rho(1))}$$



Gravity: $$|\tilde{\mathbf{h}}|^8 \, \det{}'(\mathbb{H}) \, \det{}'(\mathbb{H}^\vee)$$

Integrands in twistor space

$$\text{Gravity:} \quad |\tilde{\mathbf{h}}|^8 \, \det{}'(\mathbb{H}) \, \det{}'(\mathbb{H}^\vee)$$

- Hodges matrix has entries for $i, j \in \mathbf{h}$ ($+$ hel.)

$$\mathbb{H}_{ij} = \frac{[\frac{\partial}{\partial \mu(\sigma_i)} \, \frac{\partial}{\partial \mu(\sigma_j)}]}{(ij)}, \qquad \mathbb{H}_{ii} = -\sum_{\substack{j \in \mathbf{h} \\ j \neq i}} \mathbb{H}_{ij} \prod_{l \in \tilde{\mathbf{h}}} \frac{(jl)}{(il)}$$

- Dual Hodges matrix has entries for $i, j \in \tilde{\mathbf{h}}$ (- hel.)

$$\mathbb{H}^\vee_{ij} = \frac{\langle \lambda(\sigma_i) \, \lambda(\sigma_j) \rangle}{(ij)}, \qquad \mathbb{H}^\vee_{ii} = -\sum_{\substack{j \in \tilde{\mathbf{h}} \\ j \neq i}} \mathbb{H}^\vee_{ij} \prod_{k \in \tilde{\mathbf{h}} \setminus \{i,j\}} \frac{(ki)}{(kj)}$$

- The reduced determinants are

$$\det{}'(\mathbb{H}) = \frac{|\mathbb{H}^b_b|}{|\tilde{\mathbf{h}} \cup \{b\}|^2}, \qquad \det{}'(\mathbb{H}^\vee) = \frac{|\mathbb{H}^{\vee\, a}_a|}{|\tilde{\mathbf{h}} \setminus \{a\}|^2}$$

Example: MHV $n = 4$

Here $\tilde{\mathbf{h}} = \{3, 4\}$ and $\mathbf{h} = \{1, 2\}$

$$\mathbb{H} = \begin{pmatrix} -\frac{t_1 t_2 [12]}{(12)} \frac{(23)(24)}{(13)(14)} & \frac{t_1 t_2 [12]}{(12)} \\ \frac{t_1 t_2 [12]}{(12)} & -\frac{t_1 t_2 [12]}{(12)} \frac{(13)(14)}{(23)(24)} \end{pmatrix}$$

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Choosing $b = 2$ in $|\mathbb{H}^b_b|$

$$\Rightarrow \det{}'(\mathbb{H}) = -\frac{t_1 t_2 [12]}{(12)} \frac{(23)(24)}{(13)(14)} / |\{\mathbf{2}, \mathbf{3}, \mathbf{4}\}|^2 = \frac{-t_1 t_2 [12]}{(12)(13)(14)(23)(24)(34)^2}$$

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And for dual Hodges

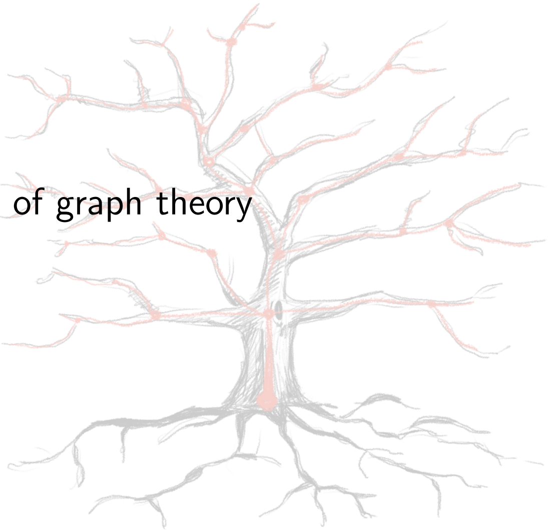
$$\mathbb{H}^\vee = \begin{pmatrix} -\frac{\langle \lambda_3\, \lambda_4 \rangle}{(34)} & \frac{\langle \lambda_3\, \lambda_4 \rangle}{(34)} \\ \frac{\langle \lambda_3\, \lambda_4 \rangle}{(34)} & -\frac{\langle \lambda_3\, \lambda_4 \rangle}{(34)} \end{pmatrix} \quad \Rightarrow \quad \det{}'(\mathbb{H}^\vee) = -\frac{\langle \lambda_3\, \lambda_4 \rangle}{(34)}$$

Integrands:

Yang-Mills: $$\frac{|\tilde{\mathbf{g}}|^4}{(\rho(1)\rho(2))\cdots(\rho(n-1)\rho(n))(\rho(n)\rho(1))}$$

Gravity: $$|\tilde{\mathbf{h}}|^8 \det{}'(\mathbb{H}) \det{}'(\mathbb{H}^\vee)$$

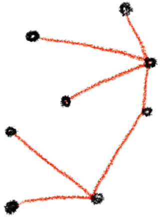
Applications of graph theory



Tree graphs

Def: A tree graph is a set of edges E over vertices V, that is connected and has no loops.

- It's possible to associate a weight w_{ij} with each possible edge $(i - j)$

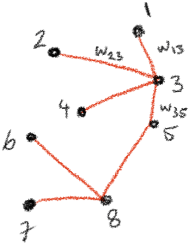


Weighted Matrix-Tree Theorem

$$\sum_{\substack{\text{tree graphs}\\ \text{on}\, V}} \left(\prod_{(i-j)} w_{ij} \right) = |W(V)_a^a|$$

where the weighted Laplacian matrix is

$$W(V)_{ij} = \begin{cases} \sum_{(k-i)} w_{ik} & \text{if } i = j \\ -w_{ij} & \text{if } i \neq j \end{cases}$$



Can rewrite the Hodges' reduced determinants in this language! For the positive helicity piece:

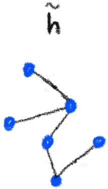
$$\det{}'(\mathbb{H}) = \frac{1}{|\tilde{\mathbf{h}}|^2} \prod_{\substack{k\in\mathbf{h}\\ l\in\tilde{\mathbf{h}}}} \frac{1}{(kl)^2} \times \underbrace{\sum_{\substack{T\\ \text{spanning } \mathbf{h}}} \prod_{(i-j)} B_{ij}}_{|B^a_a|}$$

where

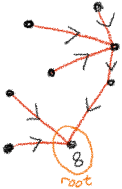
$$B_{ij} = \frac{\left[\frac{\partial}{\partial\mu(\sigma_i)}\,\frac{\partial}{\partial\mu(\sigma_j)}\right]}{(ij)} \prod_{l\in\tilde{h}} (il)(jl)$$

and similarly for the negative helicity piece

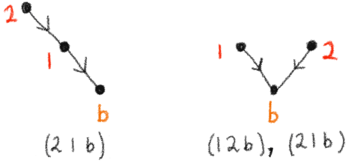
$$\det{}'(\mathbb{H}^\vee) = |\tilde{\mathbf{h}}|^2 \times \sum_{\substack{\tilde{T}\\ \text{spanning } \tilde{\mathbf{h}}}} \prod_{(i-j)} B^\vee_{ij}$$



Orderings and tree graphs



- It's possible to direct a tree graph by giving it a root. The edges now obtain a direction $(i \to j)$
- Taking b as the root, we can associate a set of orderings to each tree graph



Proposition (Frost '21)

For a directed tree T on vertices V and generic $x \in \mathbb{CP}^1$

$$\prod_{(i\to j)\in T} \frac{(jx)}{(ij)(ix)} = \sum_{\substack{\text{compatible} \\ \text{ords. } \rho^T b \text{ of } T}} \mathrm{PT}(b\rho x)(bx)^2$$

$$\det{}'(\mathbb{H}) = \frac{1}{|\tilde{\mathbf{h}}|^2} \prod_{l\in\tilde{\mathbf{h}}} \frac{1}{(bl)^2} \times \sum_{\substack{T_b \\ \text{spanning } \mathbf{h}}} \prod_{(i\to j)} B_{ij}$$

For each specific tree T_b

$$\prod_{(i\to j)} B_{ij} \equiv \prod_{(i\to j)} \frac{[\partial_\mu(\sigma_i)\,\partial_\mu(\sigma_j)]}{(ij)} \prod_{l\in\tilde{\mathbf{h}}} \frac{(jl)}{(il)}$$

$$= \sum_{b\rho} \mathrm{PT}(b\rho x)(bx)^2 \prod_{(i\to j)} [\partial_\mu(\sigma_i)\,\partial_\mu(\sigma_j)] \prod_{l\in\tilde{\mathbf{h}}\setminus\{x\}} \frac{(jl)}{(il)}$$

$$= \sum_{b\rho, b\sigma} \mathrm{PT}(b\rho x)\mathrm{PT}(b\sigma y)(bx)^2(by)^2 \prod_{(i\to j)}[\partial_\mu(\sigma_i)\,\partial_\mu(\sigma_j)](ij) \prod_{l\in\tilde{\mathbf{h}}\setminus\{x,y\}} \frac{(jl)}{(il)}$$

$$\det{}'(\mathbb{H}) = \frac{1}{|\tilde{\mathbf{h}}|^2} \prod_{l\in\tilde{\mathbf{h}}} \frac{1}{(bl)^2} \times \sum_{\substack{T_b \\ \text{spanning } \mathbf{h}}} \prod_{(i\to j)} B_{ij}$$

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Weighted tree $\to$ (Parke-Taylor)^2 $\times$ something

Weighted tree $\rightarrow$ (Parke-Taylor)^2 $\times$ something

Doing the sum over the weighted trees

$$\sum_{\substack{T_b \\ \text{spanning } \mathbf{h}}} \prod_{(i \to j)} \tilde{B}_{ij} = \sum_{\substack{T_b \\ \text{spanning } \mathbf{h}}} \sum_{\substack{b\rho, b\sigma \\ \text{comp. } T_b}} \mathrm{PT}(b\rho x)\mathrm{PT}(b\sigma y) \times$$

$$= \sum_{b\rho, b\sigma} \mathrm{PT}(b\rho x)\mathrm{PT}(b\sigma y) \sum_{\substack{\text{trees compatible} \\ \text{with } b\rho, b\sigma}}$$

So we've found that

$$\det{}'(\mathbb{H}) = \sum_{b\rho, b\sigma} \mathrm{PT}(b\rho x)\mathrm{PT}(b\sigma y) \times \text{something}[\rho|\sigma]$$

Repeat the same for $\det{}'(\mathbb{H}^\vee)$

KLT kernel in twistor space [Adamo, SK; '24]

- Combining the contributions from trees over $\mathbf{h}$ and $\tilde{\mathbf{h}}$ (gluing together the PT factors) the gravity integrand can be rewritten as

$$\sum_{\substack{a\tilde{\rho}b\rho \\ \tilde{\omega}^T ab\omega}} |\tilde{\mathbf{h}}|^8 \mathrm{PT}[a\tilde{\rho}b\rho] \underbrace{S_{n,d}[\rho,\tilde{\rho}|\omega,\tilde{\omega}]}_{\substack{\text{KLT kernel} \\ \text{in twistor space}}} \mathrm{PT}[\tilde{\omega}^T ab\omega]$$

where

$$S_{n,d}[\rho,\tilde{\rho}|\omega,\tilde{\omega}] = \mathcal{D}[\omega,\tilde{\omega}] \left[\sum_{\tilde{T}\in\mathcal{T}^a_{\tilde{\rho},\tilde{\omega}}} \prod_{(i\to j)} \tilde{\phi}_{ij} \right] \times \left[\sum_{T\in\mathcal{T}^b_{\rho,\omega}} \prod_{(i\to j)} \phi_{ij} \right]$$

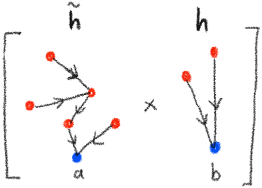
- The weights on each of the sets of trees are

$$\phi_{ij} := [\partial_\mu(\sigma_i)\,\partial_\mu(\sigma_i)](ij) \prod_{l\in\tilde{\mathbf{h}}\setminus\{a,y\}} (il)(jl), \qquad i,j\in\mathbf{h},$$

$$\tilde{\phi}_{ij} := \frac{\langle\lambda(\sigma_i)\,\lambda(\sigma_j)\rangle}{(ij)} \prod_{k\in(\tilde{\mathbf{h}}\cup\{b,t\})\setminus\{i,j\}} \frac{1}{(ki)(kj)} \qquad i,j\in\tilde{\mathbf{h}}$$

$$S_{n,d}[\rho, \tilde{\rho}|\omega, \tilde{\omega}] = \mathcal{D}[\omega, \tilde{\omega}] \left[\sum_{\tilde{T} \in \mathcal{T}^a_{\tilde{\rho},\tilde{\omega}}} \prod_{(i \to j)} \tilde{\phi}_{ij} \right] \times \left[\sum_{T \in \mathcal{T}^b_{\rho,\omega}} \prod_{(i \to j)} \phi_{ij} \right]$$

$$= \mathcal{D}(\omega, \tilde{\omega}) \sum_{\tilde{T}, T}$$



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$$\mathcal{M}_{n,d}^{\color{red}\text{GR}} = \int \mathrm{d}\mu_d \sum_{\substack{a\tilde{\rho}b\rho \\ \tilde{\omega}^T ab\omega}} |\tilde{\mathbf{h}}|^8 \color{blue}\mathrm{PT}[a\tilde{\rho}b\rho]\color{black} \underbrace{S_{n,d}[\rho,\tilde{\rho}|\omega,\tilde{\omega}]}_{\substack{\text{KLT kernel} \\ \text{in twistor space}}} \color{blue}\mathrm{PT}[\tilde{\omega}^T ab\omega]\color{black} \prod_i h_i^{\pm}(Z)$$

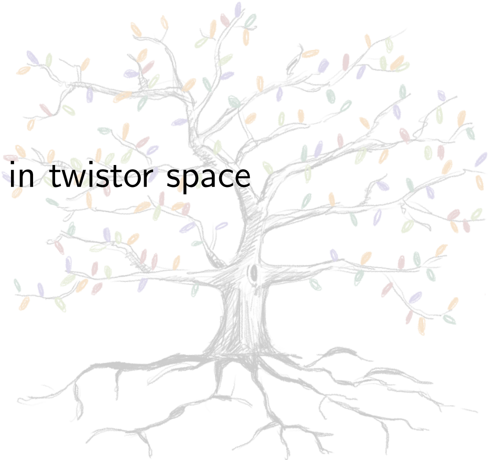
$\mathbb{PT}$ formulae

$$\downarrow \text{integrands!}$$

$$\color{red}\det{}'(\mathbb{H})\det{}'(\mathbb{H}^{\vee})\color{black} = \sum_{\alpha,\beta} \color{blue}\mathrm{PT}[\alpha]\color{black} \underbrace{\otimes}_{\text{kernel}} \color{blue}\mathrm{PT}[\beta]$$

Helicity graded double copy kernel!

Double copy in twistor space



Interpretation of KLT kernel

$$\mathcal{M}_{n,d}^{\text{GR}} = \int \mathrm{d}\mu_d \sum_{\substack{a\tilde{\rho}b\rho \\ \tilde{\omega}^T ab\omega}} |\tilde{\mathbf{h}}|^8 \mathrm{PT}[a\tilde{\rho}b\rho] \underbrace{S_{n,d}[\rho,\tilde{\rho}|\omega,\tilde{\omega}]}_{\substack{\text{KLT kernel} \\ \text{in twistor space}}} \mathrm{PT}[\tilde{\omega}^T ab\omega] \prod_i h_i^{\pm}(Z)$$

- A matrix on orderings of $\mathbf{h}$ and $\tilde{\mathbf{h}}$: basis has $(n-d-2)! \times (d)!$ elements
- Graded by helicity, where # negative gravitons $= d+1$
- Contrast with CHY kernel: 1 basis element
  Spacetime KLT kernel: $(n-3)!$ basis elements

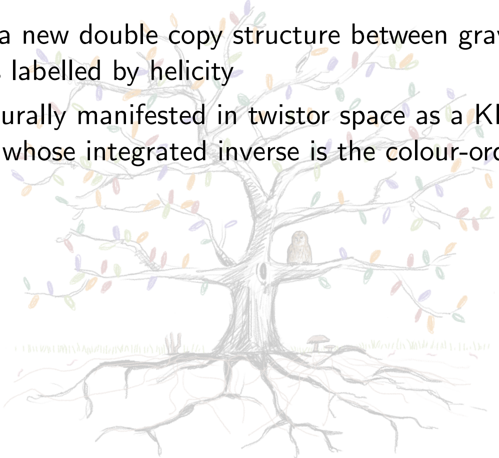
Inverse of the KLT kernel in twistor space

- It has been proven [CHY:'13;Mizera:'16; Mafra:'20;Frost,Mafra,Mason:'21] that the matrix inverse of the usual field theory kernel is equal to the scattering amplitudes of bi-adjoint scalar theory (BAS)
- We prove in [Adamo, SK: '24] (using amplitude recursion relations in twistor space) a new representation of BAS amplitudes in twistor space:

$$m_n(a\tilde{\rho}b\rho|\tilde{\omega}^T ab\omega) = \int \mathrm{d}\mu_d \, S^{-1}_{n,d}[\rho,\tilde{\rho}|\omega,\tilde{\omega}] \prod_i \phi_i(Z)$$

Summary and outlook

- We found a new double copy structure between gravity and YM tree amplitudes labelled by helicity
- This is naturally manifested in twistor space as a KLT kernel integrand, whose integrated inverse is the colour-ordered BAS amplitude



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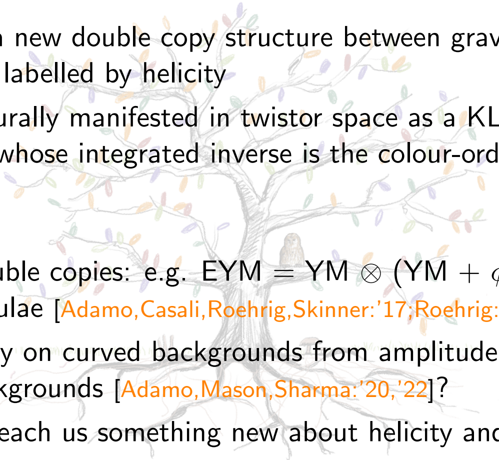

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Thank you!