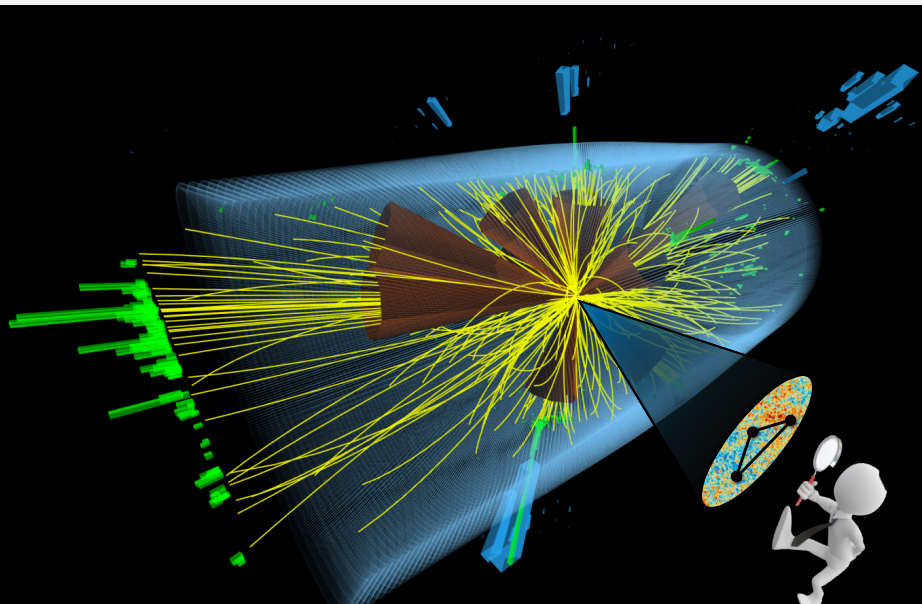


Conformal Colliders Meet the LHC

Ian Moul
Yale

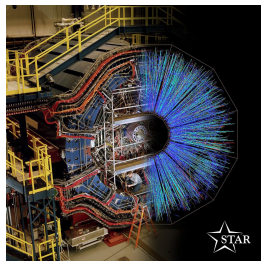
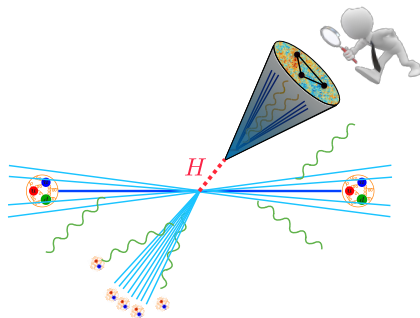


Asymptotic Correlators $\rightarrow$ Jet Substructure!



Jet Substructure

- **Jet Substructure** uses correlations in energy flux to provide **qualitatively new** ways to study physics at the LHC.



- Requires development of theoretical techniques for asymptotic observables characterizing high multiplicity states.

Energy Correlators in Data

- Significant recent progress bridging theory and experiment!



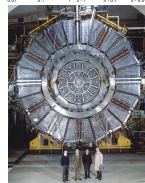
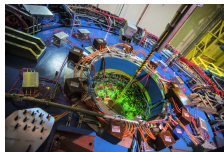
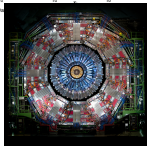
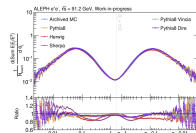
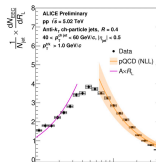
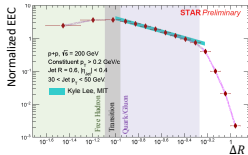
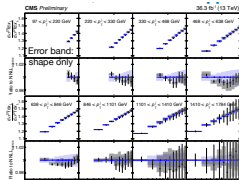
JETS ELUCIDATE HOW PARTONS
EVOLVE INTO HADRONS



STRONG INTERACTIONS | NEWS

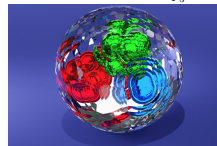
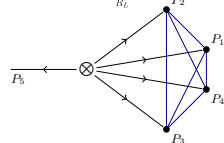
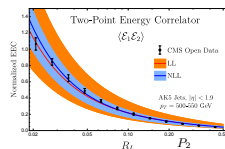
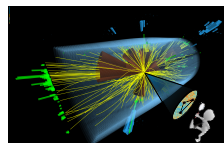
Measuring energy correlators inside jets

3 November 2023

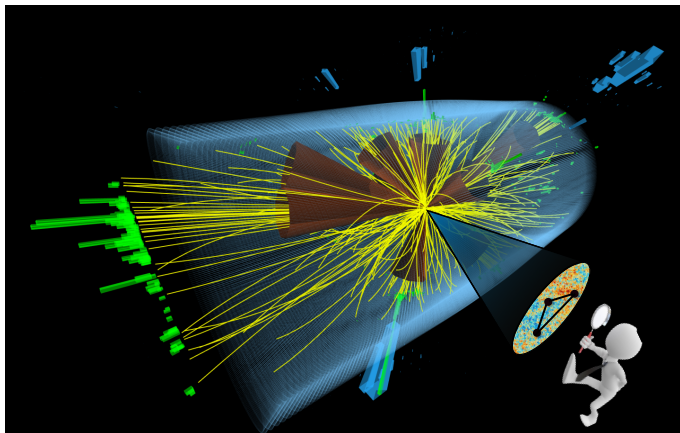


Outline

- Asymptotic Observables at Colliders
- Scaling Behavior of Quarks and Gluons
- A Perturbative Playground for Physical Observables
- Applications

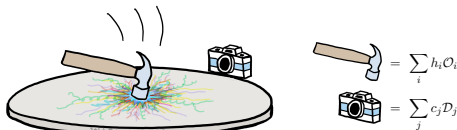


Asymptotic Observables at Colliders

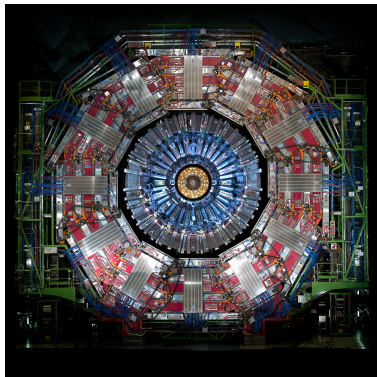


Detectors

- What is a detector?



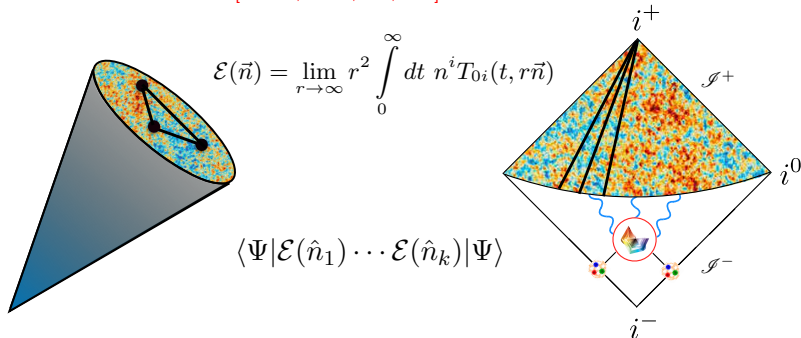
[Caron Huot, Kologlu, Kravchuk, Meltzer, Simmons Duffin]



- To be able to understand subtle signals in energy flux, we must understand what a detector is in Quantum Field Theory.

Calorimeter Cells in Field Theory

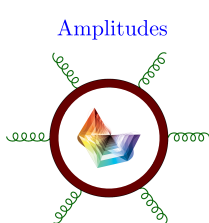
- Calorimeter cells can be given a field theoretic definition in terms of light-ray operators.
 [Hofman, Maldacena], [Belitsky, Hohenegger, Korchemsky, Sokatchev, Zhiboedov]
 [Korchemsky, Sterman]
 [Ore, Sterman]
 [Basham, Brown, Ellis, Love]



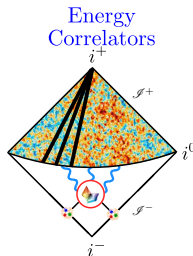
- From the perspective of QFT, jet substructure is the study of correlation functions of light ray operators.

Energy Correlators

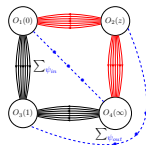
- Correlation functions of energy flow operators take an interesting intermediate position between amplitudes and correlation functions.



Asymptotic
States



Correlation
Functions



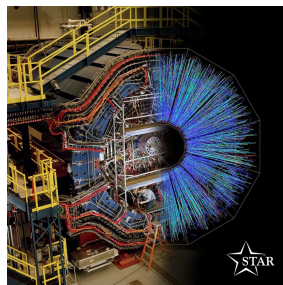
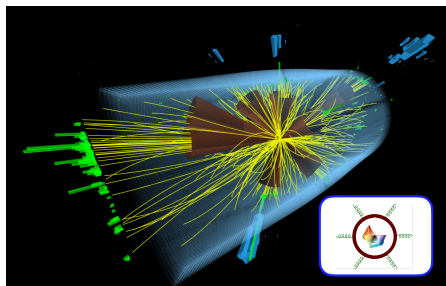
IR Finite



- Despite their physical importance, much less explored.

Towards the World of Hadron Colliders

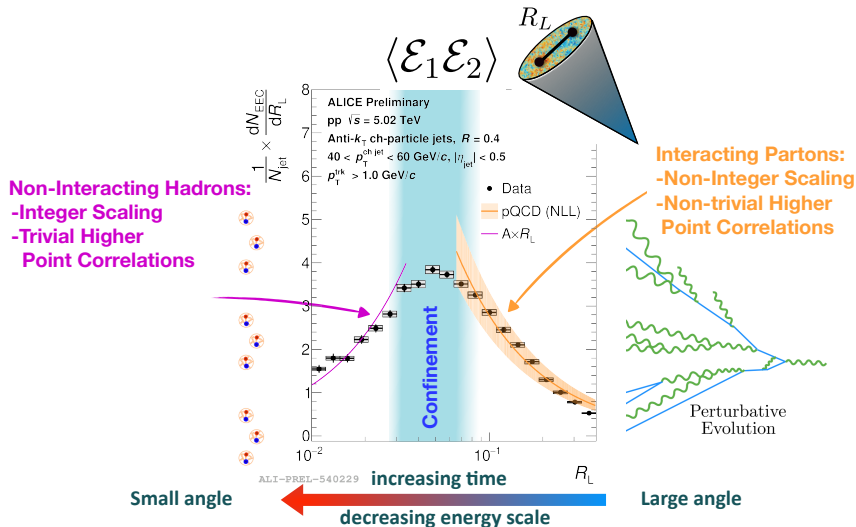
- Can this theoretical idealization possibly work in the messy world of hadron colliders?



- Can it provide new ways of understanding these complex collisions?

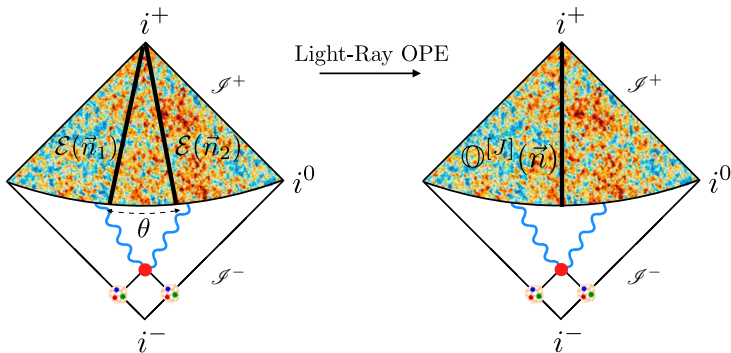
Energy Correlators: Present

Figure: Wenqing Fan



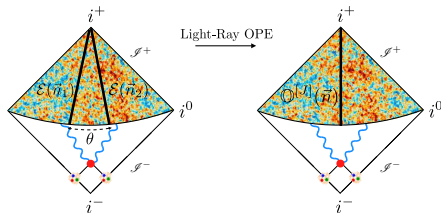
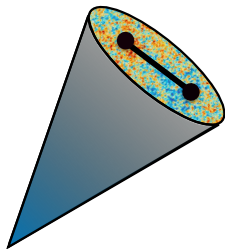
- Asymptotic Freedom and Confinement in one plot!

The Strong Coupling Constant from Scaling



The OPE Limit of Lightray Operators

- Energy flow operators admit a Lorentzian OPE: “the lightray OPE”



$$\mathcal{E}(\hat{n}_1)\mathcal{E}(\hat{n}_2) \sim \sum \theta^{\tau_i-4} \mathcal{O}_i(\hat{n}_1)$$

[Hofman, Maldacena]

[Chang, Kologlu, Kravchuk, Simmons Duffin, Zhiboedov]

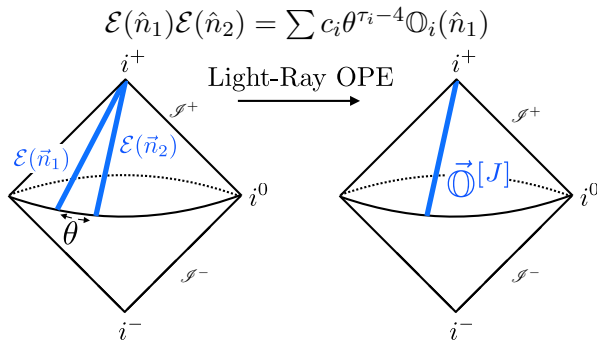
QCD: [Dixon, Moul, Zhu]

See early work by [Konishi, Ukawa, Veneziano]

- Predicts universal scaling behavior in correlations of energy flux at energies $E \gg \Lambda_{\text{QCD}}$.

The Lightray OPE

- In CFTs, the lightray OPE is a convergent, and rigorous expansion.
[Hofman, Maldacena; Chang, Kologlu, Kravchuk, Simmons Duffin, Zhiboedov]



- To describe the leading scaling at the LHC, we can restrict to the leading term in the OPE $\implies$ **twist-2 light ray operators**.
- To understand what we can hope to see at the LHC, must look at the structure of these operators.

The Leading Twist Lightray OPE

[Hofman, Maldacena]
[Chen, IM, Zhu]

- The twist-2 operators in QCD are characterized by a **spin- J** and a **transverse spin $j = 0, 2$** .
- These can be light-transformed to obtain a vector of twist-2 lightray operators parametrized by spin- J :

Local Operators [Kravchuk, Simmons Duffin]

transverse
spin-0

$$\mathcal{O}_q^{[J]} = \frac{1}{2^J} \bar{\psi} \gamma^+ (iD^+)^{J-1} \psi$$

$$\mathcal{O}_g^{[J]} = -\frac{1}{2^J} F_a^{\mu+} (iD^+)^{J-2} F_a^{\mu+}$$

$\xrightarrow{\lim_{r \rightarrow \infty} r^2 \int_0^\infty dt}$

$\vec{\mathcal{O}}^{[J]}(\vec{n}) =$

$\mathcal{O}_q^{[J]}(\vec{n})$
 $\mathcal{O}_g^{[J]}(\vec{n})$

unpolarized

$\mathcal{O}_{g,+}^{[J]}(\vec{n})$
 $\mathcal{O}_{g,-}^{[J]}(\vec{n})$

polarized

transverse
spin-2

$$\mathcal{O}_{g(\lambda)}^{[J]} = -\frac{1}{2^J} F_a^{\mu+} (iD^+)^{J-2} F_a^{\nu+} \epsilon_{\lambda,\mu} \epsilon_{\lambda,\nu}$$

helicity $\pm$

- The anomalous dimensions of these operators,

$$\frac{d}{d \ln \mu^2} \vec{\mathcal{O}}^{[J]}(\hat{n}_1) = \hat{\gamma}(J) \vec{\mathcal{O}}^{[J]}(\hat{n}_1)$$

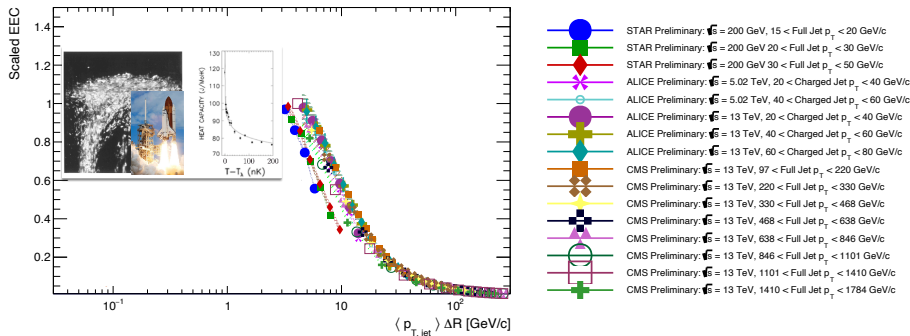
determines the leading behavior of jet substructure.

Scaling Behavior in Jets

Thanks to Helen Caines, Meng Xiao, ChenFeng Lu,

Andrew Tamis, Ananya Rai.

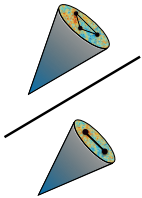
- An experimental realization of the lightray OPE:
The $\mathcal{E}(\hat{n}_1)\mathcal{E}(\hat{n}_2)$ OPE inside high-energy jets!



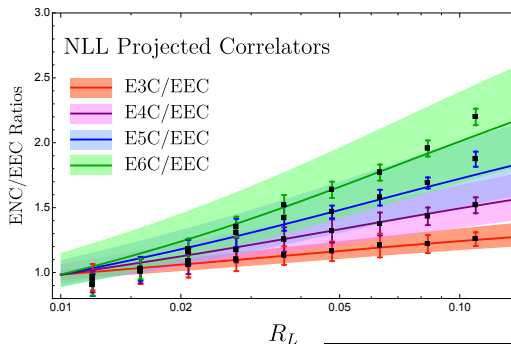
- Universal scaling in a hadron collider environment.
- Dominated by classical scaling. Can we accurately measure anomalous scaling?

The Spectrum of a Jet

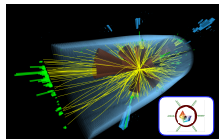
- The light-ray OPE predicts that the N -point correlators develop an **anomalous scaling that depends on N** .



$$\frac{\langle \mathcal{E}_1 \mathcal{E}_2 \cdots \mathcal{E}_{J-1} \rangle}{\langle \mathcal{E}_1 \mathcal{E}_2 \rangle} \sim \frac{\langle \mathcal{O}^{[J]} \rangle}{\langle \mathcal{O}^{[3]} \rangle}$$



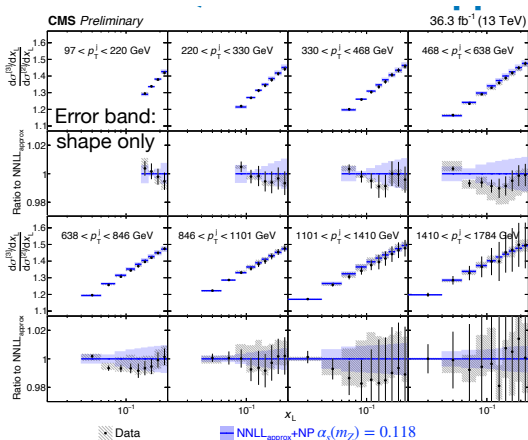
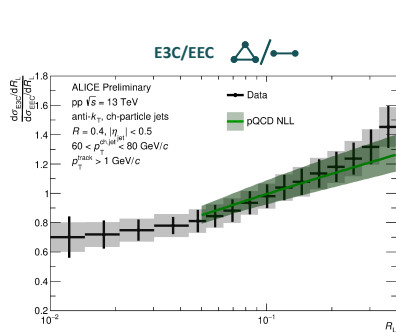
- Directly probes the spectrum of (twist-2) light-ray operators from asymptotic energy flux.



Anomalous Scaling of 3/2 Ratio

- Anomalous scaling measured from 15 GeV to 1784 GeV!

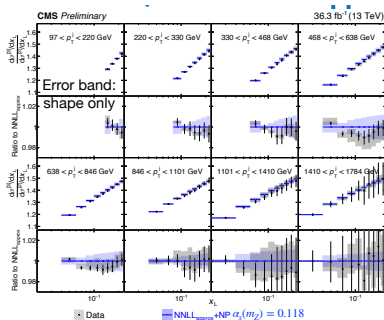
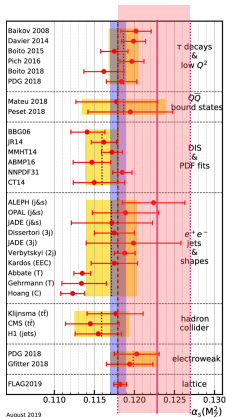
$$\frac{\langle \mathcal{E}_1 \mathcal{E}_2 \mathcal{E}_3 \rangle}{\langle \mathcal{E}_1 \mathcal{E}_2 \rangle} \sim \frac{\langle \mathcal{O}^{[4]} \rangle}{\langle \mathcal{O}^{[3]} \rangle} \sim R_L^{\gamma(4)-\gamma(3)}$$



Using [Chen, Gao, Li, Xu, Zhang, Zhu]

The Strong Coupling

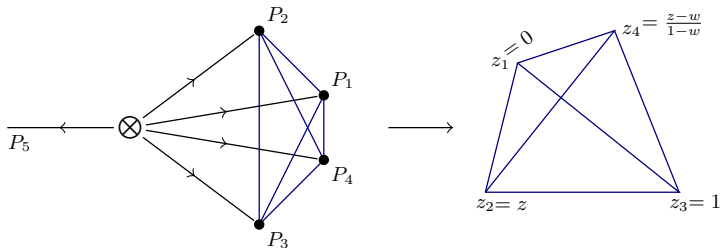
- Proof of principle α_s can be extracted from jet substructure in complicated hadron collider environment: 4% accuracy.
 - Hope to use high energies of the LHC to resolve previous tensions in α_s extractions.
- CMS Preliminary 36.3 fb⁻¹ (13 TeV)



$$\alpha_s(m_Z) = 0.1229^{+0.0040}_{-0.0050}$$

$$= 0.1229^{+0.0014(stat.)+0.0030(theo.)+0.0023(exp.)}_{-0.0012(stat.)-0.0033(theo.)-0.0036(exp.)}$$

A Perturbative Playground for Physical Observables

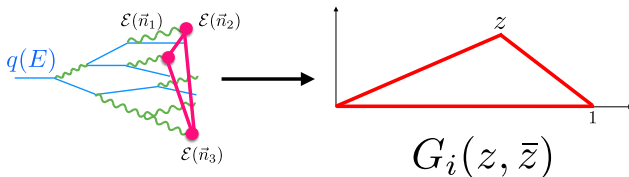


Multipoint Correlators

- The only explicit results for correlators with $N > 2$ are the remarkable strong coupling results of [Hofman and Maldacena](#):

$$\langle \mathcal{E}(\vec{n}_1) \cdots \mathcal{E}(\vec{n}_n) \rangle = \left(\frac{q}{4\pi} \right)^n \left[1 + \sum_{i < j} \frac{6\pi^2}{\lambda} [(\vec{n}_i \cdot \vec{n}_j)^2 - \frac{1}{3}] + \frac{\beta}{\lambda^{3/2}} \left[\sum_{i < j < k} (\vec{n}_i \cdot \vec{n}_j)(\vec{n}_j \cdot \vec{n}_k)(\vec{n}_i \cdot \vec{n}_k) + \cdots \right] + o(\lambda^{-2}) \right]$$

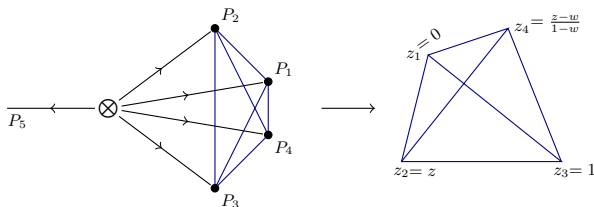
- The wealth of techniques developed to compute perturbative scattering amplitudes can be applied to multi-point correlators at weak coupling.



Multi-point Correlators at Weak Coupling

- Energy correlators define an interesting class of *finite* integrals.
- Provide a playground for the exploration of the perturbative structure of *physical observables*.

[Chicherin, Moult, Sokatchev, Yan, Zhu]

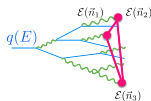


- Allow modern integration techniques to be used for directly measurable *physical observables*.

Example Perturbative Calculation

- To compute in perturbation theory, one integrates over the energy fraction of particles, with the angles of observed particles fixed.
- At lowest order in perturbation theory, one has:

$$\int d\omega_1 d\omega_2 d\omega_3 \delta(1 - \omega_1 - \omega_2 - \omega_3) (\omega_1 \omega_2 \omega_3) P_{1 \rightarrow 3}$$



- Consider for illustration a simple Mandelstam invariant in the splitting function $P_{1 \rightarrow 3} \supset \frac{1}{s_{123}}$.
- One obtains integrals of the form:

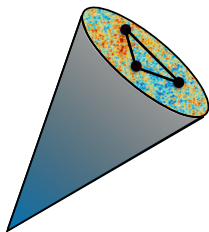
$$\int d\omega_1 d\omega_2 d\omega_3 \delta(1 - \omega_1 - \omega_2 - \omega_3) \frac{\omega_1 \omega_2 \omega_3}{\omega_1 \omega_2 z_{12}^2 + \omega_1 \omega_3 z_{13}^2 + \omega_2 \omega_3 z_{23}^2}$$

- This is immediately recognized as a Feynman parameter integral, where the $|z_{ij}|^2$ are the dual coordinates:

$$x_i^\mu - x_{i+1}^\mu = p_i^\mu, x_{ij}^2 = (x_i - x_j)^2 = (p_i + \cdots p_{j-1})^2, \\ x_{ij}^2 \leftrightarrow |z_{ij}|^2$$

Three-Point Correlator at Weak Coupling

- Turn out to have an elegant perturbative structure. e.g. in $\mathcal{N} = 4$



[Chen, Luo, Moulton, Yang, Zhang, Zhu]

$$G_{\mathcal{N}=4}(z) = \frac{1+u+v}{2uv}(1+\zeta_2) - \frac{1+v}{2uv}\log(u) - \frac{1+u}{2uv}\log(v) \\ - (1+u+v)(\partial_u + \partial_v)\Phi(z) + \frac{(1+u^2+v^2)}{2uv}\Phi(z) + \frac{(z-\bar{z})^2(u+v+u^2+v^2+u^2v+uv^2)}{4u^2v^2}\Phi(z) \\ + \frac{(u-1)(u+1)}{2uv^2}D_2^+(z) + \frac{(v-1)(v+1)}{2u^2v}D_2^+(1-z) + \frac{(u-v)(u+v)}{2uv}D_2^+\left(\frac{z}{z-1}\right)$$

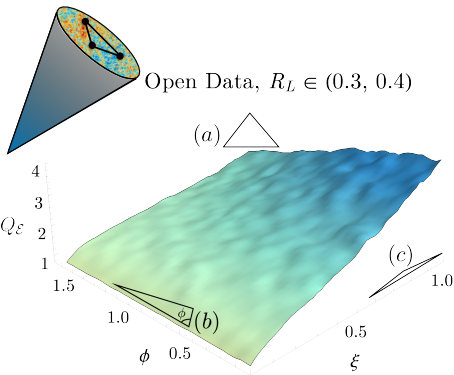
- Here Φ and D_2^+ are polylogarithmic functions

$$\Phi(z) = \frac{2}{z-\bar{z}} \left(\text{Li}_2(z) - \text{Li}_2(\bar{z}) + \frac{1}{2} (\log(1-z) - \log(1-\bar{z})) \log(z\bar{z}) \right) \\ D_2^+(z) = \text{Li}_2(1-|z|^2) + \frac{1}{2} \log(|1-z|^2) \log(|z|^2)$$

- Real world QCD involves more complicated polynomials, but is otherwise similar.

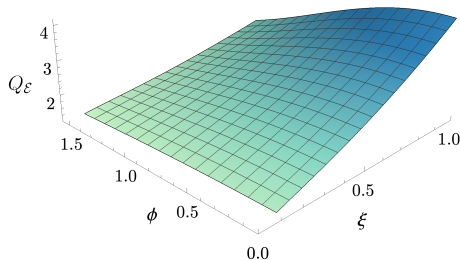
Shape Dependence of Non-Gaussianities

- Can directly study the three-point correlator of lightray operators inside high energy jets.



$$G_{N,LL}(z) = \frac{1+u+v}{2uv}(1+\zeta_2) - \frac{1+u}{2uv}\log(u) - \frac{1+v}{2uv}\log(v) \\ - (1+u+v)(\partial_u + \partial_v)\Phi(z) + \frac{(1+u^2+v^2)}{2uv}\Phi(z) + \frac{(z-z^2)(u+v+u^2+v^2+u^2v+uv^2)}{4u^2v^2}\Phi(z) \\ + \frac{(u-1)(u+1)}{2uv^2}D_2^*(z) + \frac{(v-1)(v+1)}{2u^2v}D_2^*(1-z) + \frac{(u-v)(u+v)}{2uv}D_2^*\left(\frac{z}{z-1}\right)$$

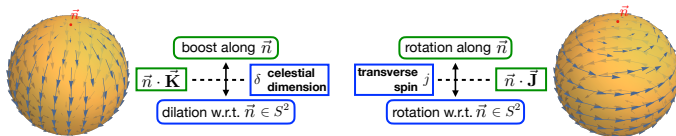
LL + LO prediction, $R_L = 0.35$



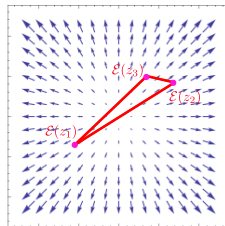
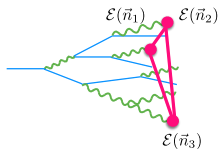
[Chen, Moul, Thaler, Zhu]

- Illustrates theoretical control over multi-point correlations!

- Can be decomposed into celestial blocks:



- In the “jet substructure” limit, reduces to living in the plane transverse to the jet:



Casimir Differential Equations

- Celestial blocks derived by solving Casimir differential equations

$$G(z_1, z_2, z_3, \bar{n}) = \frac{1}{(z_1 \cdot z_2)^3} \frac{1}{(z_3 \cdot \bar{n})^4} \left(\frac{z_1 \cdot z_3}{z_1 \cdot \bar{n}} \right) g(z, \bar{z})$$

Symmetry: Lorentz Group

Representation labels:

δ	celestial dimension		$\vec{n} \cdot \vec{K}$
j	transverse spin		$\vec{n} \cdot \vec{J}$

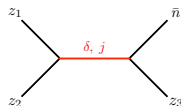
Quadratic Casimir: $\frac{1}{2} M_{\mu\nu} M^{\mu\nu}$ $\xrightarrow{\text{eigenvalue}}$ $-(\delta(\delta - 2) + j^2)$

Casimir Equation: acting Casimir operator on z_1, z_2

$$\mathcal{L}^{\mu\nu}(z_1, z_2) \mathcal{L}_{\mu\nu}(z_1, z_2) \boxed{G_{\delta,j}} = -(\delta(\delta - 2) + j^2) \boxed{G_{\delta,j}}$$

[Dolan, Osborn, 2003]

$$\mathcal{L}^{\mu\nu}(z_1, z_2) \equiv \sum_{i=1,2} \left(z_i^\mu \frac{\partial}{\partial z_{i\nu}} - z_i^\nu \frac{\partial}{\partial z_{i\mu}} \right)$$



$$g(z, \bar{z}) = \sum_{\delta,j} c_{\delta,j} g_{\delta,j}(z, \bar{z})$$

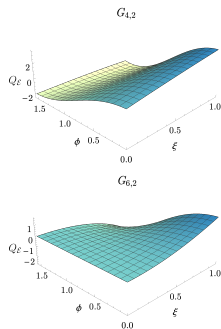
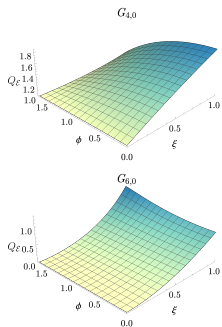
Solutions (2D Conformal Blocks):

$$g_{\delta,j}(z, \bar{z}) = \frac{1}{1 + \delta_{j,0}} (k_{\delta-j}(z) k_{\delta+j}(\bar{z}) + k_{\delta+j}(z) k_{\delta-j}(\bar{z}))$$

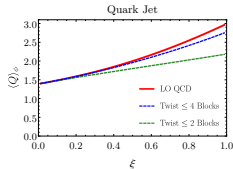
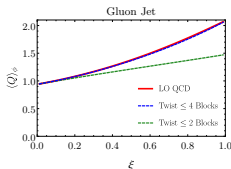
[Notations] In our case, $a = 0, b = -1$
 $k_\beta(x) \equiv x^{\beta/2} {}_2F_1\left(\frac{\beta}{2} + a, \frac{\beta}{2} + b, \beta, x\right)$

Celestial Partial Waves

- These are partial waves living on the detector:

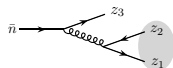


- Celestial block expansion converges rapidly.



Perturbative Data from Lightray OPE

- Interesting interplay with structure of Feynman diagrams.
- e.g. In QCD can choose color structures to isolate internal parton states.
- Only $j = 0, 2$ contribute. Leading twist $j = 2$ block uncontaminated by higher twist contributions.



Contributing Operator
 $F_a^{\mu+}(iD^+)F_a^{\nu+}\epsilon_{\lambda,\mu}\epsilon_{\lambda,\nu}$
 twist-2, transverse spin-2
 gluonic operator

highest transverse spin series

$$-z^3 \bar{z} {}_2F_1(3, 2, 6, z)$$

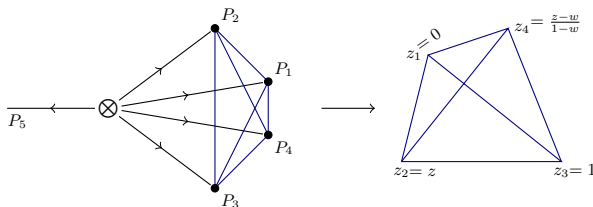
$$\begin{aligned} & -z^3 \bar{z} \cos 2\phi + \frac{39}{10} z^2 \bar{z}^2 - z \bar{z}^3 \\ & -z^4 \bar{z} \cos 3\phi + \frac{39}{20} z^3 \bar{z}^2 + \frac{39}{20} z^2 \bar{z}^3 - z \bar{z}^4 \\ & -\frac{6}{7} z^5 \bar{z} \cos 4\phi + \frac{229}{140} z^4 \bar{z}^2 - \frac{211}{140} z^3 \bar{z}^3 + \frac{229}{140} z^2 \bar{z}^4 - \frac{6}{7} z \bar{z}^5 \\ & -\frac{5}{7} z^6 \bar{z} \cos 5\phi + \frac{207}{140} z^5 \bar{z}^2 - \frac{233}{140} z^4 \bar{z}^3 - \frac{233}{140} z^3 \bar{z}^4 + \frac{207}{140} z^2 \bar{z}^5 - \frac{5}{7} z \bar{z}^6 \\ & \dots \end{aligned}$$

- Non-trivial interplay with structure of transcendental function space.

Four-point Correlator at Weak Coupling

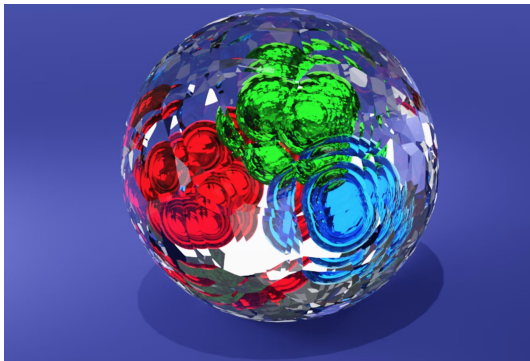
- Recently extended this to the four-point energy correlator in $\mathcal{N} = 4$ SYM.
- Obtained a compact expression in terms of weight-3 polylogarithms.

[Chicherin, Moult, Sokatchev, Yan, Zhu]



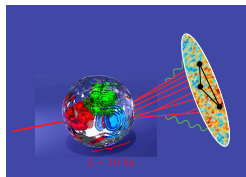
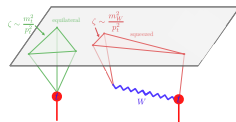
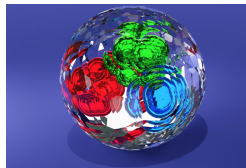
- Much still to be learned about perturbative structure of physical observables.

Applications

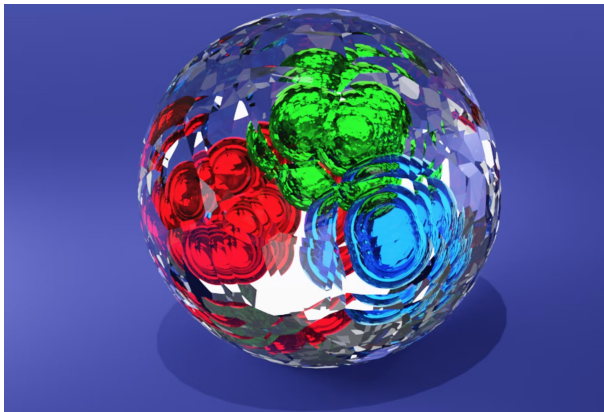


Three Applications:

- The Confinement Transition
- Weighing the Top Quark
- Resolving the Scales of the Quark Gluon Plasma

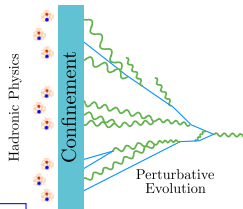
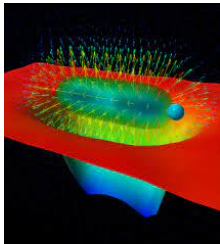
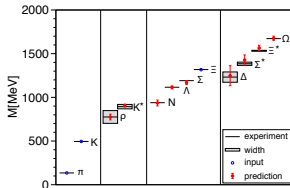


The Confinement Transition



Dynamics of Hadronization

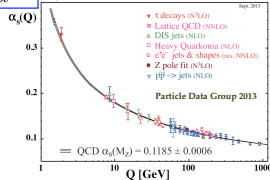
- What are the dynamics of the hadronization process?



Long Time Scale
Low Energy

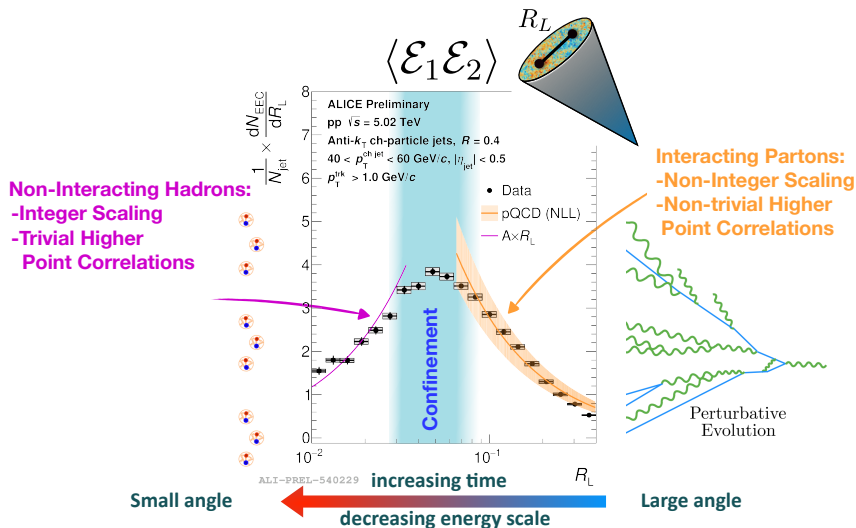
$$\alpha_s(Q)$$

Short Time Scale
High Energy

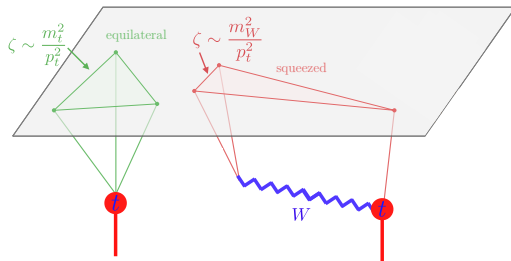


$$\hat{O} = \alpha_s + \alpha_s^2 + \dots$$

The Confinement Transition



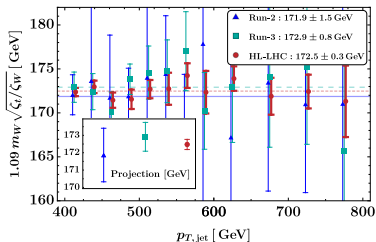
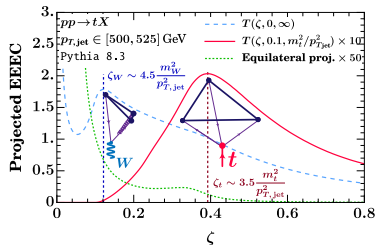
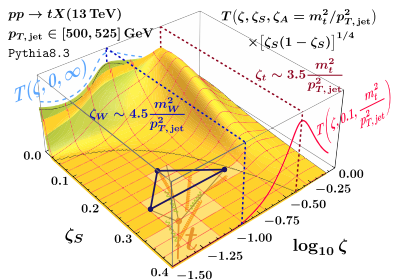
Weighing the Top Quark



[Holguin, Moul, Pathak, Procura, Schofbeck, Schwarz]

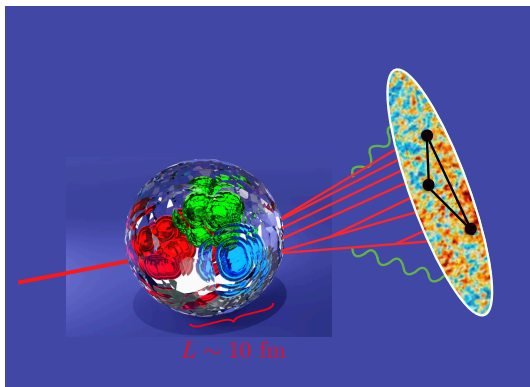
Top Quark Mass Measurement Proposal

- Use the W to convert between angular and mass scales.



- Optimistic for a precision top quark mass extraction at the LHC.

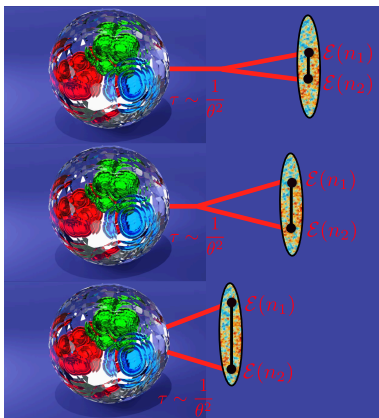
Resolving the Scales of the Quark Gluon Plasma



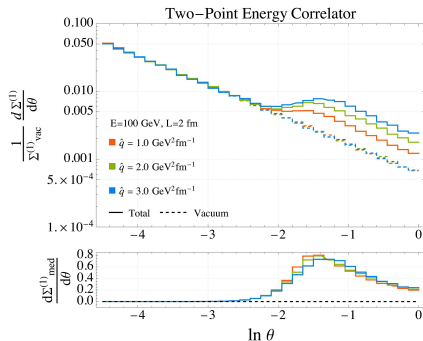
[Andres, Dominguez, Holguin, Kunnawalkam Elayavalli, Marquet, Moul]t

Resolving the Scales of the QGP

- QGP scales cleanly imprinted in two-point correlation!



Increasing θ

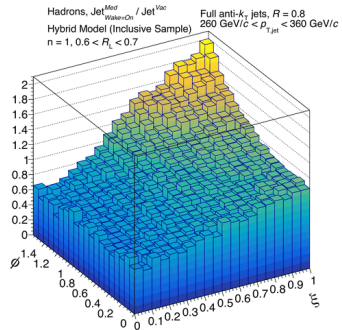
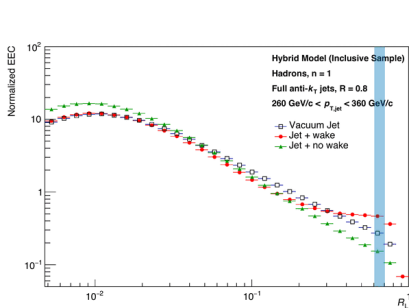


Increasing θ

- Resolve Femtometer scales from asymptotic energy flux!

Resolving the Scales of the QGP

- Higher point correlators allow us to probe the “shape” of the wake.



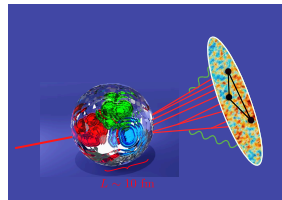
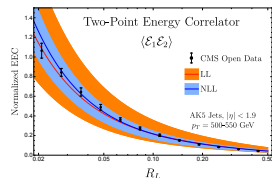
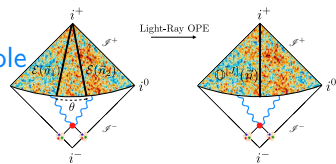
[Bossi, He, Kudinoor, Moul, Pablos, Rai, Rajagopal]

Questions

- Would be interesting to better develop connections between the study of lightray operators, and celestial holography.
- Many objects appear in both:
 - OPE
 - Celestial blocks
 - $w_{1+\infty}$, W_∞ algebras
 - ...
- Structure of energy correlators in gravity essentially unexplored...
- Improved understanding of the structure of asymptotic observables will have significant impact on experiment!

Summary

- Asymptotic observables play an important role in collider experiments.
- Jet Substructure provides a physical realization of the OPE limit of lightray operators $\Rightarrow$ direct bridge between recent field theory developments and QCD phenomenology.
- Better understanding of asymptotic observables will have significant impact on improving our understanding of the Standard Model.



Thanks!