Carrollian Amplitudes and the Flat Limit of AdS

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How to formulate flat space holography?

- Correspondence between **gravity in asymptotically flat spacetimes** and a **lower-dimensional field theory** without gravity.
- **Bottom-up** approaches to build candidates for **holographic duals**.
- Two proposals for flat space holography in 4d:

  $\Longrightarrow$ **Celestial holography**: the dual theory is a **2d CFT** living on the **celestial sphere** S^2.

  [de Boer-Solodukhin '03] [He-Mitra-Strominger '15] [Kapec-Mitra-Raclariu-Strominger '16] [Cheung-de la Fuente-Sundrum '16] [Pasterski-Shao-Strominger '17] [Pasterski-Shao '17] [Donnay-Puhm-Strominger '18] [Stieberger-Taylor '18] [Pate-Raclariu-Strominger-Yuan '19] [Adamo-Mason-Sharma '21] ...

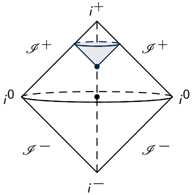
  $\Longrightarrow$ **Carrollian holography**: the dual theory is a **3d Carrollian CFT** living at **null infinity** $\mathscr{I} \simeq \mathbb{R} \times S^2$.

  [Arcioni-Dappiaggi '03] [Dappiaggi-Moretti-Pinamonti '06] [Barnich-Compère '07] [Bagchi '10] [Barnich '12] [Bagchi-Detournay-Fareghbal-Simon '12] [Barnich-Gomberoff-Gonzalez '12] [Bagchi-Basu-Grumiller-Riegler '15] [Ciambelli-Marteau-Petkou-Petropoulos-Siampos '18] [Donnay-Fiorucci-Herfray-Ruzziconi '22] ...
- **Objectives** of this talk:

  $\Longrightarrow$ The two proposals are related [Donnay-Fiorucci-Herfray-Ruzziconi '22] [Bagchi-Banerjee-Basu-Dutta '22].

  $\Longrightarrow$ Define Carrollian amplitudes [Mason-Ruzziconi-Yelleshpur Srikant '23].

  $\Longrightarrow$ Flat limit of AdS.

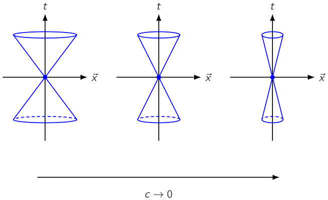


Carrollian algebra

- "Carroll" refers to the limit $c \to 0$ where c is the speed of light [Lévy-Leblond '65].
  $\Longrightarrow$ Opposite to the usual Galilean limit ($c \to \infty$).
- Carrollian algebra (Inonu-Wigner contraction of Poincaré algebra when $c \to 0$):

$$[B_i, H] = 0, \quad [B_i, B_j] = 0, \quad [B_i, P_j] = \delta_{ij} H, \quad [B_k, J_{ij}] = \delta_{k[i} B_{j]}, \quad [P_k, J_{ij}] = \delta_{k[i} P_{j]}$$

- (Global) conformal Carrollian algebra (Inonu-Wigner contraction of the conformal algebra $SO(d,2)$ when $c \to 0$):
  $\Longrightarrow$ Add the dilatation: $D = (t\partial_t + x^i \partial_i)$, and the Carrollian special conformal generators: $K = x^2 \partial_u$ and $K_i = x^2 \partial_i - 2x_i x^j \partial_j - 2x_i t \partial_t$.



Carrollian geometry

- Metric degenerates to spatial metric in the limit $c \to 0$:

$$ds^2 = \eta_{ab} dx^a dx^b = -c^2 dt^2 + \delta_{ij} dx^i dx^j \quad \xrightarrow{c \to 0} \quad ds^2 = \delta_{ij} dx^i dx^j$$

- Inverse metric degenerates to temporal bi-vector:

$$-c^2 \eta^{ab} = \begin{pmatrix} 1 & 0 \\ 0 & -c^2 \delta^{ij} \end{pmatrix} \quad \xrightarrow{c \to 0} \quad n^a n^b \text{ with } n^a = \delta^a_t$$

- Carrollian geometry: degenerate metric q_{ab} and vector field n^a such that $q_{ab} n^b = 0$. [Henneaux '79] [Duval-Gibbons-Horvathy-Zhang '14]
- Why is it relevant at null infinity?

$\Longrightarrow$ $\mathscr{I}$ being a null hypersurface, the induced metric is degenerate!

$\Longrightarrow$ Conformal Carrollian structure at $\mathscr{I}$ induced by conformal compactification [Geroch '77] [Ashtekar '14]

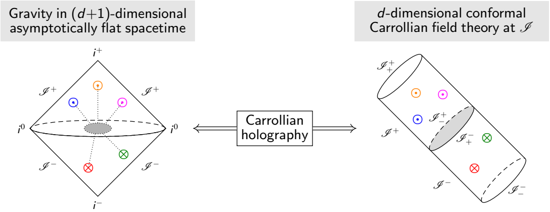
$$q_{ab} \sim \omega^2 q_{ab}, \quad n^a \sim \omega^{-1} n^a$$

$\Longrightarrow$ BMS symmetries = Asymptotic symmetries in flat space [Bondi-van der Burg-Metzner '62] [Sachs '62]
= Conformal symmetries of the Carrollian structure at $\mathscr{I}$.

$$\mathcal{L}_{\bar{\xi}} q_{ab} = 2\alpha q_{ab}\,, \qquad \mathcal{L}_{\bar{\xi}} n^a = -\alpha n^a$$

- Isomorphism: $\boxed{\mathfrak{bms}_{d+1} \simeq \mathfrak{CCarr}_d.}$ [Duval-Gibbons-Horvathy '14] and Poincaré$_{d+1}$ $\simeq$ Global $\mathfrak{CCarr}_d$

Carrollian holography



- Asymptotic symmetries of the bulk theory = global symmetries in the dual theory.
- The dual theory is a d-dimensional Carrollian CFT (= theory exhibiting conformal Carroll/BMS spacetime symmetries)
  $\Longrightarrow$ Can be constructed by taking $c \to 0$ of standard relativistic CFTs, see e.g.
  [Schild '77] [Isberg-Lindstrom-Sundborg-Theodoridis '94] [Barnich-Gomberoff-Gonzalez '12] [Duval-Gibbons-Horvathy '14] [Bagchi-Mehra-Nandi '19] ...
- Carrollian holography follows a similar pattern than AdS/CFT correspondence: $(d+1)$-dimensional bulk / d-dimensional boundary duality.
  $\Longrightarrow$ Naturally arises from a flat limit procedure ($\ell \to \infty$).
  $\Longrightarrow$ The flat limit in the bulk induces a Carrollian limit ($c \to 0$) at the boundary.

  [Bagchi '10] [Barnich-Gomberoff-Gonzalez '12] [Ciambelli-Marteau-Petkou-Petropoulos-Siampos '18] [Compère-Fiorucci-Ruzziconi '19] [Campoleoni-Delfante-Pekar-Petropoulos-Rivera Betancour '23]

An overview of previous works...

- Flat limit successfully exploited in 3d gravity:
  1. Gravitational solution space and symmetries (Witt$\oplus$Witt $\Longrightarrow$ $\mathfrak{bms}_3$) [Barnich-Gomberoff-Gonzalez '12] [Bagchi-Fareghbal '12]
  2. Entropy matching between flat space cosmologies and Carrollian CFT [Barnich '12] [Bagchi-Detournay-Fareghbal-Simon '13]
  3. Entanglement entropy formulae [Li-Takayanagi '11] [Bagchi-Basu-Grumiller-Riegler '14] [Jiang-Song-Wen '17]
  4. Holographic computation of boundary Carrollian stress tensor correlators [Detournay-Grumiller-Scholler-Simon '14] [Bagchi-Grumiller-Merbis '15] [Hartong '16]
  5. Effective Carrollian CFT action at $\mathscr{I}$ [Barnich-Gomberoff-Gonzalez '13]
  6. Holographic anomaly in flat space [Campoleoni-Ciambelli-Delfante-Marteau-Petropoulos-Ruzziconi '22]

  ...
- Flat limit of the gravitational phase space and symmetries also works in 4d.

  [Poole-Skenderis-Taylor '18] [Compère-Fiorucci-Ruzziconi '19] [Compère-Fiorucci-Ruzziconi '20] [Geiller-Zwikel '22]
- Flat limit in the fluid/gravity correspondence. [Ciambelli-Marteau-Petkou-Petropoulos-Siampos '18] [Freidel-Jai-akson '22] [Campoleoni-Delfante-Pekar-Petropoulos-Rivera Betancour-Vilatte '23]
- Carrollian CFT actions at the boundary of 4d flat space. [Adamo-Casali-Skinner '14] [Barnich-Nguyen-Ruzziconi '22]
- Important missing piece: relation with 4d $\mathcal{S}$-matrix and celestial amplitudes?

  [Donnay-Fiorucci-Herfray-Ruzziconi '22] [Bagchi-Banerjee-Basu-Dutta '22] [Mason-Ruzziconi-Yelleshpur Srikant '23] [Liu-Long-Ye '24] [Have-Nguyen-Prohazka-Salzer '24] [Stieberger-Taylor-Zhu '24]

  $\Longrightarrow$ To be clarified in this talk.

Bondi coordinates

- Flat Bondi coordinates $\{u, r, z, \bar{z}\}$ ($u, r \in \mathbb{R}$, $z \in \mathbb{C}$):

$$X^\mu = u\,\partial_z\partial_{\bar{z}} q^\mu(z,\bar{z}) + r\,q^\mu(z,\bar{z}), \qquad q^\mu(z,\bar{z}) \equiv \frac{1}{\sqrt{2}}\Big(1+z\bar{z}, z+\bar{z}, -i(z-\bar{z}), 1-z\bar{z}\Big).$$

- Minkowski metric:

$$ds^2 = -2dudr + 2r^2 dz d\bar{z}.$$

- Induced Carrollian structure at future/past null infinity $\mathscr{I}^\pm = \{r \to \pm\infty\}$:

$$ds^2_{\mathscr{I}} = q_{ab}dx^a dx^b = 0du^2 + 2dzd\bar{z}, \qquad n^a\partial_a = \partial_u$$

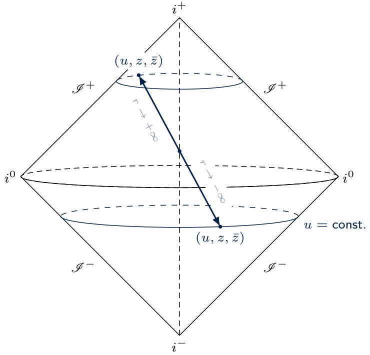
with $x^a = (u, z, \bar{z})$ the boundary coordinates. [Penrose '63] [Geroch '77] [Ashtekar '14]

- Natural (antipodal) identification between $\mathscr{I}^+$ and $\mathscr{I}^-$.
- BMS/conformal Carroll symmetries: [Bondi-van der Burg-Metzner '62] [Sachs '62]

$$\bar{\xi}^a\partial_a = \left[\mathcal{T} + \frac{u}{2}(\partial\mathcal{Y} + \bar{\partial}\bar{\mathcal{Y}})\right]\partial_u + \mathcal{Y}\partial + \bar{\mathcal{Y}}\bar{\partial}$$

with defining property: $\mathcal{L}_{\bar{\xi}} q_{ab} = 2\alpha q_{ab}$, $\mathcal{L}_{\bar{\xi}} n^a = -\alpha n^a$, $\alpha = \frac{1}{2}(\partial\mathcal{Y} + \bar{\partial}\bar{\mathcal{Y}})$.

1. $\mathcal{T} = \mathcal{T}(z,\bar{z})$ is the supertranslation parameter;
2. $\mathcal{Y} = \mathcal{Y}(z)$, $\bar{\mathcal{Y}} = \bar{\mathcal{Y}}(\bar{z})$ are the superrotation parameters satisfying the conformal Killing equation. [Barnich-Troessaert '10]



Massless scattering in flat space

Can we encode the bulk $\mathcal{S}$-matrix into boundary Carrollian CFT correlators?

- Strategy: start form the bulk operators, and deduce the boundary operators at $\mathscr{I}$. [Ashtekar '81] [Arcioni-Dappiaggi '03] [Strominger '17] [Donnay-Fiorucci-Herfray-Ruzziconi '22]
- Consider a spin-s ($s = 0, 1, 2, \dots$) massless field in flat space:

$$\phi^{(s)}_I(X) = \frac{K_{(s)}}{16\pi^3} \sum_{\alpha=\pm} \int \omega\, d\omega\, d^2 w \left[a^{(s)}_\alpha(\omega, w, \bar{w}) \varepsilon^{*\alpha}_I(w,\bar{w})\, e^{i\omega q^\mu X_\mu} + a^{(s)}_\alpha(\omega, w, \bar{w})^\dagger \varepsilon^\alpha_I(w,\bar{w})\, e^{-i\omega q^\mu X_\mu} \right]$$

with $I = (\mu_1 \mu_2 \dots \mu_s)$ and

$$p^\mu(\omega, w, \bar{w}) = \omega\, q^\mu(w,\bar{w}), \quad q^\mu(w,\bar{w}) = \frac{1}{\sqrt{2}}\Big(1 + w\bar{w}, w + \bar{w}, -i(w - \bar{w}), 1 - w\bar{w}\Big),$$

$$\varepsilon^\pm_{\mu_1 \dots \mu_s}(\vec{q}) = \varepsilon^\pm_{\mu_1}(\vec{q})\varepsilon^\pm_{\mu_2}(\vec{q}) \dots \varepsilon^\pm_{\mu_s}(\vec{q}), \qquad \varepsilon^+_\mu(\vec{q}) = \partial_w q_\mu = \frac{1}{\sqrt{2}}(-\bar{w}, 1, -i, -\bar{w}), \qquad \varepsilon^-_\mu(\vec{q}) = [\varepsilon^+_\mu(\vec{q})]^*.$$

- Taking $r \to \infty$ (stationary phase approximation), we find the boundary values:

$$\bar{\phi}^{(s)}_{z\dots z}(u,z,\bar{z})^{\text{out}} = \lim_{r\to+\infty} \left(r^{1-s} \phi^{(s)}_{z\dots z}(u,r,z,\bar{z}) \right) = -\frac{iK_{(s)}}{8\pi^2} \int_0^{+\infty} d\omega \left[a^{(s)\,\text{out}}_+(\omega,z,\bar{z}) e^{-i\omega u} - a^{(s)\,\text{out}}_-(\omega,z,\bar{z})^\dagger e^{i\omega u} \right] \quad \text{at } \mathscr{I}^+,$$

$$\bar{\phi}^{(s)}_{z\dots z}(u,z,\bar{z})^{\text{in}} = \lim_{r\to-\infty} \left(r^{1-s} \phi^{(s)}_{z\dots z}(u,r,z,\bar{z}) \right) = -\frac{iK_{(s)}}{8\pi^2} \int_0^{+\infty} d\omega \left[a^{(s)\,\text{in}}_+(\omega,z,\bar{z}) e^{-i\omega u} - a^{(s)\,\text{in}}_-(\omega,z,\bar{z})^\dagger e^{i\omega u} \right] \quad \text{at } \mathscr{I}^-.$$

$\Longrightarrow$ Insertion operators for a massless scattering between $\mathscr{I}^-$ (in) and $\mathscr{I}^+$ (out).

Carrollian primaries

- Conformal Carrollian primary field $\Phi_{(k,\bar{k})}(u,z,\bar{z})$:

$$\delta_{\bar{\xi}}\Phi_{(k,\bar{k})} = \left[\left(\mathcal{T} + \frac{u}{2}(\partial\mathcal{Y} + \bar{\partial}\bar{\mathcal{Y}})\right)\partial_u + \mathcal{Y}\partial + \bar{\mathcal{Y}}\bar{\partial} + k\,\partial\mathcal{Y} + \bar{k}\,\bar{\partial}\bar{\mathcal{Y}}\right]\Phi_{(k,\bar{k})}, \qquad (k,\bar{k})\text{: Carrollian weights.}$$

  (analogue of primary field in CFT)

  $\Longrightarrow$ Can be deduced from Carrollian highest-representation [Bagchi-Dhivakar-Dutta '23]:

$$[L_0, \Phi(0,0,0)] = k\Phi(0,0,0), \quad [\bar{L}_0, \Phi(0,0,0)] = \bar{k}\Phi(0,0,0)$$
$$[L_n, \Phi(0,0,0)] = 0 = [\bar{L}_n, \Phi(0,0,0)], \quad [M_{r,s}, \Phi(0,0,0)] = 0, \quad \forall n,r,s > 0$$

  where

$$L_n = z^{n+1}\partial_z + \frac{1}{2}(n+1)z^n u\partial_u, \qquad \bar{L}_n = \bar{z}^{n+1}\partial_{\bar{z}} + \frac{1}{2}(n+1)\bar{z}^n u\partial_u, \qquad M_{r,s} = z^r\bar{z}^s\partial_u, \qquad n,r,s \in \mathbb{Z}$$

  $\Longrightarrow$ The Carrollian primaries transform in unitary representations of global $\mathfrak{CCarr}_3 \simeq$ Poincaré$_4$ [Nguyen-West '23] [Nguyen '23].

- Carrollian correlators living at $\mathscr{I}$: $\langle \Phi_{(k_1,\bar{k}_1)}(u_1,z_1,\bar{z}_1)\Phi_{(k_2,\bar{k}_2)}(u_2,z_2,\bar{z}_2)\ldots\rangle$.

- Remark: if $\Phi_{(k,\bar{k})}(u,z,\bar{z})$ is a Carrollian primary, then $\partial_u\Phi_{(k,\bar{k})}(u,z,\bar{z})$ is also a Carrollian primary with shifted weights $(k+\frac{1}{2}, \bar{k}+\frac{1}{2})$.

Carrollian holography identification

- Carrollian operators = boundary value of bulk operators:
  [Arcioni-Dappiaggi '03] [Dappiaggi-Moretti-Pinamonti '05] [Donnay-Fiorucci-Herfray-Ruzziconi '22]

$$\Phi^{\epsilon=+1}_{(k,\bar{k})}(u,z,\bar{z}) = \bar{\phi}^{(s)\,\text{out}}_{z\dots z}(u,z,\bar{z}), \qquad \Phi^{\epsilon=-1}_{(k,\bar{k})}(u,z,\bar{z}) = \bar{\phi}^{(s)\,\text{in}}_{z\dots z}(u,z,\bar{z})^\dagger$$

$\Longrightarrow$ This implies $k = \frac{1+\epsilon J}{2}$, $\bar{k} = \frac{1-\epsilon J}{2}$ with $\epsilon = \pm 1$ for out/in.

$\Longrightarrow$ For gravity ($s=2$), $\Phi^{\epsilon=+1}_{(k,\bar{k})}(u,z,\bar{z}) \equiv C_{zz}(u,z,\bar{z})$ ($J=2$ and $(k,\bar{k}) = (\frac{3}{2}, -\frac{1}{2})$).

- Carrollian correlators = scattering amplitudes in position space at $\mathscr{I}$:

$$\begin{aligned}\langle \Phi^{\epsilon_1}_{(k_1,\bar{k}_1)}(u_1,z_1,\bar{z}_1)\dots\Phi^{\epsilon_n}_{(k_n,\bar{k}_n)}(u_n,z_n,\bar{z}_n)\rangle &= \prod_{i=1}^{n}\left(\int_0^{+\infty}\frac{d\omega_i}{2\pi}\,e^{i\epsilon_i\omega_i u_i}\right)\mathcal{A}_n\left(\{\omega_1,z_1,\bar{z}_1\}^{\epsilon_1}_{J_1},\dots,\{\omega_n,z_n,\bar{z}_n\}^{\epsilon_n}_{J_n}\right)\\ &= \mathcal{C}_n\left(\{u_1,z_1,\bar{z}_1\}^{\epsilon_1}_{J_1},\dots,\{u_n,z_n,\bar{z}_n\}^{\epsilon_n}_{J_n}\right)\end{aligned}$$

$\Longrightarrow$ Amplitudes in position space at $\mathscr{I}$ = Carrollian amplitudes. [Donnay-Fiorucci-Herfray-Ruzziconi '22] [Mason-Ruzziconi-Yelleshpur Srikant '23]

- Extrapolate dictionary for Carrollian holography:

$$\langle \Phi^{\epsilon_1}_{(k_1,\bar{k}_1)}(u_1,z_1,\bar{z}_1)\dots\Phi^{\epsilon_n}_{(k_n,\bar{k}_n)}(u_n,z_n,\bar{z}_n)\rangle = \lim_{r\to\epsilon\infty}\langle r^{1-s_1}\phi^{(s_1)}(u_1,r_1,z_1,\bar{z}_1)\dots r^{1-s_n}\phi^{(s_n)}(u_n,r_n,z_n,\bar{z}_n)\rangle$$

Relation between Carrollian CFT and celestial CFT

- Celestial amplitudes obtained by Mellin transform [de Boer-Solodukhin '03] [Pasterski-Shao-Strominger '17] [Pasterski-Shao '17]:

$$\mathcal{M}_n\left(\{\Delta_1, z_1, \bar{z}_1\}^{\epsilon_1}_{J_1}, \dots, \{\Delta_n, z_n, \bar{z}_n\}^{\epsilon_n}_{J_n}\right) = \prod_{i=1}^n \left(\int_0^{+\infty} d\omega_i\, \omega_i^{\Delta_i - 1}\right) \mathcal{A}_n\left(\{\omega_1, z_1, \bar{z}_1\}^{\epsilon_1}_{J_1}, \dots, \{\omega_n, z_n, \bar{z}_n\}^{\epsilon_n}_{J_n}\right)$$
$$\equiv \langle \mathcal{O}^{\epsilon_1}_{\Delta_1, J_1}(z_1, \bar{z}_1) \dots \mathcal{O}^{\epsilon_n}_{\Delta_n, J_n}(z_n, \bar{z}_n)\rangle$$

- Relation between Carrollian and celestial amplitudes [Donnay-Fiorucci-Herfray-Ruzziconi '22]:

$$\mathcal{M}_n\left(\{\Delta_1, z_1, \bar{z}_1\}^{\epsilon_1}_{J_1}, \dots, \{\Delta_n, z_n, \bar{z}_n\}^{\epsilon_n}_{J_n}\right) = \prod_{i=1}^n \left((-i\epsilon_i)^{\Delta_i}\Gamma[\Delta_i]\int_{-\infty}^{+\infty} \frac{du_i}{(u_i - i\epsilon_i\varepsilon)^{\Delta_i}}\right) \mathcal{C}_n\left(\{u_1, z_1, \bar{z}_1\}^{\epsilon_1}_{J_1}, \dots, \{u_n, z_n, \bar{z}_n\}^{\epsilon_n}_{J_n}\right)$$

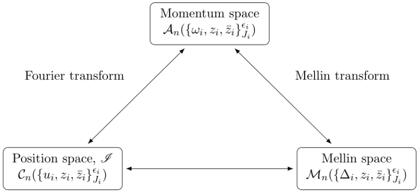
- Relation between Carrollian and celestial operators:

$$\boxed{\mathcal{O}^{\epsilon}_{\Delta_i, J_i}(z_i, \bar{z}_i) = (-i\epsilon_i)^{\Delta_i}\Gamma[\Delta_i]\int_{-\infty}^{+\infty} \frac{du_i}{(u_i - i\epsilon_i\varepsilon)^{\Delta_i}}\, \Phi^{\epsilon}_{(k_i, \bar{k}_i)}(u_i, z_i, \bar{z}_i)}$$

$\Longrightarrow$ Exchange between time and conformal dimension.

$\Longrightarrow$ Three scattering bases $(\omega,\ u,\ \Delta)$ [Donnay-Pasterski-Puhm '22] [Freidel-Pranzetti-Raclariu '22].

$\Longrightarrow$ Extrapolate dictionary in celestial holography [Pasterski-Puhm-Trevisani '21].



Carrollian Ward identities

- Consistent with the (global) conformal Carrollian Ward identities:

$$\sum_{i=0}^{n} \Big[\Big(\mathcal{T}(z_i, \bar{z}_i) + \frac{u_i}{2} (\partial_{z_i} \mathcal{Y}(z_i) + \partial_{\bar{z}_i} \bar{\mathcal{Y}}(\bar{z}_i)) \Big) \partial_{u_i} + \mathcal{Y}(z_i) \partial_{z_i} + \bar{\mathcal{Y}}(\bar{z}_i) \partial_{\bar{z}_i} + k_i \partial_{z_i} \mathcal{Y}(z_i) + \bar{k}_i \partial_{\bar{z}_i} \bar{\mathcal{Y}}(\bar{z}_i) \Big] \langle \Phi_{(k_1, \bar{k}_1)}(u_1, z_1, \bar{z}_1) \dots \Phi_{(k_n, \bar{k}_n)}(u_n, z_n, \bar{z}_n) \rangle = 0$$

where

$$\mathcal{T}(z, \bar{z}) = 1, z, \bar{z}, z\bar{z}, \qquad \mathcal{Y}(z) = 1, z, z^2, \qquad \bar{\mathcal{Y}}(\bar{z}) = 1, \bar{z}, \bar{z}^2$$

$\Longrightarrow$ The low-point correlation functions are completely fixed by the conformal Carrollian symmetries.

- In particular, for the 2-point function [Chen-Liu-Zheng, '21]:

$$\langle \Phi_{(k_1, \bar{k}_1)}(u_1, z_1, \bar{z}_1) \Phi_{(k_2, \bar{k}_2)}(u_2, z_2, \bar{z}_2) \rangle = \begin{cases} \dfrac{\alpha}{(u_1 - u_2)^{k_1 + k_2 + \bar{k}_1 + \bar{k}_2 - 2}} \delta^{(2)}(z_1 - z_2) \delta_{k_1 + k_2, \bar{k}_1 + \bar{k}_2} & \text{(Electric branch)} \\ \dfrac{\beta}{(z_1 - z_2)^{k_1 + k_2} (\bar{z}_1 - \bar{z}_2)^{\bar{k}_1 + \bar{k}_2}} \delta_{k_1, k_2} \delta_{\bar{k}_1, \bar{k}_2} & \text{(Magnetic branch)} \end{cases}$$

$\Longrightarrow$ Electric branch relevant for massless scattering.

[Donnay-Fiorucci-Herfray-Ruzziconi '22] [Bagchi-Banerjee-Basu-Dutta '22] [Mason-Ruzziconi-Yelleshpur Srikant '23]

$\Longrightarrow$ The $\delta^{(2)}(z_1 - z_2)$ distribution is a standard feature of Carrollian CFT (light cones shrink into lines), not undesirable.

Two-point Carrollian amplitude

- Two-point amplitude (one incoming and one outgoing particle):

$$\mathcal{A}_2(\{\omega_1, z_1, \bar{z}_1\}^-_{J_1}, \{\omega_2, z_2, \bar{z}_2\}^+_{J_2}) = \kappa^2_{J_1,J_2}\, \pi\, \frac{\delta(\omega_1 - \omega_2)}{\omega_1}\, \delta^{(2)}(z_1 - z_2)\, \delta_{J_1,J_2},$$

- Carrollian two-point amplitude obtained by Fourier transform [Liu-Long '22] [Donnay-Fiorucci-Herfray-Ruzziconi '22]:

$$\begin{aligned}\mathcal{C}_2(\{u_1, z_1, \bar{z}_1\}^-_{J_1}, \{u_2, z_2, \bar{z}_2\}^+_{J_2}) &= \frac{1}{4\pi^2}\int_0^{+\infty} d\omega_1 \int_0^{+\infty} d\omega_2\, e^{-i\omega_1 u_1} e^{i\omega_2 u_2}\, \mathcal{A}_2(\{\omega_1, z_1, \bar{z}_1\}^-_{J_1}, \{\omega_2, z_2, \bar{z}_2\}^+_{J_2}) \\ &= \frac{\kappa^2_{J_1,J_2}}{4\pi}\int_0^{+\infty} \frac{d\omega}{\omega}\, e^{-i\omega(u_1 - u_2)}\, \delta^{(2)}(z_1 - z_2)\, \delta_{J_1,J_2}\,.\end{aligned}$$

- The divergent integral in the last line can be regularized. Instead, let us consider the correlator of ∂_u-descendants:

$$\tilde{\mathcal{C}}_2(\{u_1, z_1, \bar{z}_1\}^-_{J_1}, \{u_2, z_2, \bar{z}_2\}^+_{J_2}) = \lim_{\varepsilon \to 0^+} \frac{\kappa^2_{J_1,J_2}}{4\pi}\, \frac{1}{(u_{12} - i\varepsilon)^2}\, \delta^{(2)}(z_{12})\, \delta_{J_1,J_2}$$

$\Longrightarrow$ Standard solution of the Carrollian Ward identities (electric branch) for operators with fixed Carrollian weights $k + \bar{k} = 2$.

- Apply the Carroll/celestial relation:

$$\mathcal{M}_2(\{\Delta_1, z_1, \bar{z}_1\}^-_{J_1}, \{\Delta_2, z_2, \bar{z}_2\}^+_{J_2}) = 2\pi^2\, \kappa^2_{J_1,J_2}\, \delta(\Delta_1 + \Delta_2 - 2)\, \delta^{(2)}(z_{12})\, \delta_{J_1,J_2},$$

[Pasterski-Shao-Strominger '17]

$\Longrightarrow$ Price to pay to go from 3d Carrollian CFT to 2d CFT: distributional low-point functions, not standard in CFT.

- Analogue results for three and four point amplitudes ✓

n-point MHV Carrollian amplitude

- Colour ordered MHV gluon amplitude (with n + 1 identified with 1):

$$\mathcal{A}_n\left(1^-,2^-,3^+,\dots,n^+\right)=\kappa^{n-2}_{1,1,-1}\frac{\langle 12\rangle^4}{\prod_{j=1}^n\langle jj+1\rangle}=\kappa^{n-2}_{1,1,-1}\frac{\omega_1\omega_2}{\prod_{j=3}^n\omega_j}\frac{z_{12}^3}{\prod_{j=2}^n z_{jj+1}}$$

(similar formula for gravitons)

- Use the decomposition of the delta distribution [Schreiber-Volovich-Zlotnikov '17]:

$$\delta^{(4)}\left(\sum_{i=1}^n p_i\right)=\frac{1}{|\mathcal{U}_{1234}|}\prod_{I=1}^4\delta\left(\omega_I-\omega_I^*\right),\quad \text{with}\quad \omega_I^*=-\frac{1}{\mathcal{U}_{1234}}\sum_{i=5}^n\omega_i\mathcal{U}_{Ii}$$

where

$$\mathcal{U}_{1234}=\det\left(q_1^\mu,\dots,q_4^\mu\right),\qquad \mathcal{U}_{Ii}=\mathcal{U}_{1234}|_{I\to i},\quad I=1,2,3,4;\, i=5,\dots,n.$$

- Applying the Fourier transform [Mason-Ruzziconi-Yelleshpur Srikant '23]:

$$\tilde{\mathcal{C}}_n\left(1^-,2^-,3^+,\dots,n^+\right)=\frac{\kappa^{n-2}_{1,1,-1}}{(2\pi)^n\,|\mathcal{U}_{1234}|}\frac{z_{12}^3}{\prod_{j=2}^n z_{jj+1}}\frac{\partial^4}{\partial u_1^2\,\partial u_2^2}I_n$$

where the integral I_n can be computed explicitly:

$$I_n=\int\prod_{j=5}^n d\omega_j e^{i\omega_j L_j}\prod_{I=1}^4\Theta\left(\omega_I^*\right)=(-1)^{n-4}\prod_{j=5}^n\frac{1}{L_j}$$

with $L_j=\left(\epsilon_j u_j-\sum_{J=1}^4\epsilon_J u_J\frac{\mathcal{U}_{Jj}}{\mathcal{U}_{1234}}\right)$

- Non-trivial dynamical constraints on the dual Carrollian CFT.
- Similar expression for the MHV graviton amplitude (same integral I_n).
- Surprisingly simpler than its celestial counterparts involving Aomoto-Gelfand hypergeometric function. [Schreiber-Volovich-Zlotnikov '17]

Collinear limit and Carrollian OPE

- Collinear limit of two outgoing particles ($\epsilon_1 = \epsilon_2 = +1$):

$$\mathcal{A}_n\left(1^{J_1}, 2^{J_2}, 3^{J_3}, \dots, n^{J_n}\right) \xrightarrow{1||2} \sum_J \mathcal{A}_3\left(1^{J_1}, 2^{J_2}, -P^{-J}\right) \frac{1}{\langle 12\rangle[21]} \mathcal{A}_{n-1}\left(P^J, 3^{J_3}, \dots, n^{J_n}\right)$$

  where J is the helicity of the exchanged particle.

- In the limit $z_{12} \to 0$, we obtain the Carrollian OPE block [Mason-Ruzziconi-Yelleshpur Srikant '23]:

$$\begin{aligned}
&\Phi_{J_1}(u_1, z_1, \bar{z}_1)\,\Phi_{J_2}(u_2, z_2, \bar{z}_2) \\
&\sim -\frac{\kappa_{J_1,J_2,-J}}{2\pi}\frac{\bar{z}_{12}^p}{z_{12}}\int_0^1 dt\, t^{J_2-J-1}(1-t)^{J_1-J-1}\left(\frac{\partial}{\partial u}\right)^p \Phi_J(u, z_2, \bar{z}_2 + t\bar{z}_{12})|_{u=u_2+tu_{12}} \\
&\sim -\frac{\kappa_{J_1,J_2,-J}}{2\pi z_{12}}\sum_{m,n=0}^{\infty} B(J_2 - J + m + n, J_1 - J)\frac{\bar{z}_{12}^{p+m}u_{12}^n}{m!n!}\left(\frac{\partial}{\partial \bar{z}_2}\right)^m\left(\frac{\partial}{\partial u_2}\right)^{p+n}\Phi_J(u_2, z_2, \bar{z}_2)
\end{aligned}$$

  with implicit sum on $p = J_1 + J_2 - J - 1$ with range determined by

$$p \geq 0, \qquad |J_1 + J_2 - p - 1| \leq 2 \quad \text{and} \quad |J_1| \leq 2, \qquad |J_2| \leq 2.$$

- Invariance under global $\mathfrak{CCarr}_3$ explicitly checked ✓
- Using the Carroll/celestial correspondence, we recover the celestial OPE block

$$\mathcal{O}_{\Delta_1,J_1}(z_1, \bar{z}_1)\,\mathcal{O}_{\Delta_2,J_2}(z_2, \bar{z}_2) \sim -\kappa_{J_1,J_2,-J}\frac{\bar{z}_{12}^p}{z_{12}}\int_0^1 dt\, t^{2\bar{h}_1+p-1}(1-t)^{2\bar{h}_2+p-1}\mathcal{O}_{\Delta_1+\Delta_2+p-1,J}.$$

  [Fan-Fotopoulos-Taylor '19] [Pate-Raclariu-Strominger-Yuan '19]

The flat limit of AdS

- AdS Witten diagrams are encoded into boundary CFT correlators in position space.
  $\Longrightarrow$ Carrollian amplitudes are natural objects obtained in the flat limit $\ell \to \infty$.
  (Related works: [Pipolo de Gioia-Raclariu '23] [Bagchi-Dhivakar-Dutta '23])
- Bondi coordinates also exist in AdS!
  1. Flat limit in the bulk: $ds^2_{AdS} = -\frac{r^2}{\ell^2} du^2 - 2dudr + 2r^2 dzd\bar{z} \quad \Longrightarrow \quad ds^2_{Flat} = -2dudr + 2r^2 dzd\bar{z}$
  2. Carrollian limit at the boundary: $ds^2_{\mathscr{I}_{AdS}} = -\frac{1}{\ell^2} du^2 + 2dzd\bar{z} \quad \Longrightarrow \quad ds^2_{\mathscr{I}} = 0du^2 + 2dzd\bar{z}$

  $\Longrightarrow$ This observation extends perfectly at the level of Einstein equations.
  [Barnich-Gomberoff-Gonzalez '12] [Poole-Skenderis-Taylor '18] [Compère-Fiorucci-Ruzziconi '19]
- Does it work at the level of the amplitudes?
- Example: four-point contact diagram:

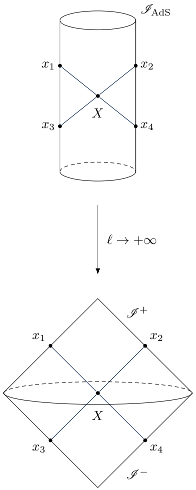
$$\langle \mathcal{O}_1(x_1)\mathcal{O}_2(x_2)\mathcal{O}_3(x_3)\mathcal{O}_4(x_4)\rangle = \int_{AdS} d^4X G^{AdS}_{B\partial}(x_1, X) G^{AdS}_{B\partial}(x_2, X) G^{AdS}_{B\partial}(x_3, X) G^{AdS}_{B\partial}(x_4, X)$$

$$\Big\downarrow c \to 0 \qquad\qquad\qquad \Big\downarrow \ell \to \infty$$

$$\langle \Phi_1(x_1)\Phi_2(x_2)\Phi_3(x_3)\Phi_4(x_4)\rangle = \int_{Flat} d^4X G^{Flat}_{B\partial}(x_1, X) G^{Flat}_{B\partial}(x_2, X) G^{Flat}_{B\partial}(x_3, X) G^{Flat}_{B\partial}(x_4, X)$$

where $G^{Flat}_{B\partial}(x, X) = \lim_{\ell\to\infty} G^{AdS}_{B\partial}(x, X) = \frac{1}{(-u_x - q_x \cdot X \pm i\varepsilon)}$.

$\Longrightarrow$ Connection between AdS/CFT and flat space holography.



Summary and perspectives

- Two complementary approaches to flat space holography:

  Carrollian holography $\Longleftrightarrow$ Celestial holography

- The two approaches are related via integral transform ✓
- Carrollian holography is a useful path:

  $\Longrightarrow$ Naturally related to AdS/CFT via $\ell \to \infty \to 0$ ✓

  $\Longrightarrow$ Successful in 3d gravity, fluid/gravity correspondence, 4d Einstein's equations ✓

  $\Longrightarrow$ The flat limit seems to work at the level of amplitudes. ✓
- Carrollian physics has applications beyond flat space holography:
  Black hole, cosmology, condensed matter, fluid...
- Perspectives:

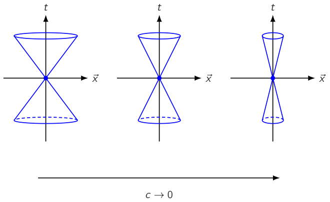
  $\Longrightarrow$ Flat limit of AdS: how far can we go?

  $\Longrightarrow$ Top-down models for Carrollian holography? via $c \to 0$ limit?

Thank you!

Carrollian algebra

- "Carroll" refers to the limit $c \to 0$ where c is the speed of light [Lévy-Leblond '65].
  $\Longrightarrow$ Opposite to the usual Galilean limit ($c \to \infty$).
- Carrollian limit of the Poincaré algebra:
  $\Longrightarrow$ Translations $H = \partial_t$, $P_i = \partial_i$ and rotations $J_{ij} = x_i \partial_j - x_j \partial_i$ are unchanged.
  $\Longrightarrow$ Boosts are affected: $B_i = c^2 t \partial_i - x_i \partial_t \xrightarrow{c \to 0} B_i = -x_i \partial_t$.
- Carrollian boosts shift time but do not affect space:
$$t' = t - \vec{b}.\vec{x}, \qquad \vec{x}' = \vec{x}$$
  $\Longrightarrow$ Space becomes absolute (see diagram).
  $\Longrightarrow$ Opposite to the usual Galilean limit ($c \to \infty$) where time becomes absolute:
$$t' = t, \qquad \vec{x}' = \vec{x} - \vec{v}t$$



- Carrollian algebra (Ionu-Wigner contraction of Poincaré algebra when $c \to 0$):
$$[B_i, H] = 0, \quad [B_i, B_j] = 0, \quad [B_i, P_j] = \delta_{ij} H, \quad [B_k, J_{ij}] = \delta_{k[i} B_{j]}, \quad [P_k, J_{ij}] = \delta_{k[i} P_{j]}$$
- (Global) conformal Carrollian algebra (Ionu-Wigner contraction of the conformal algebra $SO(d,2)$ when $c \to 0$):
  $\Longrightarrow$ Add the dilatation: $D = (t\partial_t + x^i \partial_i)$, and the Carrollian special conformal generators: $K = x^2 \partial_u$ and $K_i = x^2 \partial_i - 2x_i x^j \partial_j - 2x_i t \partial_t$.

Modified Mellin transform

- If $\Phi_{(k,\bar{k})}(u,z,\bar{z})$ is a Carrollian primary, then $\partial_u \Phi_{(k,\bar{k})}(u,z,\bar{z})$ is also a Carrollian primary with shifted weights $(k+\frac{1}{2}, \bar{k}+\frac{1}{2})$.
- Remark: the leading order of the Peeling is $\partial_u^{|J|}\Phi$. For gravity, $\Psi_4^0 = \partial_u^2 C_{zz} \implies$ Transforms as primary under the full $\mathfrak{CCarr}_4$ algebra. [Mason-Ruzziconi-Yelleshpur Srikant '23]
- Descendants:

$$\mathcal{C}_n^{m_1\dots m_n}\left(\{u_1,z_1,\bar{z}_1\}_{J_1}^{\epsilon_1},\dots,\{u_n,z_n,\bar{z}_n\}_{J_n}^{\epsilon_n}\right) = \partial_{u_1}^{m_1}\dots\partial_{u_n}^{m_n}\mathcal{C}_n\left(\{u_1,z_1,\bar{z}_1\}_{J_1}^{\epsilon_1},\dots,\{u_n,z_n,\bar{z}_n\}_{J_n}^{\epsilon_n}\right)$$
$$= \prod_{i=1}^n\left(\int_0^{+\infty}\frac{d\omega_i}{2\pi}(i\epsilon\omega_i)^{m_i}e^{i\epsilon_i\omega_i u_i}\right)\mathcal{A}_n\left(\{\omega_1,z_1,\bar{z}_1\}_{J_1}^{\epsilon_1},\dots,\{\omega_n,z_n,\bar{z}_n\}_{J_n}^{\epsilon_n}\right)$$
$$= \langle \partial_{u_1}^{m_1}\Phi^{\epsilon_1}_{(k_1,\bar{k}_1)}(u_1,z_1,\bar{z}_1)\dots\partial_{u_n}^{m_n}\Phi^{\epsilon_n}_{(k_n,\bar{k}_n)}(u_n,z_n,\bar{z}_n)\rangle$$

- In particular, $\tilde{\mathcal{C}}_n \equiv \mathcal{C}_n^{1\dots 1}$.
- Redundant to encode the $\mathcal{S}$-matrix, but can be useful to get rid of IR divergences (see example of the 2-point function).
- Remark: analytic continuation $m_i = \delta_i - 1$ ($\delta_i \in \mathbb{C}$) gives

$$\prod_{i=1}^n\left(\int_0^{+\infty}\frac{d\omega_i}{2\pi}(i\epsilon\omega_i)^{\delta_i-1}e^{i\epsilon_i\omega_i u_i}\right)\mathcal{A}_n\left(\{\omega_1,z_1,\bar{z}_1\}_{J_1}^{\epsilon_1},\dots,\{\omega_n,z_n,\bar{z}_n\}_{J_n}^{\epsilon_n}\right),$$

$\implies$ Modified Mellin transform used to regularize Mellin transform of graviton amplitudes. [Banerjee '18] [Banerjee-Ghosh-Pandey-Saha '20]

$\implies$ Clarifies the link with the alternative approach to Carrollian amplitudes. [Bagchi-Banerjee-Basu-Dutta '22]

Three-point Carrollian amplitude

- The three-point amplitude generically vanishes in Lorentzian signature $\Longrightarrow$ Go to split $(2,2)$ signature.

$$p_i^\mu = \epsilon_i q_i^\mu = \epsilon_i \omega_i \left(1 + z_i \bar{z}_i, z_i + \bar{z}_i, z_i - \bar{z}_i, 1 - z_i \bar{z}_i\right).$$

  Here $(z_i, \bar{z}_i)$ are coordinates on a Poincaré patch of $\mathcal{LT}_2$ and $\epsilon_i = \pm 1$ labels the Poincaré patches.

- Using spinor-helicity notations, $p_{\alpha\dot{\alpha}} \equiv \sigma^\mu_{\alpha\dot{\alpha}} p_\mu = \kappa_\alpha \tilde{\kappa}_{\dot{\alpha}}$ and $[ij] = \tilde{\kappa}_{i\dot{\alpha}} \tilde{\kappa}_j^{\dot{\alpha}}$, at tree-level, the three-point amplitude reads as

$$\mathcal{A}_3(1^{J_1}, 2^{J_2}, 3^{J_3}) = \kappa_{J_1,J_2,J_3} [12]^{J_1+J_2-J_3} [23]^{J_2+J_3-J_1} [31]^{J_3+J_1-J_2} \delta^{(4)}(p_1 + p_2 + p_3), \text{ if } J_1 + J_2 + J_3 > 0$$

  A similar expression exists for $J_1 + J_2 + J_3 + 2 < 0$.

- Three-point Carrollian amplitude (still determined by Ward identities):
  [Banerjee-Ghosh-Pandey-Saha '20] [Salzer '23] [Mason-Ruzziconi-Yelleshpur Srikant '23]

$$\tilde{C}_3 = \kappa_{J_1,J_2,J_3} \frac{-i\epsilon_1\epsilon_2\epsilon_3\, \delta(z_{12})\, \delta(z_{23})}{4(2\pi)^3} \Theta\left(-\frac{\bar{z}_{13}}{\bar{z}_{23}} \epsilon_1 \epsilon_2\right) \Theta\left(\frac{\bar{z}_{12}}{\bar{z}_{23}} \epsilon_1 \epsilon_3\right) |\bar{z}_{12}|^{J_1+J_2} |\bar{z}_{23}|^{J_2+J_3} |\bar{z}_{31}|^{J_3+J_1}$$
$$\times (\text{sign } \bar{z}_{12})^{J_1+J_2-J_3+1} (\text{sign } \bar{z}_{23})^{J_2+J_3-J_1+1} (\text{sign } \bar{z}_{13})^{J_1+J_3-J_2+1} \frac{(i\,\epsilon_1 \text{ sign}(\bar{z}_{23}))^{J_1+J_2+J_3+2}\, \Gamma(J_1 + J_2 + J_3 + 2)}{(\bar{z}_{23} u_1 - \bar{z}_{13} u_2 + \bar{z}_{12} u_3 + i\epsilon_1 \text{ sign}(\bar{z}_{23})\varepsilon)^{J_1+J_2+J_3+2}}$$

  for $J_1 + J_2 + J_3 + 2 > 0$ (similarly for $J_1 + J_2 + J_3 + 2 > 0$).

- Using Carroll/celestial correspondence ($\bar{h}_k = \frac{\Delta_k - J_k}{2}$):

$$\mathcal{M}_3 = \frac{(-i)^{J_1+J_2+J_3}\, \pi}{2} \kappa_{J_1,J_2,J_3} \delta(z_{12})\, \delta(z_{23})\, \Theta\left(-\frac{\bar{z}_{13}}{\bar{z}_{23}} \epsilon_1 \epsilon_2\right) \Theta\left(\frac{\bar{z}_{12}}{\bar{z}_{23}} \epsilon_1 \epsilon_3\right) \frac{1}{\bar{z}_{12}^{\bar{h}_1+\bar{h}_2-\bar{h}_3} \bar{z}_{23}^{\bar{h}_2+\bar{h}_3-\bar{h}_1} \bar{z}_{12}^{\bar{h}_3+\bar{h}_1-\bar{h}_2}}$$
$$\times (\text{sign } \bar{z}_{12})^{J_1+J_2-J_3} (\text{sign } \bar{z}_{23})^{J_2+J_3-J_1} (\text{sign } \bar{z}_{13})^{J_1+J_3-J_2} (\epsilon_1)^{\Delta_1} (\epsilon_2)^{\Delta_2} (\epsilon_3)^{\Delta_3} \delta(\Delta_1 + \Delta_2 + \Delta_3 + J_1 + J_2 + J_3 - 4).$$

  [Pasterski-Shao-Strominger '17]

Four-point Carrollian amplitude

- At tree-level the 4-point gluon MHV amplitude is given by

$$\mathcal{A}_4\left(1^{+1},2^{-1},3^{-1},4^{+1}\right)=\kappa^2_{1,1,-1}\frac{\langle 23\rangle^4}{\langle 12\rangle\langle 23\rangle\langle 34\rangle\langle 41\rangle}=\kappa^2_{1,1,-1}\frac{\omega_2\omega_3}{\omega_1\omega_4}\frac{z_{23}^3}{z_{12}z_{34}z_{41}},$$

- Applying the Fourier transform yields the corresponding Carrollian amplitude (very similar computation for gravitons):
[Banerjee-Ghosh-Pandey-Saha '20] [Mason-Ruzziconi-Yelleshpur Srikant '23]

$$\tilde{\mathcal{C}}_4\left(1^{+1},2^{-1},3^{-1},4^{+1}\right)=\frac{\kappa^2_{1,1,-1}}{(2\pi)^4}\frac{z_{34}^2\bar{z}_{14}^4\bar{z}_{34}^2}{z^3(1-z)z_{13}^3z_{24}\bar{z}_{13}^5\bar{z}_{24}^3}\delta\left(z-\bar{z}\right)\Theta\left(-z\left|\frac{z_{24}}{z_{12}}\right|^2\epsilon_1\epsilon_4\right)\Theta\left(\frac{1-z}{z}\left|\frac{z_{34}}{z_{23}}\right|^2\epsilon_2\epsilon_4\right)$$
$$\Theta\left(-\frac{1}{1-z}\left|\frac{z_{14}}{z_{13}}\right|^2\epsilon_3\epsilon_4\right)\times\frac{3!}{\left(u_4-u_1z\left|\frac{z_{24}}{z_{12}}\right|^2+u_2\frac{1-z}{z}\left|\frac{z_{34}}{z_{23}}\right|^2-u_3\frac{1}{1-z}\left|\frac{z_{14}}{z_{13}}\right|^2\right)^4},$$

- Using Carroll/celestial correspondence:

$$\mathcal{M}_4=\prod_{i=1}^{4}\left(\int_{-\infty}^{+\infty}du_i\,(-i\epsilon_i)^{\Delta_i}\Gamma(\Delta_i-1)\,u_i^{1-\Delta_i}\right)\tilde{\mathcal{C}}_4$$
$$=\prod_{i=1}^{4}(-i\epsilon_i)^{\Delta_i}z^{-\frac{1}{3}}\,(1-z)^{\frac{5}{3}}\prod_{i<j}z_{ij}^{\frac{h}{3}-h_i-h_j}\bar{z}_{ij}^{\frac{\bar{h}}{3}-\bar{h}_i-\bar{h}_j}(-1)^{\Delta_2+\Delta_4+1}2\pi\delta\left(\Delta_1+\Delta_2+\Delta_3+\Delta_4-4\right)$$
$$\delta\left(z-\bar{z}\right)\Theta\left(-z\left|\frac{z_{24}}{z_{12}}\right|^2\epsilon_1\epsilon_4\right)\Theta\left(\frac{1-z}{z}\left|\frac{z_{34}}{z_{23}}\right|^2\epsilon_2\epsilon_4\right)\Theta\left(-\frac{1}{1-z}\left|\frac{z_{14}}{z_{13}}\right|^2\epsilon_3\epsilon_4\right).$$

[Pasterski-Shao-Strominger '17]

Celestial symmetries and twistor space

- Action of $Lw_{1+\infty}$ at $\mathscr{I}$? $\Longrightarrow$ Can be deduced from Carrollian OPEs.
- Definite the (outgoing) soft operators [Guevara-Himwich-Pate-Strominger '21]

$$H_J^k \equiv \lim_{\Delta\to k}(\Delta - k)\Gamma(\Delta-1)(-i)^\Delta \int_{-\infty}^{\infty} du\, u^{1-\Delta}\partial_u\Phi_J(u,z,\bar z)\,, \qquad k = 1,0,-1,-2,\dots$$

- Action of $Lw_{1+\infty}$ on gravitons [Mason-Ruzziconi-Yelleshpur Srikant '23]:

$$H_2^k(z_1,\bar z_1)\,\partial_{u_2}\Phi_2(u_2,z_2,\bar z_2) \sim -\frac{\kappa_{2,2,-2}}{z_{12}}\sum_{m=0}^{1-k}\frac{\bar z_{12}^{m+1}}{m!}\frac{(-iu_2)^{1-k-m}}{(1-k-m)!}\left(-i\partial_{u_2}\right)^{2-m}\partial_{\bar z_2}^m\Phi_2(u_2,z_2,\bar z_2)\,.$$

$\Longrightarrow$ Local for $k \geq -1$ corresponding to universal soft theorems.

- The action of $Lw_{1+\infty}$ on twistor space is local and has a geometric interpretation (Poisson diffeomorphisms). [Adamo-Mason-Sharma '21] [Bu-Heuveline-Skinner '22] [Mason '22]
- Natural to relate Carrollian CFT at $\mathscr{I}$ and twistor space:

$$\Phi_J(u,\lambda,\tilde\lambda) = \partial_u^{|J|}\int d^2\mu\,\delta(u-[\mu\tilde\lambda])f(\lambda_\alpha,\mu^{\dot\alpha})$$

where we introduced homogeneous coordinates $(u,\lambda_\alpha,\tilde\lambda_{\dot\alpha}) = \left(\lambda_0\tilde\lambda_{\dot 0}u_B,\lambda_0 z_\alpha\,,\tilde\lambda_{\dot 0}\bar z_{\dot\alpha}\right)$.

$\Longrightarrow$ Upcoming work: what is the structure preserved by $Lw_{1+\infty}$ at $\mathscr{I}$?