QCD amplitudes in self-dual backgrounds from large N chiral algebras

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One difficulty with celestial holography is that top-down models are hard to build.

With Paquette and Sharma we proposed a model based on Burns space. Slightly unsatisfying features of this:

1. The $4d$ theory is the WZW_4 model. It is non-Lorentz invariant, and perhaps a little unphysical.
2. Burns space is an unusual asymptotically flat manifold.

Here we will give a new class of top-down models. We will define a defect chiral CFT such that

1. The large N algebra of operators in the chiral defect CFT is the celestial chiral algebra of self-dual QCD.
2. We can engineer a wide variety of self-dual backgrounds
3. The simplest background is flat space with a background axion.
4. The construction works at loop level.

Background

Self-dual QCD has the action

$$\int \mathrm{tr}(B \wedge F(A)) + \psi \bar{\partial}_A \psi.$$

for $B \in \Omega^2_-(\mathbb{R}^4, \mathfrak{g})$.

1. It can be deformed to full QCD by adding $g_{MY}^2 \,\mathrm{tr}(B^2)$.
2. Form factors in SDQCD match certain amplitudes/form factors in QCD at low loop number. E.g. $\mathrm{tr}(B^2)$ form factors in SDQCD capture certain QCD amplitudes up to two loops.

Theorem (KC, Natalie Paquette)

If the matter representation R satisfies

$$\mathrm{tr}_R(X^4) = \mathrm{tr}_{\mathfrak{g}}(X^4)$$

then there exists a chiral algebra whose correlation functions are form factors of SDQCD.

Today's talk:

We will show how to build this chiral algebra as a large N limit of a 2d defect CFT, embedded in string theory.

Anomaly free SDQCD comes from a holomorphic theory on twistor space

$$\mathbb{PT} = \mathscr{O}(1) \oplus \mathscr{O}(1) \to \mathbb{CP}^1$$

We are interested in the theory gauge group $G = \mathrm{Sp}(K)$, matter in $2 \wedge^2 F + 16F$.

Easy observation

The twistor uplift of this gauge theory is the holomorphic twist of the theory on $D5$ branes wrapping $\mathbb{PT}$ in the type I string, on the geometry

$$X = \mathscr{O}(-1) \oplus \mathscr{O}(-3) \to \mathbb{PT}.$$

with non-trivial SO(32) bundle (symmetry broken to $U(16)$).

Holomorphic twist = 1/8 BPS subsector. This means we can hope to understand SDQCD by thinking about the type I string.

Variants:

1. $D5$ in type IIB gives $N = 1$ SQCD.
2. $M5$ branes wrapping $\mathbb{PT}$ in $\mathbb{R} \times X$ give a $4d$ non-Lagrangian non-unitary CFT. (No idea what this is! Except for a single $M5$ brane).
3. Many more constructions involving orientifolds, F-theory, etc. etc.

We would like to use this to understand scattering amplitudes / form factors of SDQCD. To do this we will use holography.

Brane set up:

	$\mathscr{O}(-1)$	$\mathscr{O}(-3)$	$\mathscr{O}(1)$	$\mathscr{O}(1)$	$\mathbb{CP}^1$
Coordinate	w_1	w_2	$v_{\dot 1}$	$v_{\dot 2}$	z
K $D5'$			$\times$	$\times$	$\times$
N $D5$	$\times$	$\times$			$\times$

On the $D5 - D5'$ intersection is a chiral algebra $\mathcal{A}_{N,K}$ (viewed as a defect in N $D5$ system).

On the N $D5$ branes is a subalgebra $\mathcal{A}_N$ of bulk operators which have trivial OPEs with everything.

Conjecture

There is a holographic duality

$$\lim_{N\to\infty} \mathcal{A}_{N,K} / \textit{ bulk operators } =$$

chiral algebra of SDQCD in a self dual background

How to understand this:

1. States in $\mathcal{A}_N$ are states of type I string on after backreacting.
2. States in $\mathcal{A}_{N,K}$ are states in type I string plus K $D5'$ in backreacted geometry.
3. Giving a VEV to states in $\mathcal{A}_N$ leaves only states for the $D5'$ system. Key point: $\mathcal{A}_N$ is in the center of the algebra.
4. This is the twistor uplift of SDQCD in a self-dual background determined by both the backreaction and the VEV given to bulk states.

We are able to determine the backreacted geometries and perfom extensive checks of the duality, and use it to compute new things in gauge theory.

The algebra $\mathcal{A}_{N,K}$

States in $\mathcal{A}_{N,K}$ are build from:

1. ψ are $D5 - D5'$ strings. These are $\mathrm{Sp}(K) \times \mathrm{Sp}(N)$ bifundamental fermions with $\psi(0)\psi(z) \simeq 1/z$.
2. Bulk fields are ϕ, $D5 - D5$ strings, charged under $\mathrm{Sp}(N)$, and b, c ghosts for $\mathrm{Sp}(N)$,
3. $\gamma, \widetilde{\gamma}$ which are $D5 - D9$ strings, bifundamental $\mathrm{Sp}(N) \times U(16)$ bosons.

BRST cohomology computation tells us that defect states in $\mathcal{A}_{N,K}$ are

$$\text{Positive helicity gluon } \mathbb{J}[\widetilde{\lambda}](z) = \left\{ \psi \exp\left(\phi_{\dot{\alpha}} \widetilde{\lambda}^{\dot{\alpha}} \right) \psi \right\}(z)$$

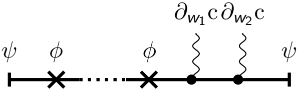
$$\text{Negative helicity gluon } \widetilde{\mathbb{J}}[\widetilde{\lambda}](z) = \left\{ \psi \partial_{w_1} \mathrm{c} \partial_{w_2} \mathrm{c} \exp\left(\phi_{\dot{\alpha}} \widetilde{\lambda}^{\dot{\alpha}} \right) \psi \right\}(z)$$

$$\cdots$$

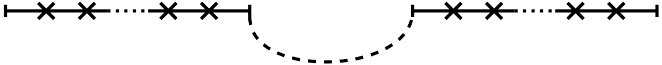
precisely matching the spectrum of SDQCD.

Pictorial representation of states:

$\mathbb{J} =$ 

$\widetilde{\mathbb{J}} =$ 

The simplest OPEs



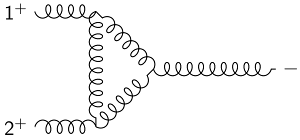
give

$$\mathbb{J}[\widetilde{\lambda}^{(1)}](z_1)\mathbb{J}[\lambda^{(2)}](z_2) \simeq \frac{1}{\langle 12 \rangle}\mathbb{J}[\lambda^{(1)} + \lambda^{(2)}](z_1)$$

the tree level collinear singularity in SDQCD.

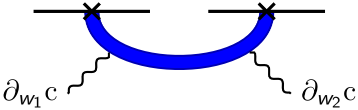
The collinear singularities of positive helicity gluons has a well-known one-loop correction

$$\mathbb{J}[\widetilde{\lambda}^{(1)}](z_1)\mathbb{J}[\widetilde{\lambda}^{(2)}](z_2) \simeq \frac{[12]}{\langle 12\rangle^2}\widetilde{\mathbb{J}}[\widetilde{\lambda}^{(1)}+\widetilde{\lambda}^{(2)}](z_1) + \text{ lower order poles}$$



To compute more complicated OPEs we need first the OPE of the ϕ fields:

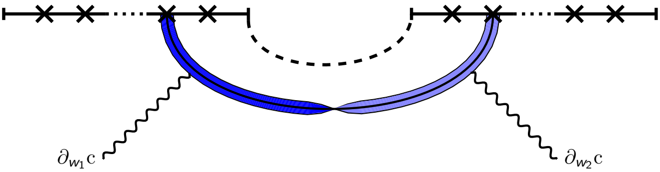
$$\phi_{\dot\alpha}(z_1)\phi_{\dot\beta}(z_2) \sim \epsilon_{\dot\alpha\dot\beta}\frac{1}{z_1 - z_2}\partial_{w_1}\mathrm{c}\partial_{w_2}\mathrm{c}$$



The $\partial\mathrm{c}$'s arise because these are bulk and not defect fields. The blue line is a bulk propagator.

We can see the loop level corrections to the tree level collinear singularities

$$\mathbb{J}[\widetilde{\lambda}^{(1)}](z_1)\mathbb{J}[\widetilde{\lambda}^{(2)}](z_2) \simeq \frac{[12]}{\langle 12\rangle^2}\widetilde{\mathbb{J}}[\widetilde{\lambda}^{(1)} + \widetilde{\lambda}^{(2)}](z_1) + \ldots$$



Subleading terms (with only $\phi - \phi$ propagator) also match.

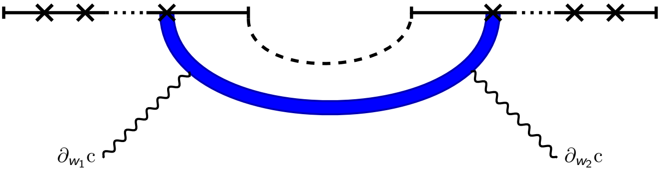
The backreaction

The limit $\lim_{N\to\infty} \mathcal{A}_{N,K}/$bulk states is states of 4d SDQCD in a backreacted geometry.

The basic backreaction (all bulk states $= 0$) is a background axion field

$$\frac{N}{2\pi \mathrm{i}} \int F \wedge F \log \|x\|^2$$

The diagram



corrects collinear limits in this background.

Other backgrounds

Giving a VEV to bulk states deforms to a new self-dual background. Examples:

$$\begin{aligned}
\mathrm{tr}(\phi^n) &\Longleftrightarrow \text{ Different background axion} \\
\gamma\widetilde{\gamma} &\Longleftrightarrow \text{ Background gauge field for } \mathfrak{gl}_{16} \text{ flavour symmet} \\
\mathrm{tr}(\partial \mathrm{c} \partial \mathrm{c}) &\Longleftrightarrow \text{ Burns metric} \\
\mathrm{tr}(\phi \partial \phi \partial \mathrm{c} \partial \mathrm{c} + \dots) &\Longleftrightarrow \text{ Another gravitational background}
\end{aligned}$$

The chiral algebra for SDQCD in any one of these (infinitely many) backgrounds has a large N description.

Gravitational backgrounds

Two basic classes of gravitational backgrounds:

1. Burns space, where $\mathrm{tr}(\partial c \partial c)$ is given a VEV. Planar computations in the chiral algebra are completely consistent with previous work of C., Paquette, and Sharma.
2. Giving $\mathrm{tr}(\phi \partial c \partial c \partial c + \dots)$ a VEV

$$\langle \mathrm{tr}(\phi \partial c \partial c \partial c + \dots)(z) \rangle = H(z)$$

introduces a source for the Weyl curvature tensor

$$W_{\alpha_1 \alpha_2 \alpha_3 \alpha_4} = \delta_{x=0} H_{\alpha_1 \alpha_2 \alpha_3 \alpha_4}$$

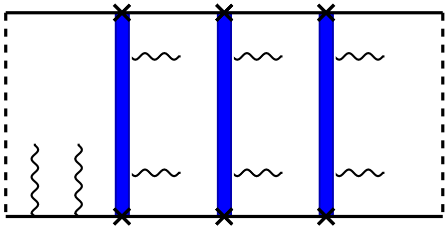
This metric is, away from $x = 0$, the double cover of Eguchi-Hansen.

Example: Burns space 2 point function

Burns space is conformally equivalent to $\mathbb{CP}^2$. From work of Gibbons-Hawking it follows that two point function of gluons on Burns space

$$\mathcal{A}(1^-2^+) = \frac{\langle 1\lambda\rangle^2 \langle 1\mu\rangle^2}{\langle 12\rangle^2} J_0\left(\langle 1\lambda\rangle \langle 1\mu\rangle \langle 2\lambda\rangle \langle 2\mu\rangle \sqrt{\frac{4[12]}{\langle 12\rangle}} \right)$$

where spinors λ, μ are parameters of the metric and J_0 is a Bessel function. This matches the chiral algebra:



Flavour symmetry backgrounds

Giving a VEV

$$\left\langle \gamma_P \widetilde{\gamma}^Q(z) \right\rangle = G_P^Q(z)$$

gives us a source for the background $U(16)$ flavour symmetry gauge field $\mathcal{A}_P^Q$ by

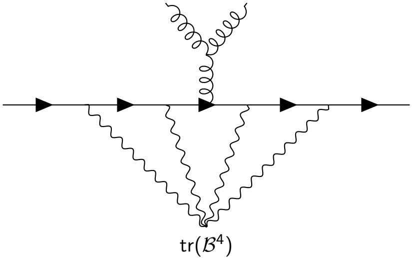
$$F_{\alpha\beta}(\mathcal{A})_P^Q = \delta_{x=0} G^Q_{\alpha\beta P}.$$

Special case:

$$F_{11}(\mathcal{A}) = N\delta_{x=0} \quad N \in U(1) \subset SU(16)$$

then amplitudes in this background are the same as form factors for the operator $e^{\mathcal{B}}$ in SDQCD coupled ot Maxwell theory for flavour symmetry.

We can use this technology to compute these form factors at arbitrary loop number (equivalently, tree level amplitudes in a strong field):



For example, the chiral algebra tells us that with two fermions and one gluon the amplitude in the strong Maxwell field sourced by

$$F^{Q}_{\alpha\beta P} = \lambda_\alpha \lambda_\beta N^{Q}_{P} \delta_{x=0}$$

for $N \in U(16)$ is

$$\frac{-\mathrm{i}}{\langle 12\rangle \langle 23\rangle} \sum_{a,b,c\geq 0} \frac{\langle 1\lambda\rangle^{a+b} \langle 2\lambda\rangle^{a+c} \langle 3\lambda\rangle^{b+c+2} [12]^a [13]^b [23]^c N^{2a+2b+2c+1}}{(a+b)!(a+c)!(b+c)!(a+b+c+1)!}$$