Dibaryons from Lattice QCD

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for HAL QCD Collaboration

HAL (Hadrons to Atomic nuclei from Lattice) QCD Collaboration

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  - NΩ, ΔΔ, ΩΩ interactions
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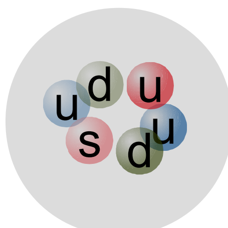
Introduction

Dibaryon candidates

- ***What is dibaryon?***

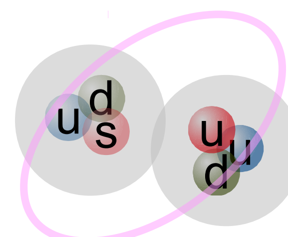
Bound and/or resonance two-baryon states

Tightly bound system



Compact exotic hadron

Molecular-like system



Deuteron-like behavior

Depend on the details of their interaction

Short range interactions

Repulsive core for general case of BB interactions?

Long range interactions

Strength of meson exchange contributions for each channel

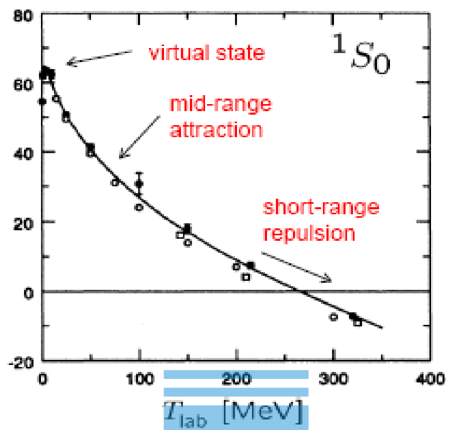
We need deep understandings of baryon-baryon interaction

How do we obtain the baryon force?

Phenomenological descriptions

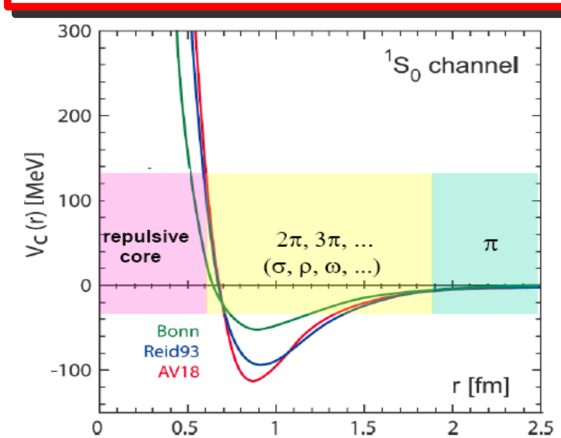
Traditional process to derive the BB interaction (potential)

Scattering observables



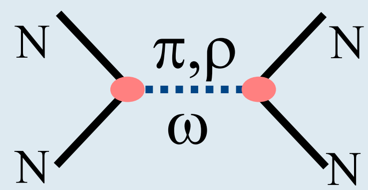
Model assumption

BB interaction (potential)



Meson exchange model

Described by hadron d.o.f.

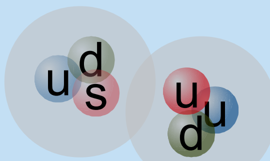


+ Phenomenological repul. core

H. Yukawa, PPMS17(1935)48
Th.A.Rijken et al, PTPS185(2010)14

Quark cluster model

Effective meson ex
+ quark anti-symmetrization

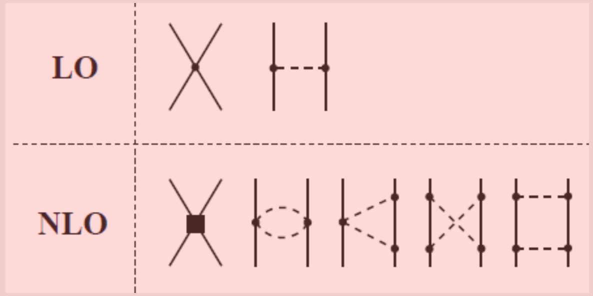


Quark Pauli effects
Color magnetic int.

M.Oka et al, PTPS137(2000)1
Y.Fujiwara et al, PPNP58(2007)439

Effective Field theory

Systematic calc. respecting with symmetry of QCD



Short range interaction is parametrized by contact term

E. Epelbaum et al, RMP81(2009)1773
R. Machleidt et al, PRept.503(2011)1
J. Haidenbauer et al, NPA954(2016)273

The models would be highly ambiguous if experimental data are scarce!

Clue to explore dibaryon candidates

We focus on the short range behavior of BB potential.

⟷ **Related to the tightly bound system.**

In view of constituent quark cluster picture

- **Short range interaction in between two baryons could be a result of Pauli principle and color-magnetic interaction for the quarks.**

- **Symmetry of constituent quarks**
  - **Assuming that all quarks are in s-orbit,**
    Flavor SU(3) x Spin SU(2) x color SU(3)
    If totally anti-symmetric : Pauli allowed state
    If not : Pauli forbidden state

- **Gluonic interaction between quarks at short range region**
  - **Gluon exchange contribution generates a color magnetic interaction**

$$V_{OGE}^{CMI} \propto \frac{1}{m_{q1} m_{q2}} \langle \lambda_1 \cdot \lambda_2 \sigma_1 \cdot \sigma_2 \rangle f(r_{ij})$$

Dibaryon candidates

Possibility of dibaryon from short range interactions

- **For octet-octet system**

$$8 \otimes 8 = 1 \oplus 8_s \oplus 27 \oplus 8_a \oplus 10 \oplus \overline{10}$$

Spin 0 (for 1): **Pauli allowed**, **Attractive color magnetic int.** → **H-dibaryon** (Coupled ΛΛ–NΞ–ΣΣ system)

- **For decuplet-octet system**

$$10 \otimes 8 = 35 \oplus 8 \oplus 10 \oplus 27$$

Spin 2 (for 8): **Pauli allowed**, **Attractive color magnetic int.** → **N-Ω system** (Ground state of coupled NΩ–ΛΞ*–ΣΞ*–ΞΣ* system)

- **For decuplet-decuplet system**

$$10 \otimes 10 = 28 \oplus 27 \oplus 35 \oplus \overline{10}$$

Spin 3 (for $\overline{10}$): **Pauli allowed**, **No color magnetic int.** → **ΔΔ system**

Spin 0 (for 28): **Pauli allowed**, **Repulsive color magnetic int.** → **ΩΩ system**

Dibaryon candidates

Some model calculations are performed for dibaryon candidates

- **H-dibaryon**
  - R.L.Jaffe PRL38(1977)
- **N-Ω system**
  - F.Wang et al. PRC51(1995)
  - Q.B.Li, P.N.Shen, EPJA8(2000)
- **ΔΔ and ΩΩ system**
  - F.J.Dyson,N-H.Xuong, PRL13(1964)
  - M.Oka, K.Yazaki, PLB 90(1980)
  - J. Haidenbauer, et al, nucl-th/1708.08071

- Predicted B.E. and structures are highly depend on the model parameters.
- Some of them are still not found in experiments.

We tackle this problem by Lattice QCD.

Baryon interaction from LQCD

BB interaction from QCD

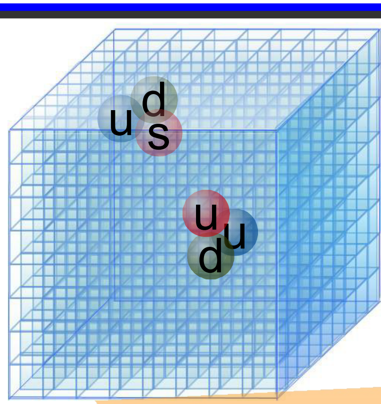
QCD is the fundamental theory of strong interactions

QCD Lagrangian

$$L_{QCD}=\bar{q}(i\gamma_\mu D^\mu - m)q + \frac{1}{4}F^a_{\mu\nu}F^{a\mu\nu}$$

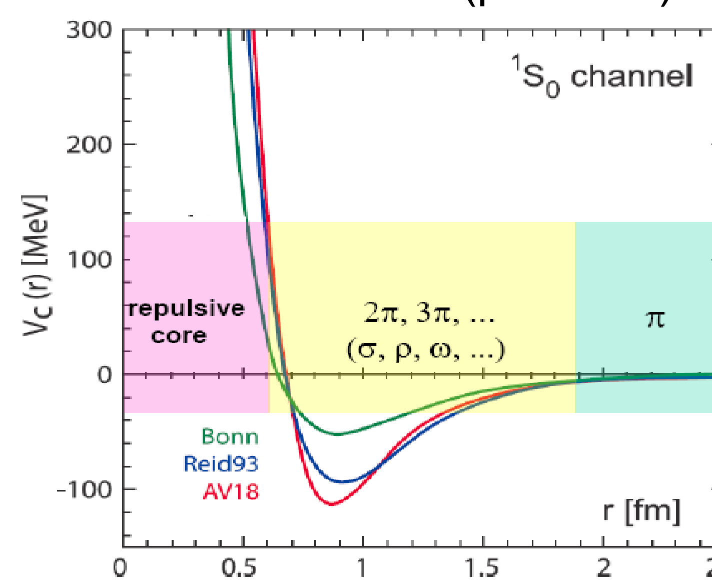
it is difficult to solve analytically the dynamics of quarks and gluons because of its non-perturbative nature at low-energies.

Lattice QCD simulation



- **Non-perturbative calculation.**
- **Huge computer resource is required.**
- **Independent of experimental situation.**

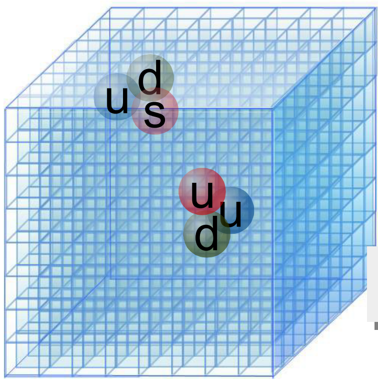
BB interaction (potential)



HAL QCD method

Hadron interaction from LQCD

Lattice QCD simulation

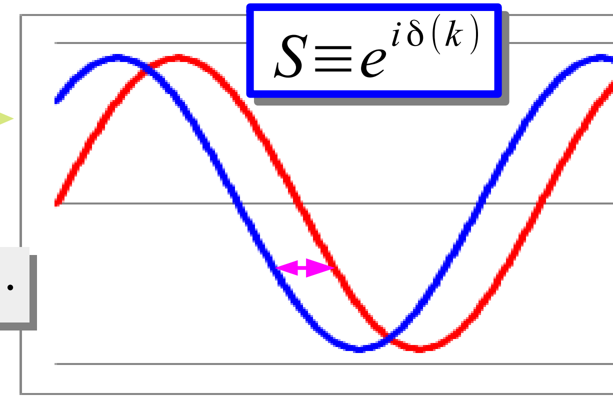


HAL QCD method

Ishii, Aoki, Hatsuda, PRL**99** (2007) 022001

Scattering S-matrix

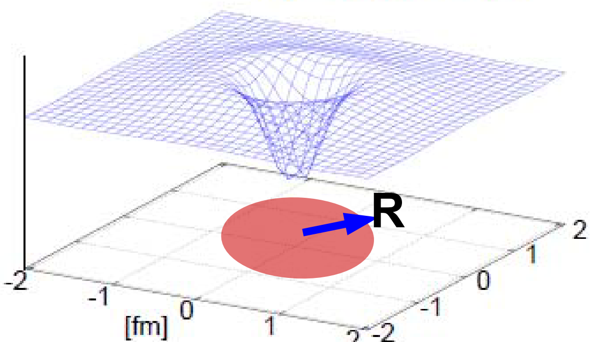
$$S \equiv e^{i\delta(k)}$$



$$\langle 0 | B_1 B_2(t, \vec{r}) \bar{I}(t_0) | 0 \rangle = A_0 \Psi(\vec{r}, E_0) e^{-E_0(t-t_0)} + \cdots$$

$$\Psi(\vec{r}, E) e^{-Et} = \sum_{\vec{x}} \langle 0 | B_1(t, \vec{x}+\vec{r}) B_2(t, \vec{x}) | E \rangle$$

NBS wave function



$$\left(p^2 + \nabla^2\right) \Psi(\vec{r}, E) = 0, (r > R)$$

$$\Psi(\vec{r}, E) \simeq A \frac{\sin(pr + \delta(E))}{pr}$$

Phase shift is embedded in NBS w.f.

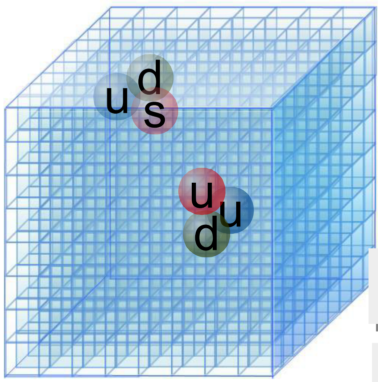
- **Inside of "interacting region"**

$$\left(p^2 + \nabla^2\right) \Psi^{\alpha}(E, \vec{x}) \equiv \int d^3 y \, U_{\alpha}^{\alpha}(\vec{x}, \vec{y}) \Psi^{\alpha}(E, \vec{y})$$

- **U(x,y) is faithful to the S-matrix.**
- **U(x,y) is not an observable.**
- **U(x,y) is energy independent but non-local.**

Hadron interaction (coupled-channel)

Lattice QCD simulation



HAL QCD method

S.Aoki et al [HAL] Proc. Jpn. Acad.,Ser.B,87 509

Scattering S-matrix

$$S(E)=\begin{pmatrix} \eta e^{2i\delta_1} & i\sqrt{1-\eta^2}\,e^{i(\delta_1+\delta_2)} \\ i\sqrt{1-\eta^2}\,e^{i(\delta_1+\delta_2)} & \eta e^{2i\delta_2} \end{pmatrix}$$

$$\langle 0|(B_1 B_2)^\alpha(t,\vec{r})\bar{I}(t_0)|0\rangle = A_0 \Psi^\alpha(\vec{r},E_0)e^{-E_0(t-t_0)}+\cdots$$

$$\langle 0|(B_1 B_2)^\beta(t,\vec{r})\bar{I}(t_0)|0\rangle = C_0 \Psi^\beta(\vec{r},E_0)e^{-E_0(t-t_0)}+\cdots$$

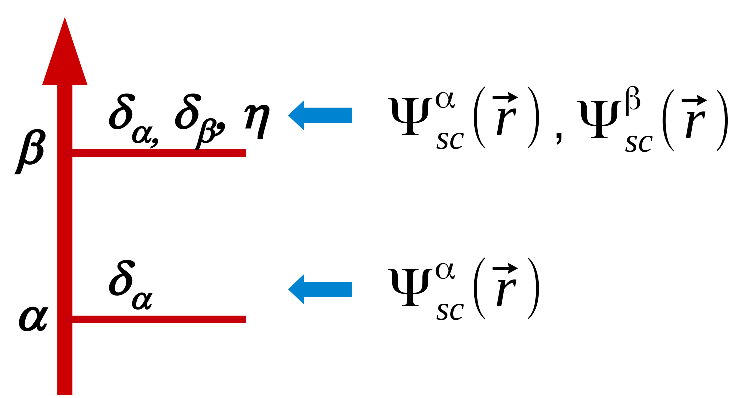
NBS wave function for each channel

$$\Psi^\alpha(\vec{r},E_i)e^{-E_i t} = \langle 0|(B_1 B_2)^\alpha(\vec{r})\,|E_i\rangle$$

$$\Psi^\beta(\vec{r},E_i)e^{-E_i t} = \langle 0|(B_1 B_2)^\beta(\vec{r})\,|E_i\rangle$$

Coupled-channel Schrödinger equation

$$\left(p_\alpha^2+\nabla^2\right)\Psi^\alpha(E,\vec{x}) \equiv \int d^3y\, U^\alpha_\beta(\vec{x},\vec{y})\,\Psi^\beta(E,\vec{y})$$



- **U(x,y) is faithful to the S-matrix beyond the threshold of channel β.**
- **U(x,y) is energy independent until the higher energy thershold opens.**
- **Derivative (velocity) expansion is used.**

S=-2 BB interaction

--- focus on the H-dibaryon ---

Keys to understand H-dibaryon state

A strongly bound state predicted by Jaffe in 1977 using MIT bag model.

H-dibaryon state is

- **SU(3) flavor singlet [uuddss], strangeness S=-2.**
- **spin and isospin equals to zero, and $J^P = 0^+$**

SU(3) classification $8 \times 8 = 27 + 8_s + 1 + \overline{10} + 10 + 8_A$

▶ **Strongly attractive interaction is expected in flavor singlet channel.**

- **Strongly attractive Color Magnetic Interaction**
- **Flavor singlet channel is free from Pauli blocking effect**

	27	8	1	$\overline{10}$	10	8
Pauli	Mixed	forbidden	allowed	Mixed	forbidden	Mixed
CMI	repulsive	repulsive	attractive	repulsive	repulsive	repulsive

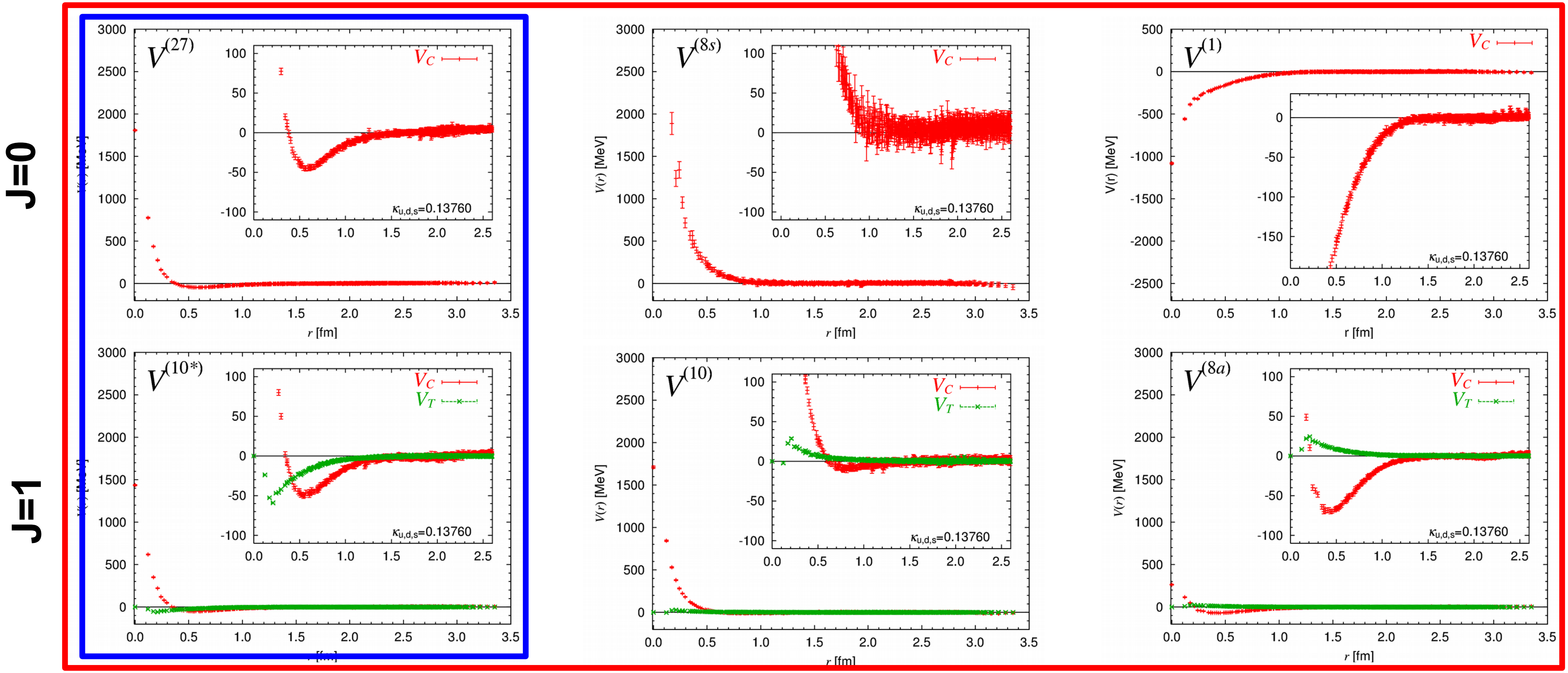
M. Oka et al NPA464 (1987)

$J^P=0^+$, I=0

$$\begin{pmatrix} \Lambda\Lambda \\ N\Xi \\ \Sigma\Sigma \end{pmatrix} = \frac{1}{\sqrt{40}} \begin{pmatrix} -\sqrt{5} & -\sqrt{8} & \sqrt{27} \\ \sqrt{20} & \sqrt{8} & \sqrt{12} \\ \sqrt{15} & -\sqrt{24} & -1 \end{pmatrix} \begin{pmatrix} 1 \\ 8 \\ 27 \end{pmatrix}$$

B-B potentials in SU(3) limit

m_π= 469MeV

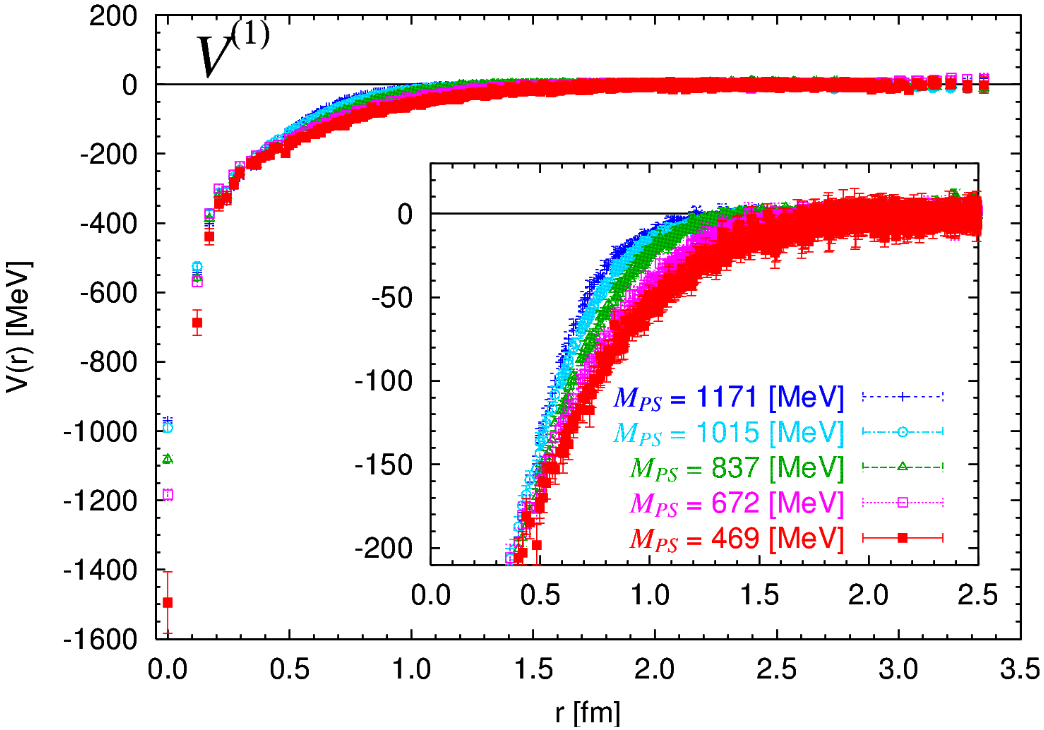


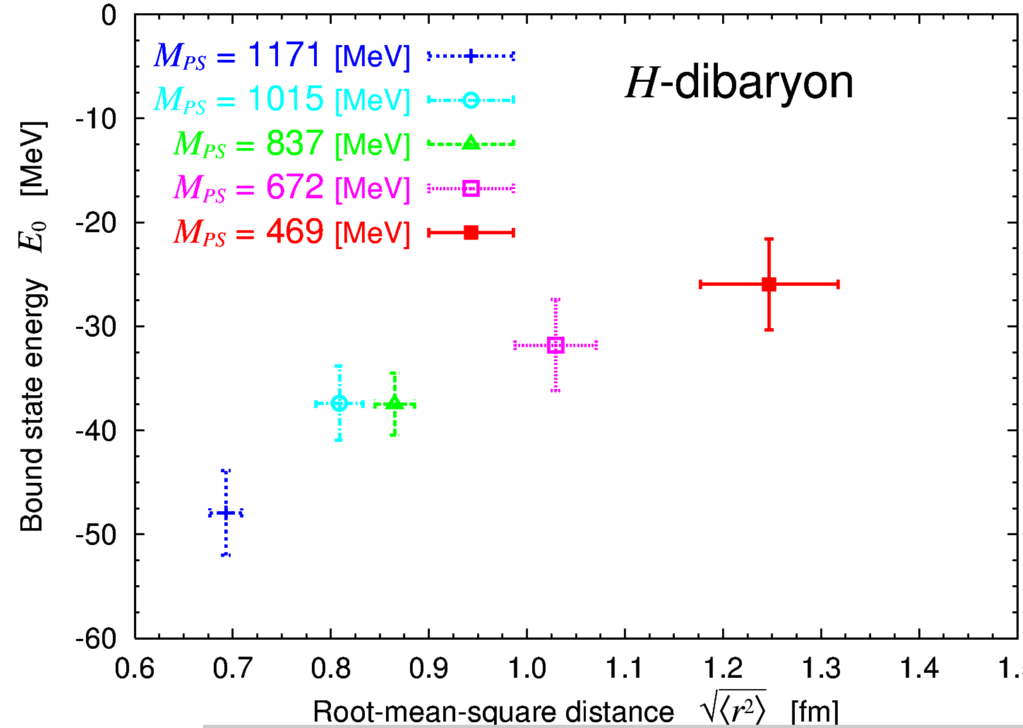
Two-flavors Three-flavors

- **Quark Pauli principle can be seen at around short distances**
  - **No repulsive core in flavor singlet state**
  - **Strongest repulsion in flavor 8s state**
- **Possibility of bound H-dibaryon in flavor singlet channel.**

H-dibaryon in SU(3) limit

Strongly attractive interaction is expected in flavor singlet channel.

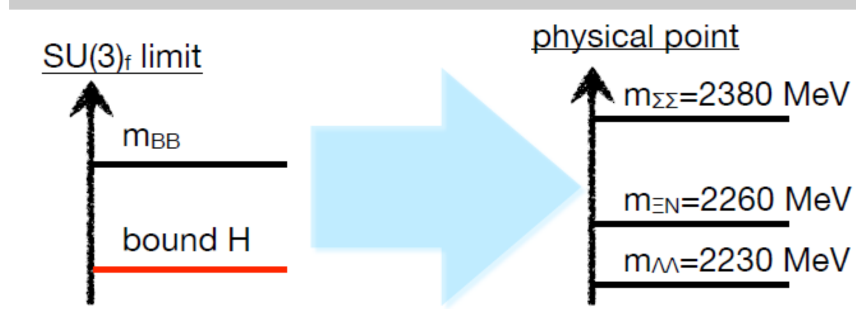




T.Inoue et al[HAL QCD Coll.] NPA881(2012) 28

m_B=1161MeV for M_{PS}=470MeV

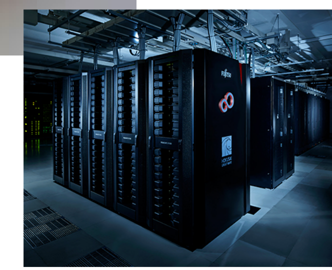
- **Strongly attractive potential was found in the flavor singlet channel.**
- **Bound state was found in this mass range with SU(3) symmetry.**



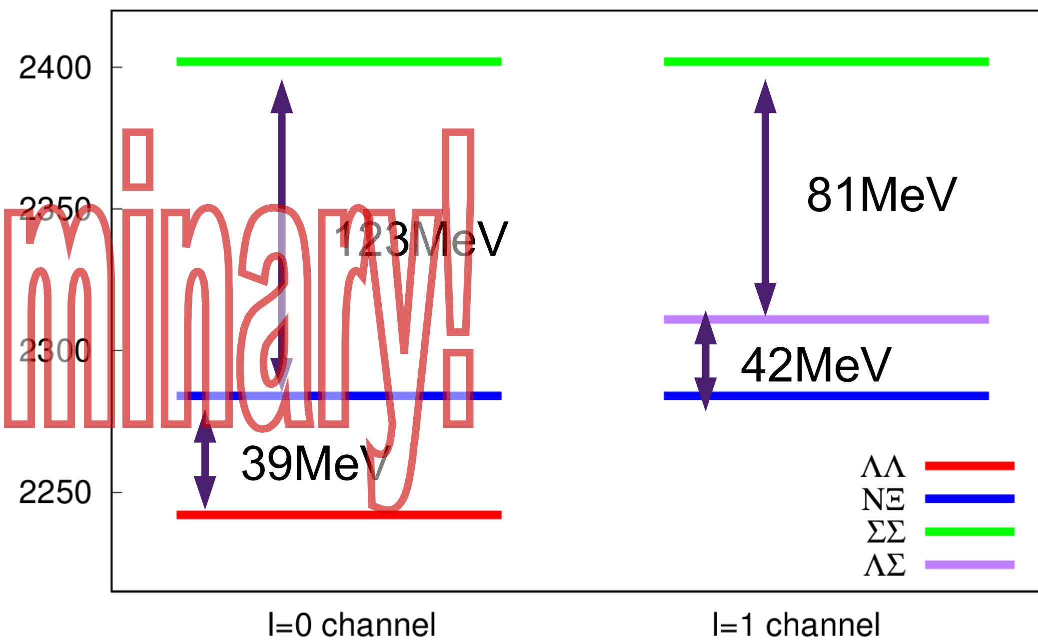
Go to the physical point simulation!

Numerical setup

- 2+1 flavor gauge configurations.
  - Iwasaki gauge action & O(a) improved Wilson quark action
  - *a* = 0.085 [*fm*], a^{-1} = 2.300 GeV.
  - *96^3x96 lattice, L* = 8.21 [*fm*].
  - 414 confs x 84 sources x 4 rotations.
- Wall source is considered to produce S-wave B-B state.



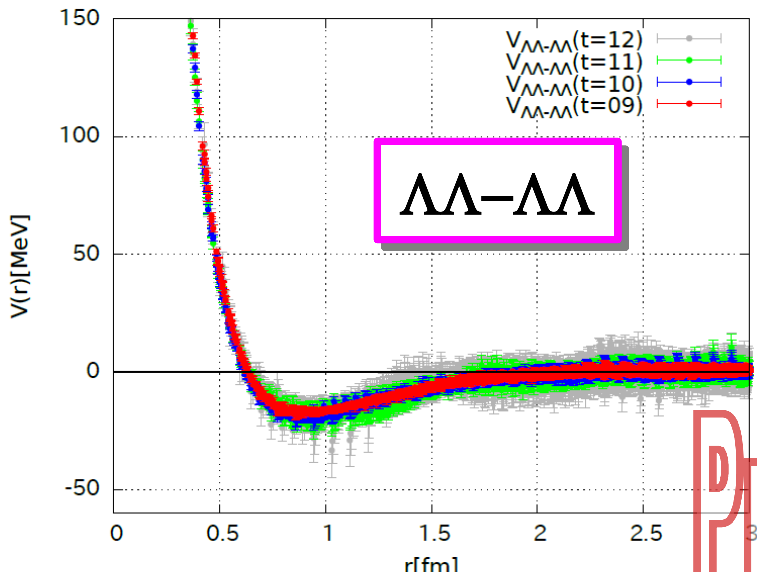
	Mass [MeV]
π	146
K	525
m_π/m_K	0.28
N	953±7
Λ	1123±3
Σ	1204±1
Ξ	1332±2

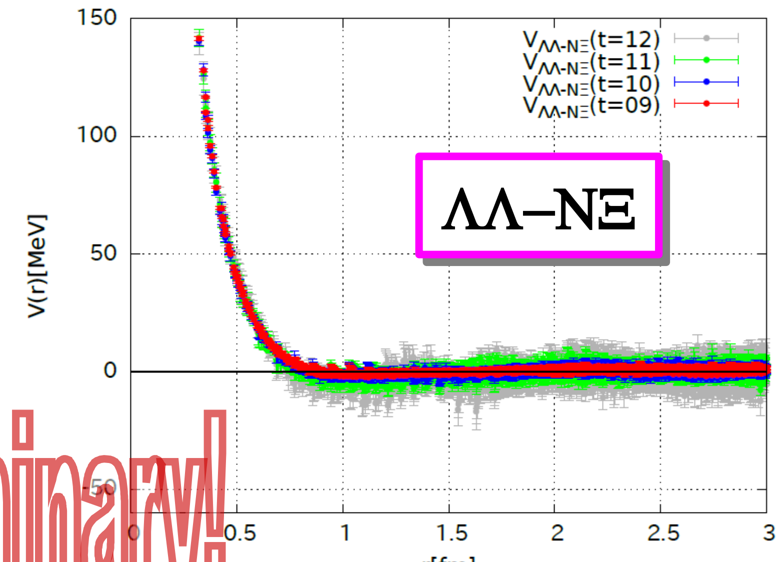


ΛΛ, NΞ (I=0) $^1S_0$ potential

t=09
t=10
t=11
t=12

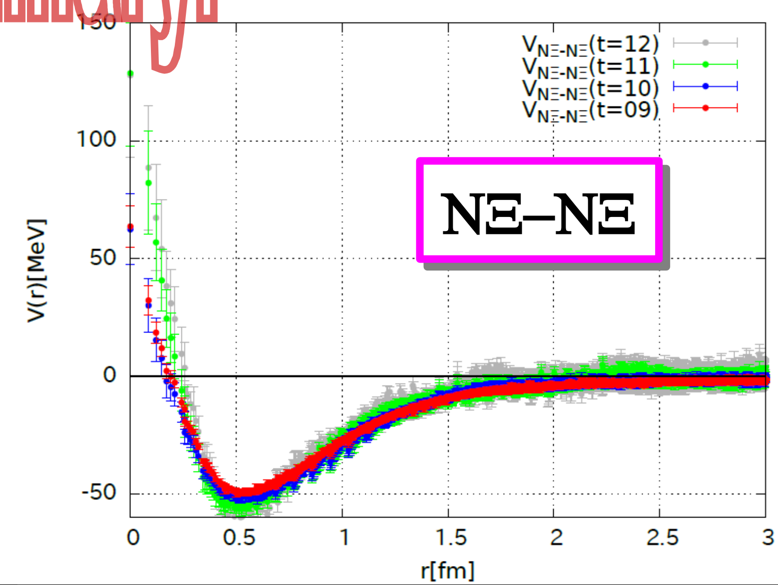
▶ N_f = 2+1 full QCD with L = 8.1fm, *m_π = 146 MeV*





Preliminary!

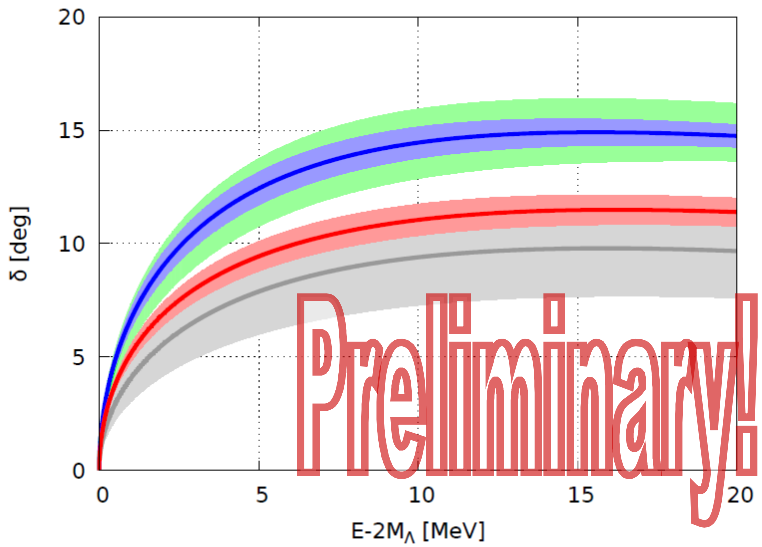
- Potential calculated by only using ΛΛ and NΞ channels.
- Long range part of potential is almost stable against the time slice.
- Short range part of NΞ potential changes as time t goes.
- ΛΛ–NΞ transition potential is quite small in r > 0.7fm region



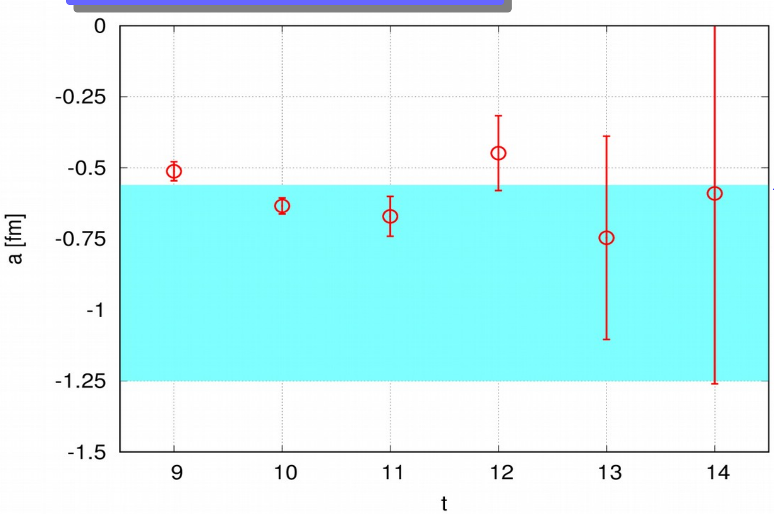
ΛΛ scattering length

t=09
t=10
t=11
t=12

Phase shift



Scattering length



Λ		NLO			
		500	550	600	650
ΛΛ	a_{1S0}	−0.62	−0.61	−0.66	−0.70
	r_{1S0}	6.95	6.06	5.05	4.56

J. Haidenbauer et al, NPA954(2016)273

$a_{\Lambda\Lambda} = -0.821$fm — Y.Fujiwara et al, PPNP58(2007)439

$a_{\Lambda\Lambda} = -0.97$fm — Th.A.Rijken et al, Few-Body Syst 54(2013)801

$-1.25 < a_{\Lambda\Lambda} < -0.56$fm (or 0fm)

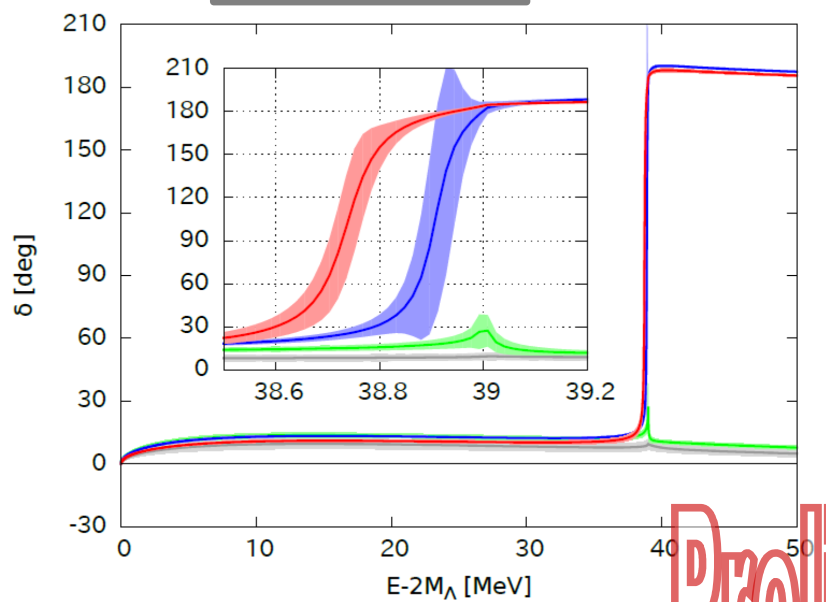
K.Morita et al, PRC91 (2015)024916
K.Morita, private communication

- **Scattering length in ΛΛ (I=0) is almost saturated.**
- **Attraction is a little bit weaker than the phenomenological values.**

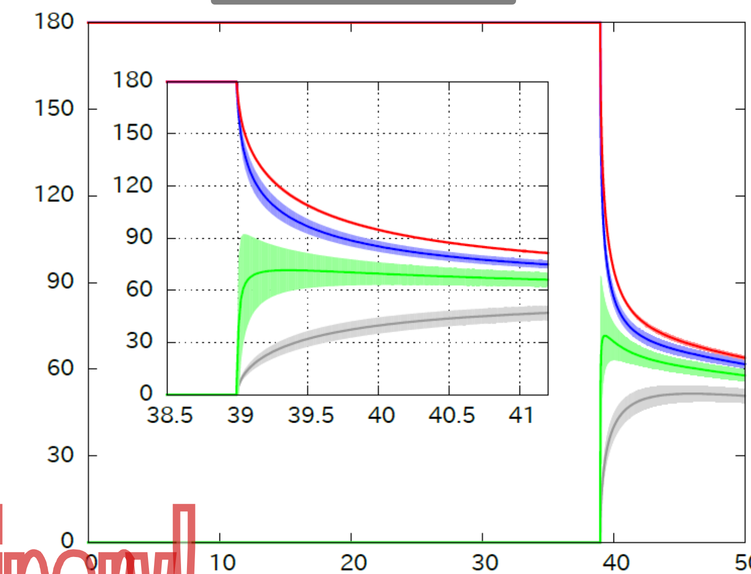
ΛΛ and *NΞ* phase shift and inelasticity

t=09
t=10
t=11
t=12

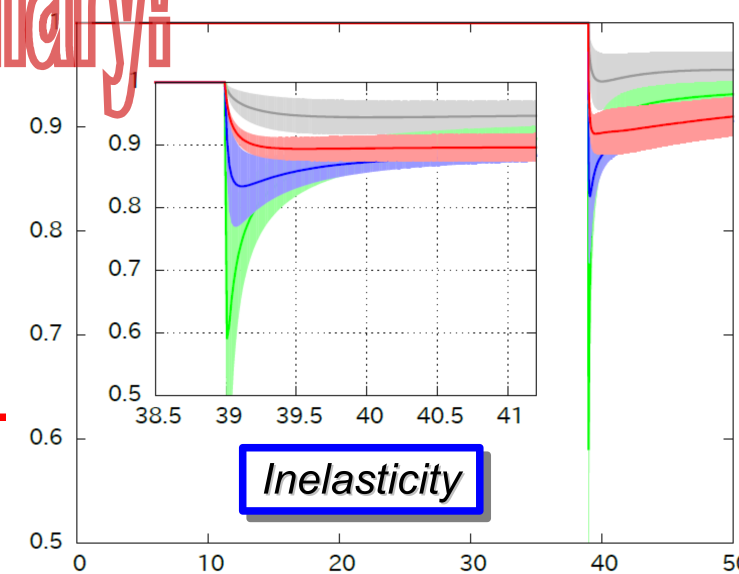
ΛΛ phase shift



NΞ phase shift



Preliminary!



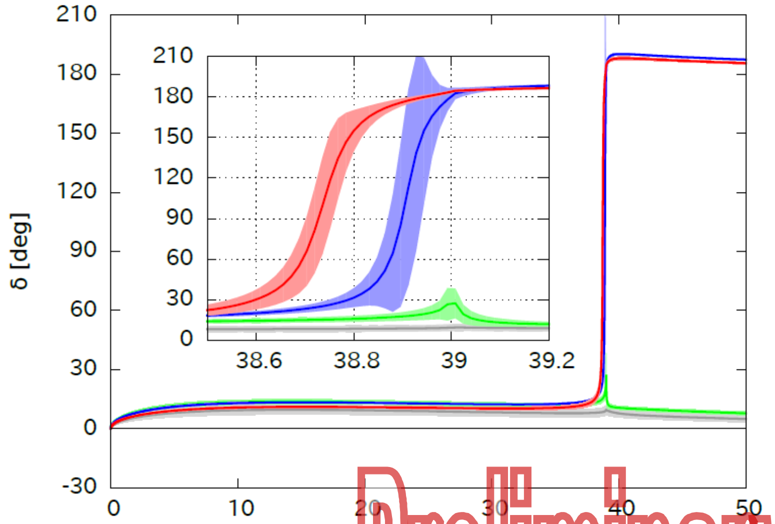
Inelasticity

- ΛΛ and NΞ phase shift is calculated
  by using 2ch effective potential.
- A sharp resonance is found
  below the NΞ threshold for t=9 and 10.

$\Lambda\Lambda$ and $N\Xi$ phase shift –comparison--

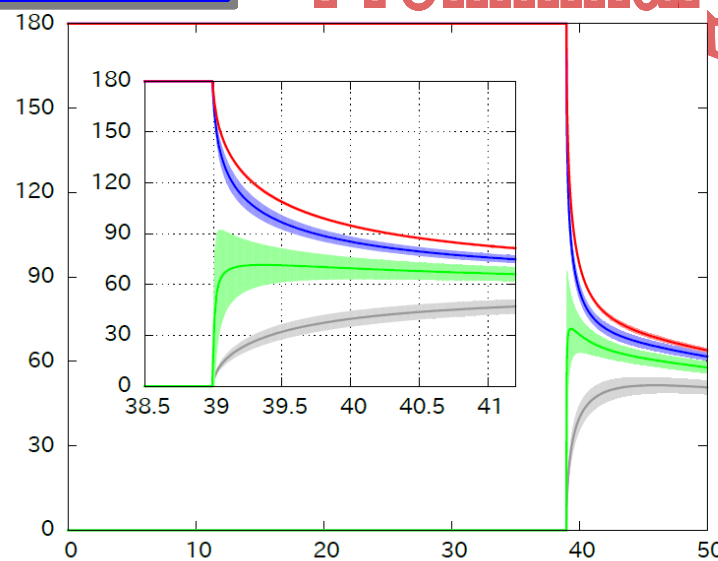
t=09
t=10
t=11
t=12

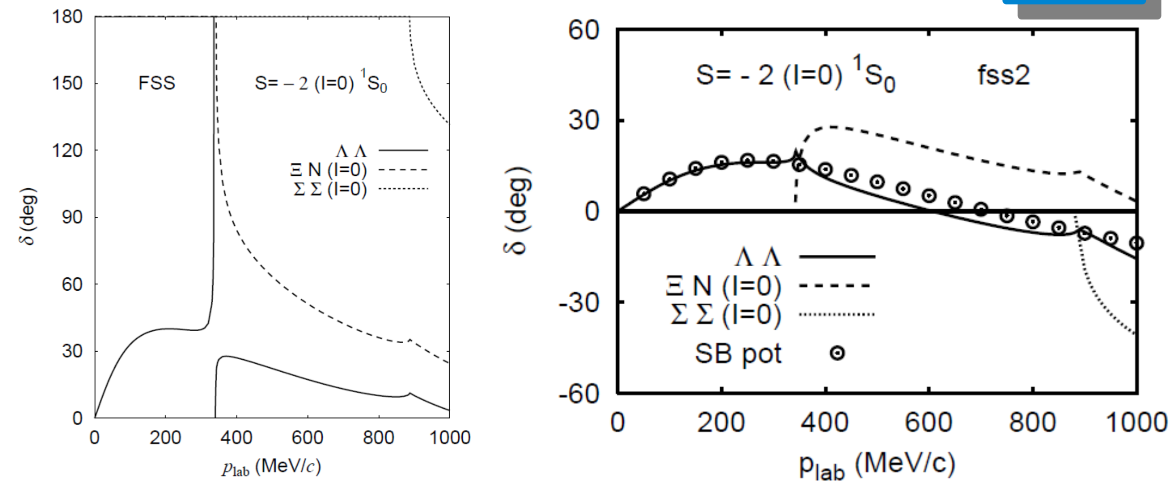
$\Lambda\Lambda$ phase shift



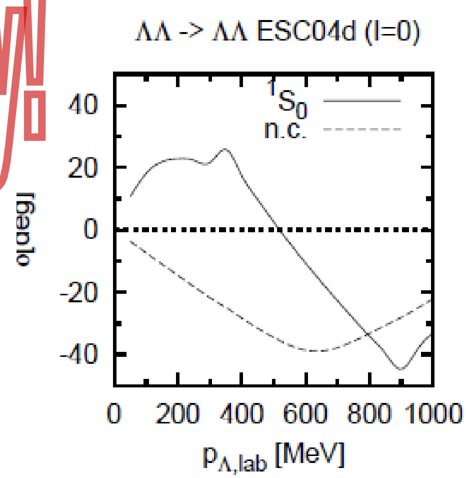
Preliminary!

$N\Xi$ phase shift

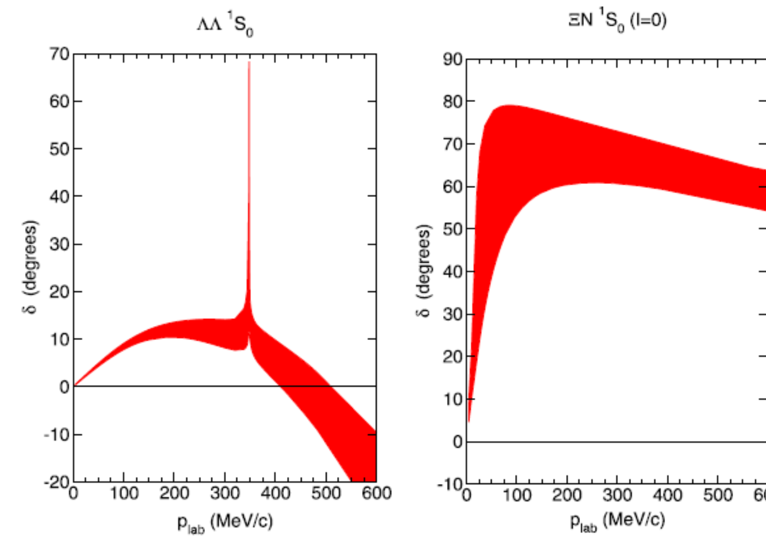




Y.Fujiwara et al, PPNP58(2007)439



Th.A. Rijken, nucl-th/060874



J. Haidenbauer et al, NPA954(2016)273

- **Our results are compatible with the phenomenological ones.**

NΩ interaction

NΩ system

NΩ system from model calculations

- One of di-baryon candidate — T.Goldman et al PRL59(1987)627
- (Quasi-)Bound state is reported with J=2, I=1/2
  - Constituent quark model — M.Oka PRD38(1988)298
    - CMI does not contribute for this system because of no quark exchange between baryons.
    - Coupled channel effect is important.
  - Chiral quark model — Q.B.Li, P.N.Shen, EPJA8(2000)
    - Strong attraction yielded by scalar exchange

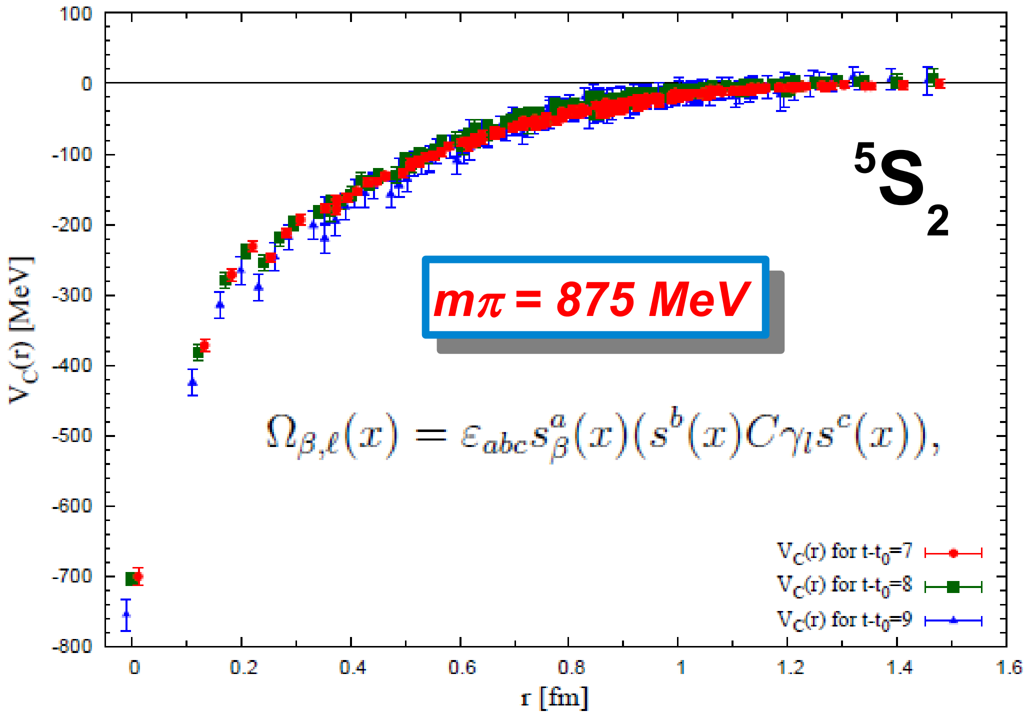
NΩ $J^p(I) = 2^+(1/2)$ is considered

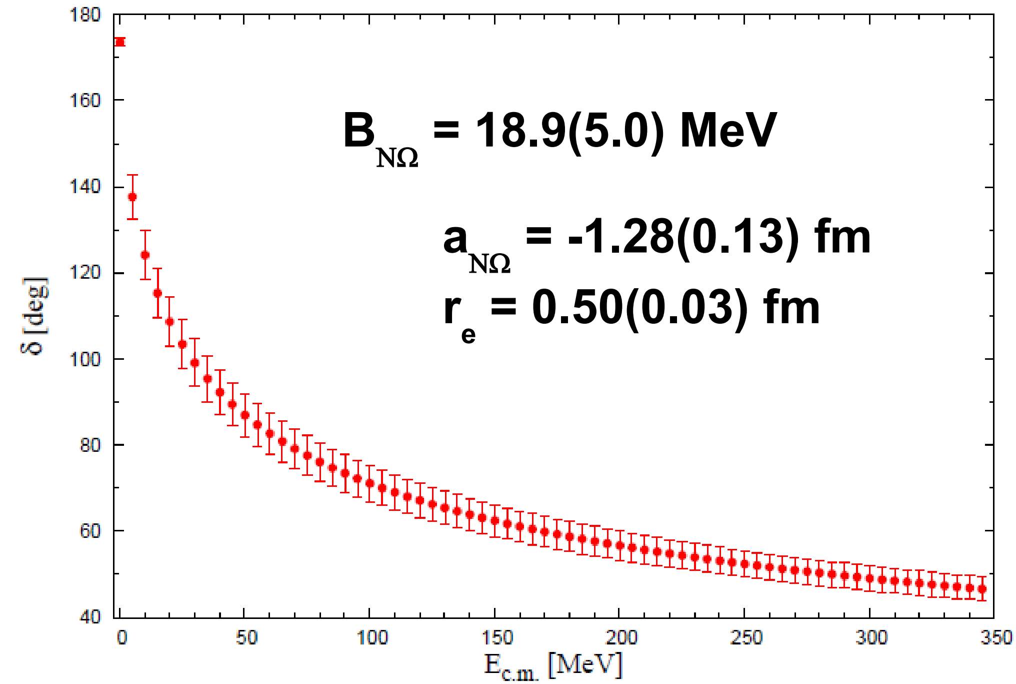
- Easy to tackle it by lattice QCD simulation
  - Lowest state in J=2 coupled channel
    - NΩ–ΛΞ*–ΣΞ*–ΞΣ*
  - Multi-strangeness reduces a statistical noise
  - Wick contraction is very simple

NΩ system $J^p(I) = 2^+(1/2)$

▶ N_f = 2+1 full QCD with L = 1.9fm

F.Etminan(HAL QCD), NPA928(2014)89





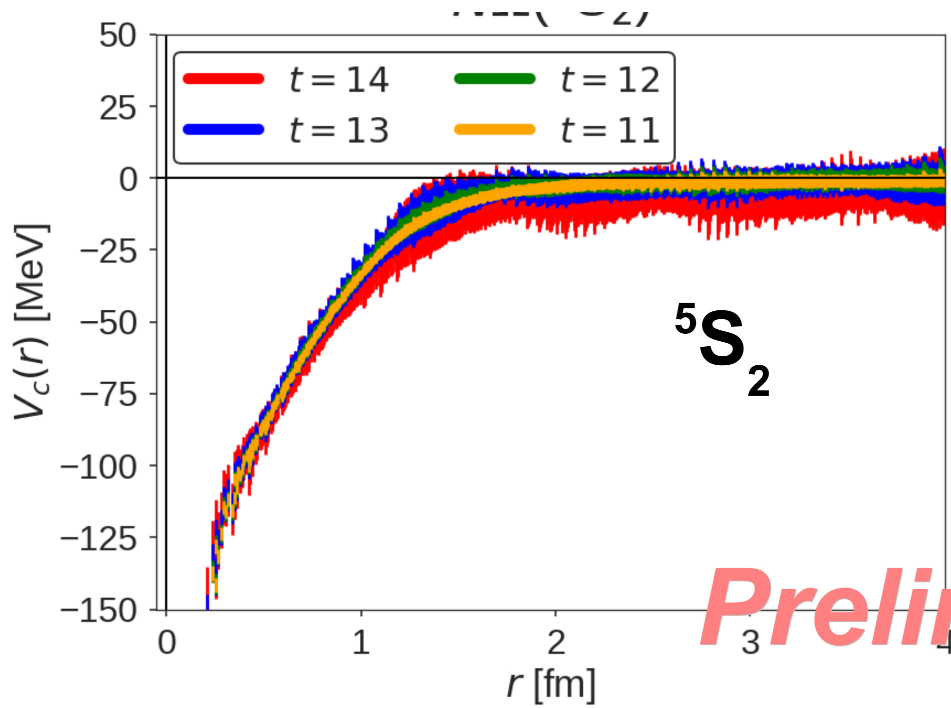
NΩ state cannot decay into ΛΞ (D-wave) state in this setup

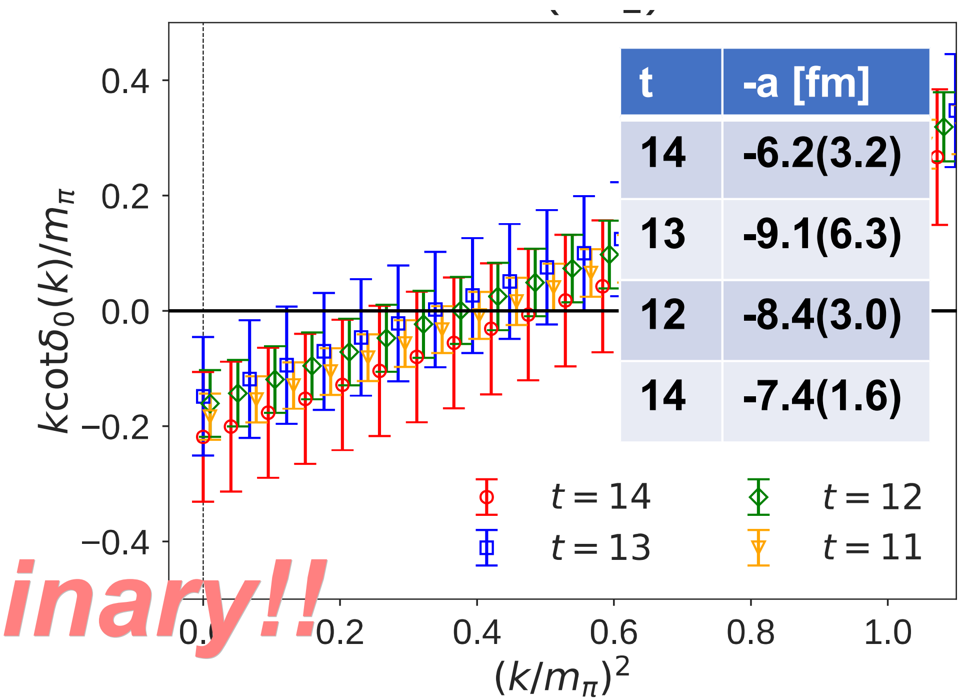
- **Strongly attractive S-wave potential in $J^p(I) = 2^+(1/2)$**
- **Bound state is found.**

→ The doorway to the S=-3 nuclear system.

NΩ system $J^p=2^+$ near the physical point

- N_f = 2+1 full QCD with L = 8fm, ***m_π = 146 MeV***





t	-a [fm]
14	-6.2(3.2)
13	-9.1(6.3)
12	-8.4(3.0)
14	-7.4(1.6)

NΩ state decay into ΛΞ (D-wave) state is suppressed.

- **The system is bound (compared to the NΩ threshold) within the errors because of the strongly attractive potential.**
- **Measurement of strong NΩ attraction at RHIC and LHC is expected.**

K.Morita et al PRC94(2016)031901

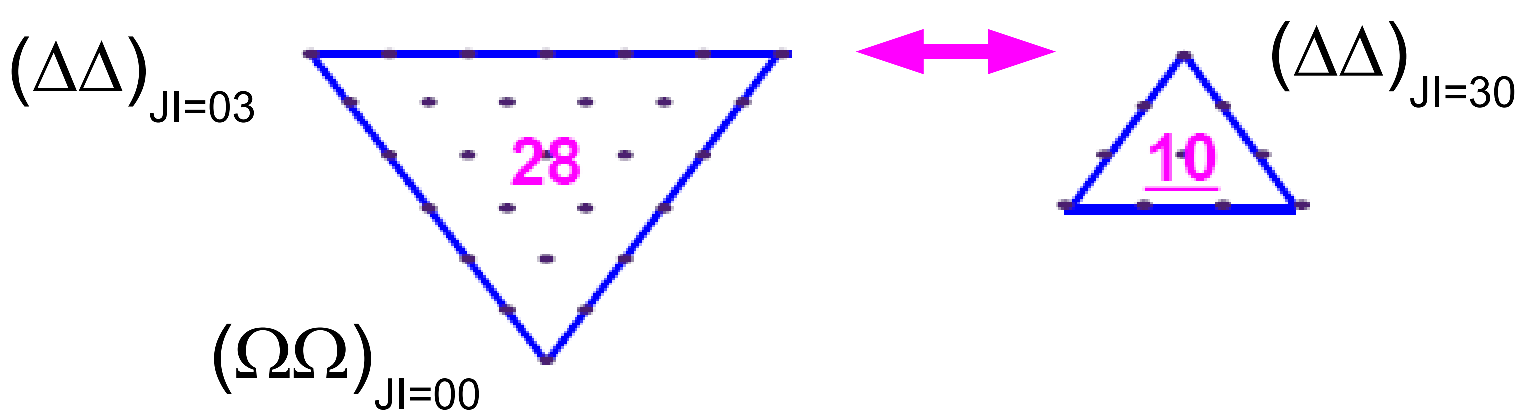
ΔΔ and ΩΩ interaction

Decuplet-Decuplet interaction

- **Flavor symmetry aspect**

Decuplet-Decuplet interaction can be classified as

$$10 \otimes 10 = 28 \oplus 27 \oplus 35 \oplus \overline{10}$$



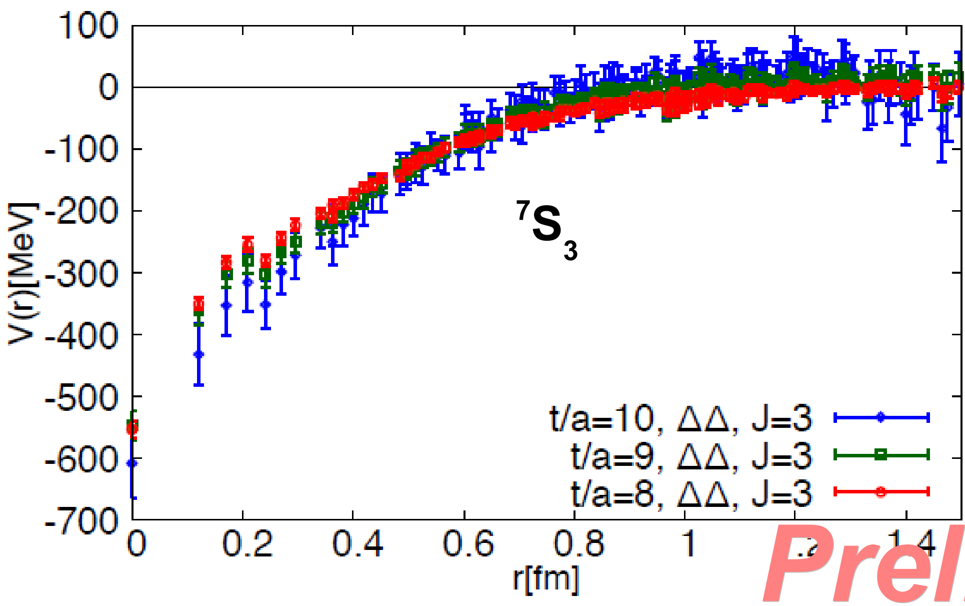
	28plet (0^+)	10*plet (3^+)
Pauli	**allowed**	**allowed**
CMI	**repulsive**	**No**

- **Δ–Δ(J=3) : Bound (resonance) state was found in experiment.**
- **Δ–Δ(J=0) [and Ω–Ω(J=0)] : Mirror of Δ–Δ(J=3) state**

S. Gongyo, K. Sasaki (HAL QCD)

Decuplet-Decuplet interaction in SU(3) limit

▶ N_f = 3 full QCD with L = 1.93fm, m_π = 1015 MeV m_Δ = 2225 MeV



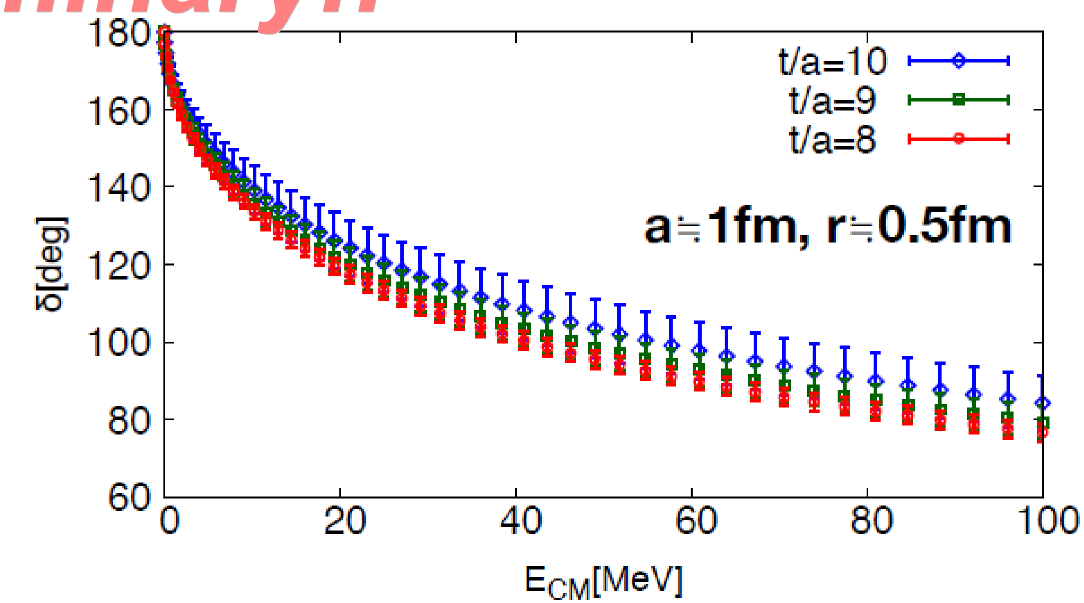
Δ−Δ(J=3) **10*plet**

- **Interaction in 10*plet [$J^P(I)=3^+(0)$] is strongly attractive.**
- **There is no repulsive core at short distances**

Preliminary!!

Bound ΔΔ state is found.

- **Decay to NN($^3D_3$) is neglected.**
- **Δ baryon can not decay into N+π in this lattice setup**



S. Gongyo, K. Sasaki (HAL QCD)

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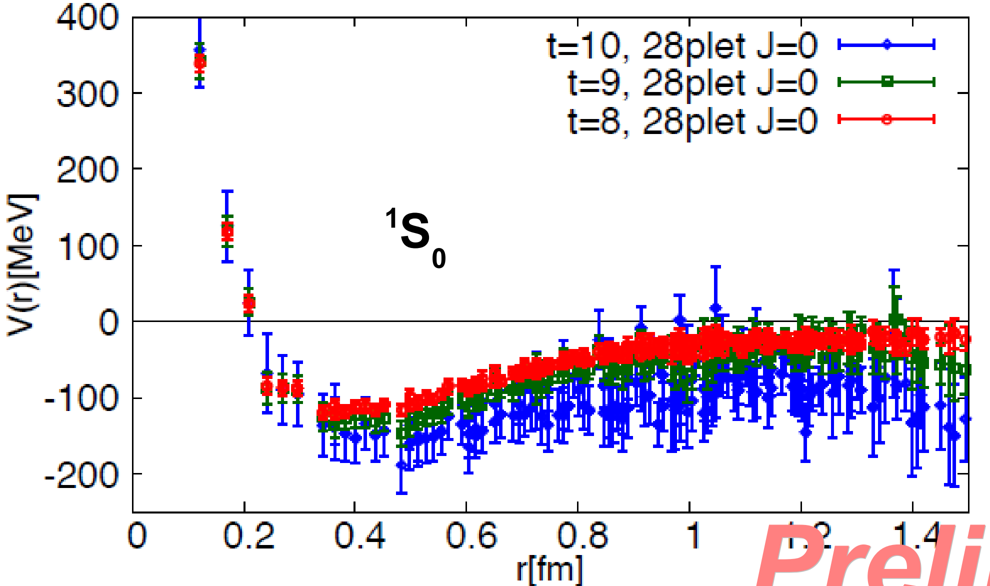


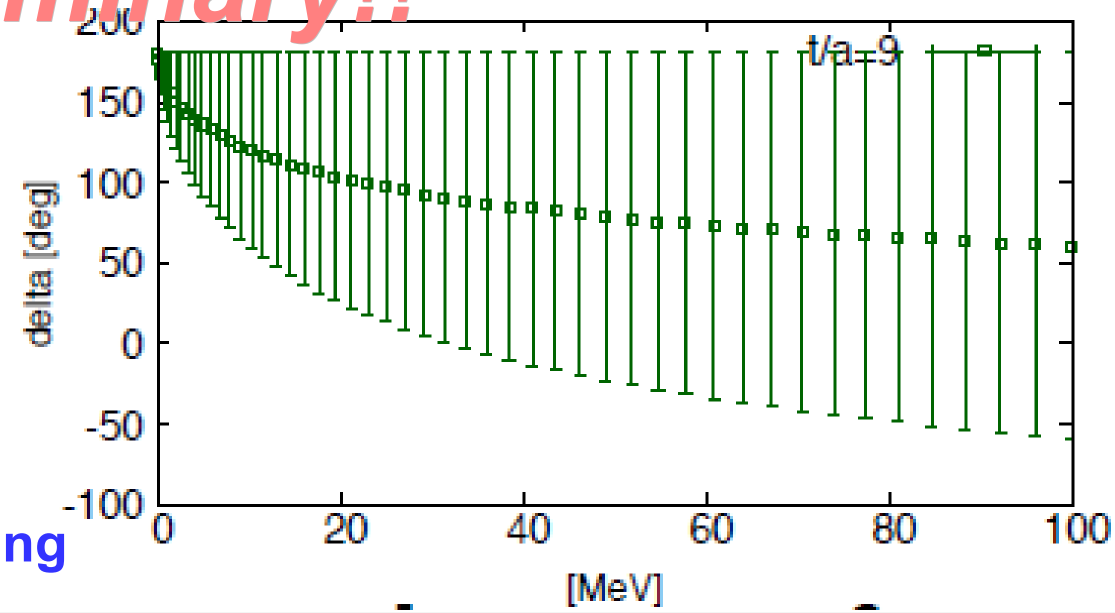
Δ−Δ(J=0)

28plet

- Repulsive core is surrounded by attractive pocket in 28plet [ΔΔ $J^p(I)=0^+(3)$] and [ΩΩ $J^p(I)=0^+(0)$].

Preliminary!!

Phase shift shows that the system is in the unitary limit.



- Δ baryon can not decay into N+π in this lattice setup

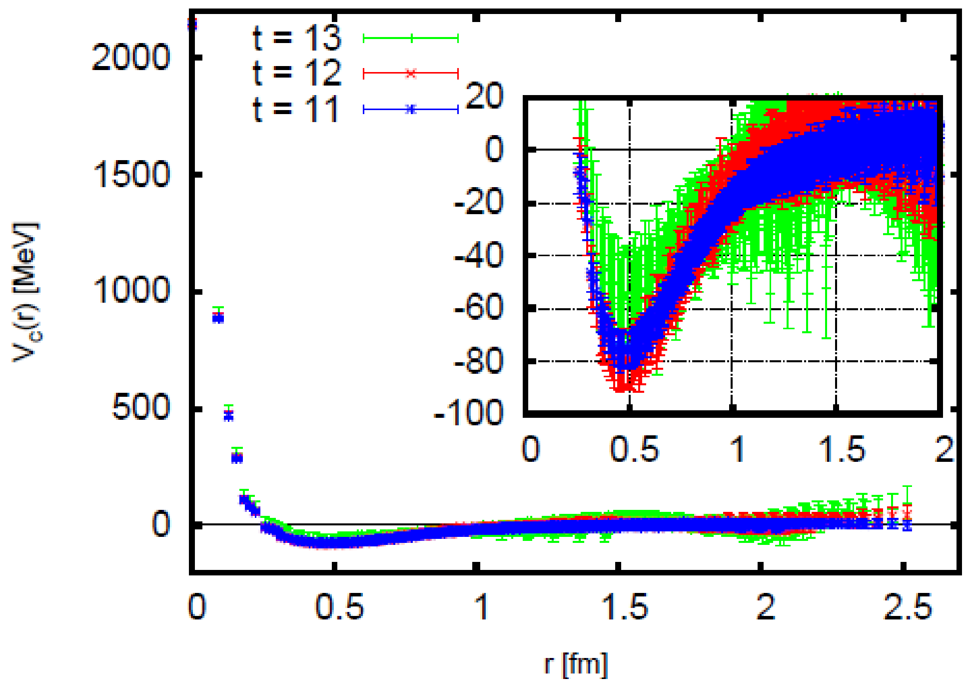
Go to the lighter quark mass region with consideration of SU(3) breaking

ΩΩ $J^p = 0^+$ state in unphysical region

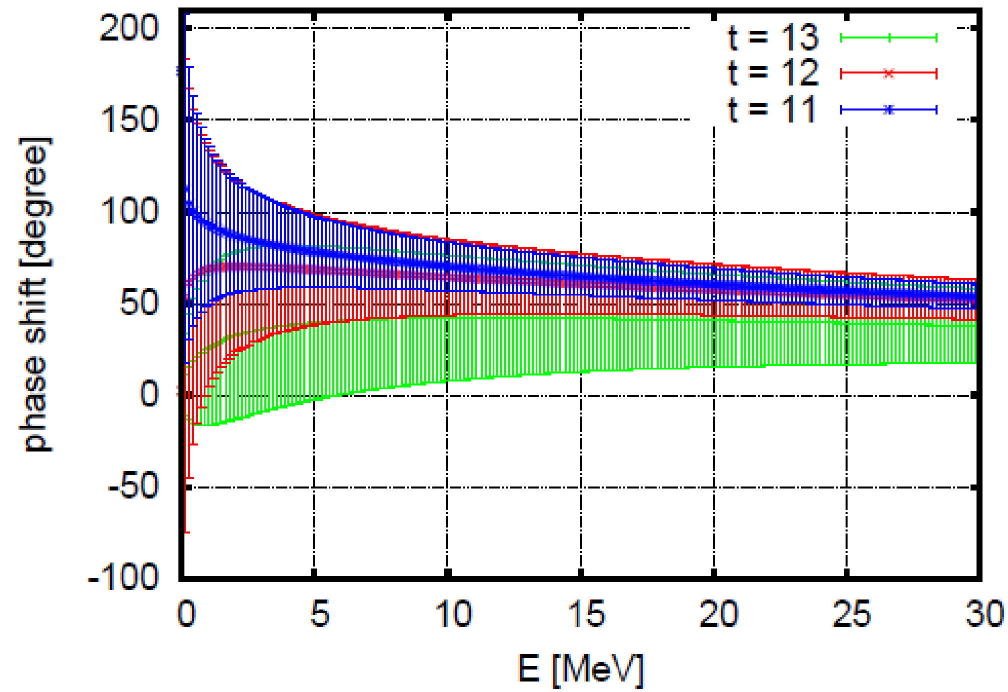
- N_f = 2+1 full QCD with L = 3fm, m_π = 700 MeV m_Ω = 1966 MeV

The ΩΩ state is stable against the strong interaction.

Potential



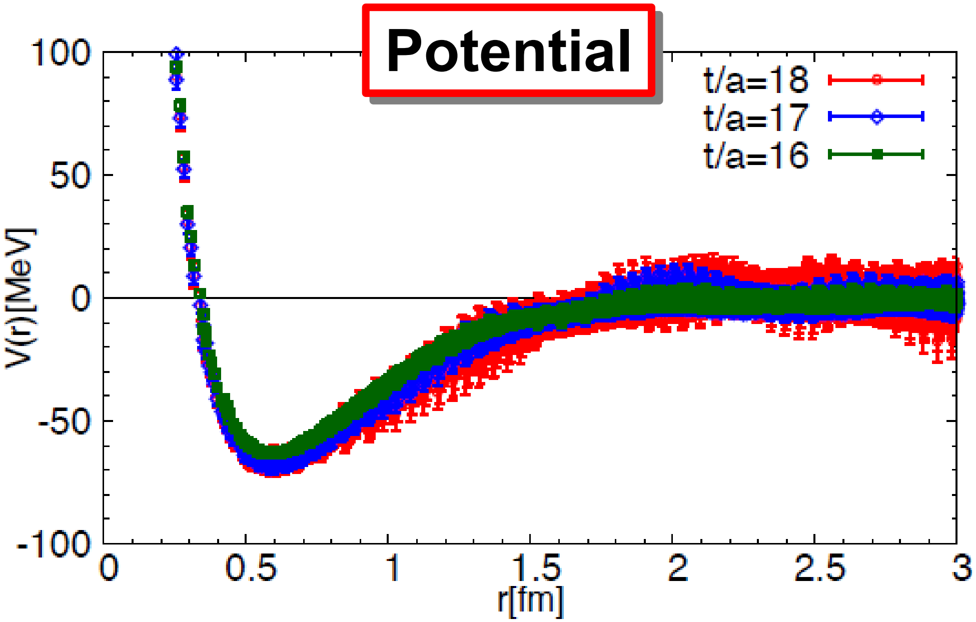
Phase shift

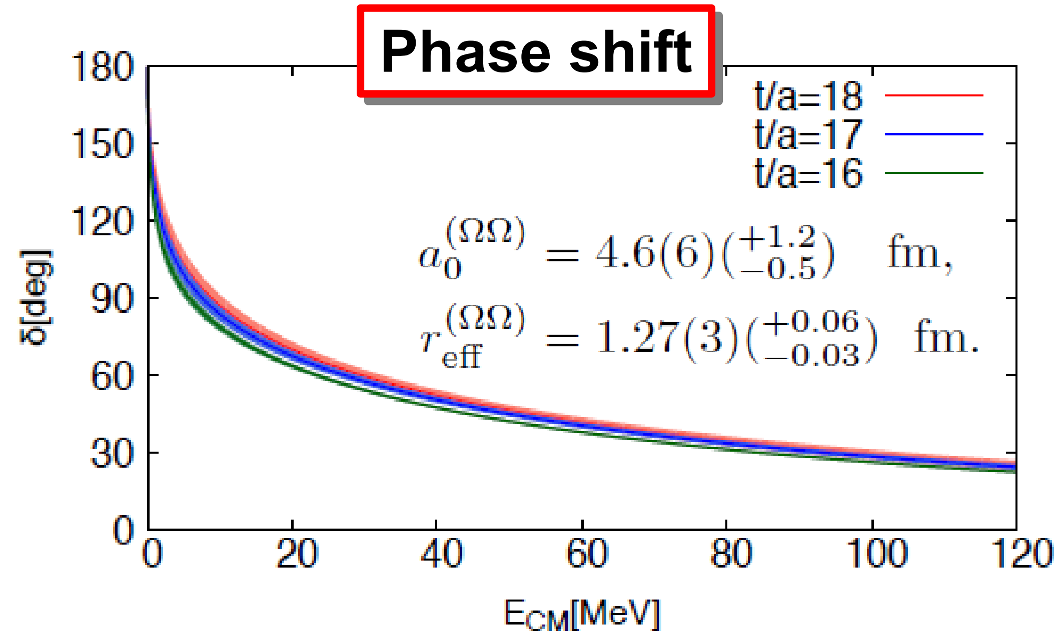


- Short range repulsion and attractive pocket are found.
- Potential is nearly independent on "t" within statistical error.
- The system may appear close to the unitary limit.

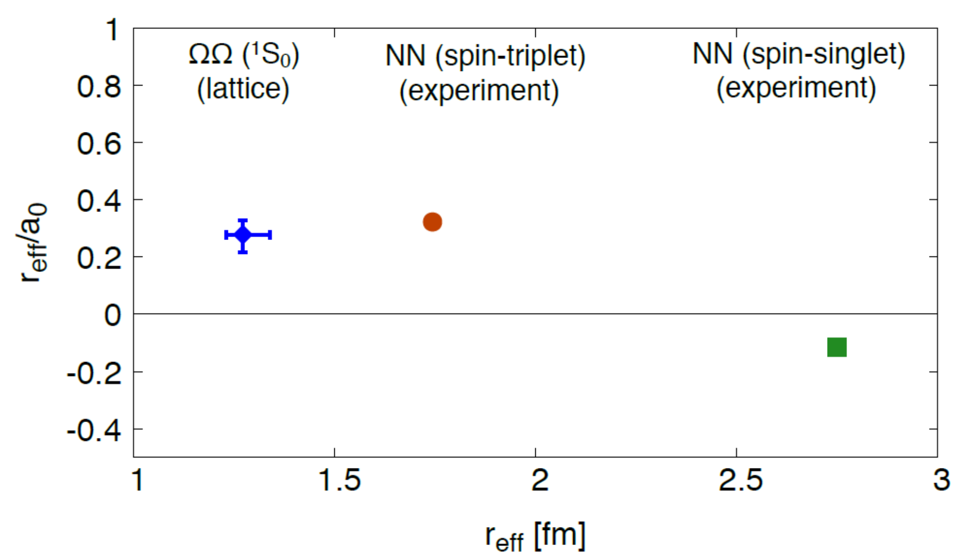
ΩΩ $J^p(I) = 0^+(0)$ state near the physical point

- **N_f = 2+1 full QCD with L = 8fm, *m_π = 146 MeV*** ***m_Ω = 1712 MeV***





- **Short range repulsion and attractive pocket is found.**
- **Calculated phase shift indicates a bound ΩΩ state** **[Most strange dibaryon].**
- **Physical ΩΩ state in $J^p(I) = 0^+(0)$ is very close to unitary region.**

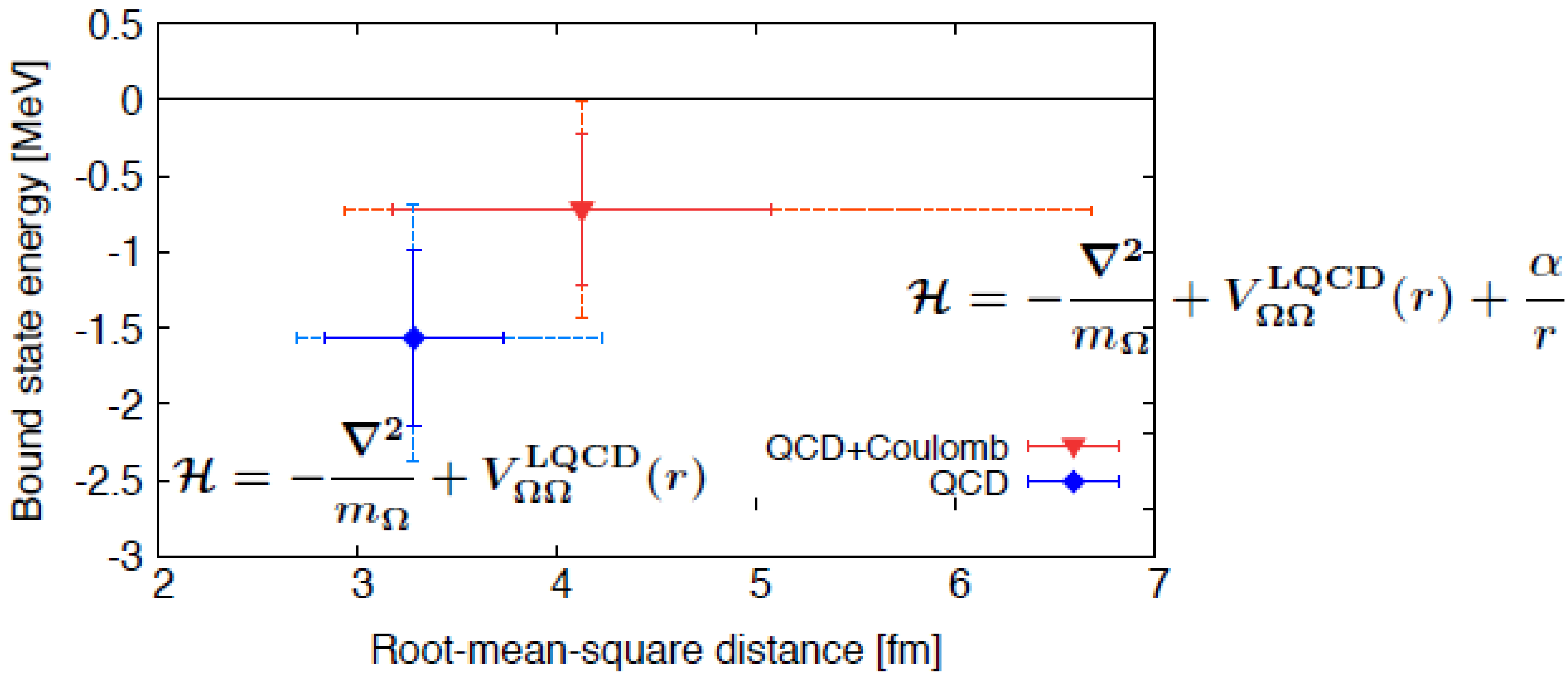


S. Gongyo, K. Sasaki (HAL QCD)arXiv:1709.00654

$\Omega\Omega$ $J^p(I) = 0^+(0)$ state near the physical point

- N_f = 2+1 full QCD with L = 8fm, *m_π = 146 MeV*

Binding energy and the Coulomb effect



Most strange dibaryon appears (within 1σ)
even if Coulomb effect is taken into accound.

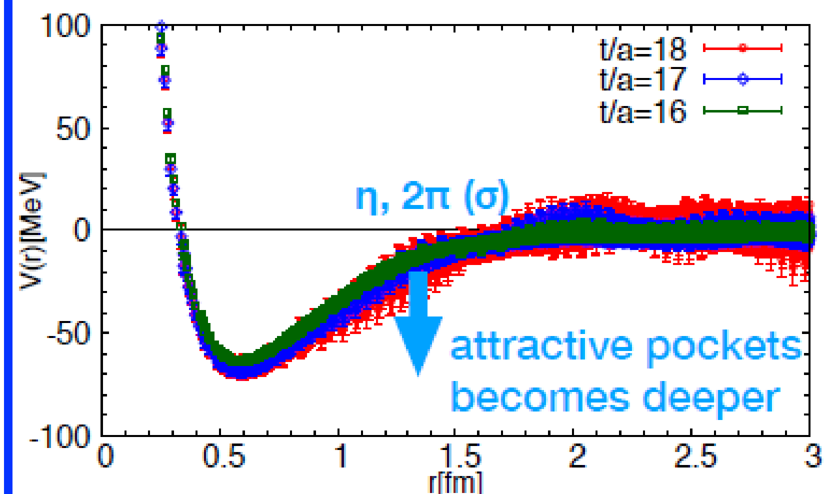
$$(B_{\Omega\Omega}^{\rm (QCD)}, B_{\Omega\Omega}^{\rm (QCD+Coulomb)}) = (1.6(6){\rm MeV}, 0.7(5){\rm MeV})$$

$\Omega\Omega$ $J^p(I) = 0^+(0)$ state at exact physical point

Conservative estimate at exact phys. pt.

m_{π}=146 MeV -> 135 MeV, m_{Ω}= 1712MeV -> 1672 MeV

B.E. ↑



V.S.

B.E. ↓

$$\mathcal{H} = -\frac{\nabla^2}{m_\Omega} + V_{\Omega\Omega}^{\mathrm{LQCD}}(r)$$

kinetic term becomes more dominant
-> B.E. is reduced

conservative estimate:
only change the mass of schroedinger eq.

$$(B_{\Omega\Omega}^{(\mathrm{QCD})}, B_{\Omega\Omega}^{(\mathrm{QCD+Coulomb})}) = (1.6(6)\mathrm{MeV}, 0.7(5)\mathrm{MeV})$$
$$\rightarrow (1.3(5)\mathrm{MeV}, 0.5(5)\mathrm{MeV})$$

These changes are well within errors

27

Summary

- We have investigated dibaryon candidate states from LQCD
  - H-dibaryon channel
    - We found a strong attraction in NΞ J=0 with I=0.
    - It is still difficult to conclude the fate of H-dibayon.
  - NΩ state with $J^p=2^+$
    - Interaction is strongly attractive and no short range repulsion.
    - It forms a bound state with about 20MeV B.E..
    - Physical point result will be open for ΩN channel.
  - ΔΔ and ΩΩ states
    - ΔΔ(I=0) have strongly attractive potential.
    - ΔΔ(I=3) and ΩΩ potential have repulsive core and attractive pocket.
    - Physical ΩΩ system in J=0 forms the most strange dibaryon (or unitary region...)

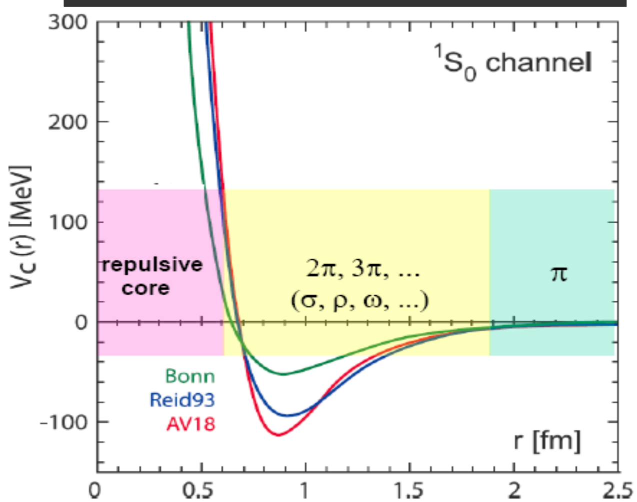
Backup

Introduction

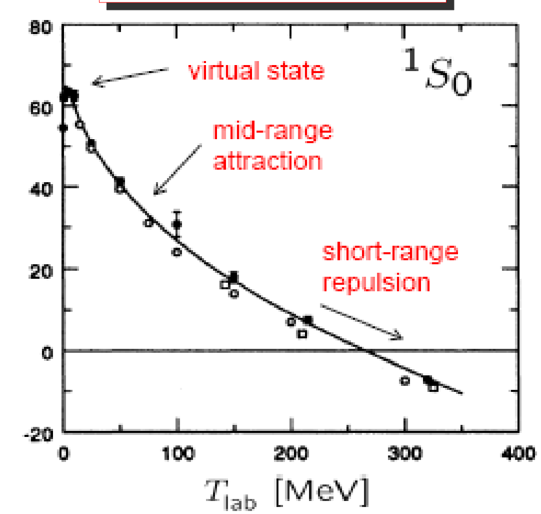
Baryon-baryon interactions are key to understand
nuclear structures and astrophysical phenomena

Traditional way to research the BB interaction / potential

BB interaction (potential)



BB phase shift



NN interaction

Large amount of scattering data
to determine theoretical parameters

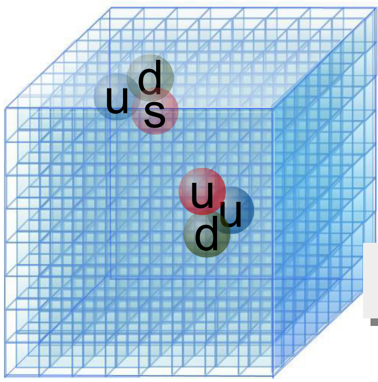
YN / YY interaction

More strangeness, more difficult to access experimentally.
Experimental data are scarce.

Lattice QCD results for YN and YY interactions are highly awaited

Hadron interaction from Lattice QCD

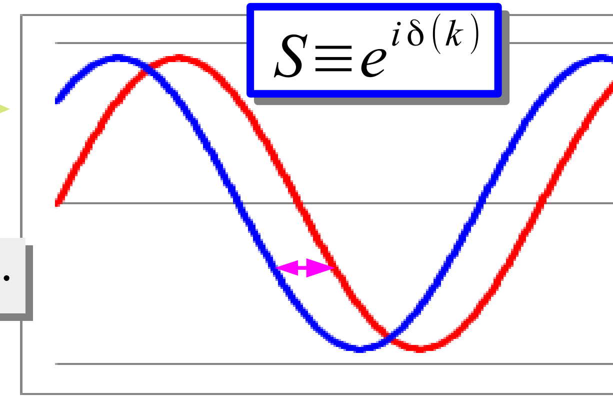
Lattice QCD simulation

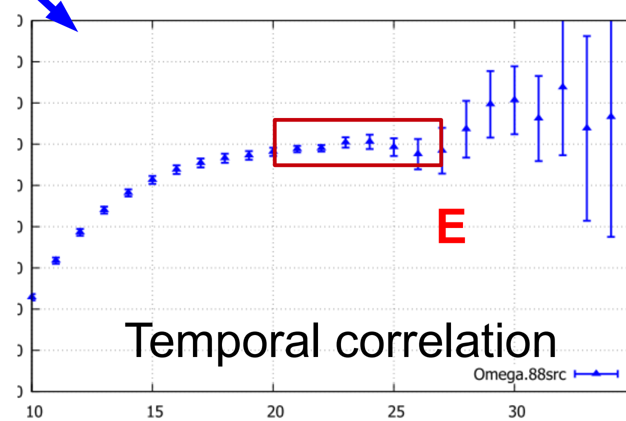


$$\langle 0 | B_1 B_2(t, \vec{r}) \bar{I}(t_0) | 0 \rangle = A_0 \Psi(\vec{r}, E_0) e^{-E_0(t-t_0)} + \cdots$$

Scattering S-matrix

$$S \equiv e^{i\delta(k)}$$





Eigen energy is extracted from plateau region

Be careful of mirage plateau

T. Iritani et al. (HAL), JHEP1610(2016)101

Lüscher's finite volume method

M. Lüscher, NPB354(1991)531

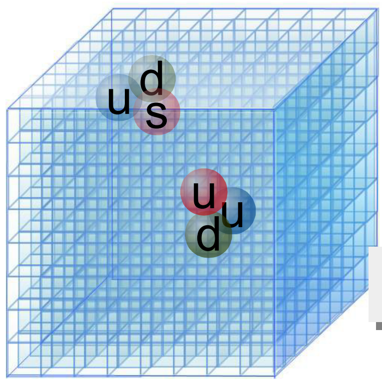
1. Measure the discrete energy spectrum, **E**
2. Put the **k** from E into the formula which connects **k** and **δ**

$$k_n \cot \delta(k_n) = \frac{4\pi}{L^3} \sum_{m \in \mathbb{Z}^3} \frac{1}{\vec{p}_m^{\,2} - k_n^2}$$

Scattering phase shift

Hadron interaction from LQCD (coupled-channel)

Lattice QCD simulation



Scattering S-matrix

$$S(E) = \begin{pmatrix} \eta e^{2i\delta_1} & i\sqrt{1-\eta^2} e^{i(\delta_1+\delta_2)} \\ i\sqrt{1-\eta^2} e^{i(\delta_1+\delta_2)} & \eta e^{2i\delta_2} \end{pmatrix}$$

$$\langle 0 | B_1 B_2(t, \vec{r}) \bar{I}(t_0) | 0 \rangle = A_0 \Psi(\vec{r}, E_0) e^{-E_0(t-t_0)} + \cdots$$

Lüscher's finite volume method

M. Lüscher, NPB354(1991)531

Two-channel S-matrix has 3-parameters

$$\delta_1(E), \quad \delta_2(E), \quad \eta(E)$$

Relations of parameters

Assumption of Interaction

Fixed form of K-matrix

These are related to the energy E by an eigenvalue equation (s-wave)

$$\cos\left(\Delta_1 + \Delta_2 - \delta_1^0 - \delta_2^0\right) = \eta \cos\left(\Delta_1 - \Delta_2 - \delta_1^0 + \delta_2^0\right)$$

$$\frac{1}{\tan \Delta_i} = \frac{4\pi}{k_i} \cdot \frac{1}{L^3} \sum_p \frac{1}{p^2 - k_i^2}$$

Unlike the single channel case,
the number of equations is less than the number of parameters in S-matrix.

Extra-information (relation) is necessary to solve coupled channel scattering

Aoki, Hatsuda, Ishii, PTP123, 89 (2010).

Potential in HAL QCD method

We define potentials which satisfy Schrödinger equation

$$\left(p^2+\nabla^2\right)\Psi^{\alpha}(E,\vec{x}) \equiv \int d^3y\, \underline{U^{\alpha}_{\alpha}(\vec{x},\vec{y})}\, \Psi^{\alpha}(E,\vec{y})$$

Energy independent potential

$$\left(p^2+\nabla^2\right)\Psi^{\alpha}(E,\vec{x}) = K^{\alpha}(E,\vec{x})$$

$$\begin{aligned} K^{\alpha}(E,\vec{x}) &\equiv \int dE'\, K^{\alpha}(E',\vec{x}) \int d^3y\, \tilde{\Psi}^{\alpha}(E',\vec{y})\Psi^{\alpha}(E,\vec{y}) \\ &= \int d^3y \left[\int dE'\, K^{\alpha}(E',\vec{x})\tilde{\Psi}^{\alpha}(E',\vec{y})\right]\Psi^{\alpha}(E,\vec{y}) \\ &= \int d^3y\, U^{\alpha}_{\alpha}(\vec{x},\vec{y})\Psi^{\alpha}(E,\vec{y}) \end{aligned}$$

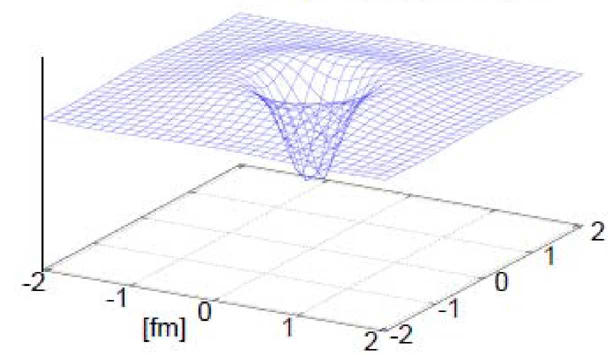
We can define an energy independent potential but it is fully non-local.

This potential automatically reproduce the scattering phase shift

Nambu-Bethe-Salpeter wave function

Definition : equal time NBS w.f.

$$\Psi^{\alpha}(E,\vec{r})e^{-Et} = \sum_{\vec{x}} \langle 0|H_1^{\alpha}(t,\vec{x}+\vec{r})H_2^{\alpha}(t,\vec{x})|E\rangle$$



E : Total energy of the system

Local composite interpolating operators

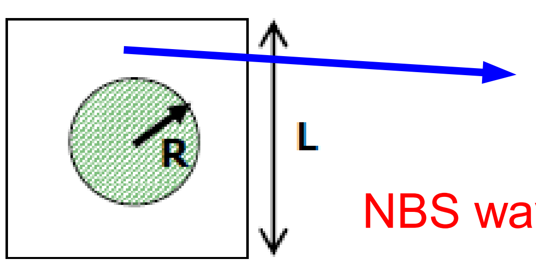
$$B_{\alpha}=\epsilon^{abc}(q_a^T C\gamma_5 q_b)q_{c\alpha} \qquad D_{\mu\alpha}=\epsilon^{abc}(q_a^T C\gamma_{\mu} q_b)q_{c\alpha}$$

$$M=(\bar{q}_a\gamma_5 q_a)$$ Etc.....

- It satisfies the Helmholtz eq. in asymptotic region : $(p^2+\nabla^2)\Psi(E,\vec{r})=0$
- Using the reduction formula,

C.-J.D.Lin et al.,NPB619 (2001) 467.

$$\Psi^{\alpha}(E,\vec{r}) = \sqrt{Z_{H_1}}\sqrt{Z_{H_2}}\left(e^{i\vec{p}\cdot\vec{r}}+\int\frac{d^3q}{2E_q}\frac{T(q,p)}{4E_p(E_q-E_p-i\epsilon)}e^{i\vec{q}\cdot\vec{r}}\right)$$



$$\Psi(E,\vec{r})\simeq A\frac{\sin(pr+\delta(E))}{pr}$$

Phase shift is defined as

$$S\equiv e^{i\delta}$$

NBS wave function has a same asymptotic form with quantum mechanics.

(NBS wave function is characterized from phase shift)

N. Ishii et al Phys. Lett. B**712**(2012)437

Time-dependent Schrödinger like equation

Start with the normalized four-point correlator.

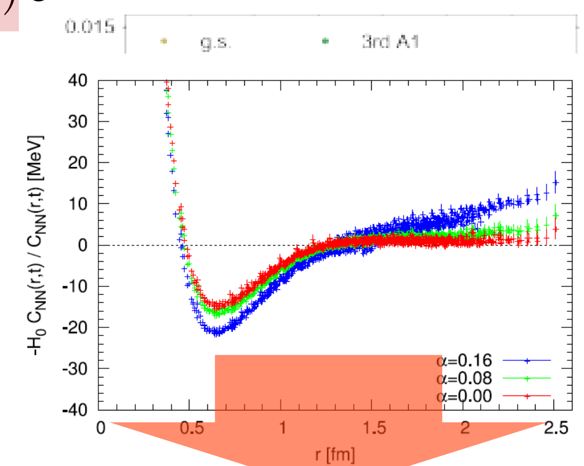
$$R_I^{B_1 B_2}(t,\vec{r}) = F^{B_1 B_2}{}_I(t,\vec{r})\, e^{(m_1+m_2)t}$$

$$= A_0 \Psi(\vec{r},E_0)\, e^{-(E_0-m_1-m_2)t} + A_1 \Psi(\vec{r},E_1)\, e^{-(E_1-m_1-m_2)t} + \cdots$$

$$\left(\frac{p_0^2}{2\mu}+\frac{\nabla^2}{2\mu}\right)\Psi(\vec{r},E_0) = \int U(\vec{r},\vec{r}\,')\Psi(\vec{r}\,',E_0)\, d^3 r'$$

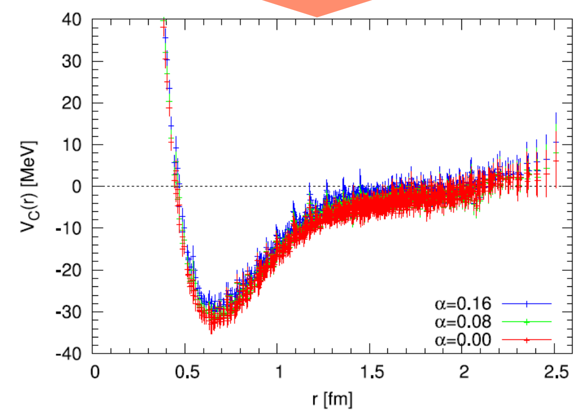
$$E_n - m_1 - m_2 \approx \frac{p_n^2}{2\mu}$$

$$\left(\frac{p_1^2}{2\mu}+\frac{\nabla^2}{2\mu}\right)\Psi(\vec{r},E_1) = \int U(\vec{r},\vec{r}\,')\Psi(\vec{r}\,',E_1)\, d^3 r'$$



A single state saturation is not required!!

$$\left(-\frac{\partial}{\partial t}+\frac{\nabla^2}{2\mu}\right) R_I^{B_1 B_2}(t,\vec{r}) = \int U(\vec{r},\vec{r}\,')\, R_I^{B_1 B_2}(t,\vec{r})\, d^3 r'$$



Derivative (velocity) expansion of U

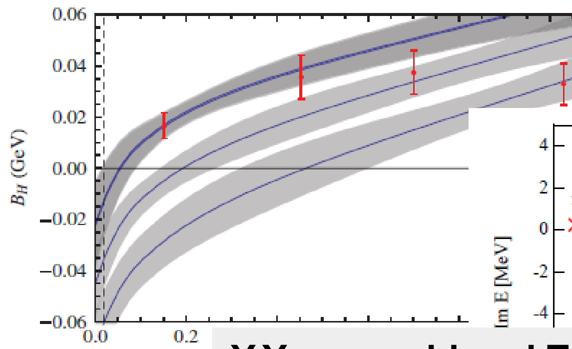
$$U(\vec{r},\vec{r}\,') = \left[V_C(r) + S_{12} V_T(r)\right] + \left[\vec{L}\cdot\vec{S}_s V_{LS}(r) + \vec{L}\cdot\vec{S}_a V_{ALS}(r)\right] + O(\nabla^2)$$

Works on H-dibaryon state

Theoretical status

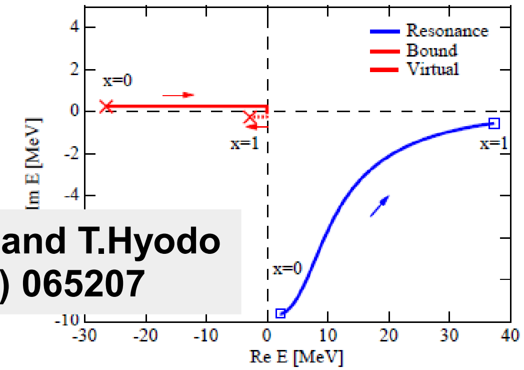
Several sort of calculations and results
(bag models, NRQM, Quenched LQCD….)

There were no conclusive result.



P. E. Shanahan et al
PRL 107(2011) 092004

Chiral extrapolations of recent LQCD data



Y.Yamaguchi and T.Hyodo
PRC 94 (2016) 065207

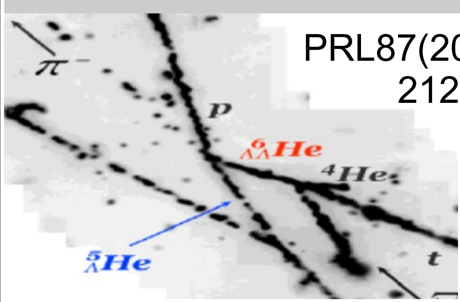
Unbound or resonance

Experimental status

"NAGARA Event"

K.Nakazawa et al
KEK-E176 & E373 Coll.

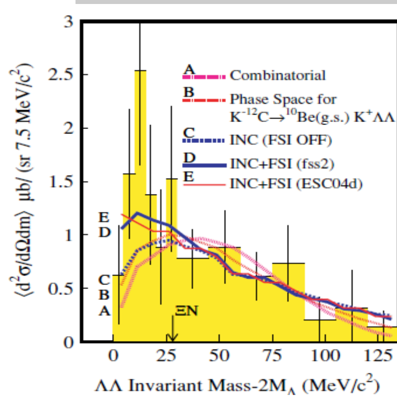
PRL87(2001) 212502



- Deeply bound dibaryon state is ruled out

"$^{12}C(K^-,K^+\Lambda\Lambda)$ reaction"

C.J.Yoon et al KEK-PS E522 Coll.



PRC75(2007) 022201(R)

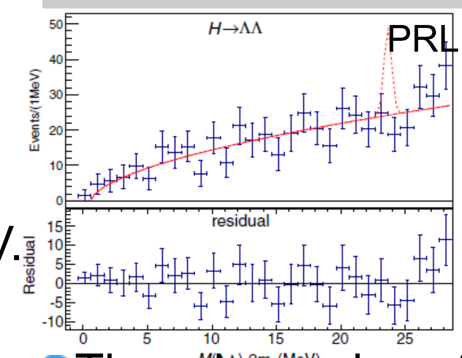
- Significance of enhancements below 30 MeV.

Larger statistics
J-PARC E42

"Y(1S) and Y(2S) decays"

B.H. Kim et al Belle Coll.

PRL110(2013) 222002



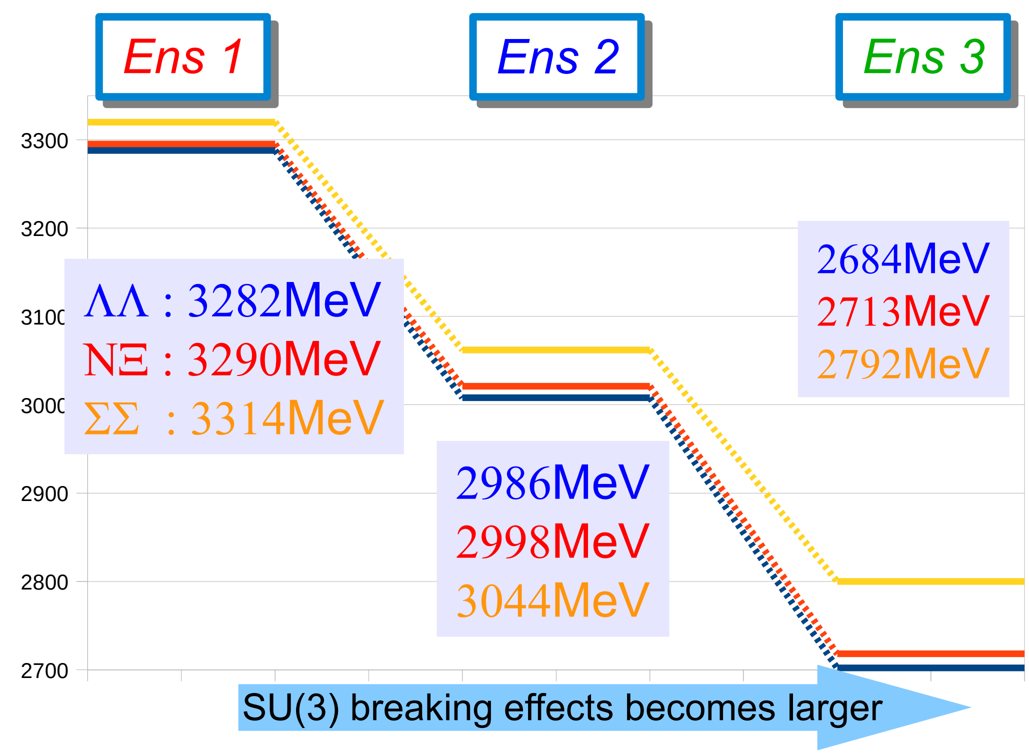
- There is no sign of near threshold enhancement.

Numerical setup

- 2+1 flavor gauge configurations by PACS-CS collaboration.
  - RG improved gauge action & O(a) improved Wilson quark action
  - β = 1.90, a^{-1} = 2.176 [*GeV*], 32^3x64 lattice, *L* = 2.902 [*fm*].
  - κ_s = 0.13640 is fixed, κ_{ud} = 0.13700, 0.13727 and 0.13754 are chosen.
- Wall source is considered to produce S-wave B-B state.

In unit of MeV	*Ens 1*	*Ens 2*	*Ens 3*
π	701±1	570±2	411±2
K	789±1	713±2	635±2
m_π/m_K	0.89	0.80	0.65
N	1581±5	1398±7	1215±8
Λ	1641±5	1493±6	1342±6
Σ	1657±5	1522±7	1394±8
Ξ	1709±4	1600±5	1498±5

u,d quark masses lighter →

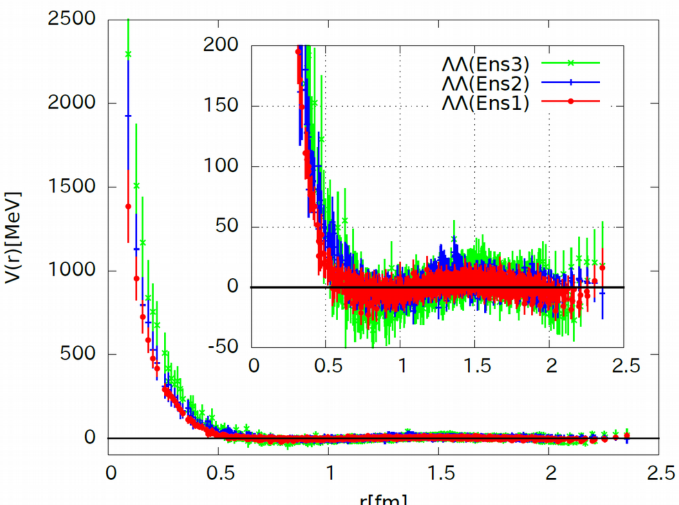


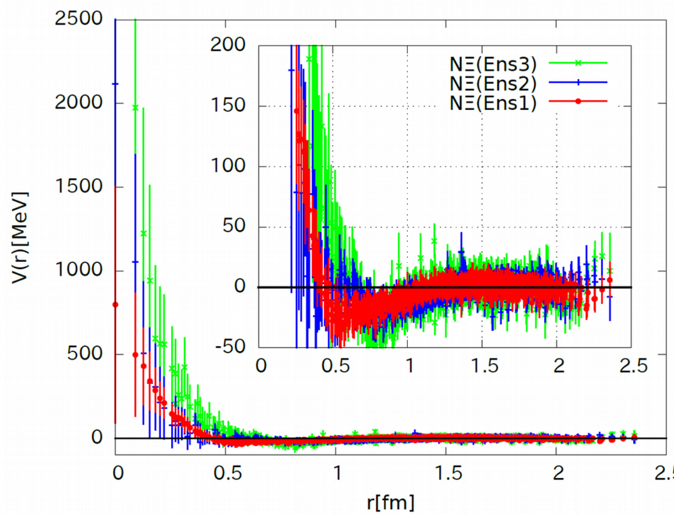
$\Lambda\Lambda$, $N\Xi$, $\Sigma\Sigma$ ($I=0$) $^1S_0$ channel

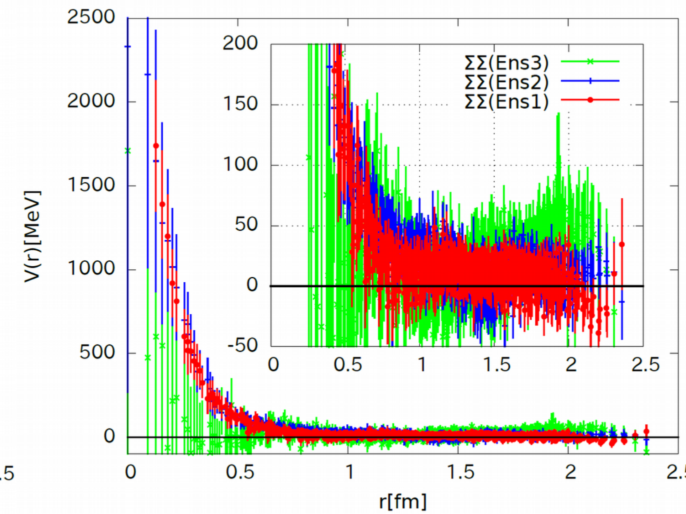
Esb1 : m_π= 701 MeV
Esb2 : m_π= 570 MeV
Esb3 : m_π= 411 MeV

- N_f = 2+1 full QCD with L = 2.9fm

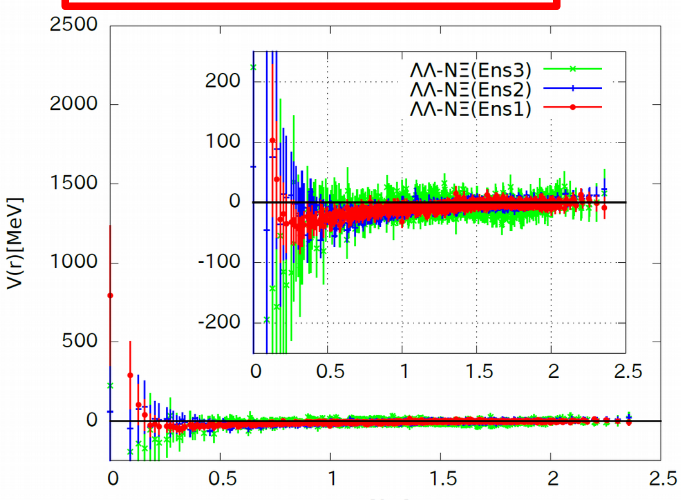
Diagonal elements

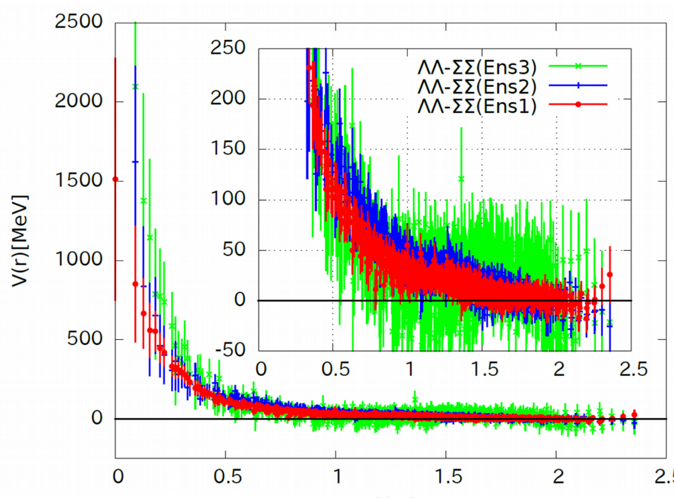


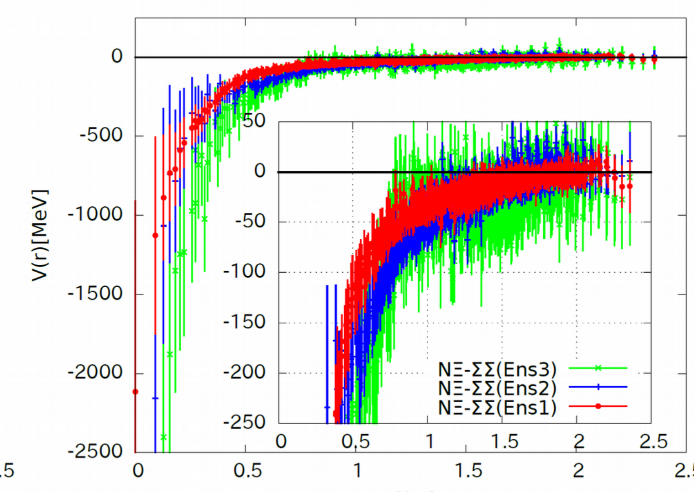




Off-diagonal elements



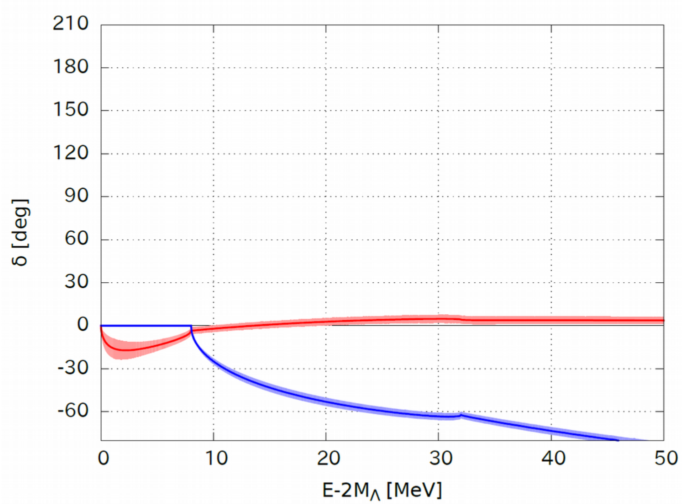




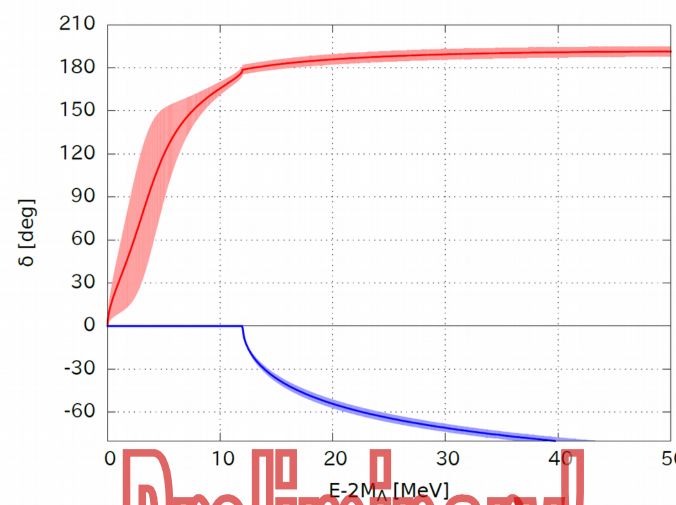
$\Lambda\Lambda$ and $N\Xi$ phase shifts

▶ N_f = 2+1 full QCD with L = 2.9fm

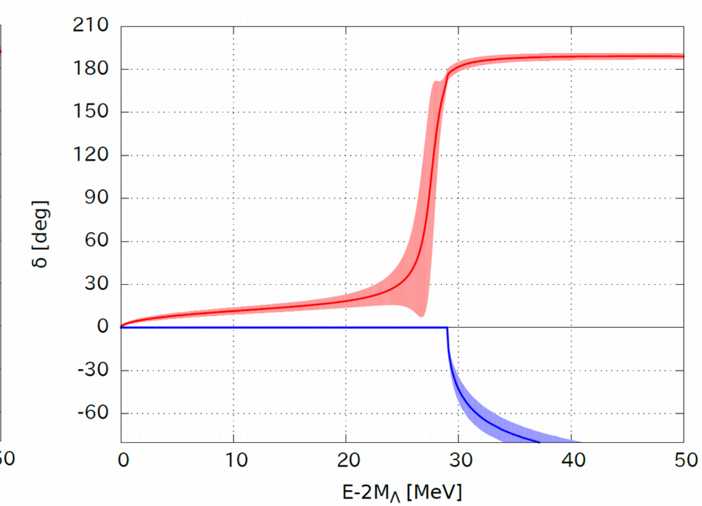
m_π = 700 MeV



m_π = 570 MeV



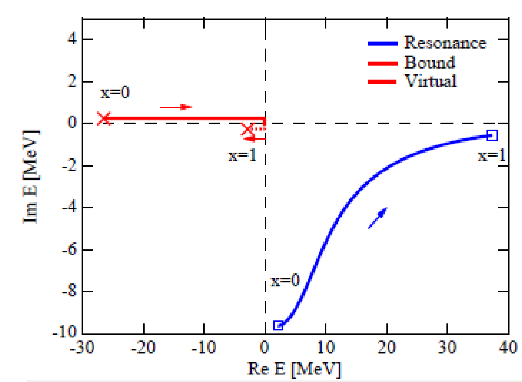
m_π = 410 MeV



Preliminary!

- m_π = 700 MeV : bound state
- m_π = 570 MeV : resonance near $\Lambda\Lambda$ threshold
- m_π = 410 MeV : resonance near $N\Xi$ threshold..

Go to the physical point simulation!



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