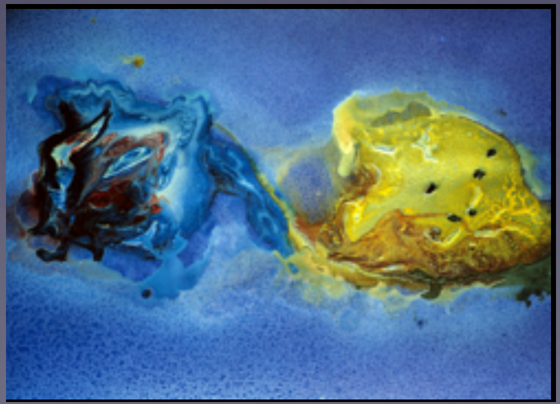
X, Y, Z with Nonrelativistic Effective Field Theories



Physik Department TUM
T30f

Nora Brambilla

- Non relativistic Effective Field Theories (plus lattice) give a QCD description of quarkonia below the strong decay threshold

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    - the hierarchy of NREFT is based on the hierarchy of scales in quarkonium

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- the hierarchy of NREFT is based on the hierarchy of scales in quarkonium

- in this framework quarkonium becomes a golden system for the extraction of SM parameters (quark masses, alphas) and the study of confinement

- Quarkonium and X Y Z at and above threshold

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- van der Waals bottomonia interaction : bound states?

Material for discussion

slides at http://itp.phy.pku.edu.cn/conference/qwg2017/

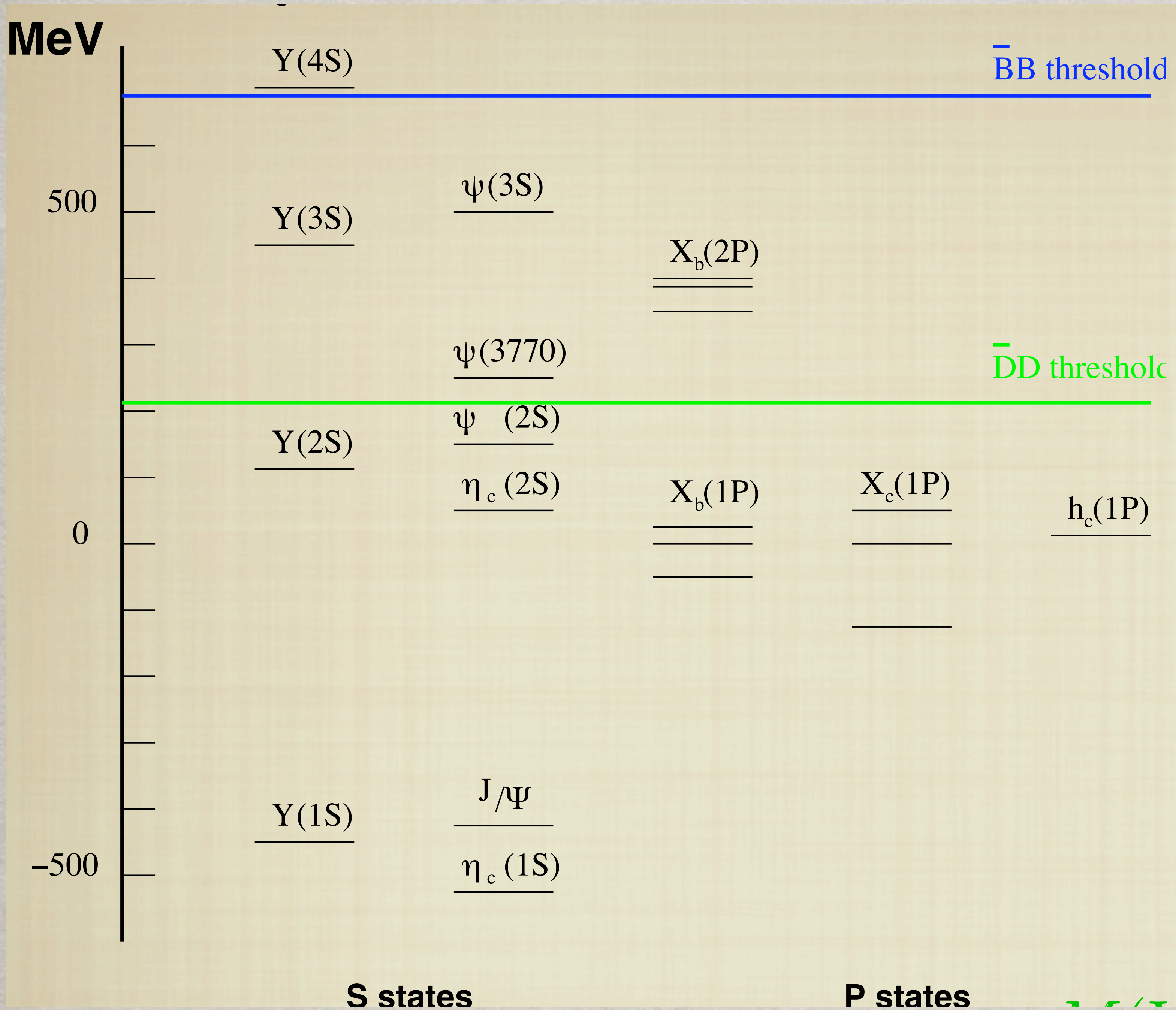
- **Heavy quarkonium: progress, puzzles, and opportunities**
  N. Brambilla (Munich, Tech. U.) *et al.*. Oct 2010. 181 pp.
  Published in **Eur.Phys.J. C71 (2011) 1534**
  e-Print: **arXiv:1010.5827 [hep-ph]- Cited by 1190 records**

- **QCD and Strongly Coupled Gauge Theories: Challenges and Perspectives**
  N. Brambilla (Munich, Tech. U.) *et al.*. Apr 2014. 241 pp.
  Published in **Eur.Phys.J. C74 (2014) no.10, 2981**
  e-Print: **arXiv:1404.3723 Cited by 264 records**

chapter on exotics

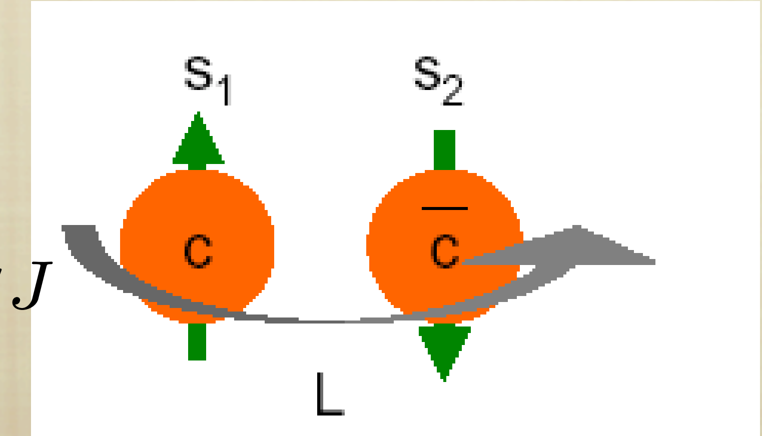
We highlight the progress, current status, and open challenges of QCD-driven physics, in theory and in experiment. We discuss how the strong interaction is intimately connected to a broad sweep of physical problems, in settings ranging from astrophysics and cosmology to strongly-coupled, complex systems in particle and condensed-matter physics, as well as to searches for physics beyond the Standard Model. We also discuss how success in describing the strong interaction impacts other fields, and, in turn, how such subjects can impact studies of the strong interaction. In the course of the work we offer a perspective on the many research streams which flow into and out of QCD, as well as a vision for future developments.

Quarkonium scales



Normalized with respect to $\chi_b(1P)$ and $\chi_c(1P)$

$^{2S+1}L_J$

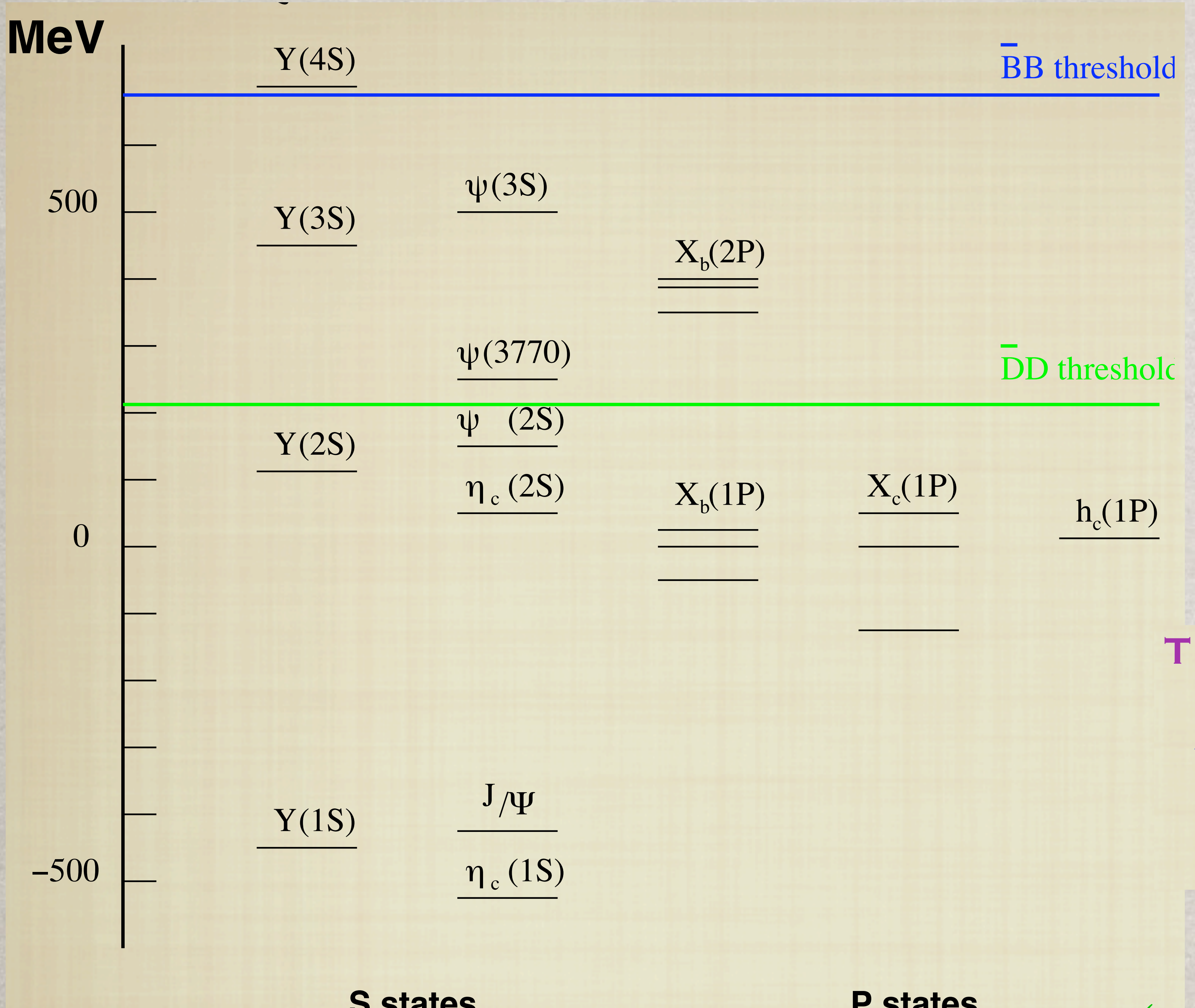


THE MASS SCALE IS PERTURBATIVE

$$m_Q \gg \Lambda_{\rm QCD}$$

$$m_b \simeq 5\,{\rm GeV}; m_c \simeq 1.5\,{\rm GeV}$$

Quarkonium scales



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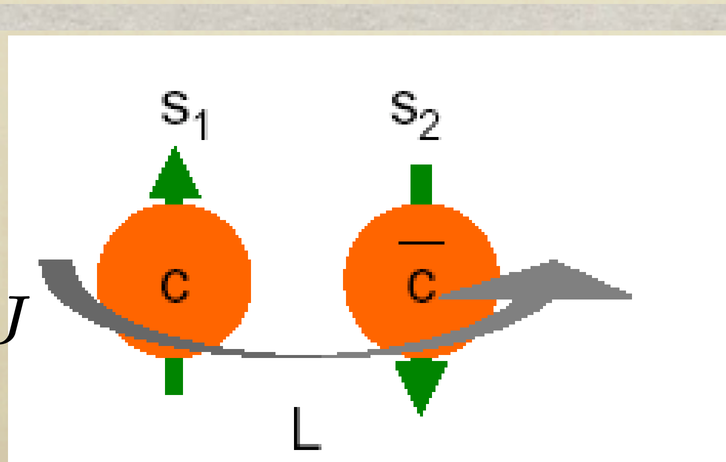

The system is nonrelativistic(NR)

$$\Delta E \sim mv^2, \Delta_{fs} E \sim mv^4$$

$$v_b^2 \sim 0.1, v_c^2 \sim 0.3$$

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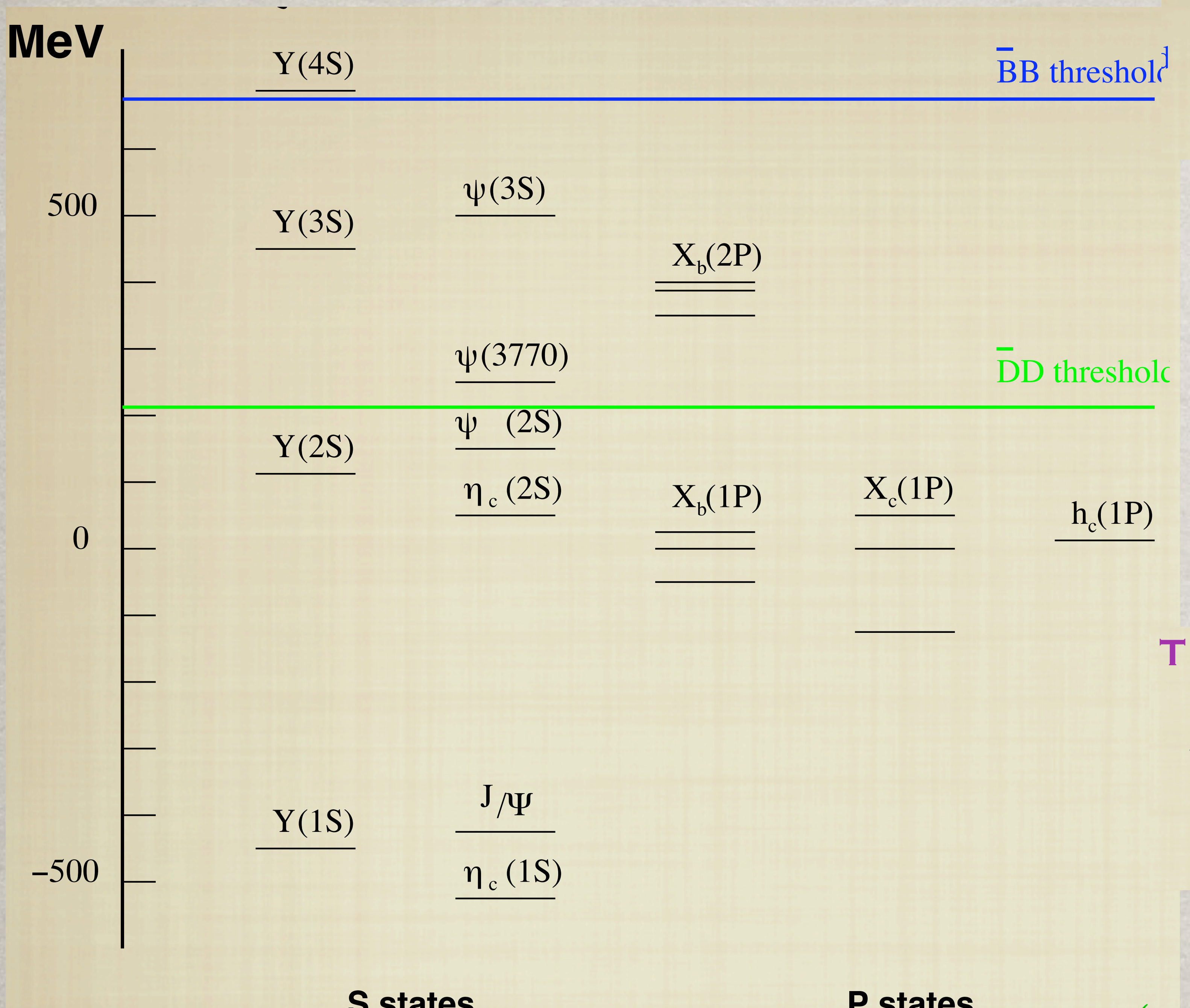

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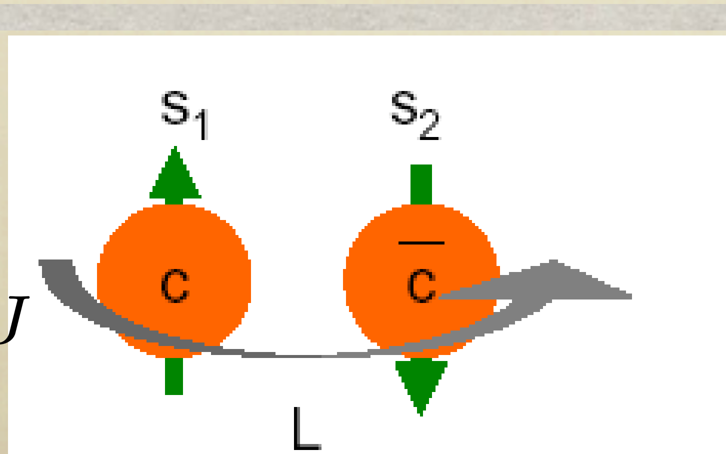

NR bound states have at least 3 scales

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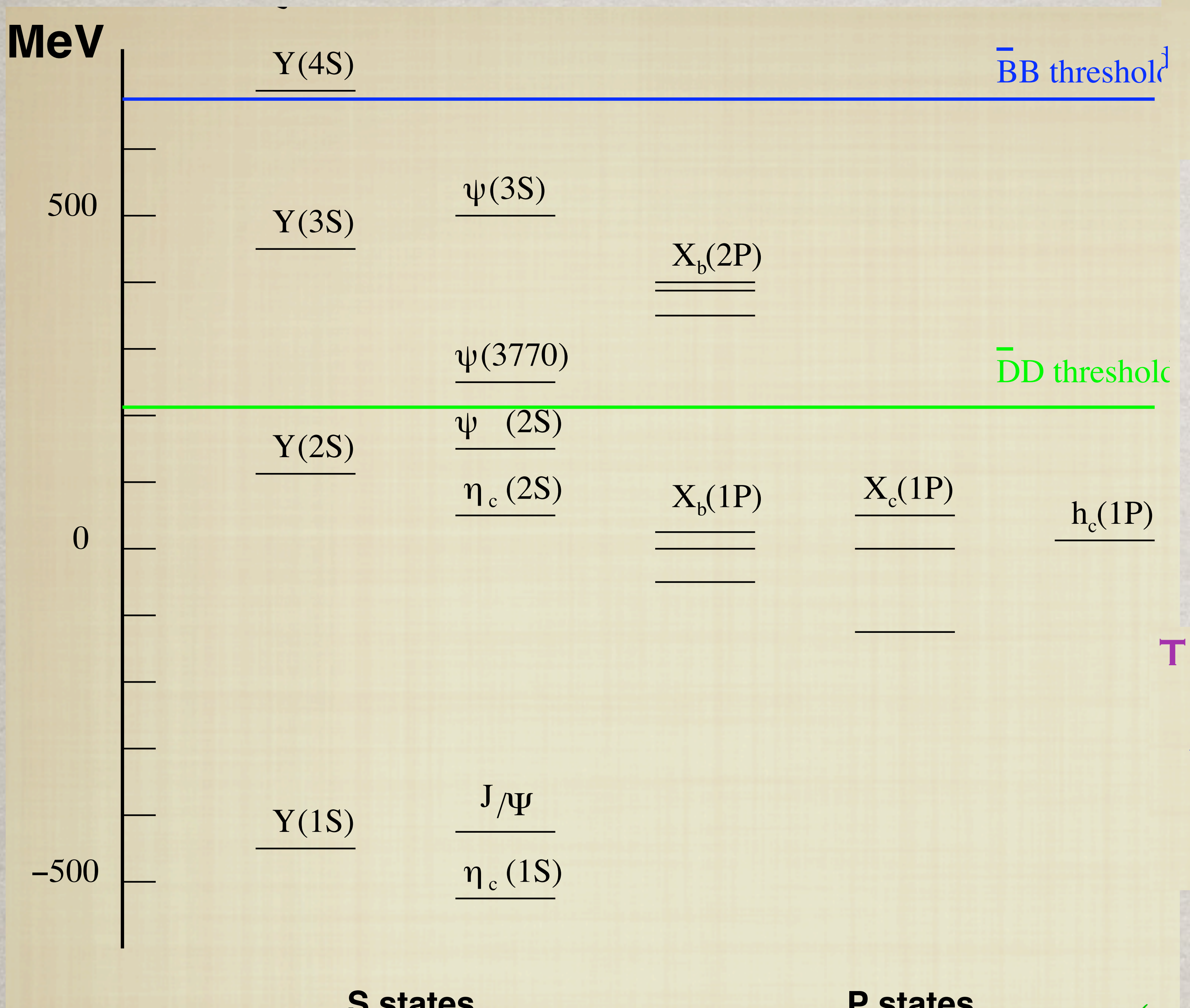

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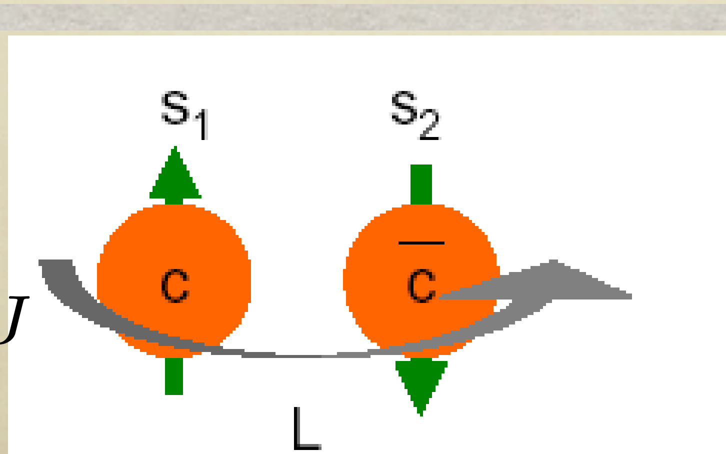

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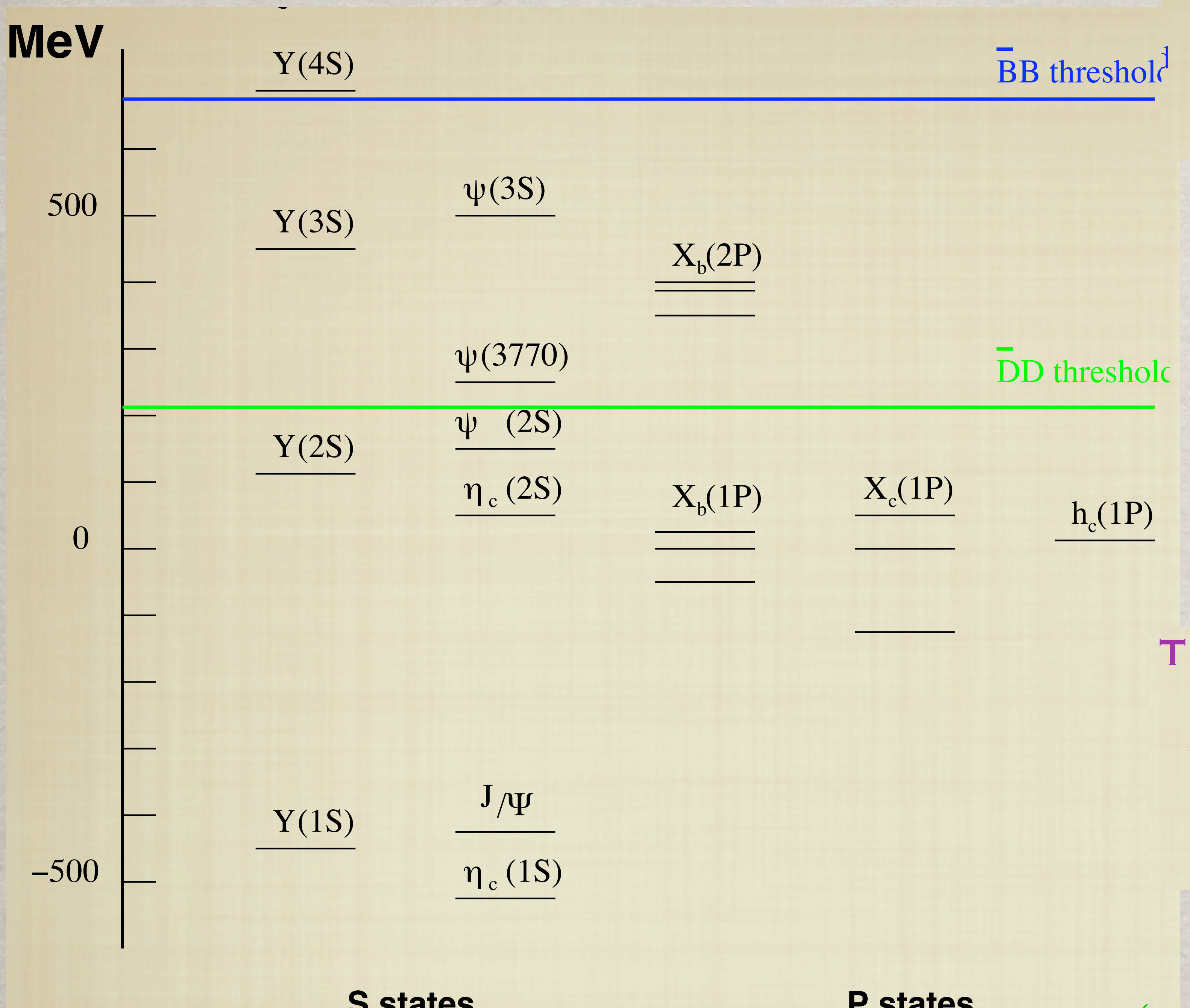

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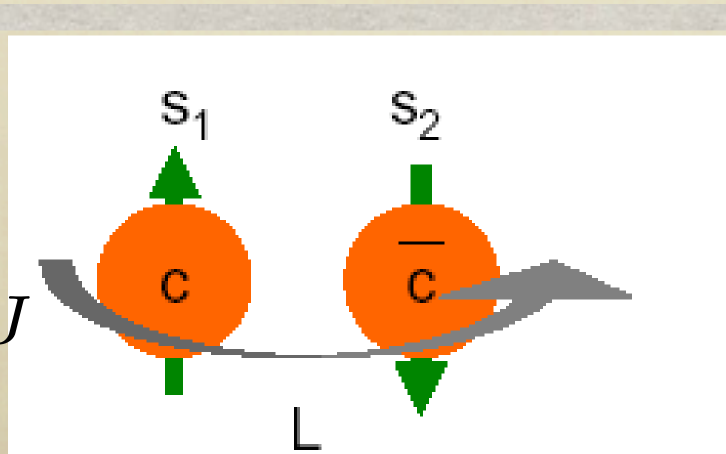

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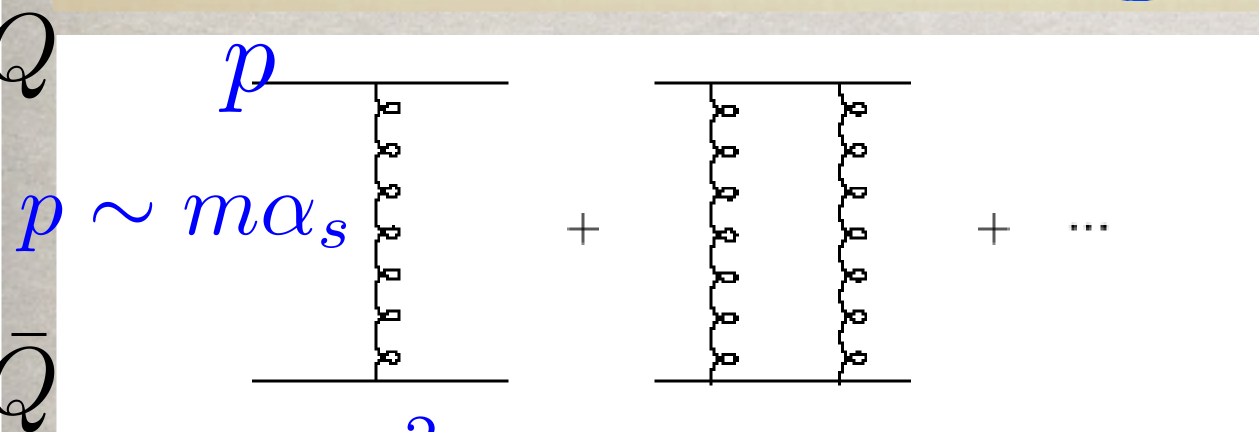
QCD theory of Quarkonium: a very hard problem

Close to the bound state $\alpha_s \sim v$



$$\frac{g^2}{p^2}\left(1+\frac{m\alpha_s}{p}\right)$$

$$\sim \frac{1}{E-\left(\frac{p^2}{m}+V\right)}$$

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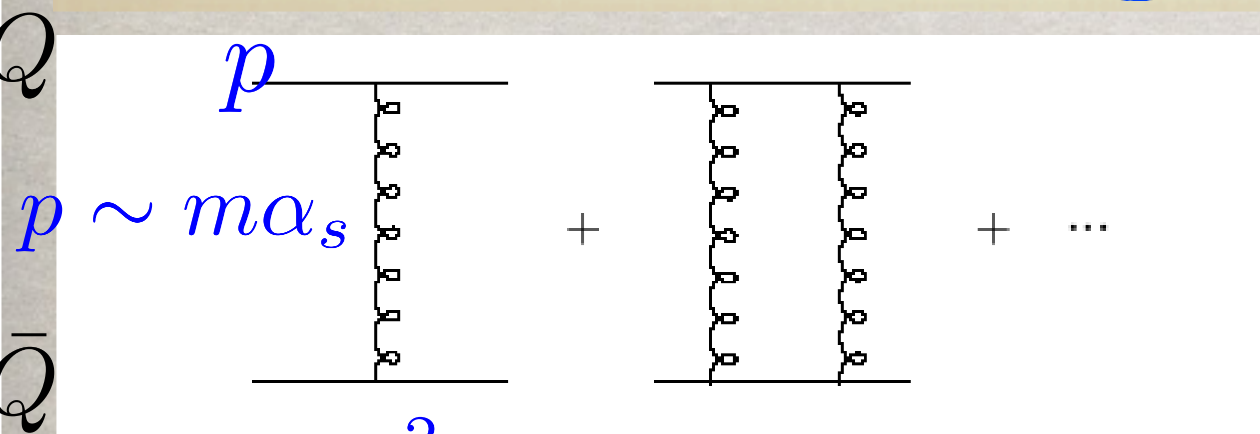

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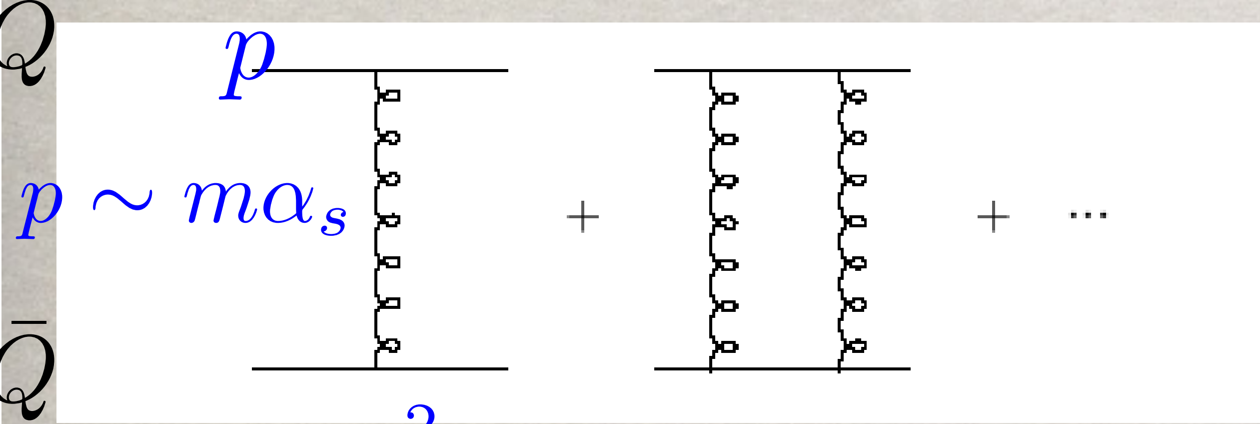

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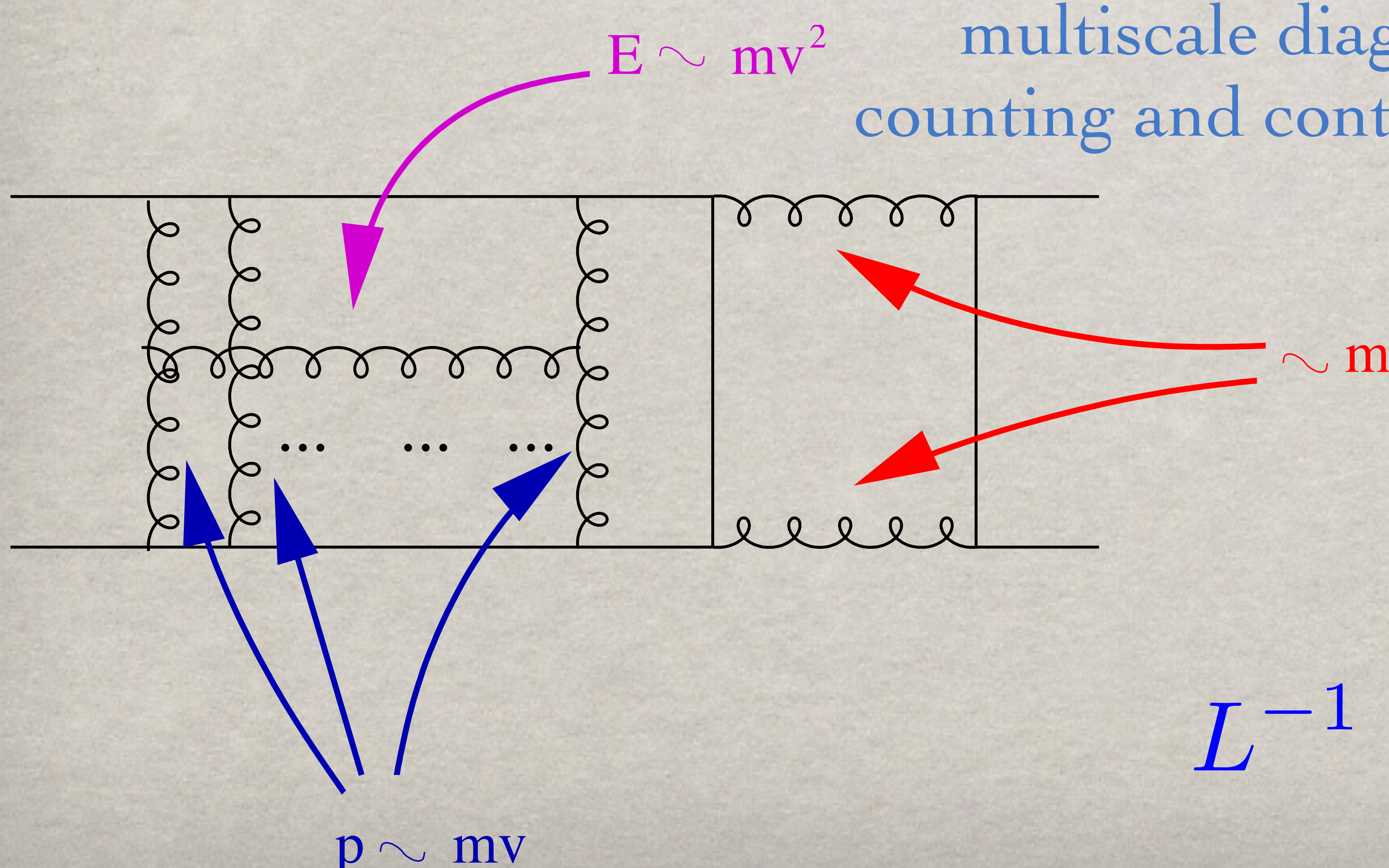
multiscale diagrams have a complicate power counting and contribute to all orders in the coupling

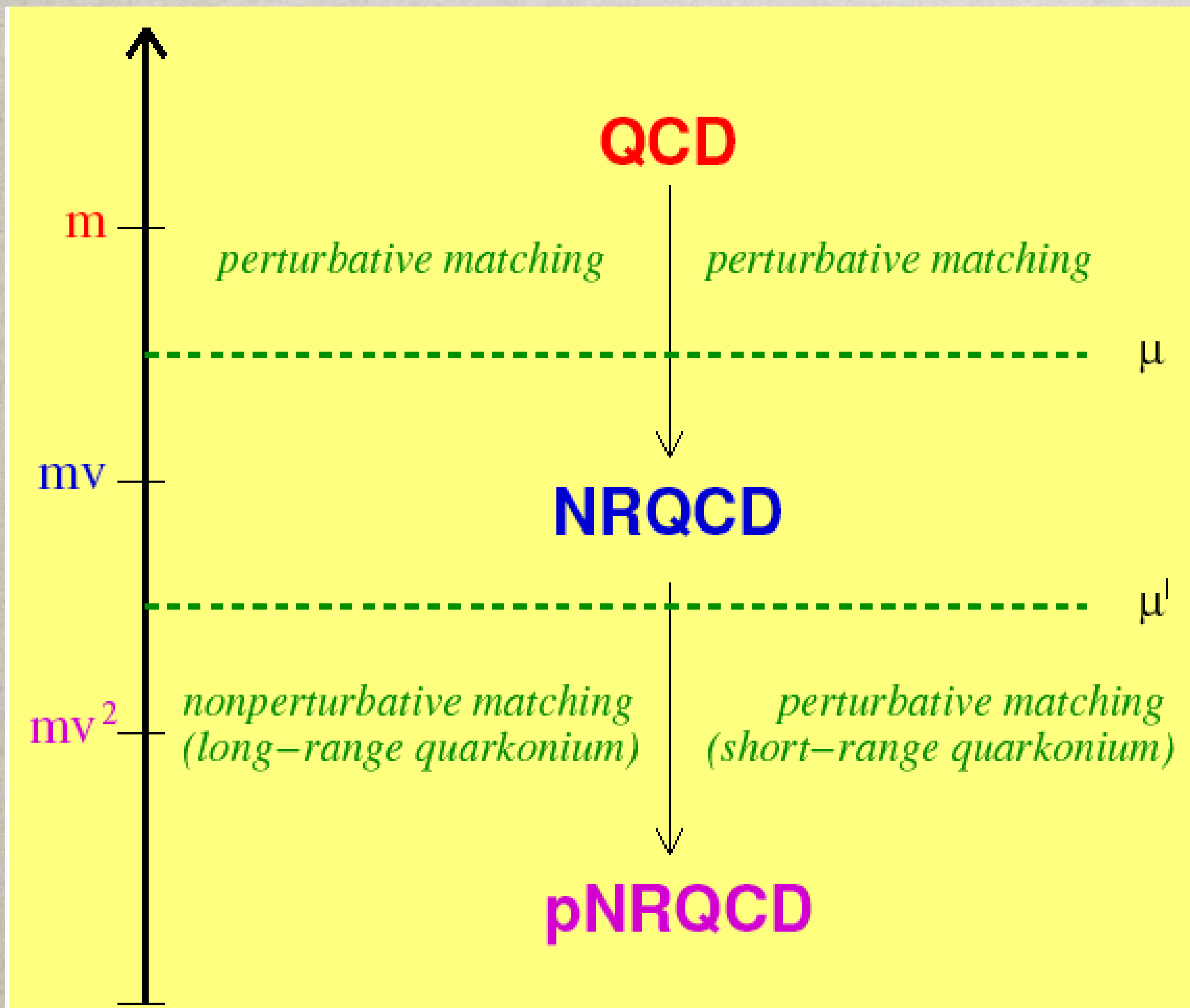


Difficult also for the lattice!

$$L^{-1} \ll \lambda \ll \Lambda \ll a^{-1}$$

Quarkonium with Non relativistic Effective Field Theories

Color degrees of freedom
3X3=1+8
singlet and octet QQbar



Hard

Soft
(relative
momentum)

Ultrasoft
(binding energy)

$$\mathcal{L}_{\rm EFT} = \sum_n c_n(E_\Lambda/\mu)\frac{O_n(\mu,\lambda)}{E_\Lambda}$$

$$\langle O_n \rangle \sim E_\lambda^n$$

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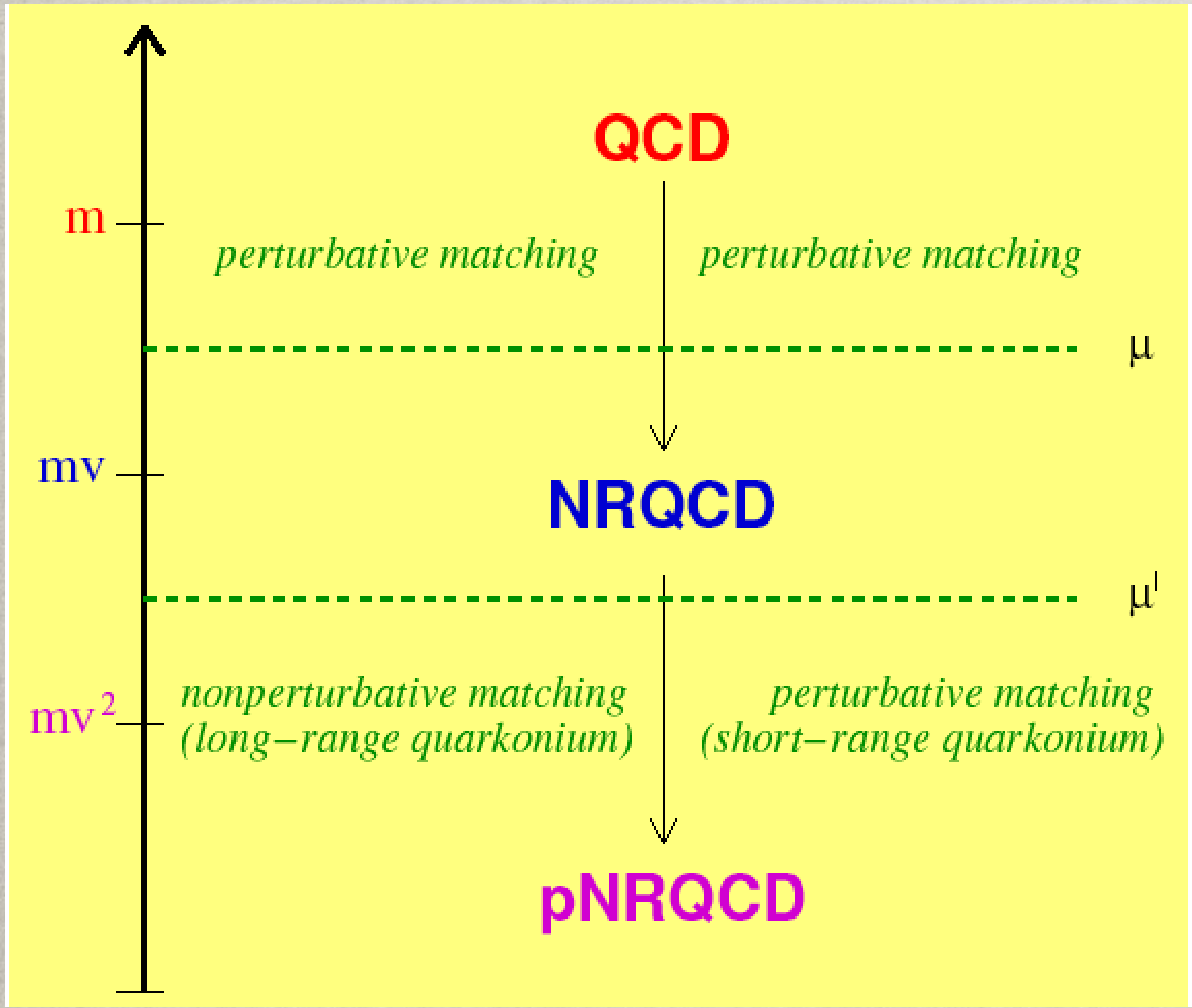

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$$\frac{E_\lambda}{E_\Lambda} = \frac{mv}{m}$$

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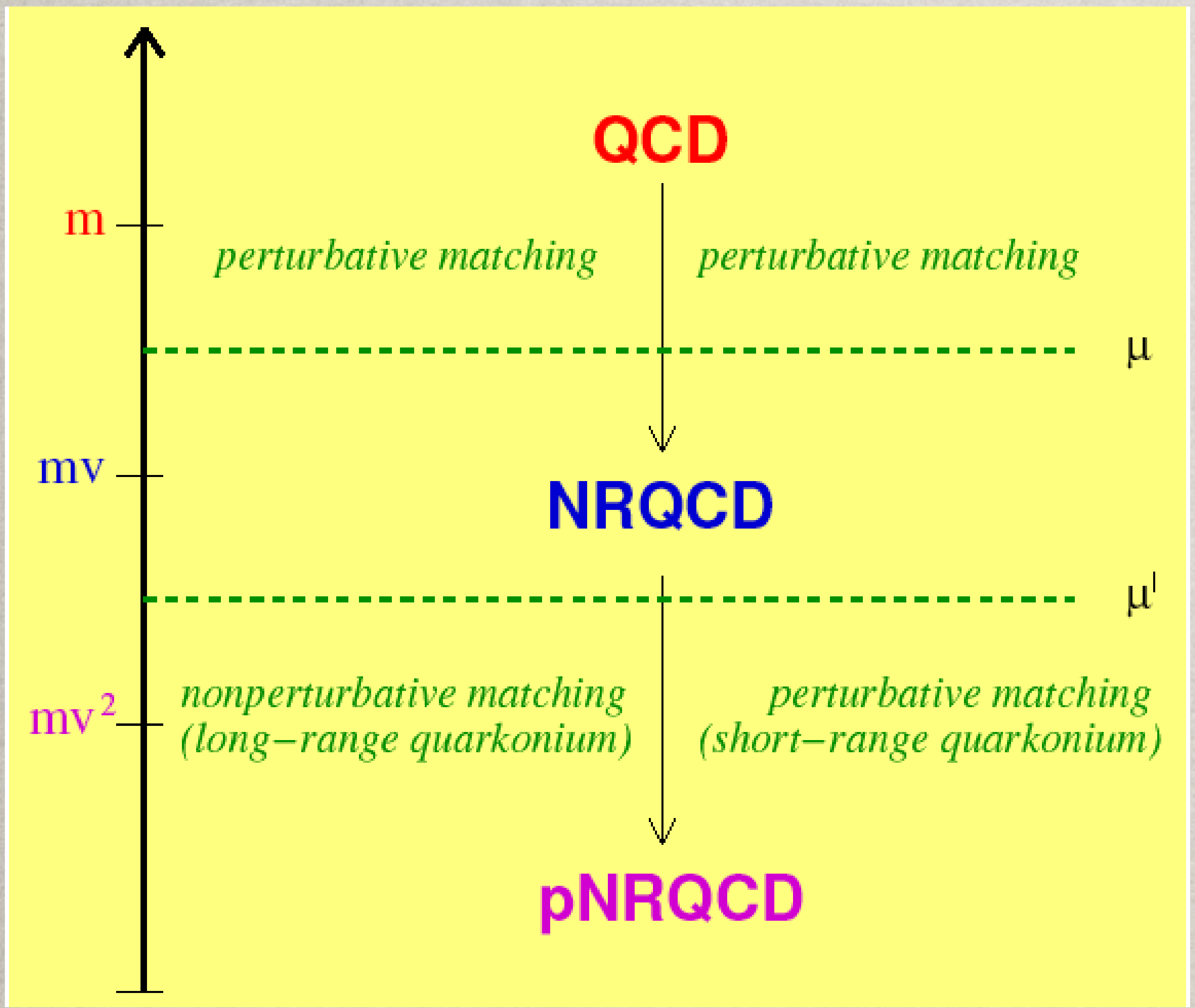

Hard

$$\frac{E_\lambda}{E_\Lambda} = \frac{mv}{m}$$

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Ultrasoft (binding energy)

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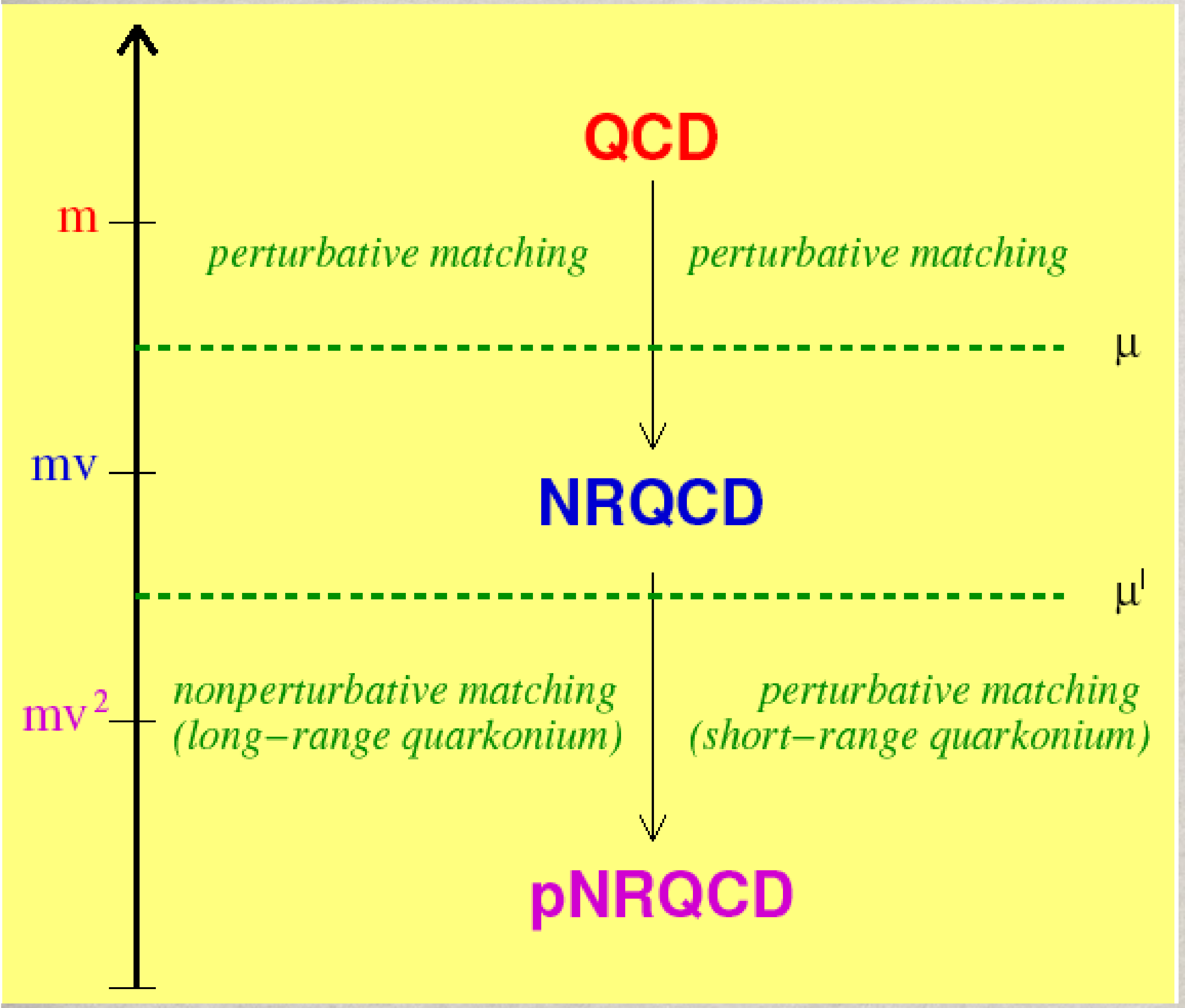
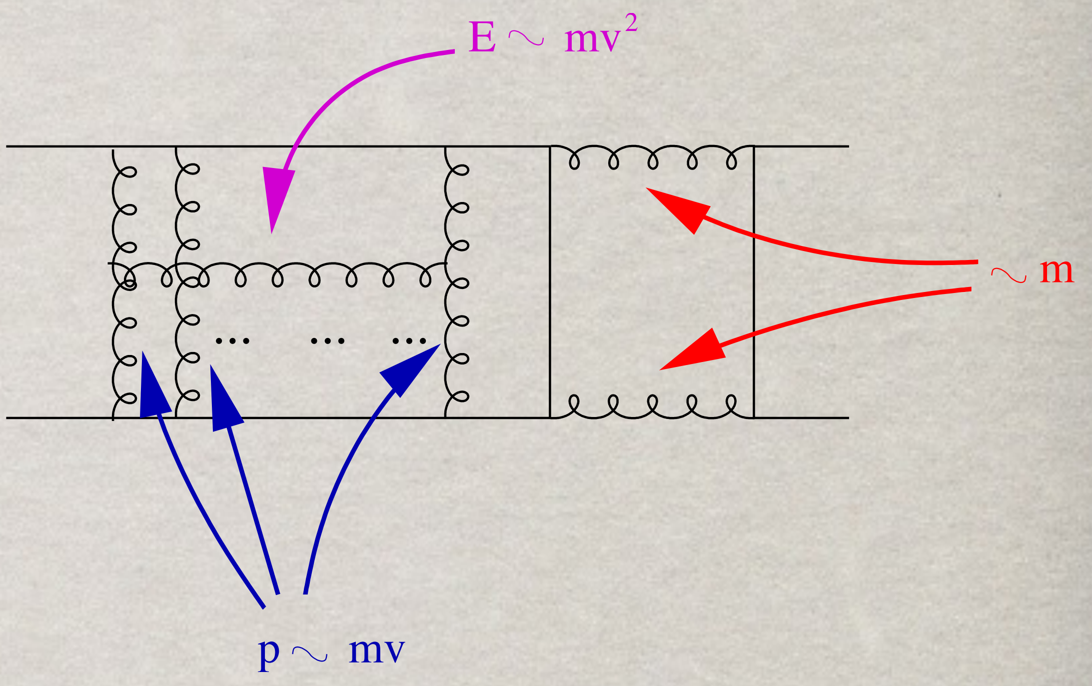
Quarkonium with NR EFT: Non Relativistic QCD (NRQCD)





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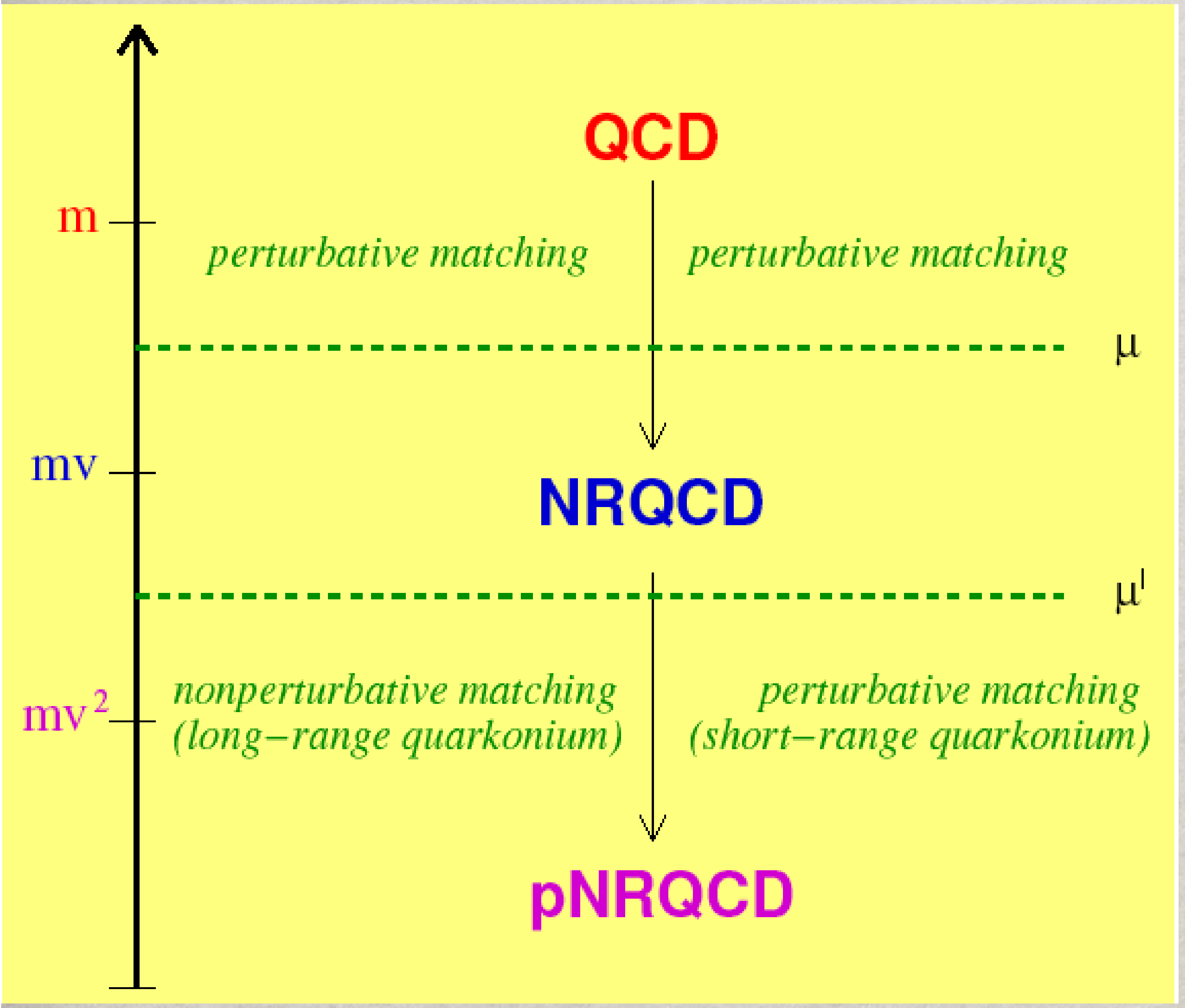

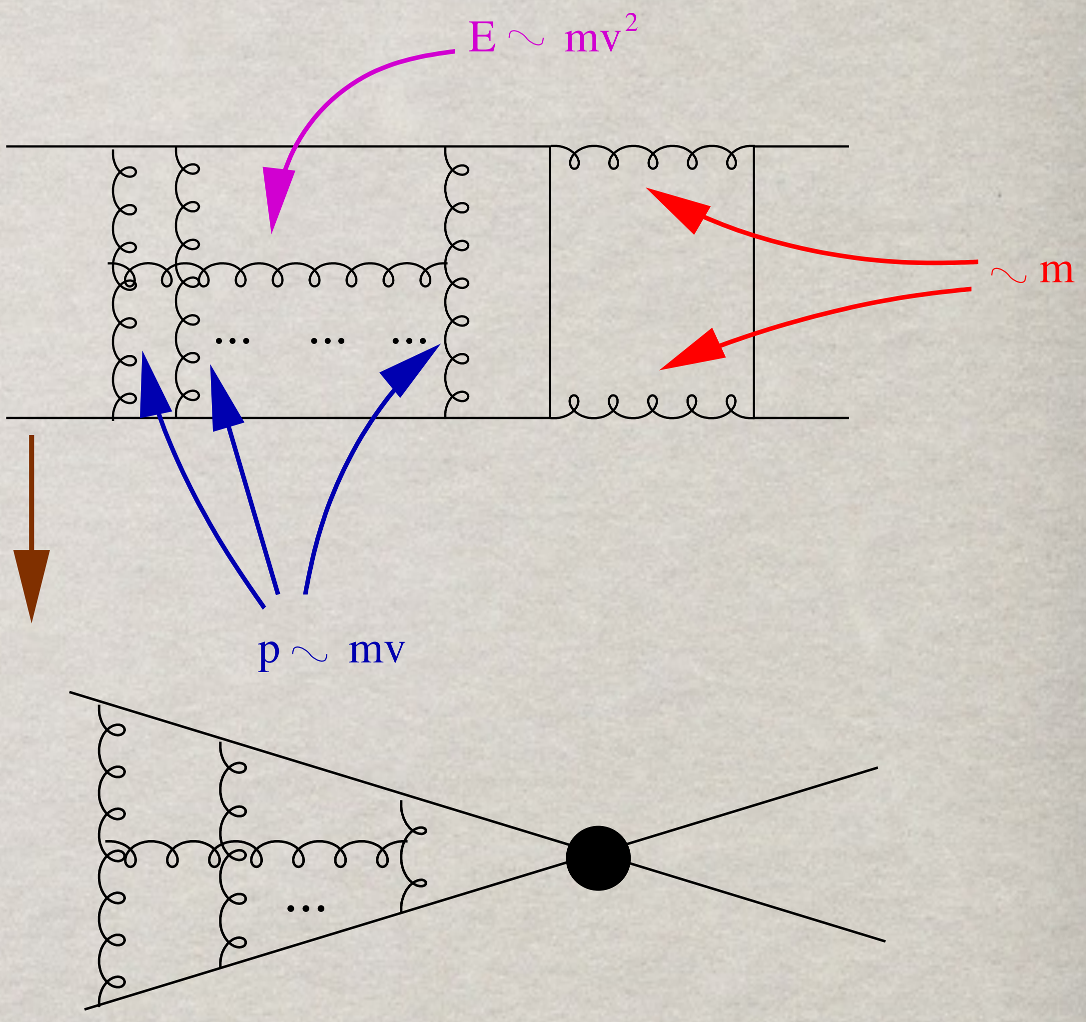

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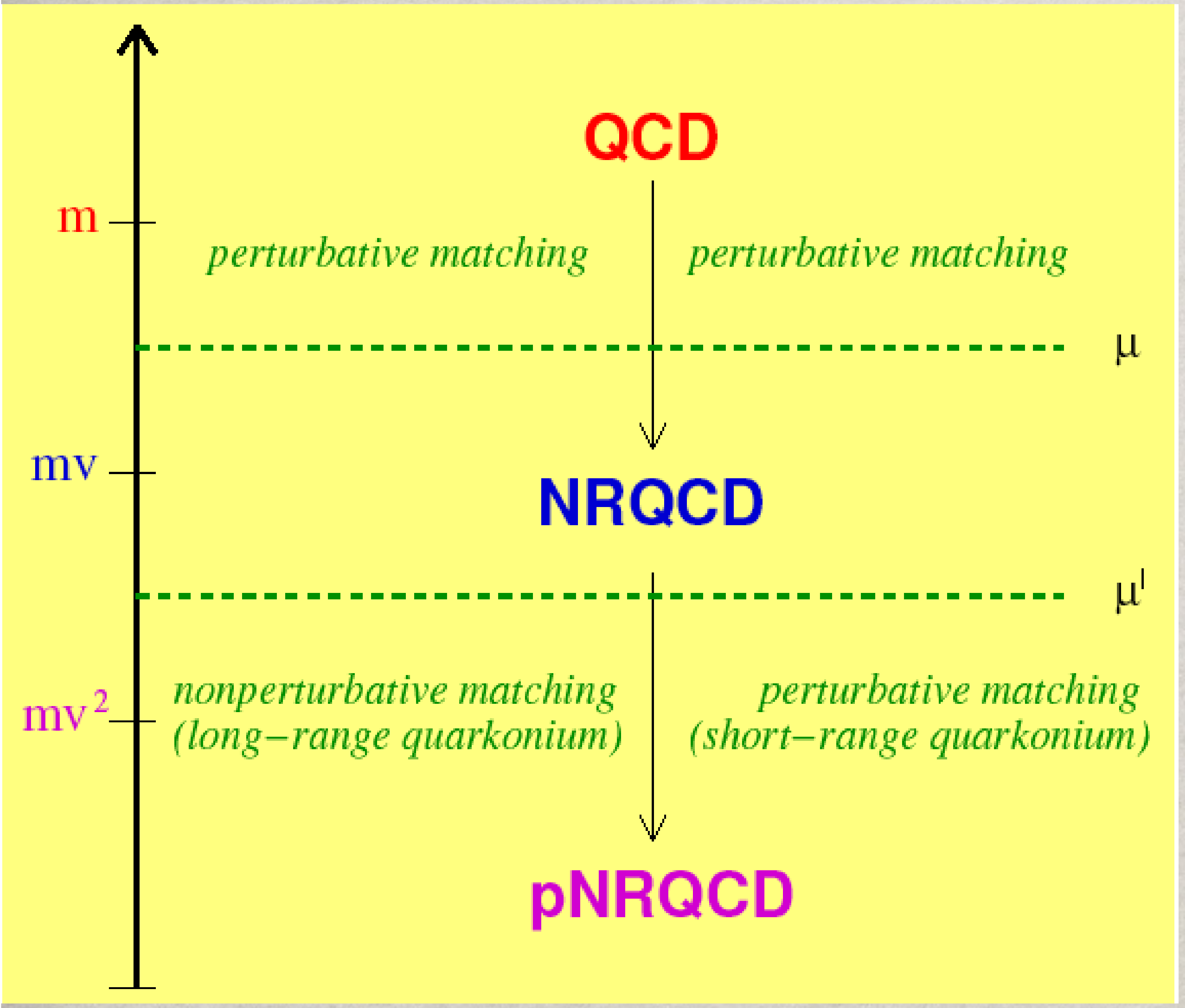
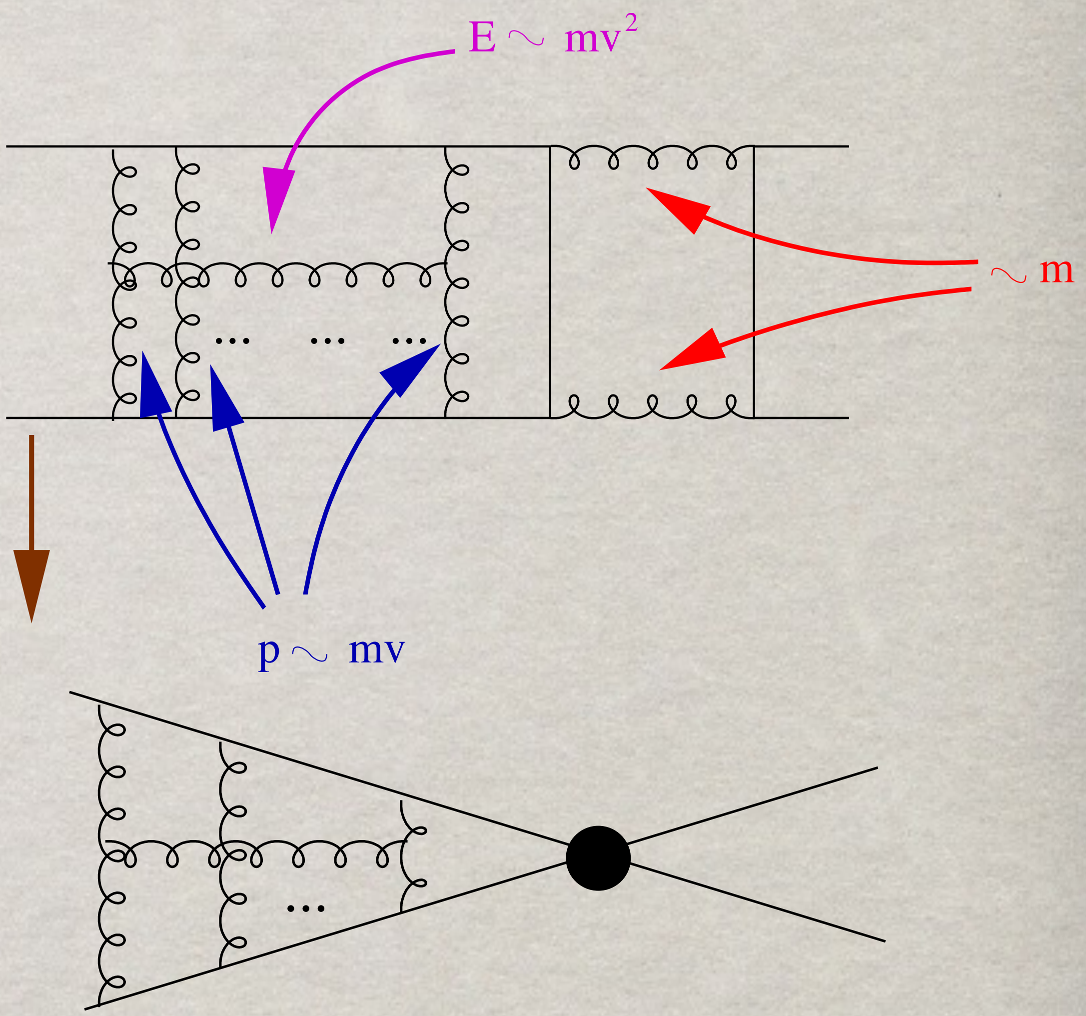
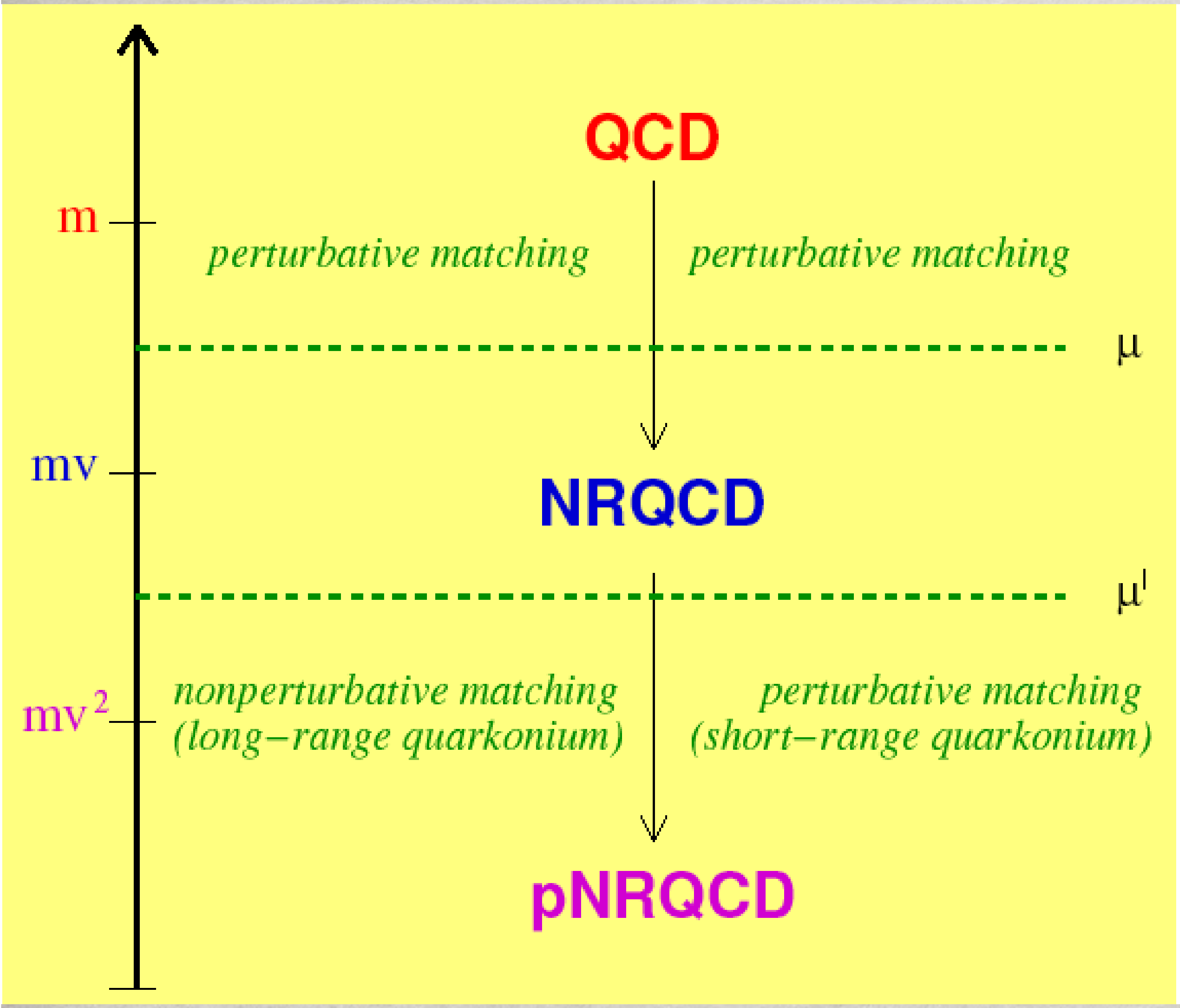
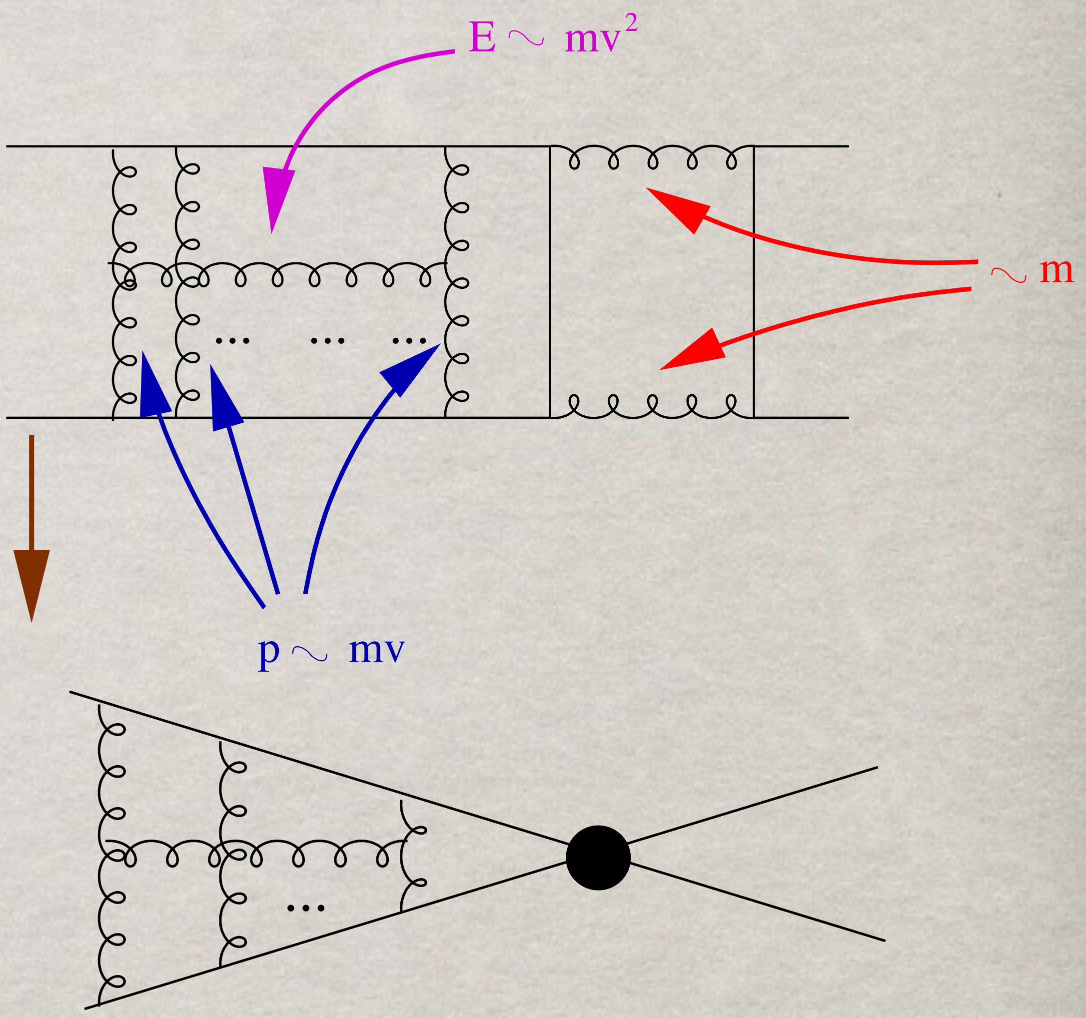




$$\mathcal{L}_{\rm NRQCD} = \sum_n c(\alpha_{\rm s}(m/\mu)) \times \frac{O_n(\mu,\lambda)}{m^n}$$

Quarkonium with NR EFT: potential NonRelativistic QCD (pNRQCD)





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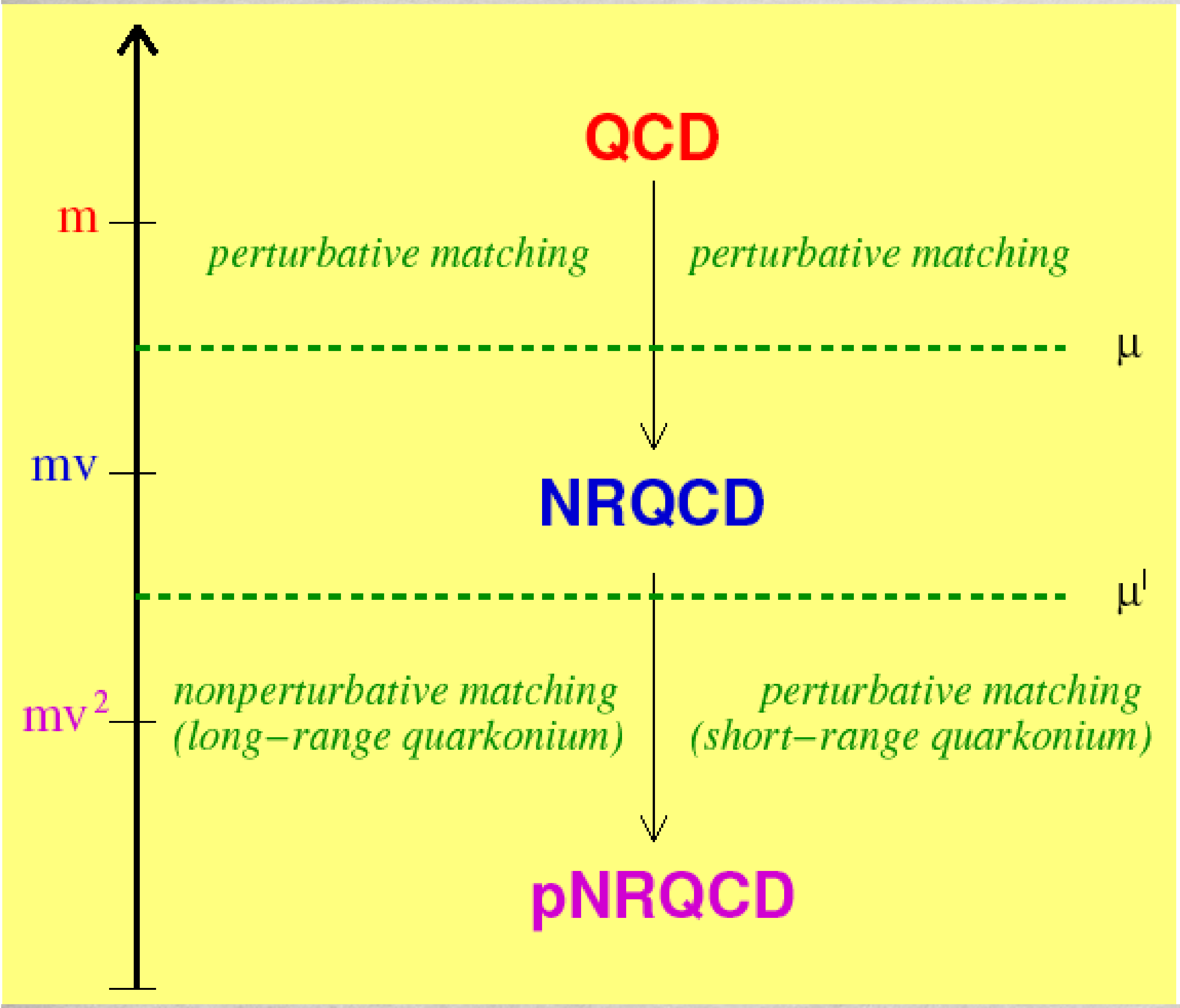

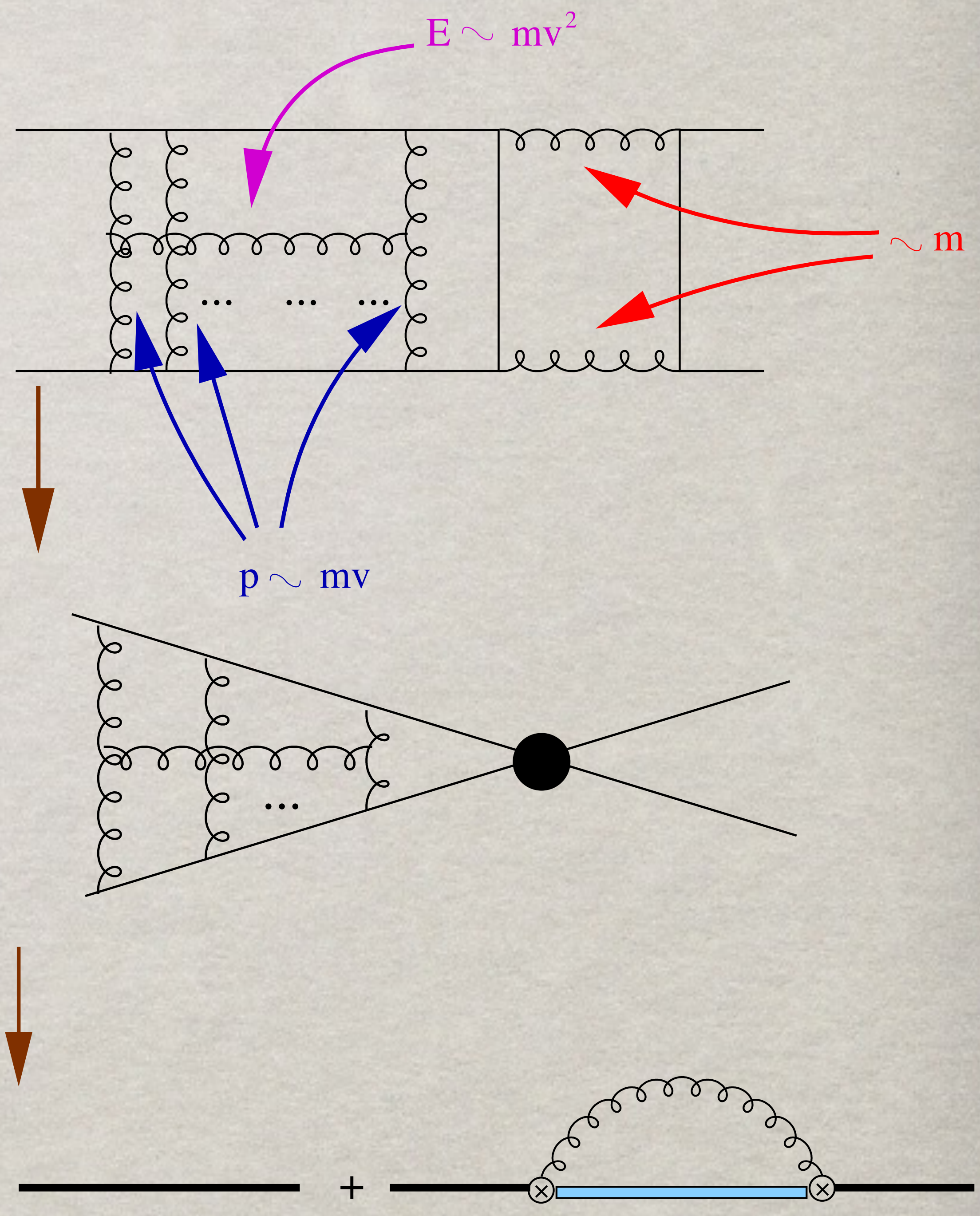

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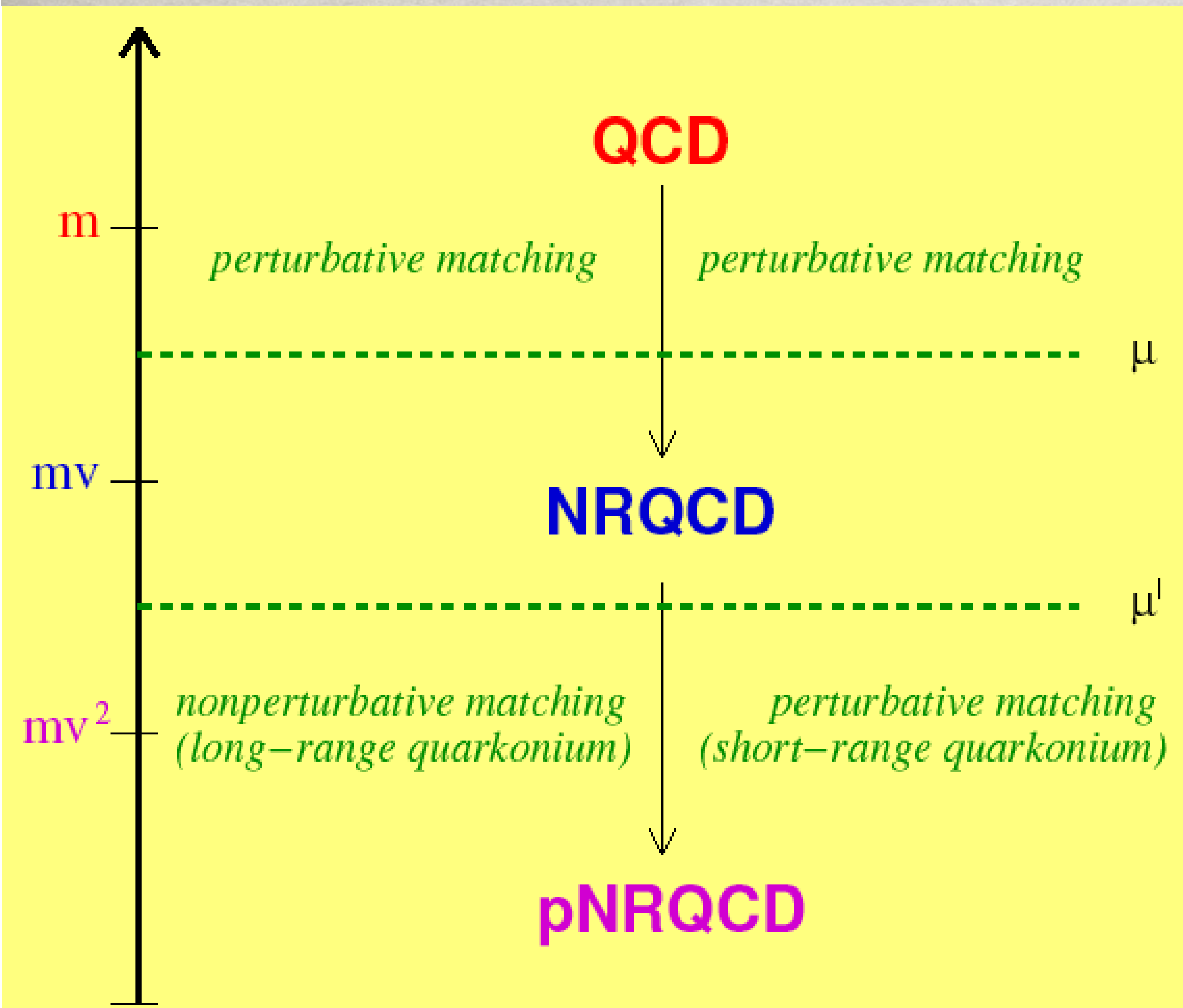

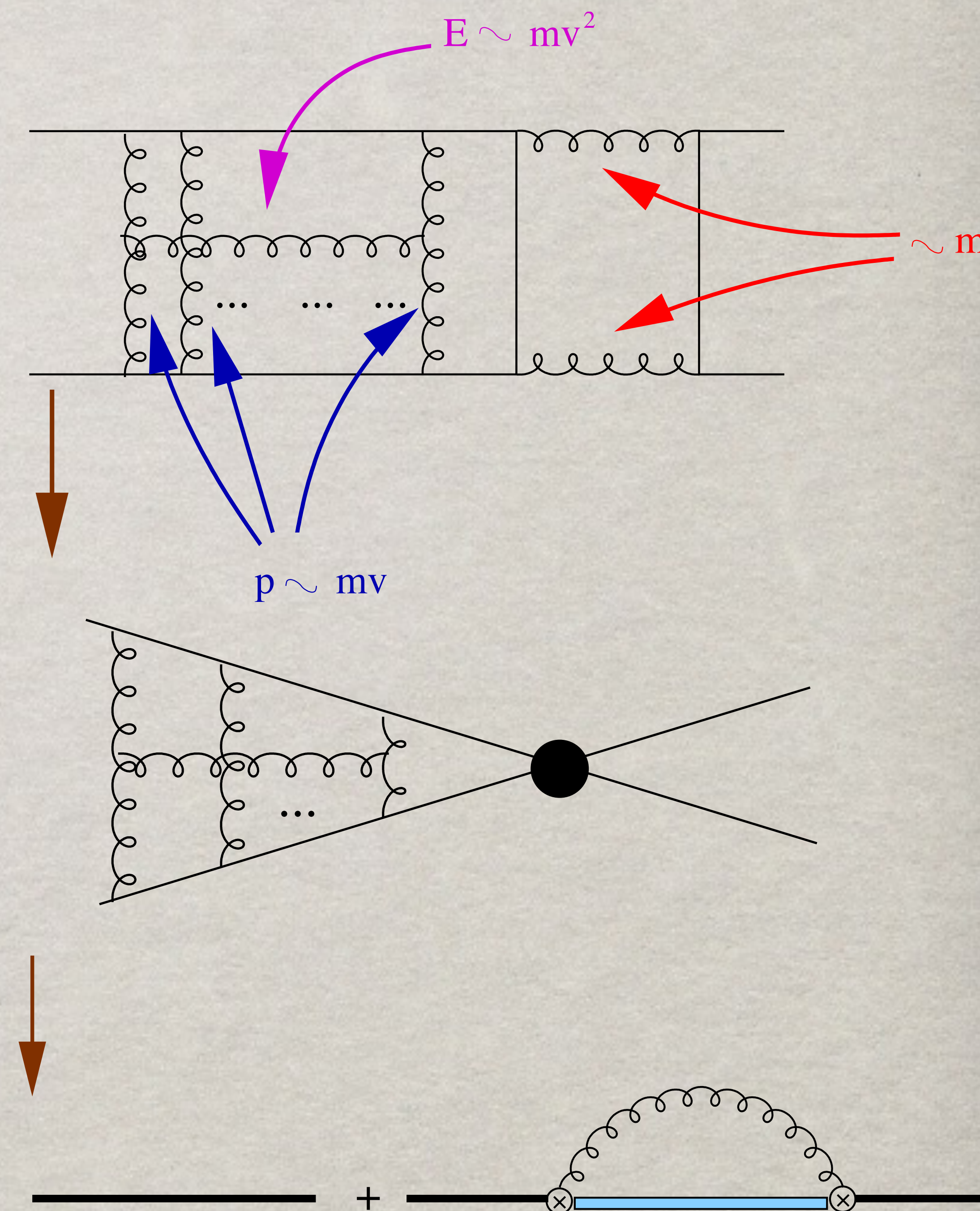

$$\mathcal{L}_{\mathrm{pNRQCD}} = \sum_k \sum_n \frac{1}{m^k} c_k(\alpha_s(m/\mu)) \times V(r\mu', r\mu) \times O_n(\mu', \lambda)\, r^n$$

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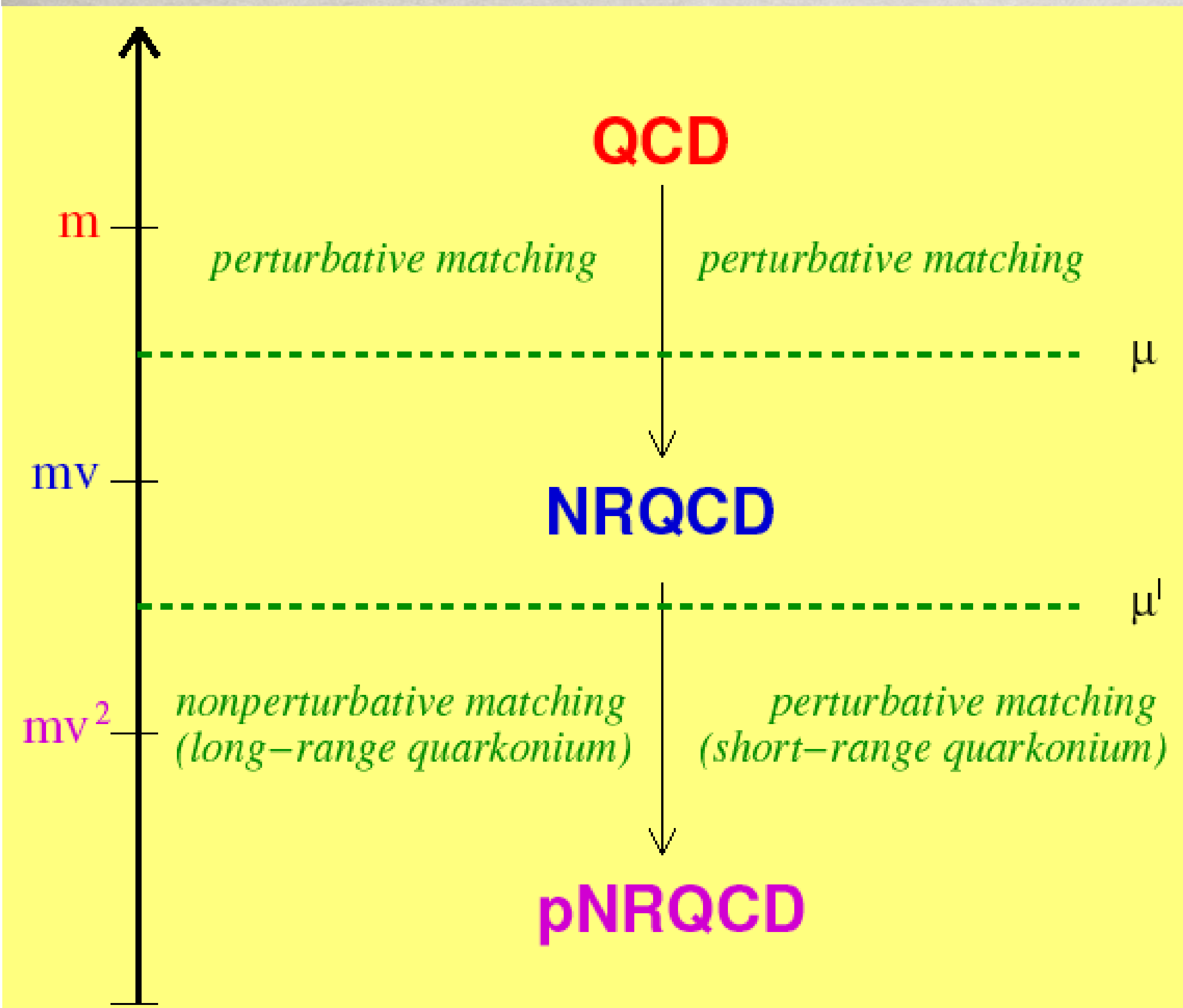

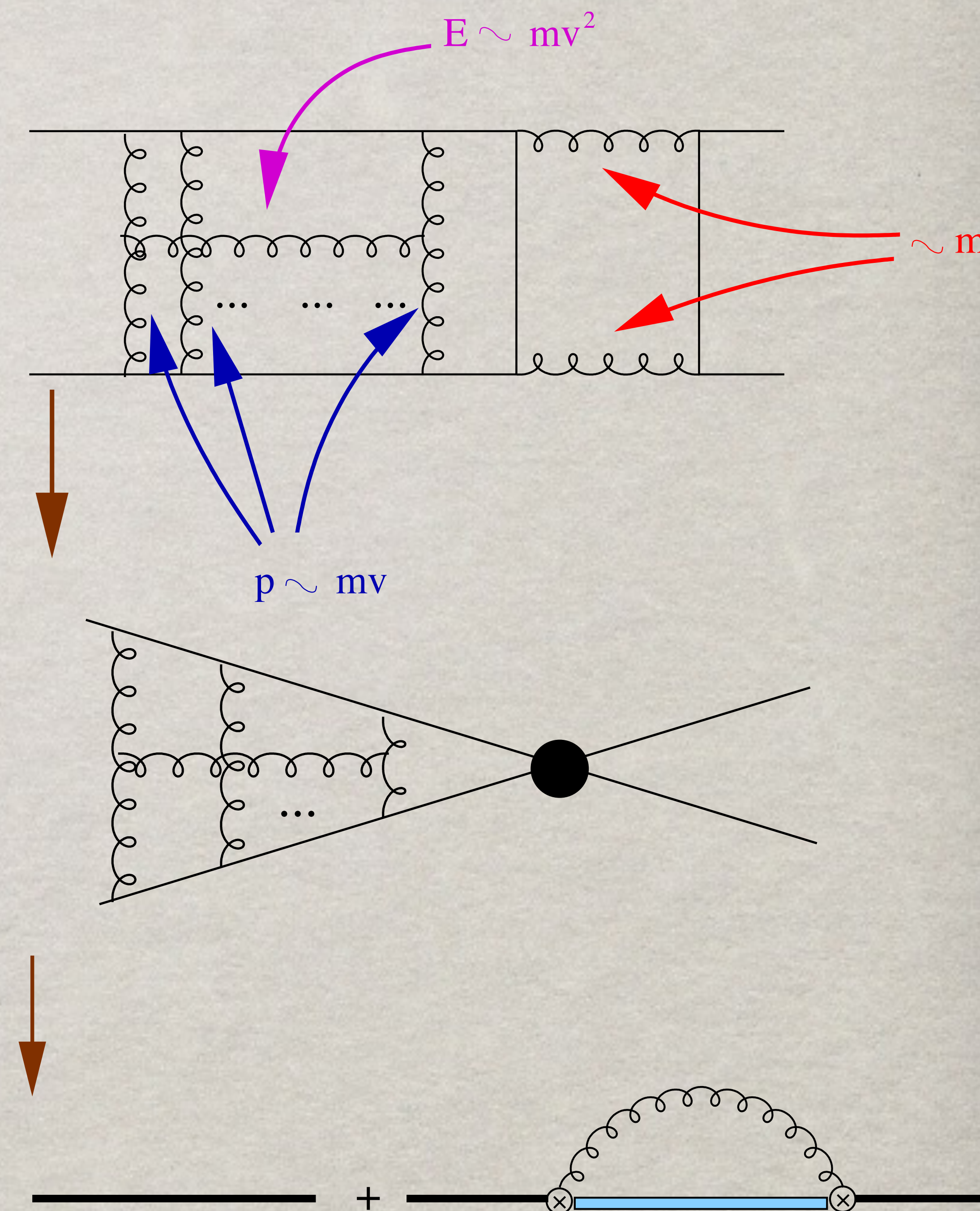

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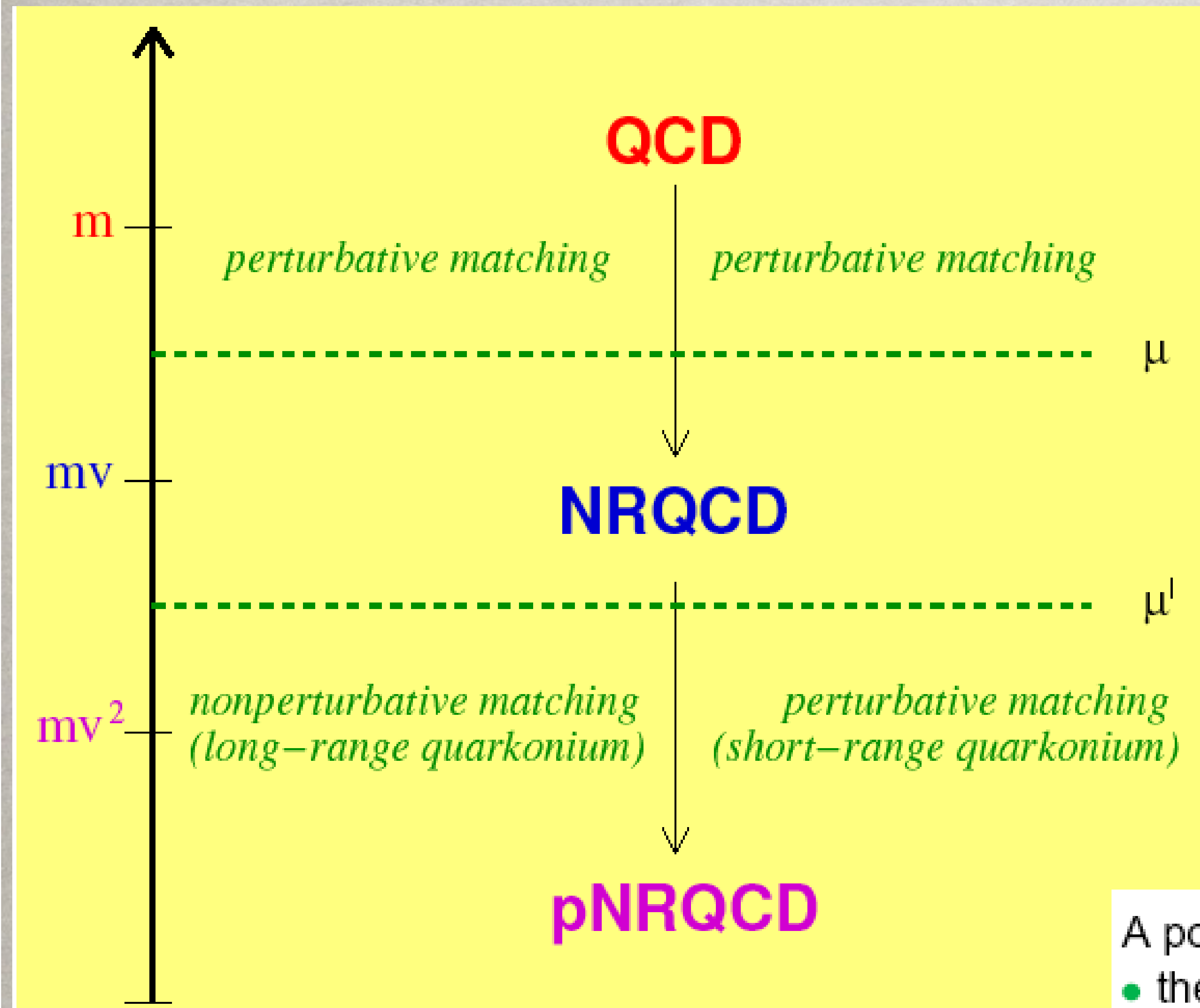
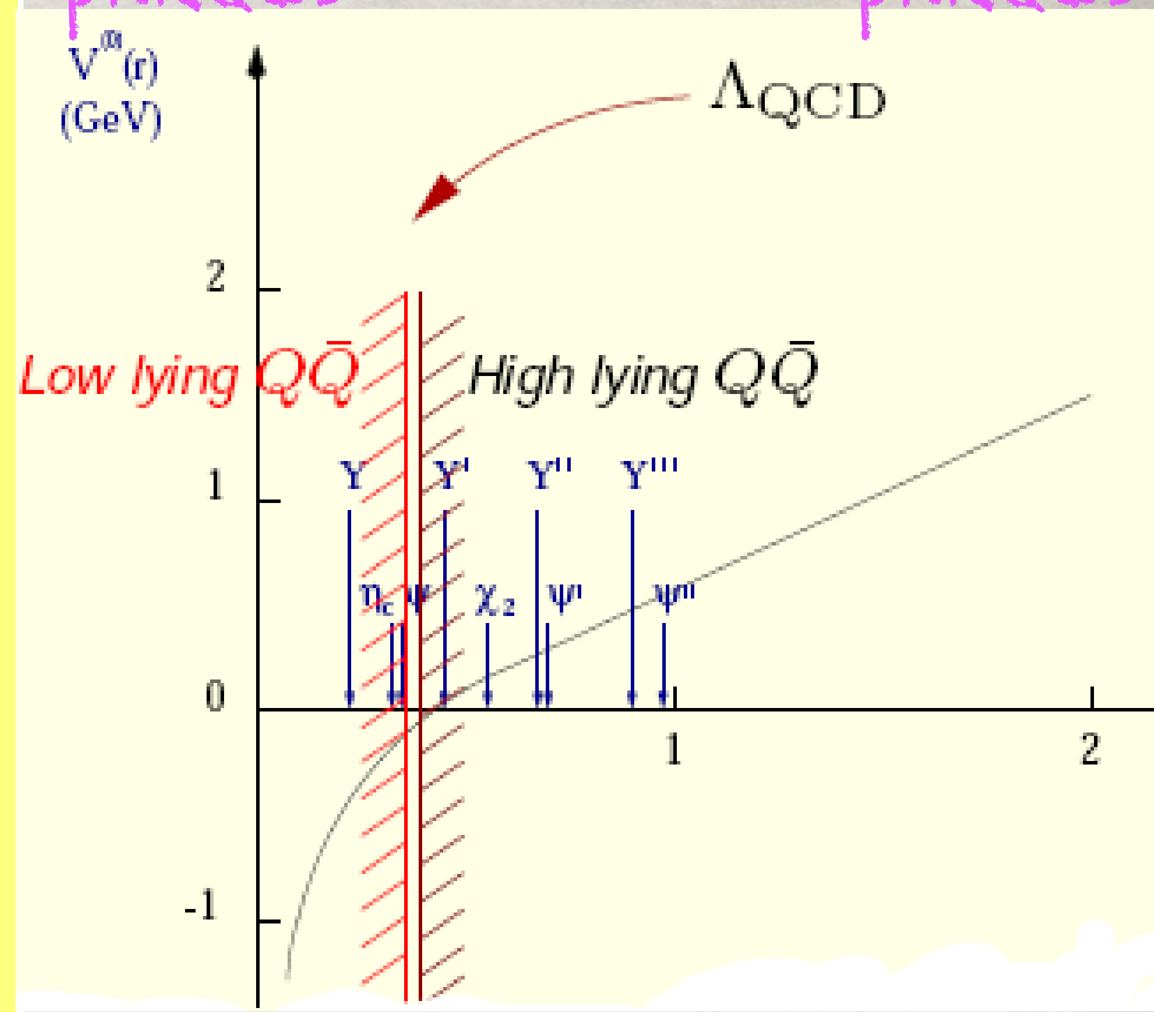
Quarkonium with NR EFT: pNRQCD

weakly coupled pNRQCD

strongly coupled pNRQCD





A potential picture arises at the level of pNRQCD:

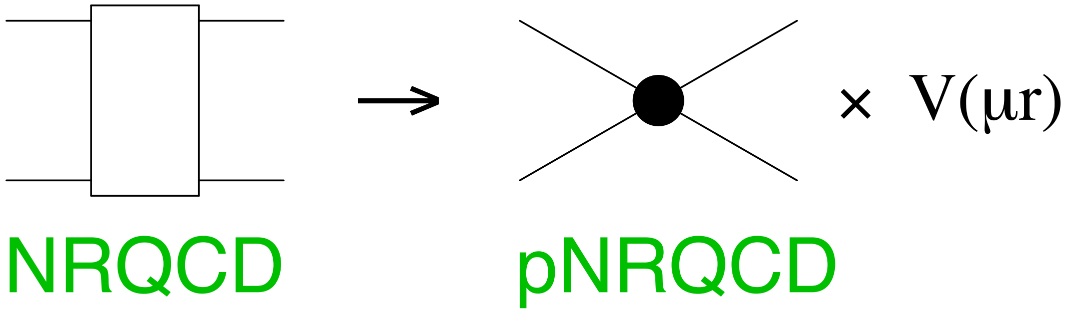
- the potential is perturbative if $mv \gg \Lambda_{QCD}$
- the potential is non-perturbative if $mv \sim \Lambda_{QCD}$

In QCD another scale is relevant Λ_{QCD}

Pineda, Soto 97, N.B., Pineda, Soto, Vairo 99
N.B. Vairo, Pineda, Soto 00--014
N.B., Pineda, Soto, Vairo Review of Modern Physis 77(2005) 1423

pNRQCD for quarkonia with small radius $r \ll \Lambda_{\rm QCD}^{-1}$

Degrees of freedom that scale like mv are integrated out:



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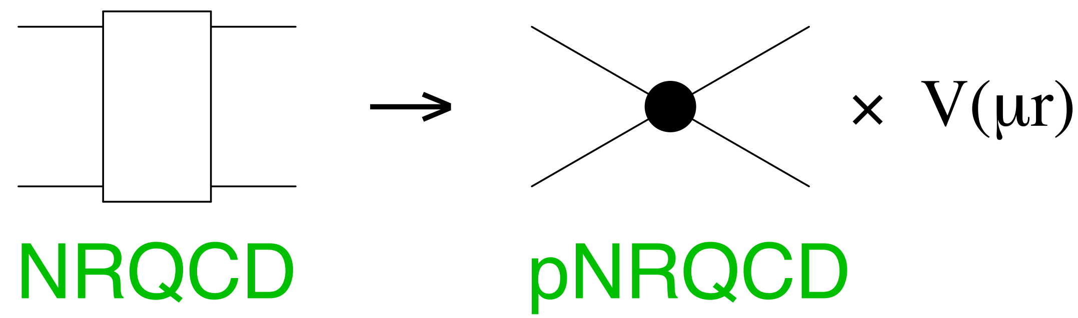

- If $mv \gg \Lambda_{\rm QCD}$, the matching is perturbative

- Degrees of freedom: quarks and gluons

  Q-$\bar{Q}$ states, with energy $\sim \Lambda_{\rm QCD}, mv^2$ and momentum $\lesssim mv$
  $\Rightarrow$ (i) singlet S (ii) octet O

  Gluons with energy and momentum $\sim \Lambda_{\rm QCD}, mv^2$

- Definite power counting: $r \sim \dfrac{1}{mv}$ and $t, R \sim \dfrac{1}{mv^2}, \dfrac{1}{\Lambda_{\rm QCD}}$

  The gauge fields are multipole expanded:
  $A(R,r,t) = A(R,t) + \mathbf{r} \cdot \boldsymbol{\nabla} A(R,t) + \ldots$

  Non-analytic behaviour in $r \to$ matching coefficients V

weak pNRQCD $\quad r \ll \Lambda_{\rm QCD}^{-1}$

$$\mathcal{L} = -\frac{1}{4} F^a_{\mu\nu} F^{\mu\nu\, a} + {\rm Tr} \left\{ {\rm S}^\dagger \left(i\partial_0 - \frac{\mathbf{p}^2}{m} - V_s \right) {\rm S} \right.$$
$$\left. + \, {\rm O}^\dagger \left(iD_0 - \frac{\mathbf{p}^2}{m} - V_o \right) {\rm O} \right\}$$

LO in r

S singlet field

O octet field

singlet propagator

octet propagator

Pineda, Soto 97; Brambilla, Pineda, Soto, Vairo 99-

weak pNRQCD $r \ll \Lambda_{\rm QCD}^{-1}$

Singlet static potential

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- At leading order in r, the singlet S satisfies the QCD Schrödinger equation.

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LO in r

Octet static potential

- At leading order in r, the singlet S satisfies the QCD Schrödinger equation.
  - The (weak coupling) static potential is the Coulomb potential:

$$V_s(r) = -C_F \frac{\alpha_{\rm s}}{r} + \dots, \qquad V_o(r) = \frac{1}{2N} \frac{\alpha_{\rm s}}{r} + \dots, \qquad N = 3,\ C_F = \frac{4}{3}$$

S singlet field — singlet propagator

O octet field — octet propagator

Pineda, Soto 97; Brambilla, Pineda, Soto, Vairo 99-

weak pNRQCD $\quad r \ll \Lambda_{\mathrm{QCD}}^{-1}$

$$\mathcal{L} = -\frac{1}{4} F^a_{\mu\nu} F^{\mu\nu\, a} + \mathrm{Tr}\left\{ \mathrm{S}^\dagger \left(i\partial_0 - \frac{\mathbf{p}^2}{m} - V_s \right) \mathrm{S} + \mathrm{O}^\dagger \left(iD_0 - \frac{\mathbf{p}^2}{m} - V_o \right) \mathrm{O} \right\}$$

LO in r

Pineda, Soto 97; Brambilla, Pineda, Soto, Vairo 99-

weak pNRQCD $\quad r \ll \Lambda_{\rm QCD}^{-1}$

$$\mathcal{L} = -\frac{1}{4} F^a_{\mu\nu} F^{\mu\nu\, a} + {\rm Tr}\left\{ {\rm S}^\dagger \left(i\partial_0 - \frac{\mathbf{p}^2}{m} - V_s \right) {\rm S} + {\rm O}^\dagger \left(iD_0 - \frac{\mathbf{p}^2}{m} - V_o \right) {\rm O} \right\}$$

LO in r

$$+ V_A {\rm Tr}\left\{ {\rm O}^\dagger \mathbf{r}\cdot g\mathbf{E}\, {\rm S} + {\rm S}^\dagger \mathbf{r}\cdot g\mathbf{E}\, {\rm O} \right\}$$
$$+ \frac{V_B}{2} {\rm Tr}\left\{ {\rm O}^\dagger \mathbf{r}\cdot g\mathbf{E}\, {\rm O} + {\rm O}^\dagger {\rm O} \mathbf{r}\cdot g\mathbf{E} \right\}$$
$$+ \cdots$$

NLO in r

Pineda, Soto 97; Brambilla, Pineda, Soto, Vairo 99-

weak pNRQCD $\quad r \ll \Lambda_{\mathrm{QCD}}^{-1}$

$$\mathcal{L} = -\frac{1}{4} F^a_{\mu\nu} F^{\mu\nu\, a} + \mathrm{Tr}\left\{ \mathrm{S}^\dagger \left(i\partial_0 - \frac{\mathbf{p}^2}{m} - V_s \right) \mathrm{S} + \mathrm{O}^\dagger \left(iD_0 - \frac{\mathbf{p}^2}{m} - V_o \right) \mathrm{O} \right\}$$

LO in r

$$+ V_A \mathrm{Tr}\left\{ \mathrm{O}^\dagger \mathbf{r} \cdot g\mathbf{E}\, \mathrm{S} + \mathrm{S}^\dagger \mathbf{r} \cdot g\mathbf{E}\, \mathrm{O} \right\}$$
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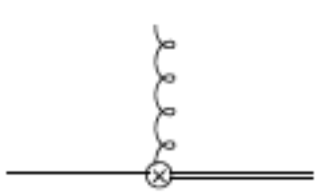
$$+ \cdots$$

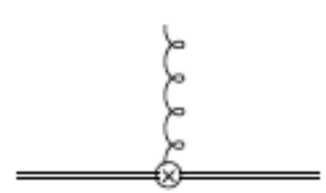
NLO in r

- Feynman rules:

 $= \theta(t)\, e^{-itH_s}$

 $= \theta(t)\, e^{-itH_o} \left(e^{-i\int dt\, A^{\mathrm{adj}}} \right)$

 $= \mathrm{O}^\dagger \mathbf{r} \cdot g\mathbf{E}\, \mathrm{S}$

 $= \mathrm{O}^\dagger \{ \mathbf{r} \cdot g\mathbf{E},\, \mathrm{O} \}$

Applications to Quarkonium physics: systems with small radius

for references see the QWG doc arXiv:1010.5827

- c and b masses at NNLO, N^{3}LO*, NNLL*;
- B_c mass at NNLO; Penin et al 04
- B_c^*, η_c, η_b masses at NLL; Kniehl et al 04
- Quarkonium 1P fine splittings at NLO;
- $\Upsilon(1S)$, η_b electromagnetic decays at NNLL;
- $\Upsilon(1S)$ and J/ψ radiative decays at NLO;
- $\Upsilon(1S) \to \gamma\eta_b$, $J/\psi \to \gamma\eta_c$ at NNLO;
- $t\bar{t}$ cross section at NNLL;
- QQq and QQQ baryons: potentials at NNLO, masses, hyperfine splitting, ... ; N. B. et al 010
- Thermal effects on quarkonium in medium: potential, masses (at $m\alpha_s^5$), widths, ...;

$$\mathcal{B}(J/\psi \to \gamma\eta_c(1S)) = (1.6 \pm 1.1)\%$$

$$\mathcal{B}(\Upsilon(1S) \to \gamma\eta_b(1S)) = (2.85 \pm 0.30) \times 10^{-4}$$

N. B. Yu Jia A. Vairo 2005

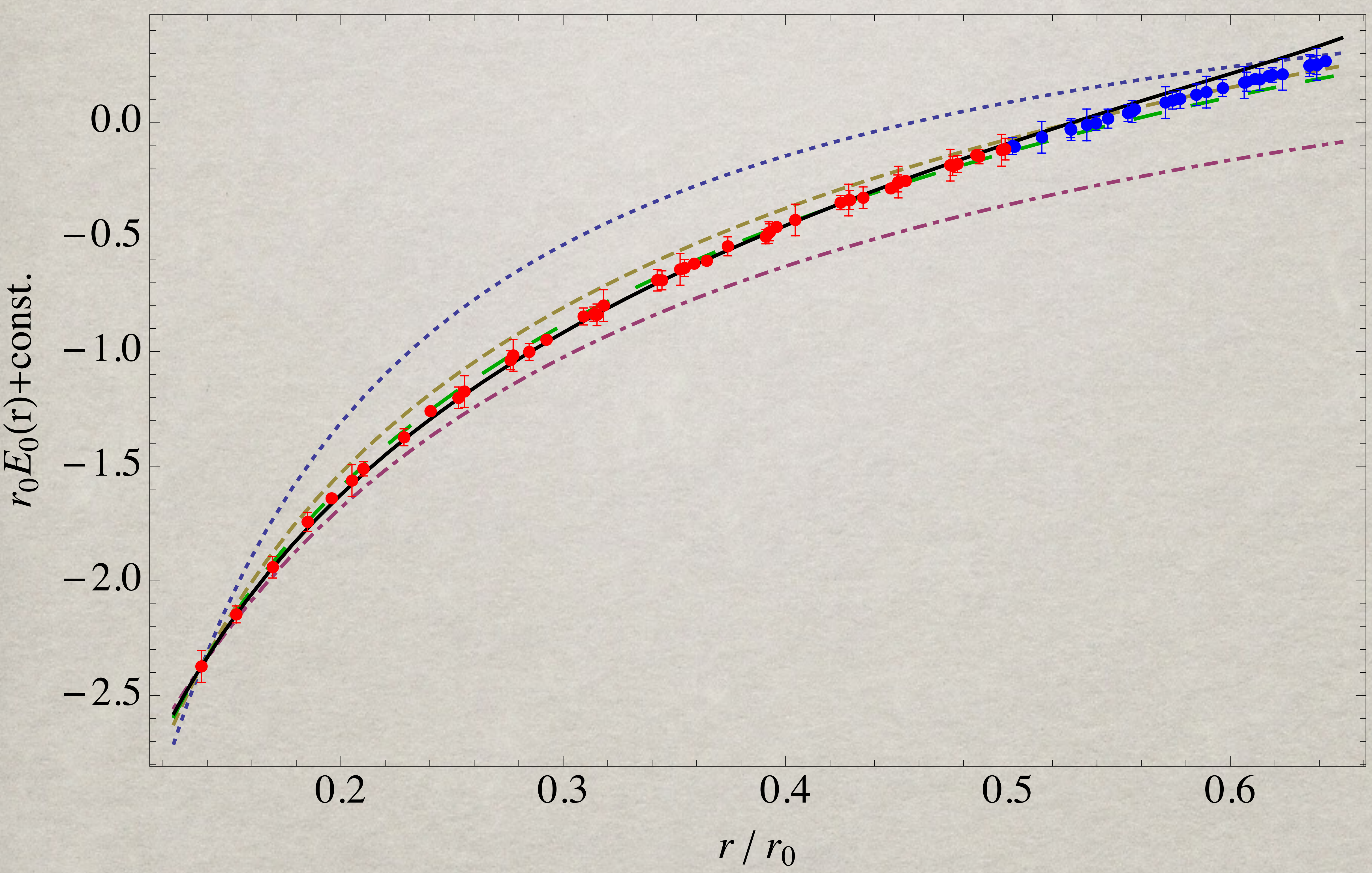
$$\Gamma(\eta_b(1S) \to \gamma\gamma) = 0.54 \pm 0.15 \text{ keV}\,.$$

$$\Gamma(\eta_b(1S) \to \text{LH}) = 7\text{-}16 \text{ MeV}$$

Y. Kiyo, A. Pineda, A. Signer 2010

QQbar singlet static energy at N^3LI in comparison with unquenched (n_f=2+1) lattice data (red points,blue points)

Bazanov, N. B., Garcia, Petreczky, Soto, Vairo , 2012, 2014



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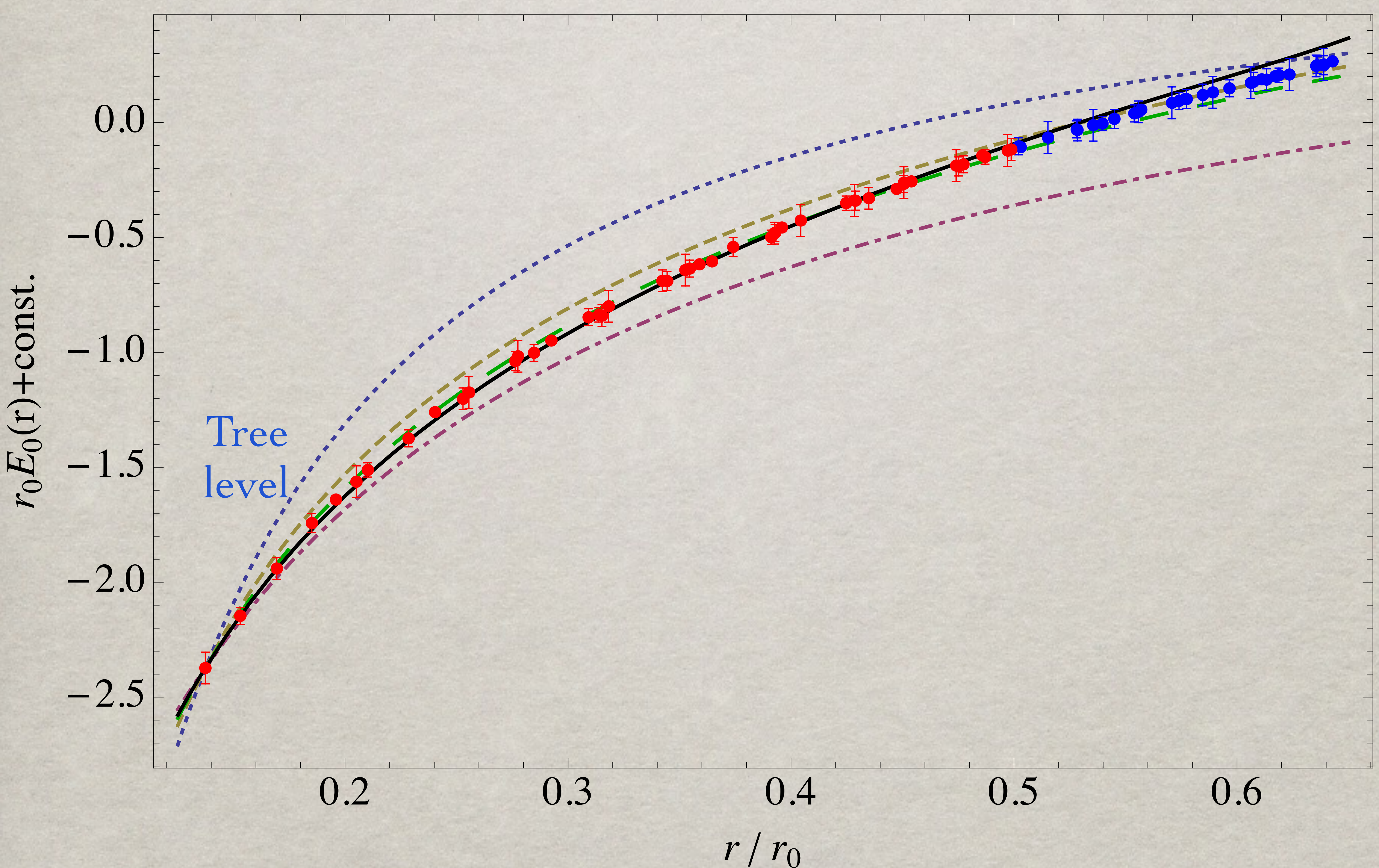
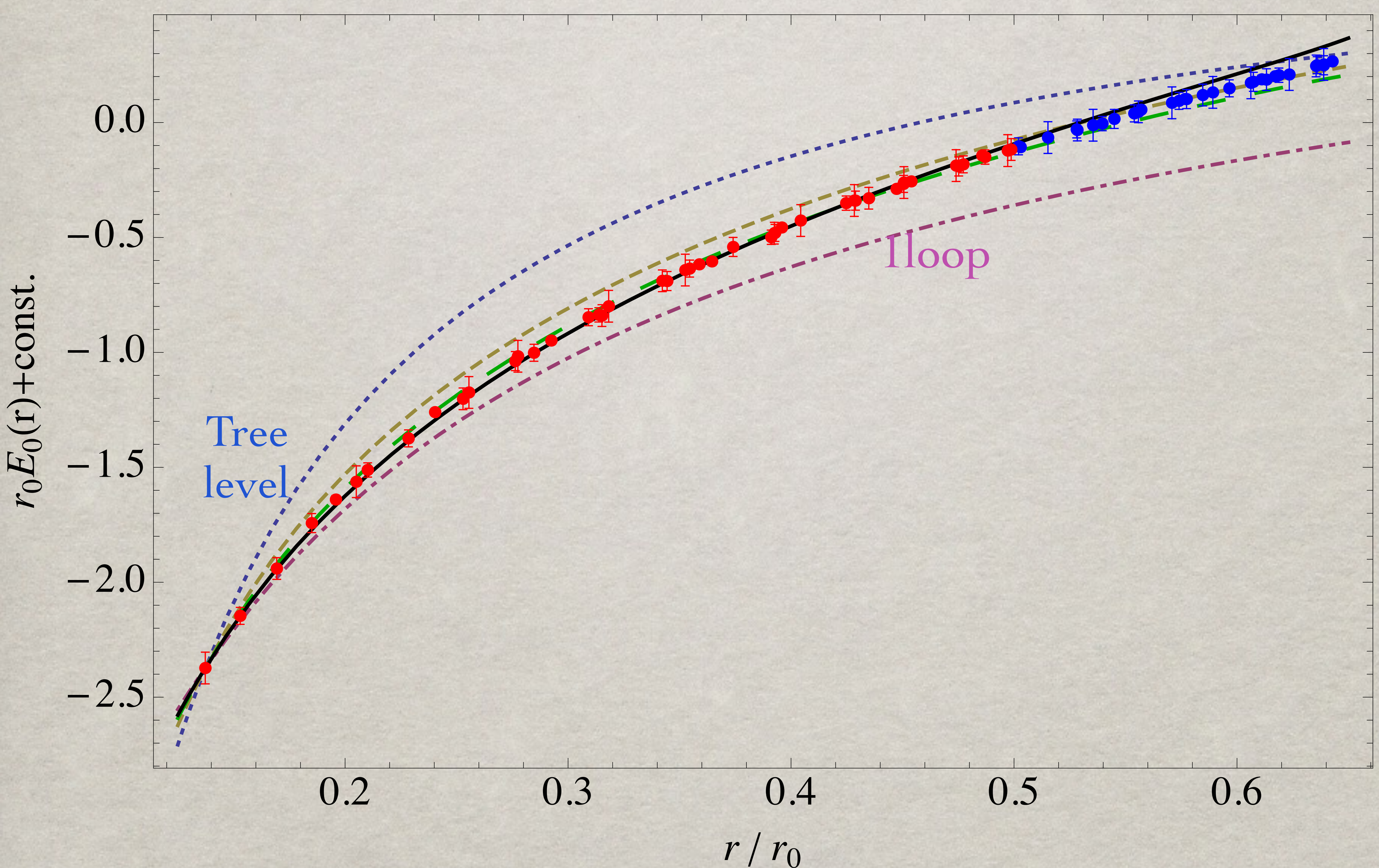
Bazanov, N. B., Garcia, Petreczky, Soto, Vairo , 2012, 2014



QQbar singlet static energy at N^3LL in comparison with unquenched (n_f=2+1) lattice data (red points,blue points)

Bazanov, N. B., Garcia, Petreczky, Soto, Vairo , 2012, 2014



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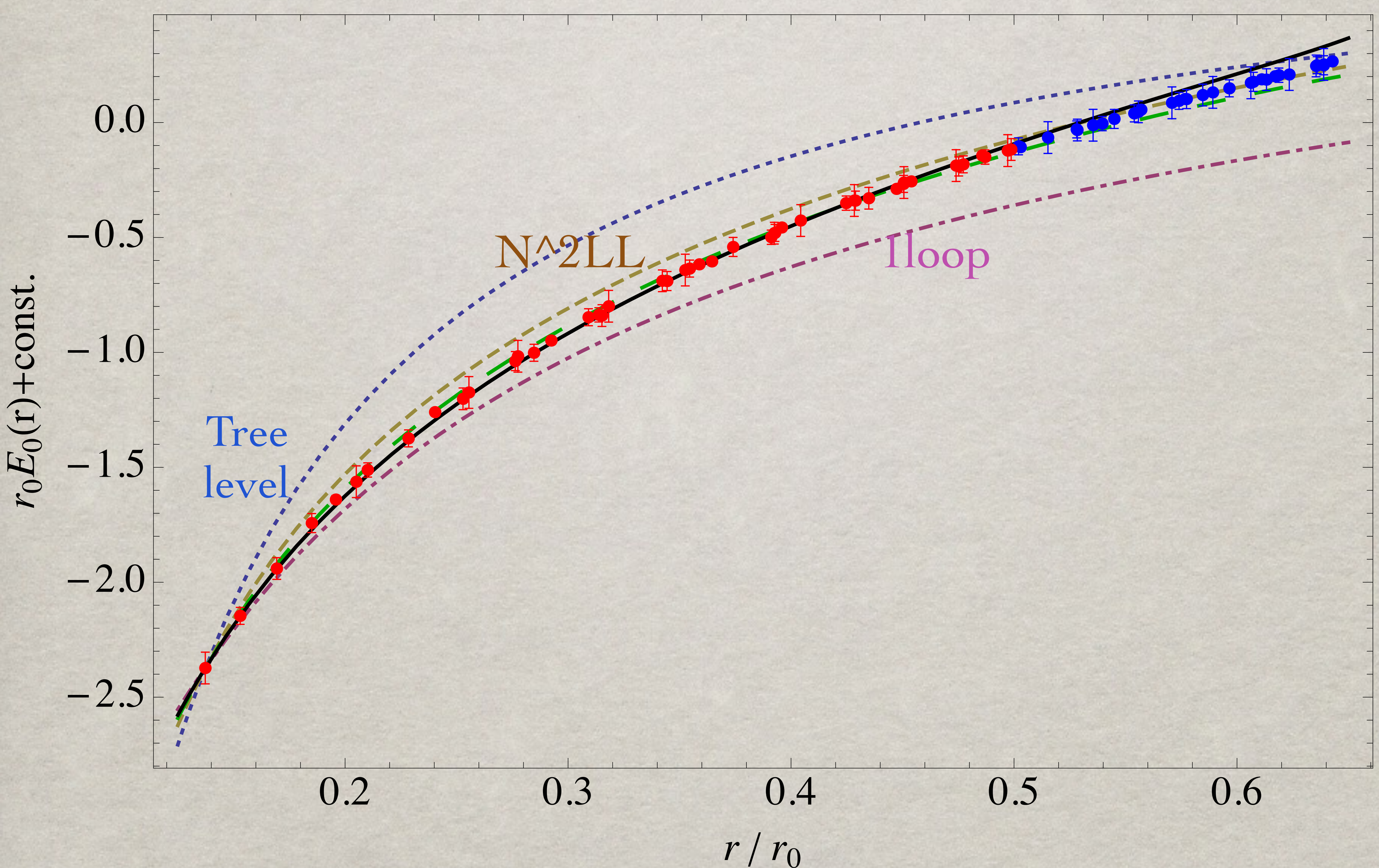
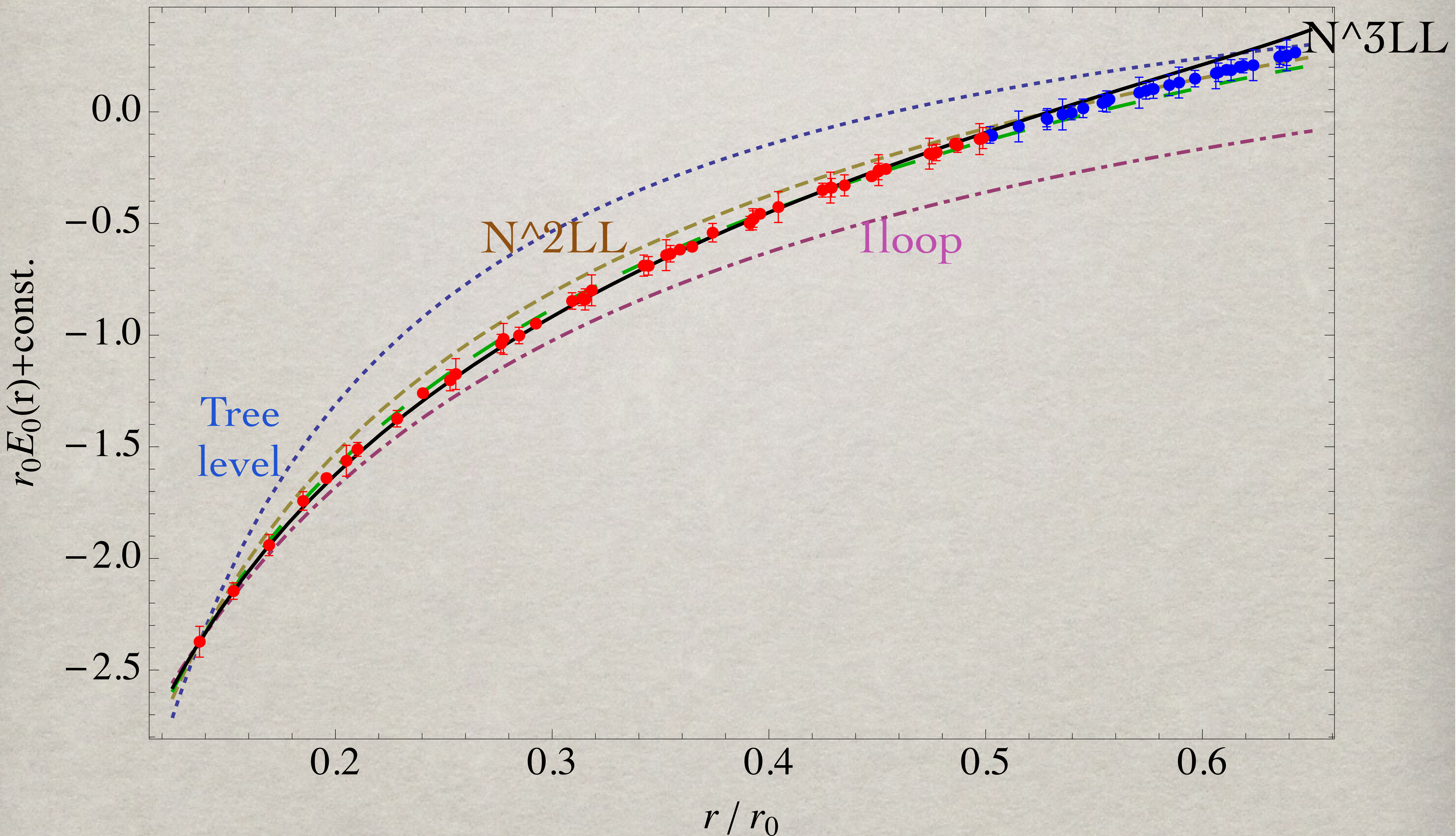
Bazanov, N. B., Garcia, Petreczky, Soto, Vairo , 2012, 2014



QQbar singlet static energy at N^3Ll in comparison with unquenched (n_f=2+1) lattice data (red points,blue points)

Bazanov, N. B., Garcia, Petreczky, Soto, Vairo , 2012, 2014



QQbar singlet static energy at N^3Ll in comparison with unquenched (n_f=2+1) lattice data (red points,blue points)

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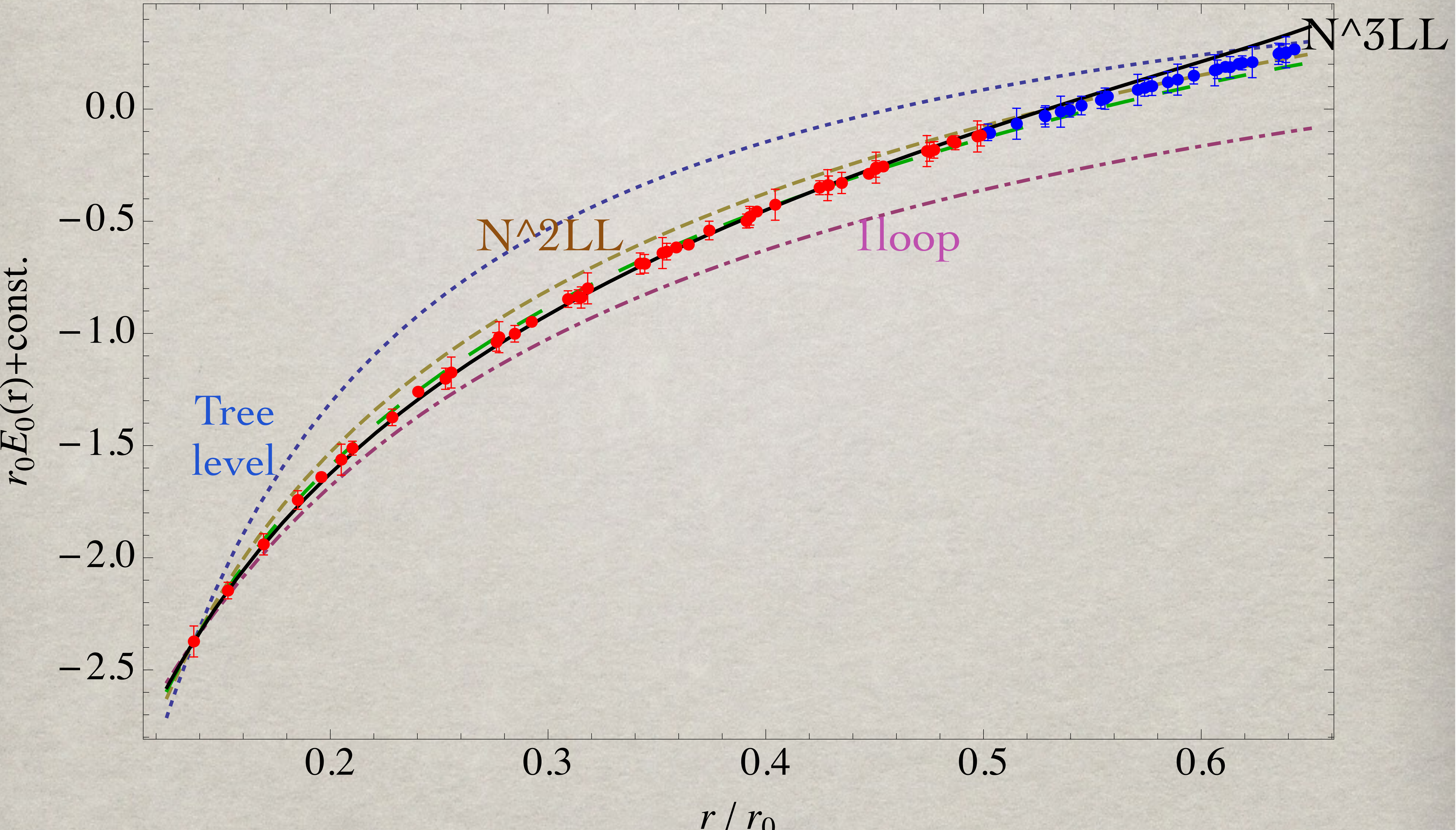

Good convergence to the lattice data

Lattice data less accurate in the unquenched case

The fit gives

The lattice scale is $r_1 = 0.3106 \pm 0.0017$ fm.

$r_1 \Lambda_{\overline{\rm MS}} = 0.495^{+0.028}_{-0.018}$ which converts to $\Lambda_{\overline{\rm MS}} = 315^{+18}_{-12}$ MeV

Bazavov Brambilla Garcia Petreczky Soto Vairo

TUMQCD coll. 2014

$$\alpha_s(1.5\ \mathrm{GeV}, n_f = 3) = 0.336^{+0.012}_{-0.008}$$

which corresponds to

$$\alpha_s(M_Z, n_f = 5) = 0.1166^{+0.0012}_{-0.0008}$$

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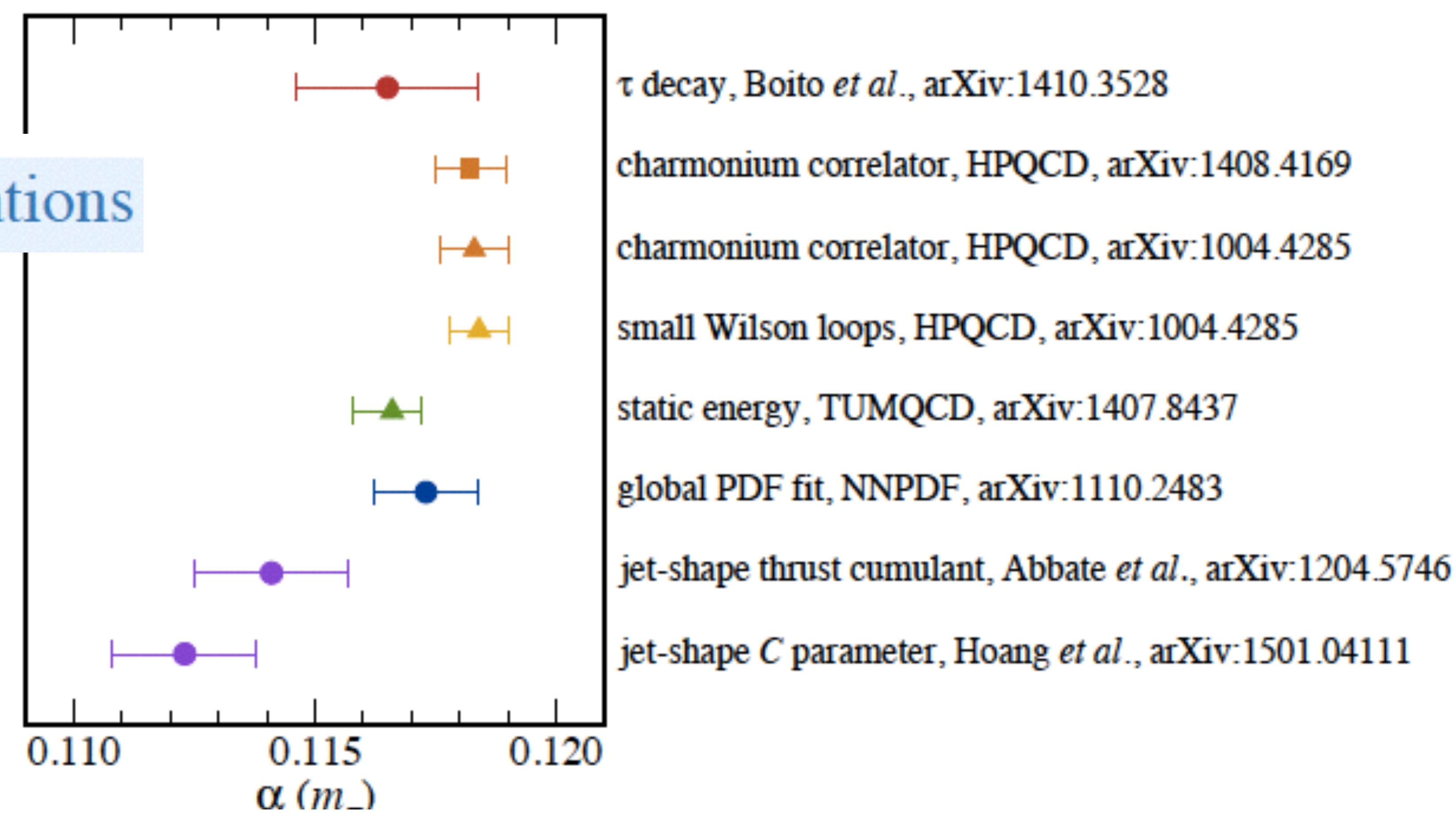
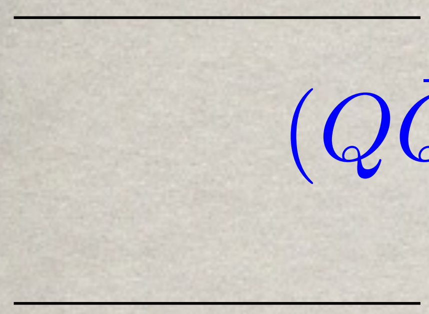
Comparison with other determinations

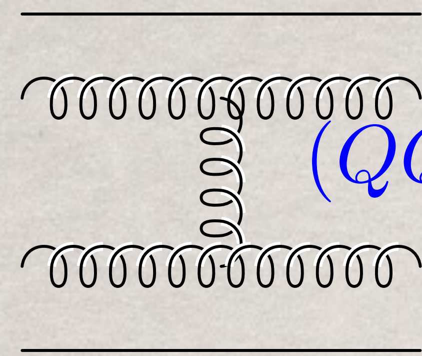


Andreas Kronfeld 2016

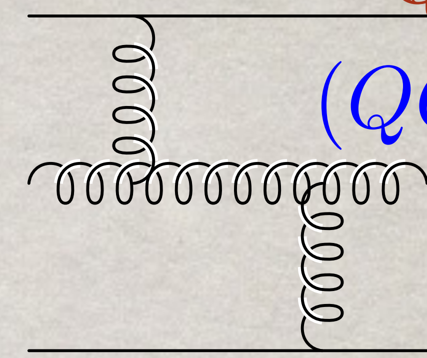
— Hitting the scale $\Lambda_{\rm QCD}$ $r \sim \Lambda_{QCD}^{-1}$



$(Q\bar{Q})_1$



$(Q\bar{Q})_1$ + Glueball

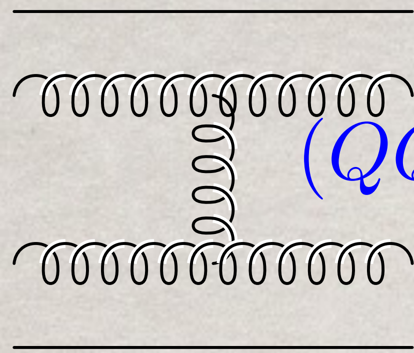


$(Q\bar{Q})_8 G$

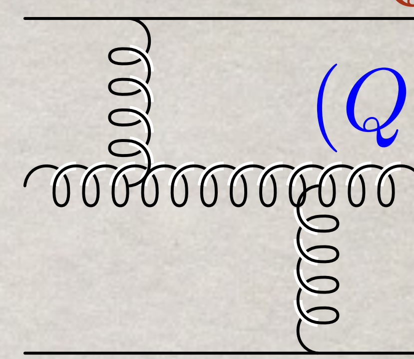
hybrid

Hitting the scale Λ_{QCD} $r \sim \Lambda_{QCD}^{-1}$

$(Q\bar{Q})_1$



$(Q\bar{Q})_1 + \text{Glueball}$



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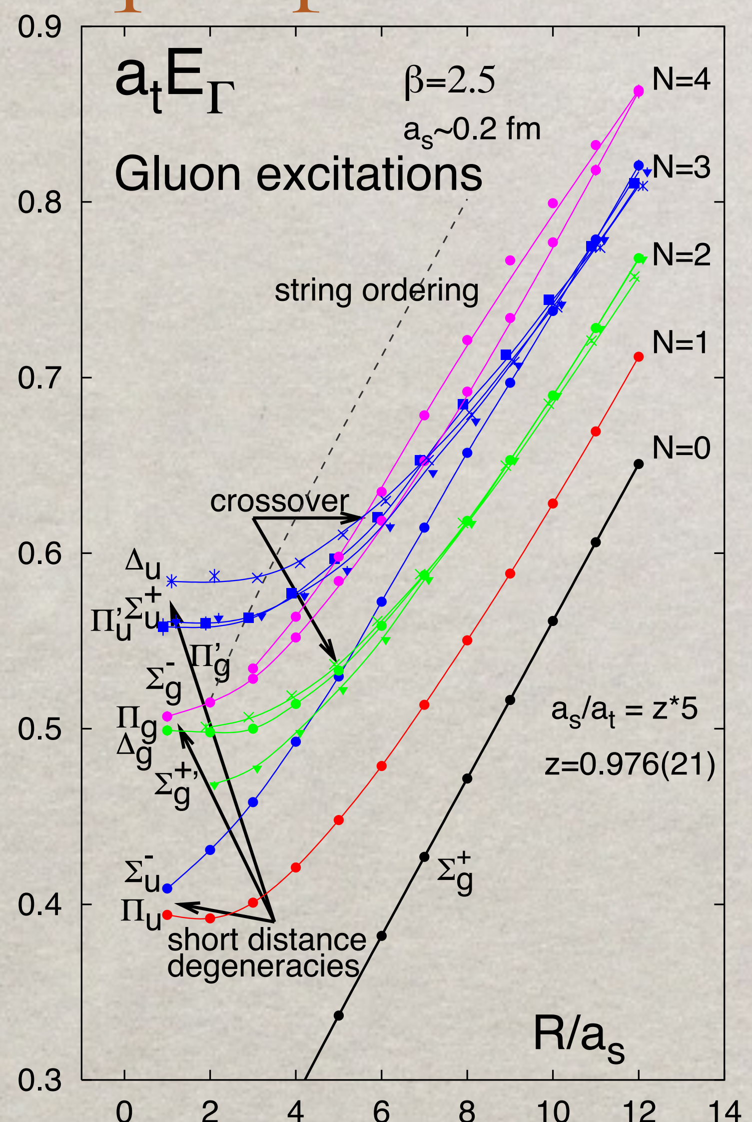
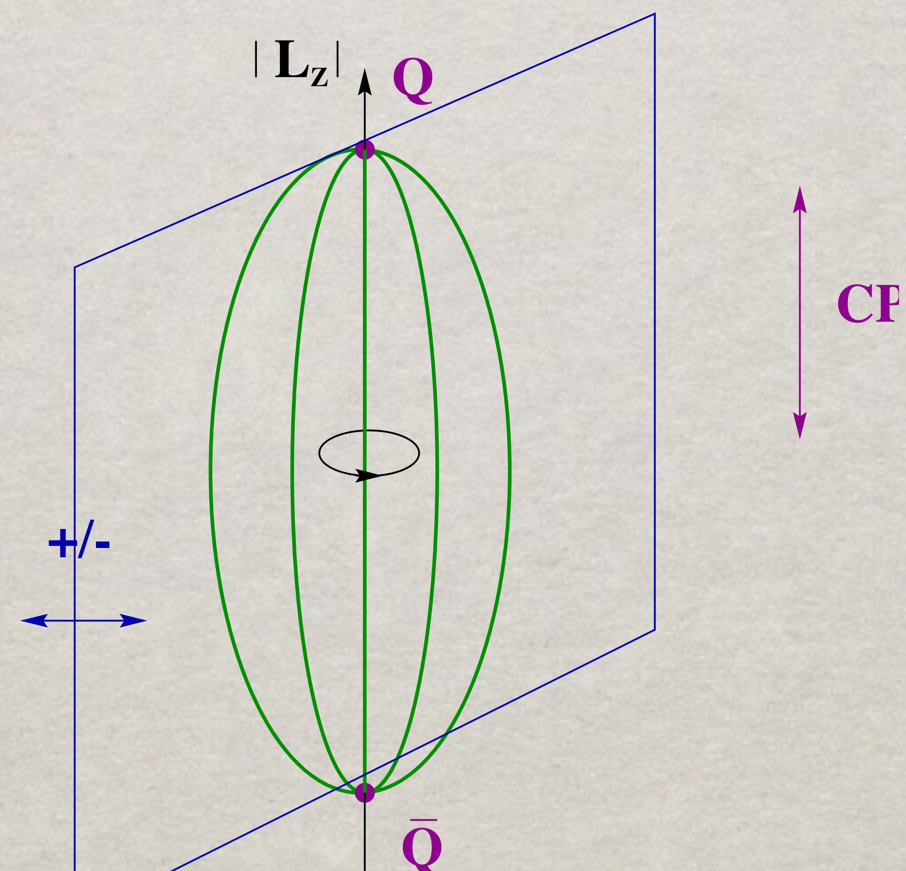
Static qcd spectrum

L a t t i c e





Symmetries of a diatomic molecule + C.C.

a) $|L_z| = 0, 1, 2, \ldots$
$= \Sigma, \Pi, \Delta \ldots$

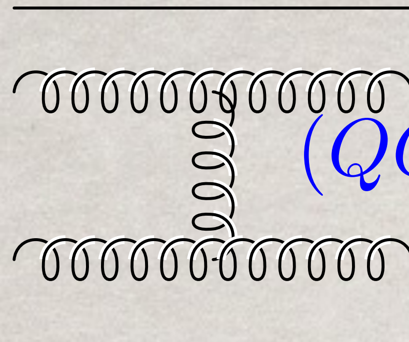
b) CP (u/g)

c) Reflection $(+/-)$
(for Σ only)

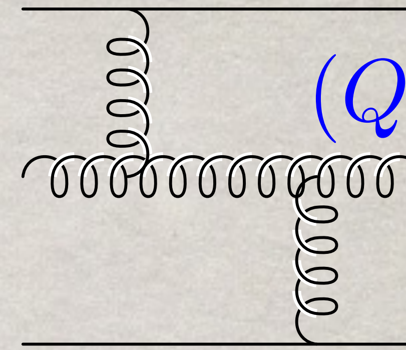
Juge Kuti Mornigstar 98-06 Static NRQCD

Hitting the scale $\Lambda_{\rm QCD}$ $\quad r \sim \Lambda_{QCD}^{-1}$

$(Q\bar{Q})_1$



$(Q\bar{Q})_1 + \text{Glueball}$



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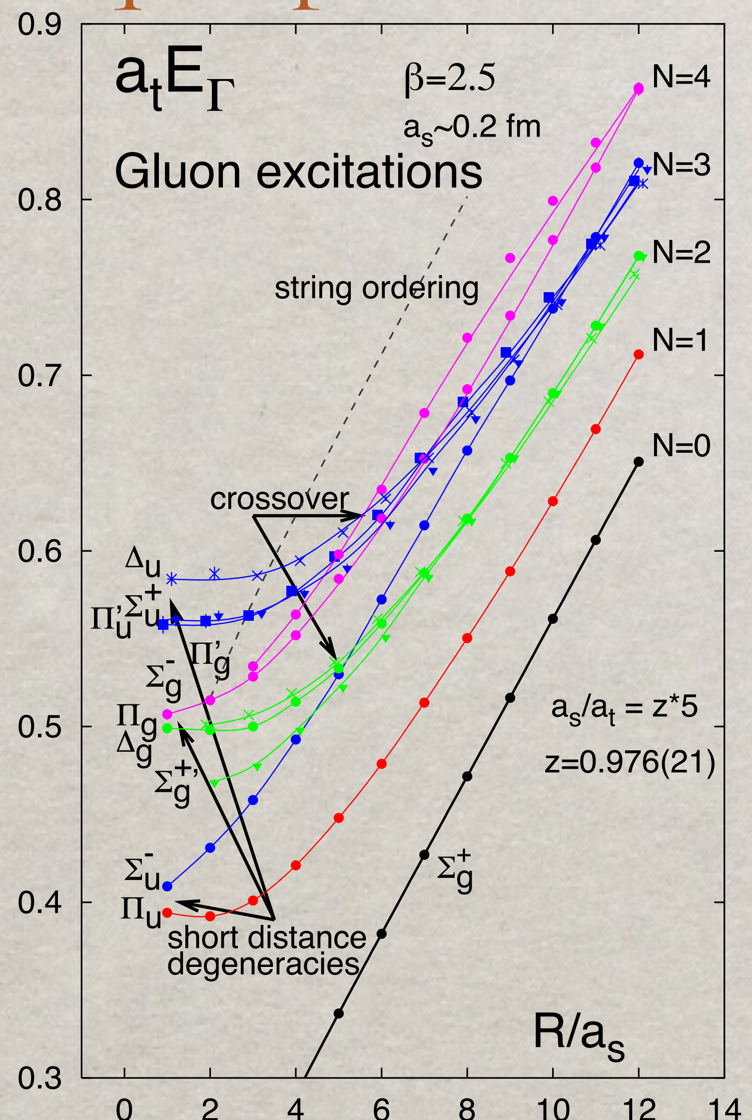
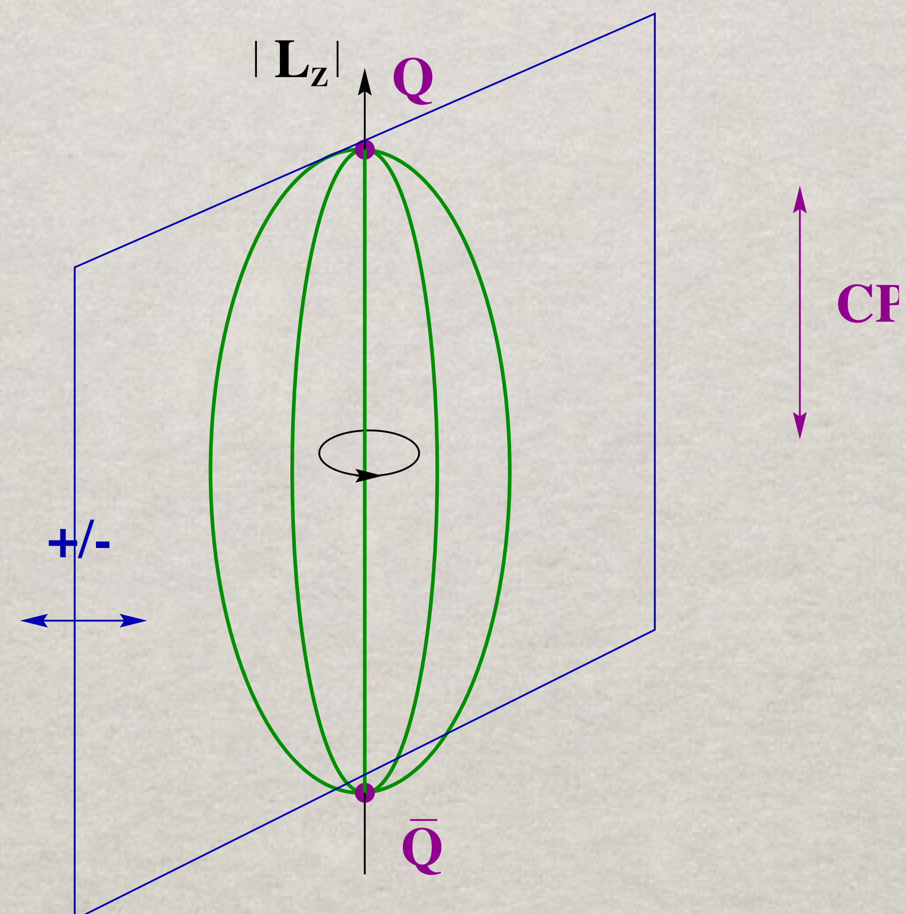
hybrid

Static qcd spectrum

Lattice



Juge Kuti Mornigstar 98-06 Static NRQCD



Symmetries of a diatomic molecule + C.C.

a) $|L_z| = 0, 1, 2, \ldots$
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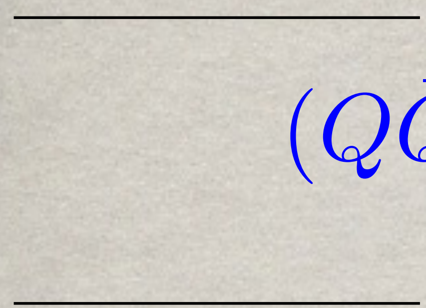
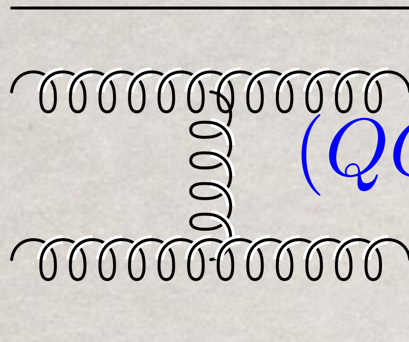
(for Σ only)

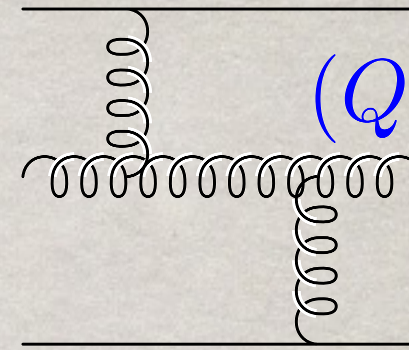
$$\mathcal{H}^{(0)} |\underline{n}; \mathbf{x}_1, \mathbf{x}_2\rangle^{(0)} = E_n^{(0)}(\mathbf{x}_1, \mathbf{x}_2) |\underline{n}; \mathbf{x}_1, \mathbf{x}_2\rangle^{(0)}$$

$$|\underline{n}; \mathbf{x}_1, \mathbf{x}_2\rangle^{(0)} = \psi^\dagger(\mathbf{x}_1)\chi(\mathbf{x}_2)|n; \mathbf{x}_1, \mathbf{x}_2\rangle^{(0)}$$

— Hitting the scale $\Lambda_{\rm QCD}$ $\quad r \sim \Lambda_{QCD}^{-1}$

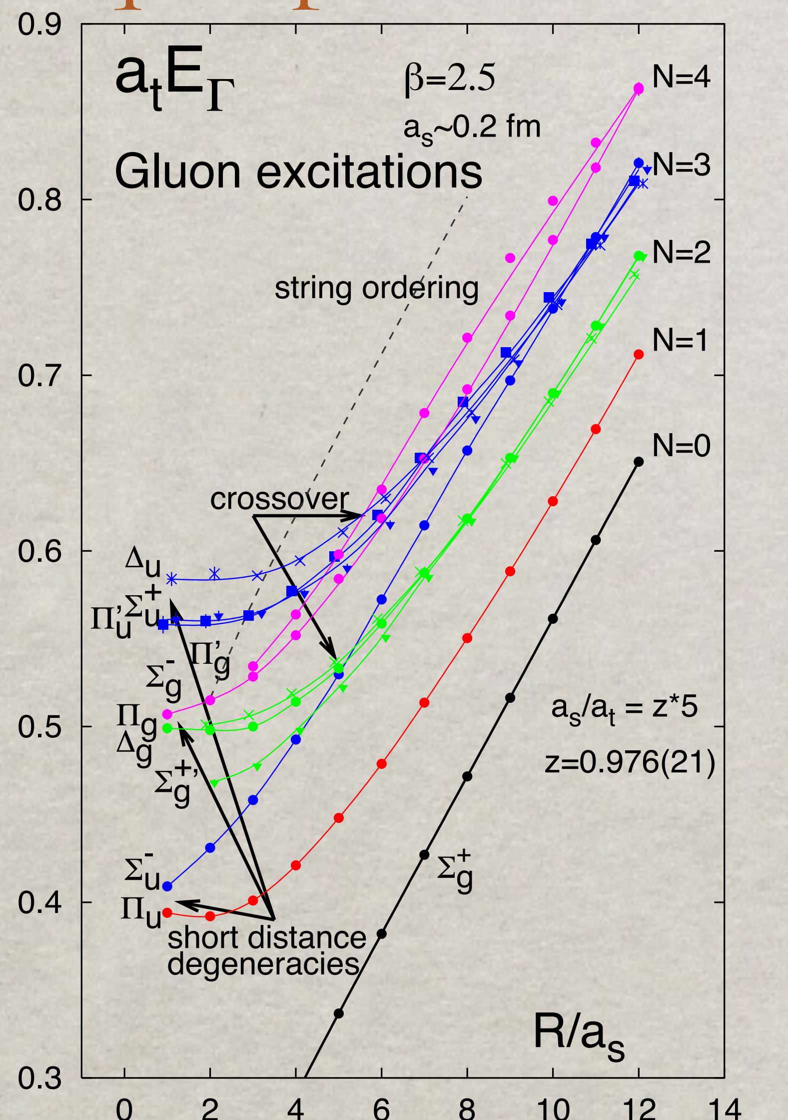


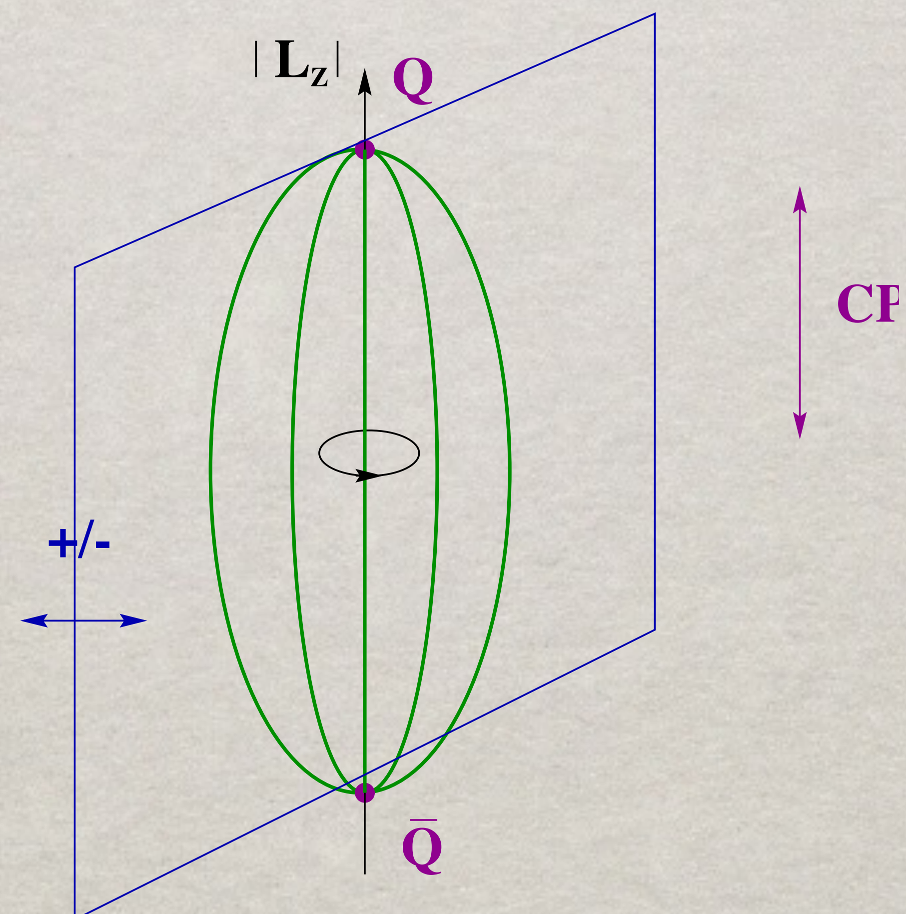




Static qcd spectrum

L a t t i c e





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Juge Kuti Mornigstar 98-06 Static NRQCD

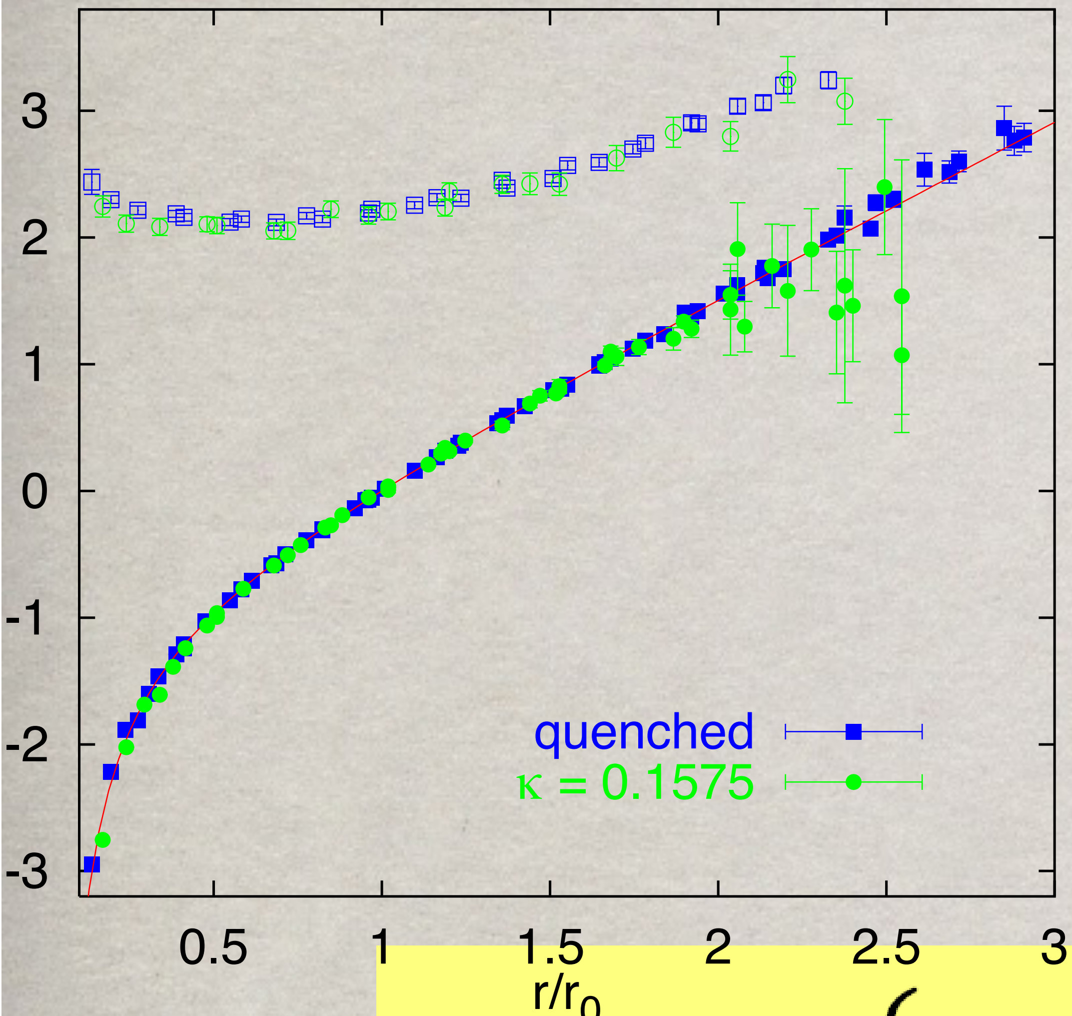
$$\mathcal{H}^{(0)}|\underline{n};\mathbf{x}_1,\mathbf{x}_2\rangle^{(0)} = E_n^{(0)}(\mathbf{x}_1,\mathbf{x}_2)|\underline{n};\mathbf{x}_1,\mathbf{x}_2\rangle^{(0)}$$

$$\ldots;\mathbf{x}_1,\mathbf{x}_2\rangle^{(0)} = \psi^\dagger(\mathbf{x}_1)\chi(\mathbf{x}_2)|n;\mathbf{x}_1,\mathbf{x}_2\rangle^{(0)}$$

$|\underline{0}\rangle^{(0)} = |(Q\bar{Q})_1\rangle \rightarrow$ Quarkonium Singlet

$|\underline{n > 0}\rangle^{(0)} = |(Q\bar{Q})g^{(n)}\rangle \rightarrow$ Higher Gluonic Excitations

strongly coupled pNRQCD $r \sim \Lambda_{QCD}^{-1}$ $mv \sim \Lambda_{QCD}$



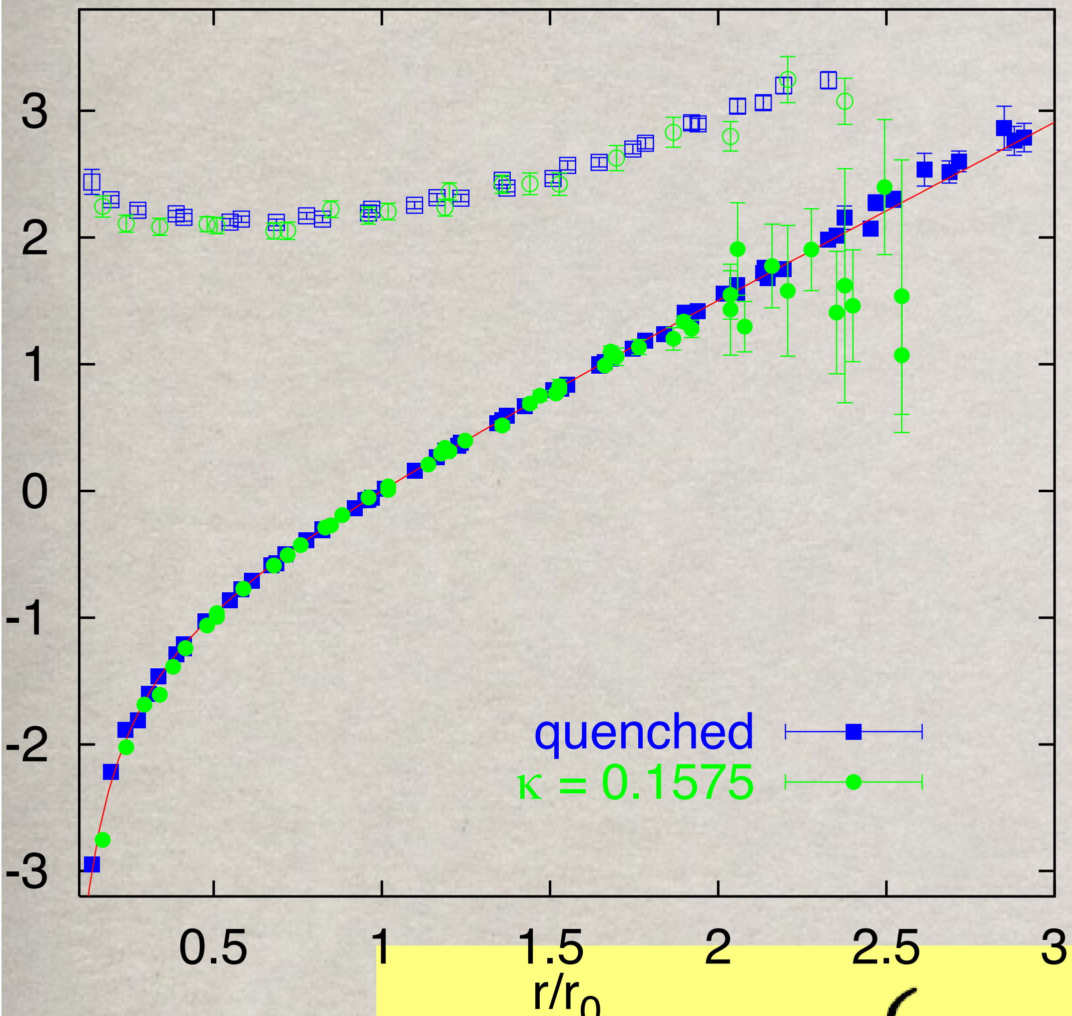
- integrate out all scales above mv^2
- gluonic excitations develop a gap Λ_{QCD} and are integrated out

⇒ The singlet quarkonium field S of energy mv^2 is the only the degree of freedom of pNRQCD (up to ultrasoft light quarks, e.g. pions).

$$\mathcal{L} = \mathrm{Tr}\left\{ \mathrm{S}^\dagger \left(i\partial_0 - \frac{\mathbf{p}^2}{m} - V_s \right) \mathrm{S} \right\}$$

Brambilla Pineda Soto Vairo 00

strongly coupled pNRQCD $r \sim \Lambda_{QCD}^{-1}$ $mv \sim \Lambda_{QCD}$



- integrate out all scales above mv^2
- gluonic excitations develop a gap $\Lambda_{\rm QCD}$ and are integrated out

⇒ The singlet quarkonium field S of energy mv^2 is the only the degree of freedom of pNRQCD (up to ultrasoft light quarks, e.g. pions).

$$\mathcal{L} = \mathrm{Tr}\left\{ \mathrm{S}^\dagger \left(i\partial_0 - \frac{\mathbf{p}^2}{m} - V_s \right) \mathrm{S} \right\}$$

- A potential description emerges from the EFT Brambilla Pineda Soto Vairo 00
- The potentials $V = \mathrm{Re}V + ImV$ from QCD in the matching: get spectra and decays
- V to be calculated on the lattice or in QCD vacuum models

The matching condition is: $\langle H | \mathcal{H} | H \rangle = \langle nljs | \frac{\mathbf{p}^2}{m} + \sum_n \frac{V_s^{(n)}}{m^n} | nljs \rangle$

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$$H_{\rm NRQCD} = H^{(0)} + \frac{1}{m_Q} H^{(1,0)} + \frac{1}{m_{\bar{Q}}} H^{(0,1)} + \dots ,$$

$$H^{(0)} = \int d^3x \frac{1}{2} (\boldsymbol{E}^a \cdot \boldsymbol{E}^a + \boldsymbol{B}^a \cdot \boldsymbol{B}^a) - \sum_{j=1}^{n_f} \int d^3\boldsymbol{x}\, \bar{q}_j\, i\boldsymbol{D} \cdot \boldsymbol{\gamma}\, q_j ,$$

$$H^{(1,0)} = -\frac{1}{2} \int d^3x\, \psi^\dagger \left(\boldsymbol{D}^2 + g c_F\, \boldsymbol{\sigma} \cdot \boldsymbol{B} \right) \psi ,$$

$$H^{(0,1)} = \frac{1}{2} \int d^3x\, \chi^\dagger \left(\boldsymbol{D}^2 + g c_F\, \boldsymbol{\sigma} \cdot \boldsymbol{B} \right) \chi ,$$

$$\mathcal{H}^{(0)} | \underline{n}; \mathbf{x}_1, \mathbf{x}_2 \rangle^{(0)} = E_n^{(0)}(\mathbf{x}_1, \mathbf{x}_2) | \underline{n}; \mathbf{x}_1, \mathbf{x}_2 \rangle^{(0)}$$

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and from this we obtain the

Quarkonium singlet static potential

$$V = V_0 + \frac{1}{m}V_1 + \frac{1}{m^2}(V_{SD} + V_{VD})$$

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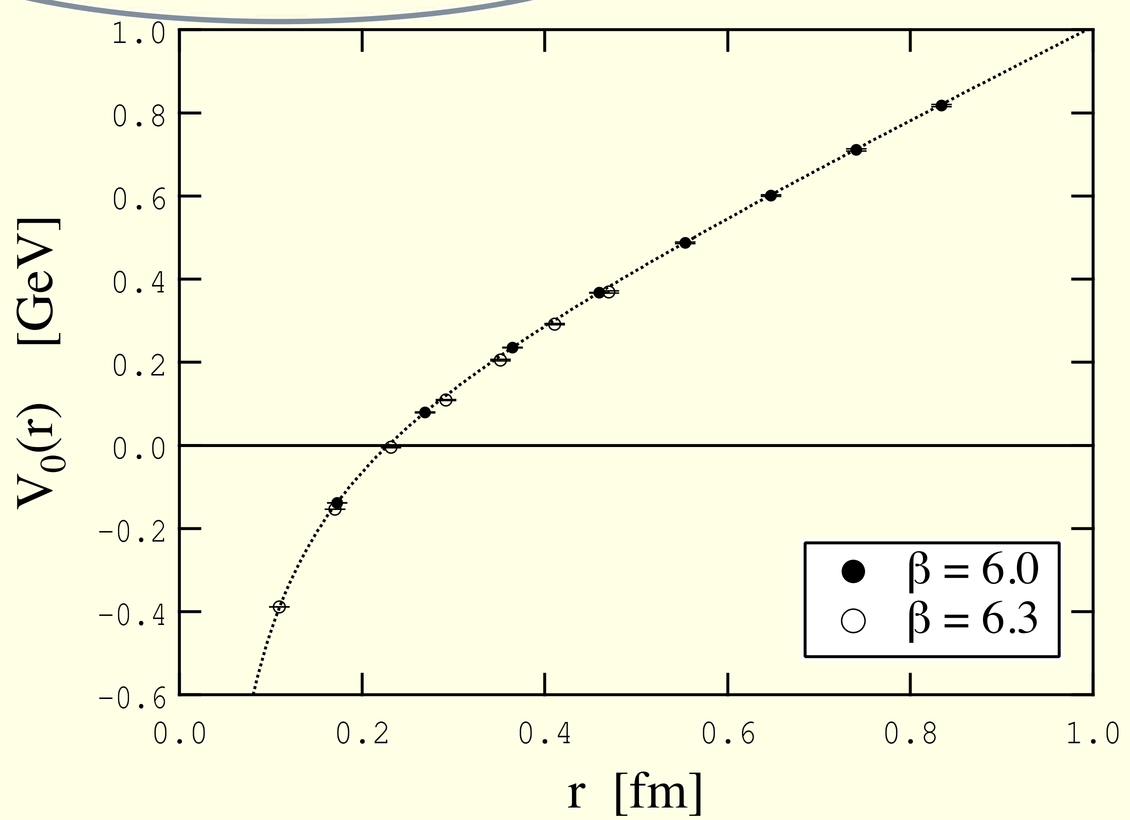
Quarkonium singlet static potential

$$V = V_0 + \frac{1}{m}V_1 + \frac{1}{m^2}(V_{SD} + V_{VD})$$

$$V_s^{(0)} = \lim_{T\to\infty} \frac{i}{T} \ln\langle W(r\times T)\rangle = \lim_{T\to\infty} \frac{i}{T} \ln\langle \square \rangle$$

$$W = \langle \exp\{ig \oint A^\mu dx_\mu\}\rangle$$



○ Koma Koma NPB 769(07)79

QCD Spin dependent potentials

$$V_{\rm SD}^{(2)} = \frac{1}{r}\left(c_F \epsilon^{kij} \frac{2r^k}{r} i \int_0^\infty dt\, t \,\langle \square \rangle - \frac{1}{2} V_s^{(0)\prime} \right) (\mathbf{S}_1 + \mathbf{S}_2)\cdot \mathbf{L}$$

$$- {c_F}^2 \hat{r}_i \hat{r}_j i \int_0^\infty dt \left(\langle \square \rangle - \frac{\delta_{ij}}{3} \langle \square \rangle \right) \times \left(\mathbf{S}_1\cdot\mathbf{S}_2 - 3(\mathbf{S}_1\cdot\hat{\mathbf{r}})(\mathbf{S}_2\cdot\hat{\mathbf{r}}) \right)$$

$$+ \left(\frac{2}{3} {c_F}^2 i \int_0^\infty dt \langle \square \rangle - 4(d_2 + C_F d_4)\delta^{(3)}(\mathbf{r}) \right) \mathbf{S}_1\cdot\mathbf{S}_2$$

Eichten Feinberg 81, Gromes 84, Chen et al. 95 Brambilla Vairo 99 Pineda, Vairo 00

-factorization: the NRQCD matching coefficients encode the physics at the large scale m, the potentials are given in terms of low energy nonperturbative Wilson loops

power counting; QM divergences absorbed NRQCD matching coefficients

EFTs (plus lattice) give a QCD description of quarkonium below threshold

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The issue here is precision physics and the study of confinement

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For states close or above the strong decay threshold the situation is much more complicated.

there is no mass gap between quarkonium and the creation of a heavy-light mesons couple

$$m_{Q\bar{q}} + m_{\bar{Q}q} = 2m + 2\Lambda_{QCD}$$

Near theshold heavy-light mesons and gluons excitations have to be included and many additional states built using the light quark quantum numbers may appear

No systematic treatment is yet available; also lattice calculations are challenging

Many phenomenological models exist

States made of two heavy and light quarks

- Pairs of heavy-light mesons: $D\bar{D}$, $B\bar{B}$, ...

- Molecular states, i.e. states built on the pair of heavy-light mesons.
  - Tornqvist PRL 67(91)556

- Pairs of heavy-light baryons.
  - Qiao PLB 639 (2006) 263

- The usual quarkonium states, built on the static potential, may also give rise to molecular states through the interaction with light hadrons (hadro-quarkonium).
  - Dubynskiy Voloshin PLB 666 (2008) 344

- Tetraquark states.

MAIANI, PICCININI, POLOSA ET AL. 2005--

  - Jaffe PRD 15(77)267
  - Ebert Faustov Galkin PLB 634(06)214

Vijande, Valcarce, Richard

Having the spectrum of tetraquark potentials, like we have for the gluonic excitations, would allow us to build a plethora of states on each of the tetraquark potentials, many of them developing a width due to decays through pion (or other light hadrons) emission. Diquarks have been recently investigated on the lattice.

  - Alexandrou et al. PRL 97(06)222002
  - Fodor et al. PoS LAT2005(06)310

States made of two heavy and light quarks

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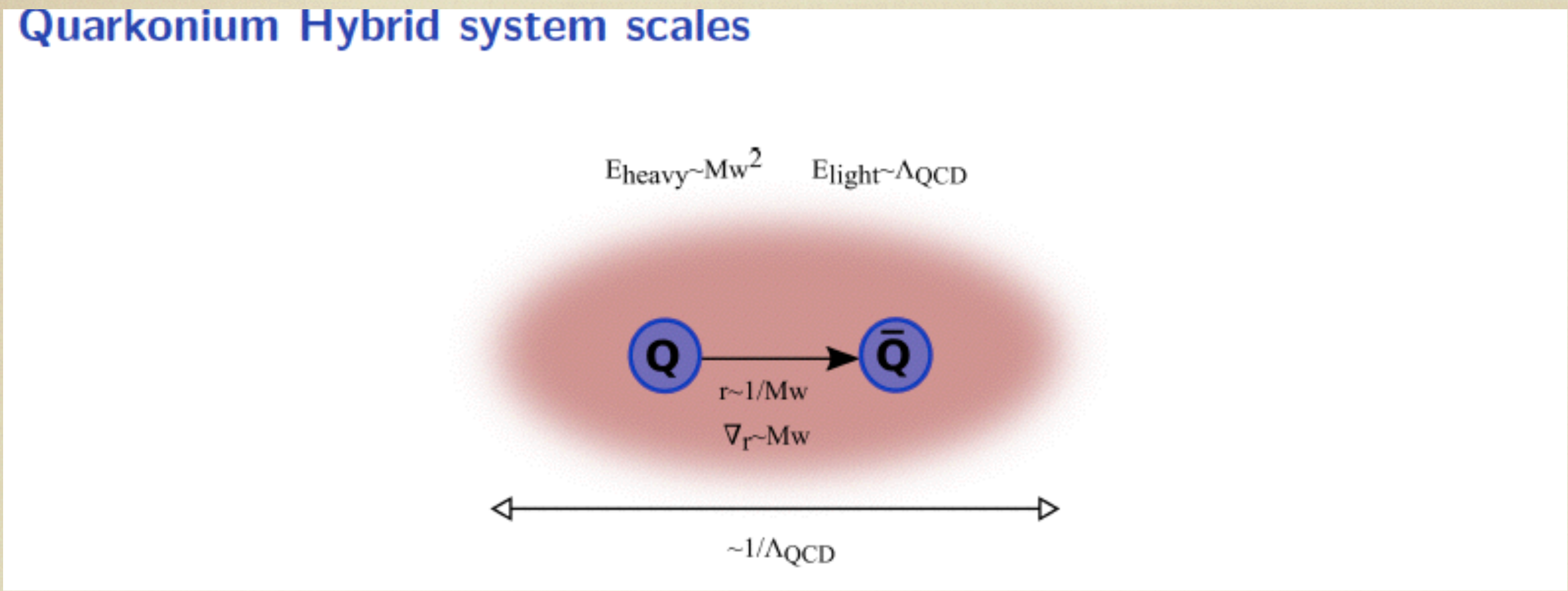
  - Jaffe PRD 15(77)267 Vijande, Valcarce, Richard
  - Ebert Faustov Galkin PLB 634(06)214

Having the spectrum of tetraquark potentials, like we have for the gluonic excitations, would allow us to build a plethora of states on each of the tetraquark potentials, many of them developing a width due to decays through pion (or other light hadrons) emission. Diquarks have been recently investigated on the lattice.

  - Alexandrou et al. PRL 97(06)222002
  - Fodor et al. PoS LAT2005(06)310

choosing one of these degrees of freedom and an interaction originates a model for exotics.

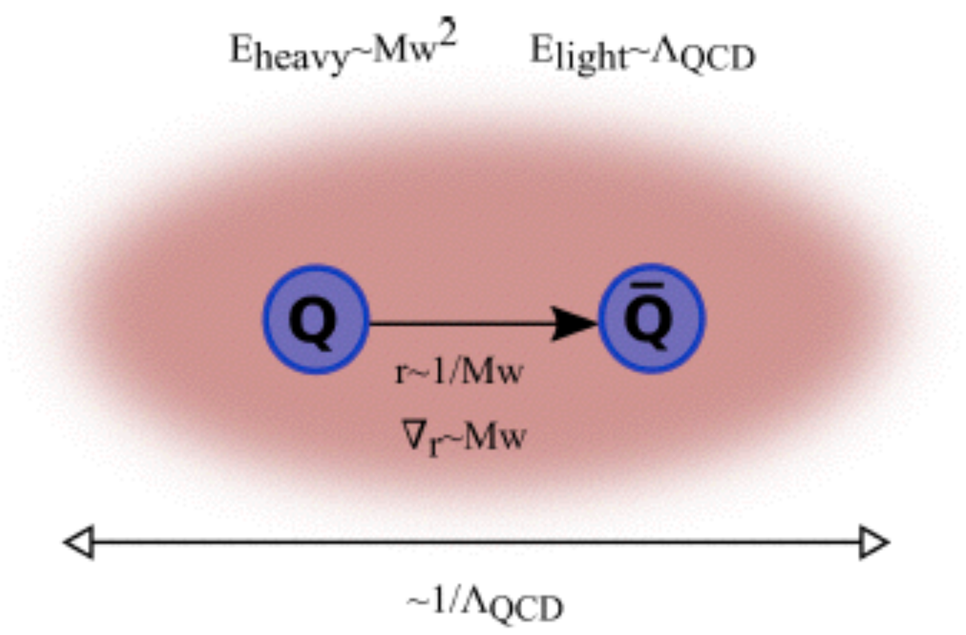
Start considering the simplified case of heavy quark, heavy antiquark plus glue



$$v = w$$

Start considering the simplified case of heavy quark, heavy antiquark plus glue

Quarkonium Hybrid system scales



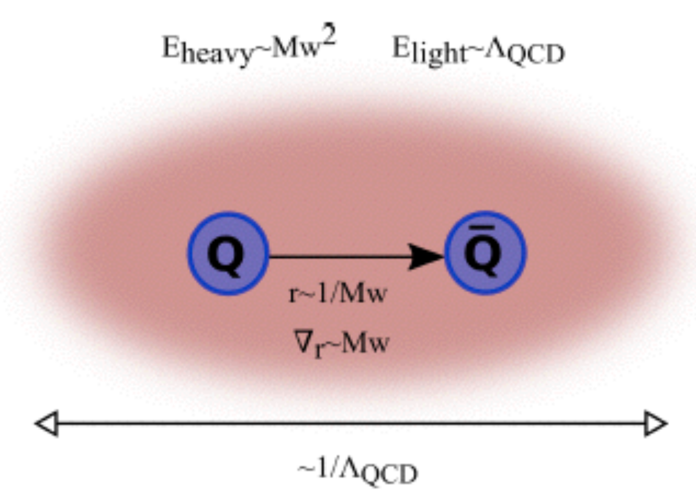
$$v = w$$

Characteristic Scales

- Heavy-quarks are non-relativistic $m_Q \gg \Lambda_{QCD}$.
- Two components with very different dynamical time scales $\Lambda_{QCD} \gg m_Q w^2$.
  - Excited gluonic state Λ_{QCD}.
  - Heavy-quark binding $m_Q w^2$ ($w \ll 1$ relative velocity).
  - Adiabatic expansion (Born-Oppenheimer approximation in atomic physics). Griffiths, Michael, Rakow 1983; Juge, Kuti, Morningstar 1998; Braaten, Langmack, Smith 2014; Meyer, Swanson 2015...

Start considering the simplified case of heavy quark, heavy antiquark plus glue

Quarkonium Hybrid system scales



$$v = w$$

Short distance regime

- Small Heavy-quark-antiquark distance $r \sim 1/(mw) < 1/\Lambda_{QCD}$.
- Factorization of perturbative and nonperturbative physics.

Heavy Hybrids EFT

- Use the hierarchy of scales to describe the system.
  - ∗ Integrate out m_Q modes: NRQCD Caswell, Lepage 1986; Bodwin, Braaten and Lepage 1995
  - ∗ Integrate out $m_Q w \sim 1/r$ modes: (weakly-coupled) pNRQCD Pineda, Soto 1998; Brambilla, Pineda, Soto, Vairo 2000
  - ∗ Integrate out Λ_{QCD}: Hybrid EFT Berwein, Brambilla, JTC, Vairo 2015; Brambilla, Krein, JTC, Vairo 2017 see also Oncala, Soto 2017

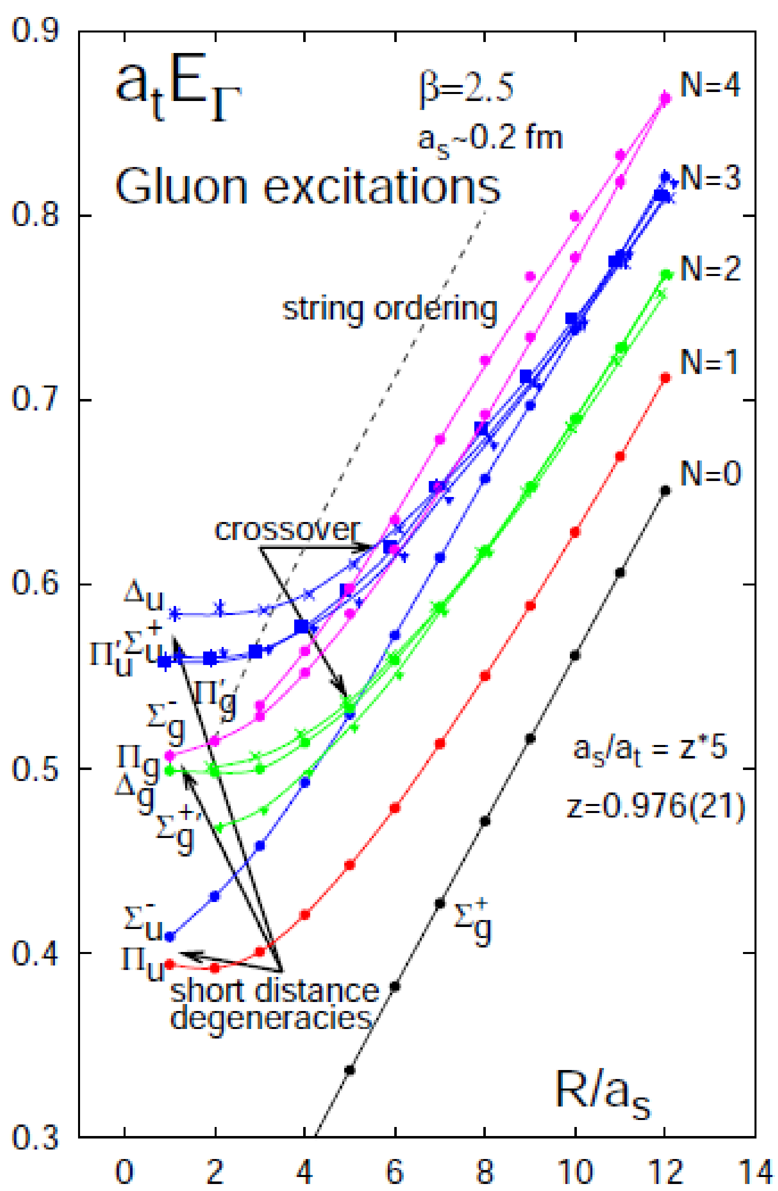
Heavy-quark heavy antiquark plus glue

Heavy-quark heavy antiquark plus glue

Define the symmetries of the system and the system static energies in NRQCD

Static

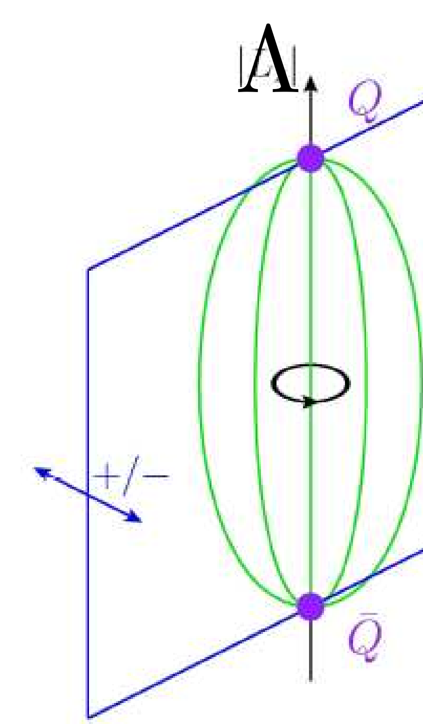
Lattice energies Juge Kuti Morningstar 2003



Symmetries

Static states classified by symmetry group $D_{\infty h}$
Representations labeled Λ^σ_η

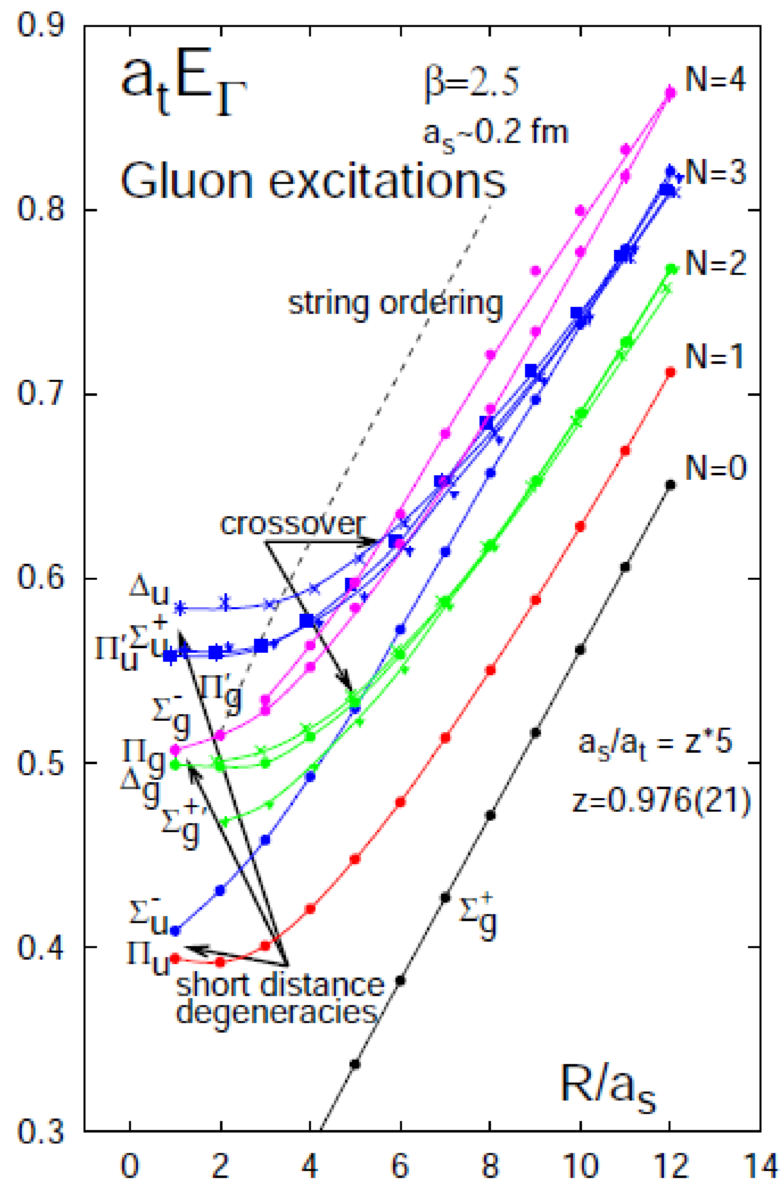
- Λ rotational quantum number $|\hat{\mathbf{n}} \cdot \mathbf{K}| = 0, 1, 2 \ldots$ corresponds to $\Lambda = \Sigma, \Pi, \Delta \ldots$
- η eigenvalue of CP: $g \hat{=} +1$ (gerade), $u \hat{=} -1$ (ungerade)
- σ eigenvalue of reflections
- σ label only displayed on Σ states (others are degenerate)



- The static energies correspond to the irreducible representations of D
- In general it can be more than one state for each irreducible represent $D_{\infty h}$, usually denoted by primes, e.g. Π_u, Π'_u, $\Pi''_u \ldots$

Heavy-quark heavy antiquark plus glue

static Lattice energies

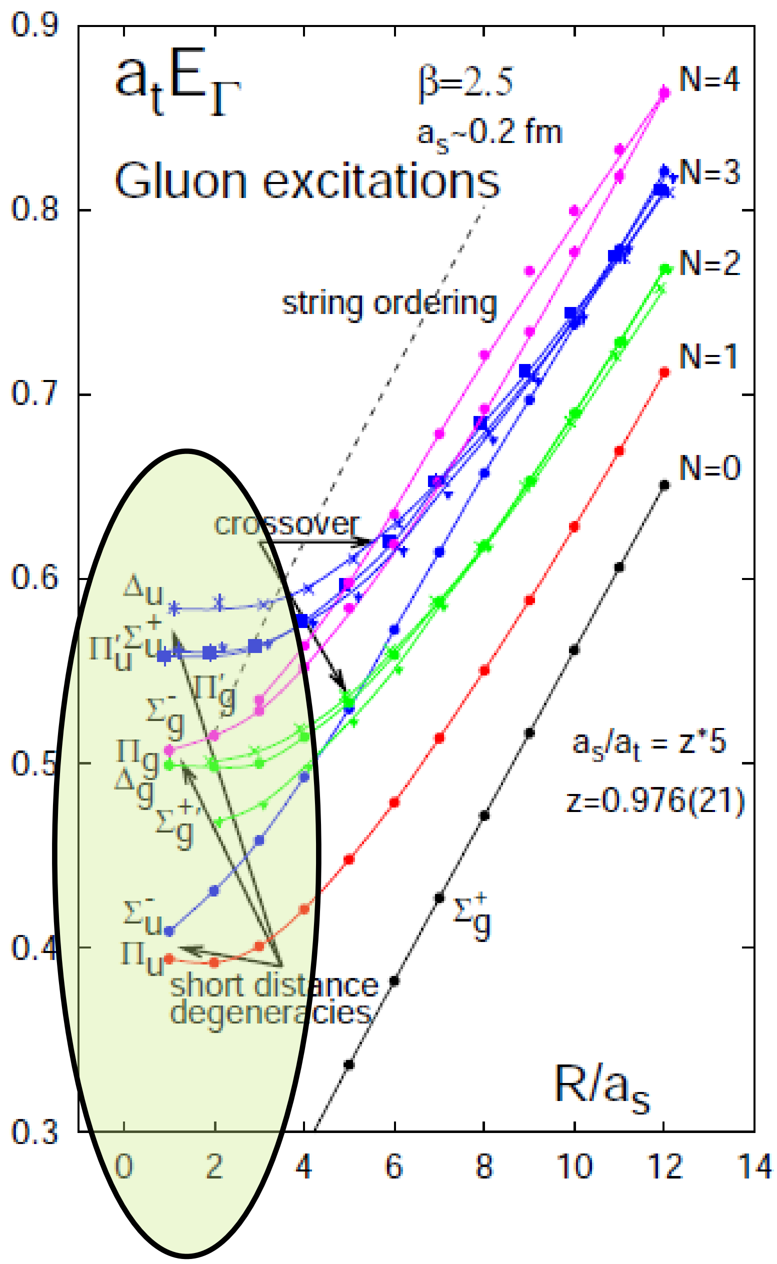


- Σ_g^+ is the ground state potential that generates the standard quarkonium states.
- The rest of the static energies correspond to excited gluonic states that generate hybrids.
- The two lowest hybrid static energies are Π_u and Σ_u^-, they are nearly degenerate at short distances.
- The static energies have been computed in quenched lattice QCD, the most recent data by Juge, Kuti, Morningstar, 2002 and Bali and Pineda 2003.
- Quenched and unquenched calculations for Σ_g^+ and Π_u were compared in Bali et al 2000 and good agreement was found below string breaking distance.

○ Juge Kuti Morningstar PRL 90 (2003) 161601

Heavy-quark heavy antiquark plus glue

static Lattice energies



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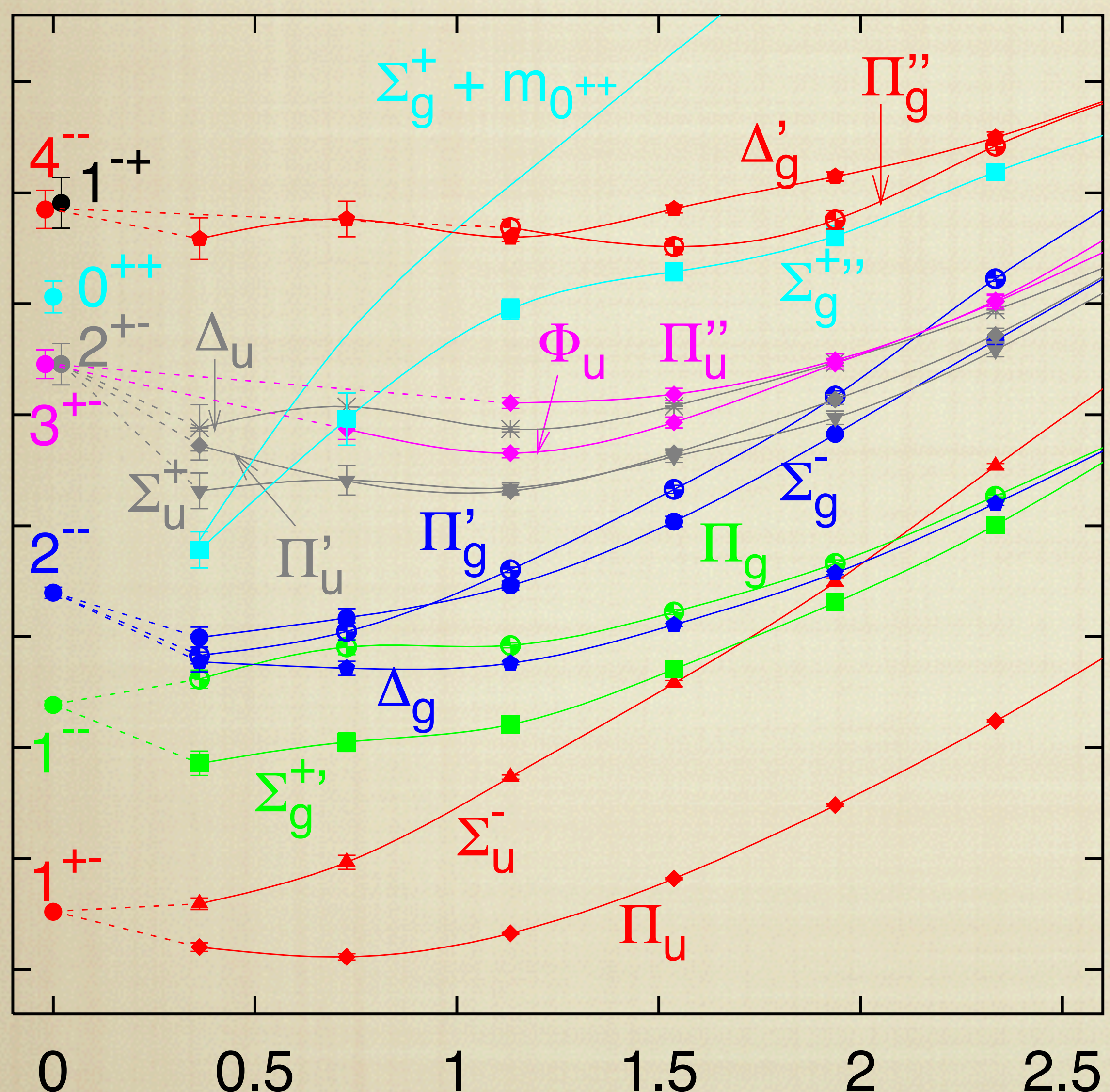
○ Juge Kuti Morningstar PRL 90 (2003) 161601

pNRQCD gives the multiplets at short distance:gluelumps

In the short-range hybrids become gluelumps, i.e., quark-antiquark octets, O^a, in the presence of a gluonic field, H^a: $H(R,r,t) = H^a(R,t)O^a(R,r,t)$.

In the limit $r \to 0$ more symmetry: $D_{\infty h} \to O(3) \times C$

- Several Λ_η^σ representations contained in one J^{PC} representation:
- Static energies in these multiplets have same $r \to 0$ limit.



Gluonic excitation operators up to dim 3

Λ_η^σ	K^{PC}	H^a
Σ_u^-	1^{+-}	$\mathbf{r}\cdot\mathbf{B}\,,\mathbf{r}\cdot(\mathbf{D}\times\mathbf{E})$
Π_u	1^{+-}	$\mathbf{r}\times\mathbf{B}\,,\mathbf{r}\times(\mathbf{D}\times\mathbf{E})$
$\Sigma_g^{+\prime}$	1^{--}	$\mathbf{r}\cdot\mathbf{E}\,,\mathbf{r}\cdot(\mathbf{D}\times\mathbf{B})$
Π_g	1^{--}	$\mathbf{r}\times\mathbf{E}\,,\mathbf{r}\times(\mathbf{D}\times\mathbf{B})$
Σ_g^-	2^{--}	$(\mathbf{r}\cdot\mathbf{D})(\mathbf{r}\cdot\mathbf{B})$
Π'_g	2^{--}	$\mathbf{r}\times((\mathbf{r}\cdot\mathbf{D})\mathbf{B}+\mathbf{D}(\mathbf{r}\cdot\mathbf{B}))$
Δ_g	2^{--}	$(\mathbf{r}\times\mathbf{D})^i(\mathbf{r}\times\mathbf{B})^j+(\mathbf{r}\times\mathbf{D})^j(\mathbf{r}\times\mathbf{B})^i$
Σ_u^+	2^{+-}	$(\mathbf{r}\cdot\mathbf{D})(\mathbf{r}\cdot\mathbf{E})$
Π'_u	2^{+-}	$\mathbf{r}\times((\mathbf{r}\cdot\mathbf{D})\mathbf{E}+\mathbf{D}(\mathbf{r}\cdot\mathbf{E}))$
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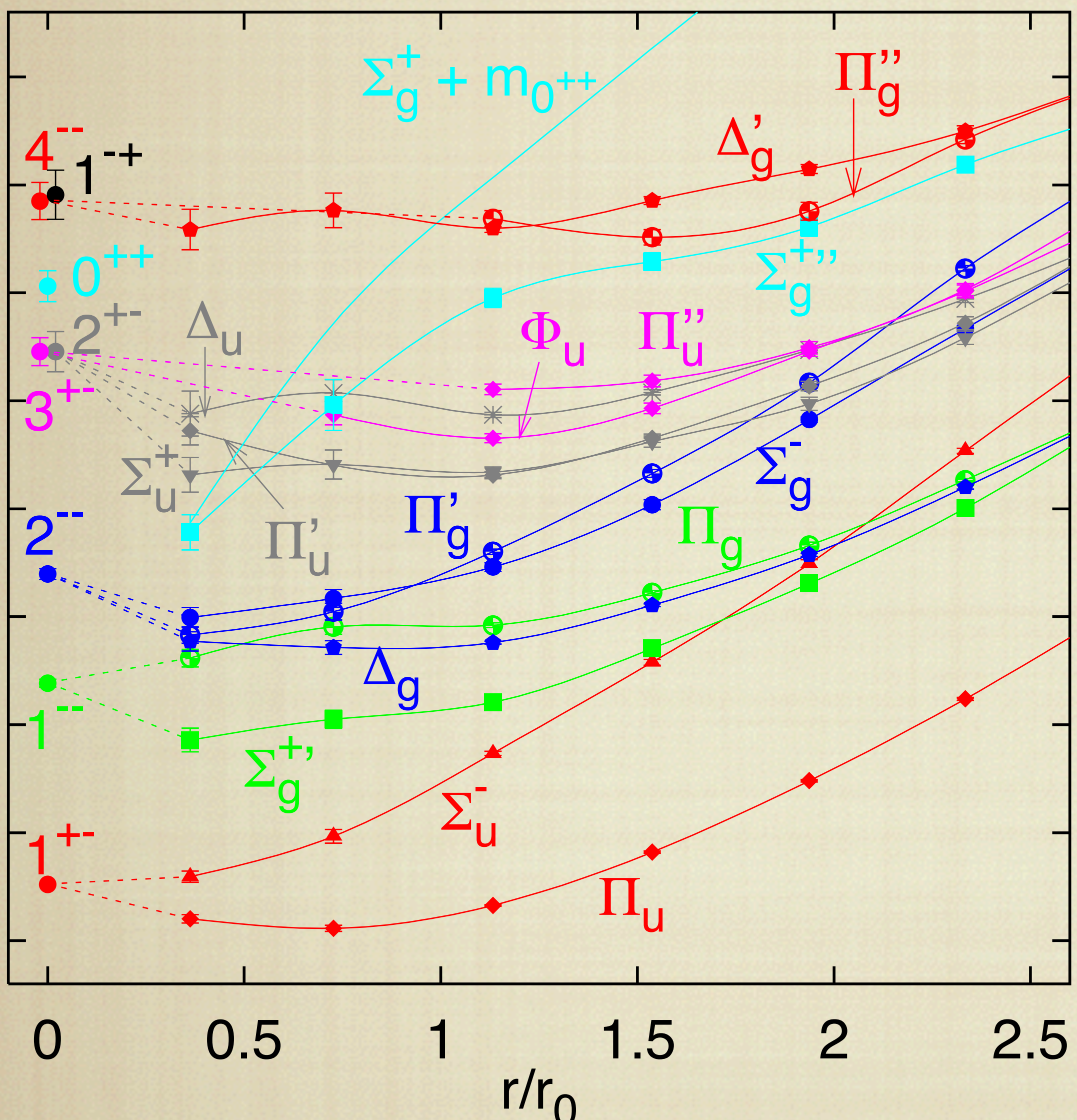
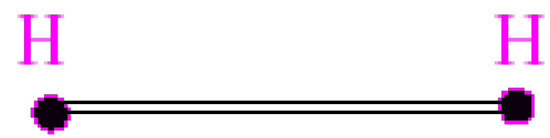
Brambilla Pineda Soto Vairo 00

The gluelump multiplets Σ_u^-, Π_u; $\Sigma_g^{+\prime}$, Π_g; Σ_g^-, Π'_g, Δ_g; Σ_u^+, Π'_u, Δ_u are degenerate.

Match to pNRQCD: one can determine the form of the potential

In the short-range hybrids become gluelumps, i.e., quark-antiquark octets, O^a, in the presence of a gluonic field, H^a: $H(R,r,t) = H^a(R,t)O^a(R,r,t)$.





$$= e^{-iTE_H}$$

$$E_H = V_o + \frac{i}{T}\ln\langle H^a(\frac{T}{2})\phi^{\rm adj}_{ab}H^b(-\frac{T}{2})\rangle$$

$$\langle H^a(\frac{T}{2})\phi^{\rm adj}_{ab}H^b(-\frac{T}{2})\rangle^{\rm np} \sim h\, e^{-iT\Lambda_H}$$

$$E_H(r) = V_o(r) + \Lambda_H + b_{\Lambda_H} r^2$$

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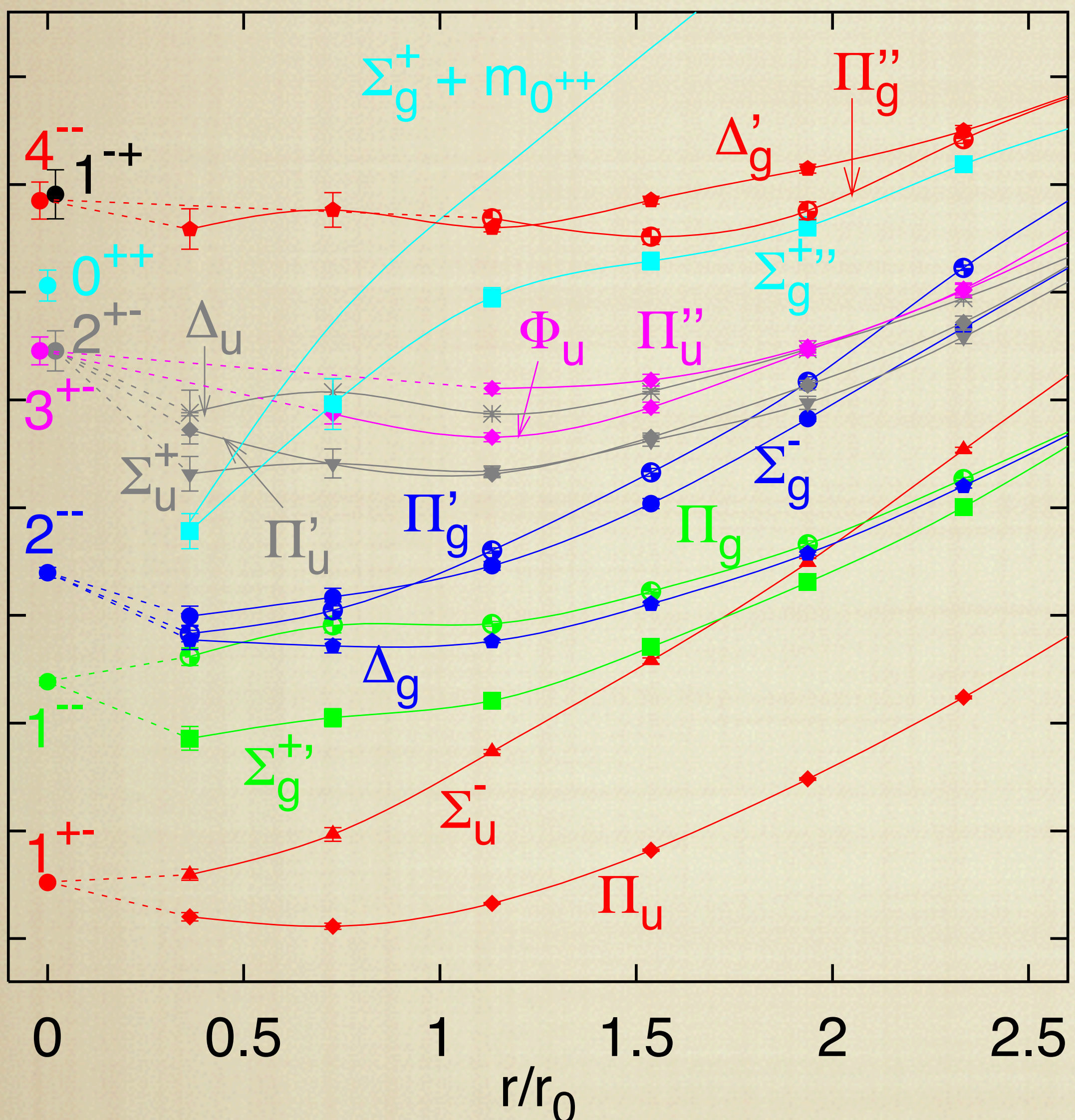

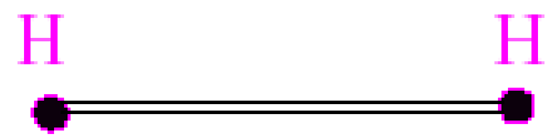

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octet potential

gluelump mass

correction softly breaking the symm

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Match to pNRQCD: one can determine the form of the potential

$$E_H(r) = V_O(r) + \Lambda_H + b_H r^2$$

Λ_H

- It is a non-perturbative quantity.
- It depends on the particular operator H^a, however it is the same for operators corresponding to different projections of the same gluonic operators.
- The gluelump masses have been determined in the lattice. Foster *et all* 1999; Bali, Pineda 2004; Marsh Lewis 2014
- At the subtraction scale $\nu_f = 1$ GeV: $\Lambda^{RS}_{1^{+-}} = 0.87(15)$ GeV.

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$$E_H(r) = V_O(r) + \Lambda_H + b_H r^2$$

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- It is a non-perturbative quantity.

- Proportional to r^2 due to rotational invariance and the multipole expansion.
- We are going to fix it through a fit to the static energies lattice data.
- Breaks the degeneracy of the potentials.

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Octet potential at two loops; renormalon subtraction realised among pole mass, octet potential and gluelump mass, use RS scheme

State multiplets

We consider hybrids that are excitations of the lowest lying static energies Π_u and Σ_u^-. In the $r \to 0$ limit Π_u and Σ_u^- are degenerate and correspond to a gluonic operator with quantum numbers 1^{+-}.

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States are organized in spin multiplets.

	l	$J^{PC}\{s=0, s=1\}$	$E_n^{(0)}$
H_1	1	$\{1^{--}, (0,1,2)^{-+}\}$	Σ_u^-, Π_u
H_2	1	$\{1^{++}, (0,1,2)^{+-}\}$	Π_u
H_3	0	$\{0^{++}, 1^{+-}\}$	Σ_u^-
H_4	2	$\{2^{++}, (1,2,3)^{+-}\}$	Σ_u^-, Π_u
H_5	2	$\{2^{--}, (1,2,3)^{-+}\}$	Π_u

Braaten PRL 111 (2013) 162003

Braaten Langmack Smith PRD 90 (2014) 014044

Berwein Brambilla Tarrus Vairo PRD 92 (2015) 114019

$$E_H(r) = V_O(r) + \Lambda_H + b_H r^2$$

Coupled radial Schrödinger equations

Projection vectors in matrix elements allow for two different solutions (coupled or uncoupled) for the Σ_u^- and Π_u radial wave functions:

1st solution

$$\left[-\frac{1}{2\mu r^2}\partial_r r^2 \partial_r + \frac{1}{2\mu r^2}\begin{pmatrix} l(l+1)+2 & 2\sqrt{l(l+1)} \\ 2\sqrt{l(l+1)} & l(l+1) \end{pmatrix} + \begin{pmatrix} E_\Sigma^{(0)} & 0 \\ 0 & E_\Pi^{(0)} \end{pmatrix}\right]\begin{pmatrix}\psi_\Sigma \\ \psi_\Pi\end{pmatrix} = \mathcal{E}\begin{pmatrix}\psi_\Sigma \\ \psi_\Pi\end{pmatrix}$$

2nd solution

$$\left[-\frac{1}{2\mu r^2}\,\partial_r\, r^2\,\partial_r + \frac{l(l+1)}{2\mu r^2} + E_\Pi^{(0)}\right]\psi_\Pi = \mathcal{E}\,\psi_\Pi$$

- energy eigenvalue $\mathcal{E}$ gives hybrid mass: $m_H = m_Q + m_{\bar{Q}} + \mathcal{E}$
- $l(l+1)$ is the eigenvalue of angular momentum $\boldsymbol{L}^2 = (\boldsymbol{L}_{Q\bar{Q}} + \boldsymbol{L}_g)^2$
- the two solutions correspond to **opposite parity** states: $(-1)^l$ and $(-1)^{l+1}$
- corresponding eigenvalues under charge conjugation: $(-1)^{l+s}$ and $(-1)^{l+s+1}$
- Schrödinger equations can be solved numerically

For $l = 0$ the off-diagonal terms vanish, so the equations for $\psi_\Sigma^{(N)}$ and $\psi_{-\Pi}^{(N)}$ decouple. There exists only one parity state, and its radial wave function is given by a Schrödinger equation with the $E_\Sigma^{(0)}$ potential and an angular part $2/mr^2$.

Berwein Brambilla Tarrus Vairo PRD 92 (2015) 114019

$$E_H(r) = V_O(r) + \Lambda_H + b_H r^2$$

The Lambda -doubling effect breaks the degeneracy between opposite parity spin-symmetry
multiplets and lowers the mass of the multiplets that get mixed contributions of different static
energies.

1st solution

$$\left[-\frac{1}{2\mu r^2}\partial_r r^2 \partial_r + \frac{1}{2\mu r^2}\begin{pmatrix} l(l+1)+2 & 2\sqrt{l(l+1)} \\ 2\sqrt{l(l+1)} & l(l+1) \end{pmatrix} + \begin{pmatrix} E_\Sigma^{(0)} & 0 \\ 0 & E_\Pi^{(0)} \end{pmatrix}\right]\begin{pmatrix}\psi_\Sigma \\ \psi_\Pi\end{pmatrix} = \mathcal{E}\begin{pmatrix}\psi_\Sigma \\ \psi_\Pi\end{pmatrix}$$

2nd solution

$$\left[-\frac{1}{2\mu r^2}\,\partial_r\, r^2\, \partial_r + \frac{l(l+1)}{2\mu r^2} + E_\Pi^{(0)}\right]\psi_\Pi = \mathcal{E}\,\psi_\Pi$$

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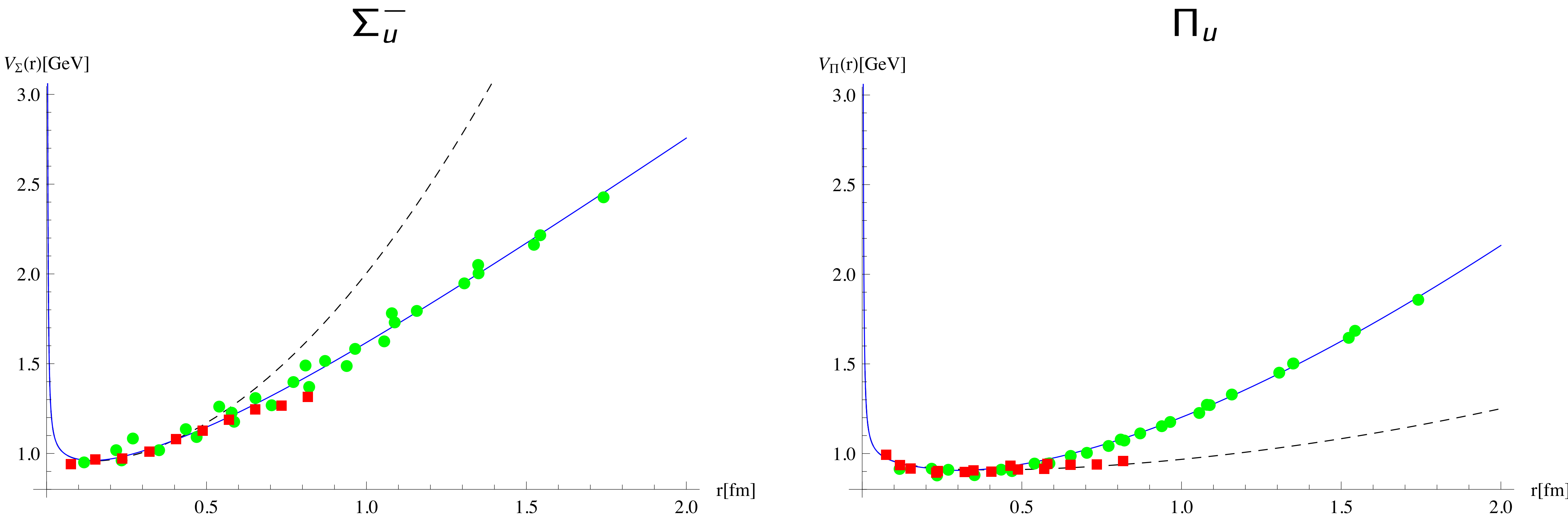


Lattice data:Bali, Pineda 2004; Juge, Kuti, Morningstar 2003, dashed line $V^{(0.5)}$, solid line $V^{(0.25)}$

$V^{(0.25)}$

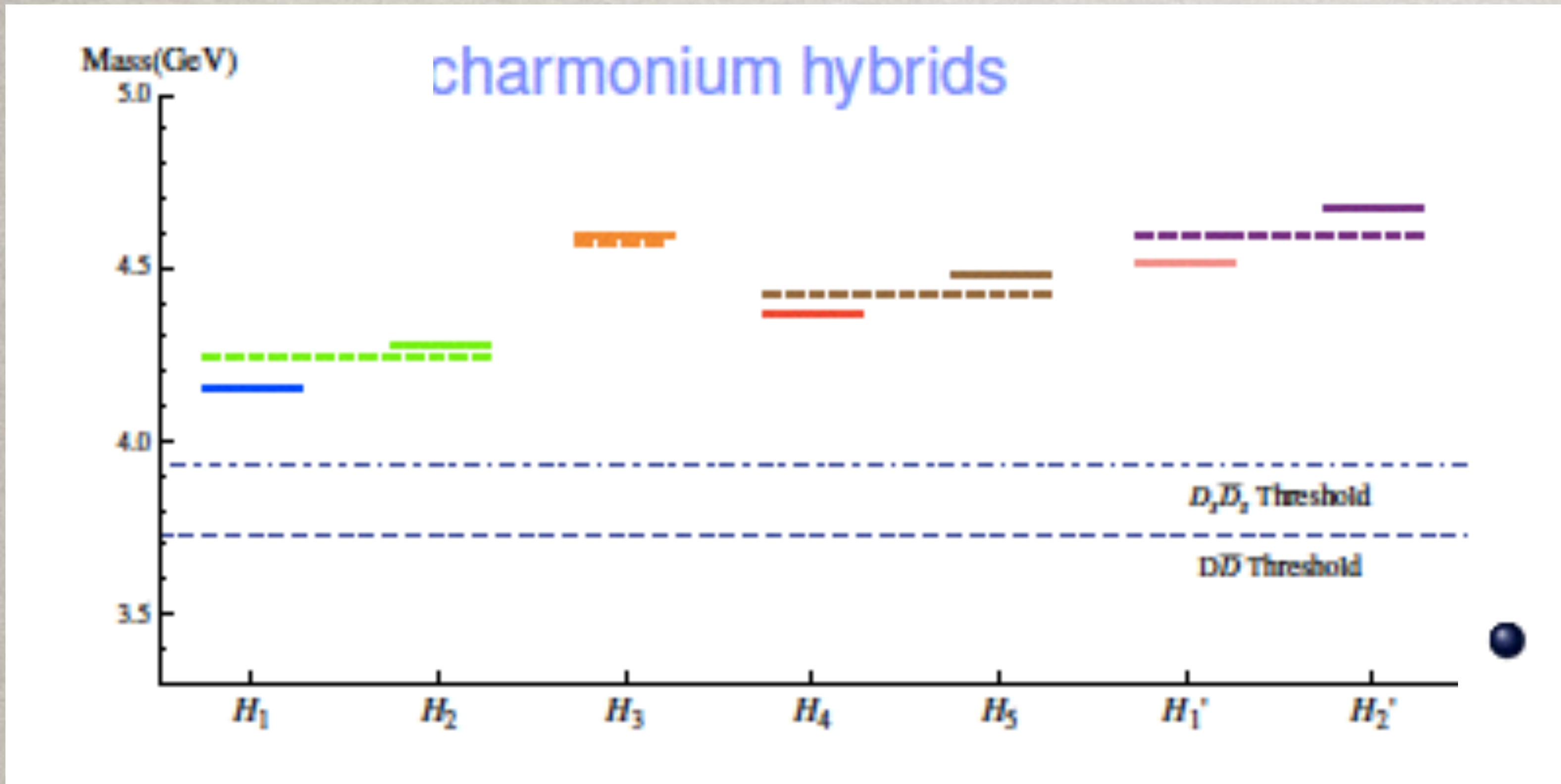
- $r \leq 0.25$ fm: pNRQCD potential.
  - Lattice data fitted for the $r = 0 - 0.25$ fm range with the same energy offsets as in $V^{(0.5)}$.

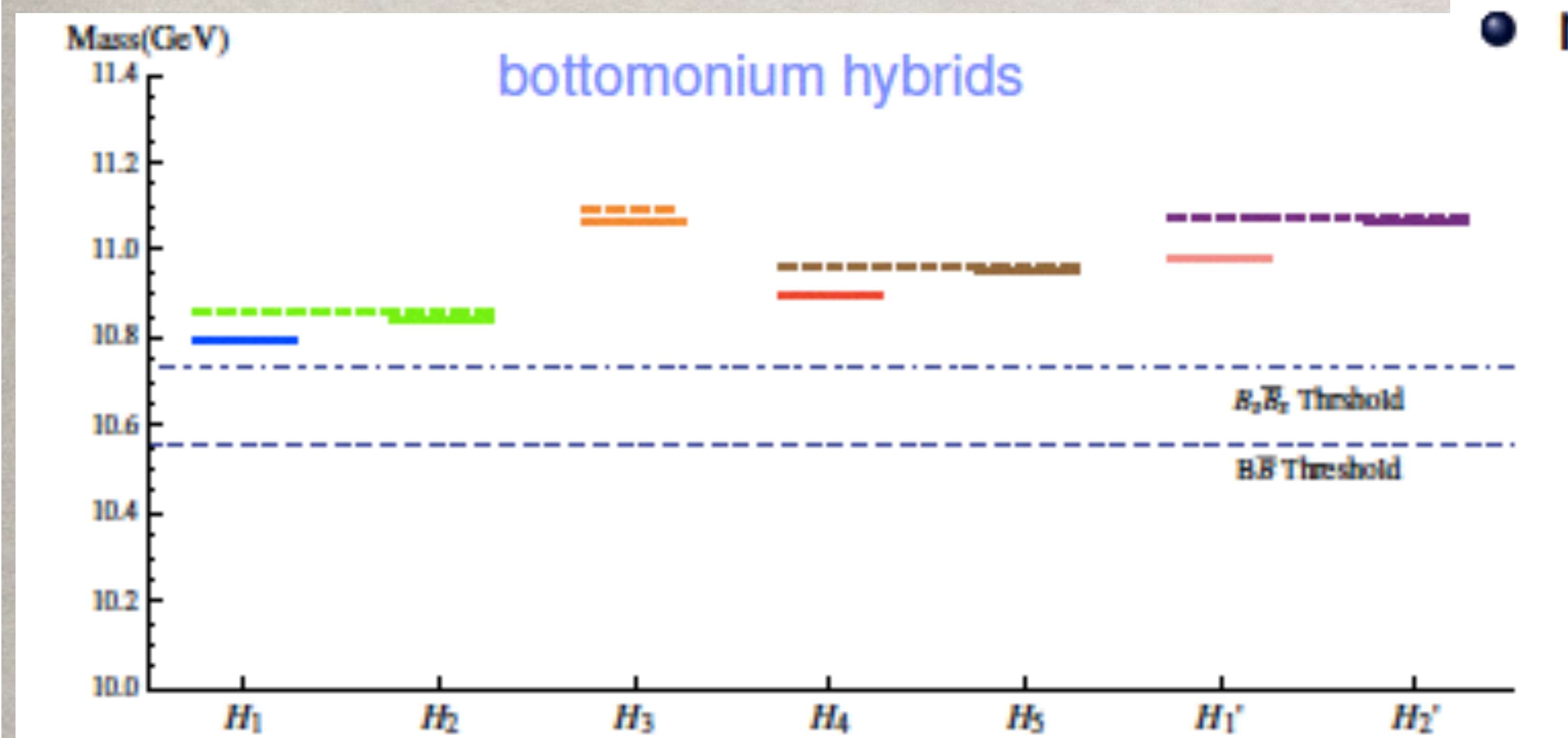
$$b_\Sigma^{(0.25)} = 1.246\,\mathrm{GeV/fm}^2, \quad b_\Pi^{(0.25)} = 0.000\,\mathrm{GeV/fm}^2\,.$$

- $r > 0.25$ fm: phenomenological potential.
  - $\mathcal{V}'(r) = \frac{a_1}{r} + \sqrt{a_2 r^2 + a_3} + a_4$.
  - Same energy offsets as in $V^{(0.25)}$.
  - *Constraint:* Continuity up to first derivatives.

Berwein,N.B. , Tarrus, Vairo arXiv:1510.04299

Λ doubling in quarkonium hybrid states

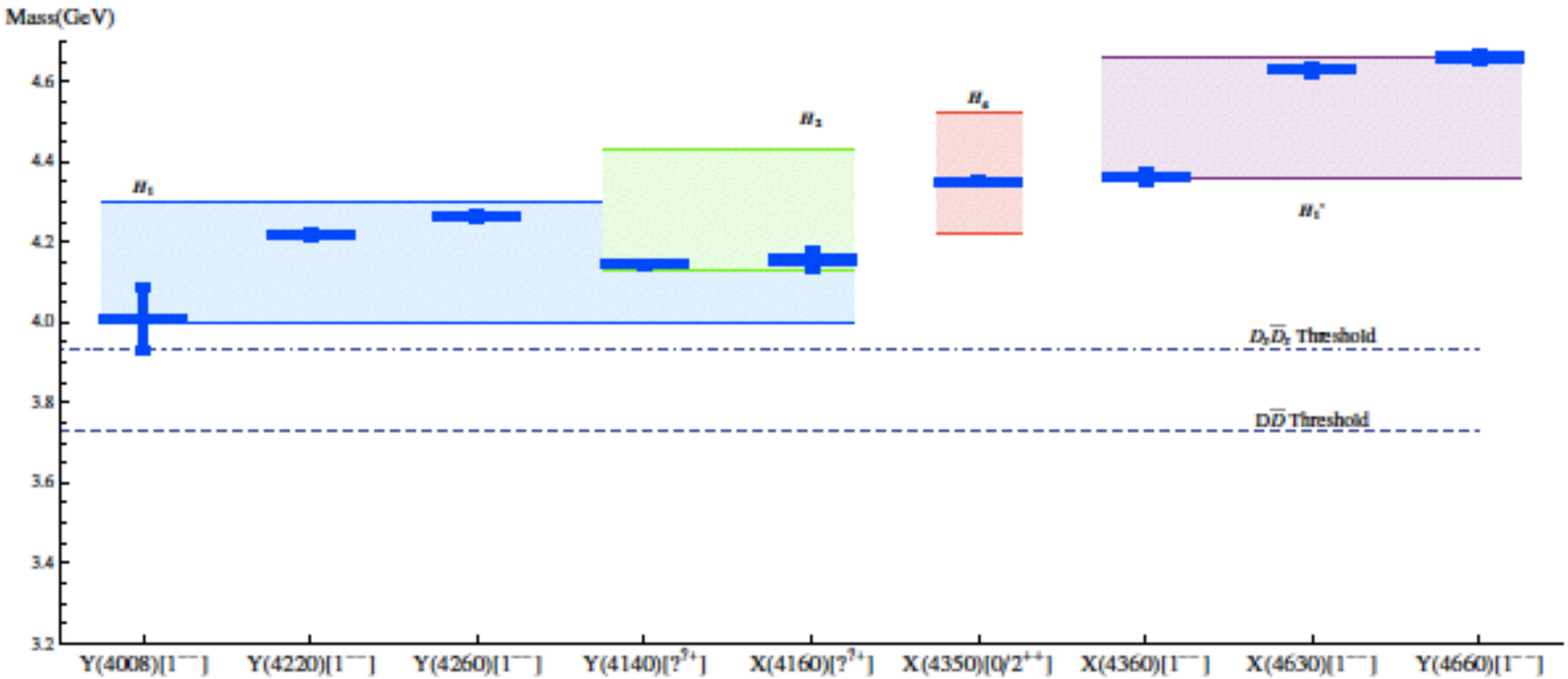




- no distinction between opposite parity states in BO
- mixed states lie lower than pure

- Berwein Brambilla Tarrus Vairo PRD 92 (2015) 114019
  data without mixing (dashed) from Braaten et al PRD 90 (2014) 114044

Charmonium states (BELLE, CDF, BESIII, BABAR):



Note: only $Y(4220) \to h_c(1P)\pi^+\pi^-$ does not violate Heavy Quark Spin Symm

Bottomonium states: $Y_b(10890)[1^{--}]$, $M_{Y_b} = (10.8884 \pm 3.0)$ GeV (BELLE).
Possible H1 candidate, $M_{H1} = (10.79 \pm 0.15)$ GeV.

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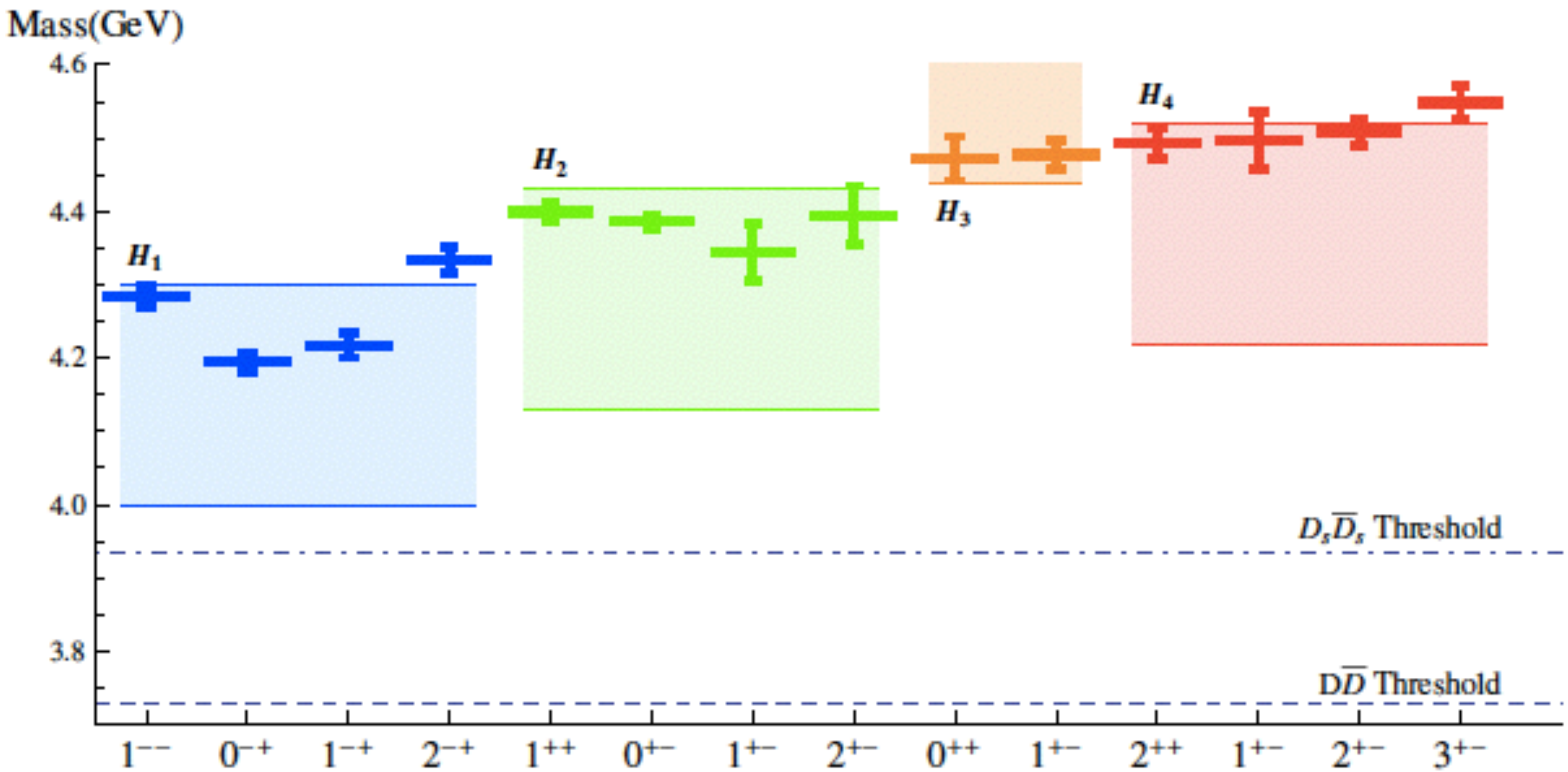
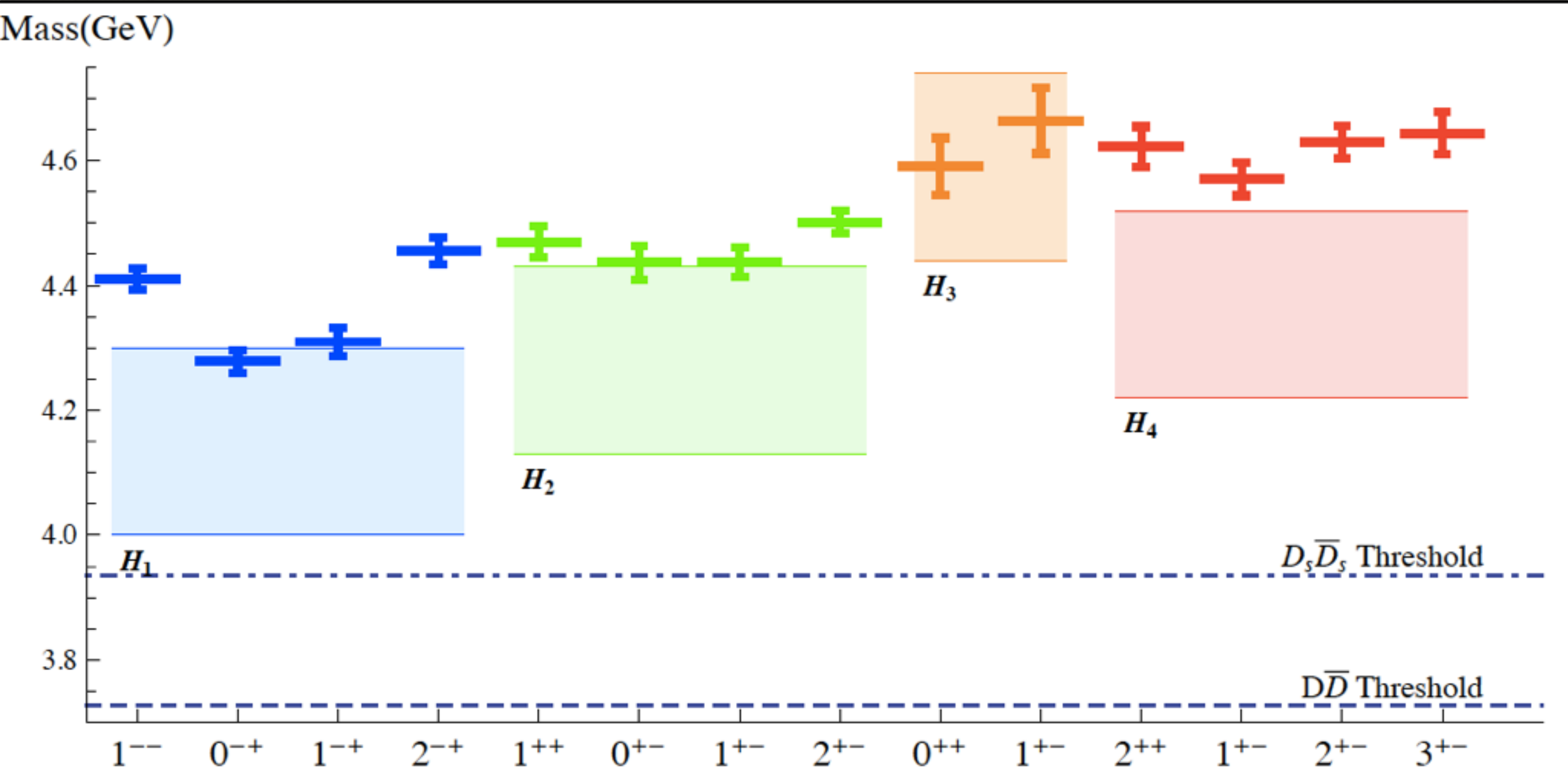
Charmonium hybrid states vs direct lattice data



- Berwein Brambilla Tarrus Vairo PRD 92 (2015) 114019
  lattice data from Liu et al JHEP 1207 (2012) 126

Comparison to direct lattice calculations



new lattice data from hadron spectrum collaboration

JHEP 1612 (2016) 089 with pion mass 240 MeV but no continuum limit

Introducing the spin of the quark

Up to now we have worked at the leading order, now we want to include the correction coming from the quark spin

We calculate the spin dependent potentials matching NRQCD and pNRQCD: we get a purely perturbative contribution in the form of spin dependent octet potential integrating out mv and then we get nonperturbative correlators depending only on glue when integrating out Lambda_QCD.

the nonperturbative correlators should be calculated on the lattice or in QCD vacuum models

we fix them on lattice data of charmonium—> we can then predict hybrids spin multiplet for bottomonium (in progress)

N.B, Wk Lai, J. Segovia, J. Tarrus, A. Vairo 2017 , in preparation

Hybrid EFT and Spin-dependent operators

$$L_{BO} = \int d^3R d^3r \sum_{\kappa} \sum_{\lambda\lambda'} \Psi^{\dagger}_{\kappa\lambda}(t, \mathbf{r}, \mathbf{R}) \left\{ i\partial_t - V_{\kappa\lambda\lambda'}(r) + P^{i\dagger}_{\kappa\lambda} \frac{\nabla_r^2}{M} P^{i}_{\kappa\lambda'} \right\} \Psi_{\kappa\lambda'}(t, \mathbf{r}, \mathbf{R}) \dots$$

The potential $V_{\kappa\lambda\lambda'}$ can be organized into an expansion in $1/m$ and spin-dependent and independent parts

$$V_{\kappa\lambda\lambda'}(r) = V^{(0)}_{\kappa\lambda}(r)\delta_{\lambda\lambda'} + \frac{V^{(1)}_{\kappa\lambda\lambda'}(r)}{m} + \frac{V^{(2)}_{\kappa\lambda\lambda'}(r)}{m^2} + \dots,$$

$$V^{(1)}_{\kappa\lambda\lambda'}(r) = V^{(1)}_{\kappa\lambda\lambda'\,SD}(r) + V^{(1)}_{\kappa\lambda\lambda'\,SI}(r),$$

$$V^{(2)}_{\kappa\lambda\lambda'}(r) = V^{(2)}_{\kappa\lambda\lambda'\,SD}(r) + V^{(2)}_{\kappa\lambda\lambda'\,SI}(r),$$

$$V^{(1)}_{1\lambda\lambda'\,SD}(r) = V_{1\,SK}(r) \left(P^{i\dagger}_{1\lambda} \mathbf{K}^{ij}_1 P^{j}_{1\lambda'} \right) \cdot \mathbf{S},$$

$$V^{(2)}_{1\lambda\lambda'\,SD}(r) = V_{1\,LSa}(r) \left(P^{i\dagger}_{1\lambda} \mathbf{L}_{Q\bar{Q}} P^{i}_{1\lambda'} \right) \cdot \mathbf{S} + V_{1\,LSb} P^{i\dagger}_{1\lambda}(r) \left(\mathbf{L}^{i}_{Q\bar{Q}} \mathbf{S}^{j} + \mathbf{S}^{i} \mathbf{L}^{j}_{Q\bar{Q}} \right) P^{j}_{1\lambda'}$$
$$+ V_{1\,S^2}(r)\mathbf{S}^2\delta_{\lambda\lambda'} + V_{1\,S_{12}a}(r)\mathbf{S}_{12}\delta_{\lambda\lambda'} + V_{1\,S_{12}b}(r) P^{i\dagger}_{1\lambda} P^{j}_{1\lambda'} \left(\mathbf{S}^{i}_1 \mathbf{S}^{j}_2 + \mathbf{S}^{i}_2 \mathbf{S}^{j}_1 \right)$$

At the pNRQCD level a basis of hybrid states is defined as

$$|\kappa, \lambda\rangle = P^{i}_{\kappa\lambda} O^{a\,\dagger}(\mathbf{r}, \mathbf{R})\, G^{i\,a}_{\kappa}(\mathbf{R})|0\rangle$$

The hybrid EFT is formulated for the subspace spanned by

$$\int d^3r d^3R \sum_{\kappa} |\kappa, \lambda\rangle\, \Psi_{\kappa\lambda}(t, \mathbf{r}, \mathbf{R})$$

$$h_0(\mathbf{R}) = \frac{1}{2}\left(\mathbf{E}^a\mathbf{E}^a - \mathbf{B}^a\mathbf{B}^a\right)$$

Λ_κ is the gluelump mass

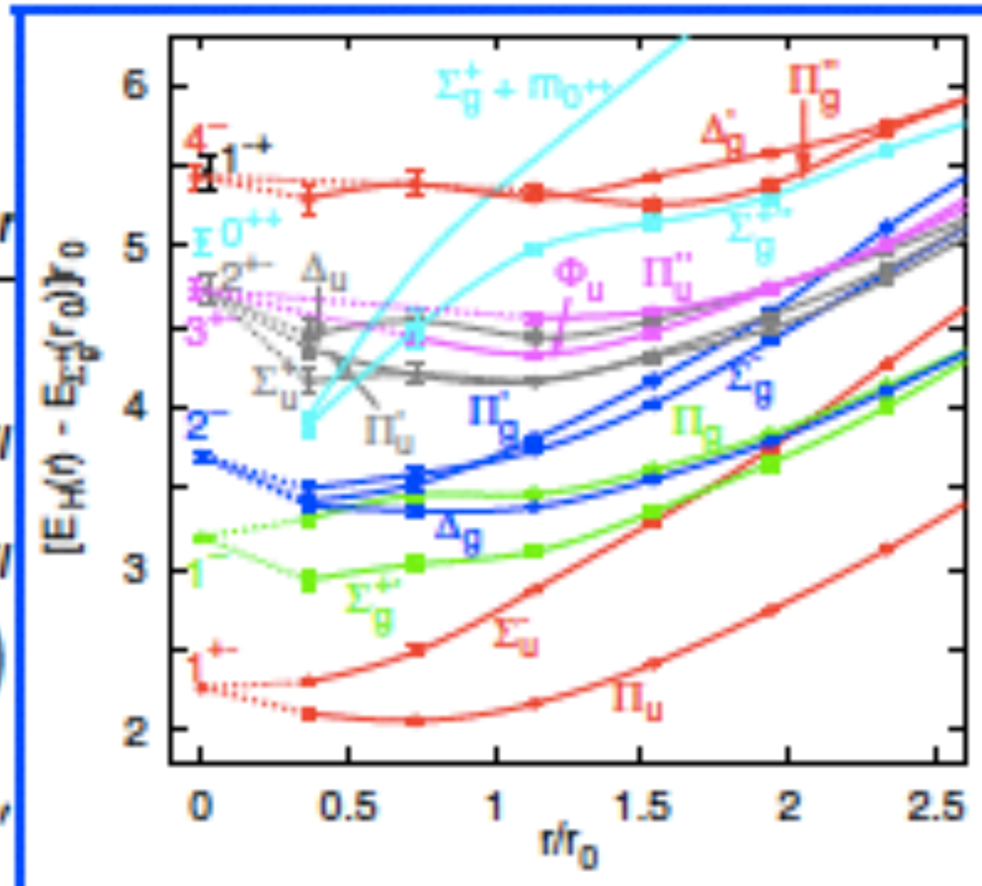
G^{ia}_{κ} are a basis of color-octet eigenstates of $h_0(\mathbf{R})$

$$h_0(\mathbf{R}) G^{i\,a}_{\kappa}(\mathbf{R})|0\rangle = \Lambda_\kappa G^{i\,a}_{\kappa}(\mathbf{R})|0\rangle$$

Hybrid EFT and Spin-dependent operators

$$L_{BO} = \int d^3R d^3r \sum_{\kappa} \sum_{\lambda\lambda'} \Psi^{\dagger}_{\kappa\lambda}(t, r, R) \left\{ i\partial_t - V_{\kappa\lambda\lambda'}(r) + P^{i\dagger}_{\kappa\lambda} \frac{\nabla_r^2}{M} P^i_{\kappa\lambda'} \right\} \Psi_{\kappa\lambda'}(t, r, R) \ldots$$

The potential $V_{\kappa\lambda\lambda'}$ can be organized into an expansion in $1/m$ and spin-dependent and independent parts



$$V_{\kappa\lambda\lambda'}(r) = V^{(0)}_{\kappa\lambda}(r)\delta_{\lambda\lambda'} + \frac{V^{(1)}_{\kappa\lambda\lambda'}(r \ldots}{m}$$

$$V^{(1)}_{\kappa\lambda\lambda'}(r) = V^{(1)}_{\kappa\lambda\lambda'\,SD}(r) + V^{(1)}_{\kappa\lambda\lambda'\,SI}$$

$$V^{(2)}_{\kappa\lambda\lambda'}(r) = V^{(2)}_{\kappa\lambda\lambda'\,SD}(r) + V^{(2)}_{\kappa\lambda\lambda'\,SI}$$

$$V^{(1)}_{1\lambda\lambda'\,SD}(r) = V_{1\,SK}(r) \left(P^{i\dagger}_{1\lambda} K^{ij}_1 P^j_{1\lambda'} \right)$$

$$V^{(2)}_{1\lambda\lambda'\,SD}(r) = V_{1\,LSa}(r) \left(P^{i\dagger}_{1\lambda} L_{Q\bar{Q}} P^i_{1\lambda'} \ldots Q \right) P^j_{1\lambda'}$$

$$+ V_{1\,S^2}(r) S^2 \delta_{\lambda\lambda'} + V_{1\,S_{12}a}(r) S_{12} \delta_{\lambda\lambda'} + V_{1\,S_{12}b}(r) P^{i\dagger}_{1\lambda} P^j_{1\lambda'} \left(S^i_1 S^j_2 + S^i_2 S^j_1 \right)$$

The static potential can be matched to the lattice static energies.
The spectrum for $\kappa = 1^{+-}$ in this framework was obtained in Berwein, Brambilla, JTC, Vairo 2015

Hybrid EFT and Spin-dependent operators

$$L_{BO} = \int d^3R d^3r \sum_{\kappa} \sum_{\lambda\lambda'} \Psi^{\dagger}_{\kappa\lambda}(t, r, R) \left\{ i\partial_t - V_{\kappa\lambda\lambda'}(r) + P^{i\dagger}_{\kappa\lambda} \frac{\nabla_r^2}{M} P^{i}_{\kappa\lambda'} \right\} \Psi_{\kappa\lambda'}(t, r, R) \ldots$$

The potential $V_{\kappa\lambda\lambda'}$ can be organized into an expansion in $1/m$ and spin-dependent and independent parts

$$V_{\kappa\lambda\lambda'}(r) = V^{(0)}_{\kappa\lambda}(r)\delta_{\lambda\lambda'} + \frac{V^{(1)}_{\kappa\lambda\lambda'}(r)}{m} + \frac{V^{(2)}_{\kappa\lambda\lambda'}(r)}{m^2} + \ldots,$$

$$V^{(1)}_{\kappa\lambda\lambda'}(r) = V^{(1)}_{\kappa\lambda\lambda'\,SD}(r) + V^{(1)}_{\kappa\lambda\lambda'\,SI}(r),$$

$$V^{(2)}_{\kappa\lambda\lambda'}(r) = V^{(2)}_{\kappa\lambda\lambda'\,SD}(r) + V^{(2)}_{\kappa\lambda\lambda'\,SI}(r),$$

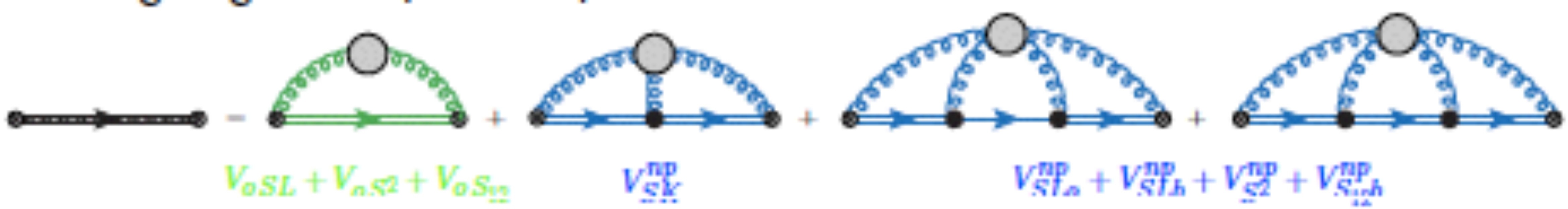
$$V^{(1)}_{1\lambda\lambda'\,SD}(r) = \boxed{V_{1\,SK}(r)\left(P^{i\dagger}_{1\lambda} K^{ij}_{1} P^{j}_{1\lambda'}\right) \cdot S}$$

$$V^{(2)}_{1\lambda\lambda'\,SD}(r) = V_{1\,LSa}(r)\left(P^{i\dagger}_{1\lambda} L_{Q\bar{Q}} P^{i}_{1\lambda'}\right) \cdot S + \boxed{V_{1\,LSb} P^{i\dagger}_{1\lambda}(r)\left(L^{i}_{Q\bar{Q}} S^{j} + S^{i} L^{j}_{Q\bar{Q}}\right) P^{j}_{1\lambda'}}$$
$$+ V_{1\,S^2}(r) S^2 \delta_{\lambda\lambda'} + V_{1\,S_{12}a}(r) S_{12} \delta_{\lambda\lambda'} + \boxed{V_{1\,S_{12}b}(r) P^{i\dagger}_{1\lambda} P^{j}_{1\lambda'} \left(S^{i}_{1} S^{j}_{2} + S^{i}_{2} S^{j}_{1}\right)}$$

New operators not present in standard Quarkonium.

Matching of the Spin-dependent operators for $\kappa = 1^{+-}$

Matching diagrams in position space:



$$V_{1\,SK} = V^{np}_{SK}\,,$$
$$V_{1\,SLa} = V^{np}_{SLa} + V_{o\,SL}\,,$$
$$V_{1\,SLb} = V^{np}_{SLb}\,,$$
$$V_{1\,S^2} = V^{np}_{S^2} + V_{o\,S^2}\,,$$
$$V_{1\,S_{12}a} = V_{o\,S_{12}}\,,$$
$$V_{1\,S_{12}b} = V^{np}_{S_{12}b}\,.$$

- The perturbative part is given by the octet quark-antiquark spin-dependent potential

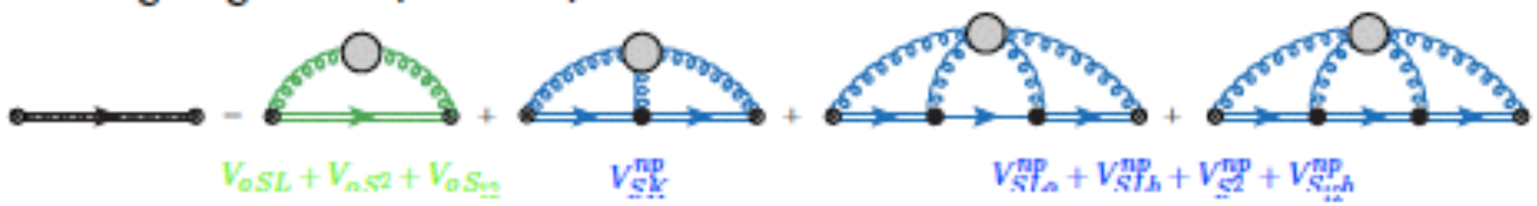
$$V_{o\,LS}(r) = \left(C_F - \frac{C_A}{2}\right)\left(\frac{c_s}{2} + c_F\right)\frac{\alpha_s(\nu)}{r^3}$$

$$V_{o\,S^2}(r) = \left[\frac{4\pi}{3}\left(C_F - \frac{C_A}{2}\right)c_F^2\alpha_s(\nu) + T_F\left(f_8({}^1S_0) - f_8({}^3S_1)\right)\right]\delta^3(r)$$

$$V_{o\,S_{12}}(r) = \left(C_F - \frac{C_A}{2}\right)\frac{\alpha_s(\nu)}{4r^3}$$

Matching of the Spin-dependent operators for $\kappa = 1^{+-}$

Matching diagrams in position space:



$$V_{1\,SK} = V^{np}_{SK}\,,$$
$$V_{1\,SLa} = V^{np}_{SLa} + V_{o\,SL}\,,$$
$$V_{1\,SLb} = V^{np}_{SLb}\,,$$
$$V_{1\,S^2} = V^{np}_{S^2} + V_{o\,S^2}\,,$$
$$V_{1\,S_{12}a} = V_{o\,S_{12}}\,,$$
$$V_{1\,S_{12}b} = V^{np}_{S_{12}b}\,.$$

- The nonperturbative part is given in terms of gluon correlators $\widetilde{U}$

$$V^{np}_{SK} = 2c_F\widetilde{U}^K_B$$
$$V^{np}_{SLa} = -\frac{3c_F}{8}\widetilde{U}^o_{Ba} + c_s\left(\widetilde{U}^s_{Ea} + \frac{N_c^2-4}{8N_c^2}\widetilde{U}^o_{Ea}\right)$$
$$V^{np}_{SLb} = -\frac{3c_F}{8}\widetilde{U}^o_{Bb} + c_s\left(\widetilde{U}^s_{Eb} + \frac{N_c^2-4}{8N_c^2}\widetilde{U}^o_{Eb}\right)$$
$$V^{np}_{S^2} = -c_F^2\left(\widetilde{U}^s_{Ba} + \frac{N_c^2-1}{2N_c^2}\widetilde{U}^o_{Ba}\right)$$
$$V^{np}_{S_{12}b} = -c_F^2\left(\widetilde{U}^s_{Bb} + \frac{N_c^2-1}{2N_c^2} \ldots\right.$$

Nonperturvative Gluon correlators

$$\widetilde{U}^K_B = \lim_{T\to\infty}\frac{ie^{i\Lambda T}}{T}\frac{1}{48T_F}\int_{-T/2}^{T/2}dt\left[\langle 0|\boldsymbol{G}^\dagger(T/2)\cdot(g\boldsymbol{B}_{\rm adj}(t)\times\boldsymbol{G}(-T/2))|0\rangle\right],$$

$$\widetilde{U}^s_{Ba} + 4\widetilde{U}^s_{Bb} = \lim_{T\to\infty}\frac{e^{i\Lambda T}}{iT}\frac{N_c}{3T_F}\int_{-T/2}^{T/2}dt\int_{-T/2}^{t}dt'\langle 0|(\boldsymbol{G}^{a\dagger}(T/2)\cdot g\boldsymbol{B}^a(t))(g\boldsymbol{B}^a(t')\cdot\boldsymbol{G}^a(-T/2))|0\rangle,$$

$$3\widetilde{U}^s_{Ba} + 2\widetilde{U}^s_{Bb} = \lim_{T\to\infty}\frac{e^{i\Lambda T}}{iT}\frac{N_c}{3T_F}\int_{-T/2}^{T/2}dt\int_{-T/2}^{t}dt'\langle 0|\boldsymbol{G}^{a\dagger}(T/2)\cdot\Big((g\boldsymbol{B}^a(t)\cdot g\boldsymbol{B}^a(t'))\boldsymbol{G}^a(-T/2)\Big)|0\rangle,$$

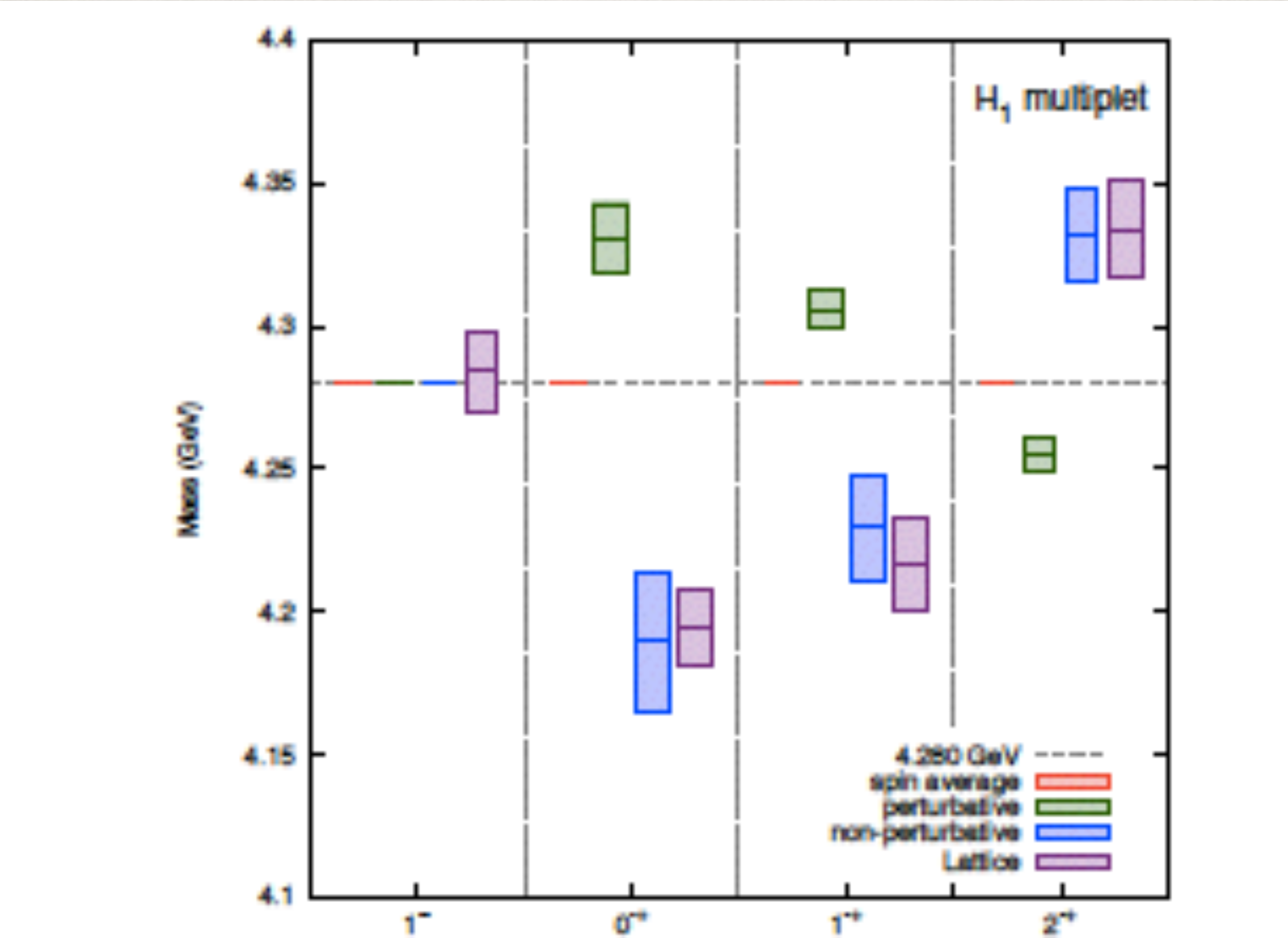
$$\widetilde{U}^o_{Ba} + 4\widetilde{U}^o_{Bb} = \lim_{T\to\infty}\frac{ie^{i\Lambda T}}{T}\frac{1}{18T_F^2}\int_{-T/2}^{T/2}dt\int_{-T/2}^{t}dt'\langle 0|(\boldsymbol{G}^\dagger(T/2)\cdot g\boldsymbol{B}_{\rm adj}(t))(g\boldsymbol{B}_{\rm adj}(t')\cdot\boldsymbol{G}^a(-T/2))|0\rangle,$$

$$3\widetilde{U}^o_{Ba} + 2\widetilde{U}^o_{Bb} = \lim_{T\to\infty}\frac{ie^{i\Lambda T}}{T}\frac{1}{18T_F^2}\int_{-T/2}^{T/2}dt\int_{-T/2}^{t}dt'\langle 0|\boldsymbol{G}^\dagger(T/2)\cdot\Big((g\boldsymbol{B}_{\rm adj}(t)\cdot g\boldsymbol{B}_{\rm adj}(t'))\boldsymbol{G}(-T/2)\Big)|0\rangle,$$

- $\widetilde{U}^o_{Ea}$ and $\widetilde{U}^o_{Eb}$ are defined by replacing $\boldsymbol{B}$ for $\boldsymbol{E}$.
- The gluon correlators $\widetilde{U}$ are independent of r and the heavy quark flavor.

Charmonium hybrids

- The contributions of the spin-dependent operators are computed in standard QM perturbation theory.
- The value of the gluon correlators is fitted to reproduce the lattice spectrum of Liu et al 2012.



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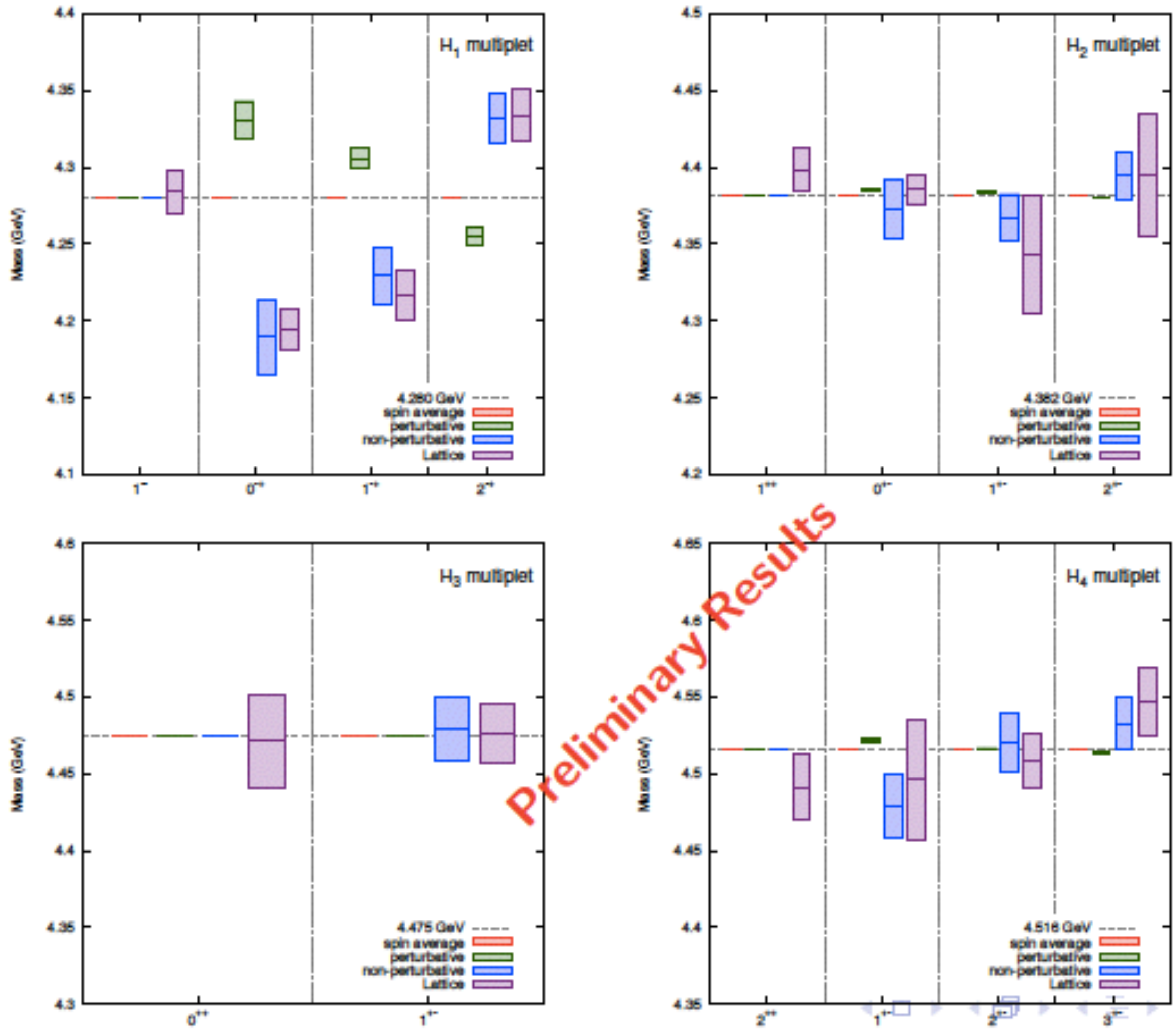
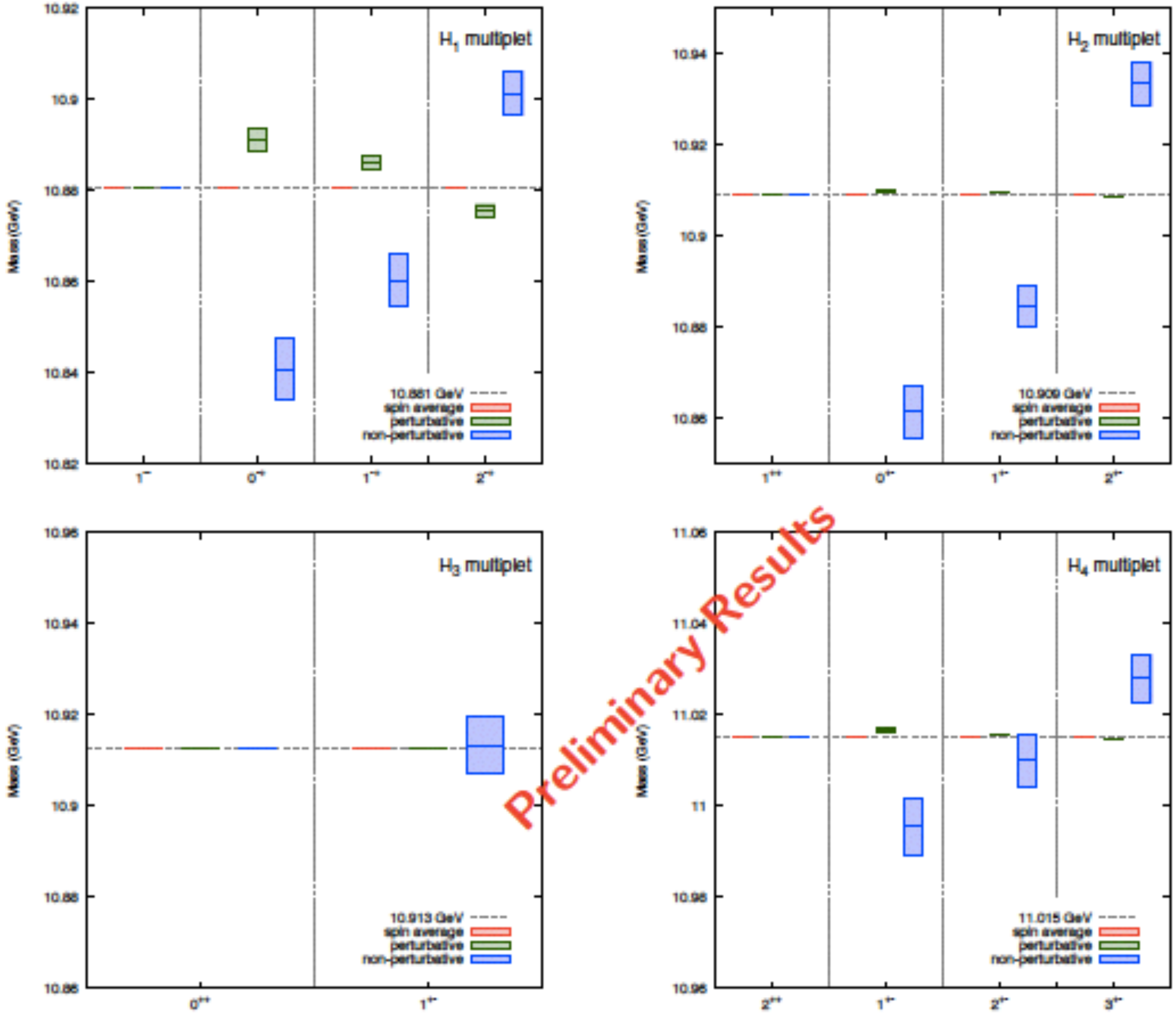


Bottomonium hybrids

- Extrapolation of the spin-splittings in the bottomonium sector.



we can consider more general eigenstates of the octet sector the pNRQCD hamiltonian

The Born-Oppenheimer approximation in effective field theory language

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$$\kappa = \{J^{PC}, f\},$$

light flavour

$$|\kappa\rangle = O^{a\dagger}(\boldsymbol{r}, \boldsymbol{R})\, G^a_{i\kappa}(\boldsymbol{R})|\mathrm{US}\rangle,$$

project on $\int d^3r d^3R \sum_{i\kappa} |\kappa\rangle\, \Psi_{i\kappa}(t, \boldsymbol{r}, \boldsymbol{R}).$

obtain

$$L_{BO} = \int d^3R d^3r \sum_{\kappa} \Psi^{\dagger}_{i\kappa}(t, \boldsymbol{r}, \boldsymbol{R})\left[(i\partial_t - h_o - \Lambda_\kappa)\,\delta^{ij} - \sum_{\lambda} P^i_{\kappa\lambda} b_{\kappa\lambda} r^2 P^j_{\kappa\lambda} + \cdots\right]\Psi_{j\kappa}(t, \boldsymbol{r}, \boldsymbol{R}),$$

gives origin to a coupled Schroedinger equation

$$i\partial_t \Psi_{\kappa\lambda}(t, \boldsymbol{r}, \boldsymbol{R}) = \left[\left(-\frac{\nabla^2_r}{M} + V_o(r) + \Lambda_\kappa + b_{\kappa\lambda} r^2\right)\delta_{\lambda\lambda'} - \sum_{\lambda'} C_{\kappa\lambda\lambda'}\right]\Psi_{\kappa\lambda'}(t, \boldsymbol{r}, \boldsymbol{R}).$$

that can describe “**tetraquarks**” —> needs lattice calculations of tetraquarks static energies

coefficients C in calculation for any J M. Berwein, N. Brambilla, Wk Lai, A. Vairo

Conclusions

Quarkonium is a golden system to study strong interactions

For states below threshold non relativistic EFTs provide a systematic tool to investigate a wide range of observables in the realm of QCD and quarkonium becomes a

NREFT Allow us to make calculations with unprecented precision, where high order perturbative calculations are possible
and to systematically factorize short from long range contributions where observables are sentitive to the nonperturbative dynamics of QCD

For states close or above the strong decay threshold the situation is much more complicated.

Many degrees of freedom show up and the absence of a clear systematic is an obstacle to a universal picture

We have presented results obtained for the hybrid masses in pNRQCD that show a very rich structure of multiplets.

Conclusions

- study of hybrids in EFT framework (NRQCD and pNRQCD)
- study of the spectrum of the static Hamiltonian with non-static corrections
- short distance degeneracy of static energies
- mixing of different static states
- breaking of degeneracy between opposite parity states

We have included spin in the hybrids multiplet structure:
—could interpret the lattice result
—make independent predictions for the bottomonium sector

Same approach can be used to include light quarks: "tetraquarks"

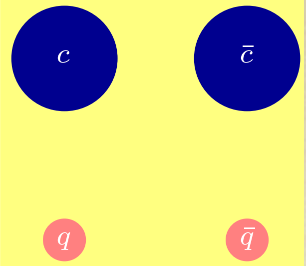
This approach holds the promise to be able to explain all exotics (including pentaquark) from QCD in the same framework

Input from the lattice is needed: more precise calculations of the gluelump masses, static energies for the hybrids and the tetra quarks, correlators of gluons fields..

Exotics may be generated also by QCD van der Waals forces: for example eta_b-eta_b bound states?

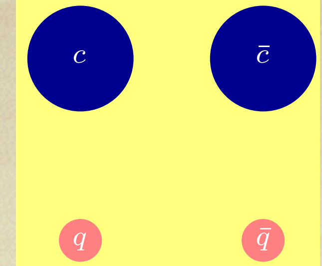
backup

$X(3872)$: interpretations

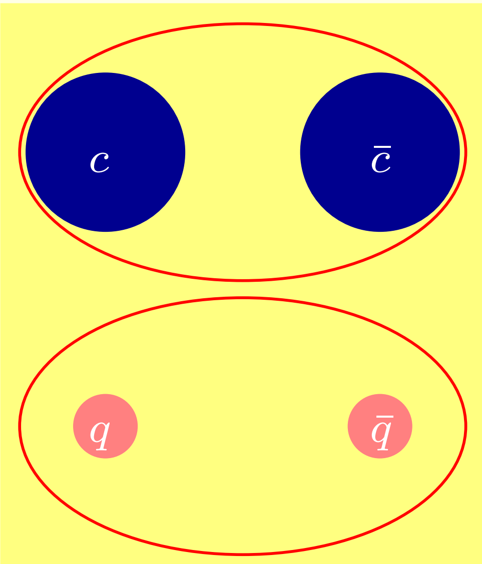


4-quark state with $J^{PC} = 1^{++}$

$X(3872)$: interpretations



4-quark state with $J^{PC} = 1^{++}$

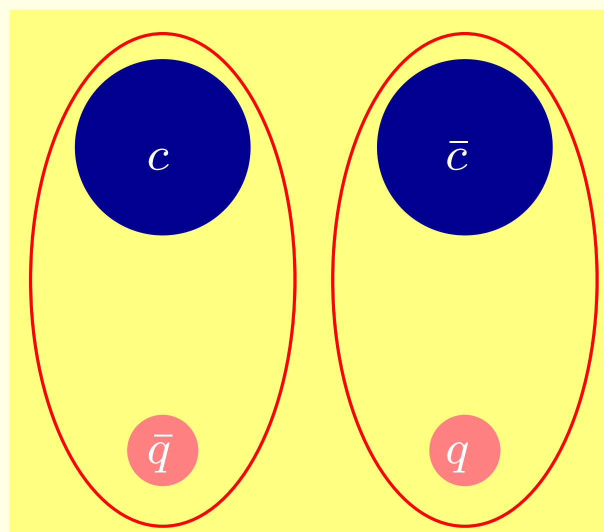


Høgassen et al 05

$$X \sim (c\bar{c})^8_{S=1} \otimes (q\bar{q})^8_{S=1}$$
$$\sim (c\bar{q})^1_{S=0} \otimes (q\bar{c})^1_{S=1} + (c\bar{q})^1_{S=1} \otimes (q\bar{c})^1_{S=0}$$

Predictions based on the phenomenological $H = -\sum_{ij} C_{ij}\, T^a \otimes T^a\, \boldsymbol{\sigma} \otimes \boldsymbol{\sigma}$;

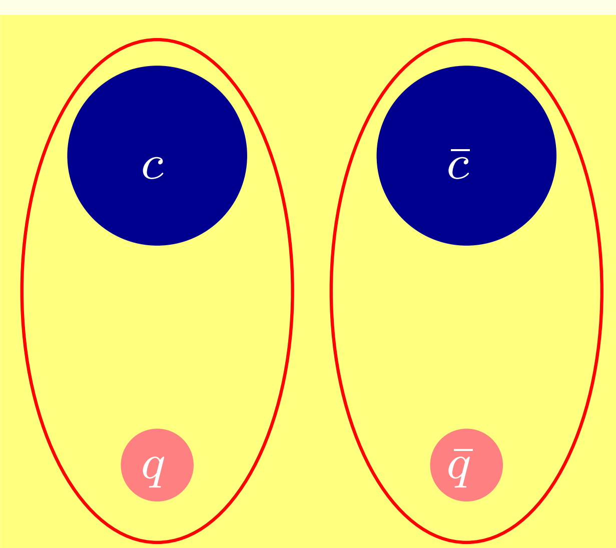
Molecular model



Törnqvist 93, Swanson 04

$$X \sim (c\bar{q})^1_{S=0} \otimes (q\bar{c})^1_{S=1} + (c\bar{q})^1_{S=1} \otimes (q\bar{c})^1_{S=0}$$
$$\sim D\,\bar{D}^* + D^*\,\bar{D}$$

This is assumed to be the dominant long-range Fock component; short-range components of the type $(c\bar{c})^1_{S=1} \otimes (q\bar{q})^1_{S=1}$ $\sim J/\psi\ \rho, \omega$ are assumed as well.



Maiani et al 04

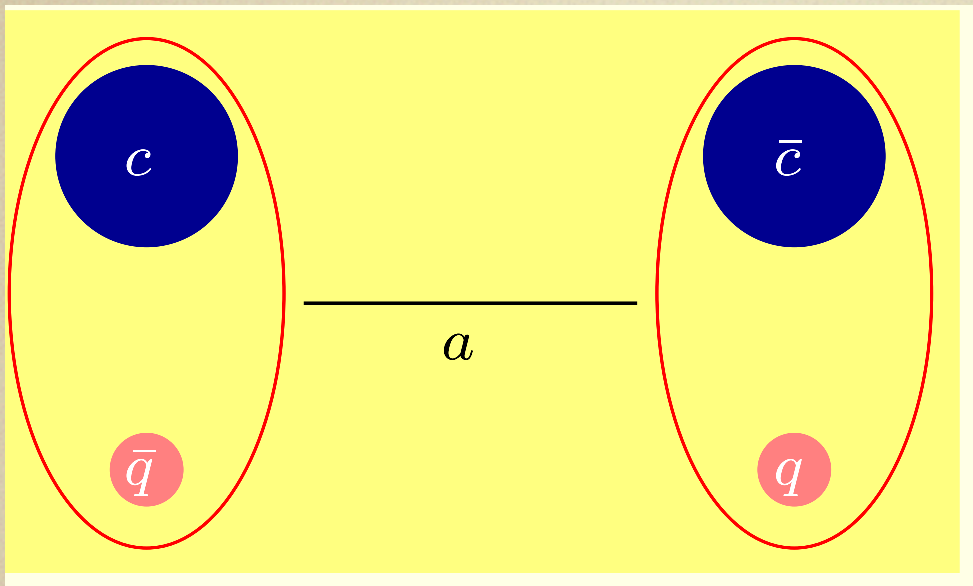
$$X \sim (cq)^{\bar{3}}_{S=1} \otimes (\bar{c}\bar{q})^3_{S=0} + (cq)^{\bar{3}}_{S=0} \otimes (\bar{c}\bar{q})^3_{S=1}$$

the dynamical assumption (there is no scale separation like in the doubly heavy baryons) is that quark pair cluster in tightly bound color triplet diquarks (see 1-gluon exchange); the difficulty in breaking the system explains the narrow width.

Tetraquark model

Predictions based on the phenomenological Hamiltonian: $H = \sum_{ij} \kappa_{ij}\, \boldsymbol{\sigma} \otimes \boldsymbol{\sigma}$; the

In some cases it is possible to develop an EFT owing to special dynamical condition



this happens if the state is sufficiently close to a threshold and if it has S-wave coupling to the threshold—> loosely bound molecule with universal properties

- An example is the $X(3872)$ intepreted as a $D^0\,\bar{D}^{*\,0}$ or $\bar{D}^0\,D^{*\,0}$ molecule. In this case, one may take advantage of the hierarchy of scales:

$$\Lambda_{\rm QCD} \gg m_\pi \gg m_\pi^2/M_{D^0} \approx 10\ \text{MeV} \gg E_{\rm binding}$$
$$\approx M_X - (M_{D^{*0}} + M_{D^0}) = (0.1 \pm 1.0)\ \text{MeV}$$

Systems with a short-range interaction and a large scattering length have universal properties that may be exploited: in particular, production and decay amplitudes factorize in a short-range and a long-range part, where the latter depends only on one single parameter, the scattering length. An universal property that fits well with the observed large branching fraction of the $X(3872)$ *decaying into* $D^0\bar{D}^0\pi^0$ *is* $\mathcal{B}(X \to D^0\bar{D}^0\pi^0) \approx \mathcal{B}(D^{*\,0} \to D^0\pi^0) \approx 60\%$.

Pakvasa Suzuki 03, Voloshin 03, Braaten Kusunoki 03 Braaten Hammer 06

$$V_{\kappa\lambda\lambda'}(r) = V^{(0)}_{\kappa\lambda}(r)\delta_{\lambda\lambda'} + \frac{V^{(1)}_{\kappa\lambda\lambda'}(r)}{m} + \frac{V^{(2)}_{\kappa\lambda\lambda'}(r)}{m^2} + \dots, \quad (3)$$

$$V^{(1)}_{\kappa\lambda\lambda'}(r) = V^{(1)}_{\kappa\lambda\lambda'\,SD}(r) + V^{(1)}_{\kappa\lambda\lambda'\,SI}(r), \quad (4)$$

$$V^{(2)}_{\kappa\lambda\lambda'}(r) = V^{(2)}_{\kappa\lambda\lambda'\,SD}(r) + V^{(2)}_{\kappa\lambda\lambda'\,SI}(r), \quad (5)$$

$$V^{(1)}_{\kappa\lambda\lambda'\,SD}(r) = V_{\kappa\,SK}(r)\left(P^{i\dagger}_{\kappa\lambda}\boldsymbol{K}^{ij}_{\kappa}P^{j}_{\kappa\lambda'}\right)\cdot\boldsymbol{S}, \quad (6)$$

$$\begin{aligned} V^{(2)}_{\kappa\lambda\lambda'\,SD}(r) = {} & V_{\kappa\,LSa}(r)\left(P^{i\dagger}_{\kappa\lambda}\boldsymbol{L}_{Q\bar{Q}}P^{i}_{\kappa\lambda'}\right)\cdot\boldsymbol{S} \\ & + V_{\kappa\,LSb}P^{i\dagger}_{\kappa\lambda}\left(\boldsymbol{L}^{i}_{Q\bar{Q}}S^{j} + S^{i}\boldsymbol{L}^{j}_{Q\bar{Q}}\right)P^{j}_{\kappa\lambda'} \\ & + V_{\kappa\,S^2}(r)\boldsymbol{S}^2\delta_{\lambda\lambda'} + V_{\kappa\,S_{12}a}(r)\boldsymbol{S}_{12}\delta_{\lambda\lambda'} \\ & + V_{\kappa\,S_{12}b}(r)P^{i\dagger}_{\kappa\lambda}P^{j}_{\kappa\lambda'}\left(\boldsymbol{S}^{i}_{1}\boldsymbol{S}^{j}_{2} + \boldsymbol{S}^{i}_{2}\boldsymbol{S}^{j}_{1}\right), \end{aligned} \quad (7)$$

with $\boldsymbol{L}_{Q\bar{Q}}$ the heavy quark angular momentum, $\boldsymbol{S} = \boldsymbol{S}_1 + \boldsymbol{S}_2$, $\boldsymbol{S}_{12} = 3(\boldsymbol{\sigma}_1\cdot\hat{r}_0)(\boldsymbol{\sigma}_2\cdot\hat{r}_0) - \boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2$ and $\boldsymbol{K}_\kappa$ the angular momentum operator in the representation determined by κ.

The hybrid spectrum generated by the lowest mass gluelump, with $\kappa = 1^{+-}$, was obtained in Ref. [35]

For $K = 1$ the projectors $P^{i}_{1\lambda}$ read as

$$P^{i}_{10} = \hat{r}^{i}, \quad (8)$$

$$P^{i}_{1\pm1} = \mp\left(\hat{\theta}^{i} \pm i\hat{\phi}^{i}\right)/\sqrt{2}, \quad (9)$$

with

$$\begin{aligned} \hat{r} &= (\sin(\theta)\cos(\phi), \sin(\theta)\sin(\phi), \cos(\theta))^T, \\ \hat{\theta} &= (\cos(\theta)\cos(\phi), \cos(\theta)\sin(\phi), -\sin(\theta))^T, \\ \hat{\phi} &= (-\sin(\phi), \cos(\phi), 0)^T, \end{aligned} \quad (10)$$

and the spin operator is $\left(\boldsymbol{K}^{ij}\right)^k = i\epsilon^{ijk}$.

For $\kappa = 1^{+-}$ the potentials read:

$$V_{SK} = V^{np}_{SK}, \quad (11)$$

$$V_{SLa} = V^{np}_{SLa} + V_{o\,SL}(r), \quad (12)$$

$$V_{SLb} = V^{np}_{SLb}, \quad (13)$$

$$V_{S^2} = V^{np}_{S^2} + V_{o\,S^2}(r), \quad (14)$$

$$V_{S_{12}a} = V_{o\,S_{12}}(r), \quad (15)$$

$$V_{S_{12}b} = V^{np}_{S_{12}b}. \quad (16)$$