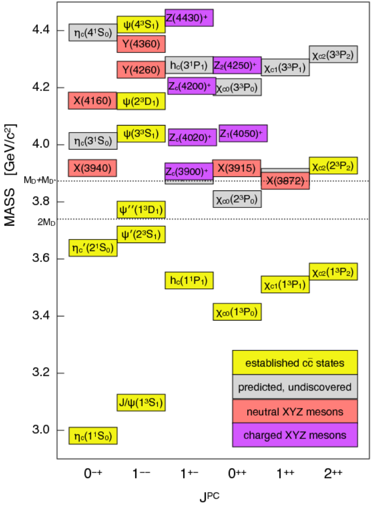
Exotics on the Lattice

Gavin Cheung
Hadron Spectrum Collaboration

DAMTP, University of Cambridge

12 December 2017

Introduction



Essential to understand this within QCD. Can we reproduce this spectrum theoretically?

S. Olsen, arxiv:1511.01589

Lattice QCD

- Lattice QCD is the only systematically improvable method for performing first principle calculations involving hadrons.

Lattice QCD

- Lattice QCD is the only systematically improvable method for performing first principle calculations involving hadrons.
- Discretisation of spacetime allows for numerical computation of two-point correlation functions $\langle \mathcal{O}(t)\mathcal{O}^\dagger(0)\rangle$ for lots of operators.

Lattice QCD

- Lattice QCD is the only systematically improvable method for performing first principle calculations involving hadrons.
- Discretisation of spacetime allows for numerical computation of two-point correlation functions $\langle \mathcal{O}(t)\mathcal{O}^\dagger(0)\rangle$ for lots of operators.

$$\begin{aligned}\langle \mathcal{O}(t)\mathcal{O}^\dagger(0)\rangle &= \langle 0|\mathcal{O}(t)\mathcal{O}^\dagger(0)|0\rangle \\ &= \sum_n |\langle 0|\mathcal{O}|n\rangle|^2 e^{-M_n t}.\end{aligned}$$

Lattice QCD

- Lattice QCD is the only systematically improvable method for performing first principle calculations involving hadrons.
- Discretisation of spacetime allows for numerical computation of two-point correlation functions $\langle \mathcal{O}(t)\mathcal{O}^\dagger(0)\rangle$ for lots of operators.

$$\begin{aligned}\langle \mathcal{O}(t)\mathcal{O}^\dagger(0)\rangle &= \langle 0|\mathcal{O}(t)\mathcal{O}^\dagger(0)|0\rangle \\ &= \sum_n |\langle 0|\mathcal{O}|n\rangle|^2 e^{-M_n t}.\end{aligned}$$

- The spectrum is contained in the two-point correlation function and can be extracted. What about $\mathcal{O}$?

Meson Operators

- We want to build good operators with the correct quantum numbers of the states we're interested in. Starting with fermion bilinears,

$$\mathcal{O}(t) \sim \bar{c}\Gamma c.$$

Meson Operators

- We want to build good operators with the correct quantum numbers of the states we're interested in. Starting with fermion bilinears,
$$\mathcal{O}(t) \sim \bar{c}\Gamma c.$$

- Γ can only probe up to $J = 1$. But we can do better! Add in some gauge-covariant derivatives to access any J^{PC}.
$$\mathcal{O}(t) \sim \bar{c}\Gamma \overleftrightarrow{D} \ldots \overleftrightarrow{D} c.$$

Meson Operators

- We want to build good operators with the correct quantum numbers of the states we're interested in. Starting with fermion bilinears,

$$\mathcal{O}(t) \sim \bar{c}\Gamma c.$$

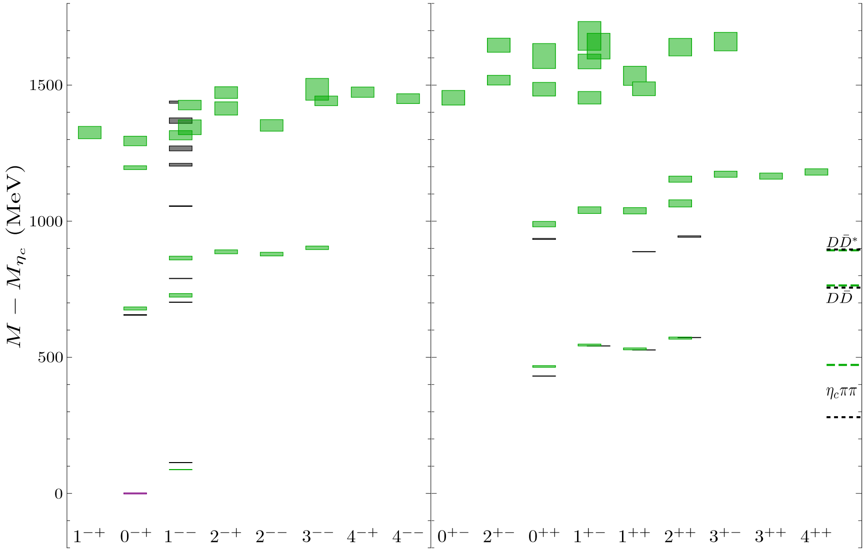
- Γ can only probe up to $J = 1$. But we can do better! Add in some gauge-covariant derivatives to access any J^{PC}.

$$\mathcal{O}(t) \sim \bar{c}\Gamma \overleftrightarrow{D} \dots \overleftrightarrow{D} c.$$

- This construction also gives 'gluey' operators $\mathcal{O}(t) \propto F_{\mu\nu}$ that resemble a hybrid meson structure.

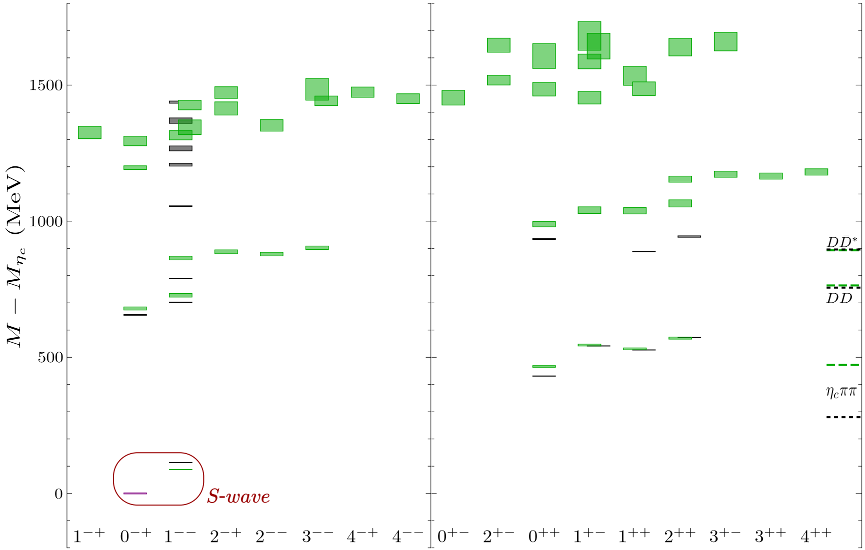
Results

$c\bar{c}$ Spectrum at $m_\pi \sim 240$ MeV



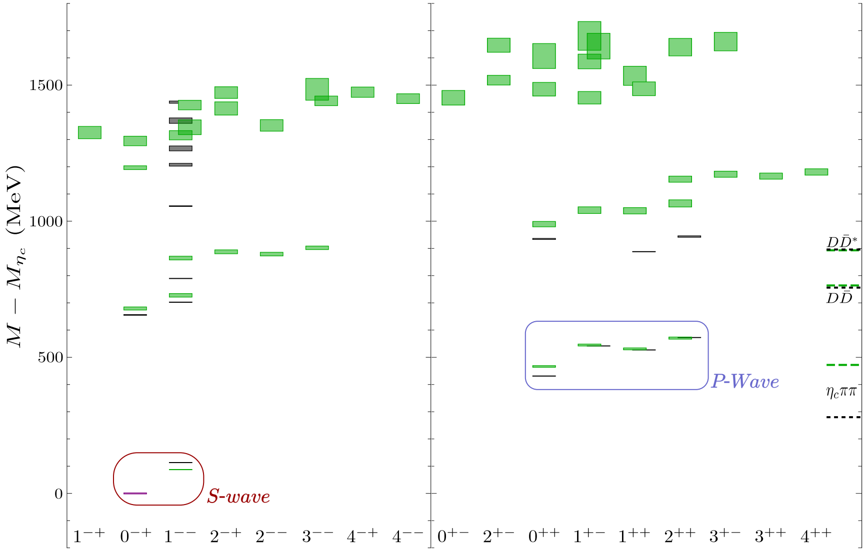
GC et al., arXiv:1610.01073

$c\bar{c}$ Spectrum at $m_\pi \sim 240$ MeV



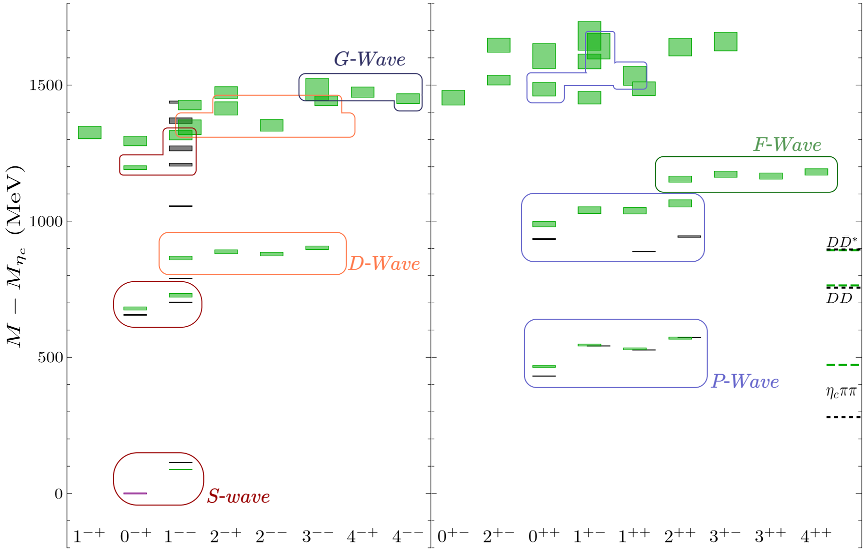
GC et al., arXiv:1610.01073

$c\bar{c}$ Spectrum at $m_\pi \sim 240$ MeV



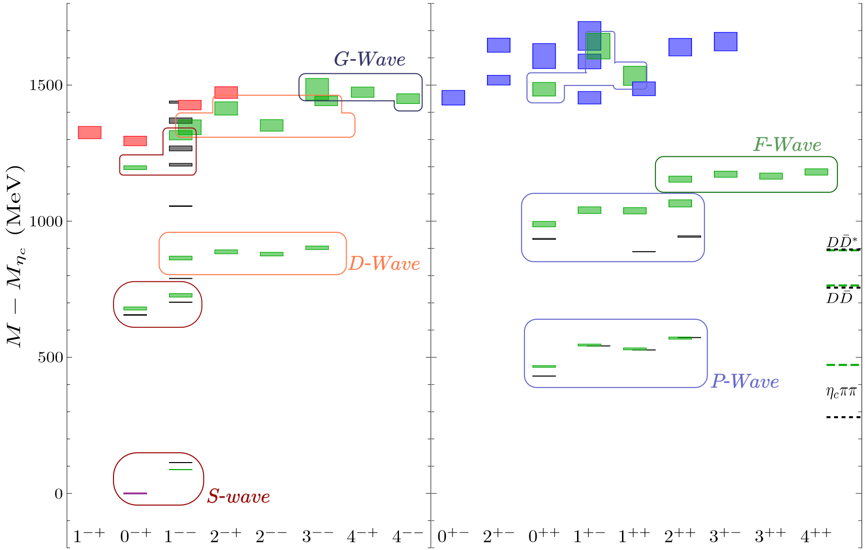
GC et al., arXiv:1610.01073

$c\bar{c}$ Spectrum at $m_\pi \sim 240$ MeV



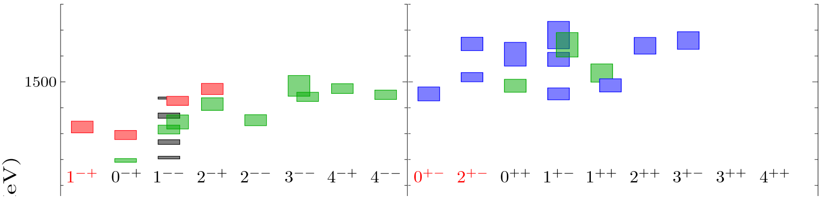
GC et al., arXiv:1610.01073

$c\bar{c}$ Spectrum at $m_\pi \sim 240$ MeV



GC et al., arXiv:1610.01073

Hybrid Mesons

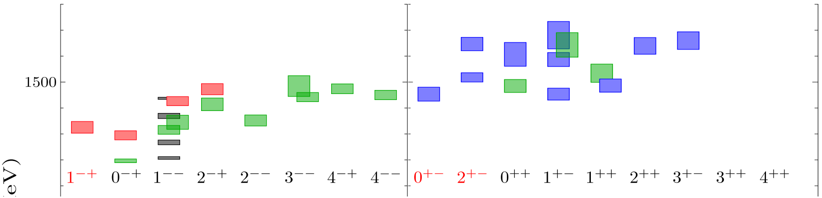


Pattern consistent with adding an effective gluonic degree of freedom $J^{PC} = 1^{+-}$ to quark model.

$q\bar{q}\ L = 0$

$$\{0^{-+}; 1^{--}\} \to \{1^{--}; 0^{-+}, 1^{-+}, 2^{-+}\}$$

Hybrid Mesons



Pattern consistent with adding an effective gluonic degree of freedom $J^{PC} = 1^{+-}$ to quark model.

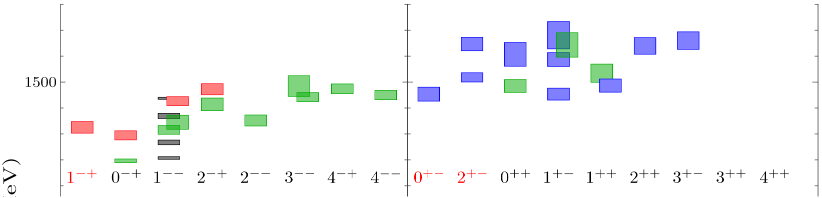
$q\bar{q}$ $L = 0$

$$\{0^{-+}; 1^{--}\} \to \{1^{--}; 0^{-+}, 1^{-+}, 2^{-+}\}$$

$q\bar{q}$ $L = 1$

$$\{1^{+-}; 0^{++}, 1^{++}, 2^{++}\} \to \{0^{++}, 1^{++}, 2^{++}; 0^{+-}, 1^{+-}(3), 2^{+-}(2), 3^{+-}\}$$

Hybrid Mesons



Pattern consistent with adding an effective gluonic degree of freedom $J^{PC} = 1^{+-}$ to quark model.

$q\bar{q}$ $L = 0$

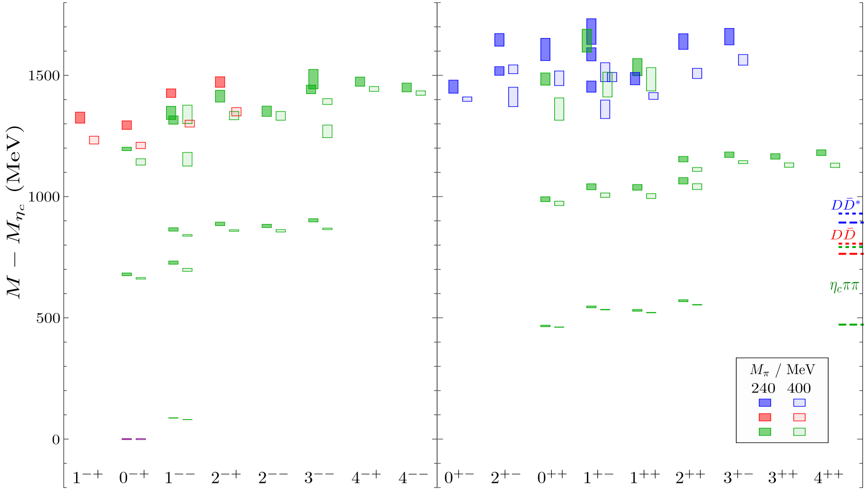
$$\{0^{-+}; 1^{--}\} \rightarrow \{1^{--}; 0^{-+}, 1^{-+}, 2^{-+}\}$$

$q\bar{q}$ $L = 1$

$$\{1^{+-}; 0^{++}, 1^{++}, 2^{++}\} \rightarrow \{0^{++}, 1^{++}, 2^{++}; 0^{+-}, 1^{+-}(3), 2^{+-}(2), 3^{+-}\}$$

These states are reliably seen when we include gluey operators in the calculation.

$m_\pi \sim 240$ MeV vs $m_\pi \sim 400$ MeV

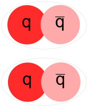


$m_\pi \sim 400$ MeV from Liu et al., arXiv:1204.5425

Four-quark Operators

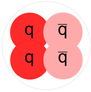
- No multi-meson states seen in the previous spectrum. Need different operators?
- Meson-meson operators (M)

$$\mathcal{O}(t) \sim (\bar{c}\Gamma q')(\vec{p}) \times (\bar{q}\Gamma' c)(-\vec{p}).$$

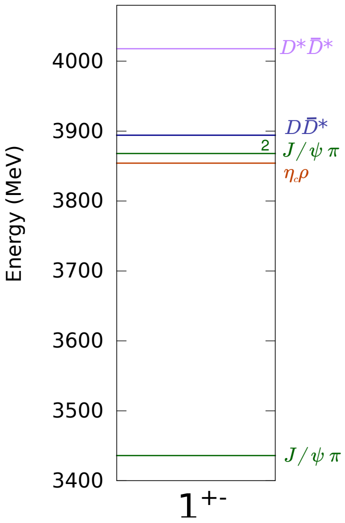


- Tetraquark operators (T)

$$\mathcal{O}(t) \sim G_{ad} \underbrace{\left(g_{abc} c_b (C\Gamma_1) q_c^T\right)}_{\text{Diquark}} \underbrace{\left(g_{def} \bar{c}_e^T (\Gamma_2 C) \bar{q}_f\right)}_{\text{Anti-diquark}}.$$

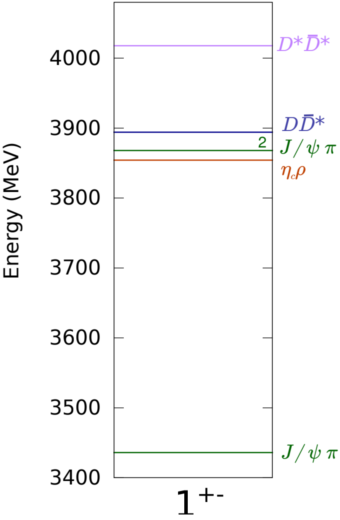


Finite-volume Spectrum ($c\bar{c}q\bar{q}$)



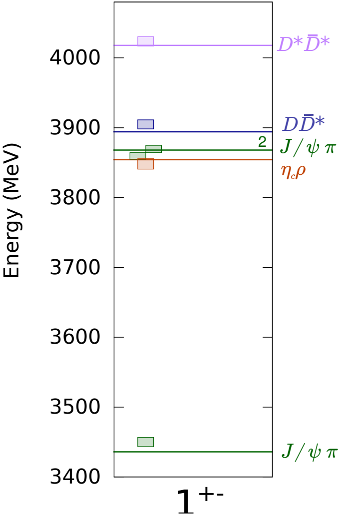
- The energy levels are discrete in a finite volume. Momentum is quantised.
- We can plot the non-interacting meson-meson levels.
- Interactions cause deviations from the non-interacting levels. Forms the basis of the Lüscher formalism to determine scattering amplitudes from a Euclidean field theory.

Finite-volume Spectrum ($c\bar{c}q\bar{q}$)



- In the non-interacting limit, we know how many meson-meson levels are in this channel. Will we see an 'extra' energy level of tetraquark origin?
- Will there be large shifts from the non-interacting levels suggesting a strong interaction? Hints of bound states or narrow resonances?

Isospin-1 $c\bar{c}q\bar{q}$ Spectrum at $m_\pi \sim 400$ MeV

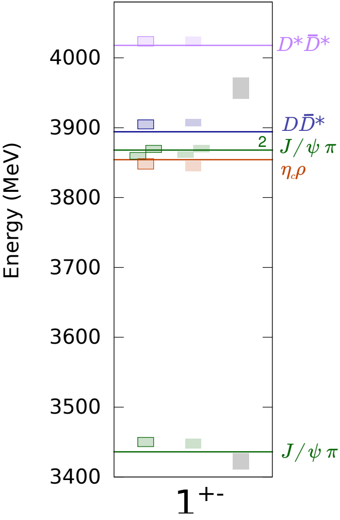


GC, C.E.Thomas, J.J.Dudek, R.G.Edwards,

arXiv:1709.01417

- The number of energy levels we find is equal to the number of expected non-interacting meson-meson levels.
- Finite-volume spectrum lies close to non-interacting meson-meson levels suggesting there are weak meson-meson interactions.
- There is no strong indication for a bound state or narrow resonance in this channel. $Z_C(3900)$?
- Tetraquark operators do not significantly affect the extraction of the spectrum.

Isospin-1 $c\bar{c}q\bar{q}$ Spectrum at $m_\pi \sim 400$ MeV

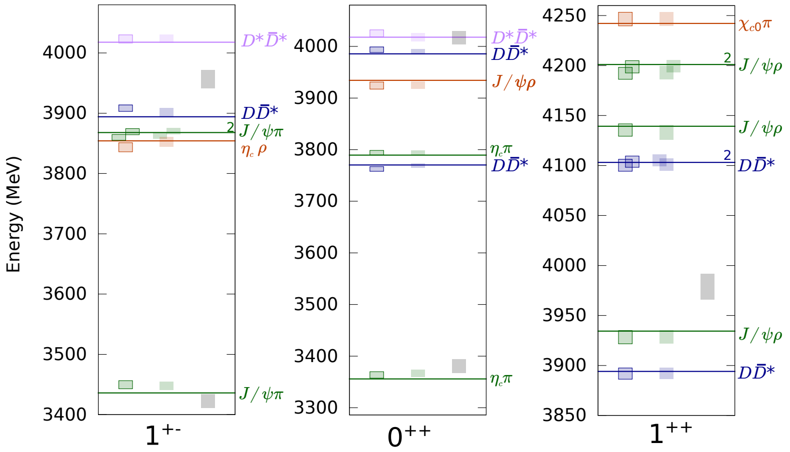


GC, C.E.Thomas, J.J.Dudek, R.G.Edwards,

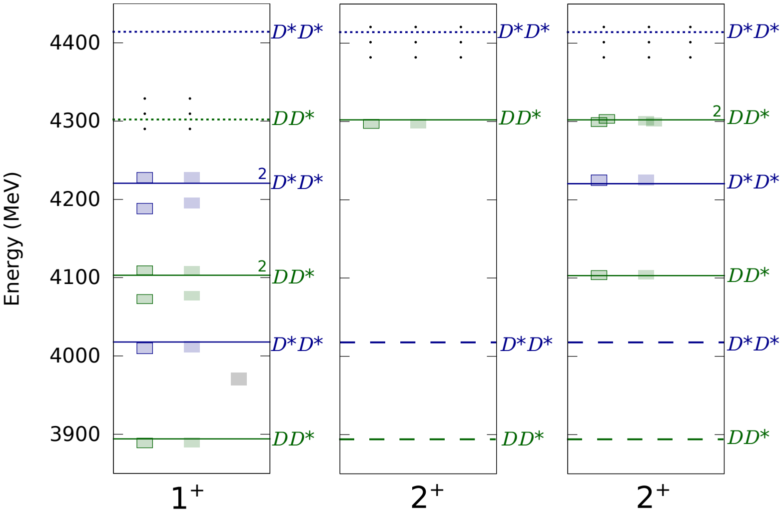
arXiv:1709.01417

- The number of energy levels we find is equal to the number of expected non-interacting meson-meson levels.
- Finite-volume spectrum lies close to non-interacting meson-meson levels suggesting there are weak meson-meson interactions.
- There is no strong indication for a bound state or narrow resonance in this channel. $Z_C(3900)$?
- Tetraquark operators do not significantly affect the extraction of the spectrum.

Isospin-1 hidden charm spectrum ($c\bar{c}q\bar{q}$) for $m_\pi \sim 400$ MeV



Doubly-charmed spectrum ($cc\bar{q}\bar{q}$) for $m_\pi \sim 400$ MeV



Conclusions and outlook

- Lattice QCD is the only ab-initio way to study exotic mesons.
- In lattice QCD, we find states with exotic J^{PC} quantum numbers and identify states that are consistent with a quark-antiquark combination coupled to a 1^{+-} gluonic excitation.
- Turning to four-quark states, we do not find significant changes to spectrum when including a class of operators resembling tetraquarks in our calculations. The extracted spectrum does not show any clear signs of bound states or narrow resonances.
- Next steps: calculations towards physical point, study other interesting channels, relate the discrete finite volume spectrum to scattering amplitudes using the Lüscher formalism.