Three and four quark systems in the DSE/BSE framework

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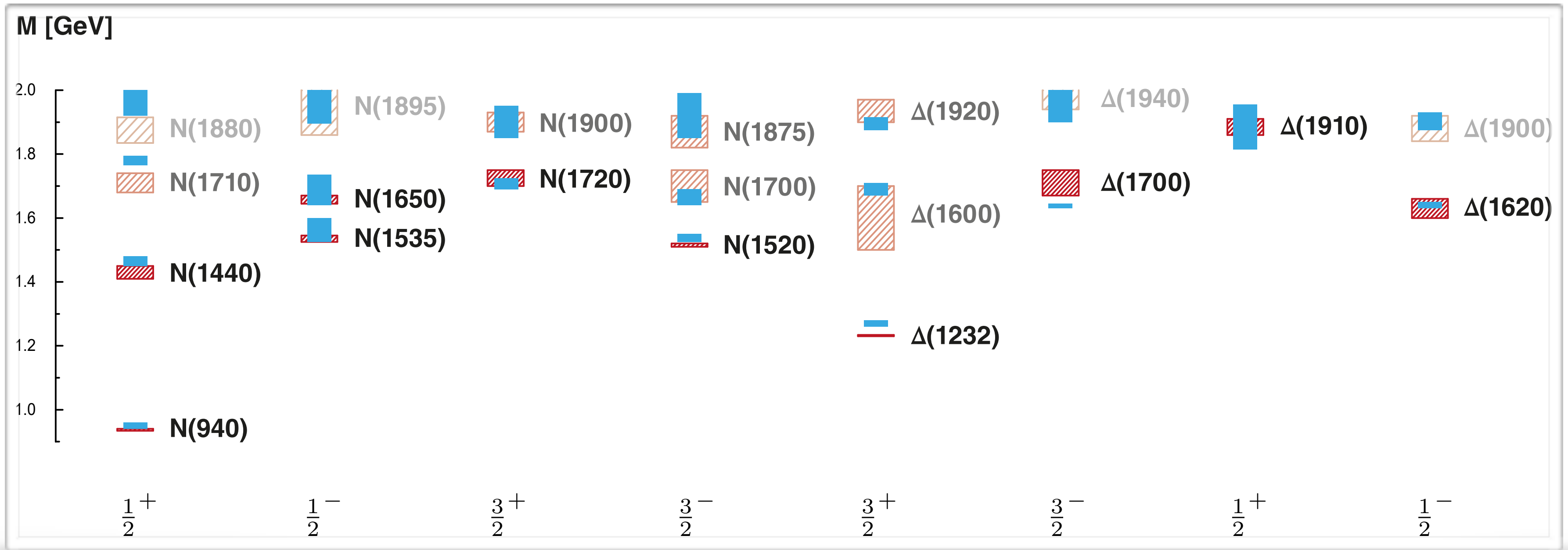
Exotic Hadron Spectroscopy 2017

Review: Eichmann, Sanchis-Alepuz, Williams, Alkofer, CF, PPNP 91, 1-100 [1606.09602]

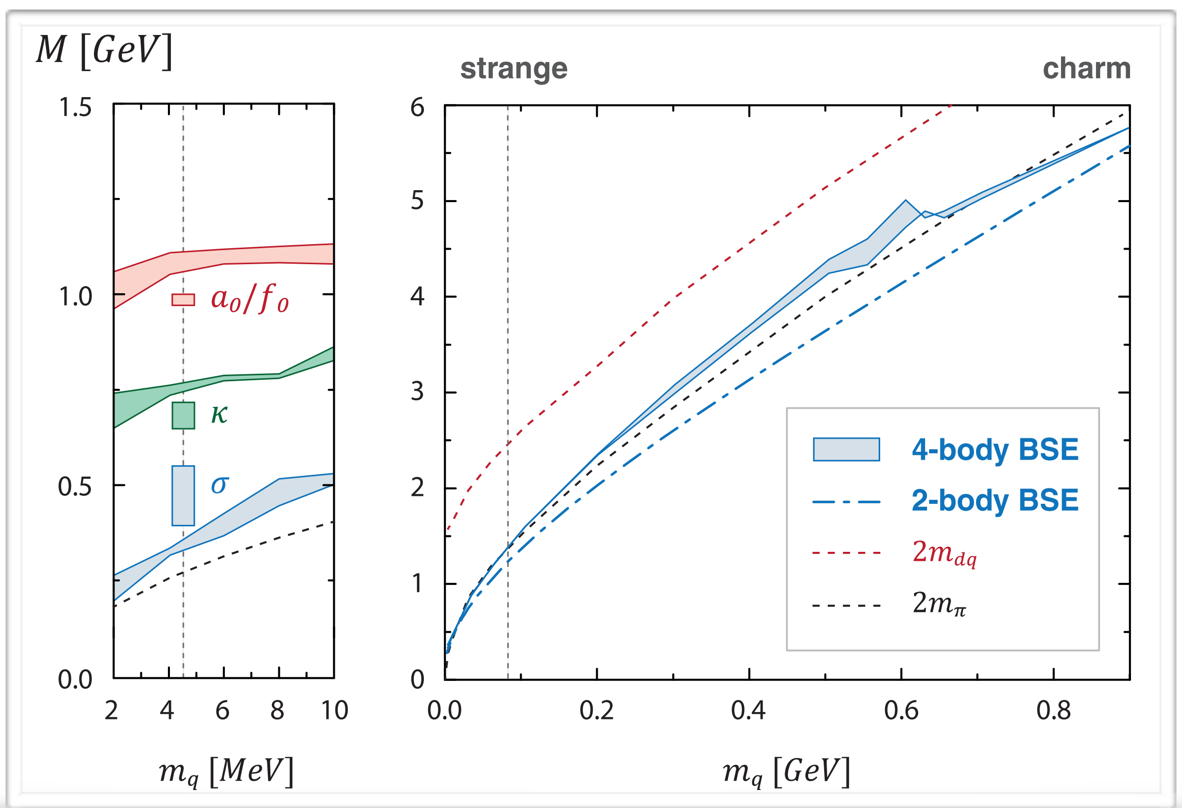
Overview - Take home messages

- Light and strange baryon spectrum:

Eichmann, CF, Sanchis-Alepuz, PRD 94 (2016) [1607.05748]

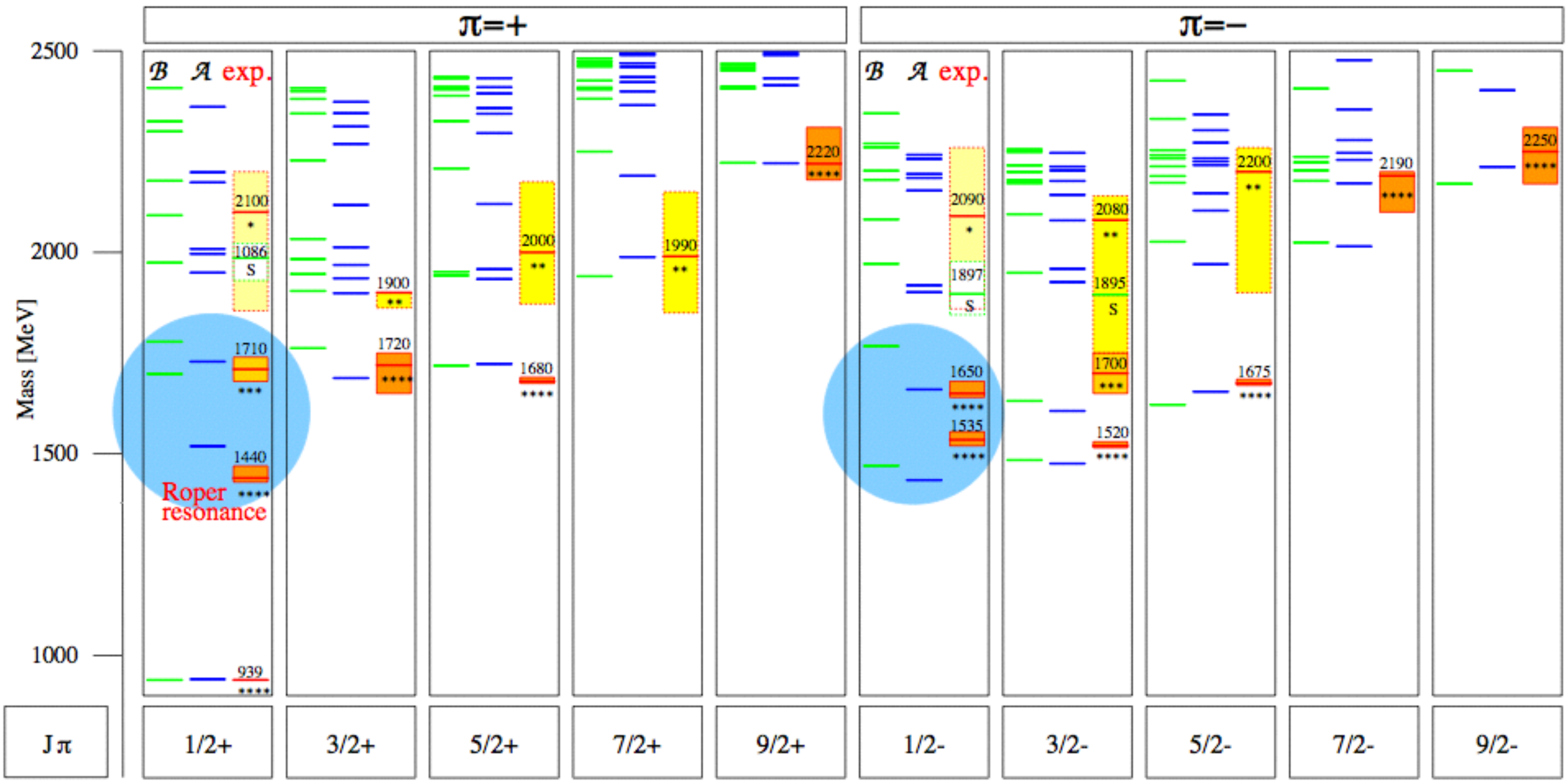


- Light tetraquarks:



Eichman, CF, Heupel, PLB 753 (2016) 282-287

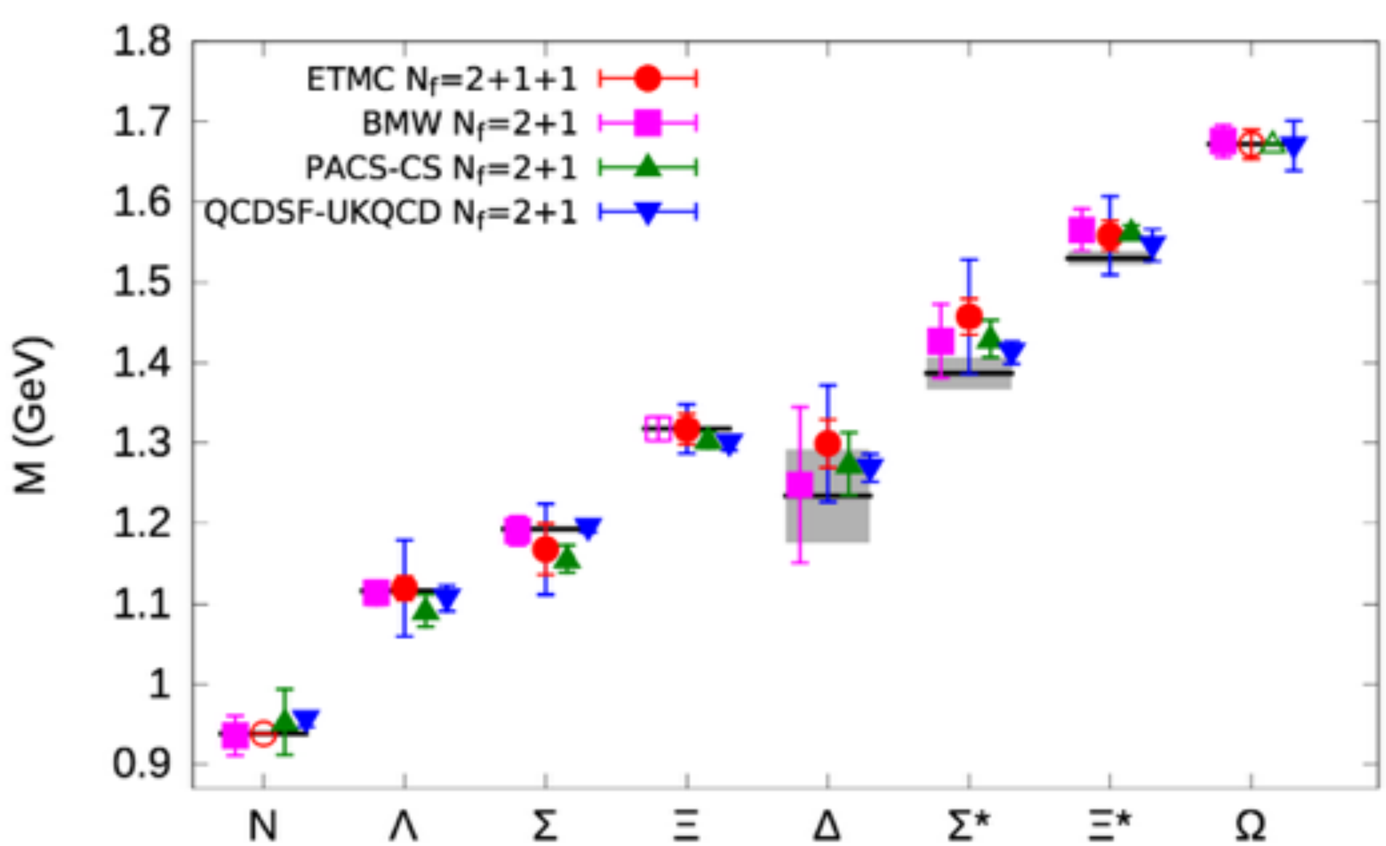
Light baryon spectrum - quark model



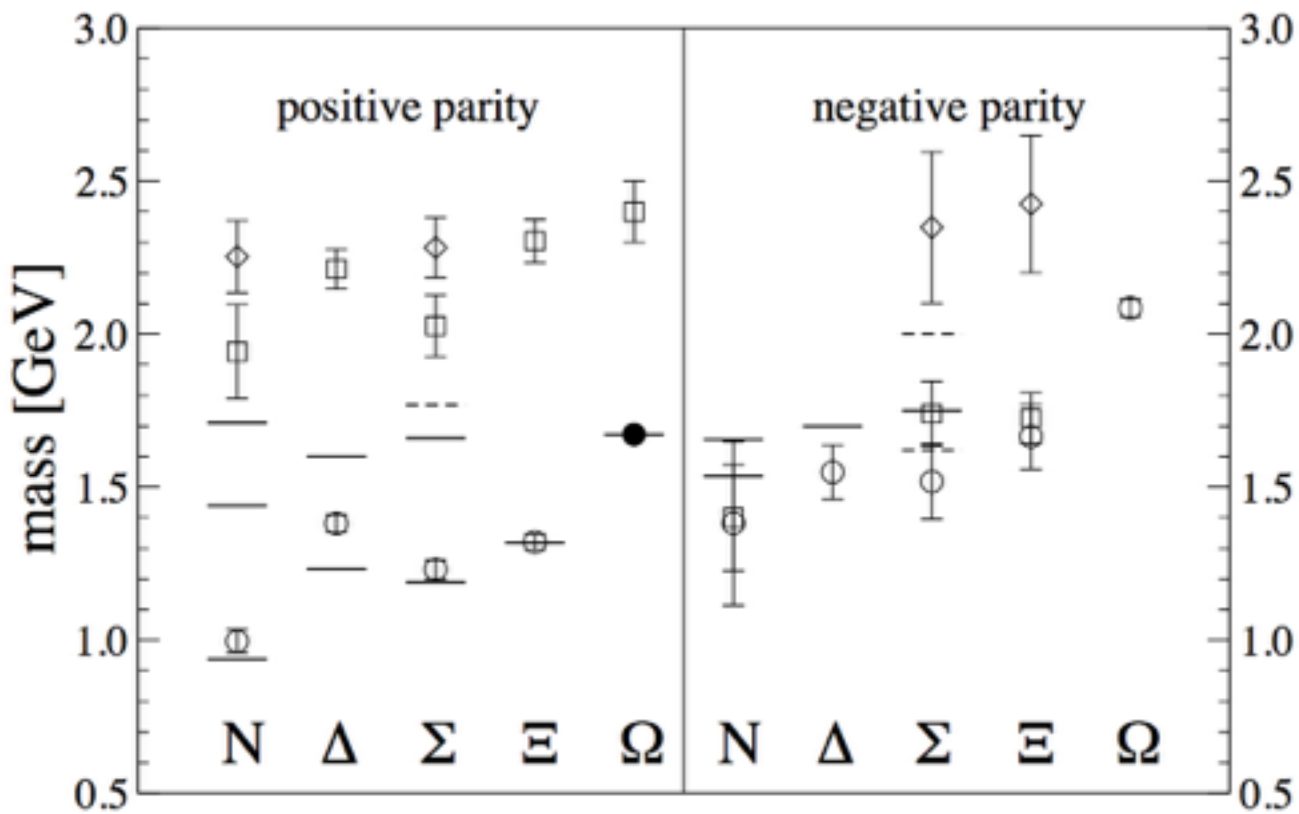
Loring, Metsch, Petry, EPJA 10 (2001) 395

- 'missing resonances': three-body vs. quark-diquark
- level ordering: $N_{\frac{1}{2}^+}$ vs. $N_{\frac{1}{2}^-}$

Lattice QCD



Alexandrou et al. (ETMC), PRD 90 (2014) 7

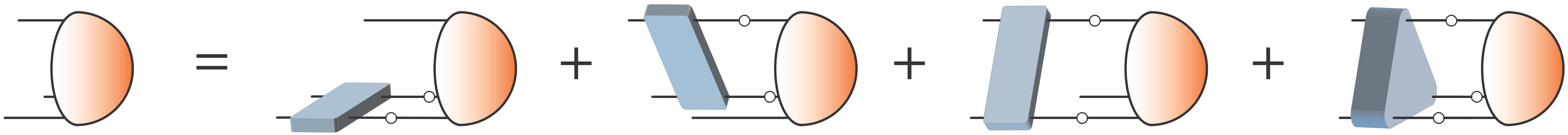


Engel et al. (BGR), PRD82 (2010) 034505

- baryon ground states well under control
- baryon excited states: very tough problem

Three-body vs. Diquark-quark approximation

Bethe-Salpeter equation for baryons:



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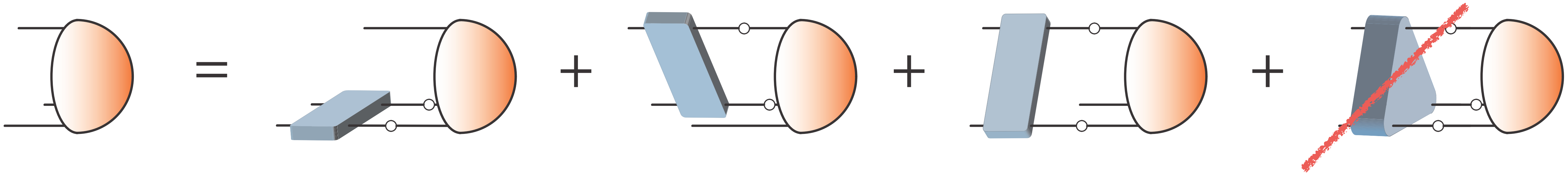

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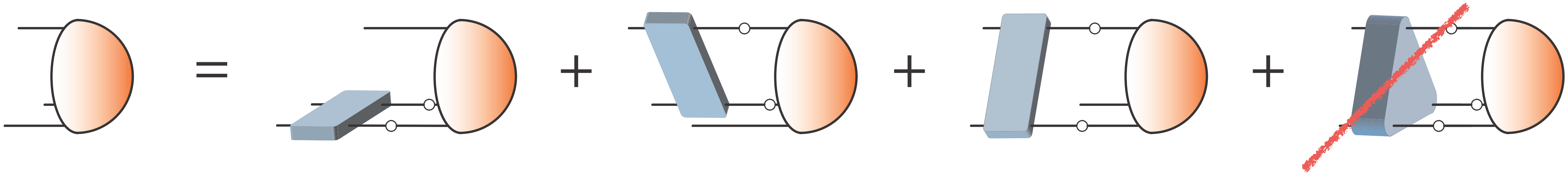
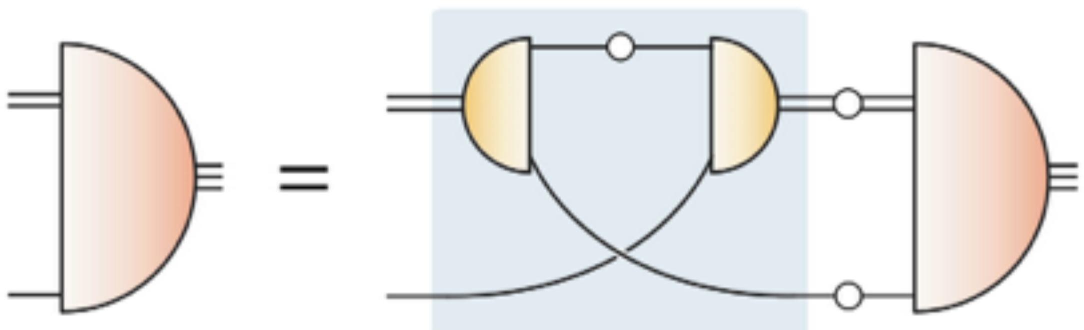
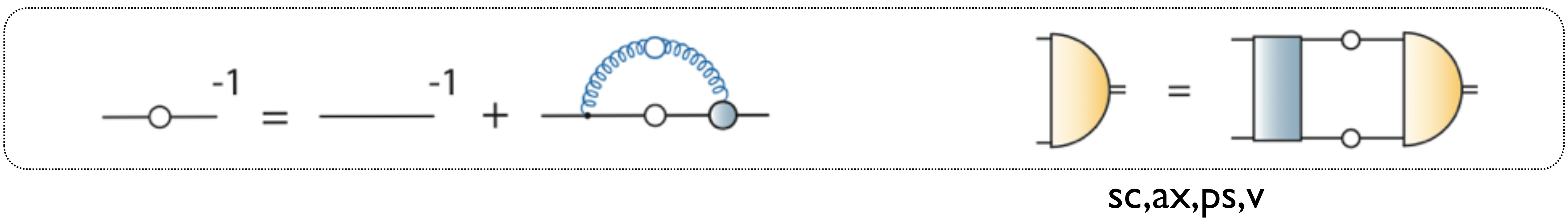


Diquark-quark approximation:





- Input: quark-gluon interaction
- Diquarks are NOT point like

Quantum numbers: non-relativistic vs relativistic

non-relativistic $q\bar{q}$

S	L	J^{PC}
0	0	0^{-+}
1	0	1^{--}
0	1	1^{+-}

$$P : (-1)^{L+1}$$

relativistic $q\bar{q}$

$$\Gamma_\pi(P,p) = \gamma_5 \left[F_1(P,p) + F_2(P,p) i \not{P} + F_3(P,p) pP i\not{p} + F_4(P,p) [\not{p}, \not{P}] \right]$$

s-wave: F_1, F_2

p-wave: F_3, F_4

~~$P : (-1)^{L+1}$~~

Quantum numbers: non-relativistic vs relativistic

non-relativistic $q\bar{q}$

S	L	J^{PC}
0	0	0^{-+}
1	0	1^{--}
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relativistic $q\bar{q}$

$$\begin{aligned}\Gamma_\pi(P,p) = \gamma_5\,[&F_1(P,p) \\ &+ F_2(P,p)\,i\not{P} \\ &+ F_3(P,p)\,pP\,i\not{p} \\ &+ F_4(P,p)\,[\not{p},\not{P}]]\end{aligned}$$

s-wave (F_1, F_2); p-wave (F_3, F_4)

$$P : (-1)^{L+1}$$ (struck through)

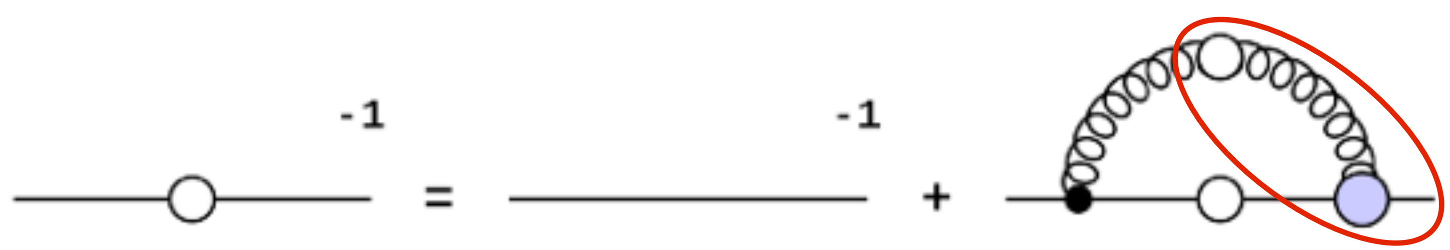
- conventional states more complicated
  - baryon octet: 64 tensors with s,p,d wave
  - decuplet: 128 tensors with s,pd,f wave
- mesons: 'exotic' quantum numbers possible: $0^{--}, 0^{+-}, 1^{-+}, 2^{+-} \dots$

The DSE for the quark propagator

Approximations:

I) NJL/contact model: 

II) Quark-diquark model: *ansatz for quark (and diquark wave function)*

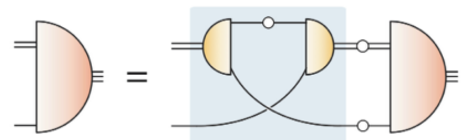
III) Rainbow-ladder (RL): 

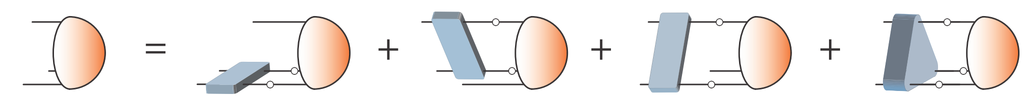
IV) Beyond rainbow-ladder (bRL): *solve DSEs for quark, gluon, vertex*

Sanchis-Alepuz, Williams, PLB 749 (2015) 592
Williams, CF, Heupel, PRD93 (2016) 034026, and refs. therein
Binosi, Chang, Papavassiliou, Qin, Roberts PRD95 (2017) 031501 and refs. therein

DSE/BSE/Faddeev landscape

level of sophistication →



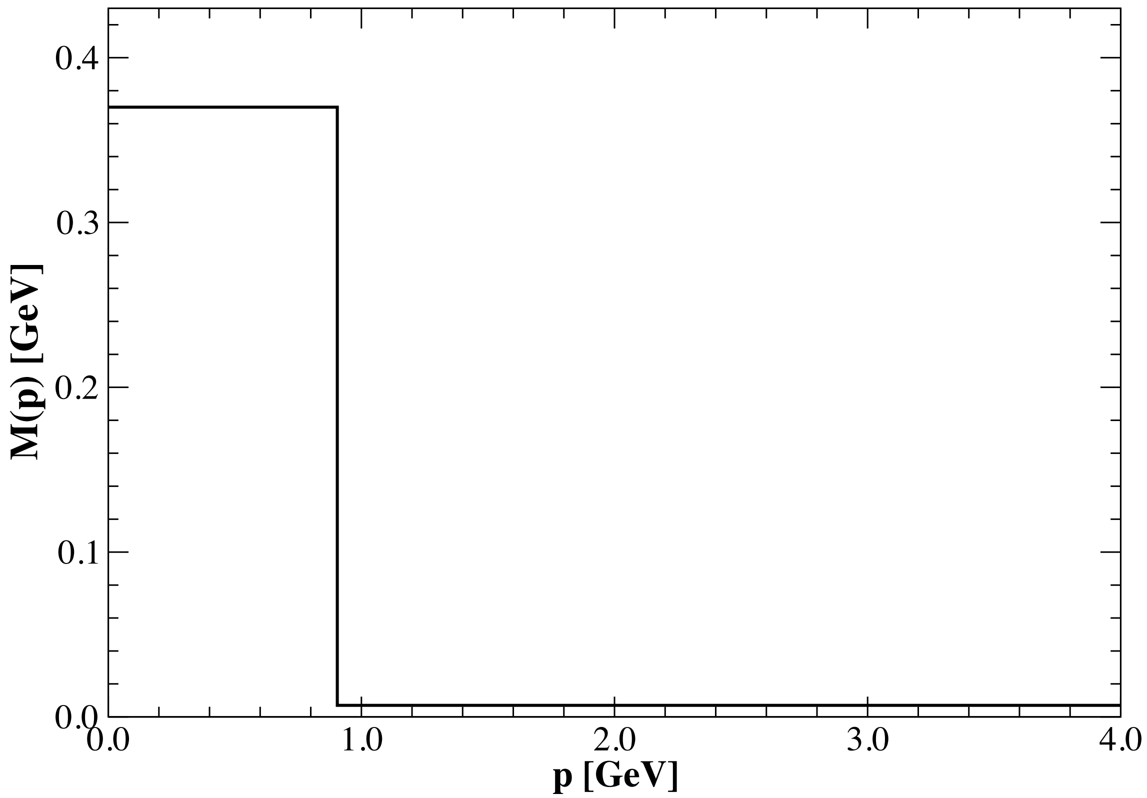


	I) NJL/contact interaction	II) Quark-diquark model	III) DSE (RL)	III) DSE (RL)	IV) DSE (bRL)
N, Δ masses	✓	✓	✓	✓	✓
N, Δ em. FFs	✓	✓	✓	✓	
$N \to \Delta\gamma$	✓	✓	✓	✓	
Roper,...	✓	✓	✓	✓	
$N \to N^*\gamma$	✓	✓			
$N^*(1535), \ldots$	✓	✓	✓	✓	
$N \to N^*\gamma$					
Σ, Ξ, Ω	✓	✓	✓	✓	
excited strange	✓		✓	✓	
Σ, Ξ, Ω em. FFs			✓	✓	
	Cloet, Thomas, Roberts, Segovia et al.	Oettel, Alkofer, Roberts, Bloch, Segovia et al.	Eichmann, Alkofer, Krassnigg, Nicmorus, Sanchis-Alepuz, CF	Eichmann, Alkofer, Sanchis-Alepuz, CF	Sanchis-Alepuz, Williams, CF

Quark mass

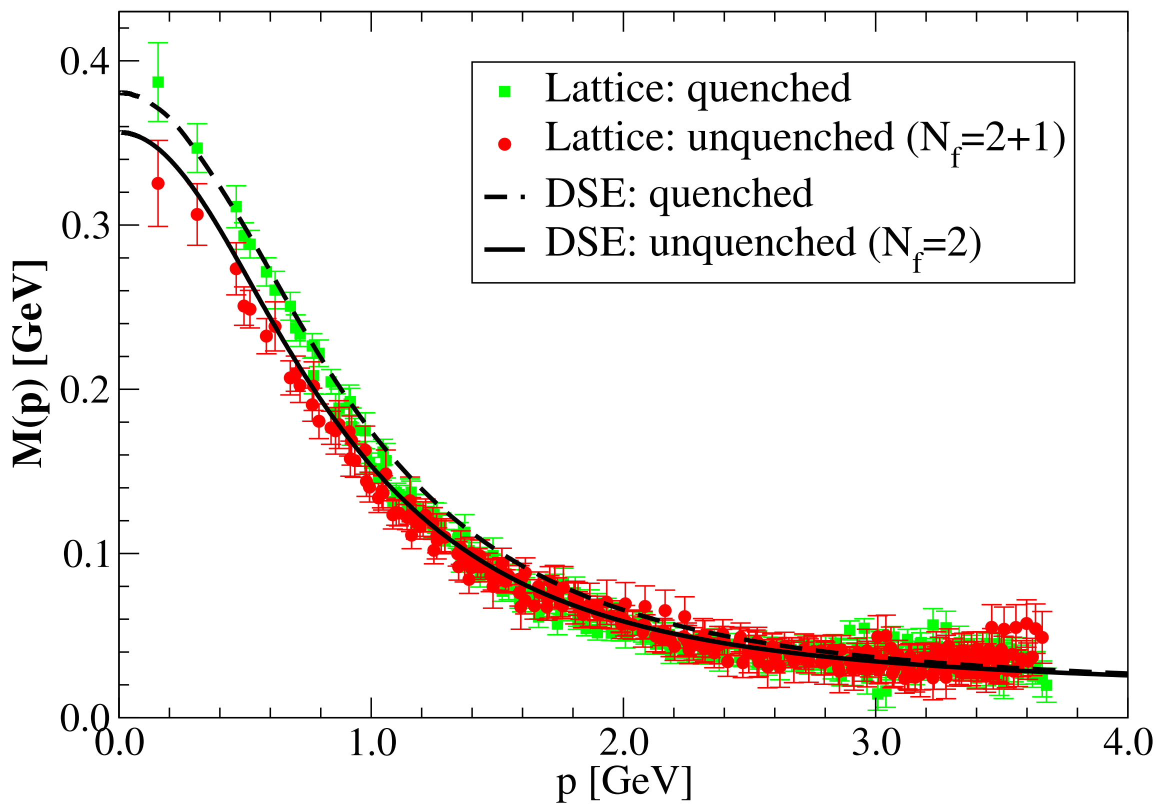
$$[S(p)]^{-1} = [-i\not{p} + M(p^2)]/Z_f(p^2)$$

NJL/contact interaction



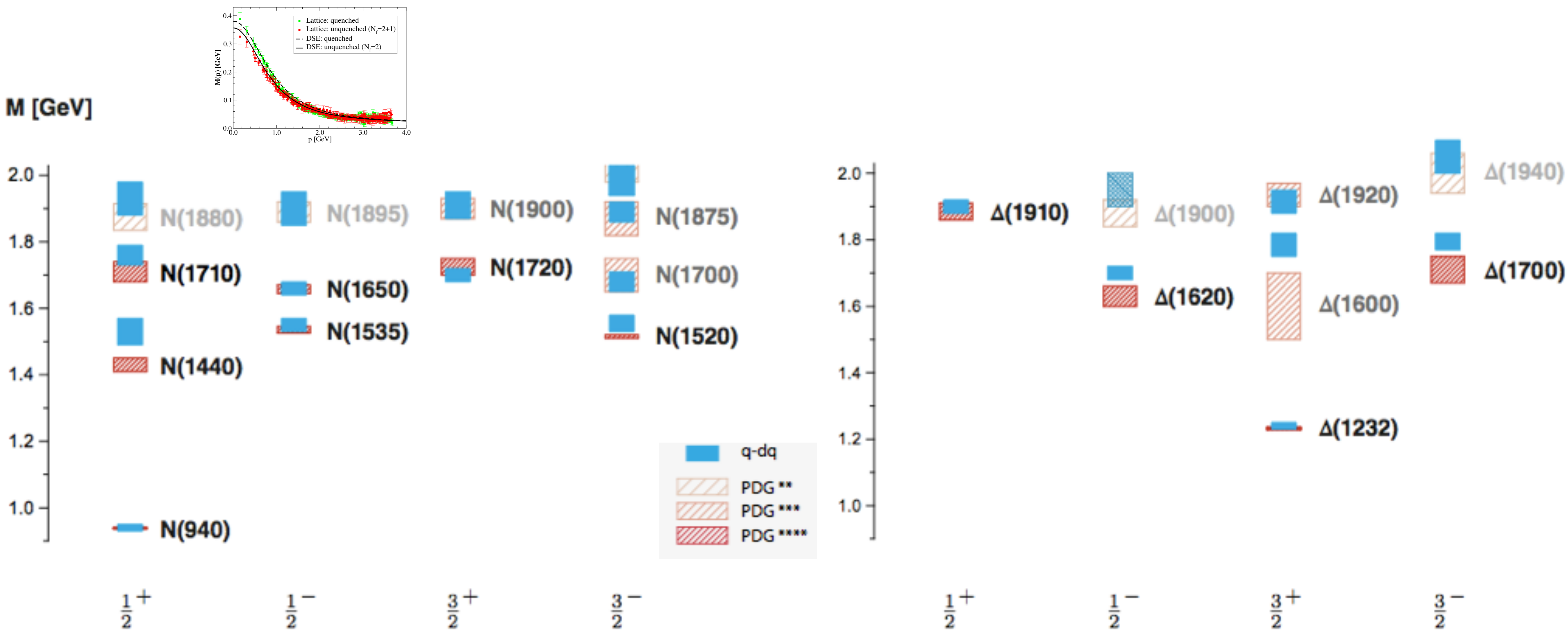
Lu, Chen, Roberts et al., PRC 96 (2017) 015208

DSE (RL)



DSE: CF, Nickel, Williams, EPJ C 60 (2009) 47
Lattice: P. O. Bowman, et al PRD 71 (2005) 054507

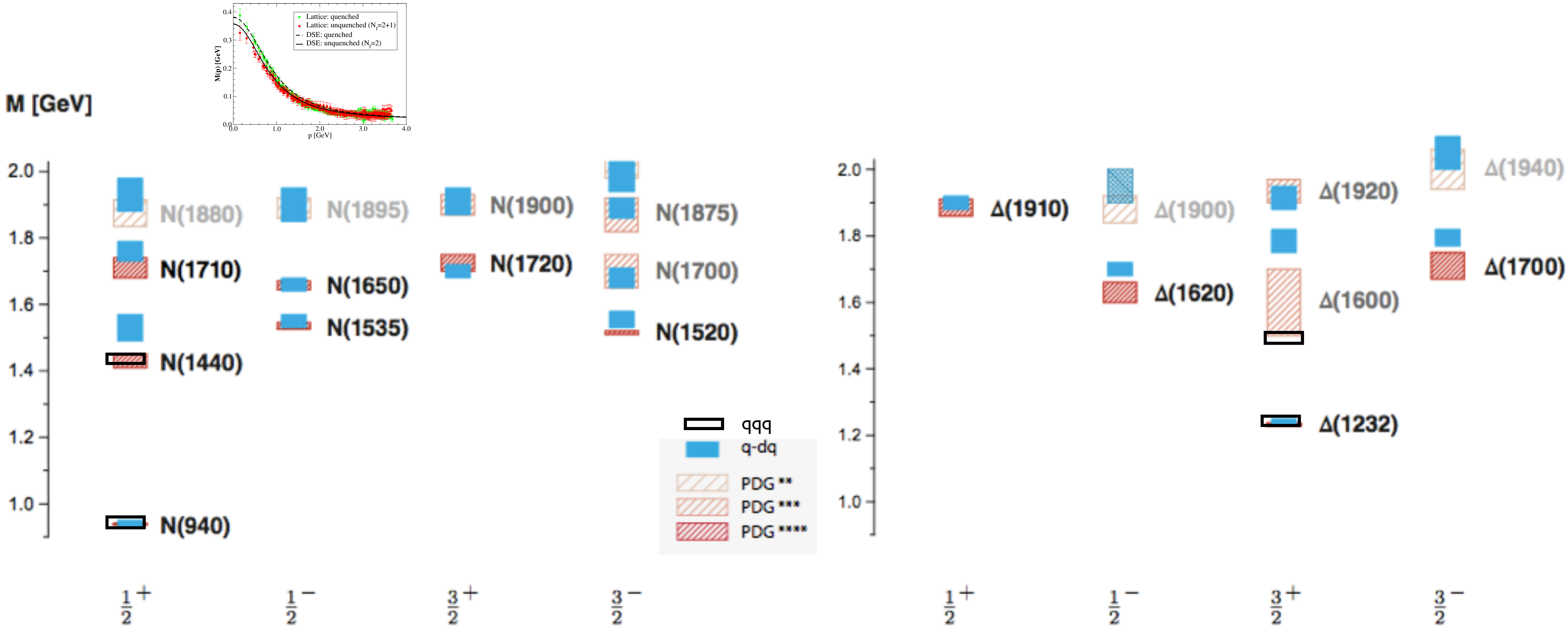
- Quark mass dynamically generated
- running important for wave functions, FFs etc.



Eichmann, CF, Sanchis-Alepuz, PRD 94 (2016) [1607.05748]
Eichmann, CF, in preparation

- spectrum in one to one agreement with experiment
- correct level ordering (without coupled channel effects…)
- sc,ax diquark always dominate over ps,v

Light baryon spectrum: DSE-RL



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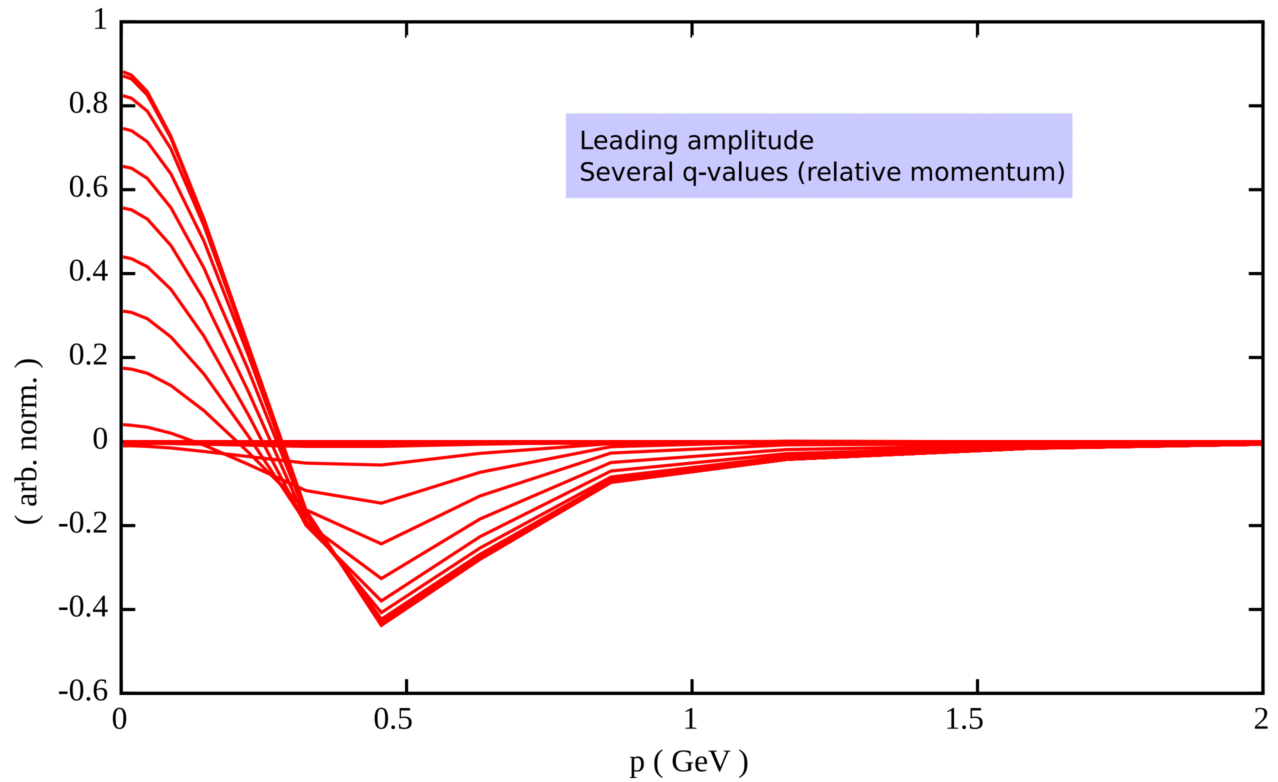
Eichmann, CF, in preparation

- spectrum in one to one agreement with experiment
- correct level ordering (without coupled channel effects…)
- sc,ax diquark always dominate over ps,v
- three-body agrees with diquark-quark where applicable

Properties of the Roper

angular mom. decomposition

%	N	$N^*(1440)$	Δ	$\Delta^*(1600)$
s wave	66	15	56	10
p wave	33	61	40	33
d wave	1	24	3	41
f wave	–	–	< 0.5	16

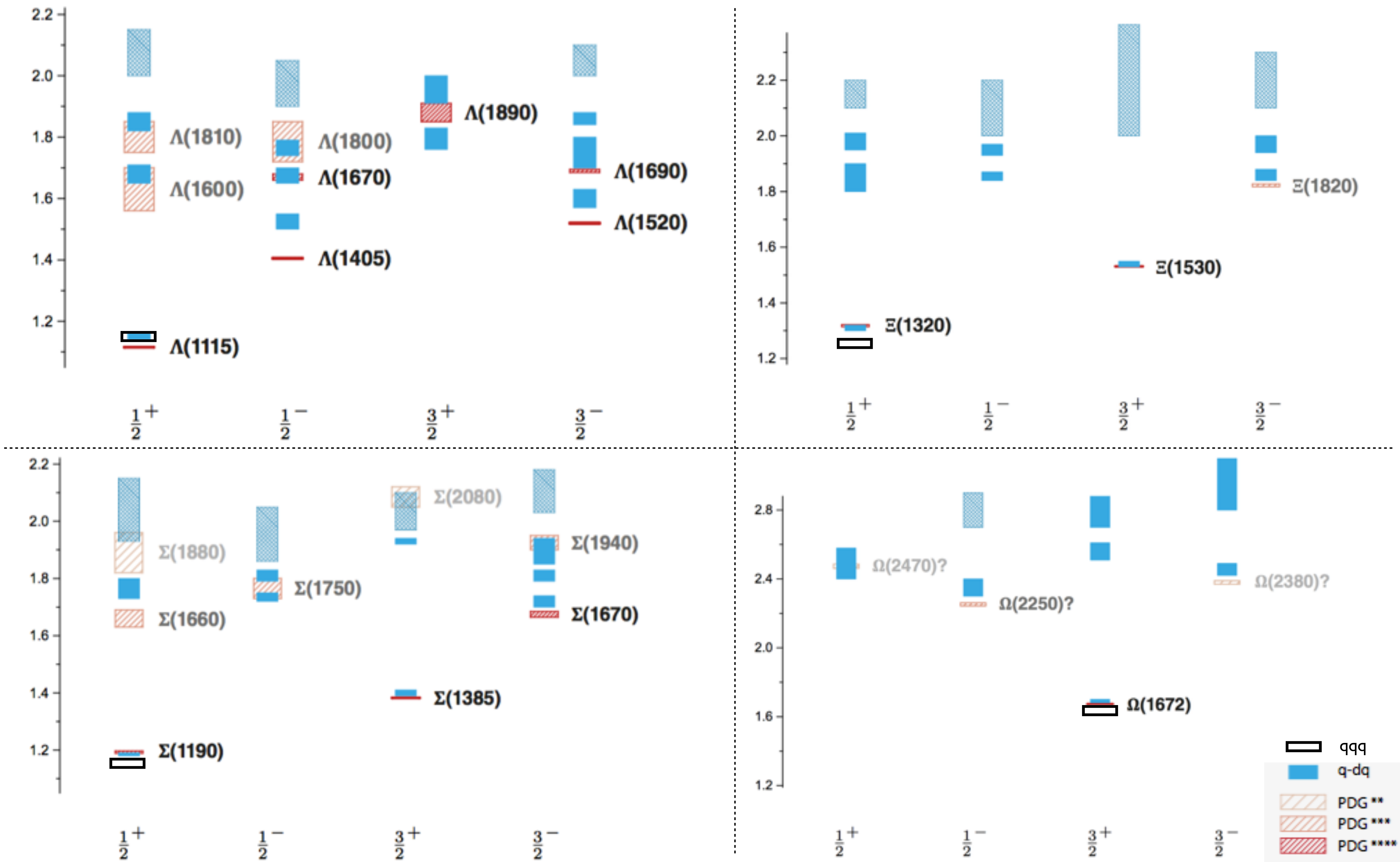


Eichmann, CF, Sanchis-Alepuz, PRD 94 (2016)

- zero crossing of wave function: 2s-state
- every state is mixture of several partial waves !
- different internal structure of radial excitations

tension with simpler calculations (‘contact interaction’, ‘QCD based model’):
Wilson, Cloet, Chang and Roberts, PRC 85 (2012) 025205,
Segovia, El-Bennich, Rojas, Cloet, Roberts, Xu and Zong, PRL 115 (2015) 17
Lu, Chen, Roberts et al., PRC 96 (2017) 015208

Strange baryon spectrum: DSE-RL (preliminary !)



Eichmann, CF, in preparation
Sanchis-Alepuz, CF, PRD 90 (2014) 096001

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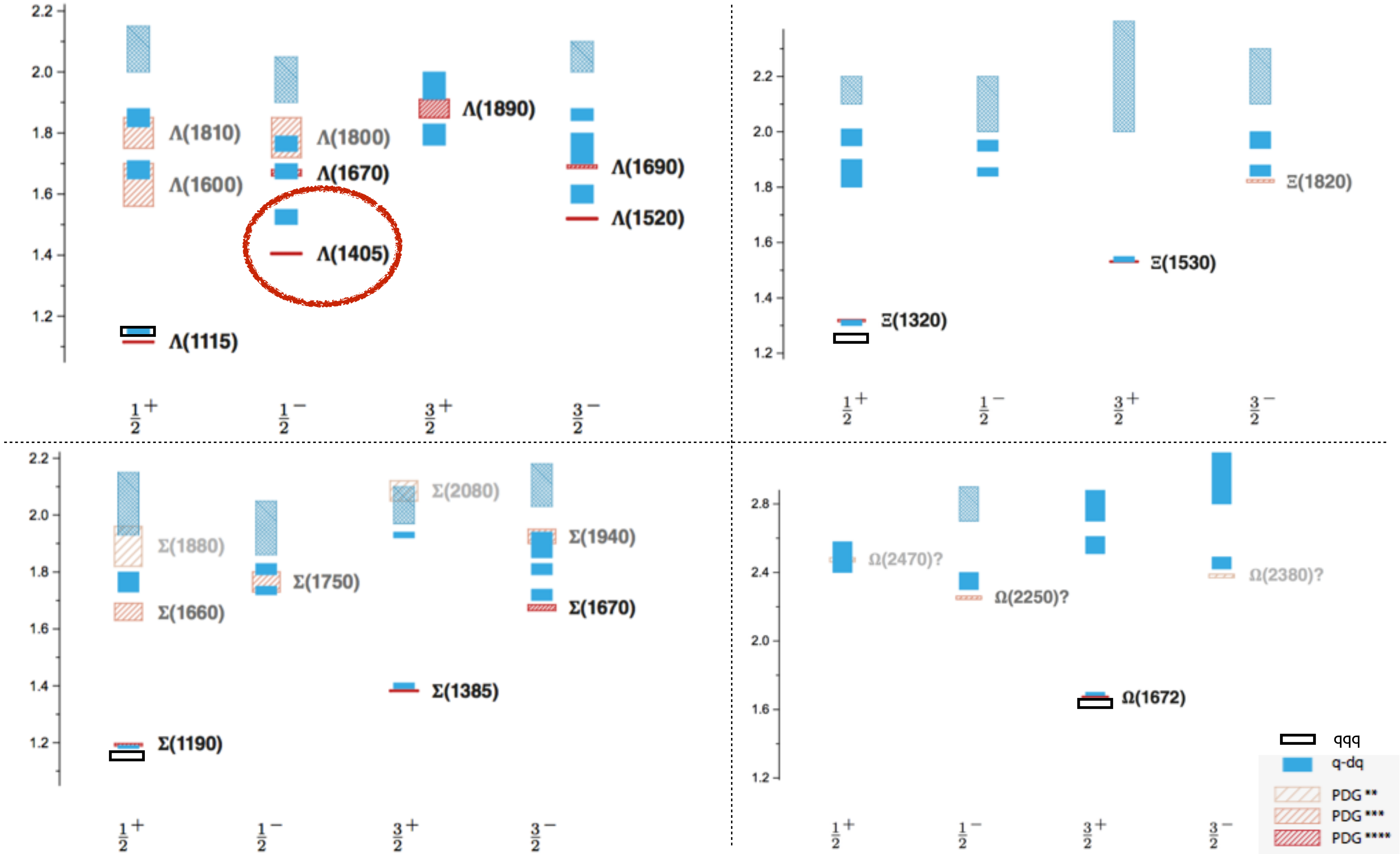

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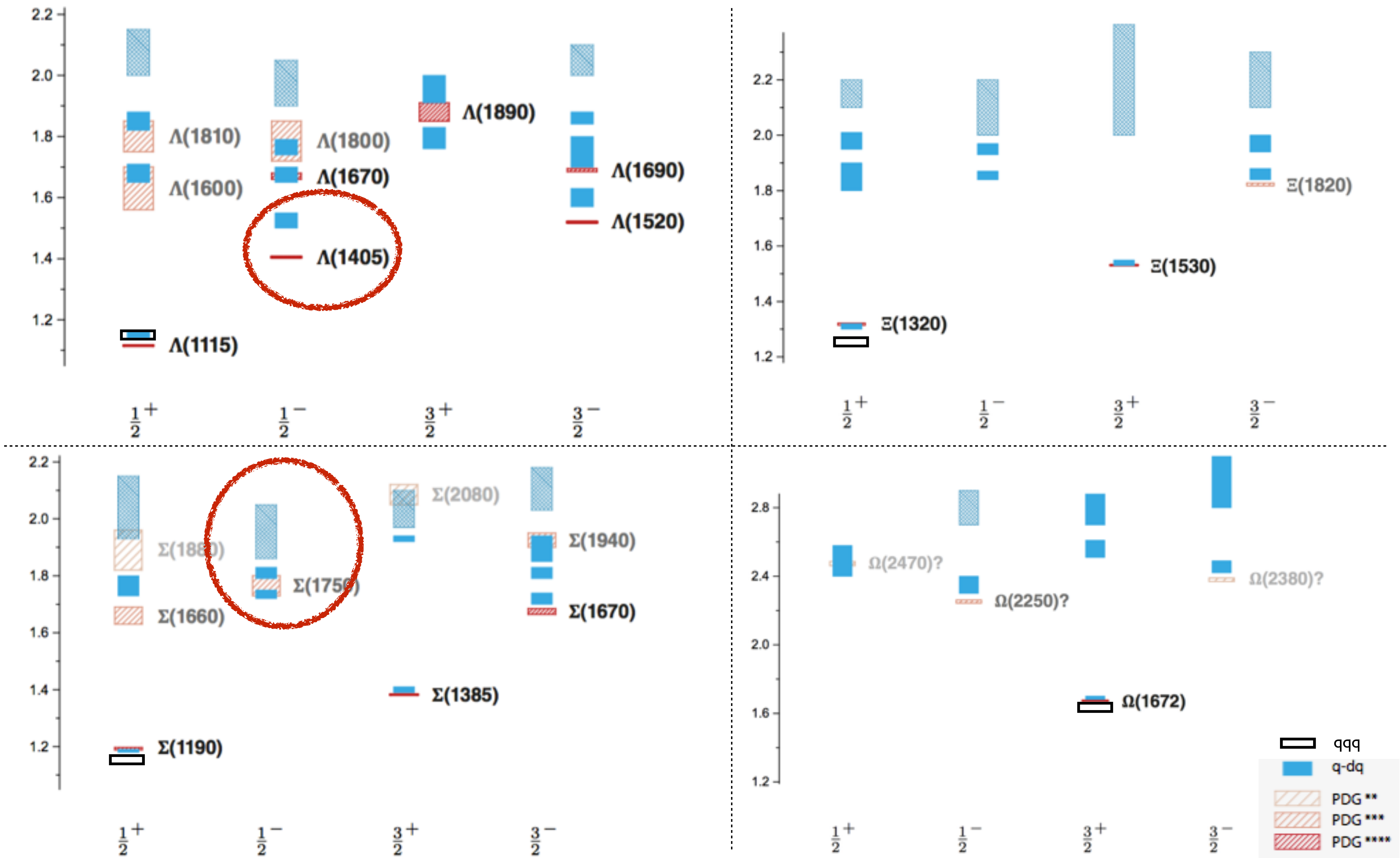

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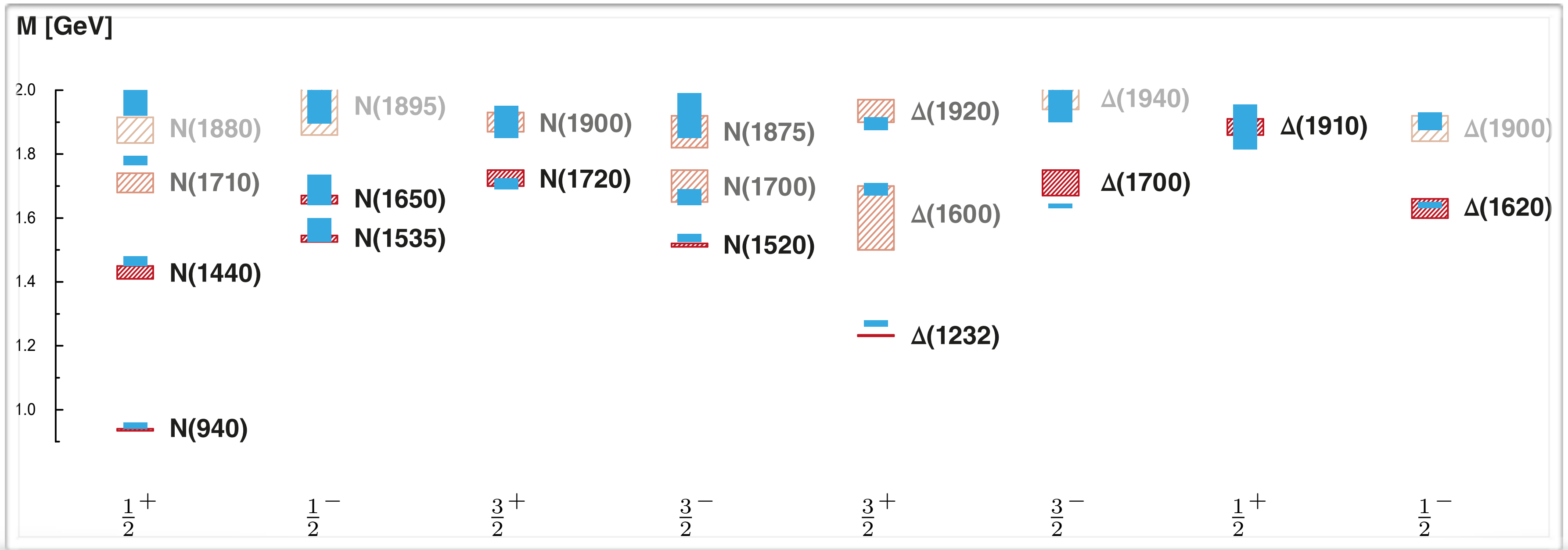
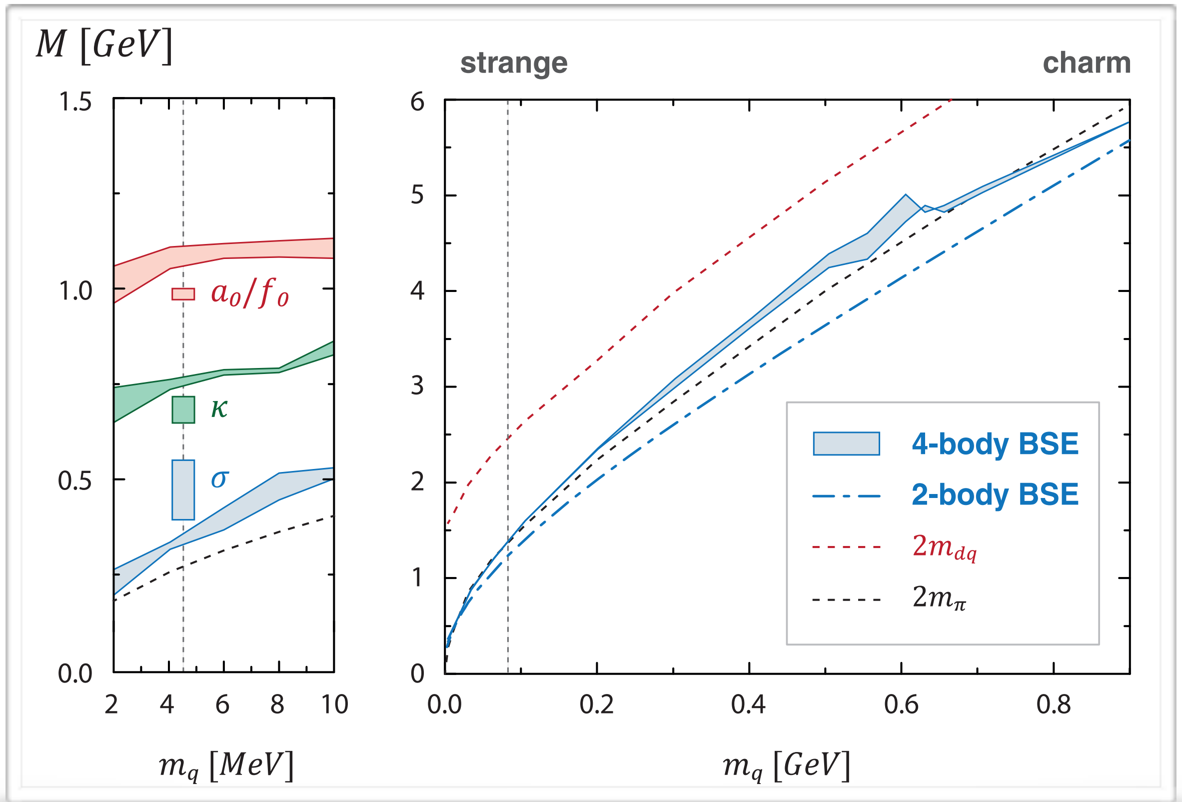
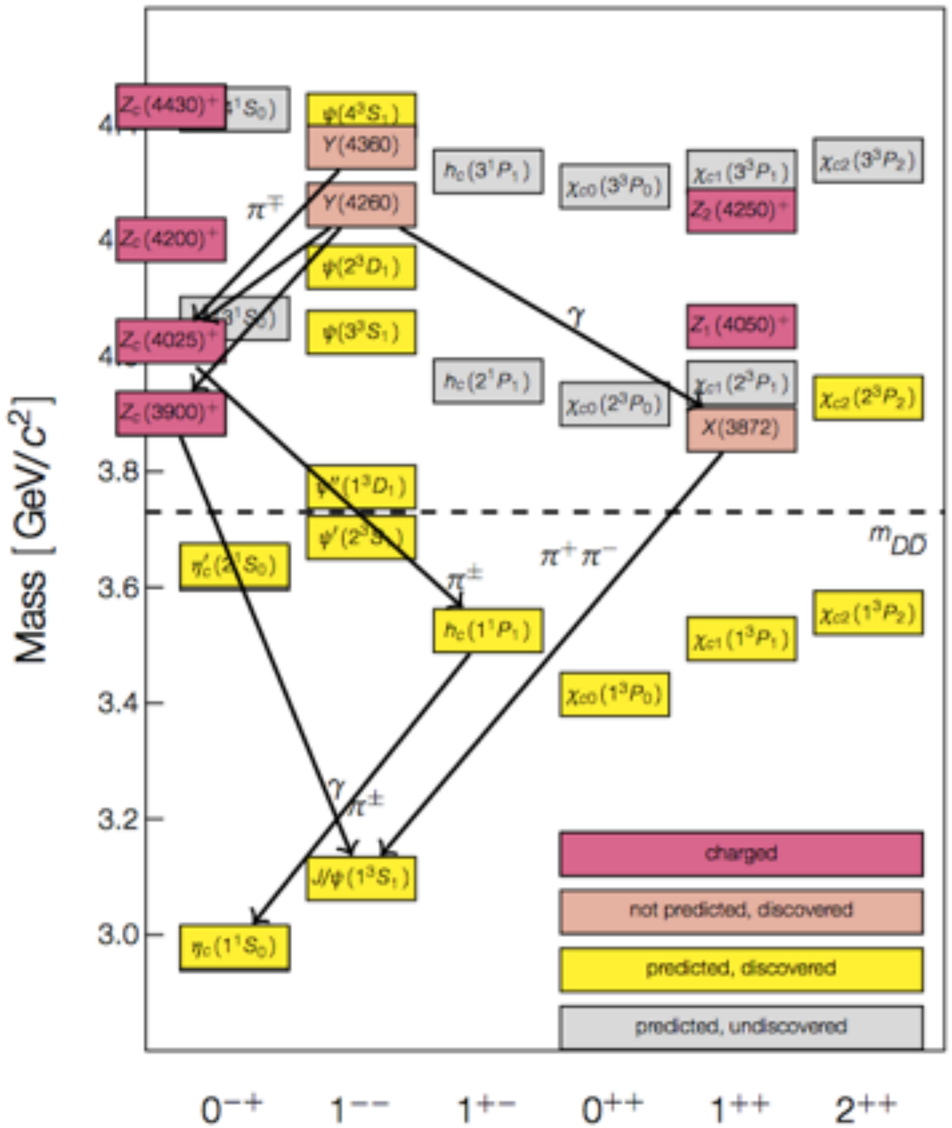


- Light tetraquarks:



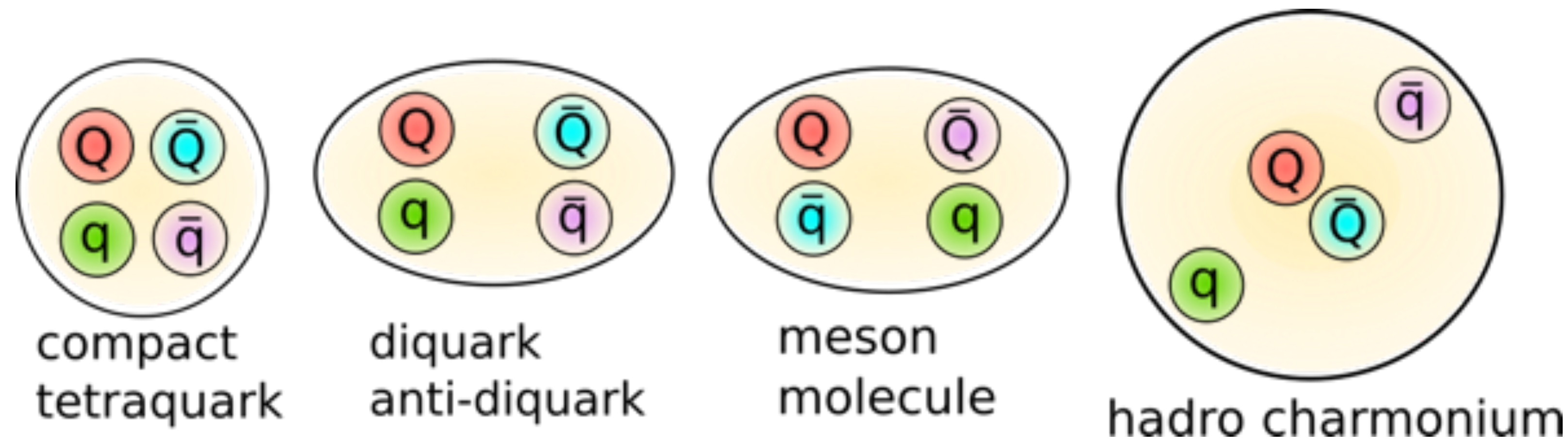
Eichman, CF, Heupel, PLB 753 (2016) 282-287

Heavy and light tetraquark

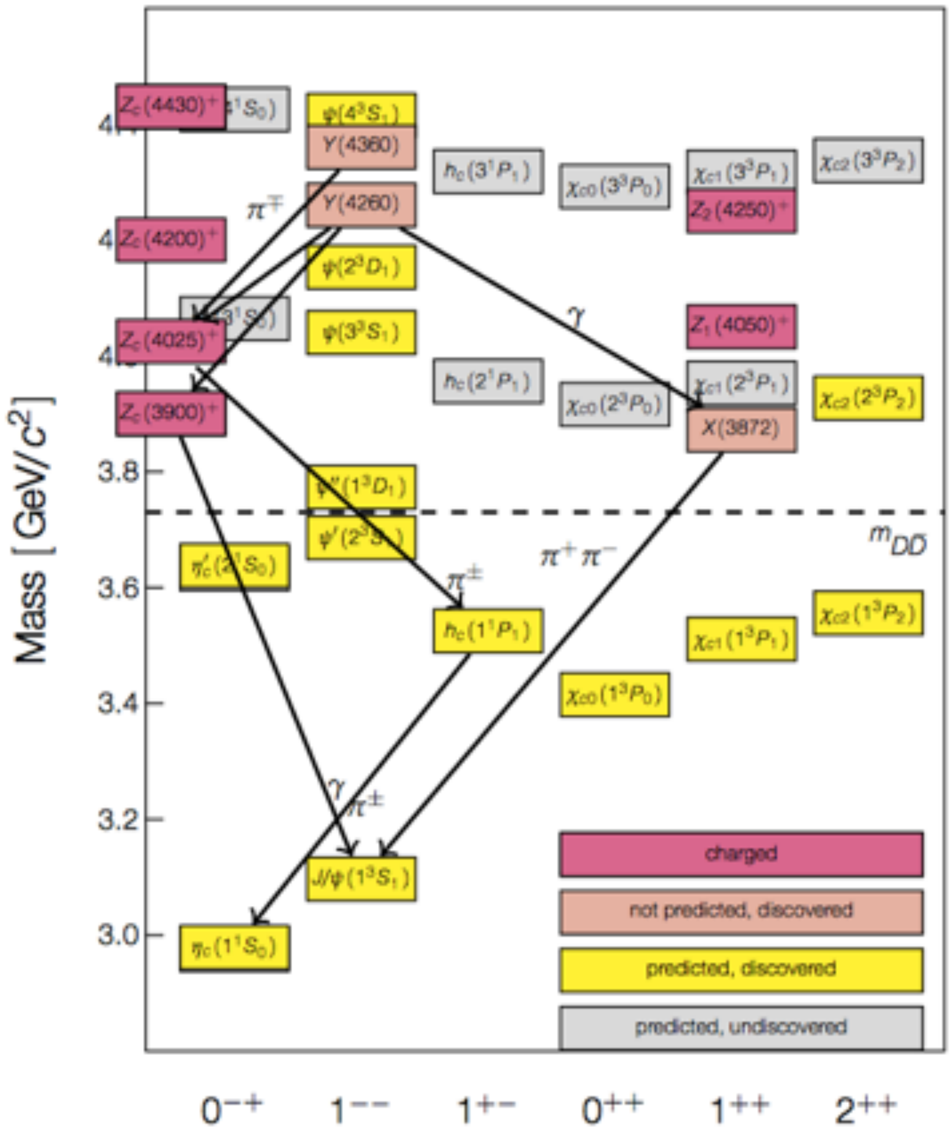


Wolfgang Gradl, BESIII, St Goar 2015

Internal structure ??



Related to details of underlying QCD forces between quarks



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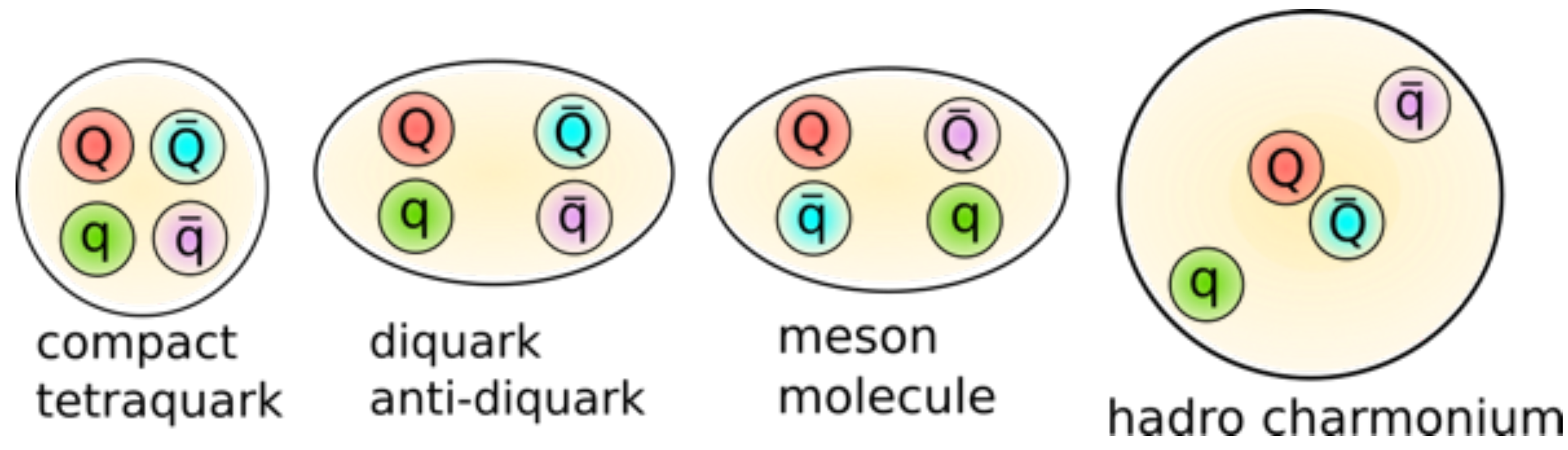
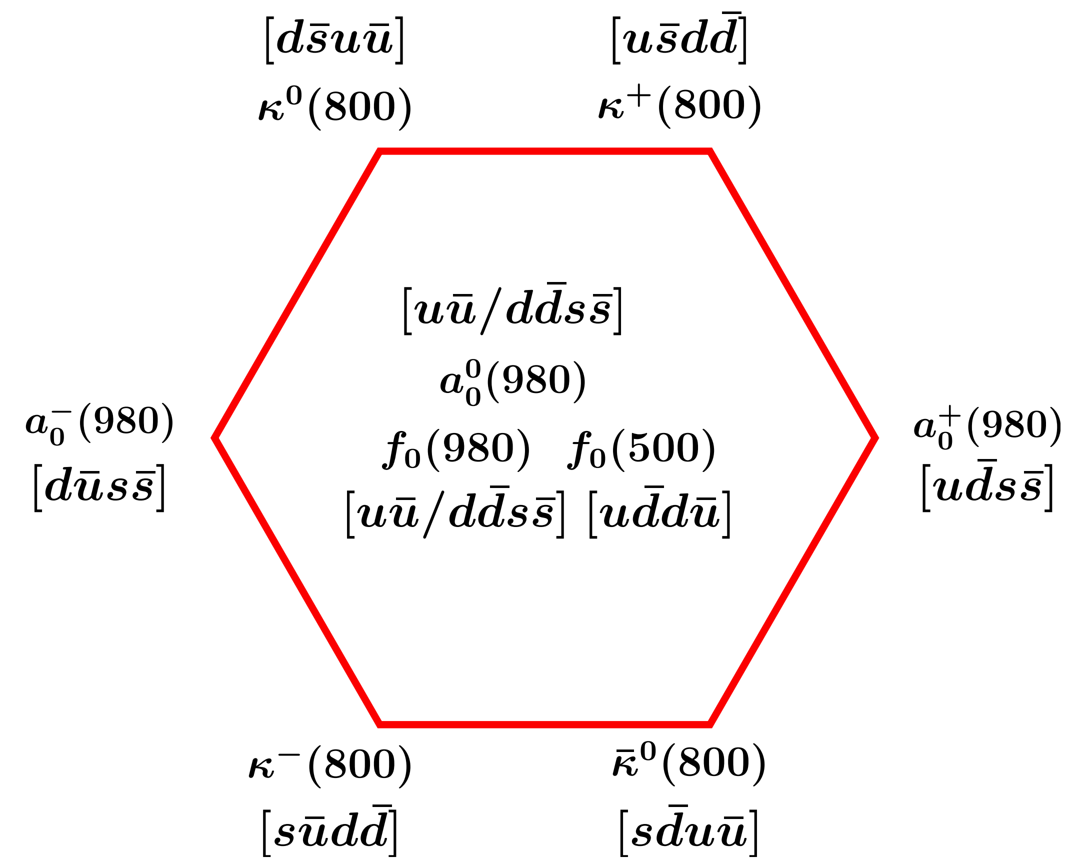
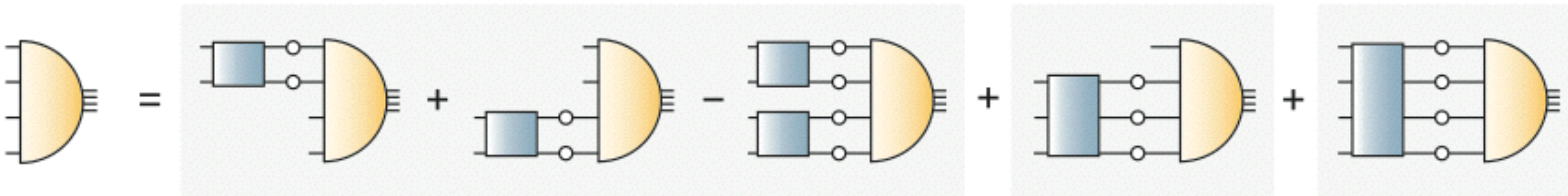




Related to details of underlying QCD forces between quarks

Tetraquarks from the four-body equation

Exact equation:



Two-body interactions

Three- and four-body interactions

Kvinikhidze & Khvedelidze, Theor. Math. Phys. 90 (1992)

Heupel, Eichmann, CF, PLB 718 (2012) 545-549

Eichmann, CF, Heupel, PLB 753 (2016) 282-287

- Basic idea:
  solve four-body equation without any assumption on internal clustering
- Key elements: quark propagator and interaction kernels

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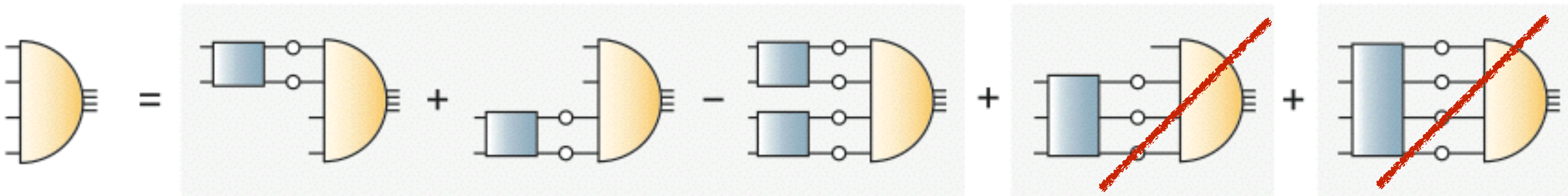

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- Basic idea:
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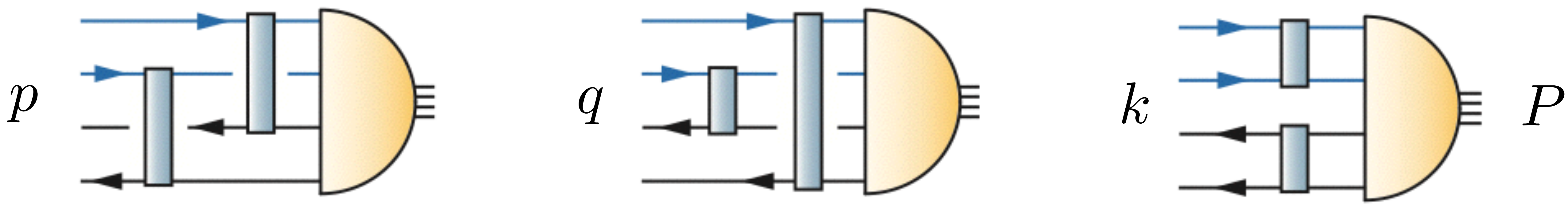
Structure of the amplitude

Scalar tetraquark:



$$\Gamma(P,p,q,k) = \sum_i f_i(s_1,\cdots,s_9) \times \tau_i(P,p,q,k) \times color \times flavor$$

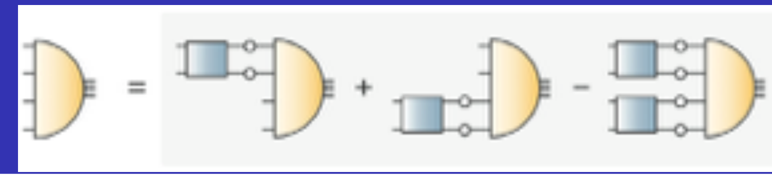
9 Lorentz scalars (built from P,p,q,k)

256 tensor structures (scalar tetra)

$3 \otimes \bar{3}, 6 \otimes \bar{6}$ or $1 \otimes 1, 8 \otimes 8$

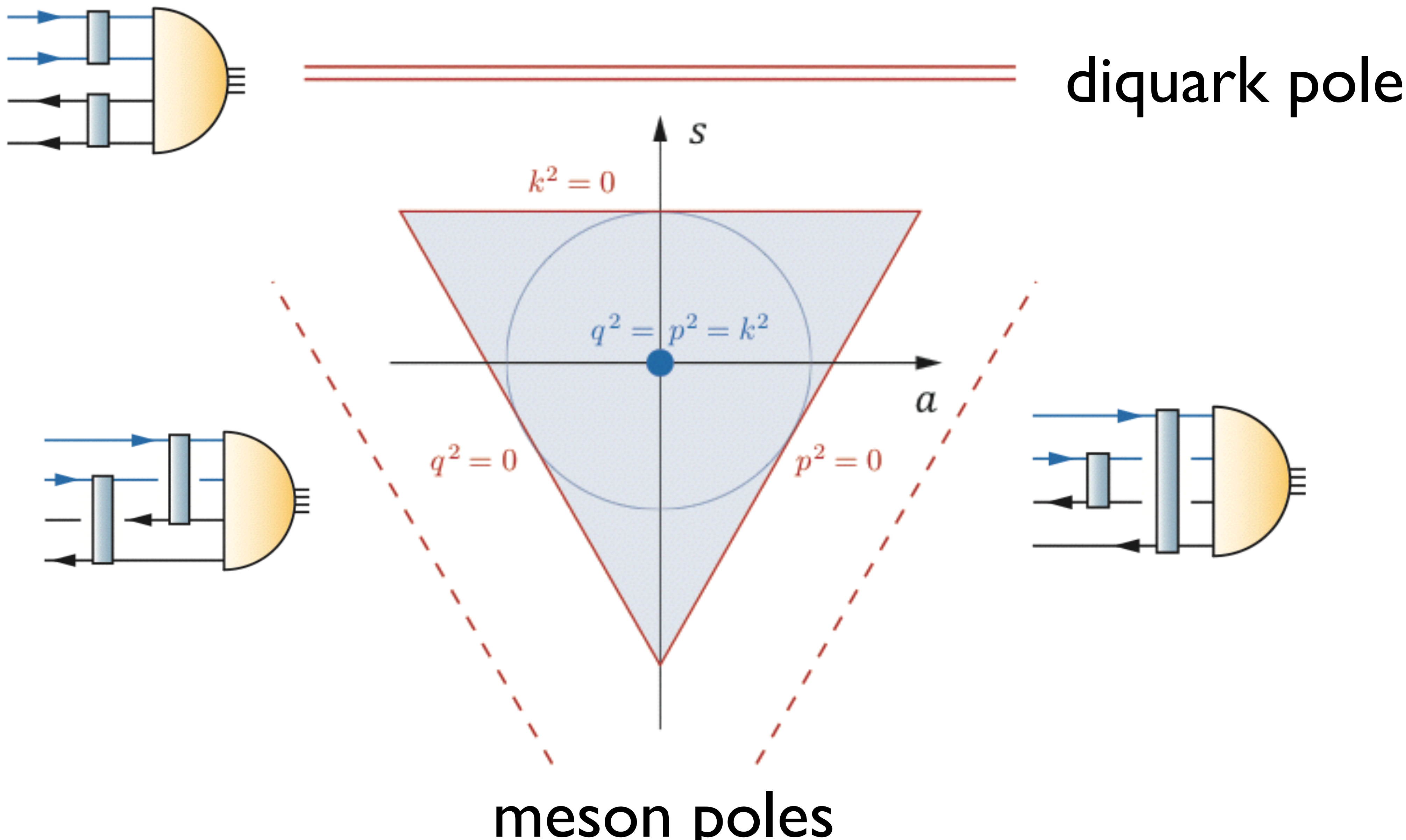
- reasonable approximation: keep s-waves only;
  → 16 tensor structures

Four-body equation:



Organise Dirac-Lorentz-tensors into multiplets of **S4**

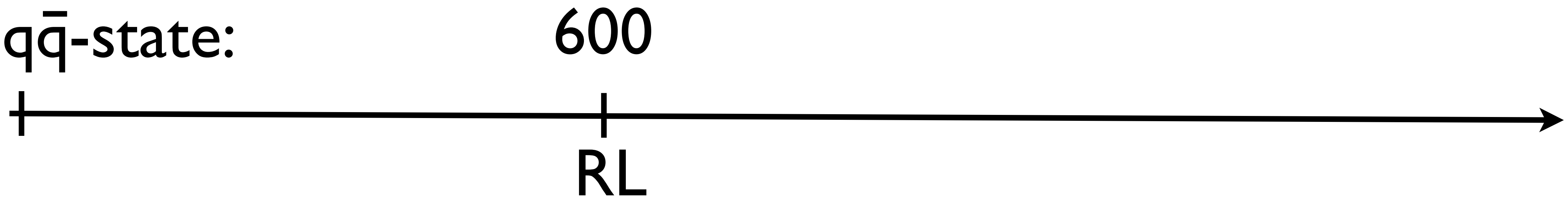
- Singlet, carries overall scale
- Doublet



- Two triplets

Eichmann, CF, Heupel, PLB 753 (2016) 282-287

Bound state vs resonance: light scalars



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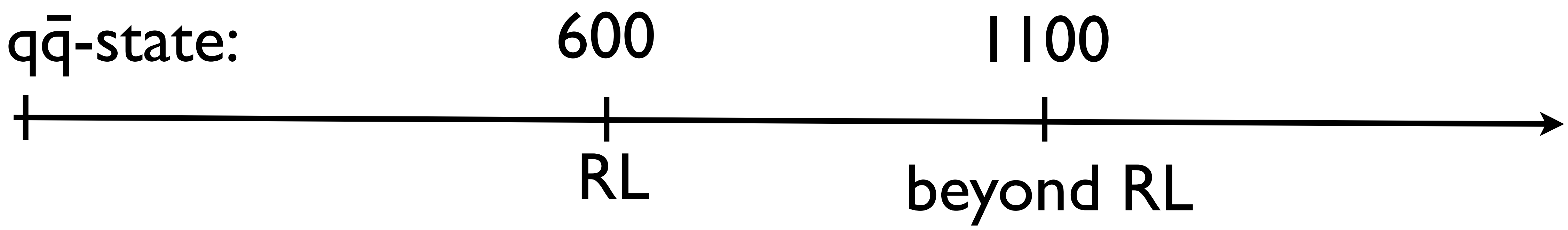

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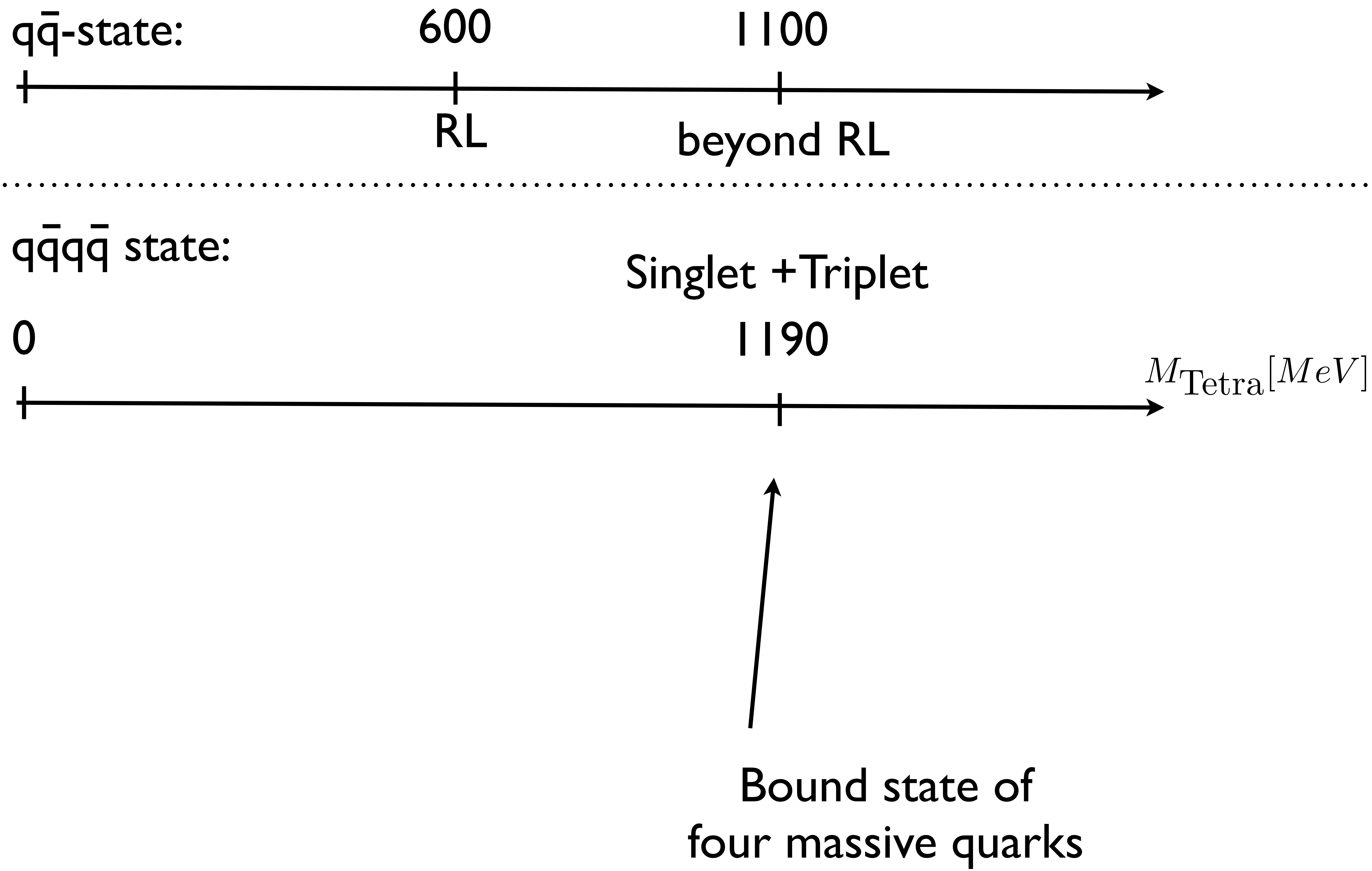


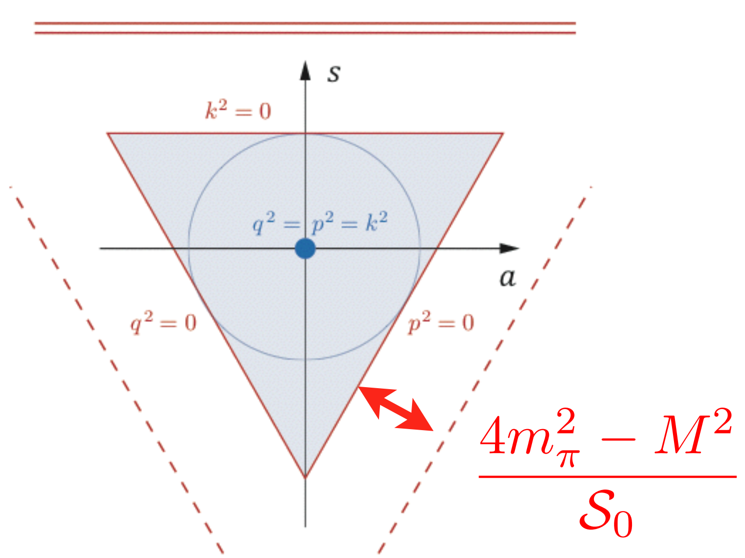
Bound state vs resonance: light scalars

$q\bar{q}$-state: 600 RL, 1100 beyond RL

$q\bar{q}q\bar{q}$ state: 0, 350 (Singlet + Doublet), 1190 (Singlet + Triplet)

$M_{\text{Tetra}} [MeV]$

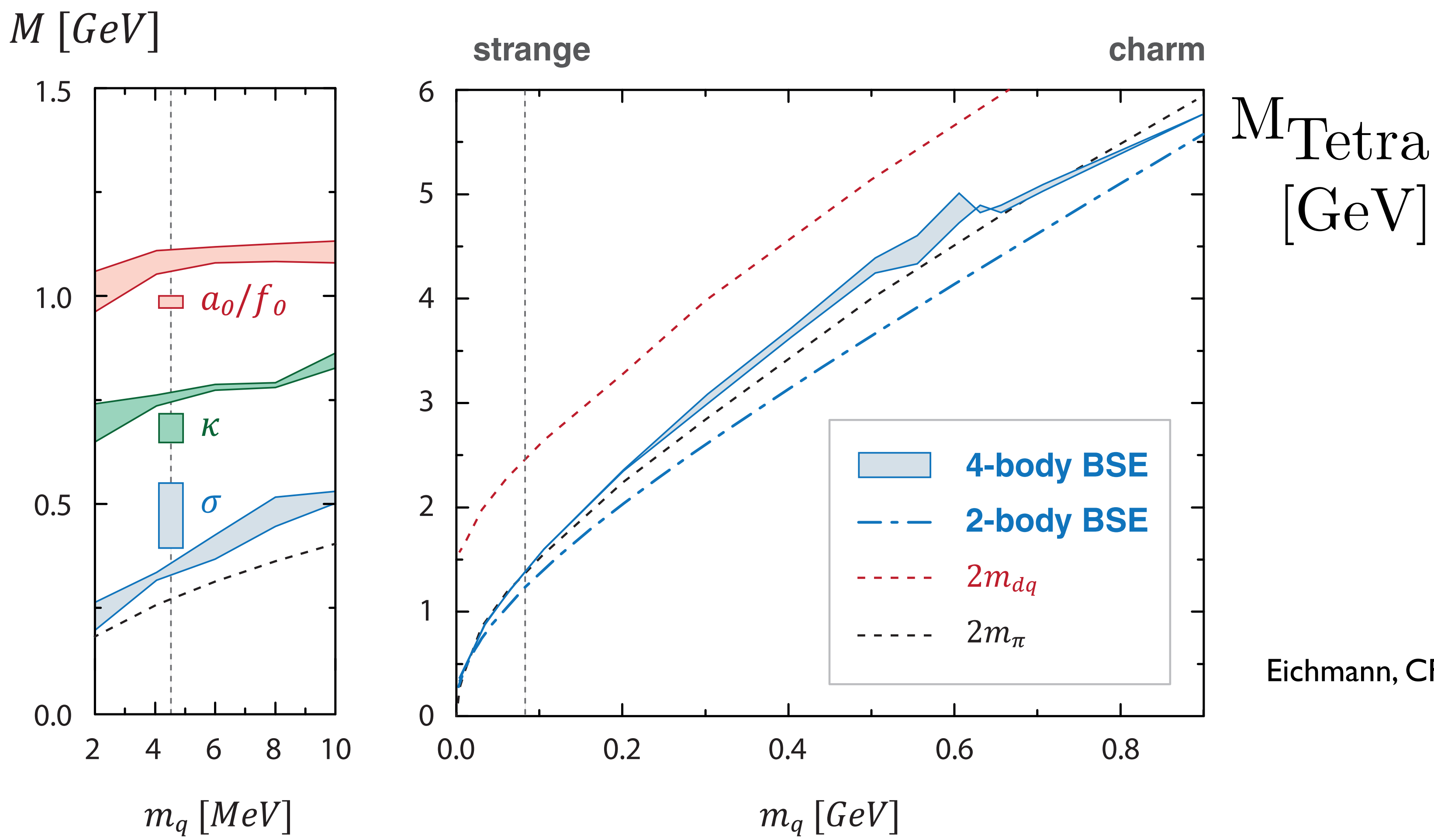
Singlet + Doublet



Two-pion resonance

Bound state of four massive quarks

Mass evolution of tetraquark



Eichmann, CF, Heupel, PLB 753 (2016) 282-287

- Resonance becomes bound state for large m_q
- Dynamical decision: meson clusters, not diquarks
- Results:

$$m_\sigma \sim 350\,\text{MeV}$$

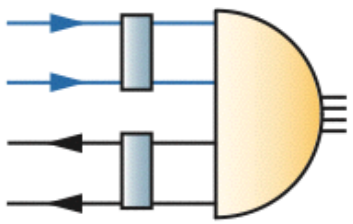
$$m_\kappa \sim 750\,\text{MeV} \qquad m_{ss\bar{s}\bar{s}} \sim 1.5\,\text{GeV}$$

$$m_{a_0,f_0} \sim 1080\,\text{MeV} \qquad m_{cc\bar{c}\bar{c}} \sim 5.7\,\text{GeV}$$

qualitatively similar to two-body framework

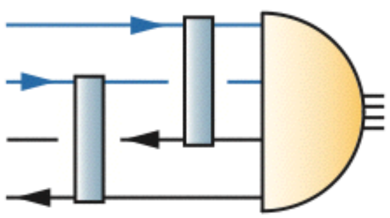
Heupel, Eichmann, CF, PLB 718 (2012) 545-549

Dynamical situation in **S4**-doublet:



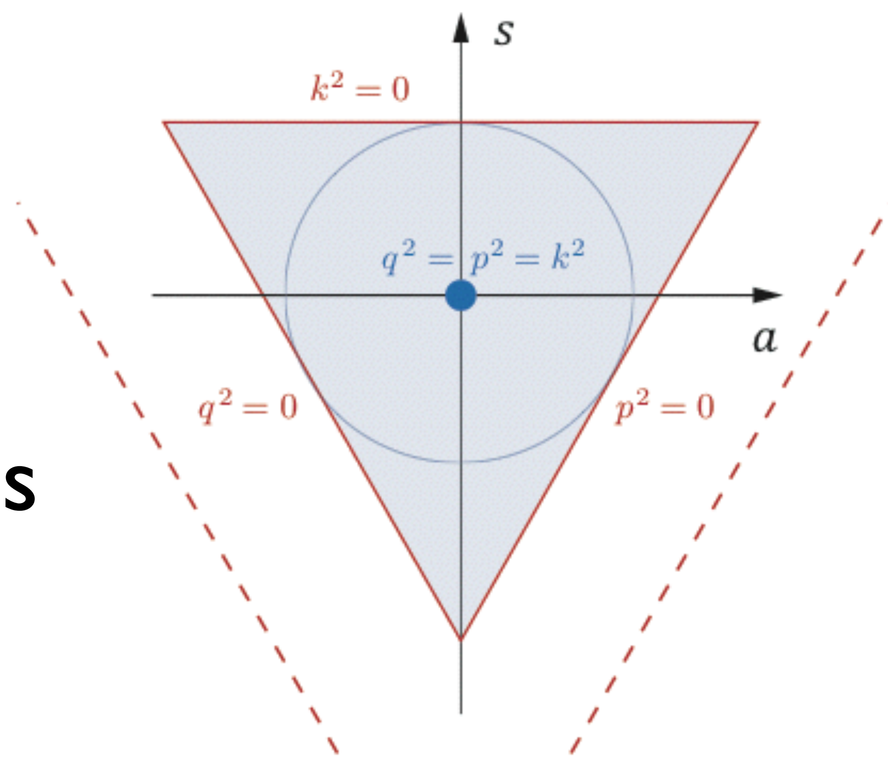
heavy-light diquarks

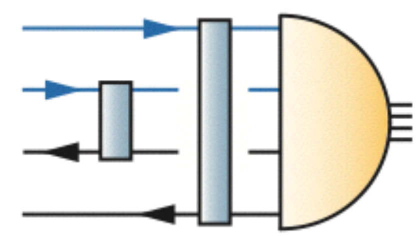
$cq - \bar{c}\bar{q}$



meson-molecules

$c\bar{q} - \bar{c}q$





hadro-charmonium

$c\bar{c} - q\bar{q}$

Dynamical decision of most important clustering!

Summary and outlook

Summary

- Baryon spectrum: good agreement with experiment!
- Three-body vs diquark-quark: fair agreement

Review: Eichmann, Sanchis-Alepuz, Williams, Alkofer, CF, PPNP 91, 1-100 [1606.09602]

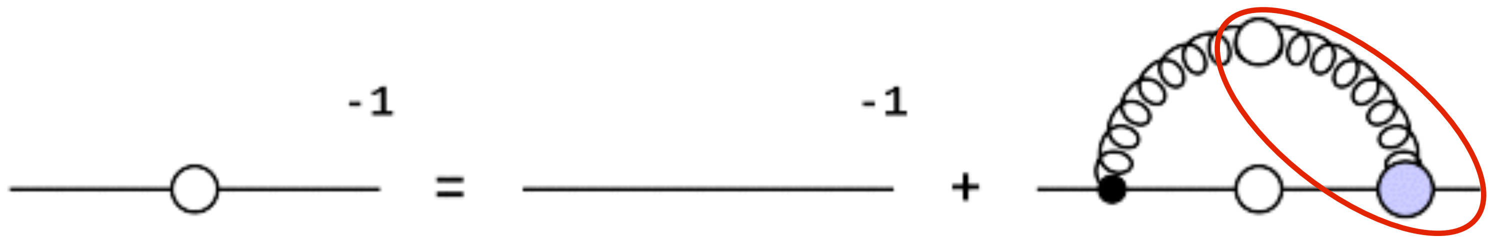
- Four-quarks states dominated by meson-meson configurations
- Dynamical description of σ as π-π resonance

Outlook

- Baryons: J=5/2 and J=7/2
- Tetraquarks: explore heavy-light systems
- Glueballs: $M(0^{++}) = 1.64\ \mathrm{GeV}$ Sanchis-Alepuz, CF, Kellermann and von Smekal, PRD 92 (2015) 3, 034001 (see also Meyers, Swanson, PRD 87 (2013) 3, 036009)
- Hybrids: work in progress

Backup

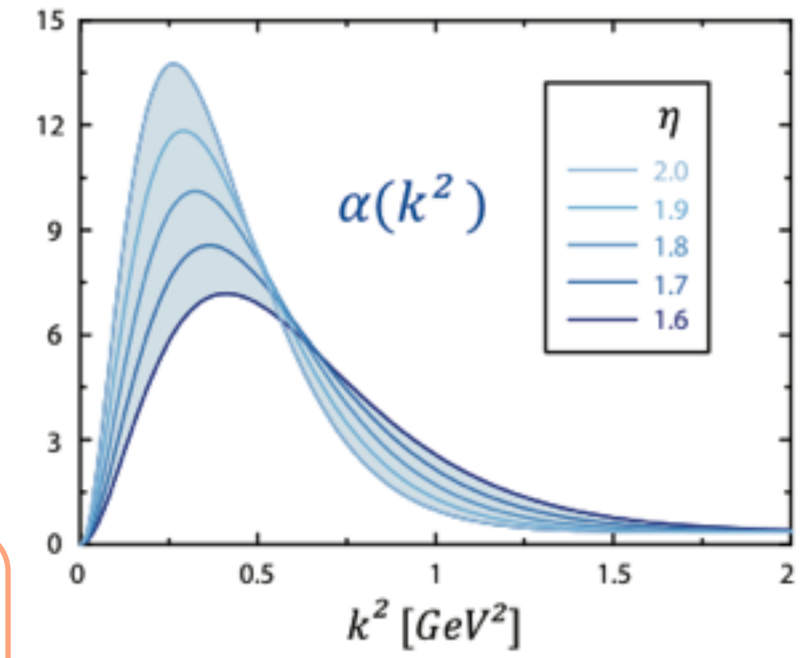
Rainbow-ladder model for quark-gluon interaction



Combine gluon with quark-gluon vertex:

effective coupling

$$\alpha(k^2) = \pi\eta^7\left(\frac{k^2}{\Lambda^2}\right)e^{-\eta^2\left(\frac{k^2}{\Lambda^2}\right)} + \alpha_{UV}(k^2)$$



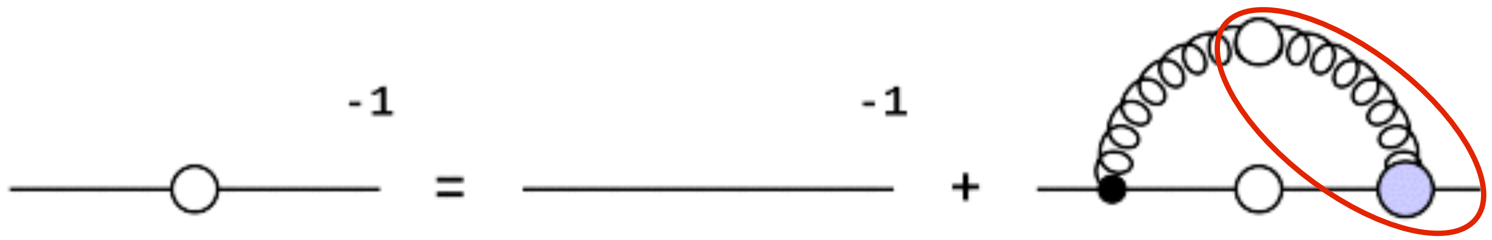
Maris, Roberts, Tandy, PRC 56 (1997), PRC 60 (1999)

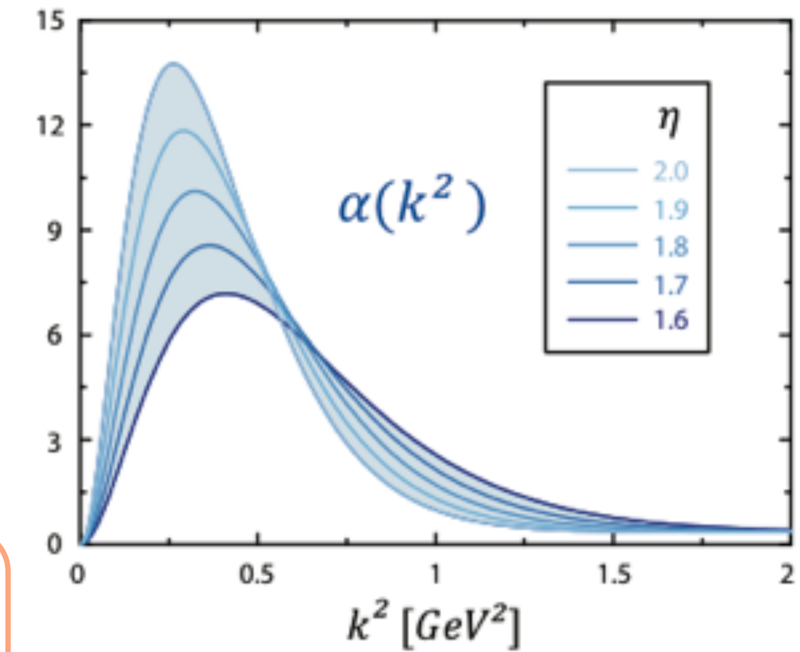
- scale Λ from f_π, masses $m_u = m_d$, m_s from m_π, m_K
- α_{UV} from perturbation theory
- parameter η : band of results

Binosi, Chang, Papavassiliou and Roberts, PLB 742 (2015) 183

Eichmann, Sanchis-Alepuz, Williams, Alkofer, CF, PPNP 91, 1-100 [1606.09602]

Rainbow-ladder model for quark-gluon interaction





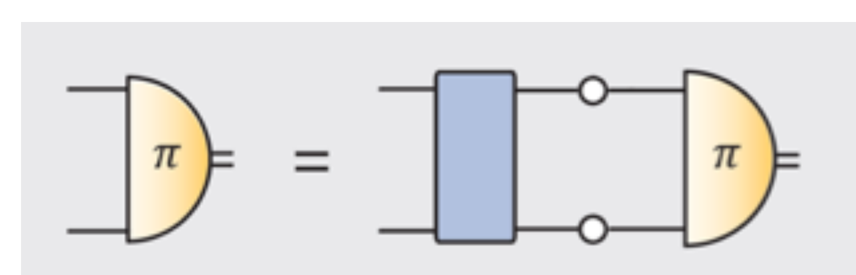
Maris, Roberts, Tandy, PRC 56 (1997), PRC 60 (1999)

Combine gluon with quark-gluon vertex:

effective coupling

$$\alpha(k^2) = \pi\eta^7 \left(\frac{k^2}{\Lambda^2}\right) e^{-\eta^2\left(\frac{k^2}{\Lambda^2}\right)} + \alpha_{UV}(k^2)$$

- scale Λ from f_π, masses $m_u = m_d$, m_s from m_π, m_K
- α_{UV} from perturbation theory
- parameter η : band of results



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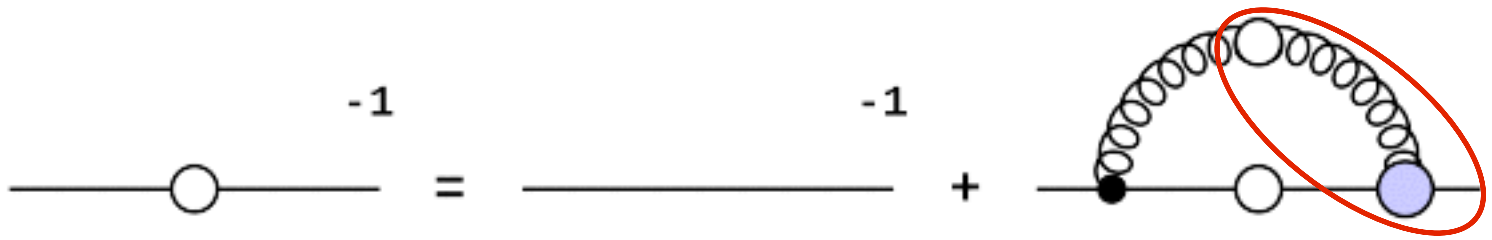

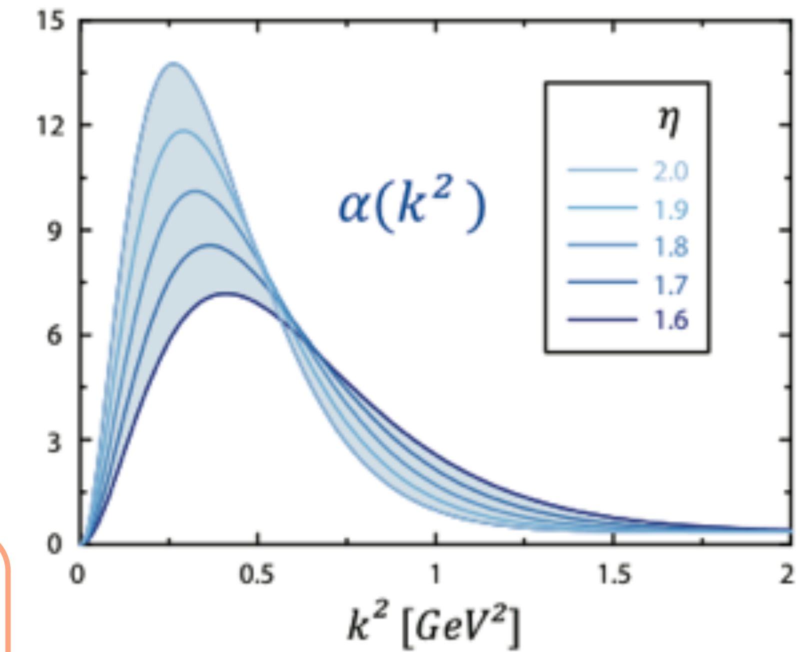

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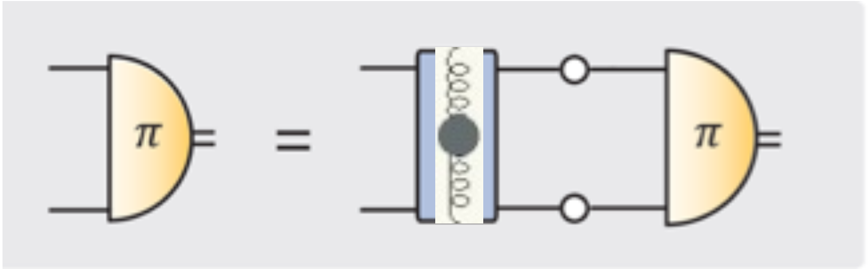
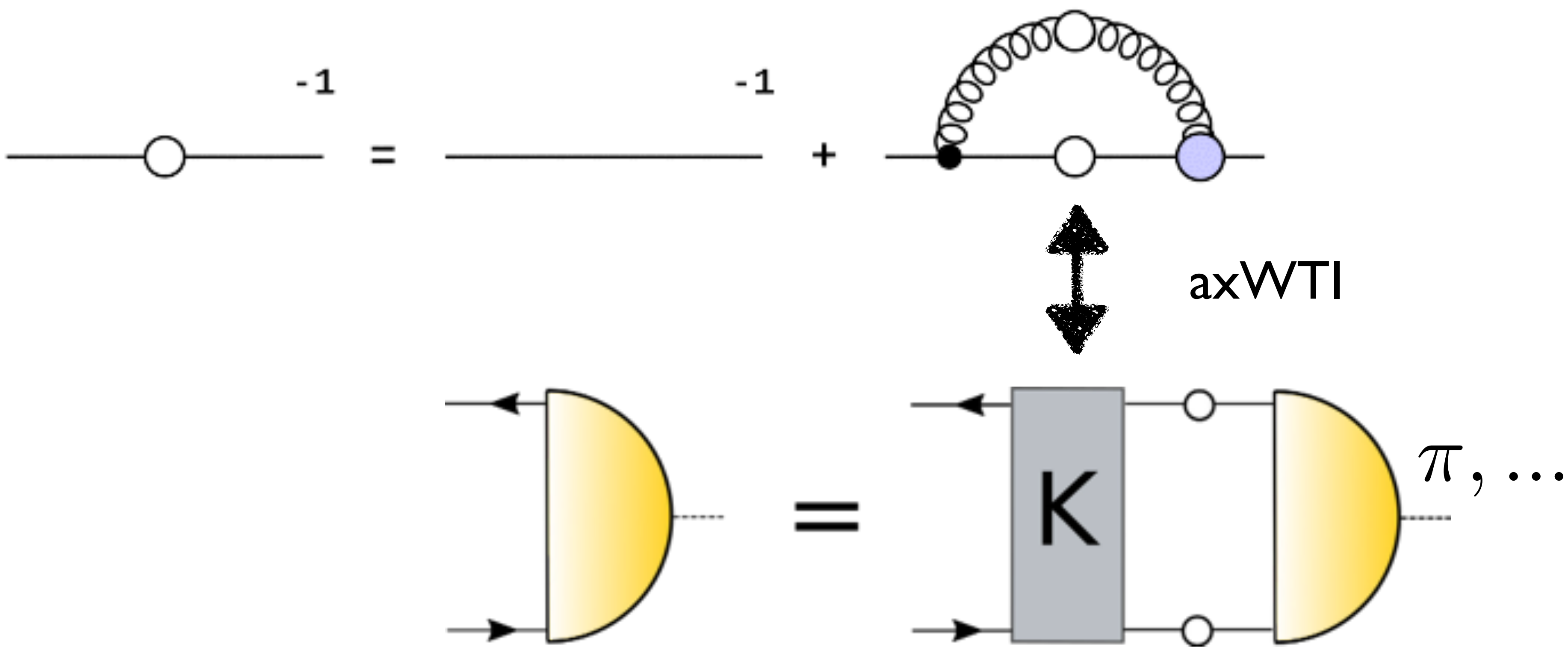


Binosi, Chang, Papavassiliou and Roberts, PLB 742 (2015) 183

Eichmann, Sanchis-Alepuz, Williams, Alkofer, CF, PPNP 91, 1-100 [1606.09602]

DSEs and Bethe-Salpeter equation



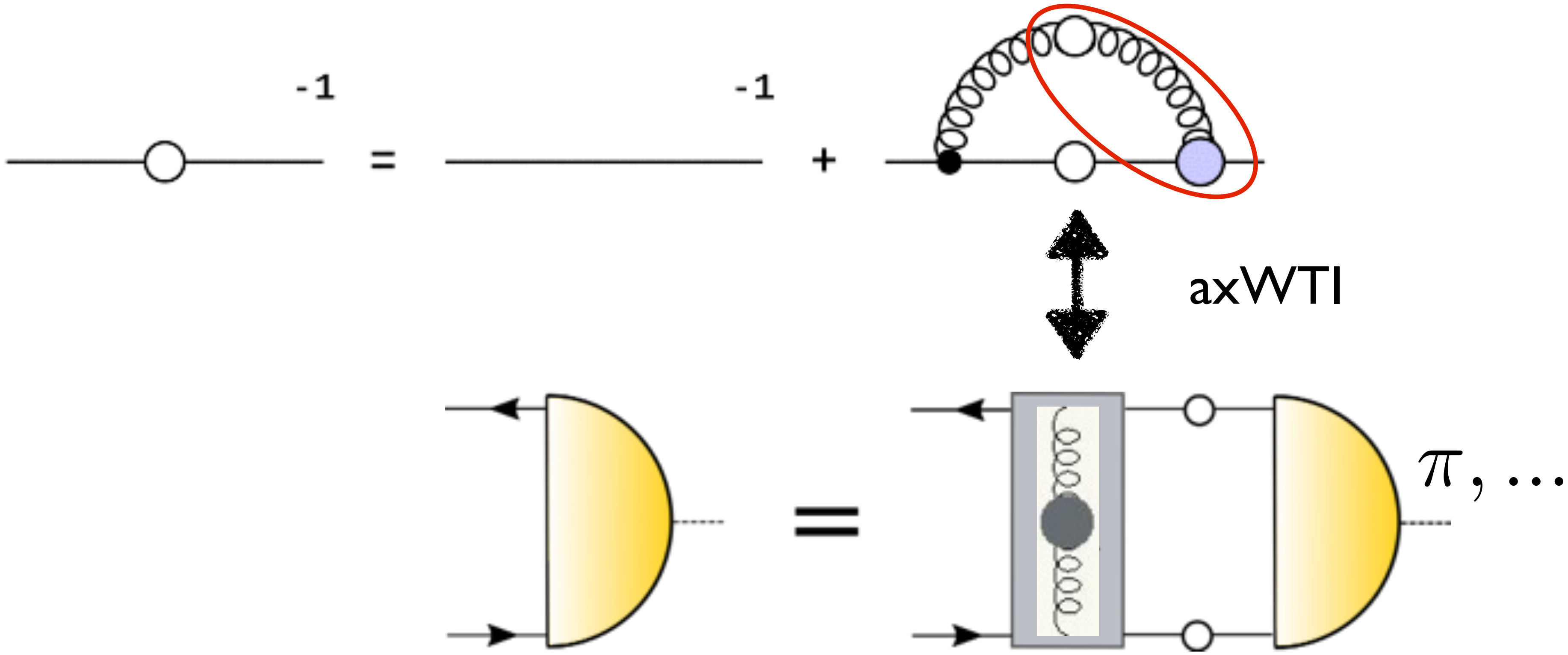
Kernel K uniquely related to quark-DSE via axialvector Ward-Takahashi-Identity (axWTI):

$$-i \int (K\gamma_5 S_- + K S_+ \gamma_5) = \int \gamma_\mu S_+ D_{\mu\nu} \Gamma_\nu \gamma_5 + \int \gamma_5 \gamma_\mu S_- D_{\mu\nu} \Gamma_\nu$$

→Pion is bound state **and** Goldstone boson

Maris, Roberts, Tandy, PLB 420 (1998) 267

DSEs and Bethe-Salpeter equation



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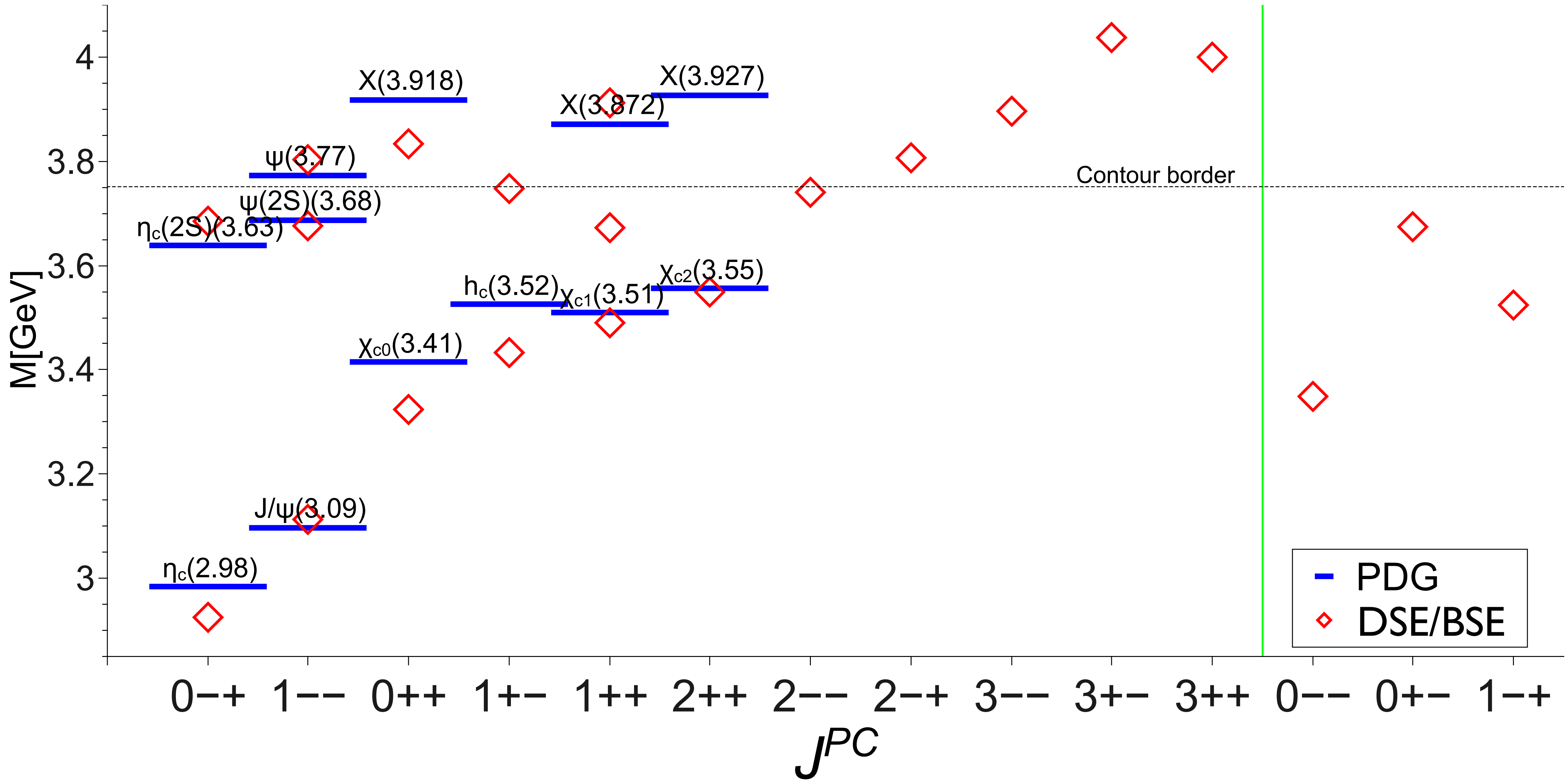
Maris, Roberts, Tandy, PLB 420 (1998) 267

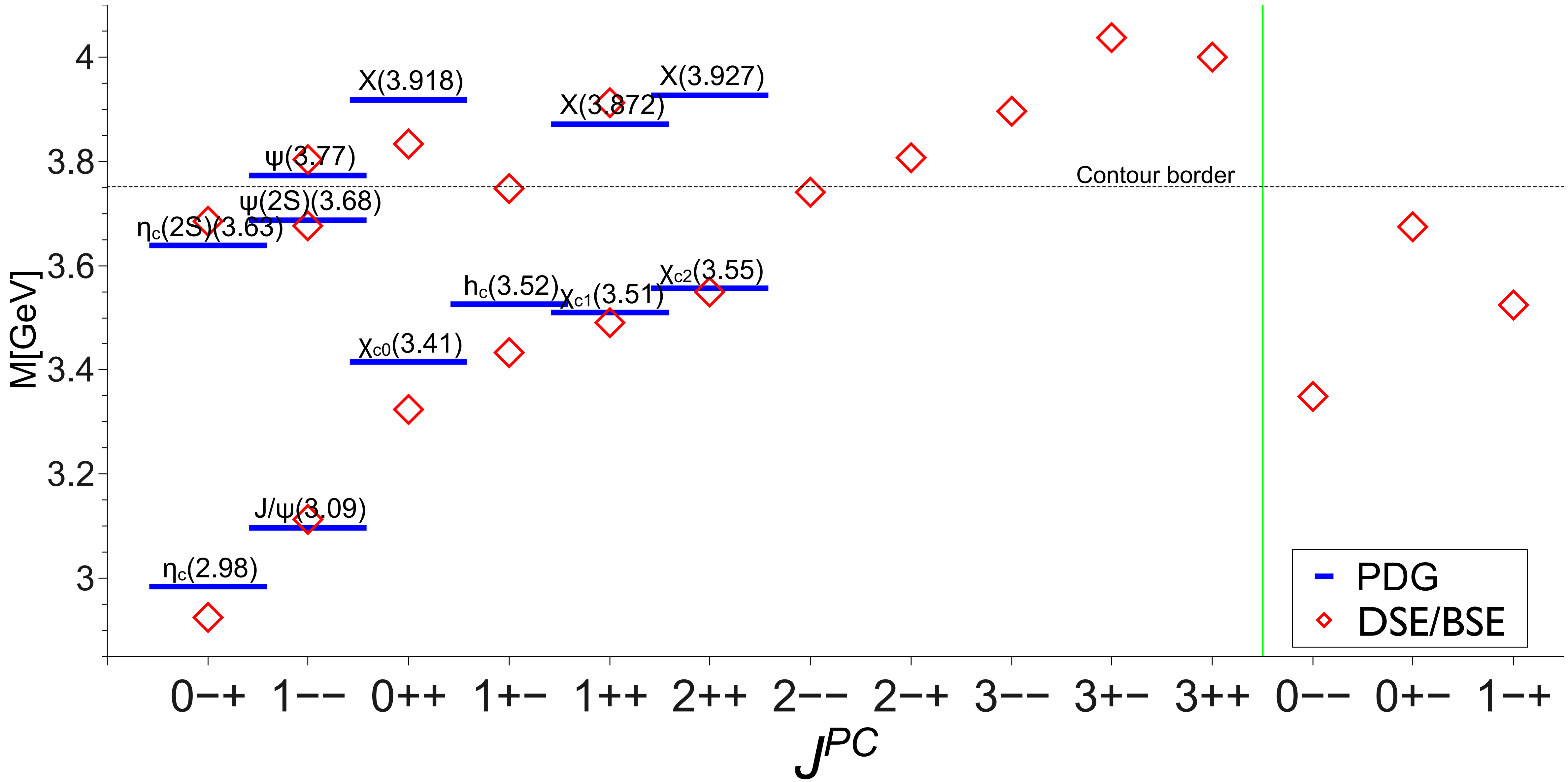
Charmonium spectrum



CF, Kubrak, Williams, EPJA 51 (2015)
Hilger et al. PRD 91 (2015)

- good channels: 1^{--}, 2^{++}, 3^{--},...
- acceptable channels: 0^{-+}
- clear deficiencies in other channels: missing spin-structure
- excited states fine ! (in good channels)

Charmonium spectrum



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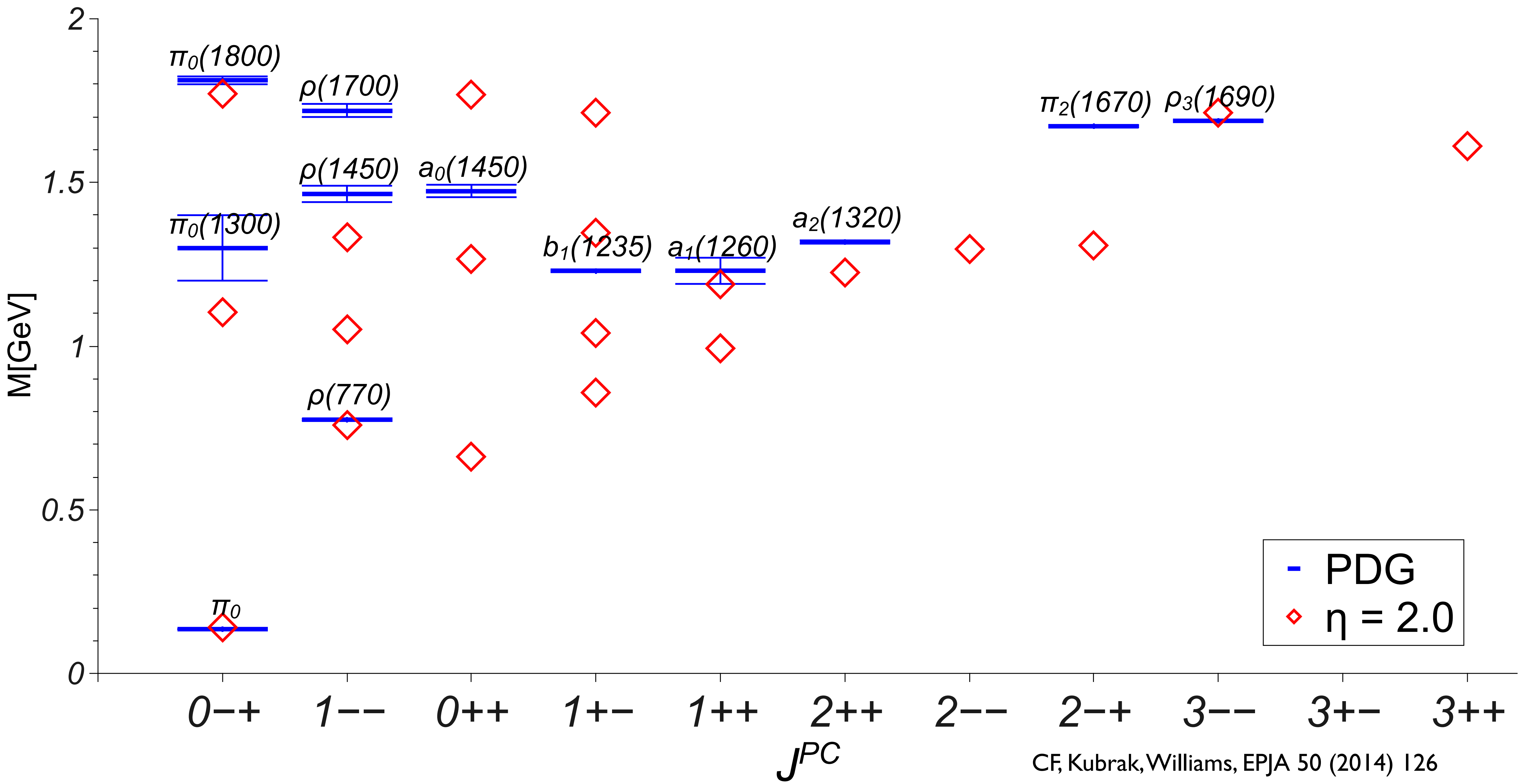

CF, Kubrak, Williams, EPJA 50 (2014) 126

Williams, CF, Heupel, PRD93 (2016) 034026

- good channels (ground state): 0^{-+}, 1^{--}
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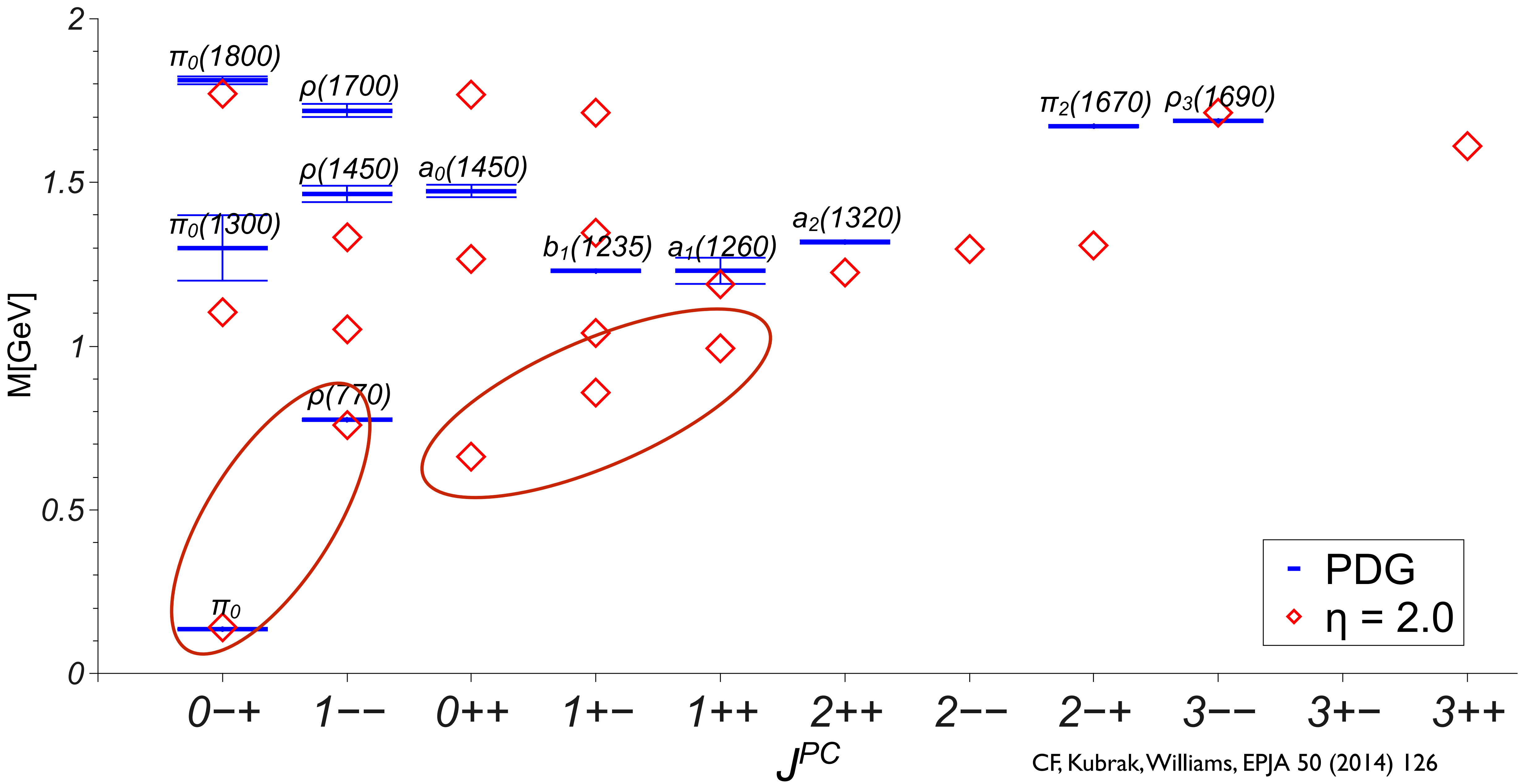

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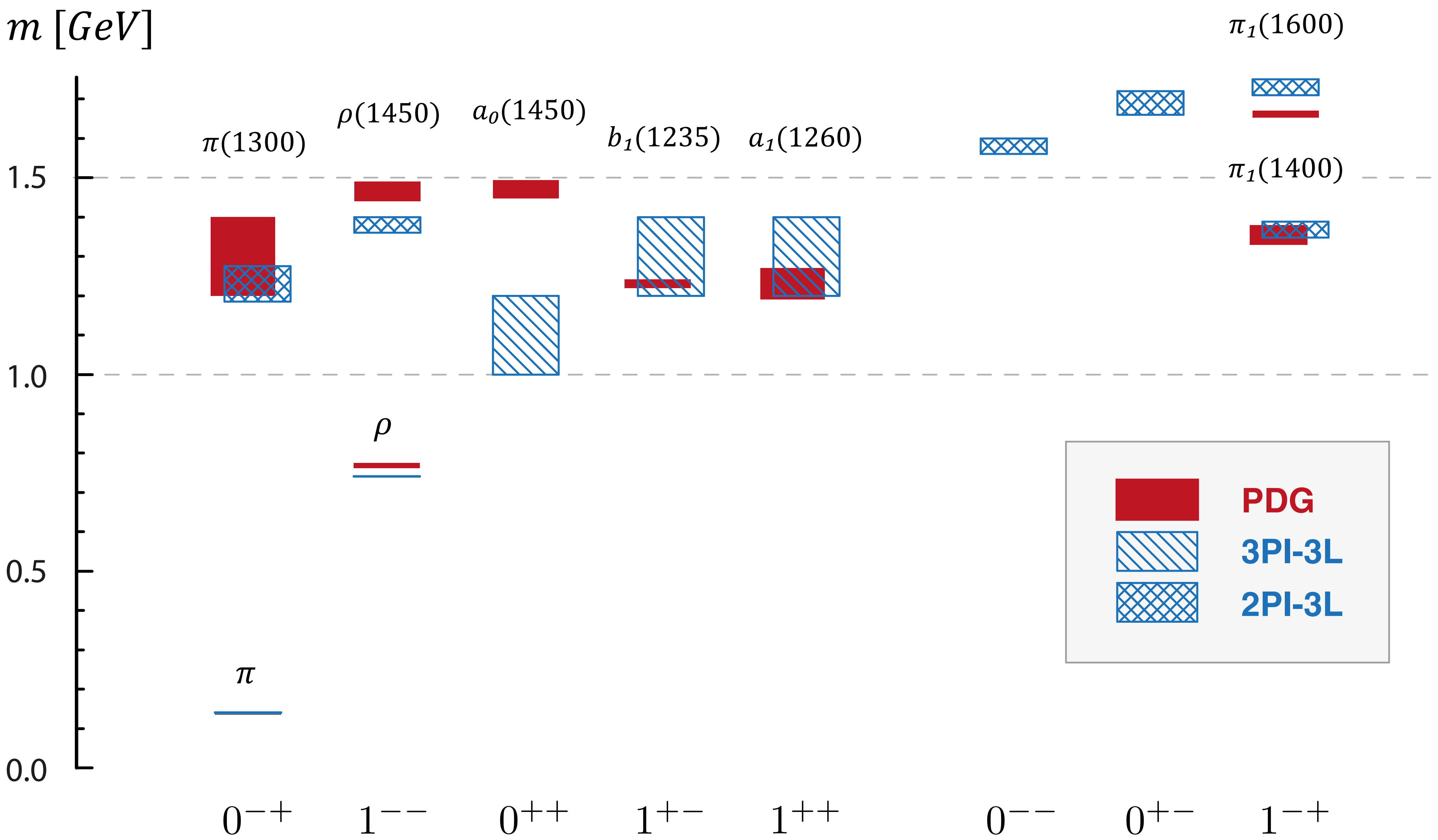

CF, Kubrak, Williams, EPJA 50 (2014) 126

Williams, CF, Heupel, PRD93 (2016) 034026

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Light meson spectrum (bRL)

CF, Kubrak, Williams, EPJA 50 (2014) 126

Williams, CF, Heupel, PRD93 (2016) 034026

- nice agreement with experiment (up to scalar)
- exotics as relativistic quark-antiquark states
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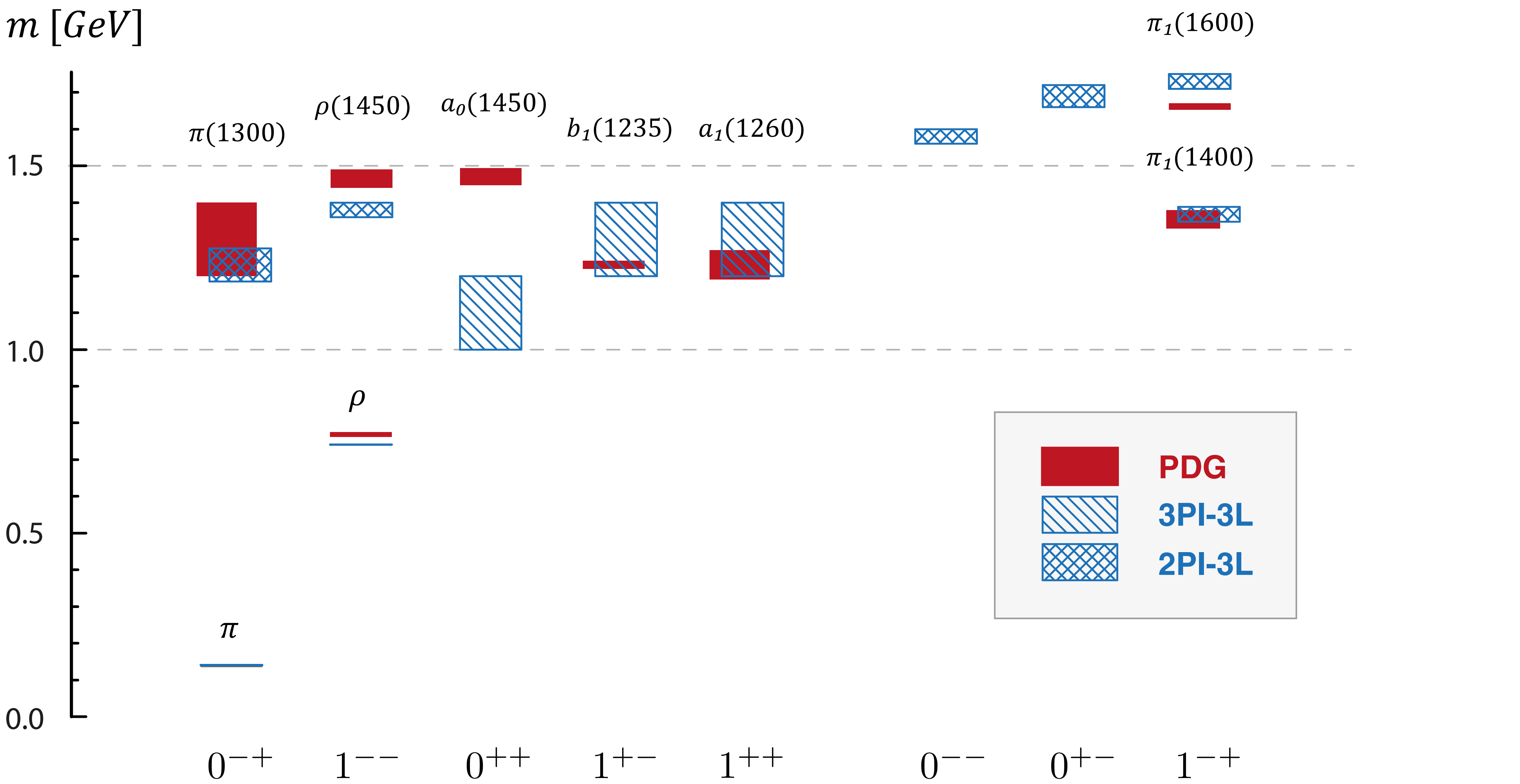
Light meson spectrum (bRL)



CF, Kubrak, Williams, EPJA 50 (2014) 126

Williams, CF, Heupel, PRD93 (2016) 034026

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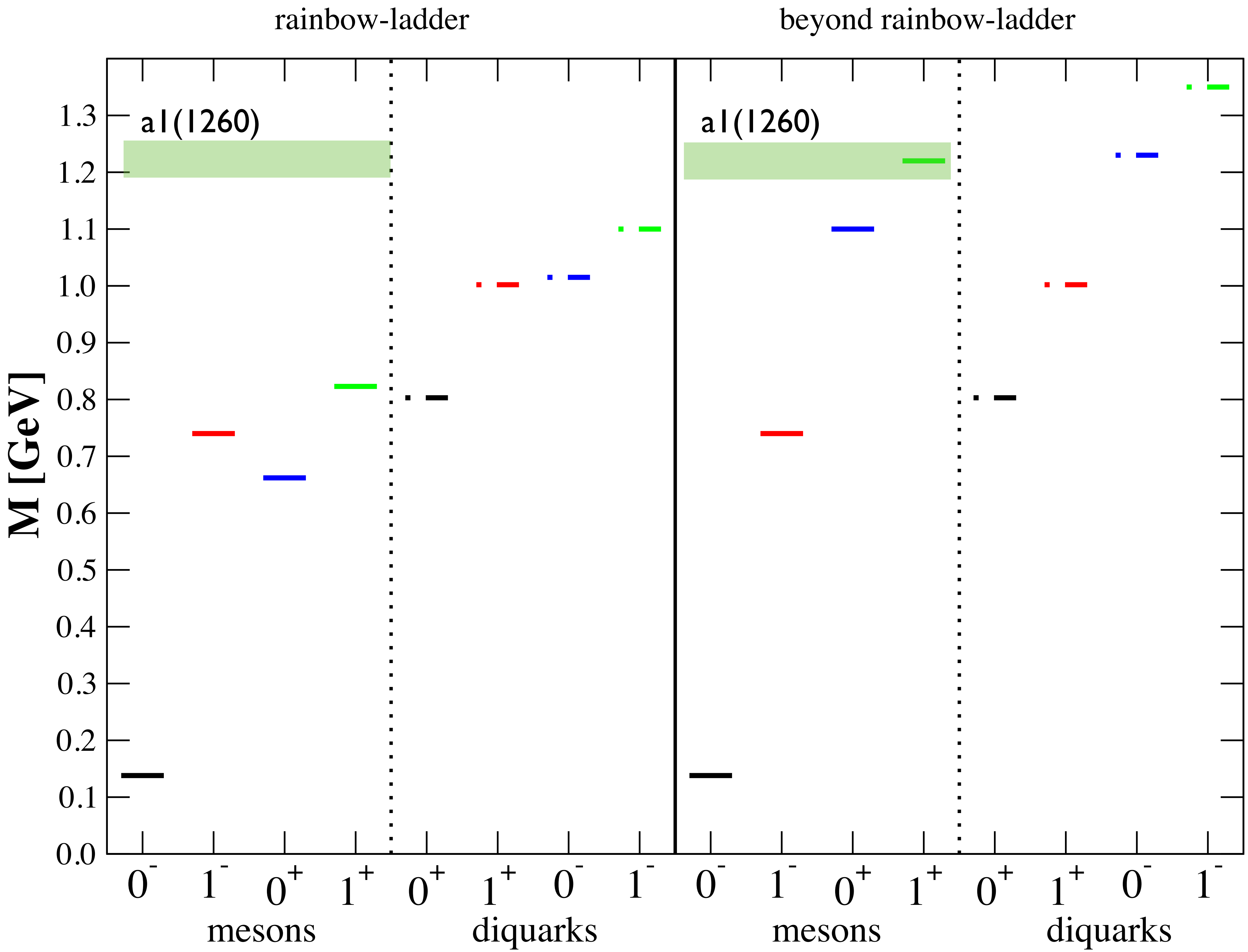
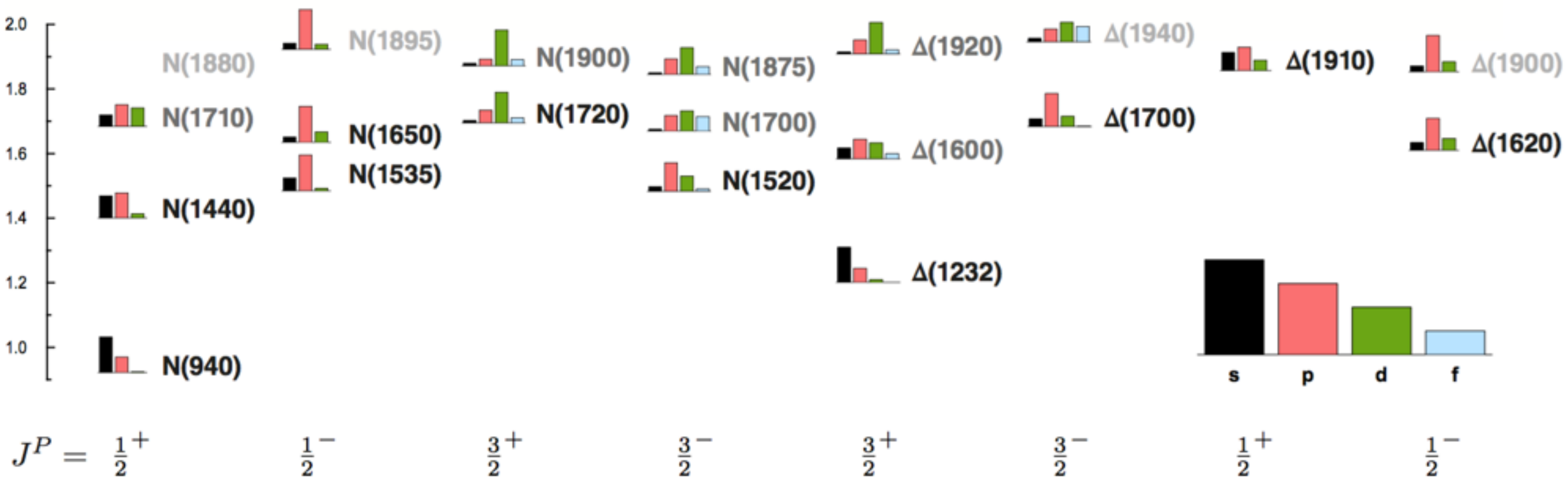
Diquarks with modified rainbow-ladder



- α multiplied with 0.35 in ‘bad’ channels

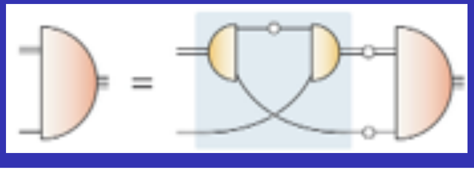
see also: Williams, CF, Heupel, PRD93 (2016) 034026

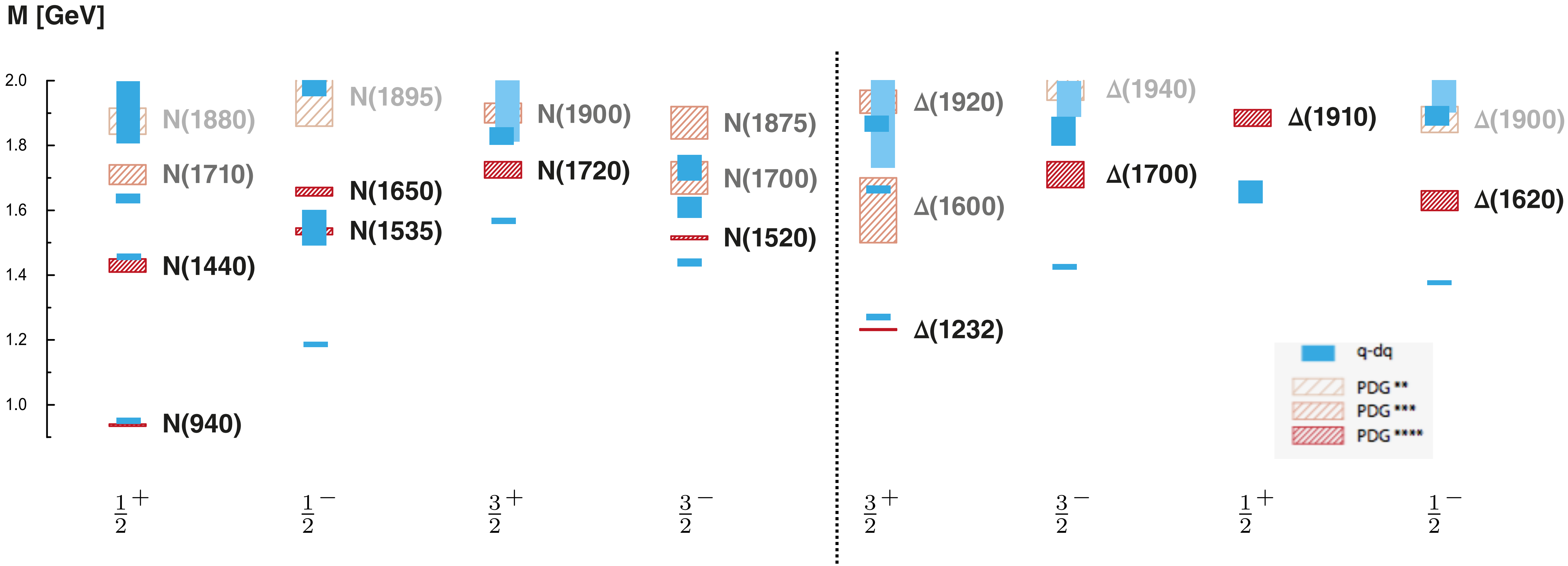
Angular momentum



- non-relativistic quark model: restriction to certain ang. mom.
- here: quark-model forbidden contributions always present

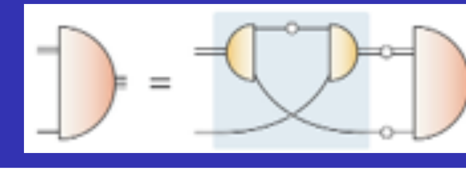
Light baryon spectrum: diquarks

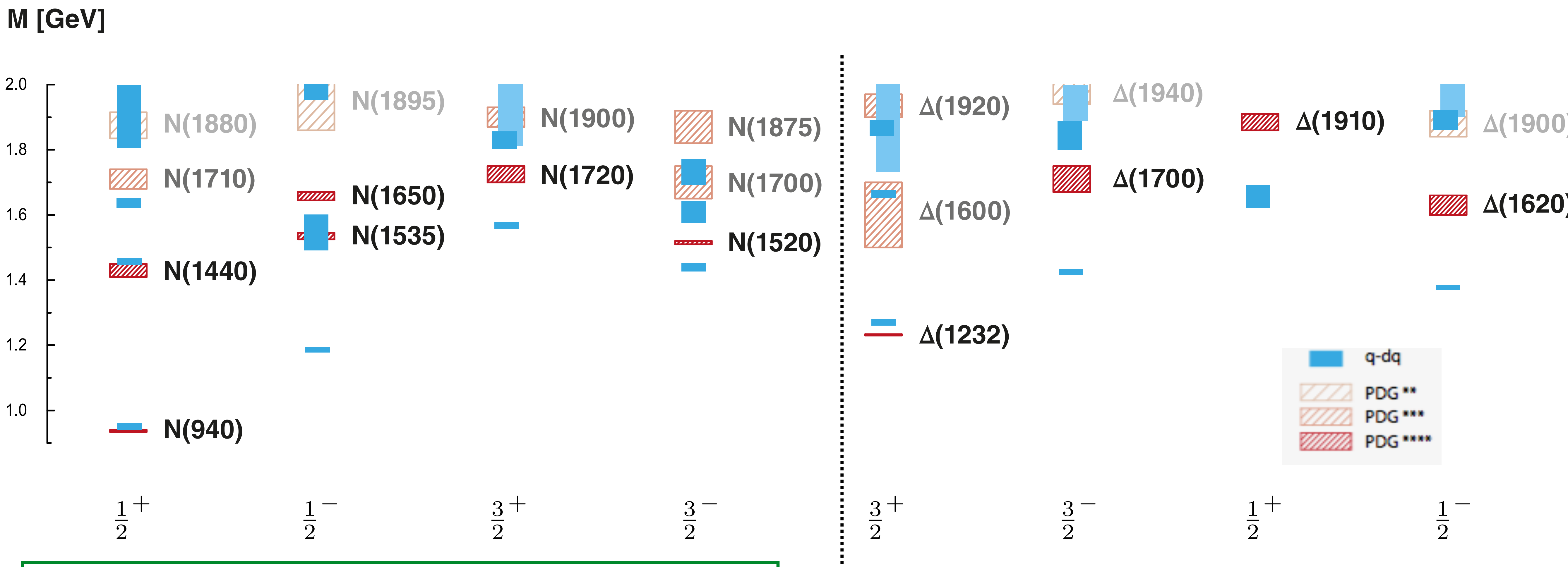




- nucleon and delta - channels: good results
- but: severe problems in all other channels

Light baryon spectrum: diquarks





sc+av	ps+v	all	all

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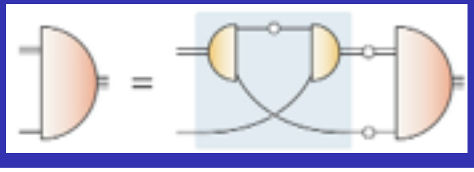

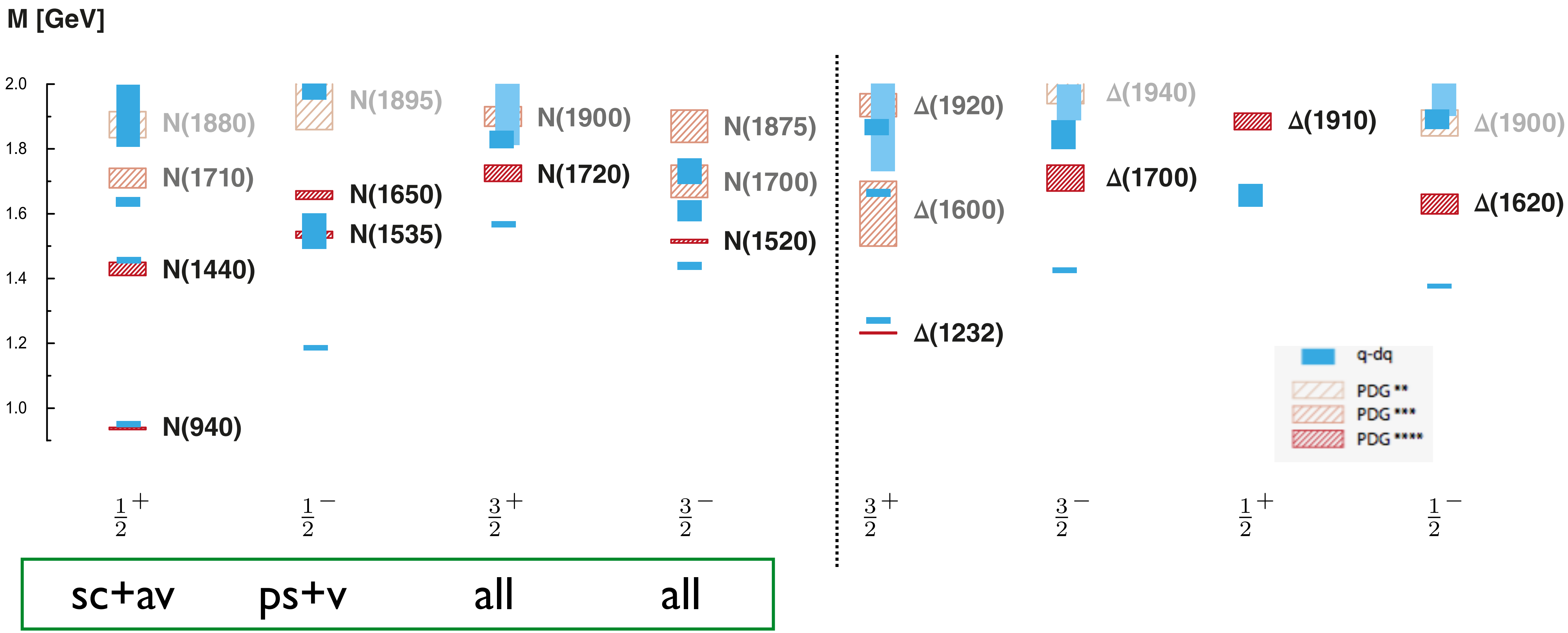

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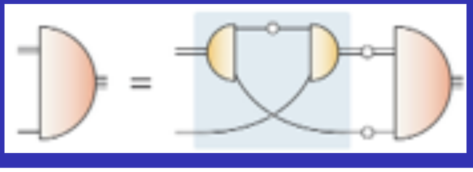
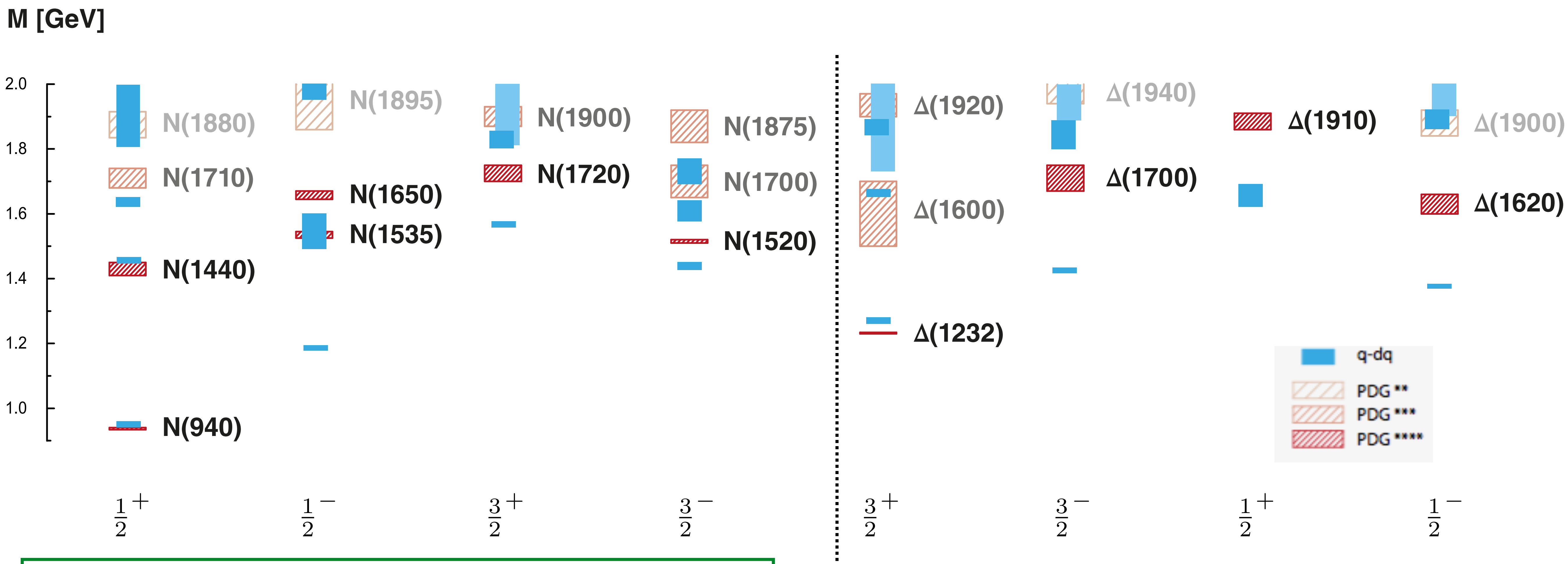
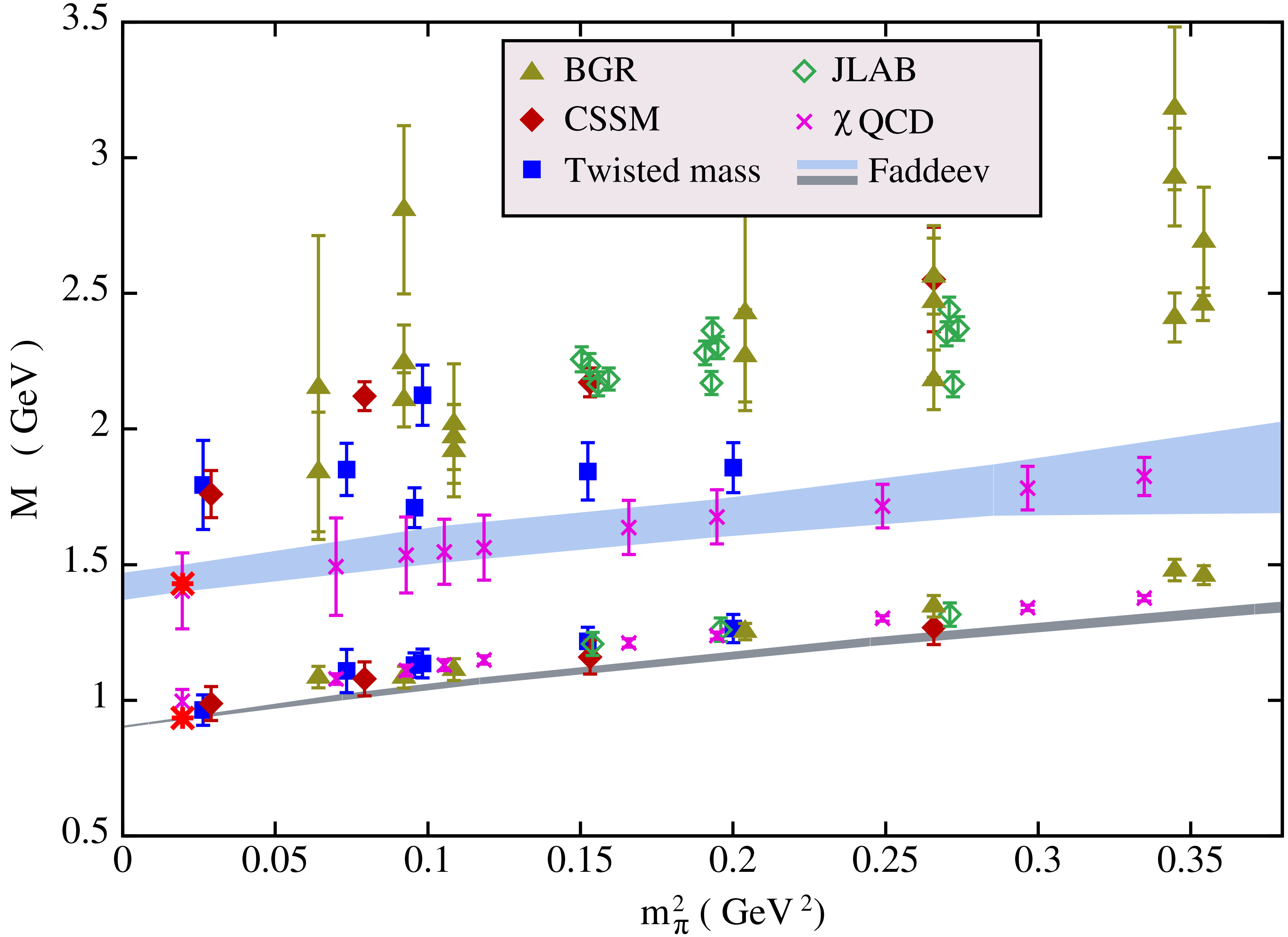




- nucleon and delta - channels: good results
- but: severe problems in all other channels
- artifact of rainbow-ladder: ps and v too strongly bound !

reduce binding in ps and v diquark channels, adjust to ρ-a_1-splitting

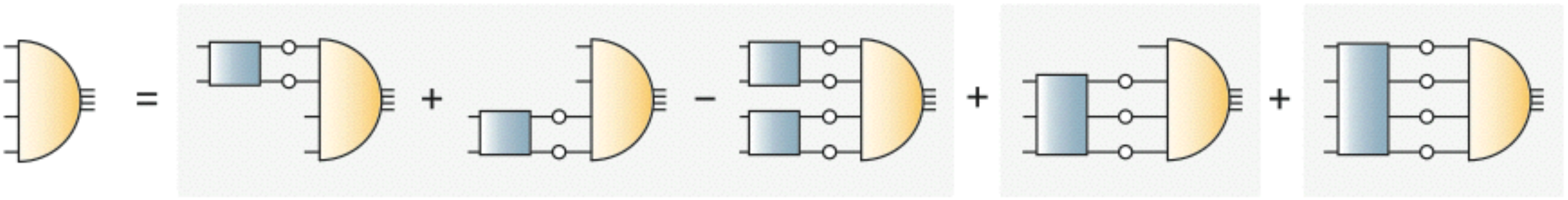
Mass evolution



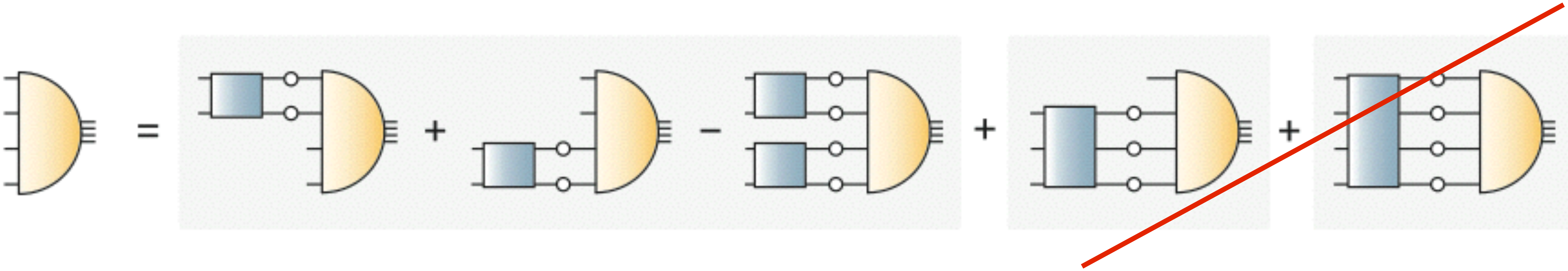
Eichmann, CF, Sanchis-Alepuz, PRD 94 (2016) [1607.05748]

- Mass evolution as expected for three-body state…

Two-body approximation



Two-body approximation



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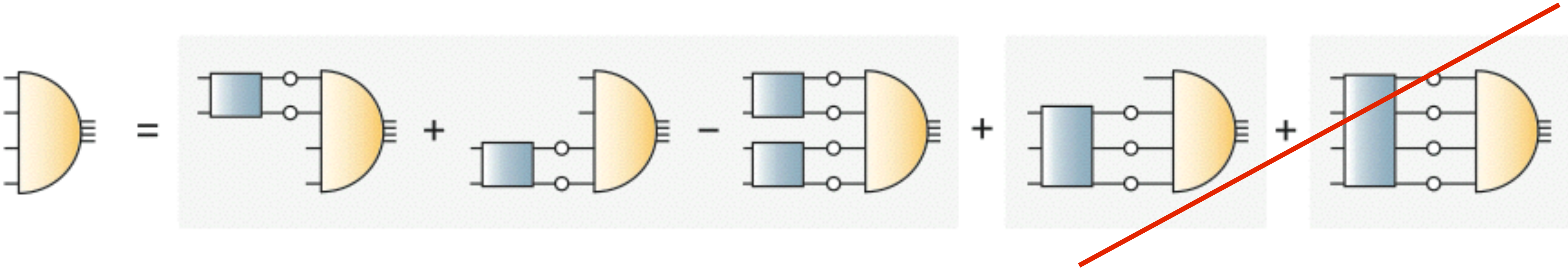
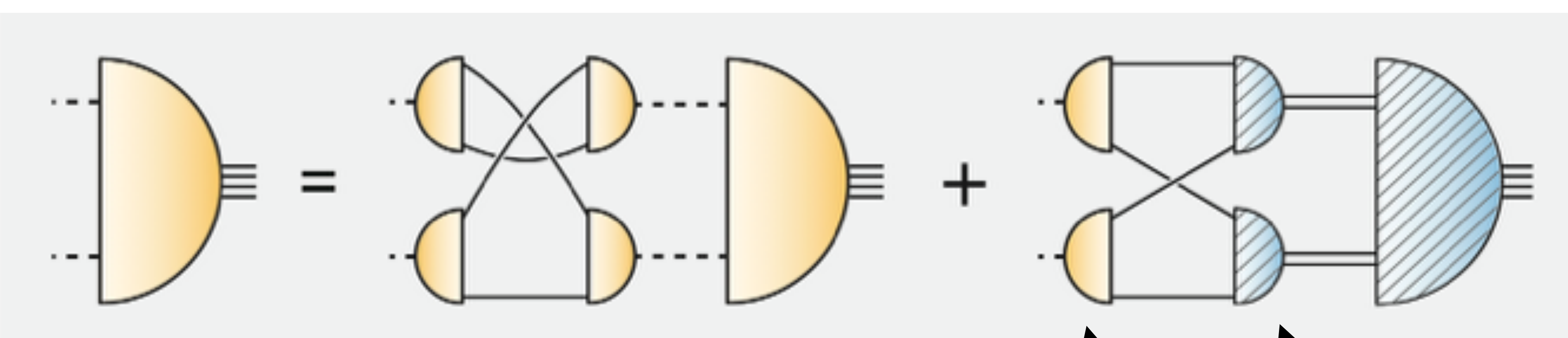
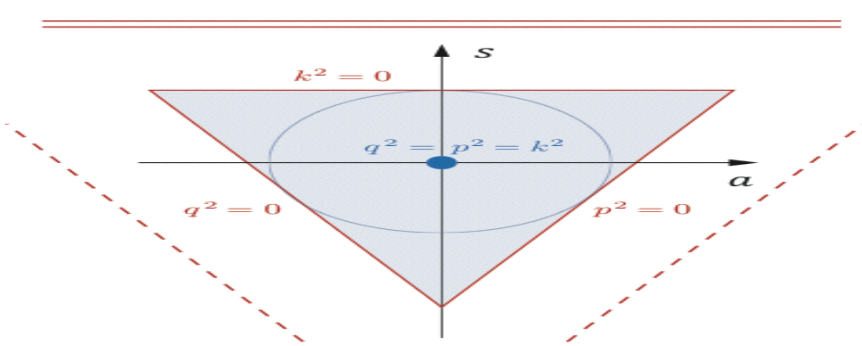


approximation: separable ansatz for interaction kernel

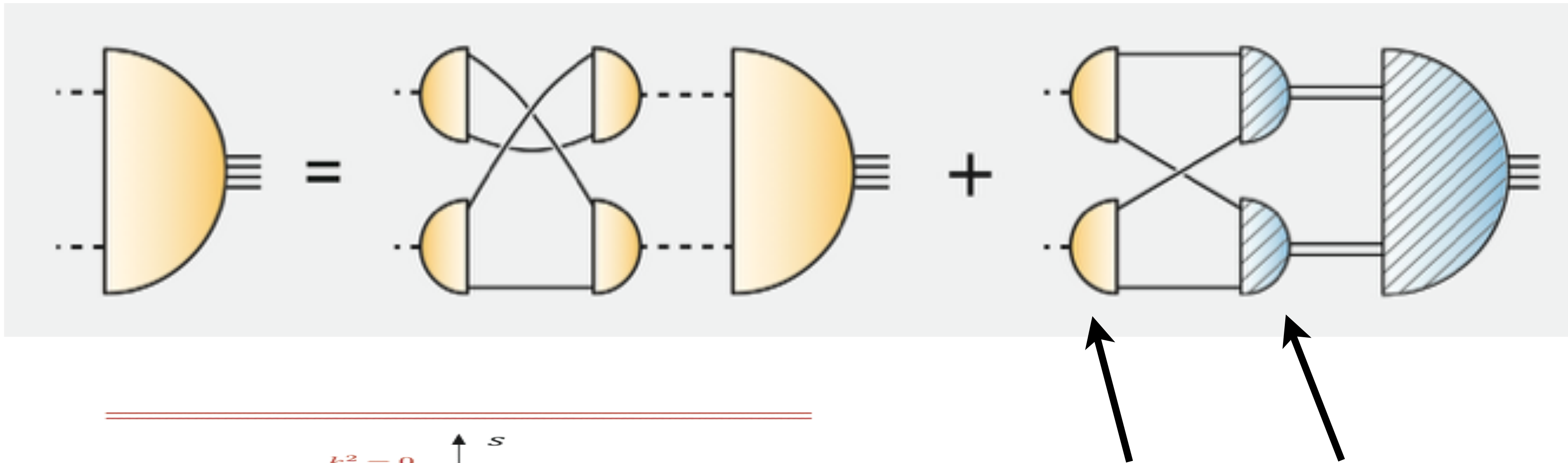
Heupel, Eichman, CF, PLB 718 (2012) 545-549



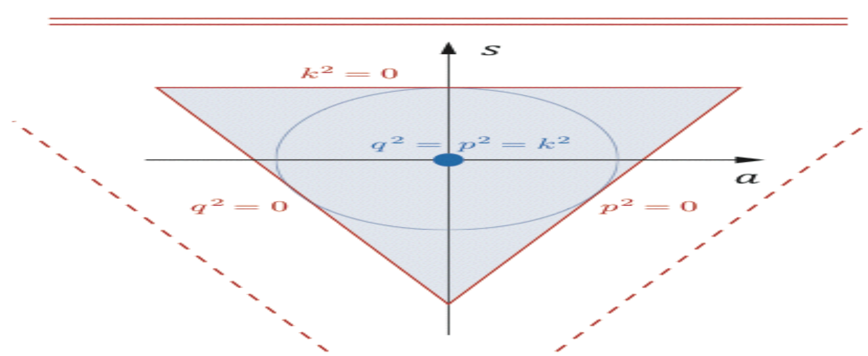


Meson Diquark

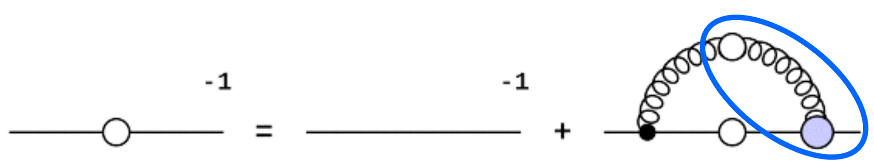
Tetraquark-BSEs - two-body equations



Meson Diquark



- Input: Covariant Quark-Gluon interaction - Maris-Tandy model



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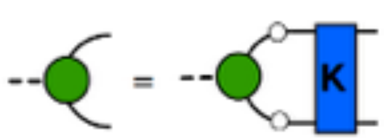
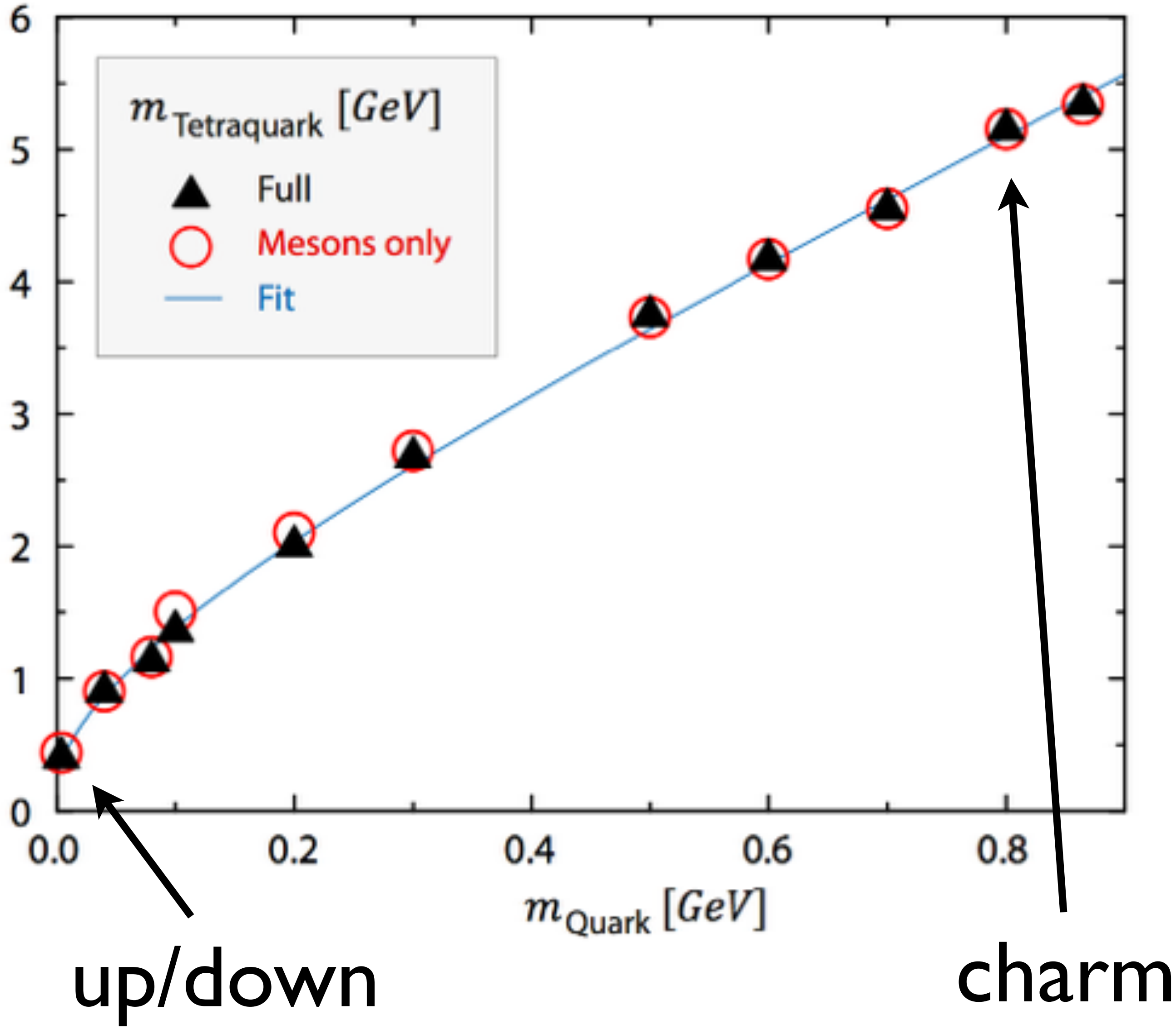
- Mesons and Diquarks via Bethe-Salpeter equation



Dynamical decision between Meson- and Diquark-configurations

Results: scalar tetraquarks



Heupel, Eichman, CF, PLB 718 (2012) 545-549

- Pion-Pion-contribution dominates !
- m(0^{++}) = 403 MeV

} $f_0(500)$

see also Caprini, Colangelo and Leutwyler, PRL. 96 (2006) 132001
Parganlija, Kovacs, Wolf, Giacosa and Rischke, PRD 87 (2013) 014011

- Narrow scalar $cc\overline{cc}$: m(0^{++}) = 5.3 ± (0.5) GeV